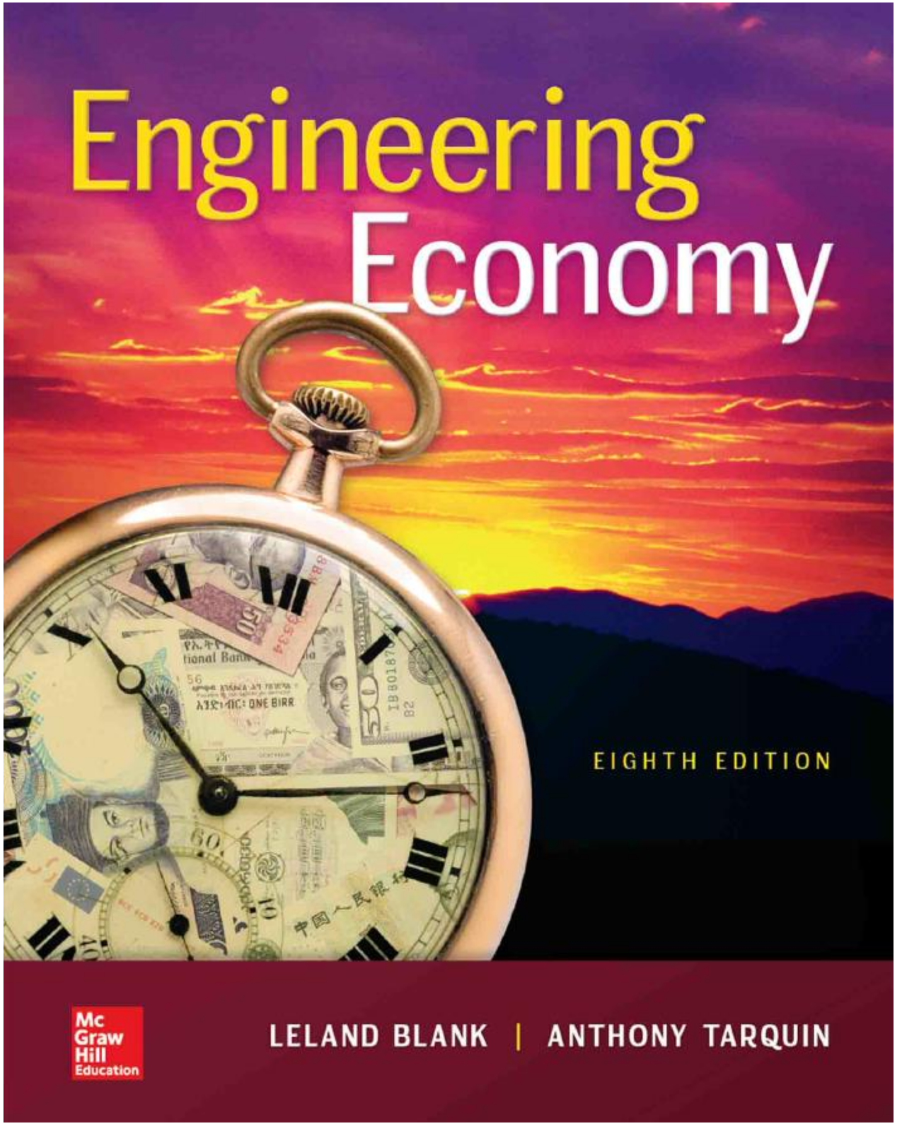


Chapter 3

Combining Factors and Spreadsheet Functions

Lecture slides to accompany

Engineering Economy

8th edition

Leland Blank

Anthony Tarquin

LEARNING OUTCOMES

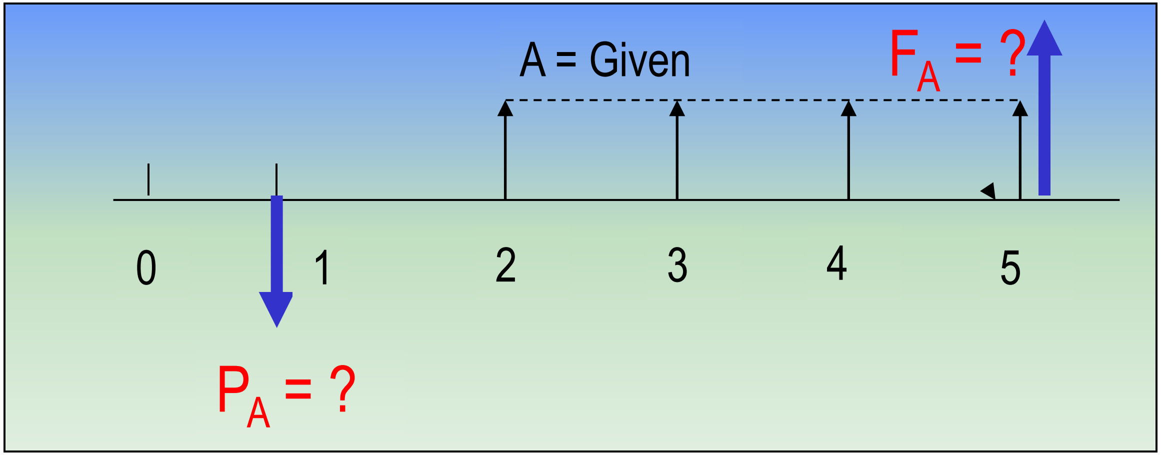
1. **Shifted uniform series**
2. **Shifted series and single cash flows**
3. **Shifted gradients**

Shifted Uniform Series

A shifted uniform series starts at a time *other than period 1*

