Assignment 2

CS41001: Theory of Computation — Autumn, 2021
Deadline: 25 October 2021, 13:00 — Total Marks: 60

Solve all problems. Stick to notation used in the classes. Write solutions on white paper, scan and then upload a single pdf file. Make sure that the file size does not exceed 20 MB. Any format other than pdf is not acceptable.

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1. 2-SAT is the language containing all satisfiable CNF-SAT formulae with each clause having at most 2 literals. Show that if 2-SAT is NP-complete then P=NP.
   Hint: Try to construct a directed graph corresponding to a 2-SAT formula. You can assume that given two vertices u, v it is possible to check if there is a directed path from u to v in polynomial time. [10]

2. Max-Cut = $\{G|G$ has an edge-cut of size at least $k\}$. Here, the graph G is simple (no loops, no multiple edges). Show that Max-Cut is NP-complete. [10]

3. Assuming $NP \neq coNP$, show the following:
   (a) No NP-complete language can be in coNP. [5]
   (b) A language exists that is not in P nor coNP-complete. [5]

4. Answer the following questions on the time hierarchy theorems.
   (a) Why is time constructibility needed in the THTs? When will the proofs break down if time constructibility is not assumed? [4]
   (b) Why is the satisfiability constraint in THT $f(n) \log n = o(g(n))$? [3]
   (c) Why is the satisfiability constraint in NTHT $f(n+1) = o(g(n))$? [3]

5. State whether the following statements are true or false with proof:
   (a) $P = DSPACE(n)$.
   Hint: Space Hierarchy Theorem. [10]
   (b) The language consisting of string with properly nested parentheses is in L. [10]