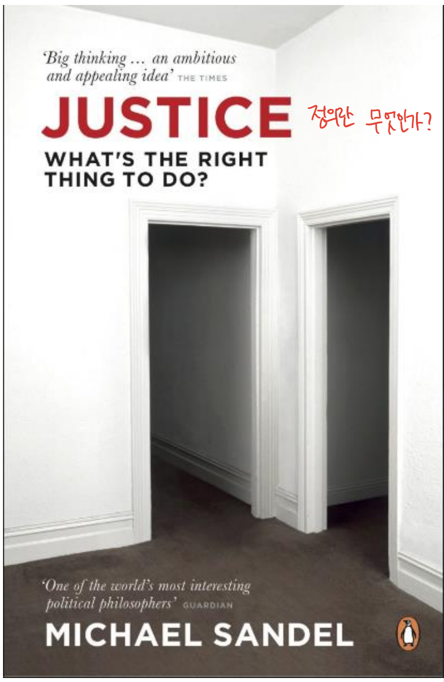
Concept of Engineering Economy

Engineering Economy

- Is about **Money** - economy
- Involves **many choices** among alternative designs, procedures, plans, and methods - engineering (e.g., build bridge)
- Asks the question "**Will it pay?**" - deserve to pay? 「지불할 가치가 있는가?」
  trade off [cost] [benefit]
- Judges whether any proposed course of action will prove **to be economical** as compared to others

for best / optimum choice
→ considering economic aspect



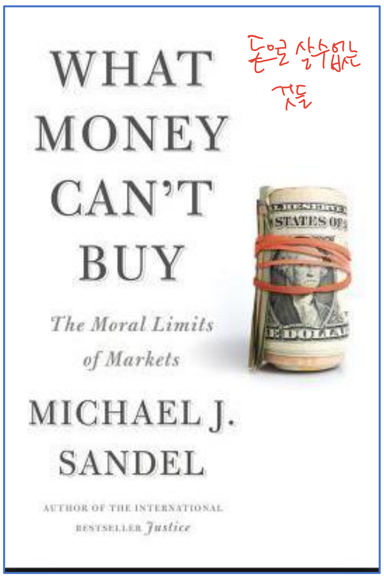
Sandel's Theories of Justice

Michael Sandel is a professor of history and philosophy at Harvard University. He is one of the most prominent thinkers and writers on the idea of justice today.

Justice

- EE's focus — Maximizing Welfare — economy related
- Promoting Virtue
- Respecting Freedom

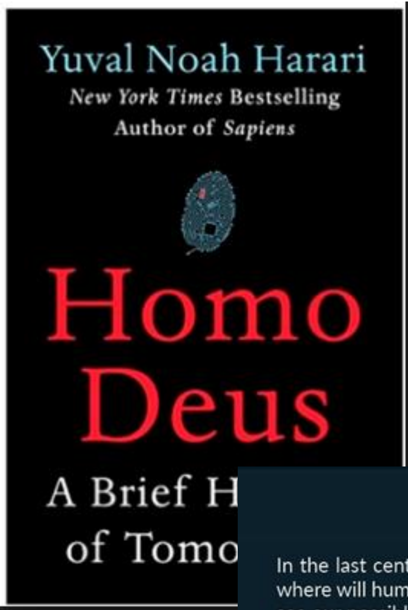
https://www.youtube.com/watch?v=0O2Rq4HJBxw&list=PL30C13C91CFFEFEA6