The cash flow diagram below is an example of a shifted series

Series starts in period 2, not period 1



Shifted series usually require the use of *multiple factors*

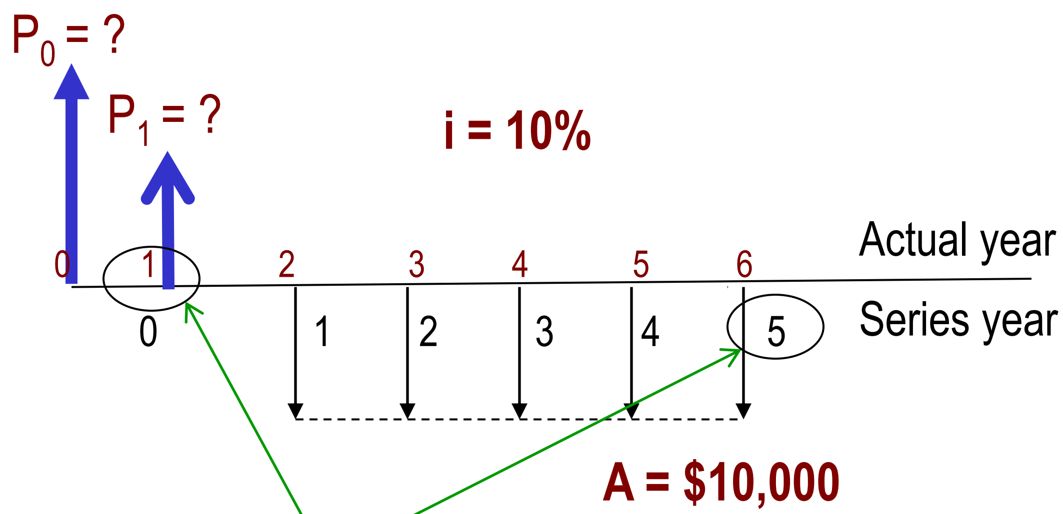
Remember: **When using P/A or A/P factor, P_A is always ________________ of first A**

When using F/A or A/F factor, F_A is in ____________ *as last A*

Example Using P/A Factor: Shifted Uniform Series

The present worth of the cash flow shown below at i = 10% is:

