Scalar arithmetic

Assigment statements are of the form

- variable_name = expression
- Scalar operators:
  - Addition, subtraction: a+b a-b
  - Multiplication, division: a*b a/b
  - Exponentiation: a^b

When many operations are combined into a single expression, it is important to know in what order Matlab will evaluate the terms.

Scalar arithmetic

Matlab has a series of rules governing the hierarchy (or order) in which operations are evaluated within an expression, as follows

1. Expressions in parentheses (innermost to outermost)
2. Exponentials (left to right)
3. Multiplications and divisions (left to right)
4. Additions and subtractions (left to right)

Thus, the expression

- distance = 0.5 * accel * time ^ 2
- is equivalent to
  - distance = 0.5 * accel * (time ^ 2)
- but not equivalent to
  - distance = (0.5 * accel * time) ^ 2

Relational operators

Relational operators:

==	equal to
~=	not equal to
>	greater than
>=	greater than or equal to
<	less than
<=	less than or equal to

For example,

- 3<4
- 3==4
- 'A' < 'B' (note strings are evaluated by ascii value)

Beware of comparisons and rounding errors, e.g.

- sin(pi) == 0

Can get around this with tolerance threshold, e.g.

- abs(a-b) < 1.0e-14

Logical operators

Can have one or two operands, giving a logical result:

&	Logical AND
|	Logical OR
xor	Exclusive XOR
~	Logical NOT

Inputs		and	or	not	xor
A	B	A & B	A | B	~A	xor(A,B)
0	0	0	0	1	0
0	1	0	1	1	1
1	0	0	1	0	1
1	1	1	1	0	0

An operand is treated as true if it is any non-zero value and false if it is zero, thus ~5 is zero (and so is ~-1) and ~0 is one.

Logic operators can also be used to compare two arrays (must be same size) or a scalar value with an array. For example,

If $a=\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $b=\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$ then $a|b=\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$

Hierarchy revisited

- Relational and logical operators are evaluated after arithmetic operators:
  - 1. Arithmetic operators as described before
  - 2. Relational operators (==, ~=, >, etc) left to right
  - 3. Logical operators as follows

Operator	Operation	Priority
~	NOT	Highest
&	Elementwise AND	
|	Elementwise OR	
&&	Short-circuit AND	
||	Short-circuit OR	Lowest

Short-circuit operators will not evaluate second operand unless necessary, e.g. 0 && f(x) will not evaluate f(x)

Exercises

- Declare: a=3; b=2; c=5; d=3 and calculate the following expressions:

$$ab + cd$$
$$a(b + c)d$$
$$(ab) + (cd)$$
$$(a^b)^d$$
$$a^{b^d}$$

For a=20, b=-2, c=0, d=1, evaluate

- a greater than b ?
- (a greater than b) and (c greater than d) ?
- (a different to b) or (b different to c) ?
- b equal to c ?

Array and Matrix operations

- Matlab has two types of operations between arrays:
  - (a) element-by-element operations where the two operands have the same number of rows and columns, for example
    - A = [1 2 ; 3 4]; B=[-1 3; -2 1]
    - Addition, Subtraction: A+B A-B
    - Multiplication, Division: A .* B A ./ B
    - Exponentiation: A .^ B
    - One operand can be scalar, the other a matrix
      - e.g. 2 .^ A
      - Although for multiplication,divsion A * 2 = A .* 2 and A/2 = A./2
  - (b) Matrix operations
    - Ordinary matrix multiplication: A * B
    - Matrix exponent: A^2 (scalar exponent)
    - Matrix left division: A \ B = inv(A) * b

Exercises

- Assign n=100, then compute $\frac{n(n+1)}{2}$.
- Compute: $\sum_{k=1}^{100} k$.
- Compute $\sum_{k=1}^{15} k^2$.
- if x=3, calculate cosh $x = \frac{e^x + e^{-x}}{2}$ (using exp)
- Generate a vector *r* of means for 1000 random number vectors of length 50. See your results with the command: hist(r,40)

Difference equations

A single-variable discrete dynamical system can be defined as:

$$x(t + 1) = f(x(t))$$

For example:

$$x(t + 1) = x(t)^2$$

Complex numbers

- Complex variables are created automatically when a complex value is assigned to a variable name

```
>> c1 = 4 + 3i
```

- Note that comparisons (>,>=,<,<=) between complex variables only compare the real parts, not the magnitudes.

Functions for complex numbers

- abs(c) Magnitude (norm)
- real(c), imag(c)
- isreal(c)
- angle(c) = atan(imag(c)/real(c))
- Standard functions such as sin(), cos(), log() will also work with complex numbers

Functions of two variables

The function [XX,YY] = meshgrid(X,Y) can be used to generate (x,y) pairs for evaluating f(x,y) efficiently

For example,

```
>> x = y = 0:0.5:4
>> [xx,yy] = meshgrid(x,y)
>> z = xx+yy
>> surf(z)
```

Exercise

- Evaluate and plot the function (using surf):

f(x,y) = sin(*r*) / *r* , where $r = \sqrt{x^2 + y^2}$

in the interval [-10,10] for both x and y.

- For the discrete dynamical system:

$$x(t+1) = x(t)^2 + c$$

with x(0)=0. Evaluate the system for values of c=a+b*i in the complex plane for a and b in the interval [-1.5,1] with interval size of 0.005 (hint: build c in matrix form using meshgrid). Evolve the system up to t=20 for all values of c **in parallel**. Use image() to display, in the complex plane, those values of c for which |x(20)| < 2.