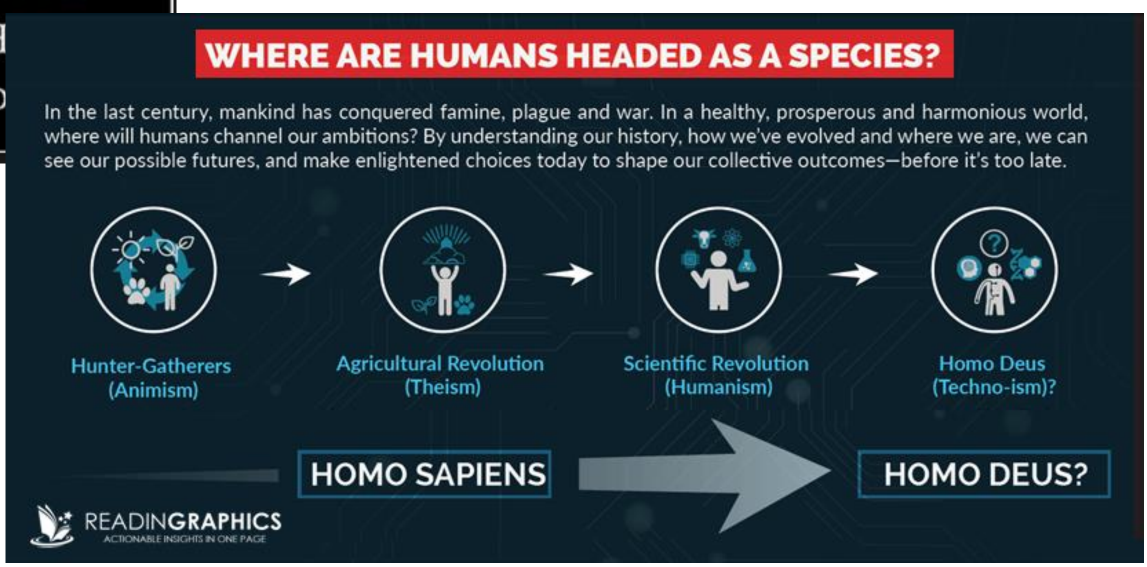


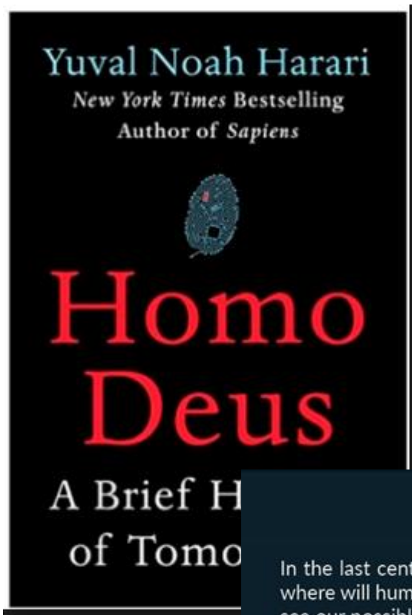
moral vs market

모든것의 상품화!
빈부격차의 심화
common life 필요해!

https://www.youtube.com/watch?v=GvDpYHyBlgc


Yuval Noah Harari
New York Times Bestselling
Author of Sapiens
Homo
Deus
A Brief H
of Tomo


WHERE ARE HUMANS HEADED AS A SPECIES?
In the last century, mankind has conquered famine, plague and war. In a healthy, prosperous and harmonious world, where will humans channel our ambitions? By understanding our history, how we've evolved and where we are, we can see our possible futures, and make enlightened choices today to shape our collective outcomes—before it's too late.
Hunter-Gatherers (Animism)
Agricultural Revolution (Theism)
Scientific Revolution (Humanism)
Homo Deus (Techno-ism)?
HOMO SAPIENS
HOMO DEUS?
READINGGRAPHICS
ACTIONABLE INSIGHTS IN ONE PAGE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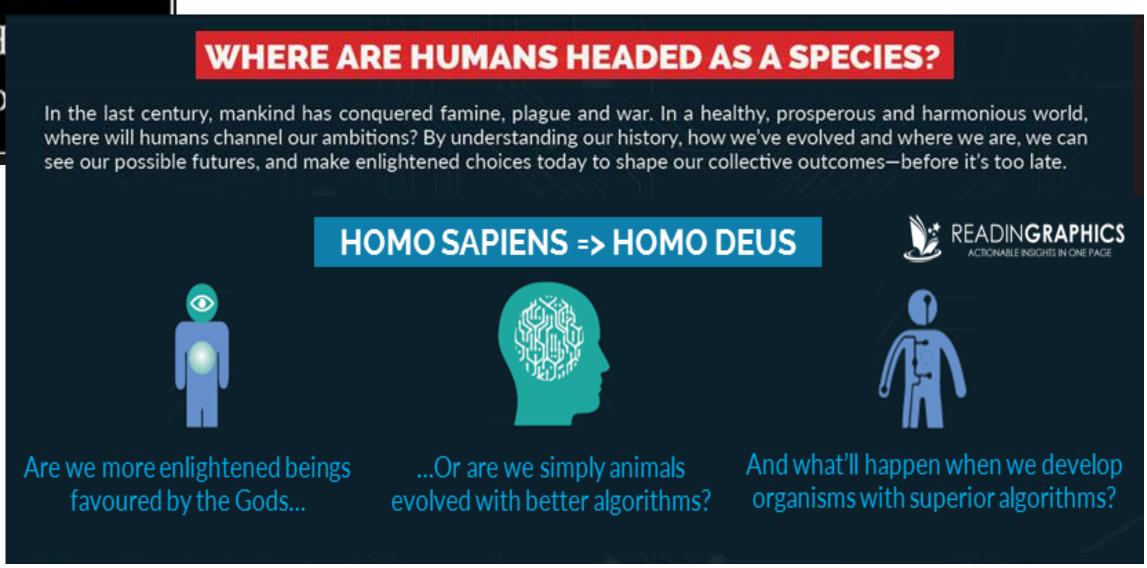
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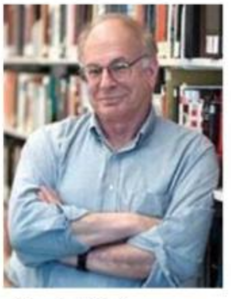
HOMO SAPIENS => HOMO DEUS
READINGGRAPHICS
ACTIONABLE INSIGHTS IN ONE PAGE
Are we more enlightened beings favoured by the Gods...
...Or are we simply animals evolved with better algorithms?
And what'll happen when we develop organisms with superior algorithms?

https://www.youtube.com/watch?v=4ChHc5jhZxs

Amos Tversky



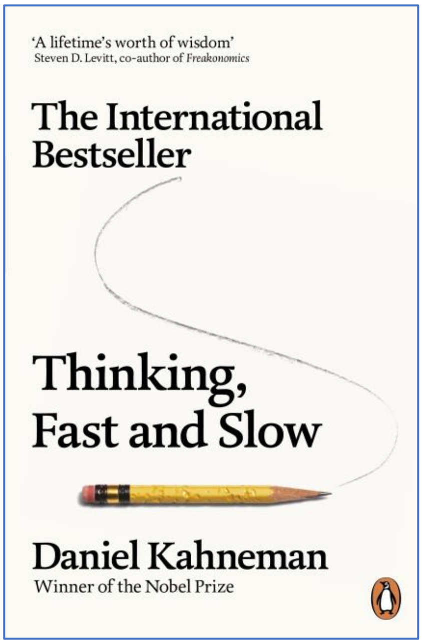
Daniel Kahneman

"Prospect Theory: An Analysis of Decisions Under Risk," *Econometrica (1979)* 47:263 – 291.