(a) \$25,304 (b) \$29,562 (c) \$34,462 (d) \$37,908



Solution: (1) Use P/A factor with n = 5 (for 5 arrows) to get P_1 **in year 1**

(2) Use P/F factor with n = 1 to move P_1 back for P_0 **in year 0**

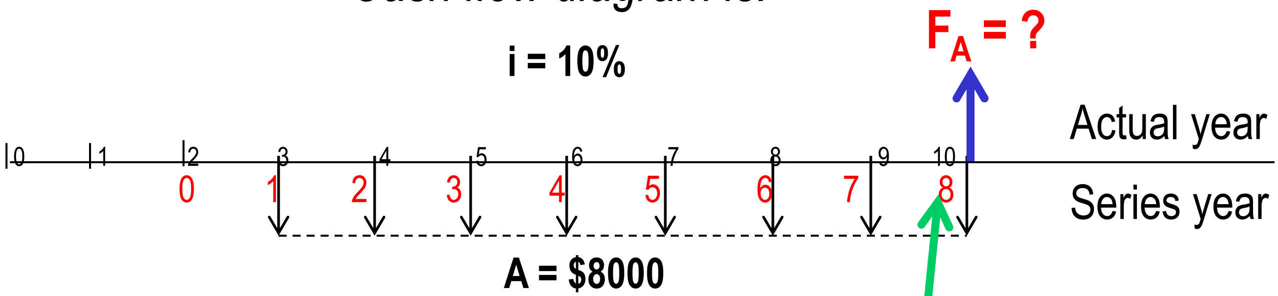
P_0 = P_1(P/F,10%,1) = A(P/A,10%,5)(P/F,10%,1) = 10,000(3.7908)(0.9091) = \$34,462

Answer is____

Example Using F/A Factor: Shifted Uniform Series

How much money would be available in year 10 if $8000 is deposited each year in years 3 through 10 at an interest rate of 10% per year?

Cash flow diagram is:



Solution: Re-number diagram to determine n = ___(number of arrows)

$$F_A = 8000(F/A,10\%,8)$$
$$= 8000(11.4359)$$
$$= \_\_\_\_\_\_\_\_\_\_\_\_$$

Shifted Series and Random Single Amounts

For cash flows that include *uniform series* **and** *randomly placed single amounts*:

Uniform series procedures **are applied to the** ***series amounts***

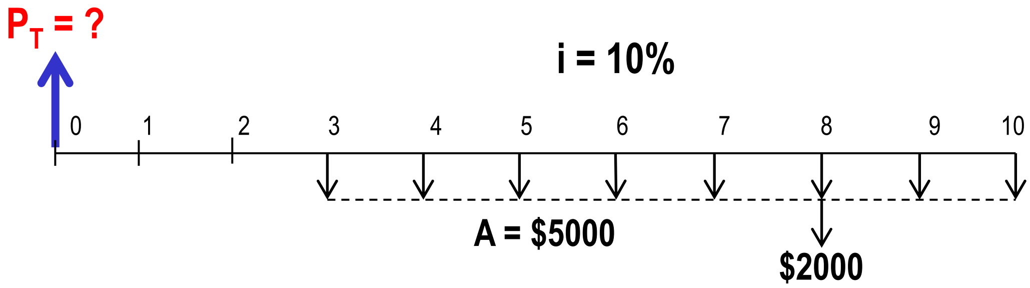
Single amount formulas **are applied to the** ***one-time cash flows***

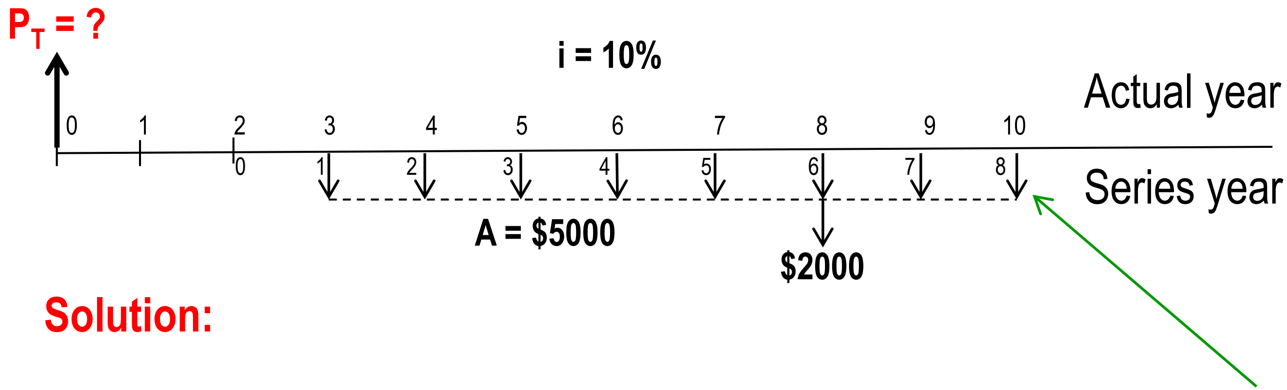
The resulting values are then _________ per the problem statement

The following slides illustrate the procedure

Example: Series and Random Single Amounts

Find the present worth in year 0 for the cash flows shown using an interest rate of 10% per year.