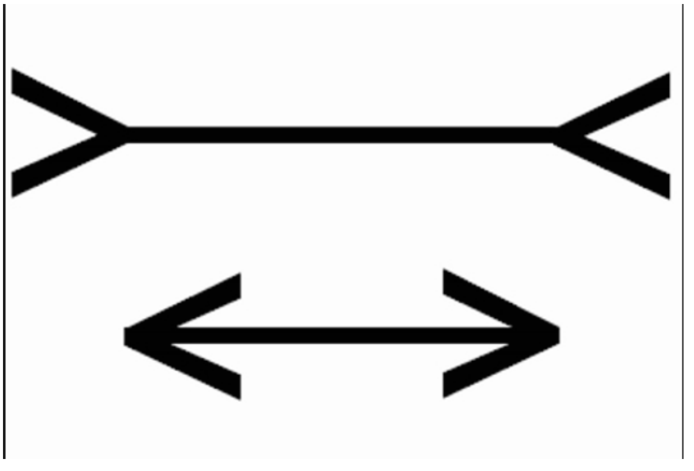
"The Framing of Decisions and the Psychology of Choice," *Science (1981)* 211:453 - 458.

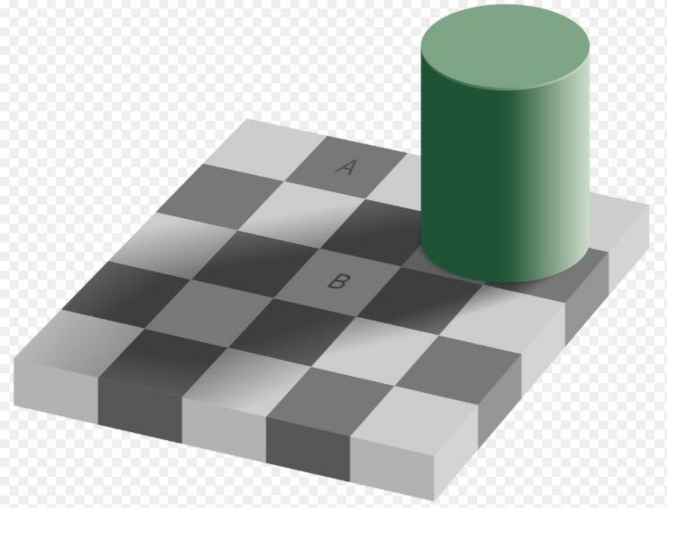
"…the research that Tversky and I conducted was guided by the idea that intuitive judgments occupy a position … between the automatic operations of perception and the deliberate operations of reasoning…we held *a two-system view, which distinguished* ① *intuition from* ② *reasoning*." -- Kahneman's Nobel Lecture

http://en.wikipedia.org/wiki/Behavioral_economics
http://nobelprize.org/nobel_prizes/economics/laureates/2002/kahnemann-lecture.pdf
http://harvardmagazine.com/2006/03/the-marketplace-of-perce.html

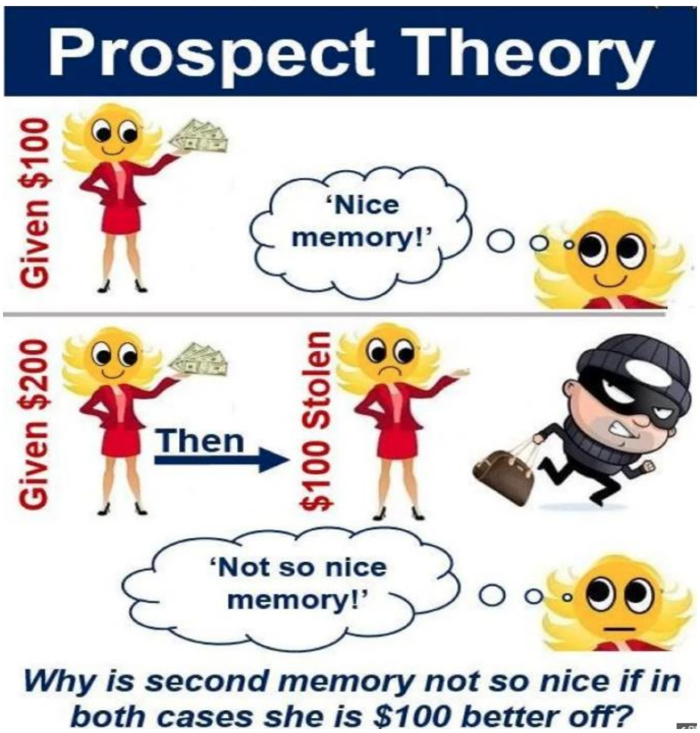

'A lifetime's worth of wisdom'
Steven D. Levitt, co-author of Freakonomics
The International Bestseller
Thinking, Fast and Slow
Daniel Kahneman
Winner of the Nobel Prize


Ri
DANIEL KAHNEMAN
THINKING FAST AND SLOW
The Role of LUCK in achievements
LIFE SATISFACTION vs EMOTIONAL HAPPINESS
MORE MONEY = MORE SATISFACTION
EMOTIONAL HAPPINESS
$70K INCOME
EMOTIONAL HAPPINESS FLATENS AT 70K PER YEAR
Intuition
dominant
FAST
SLOW
logic
SYSTEM 1
INTUITION, INSTICT
AUTOMATIC... YOU DON'T HAVE TO DO ANYTHING ACTIVELY
NEEDS MENTAL EFFORT + SEVERAL STEPS TO RETRIEVE INFORMATION
4!
2+2
DISGUSTING NAUSEA UHHH
VOMIT
HMMM...
DEMANDS ATTENTION + EFFORT
SYSTEM 2
SELF CONTROL
NO DOUBT
TRIES TO ESTABLISH WHAT IS NORMAL
ASSOCIATIVE REACTION
→ other words come to mind
→ emotional + physical reaction (disgusted face)
CREATES BIAS by quick reactions
monitors + controls SYSTEM 1 (usually endorses what System 1 comes up with)
OR
When System 2 is at work, self control goes down
when tasked with mathematical problems people are more likely to choose chocolate cake over fruit...)
IS THAT THE REASON FOR STRESS EATING??
DISGUSTING
PHYSICAL REACTION REINFORCES EMOTION
BETTER!
BIASES INFLUENCE "RATIONAL DECISIONS"
"WHEN PEOPLE ARE FACED WITH A PROBLEM THEY CAN'T SOLVE, THEY COME UP WITH AN ANSWER NEVERTHELESS"
EMOTION CREATES PHYSICAL RESPONSE
90% FAT FREE
10% FAT
FORMS STEREO-TYPES
BASED ON PREVIOUS EXPERIENCE
OPERATES IN THE NOW
SAME MEAT, DISCRIBED IN 2 DIFFERENT WAYS. WE PREFER THE 90%. FAT-FREE ONE
OH, DAMN WHATEVER, GIVE ME THE CAKE!
I'M STRESSED!

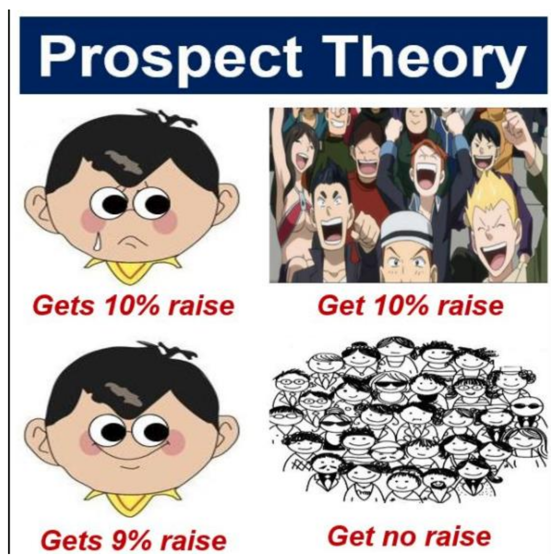



A
B

전망이론

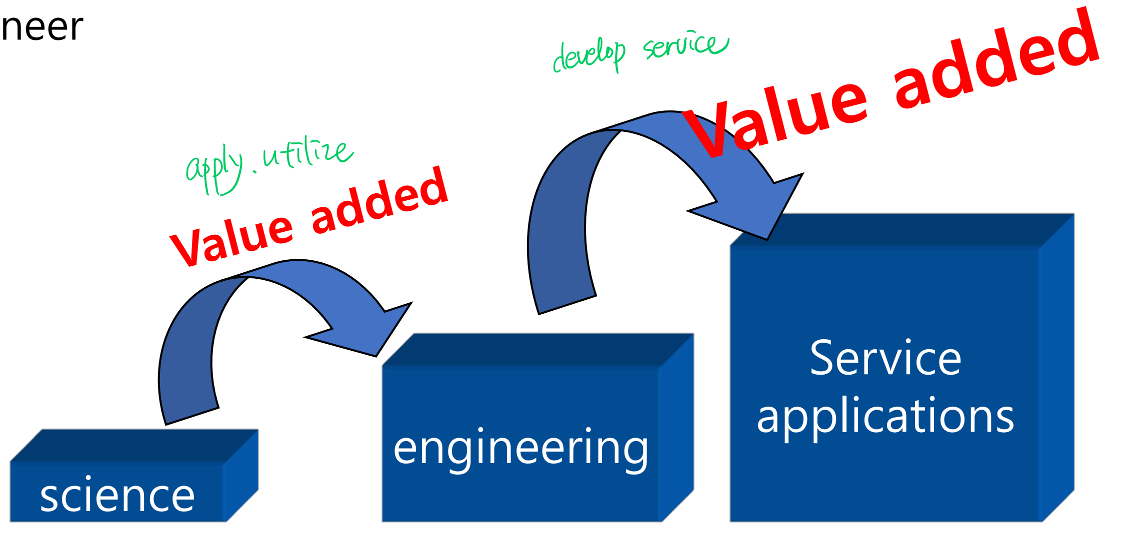


↳ focusing on "LOST"