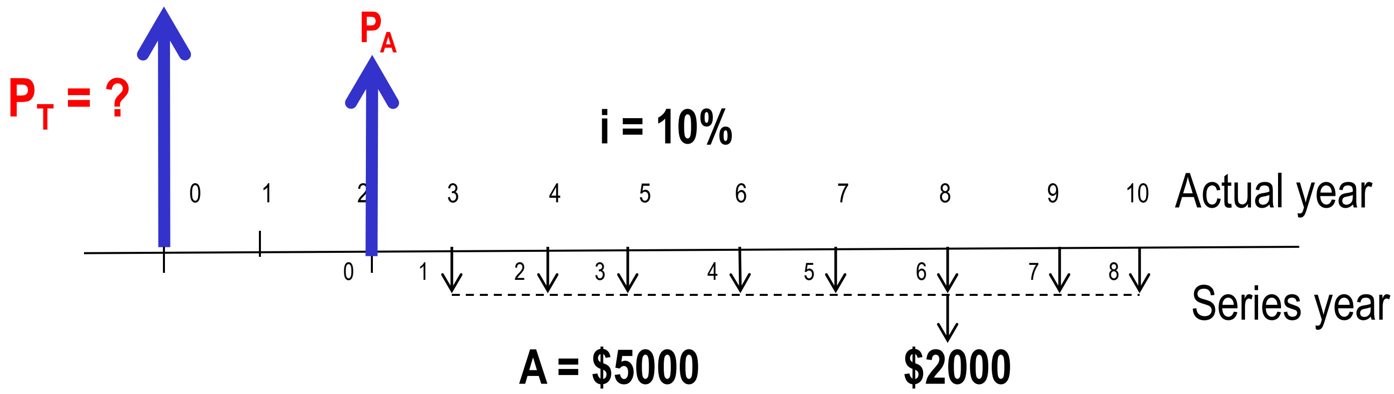




Solution:

First, re-number cash flow diagram to get n for uniform series: n =__

Example: Series and Random Single Amounts



Use P/A to get P_A in year 2: P_A = 5000(P/A,10%,8) = 5000(5.3349) = $26,675

Move P_A back to year 0 using P/F: P_0 = 26,675(P/F,10%,2) = 26,675(0.8264) = $22,044

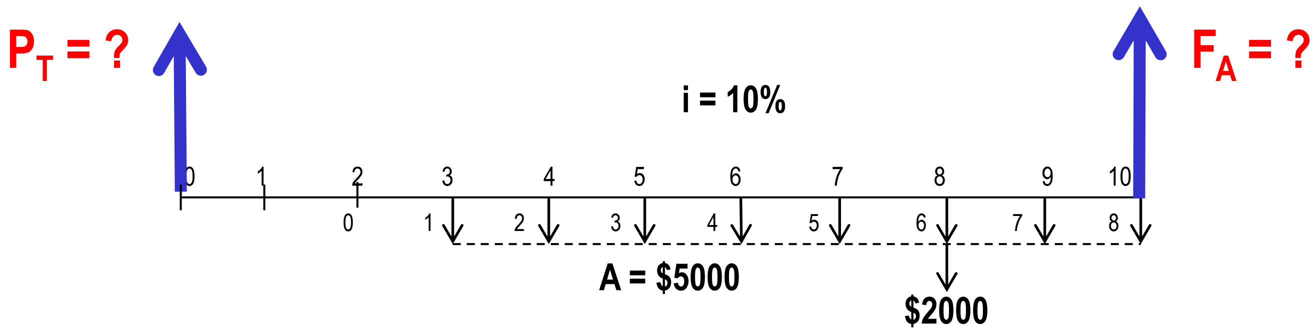
Move $2000 single amount back to year 0: P_{2000} = 2000(P/F,10%,8) = 2000(0.4665) = $933

Now, add P_0 and P_{2000} to get P_T: P_T =__________________________

Example Worked a Different Way

(Using F/A instead of P/A for uniform series)

The same re-numbered diagram from the previous slide is used



Solution: Use F/A to get F_A in actual year 10: $F_A = 5000(F/A,10\%,8) = 5000(11.4359) = \$57,180$

Move F_A back to year 0 using P/F: $P_0 = 57,180(P/F,10\%,10) = 57,180(0.3855) = \$22,043$

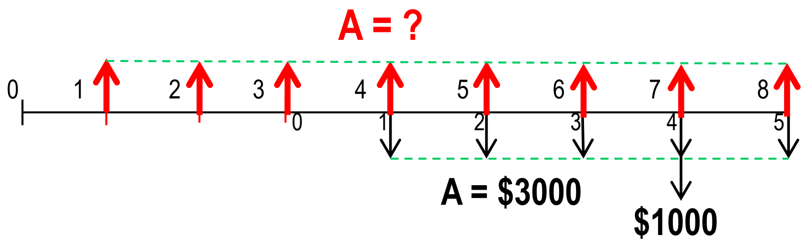
Move $2000 single amount back to year 0: $P_{2000} = 2000(P/F,10\%,8) = 2000(0.4665) = \$933$

Now, add two P values to get P_T: $P_T = 22,043 + 933 = \$22,976$ **Same as before**

As shown, there are usually ___________ to work equivalency problems

Example: Series and Random Amounts

Convert the cash flows shown below (black arrows) into an equivalent annual worth A in years 1 through 8 (red arrows) at i = 10% per year.



i = 10%

Approaches:

1. Convert all cash flows into P in year 0 and use A/P with n = 8

2. Find F in year 8 and use A/F with n = 8

Solution:

Solve for F: **F = 3000(F/A,10%,5) + 1000(F/P,10%,1)**

= 3000(6.1051) + 1000(1.1000)

= ______________

Find A: **A = 19,415(A/F,10%,8)**

= 19,415(0.08744)

= ____________

Shifted Arithmetic Gradients

Shifted gradient begins at a time other than between periods 1 and 2

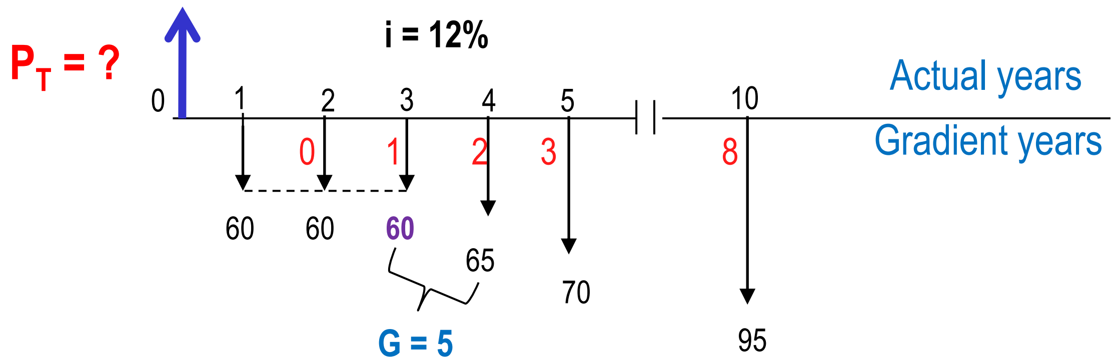
Present worth P_G is located ______________ gradient starts

Must use multiple factors to find P_T in actual year 0

To find equivalent A series, find P_T at actual time 0 and apply (___, __, __)

Example: Shifted Arithmetic Gradient

John Deere expects the cost of a tractor part to increase by $5 per year beginning 4 years from now. If the cost in years 1-3 is $60, determine the *present worth in year 0* of the cost through year 10 at an interest rate of 12% per year.



Solution: **First find P_2 for G = $5 *and* base amount ($60) in actual year 2**

$$P_2 = 60(P/A,12\%,8) + 5(P/G,12\%,8) = \$370.41$$

Next, move P_2 back to year 0

$$P_0 = P_2(P/F,12\%,2) = \$295.29$$

Next, find P_A for the $60 amounts of years 1 and 2

$$P_A = 60(P/A,12\%,2) = \$101.41$$

Finally, add P_0 and P_A to get P_T in year 0

$P_T = P_0 + P_A =$ ___________

Shifted Geometric Gradients

Shifted gradient begins at a time other than between periods 1 and 2

Equation yields P_g for *all* cash flows (base amount A_1 is included)