Engineering vs. Eng. Economy

- Engineer



Science

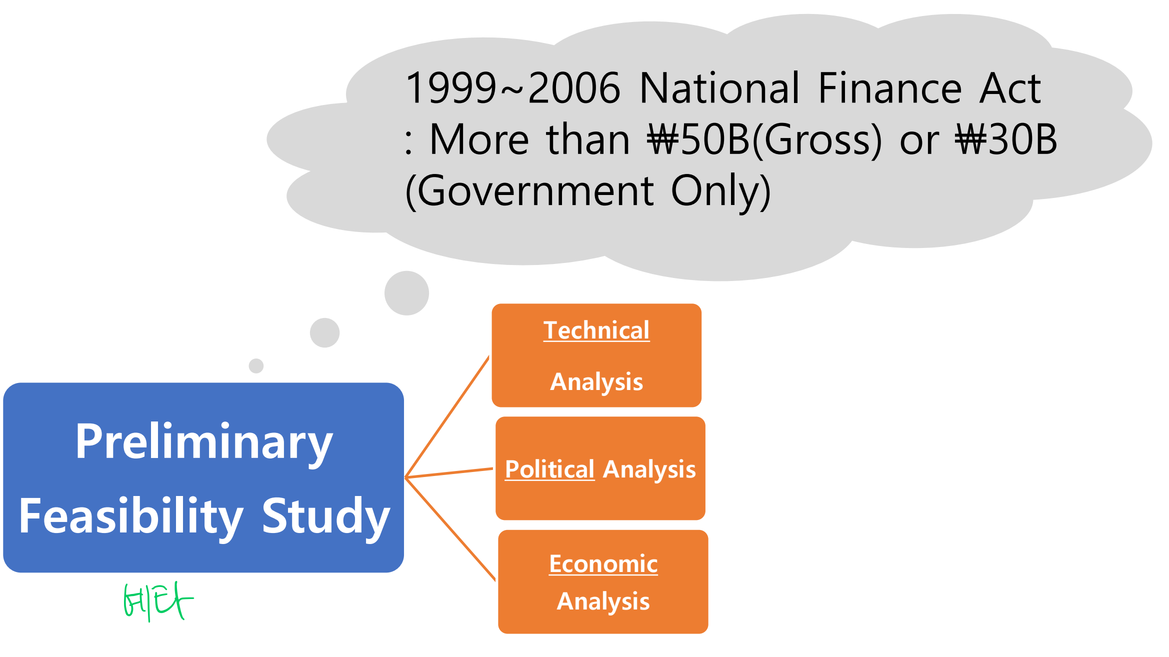
- Engineering (ABET)
  - The Profession in which a knowledge of the mathematical and natural sciences gained by study, experience, and practice is applied with judgement to develop ways to utilize, ***economically**, the materials and forces of nature **for the benefit of mankind**

compare vs input

Engineering vs. Eng. Economy

- Economic Efficiency
  - **System Worth / System Cost > 1** ⇒ Worth > Cost
    - ♣ cf. Physical Efficiency = output / input (**<** **1**) → 에너지 효율은 1을 넘지 못한다.

- Engineering Economy
  - A systematic evaluation of the economic merits (benefit-cost) of proposed solutions to engineering projects
  - Provides technical solutions with economic justification



예타

Eng. Economy Procedures

1. Define the problem(Engineer)

2. Find the solutions (Engineer)

3. Decision Making (E. economist)
    - Quantitative: B/C ratio, NPV,..
    - Qualitative: AHP,Delphi,..
    - Economic Evaluation

4. Presentation (E. economist)

Preliminary Feasibility Analysis

Provide **Rational Principles** and **Systematic Analysis Methods**

to solve problems in 3. 4.

Decision making

presentation

Engineering Economy

1. Objective – Evaluation

✓ **How to compare the economic value of alternative design options?**

2. Basis – Cash Flow Analysis

✓ **Equivalence occurs when one is indifferent between two sets of cash flows**

각 프로젝트마다 현금흐름 다르다.

각 현금흐름 fairly 비교위한 방법을 필요.

3. Key Issues

✓ **Time Value of Money** 시간에 따라 돈의 가치 변화

✓ **Cash flows occurring at different times** → 어떻게 처리할 것인가?

✓ **Designs with different durations** → 어떻게 처리할 것인가?

Time Value of Money

- **Value of money changes over time!!!**

- Interest & Interest Rate
  - Interest : Manifestation (발현) of TVM Time Value of Money
  - Interest Rate = Interest / Principal (원금)

- How to determine Interest Rate ?
  - risk cost, opportunity cost, inflation, financing cost
  - ……

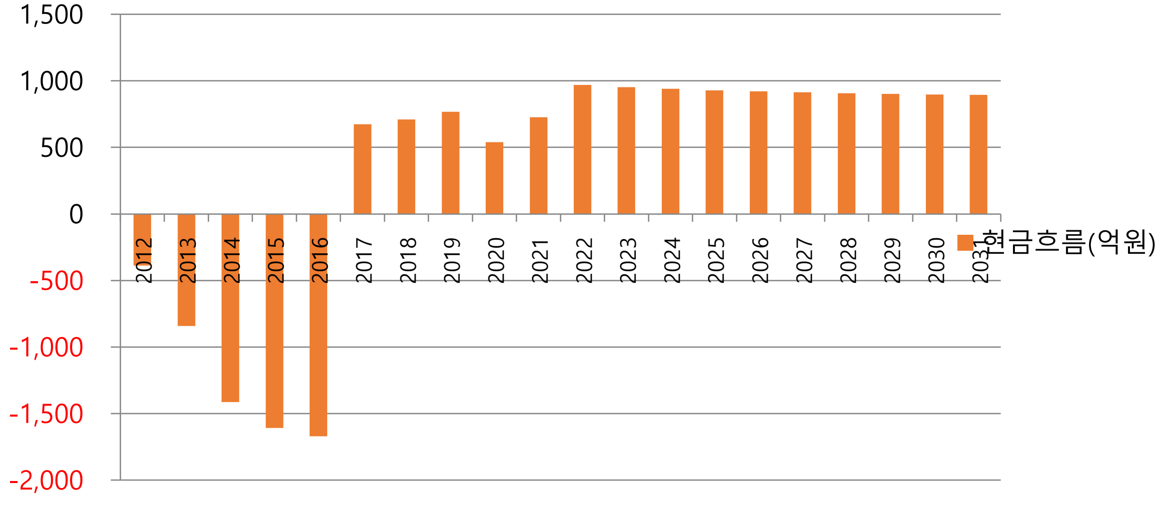
Examples

통신위성 시스템 cost vs 통신위성으로 얻는 benefit

compare ⇒ Merit 계산.