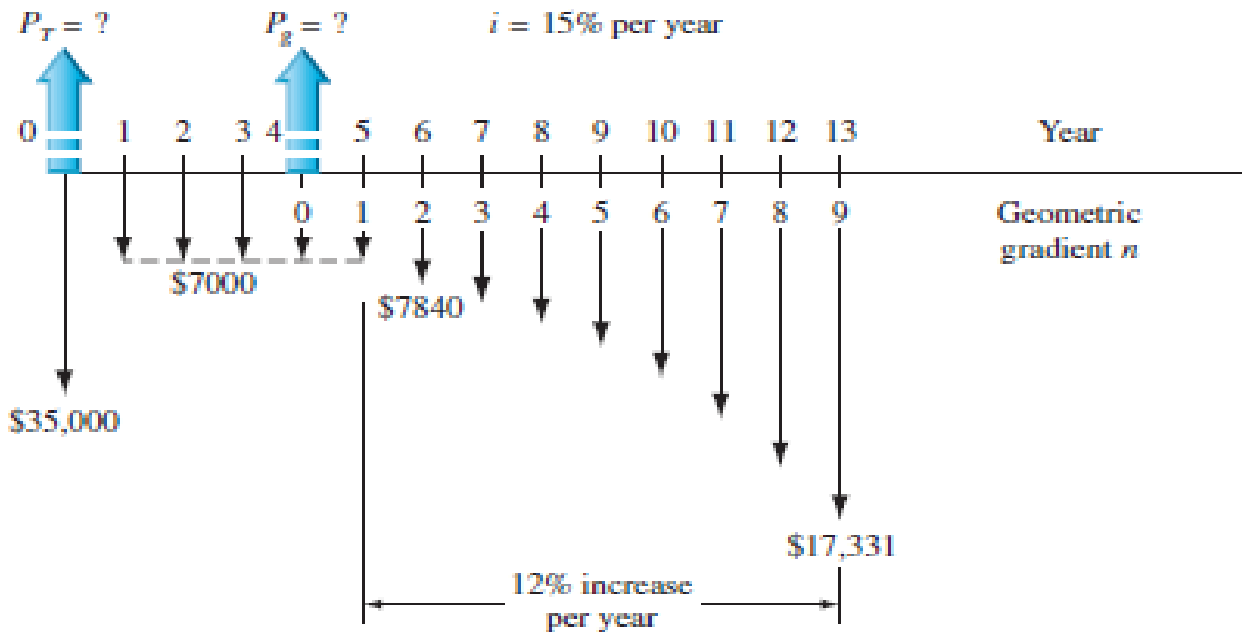
Equation ($i \neq g$): $P_g = A_1\{1 - [(1+g)/(1+i)]^n/(i-g)\}$

For negative gradient, change signs on both g values

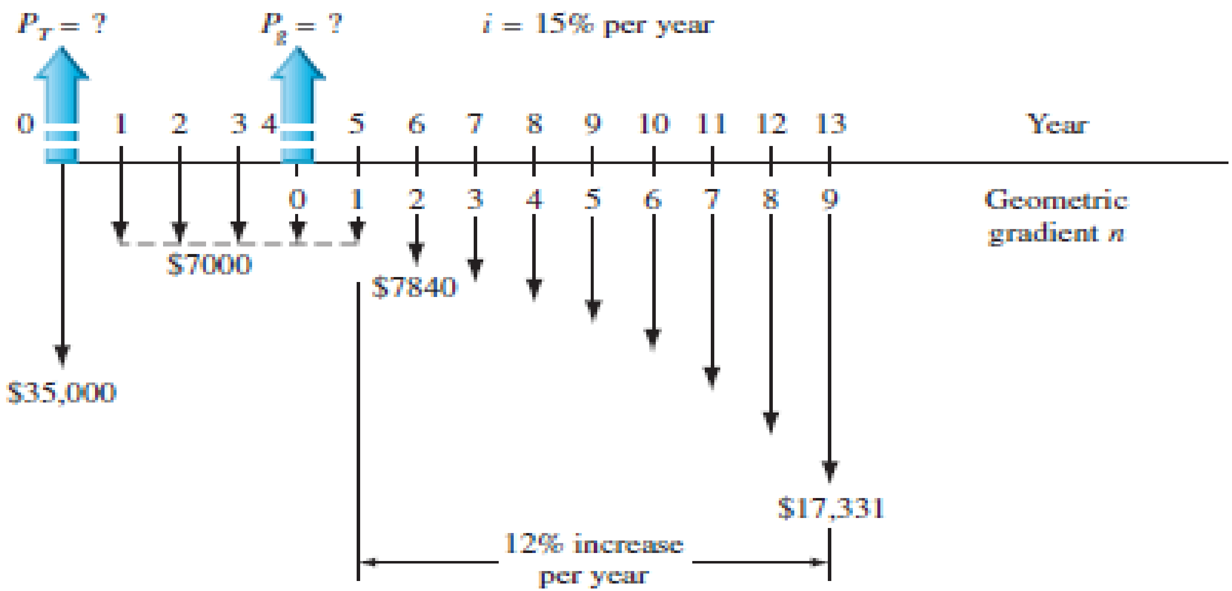
There are ___ _________ for geometric gradient factors

Example: Shifted Geometric Gradient

Weirton Steel signed a 5-year contract to purchase water treatment chemicals from a local distributor for $7000 per year. When the contract ends, the cost of the chemicals is expected to increase by 12% per year for the next 8 years. If an initial investment in storage tanks is $35,000, determine the equivalent present worth in year 0 of all of the cash flows at i = 15% per year.



Example: Shifted Geometric Gradient



Gradient starts between actual years 5 and 6; these are gradient years 1 and 2. P_g is located in gradient year 0, which is actual year 4

$P_g = 7000\{1-[(1+0.12)/(1+0.15)]^9/(0.15-0.12)\} = \$49,401$

Move P_g and other cash flows to year 0 to calculate P_T

P_T **= 35,000 + 7000(P/A,15%,4) + 49,401(P/F,15%,4) = ____________**

Negative Shifted Gradients

For negative arithmetic gradients, change sign on G term from + to -

General equation for determining P: **P = present worth of base amount - P_G**

Changed from + to -

For negative geometric gradients, change signs on both g values

Changed from + to -

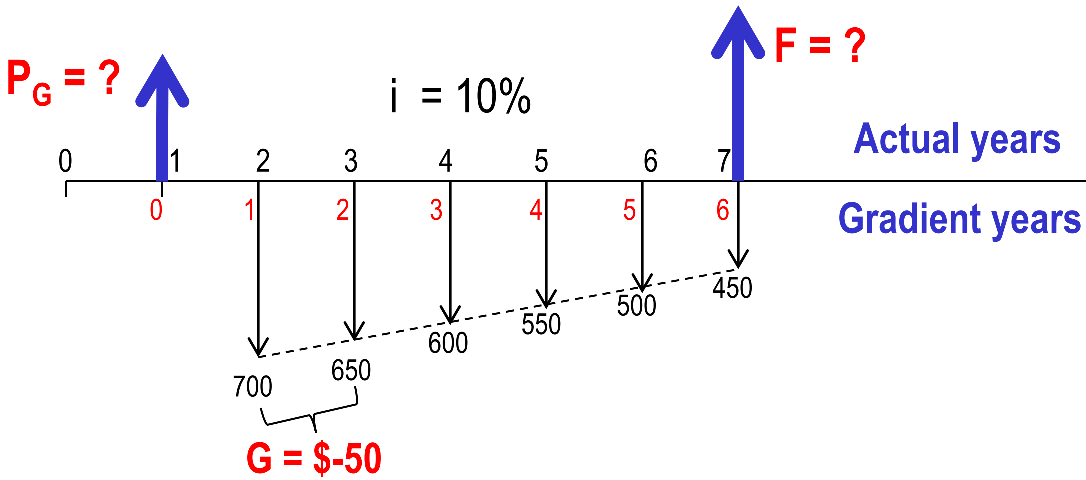
$$P_g = A_1\{1-[(1-g)/(1+i)]^n/(i+g)\}$$