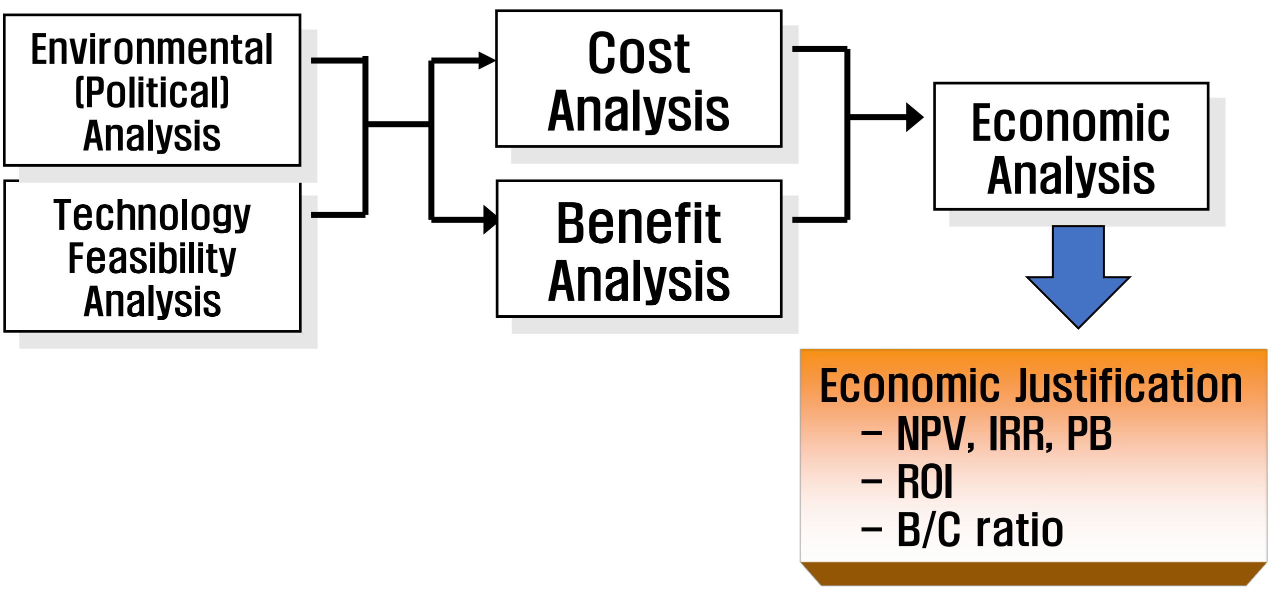
EE의 역할

An Example of Cash Flow



Procedures for Economic Analysis

메타만 유사



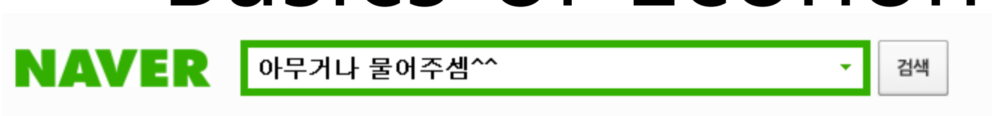
E.E., Accounting, Finance

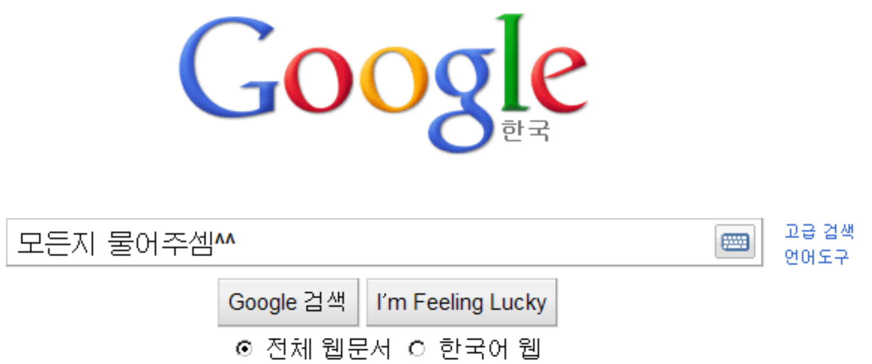
- Accounting vs. E.E.
  - Past Cashier Records vs. Future Investment Estimation
  - Ex-Post (사후) vs. Ex-Ante (사전)
  - Integrity vs. Accuracy
  - Historian vs. Fortune teller

- Finance vs. E.E.
  - Fund Mgmt & Financial Investment vs. Project Investment
  - Financing, Stock& Bond Issue (사채발행), M&A vs. New Technology & Business Investment

Need All the Answers!

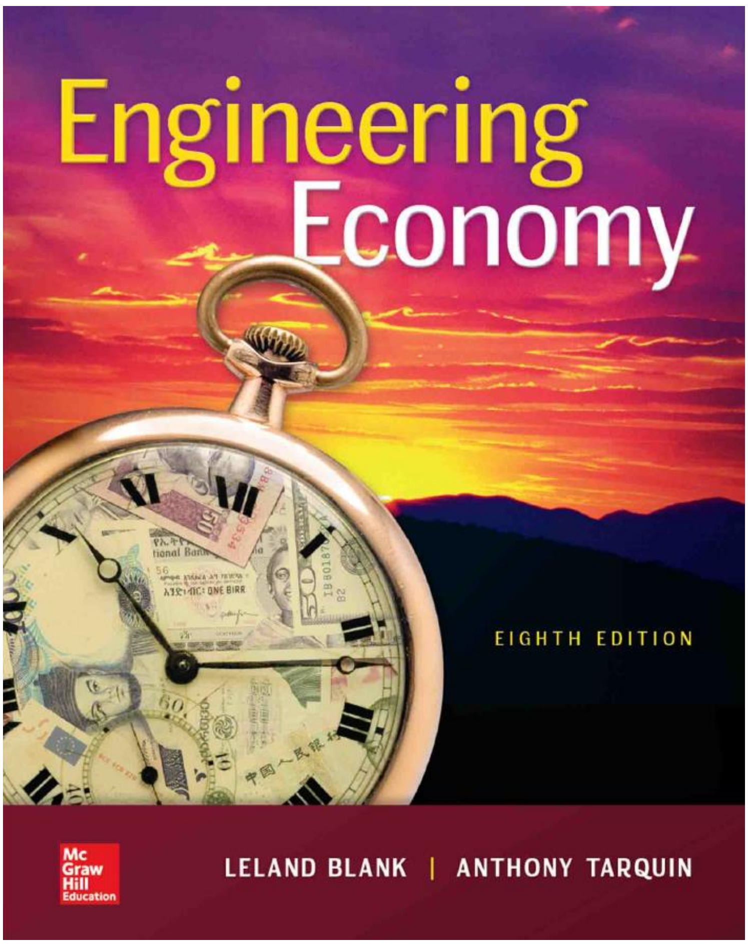
Basics of Economics





거인의 어깨에 올라서서 더 넓은 세상을 바라보라 - 아이작 뉴턴

Thank you !



Chapter 1

Foundations Of Engineering Economy

Engineering Economy

8th edition

Leland Blank

Anthony Tarquin

McGraw Hill Higher Education

LEARNING OUTCOMES

1. Role in decision making
2. Study approach
3. Ethics and economics
4. Interest rate
5. Terms and symbols
6. Cash flows

7. Economic equivalence
8. Simple and compound interest
9. Minimum attractive rate of return(MARR)
10. Spreadsheet functions excel

build a bridge

#1 alternative

Simple

not enough money
only functional property

Vs.

#2 alternative

Sophisticated

enough money
multi-purpose
(function, landscape, ...)

⇒ consider various aspects of making something

Why Engineering Economy is Important to Engineers

- Engineers design and create - something
- Designing involves economic decision (simple, cheap VS expensive, sophisticated)
- Engineers must be able to incorporate economic analysis into their creative efforts (depending on objective)
- Often engineers must select and implement from multiple alternatives
- Understanding and applying time value of money, economic equivalence, and cost & revenue estimation are vital for engineers
- A proper economic analysis for selection and execution is a fundamental task of engineering

Time Value of Money (TVM)

<$100 now> ≠ ∵ TVM
<$100 next year>

Description: TVM explains the change in the amount of money over time for funds owed 빚지다 by or owned by a corporation (or individual)

Because
- Corporate investments are expected to earn a return
- Investment involves money
- Money has a ' time value '

The time value of money is the most important concept in engineering economy

Engineering Economy

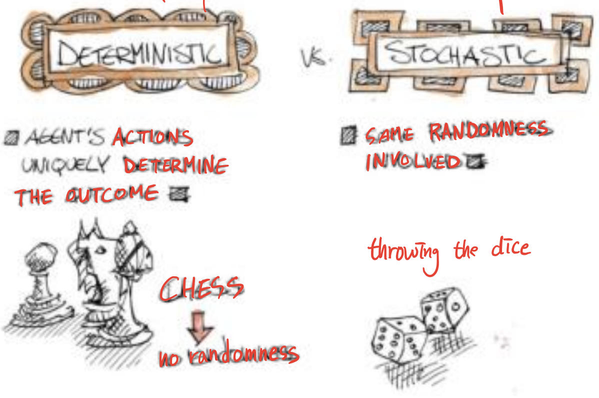
- Engineering Economy involves
  - Formulating 공식화 (정보 시스템화. 단순화 ⇒ 문제 해결 쉽게 하기위해)
  - Estimating, and 추정. 추산
  - Evaluating 평가

  expected economic outcomes of alternatives designed to accomplish a defined purpose
- Estimates of economic outcomes can be deterministic 결정론적 or stochastic 확률론적 in nature
- Easy-to-use math techniques simplify the evaluation

(easy to solve)