Changed from - to +

All other procedures are the same as for positive gradients

Example: Negative Shifted Arithmetic Gradient

For the cash flows shown, find the future worth in year 7 at i = 10% per year



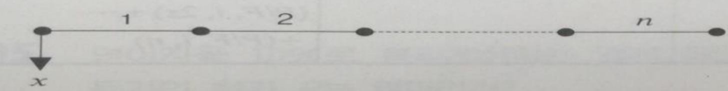
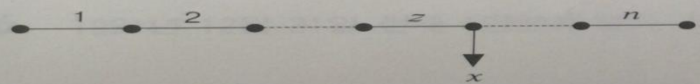
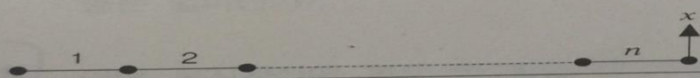
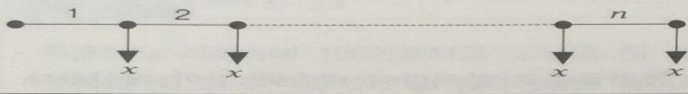
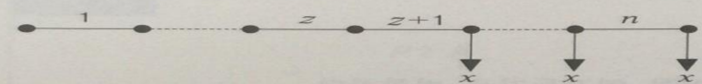
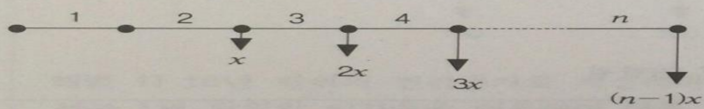
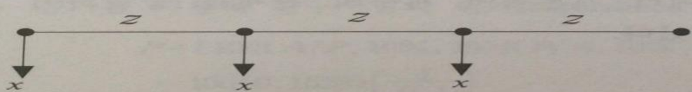
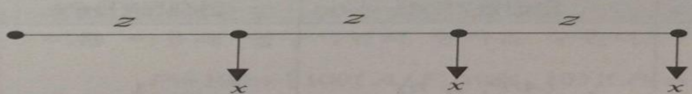
Solution: Gradient G first occurs between actual years 2 and 3; these are gradient years 1 and 2

P_G is located in gradient year 0 (actual year 1); base amount of $700 is in gradient years 1-6

$$P_G = 700(P/A,10\%,6) - 50(P/G,10\%,6) = 700(4.3553) - 50(9.6842) = \_\_\_\_\_\_$$

$$F = P_G(F/P,10\%,6) = 2565(1.7716) = \_\_\_\_\_\_\_\_$$

그림 3－7 선택된 현금흐름 X를 연간등가와 현재등가로 변환시 곱하는 인자

현금흐름의 형태	사례	연간등가(A)	현재등가(P)
1. 초기비용 	건물	$(A/P,\ i,\ n)$	1
2. 중간발생비용 	건물의 증축	$(P/F,\ i,\ z)\times(A/P,\ i,\ n)$	$(P/F,\ i,\ z)$
3. 잔존가치 	건물의 재판매	$(A/F,\ i,\ n)$	$(P/F,\ i,\ n)$
4. 연간등가 	재산세, 보험료	1	$(P/A,\ i,\ n)$
5. z＋1년에서 시작하는 연간등가 	증가된 노무비	$(F/A, i, n-z)\times$ $(A/F, i, n)$ 또는, $[(P/A, i, n)-$ $(P/A, i, z)]\cdot(A/P, i, n)$	$(P/A,\ i,\ n-z)\times$ $(P/F,\ i,\ z)$ 또는, $(P/A,\ i,\ n)-$ $(P/A,\ i,\ z)$
6. 산술적 증가 	유지비	$(A/G,\ i,\ n)$	$(P/G,\ i,\ n)$
7. 주기적비용(0년부터 각 z년마다 발생) 	대수리비	$(A/P,\ i,\ z)$	$(A/P, i, z)\times(P/A, i, n)$ 또는 $1+(P/F,\ i,\ z)$ $+\cdots+(P/F, i, n-z)$
8. 주기적비용(z년부터 각 z년마다 발생) 	대수리비	$(A/F,\ i,\ z)$	$(A/F, i, z)\times(P/A, i, n)$ 또는 $(P/F, i, z)+$ $(P/F, i, 2z)+\cdots$ $+(P/F, i, n)$

Summary of Important Points

P for shifted uniform series is ________________ of first A;
n is equal to number of A values

F for shifted uniform series is in __________________ as last A;
n is equal to number of A values

For gradients, *first change* equal to G or g occurs
between gradient years 1 and 2

For ________arithmetic gradients, change sign on G from + to -

For negative _________gradients, change sign on g from + to -

HOMEWORK

1. Please solve every Examples in your textbook. You do not have to submit your works.

2. Please upload following “PROBLEMS” solution file on “Assignment” menu in e-Class.
   ① 3.10
   ② 3.12
   ③ 3.28
   ④ 3.33
   ⑤ 3.42
   ⑥ 3.60
   ⑦ 3.75
   ⑧ 3.77