결정론적

확률론적



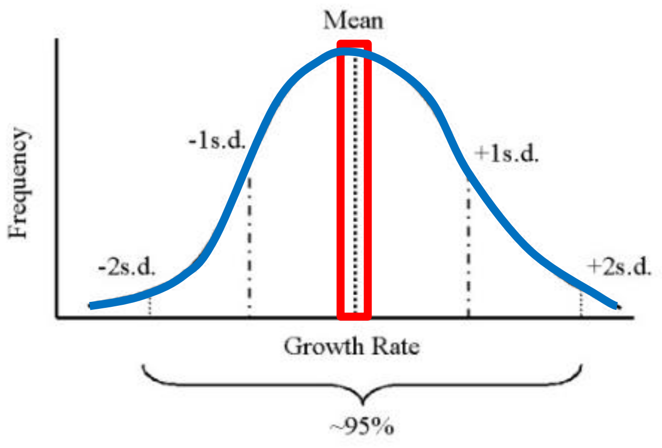
CHESS
⇩
no randomness

throwing the dice

prediction & forecasting in real life

⇒ $\boxed{\text{STOCHASTIC}}$

Deterministic vs. Stochastic Factors

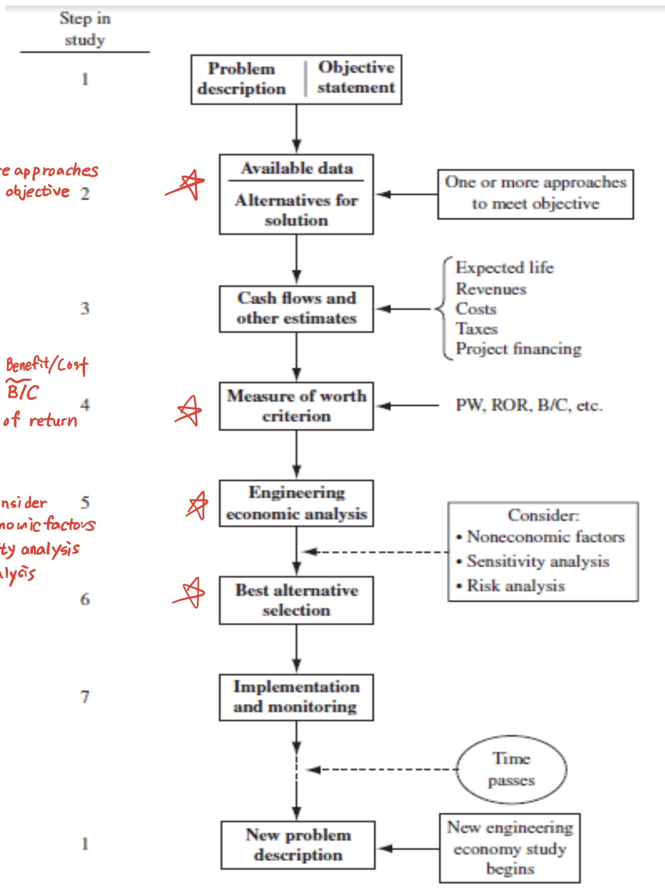


General Steps for Decision Making Processes

formulation

1. Understand the problem – Define objectives
2. Collect relevant information
3. Define the set of feasible 실현가능한 alternatives
4. Identify the criteria for decision making
5. Evaluate the alternatives and apply sensitivity analysis
6. Select the "best" alternative
7. Implement the alternative and monitor results

Steps in an Engineering Economy Study



Available data

Alternatives for solution

One or more approaches to meet objective

Measure of worth criterion

present worth Benefit/Cost

PW ROR B/C

rate of return

Engineering economic analysis

Consider

Noneconomic factors

Sensitivity analysis

Risk analysis

Best alternative selection

Ethics – Different Levels

- **Universal morals or ethics** – Fundamental beliefs: stealing, lying, harming or murdering another are wrong
- **Personal morals or ethics** – Beliefs that an individual has and maintains over time; how a universal moral is interpreted and used by each person
- **Professional or engineering ethics** – Formal standard or code that guides a person in work activities and decision making

Code of Ethics for Engineers

All disciplines have a formal code of ethics. National Society of Professional Engineers (NSPE) maintains a code specifically for engineers; many engineering professional societies have their own code

National Society of Professional Engineers®

Code of Ethics for Engineers

Preamble

Engineering is an important and learned profession. As members of this profession, engineers are expected to exhibit the highest standards of honesty and integrity. Engineering has a direct and vital impact on the quality of life for all people. Accordingly, the services provided by engineers require honesty, impartiality, fairness, and equity, and must be dedicated to the protection of the public health, safety, and welfare. Engineers must perform under a standard of professional behavior that requires adherence to the highest principles of ethical conduct.

I. Fundamental Canons

Engineers, in the fulfillment of their professional duties, shall:

1. Hold paramount the safety, health, and welfare of the public.
2. Perform services only in areas of their competence.
3. Issue public statements only in an objective and truthful manner.
4. Act for each employer or client as faithful agents or trustees.
5. Avoid deceptive acts.
6. Conduct themselves honorably, responsibly, ethically, and lawfully so as to enhance the honor, reputation, and usefulness of the profession.

II. Rules of Practice

1. Engineers shall hold paramount the safety, health, and welfare of the public.

4. Engineers shall act for each employer or client as faithful agents or trustees.
   a. Engineers shall disclose all known or potential conflicts of interest that could influence or appear to influence their judgment or the quality of their services.
   b. Engineers shall not accept compensation, financial or otherwise, from more than one party for services on the same project, or for services pertaining to the same project, unless the circumstances are fully disclosed and agreed to by all interested parties.
   c. Engineers shall not solicit or accept financial or other valuable consideration, directly or indirectly, from outside agents in connection with the work for which they are responsible.
   d. Engineers in public service as members, advisors, or employees of a governmental or quasi-governmental body or department shall not participate in decisions with respect to services solicited or provided by them or their organizations in private or public engineering practice.
   e. Engineers shall not solicit or accept a contract from a governmental body on which a principal or officer of their organization serves as a member.
5. Engineers shall avoid deceptive acts.
   a. Engineers shall not falsify their qualifications or permit misrepresentation of their or their associates' qualifications. They shall not misrepresent or exaggerate their responsibility in or for the

Interest and Interest Rate

- **Interest** – the manifestation (징후) of the time value of money
  - Fee that one pays to use someone else's money
  - Difference between an ending amount of money and a beginning amount of money
    - ➤ **Interest =** amount owed now − principal (원금)

- **Interest rate** – Interest paid over a time period expressed as a percentage of principal
    - ➤ $$\text{Interest rate (\%)} = \frac{\text{interest accrued per time unit}}{\text{principal}} \times 100\%$$

누적,축적

Rate of Return (R.R) 수익률

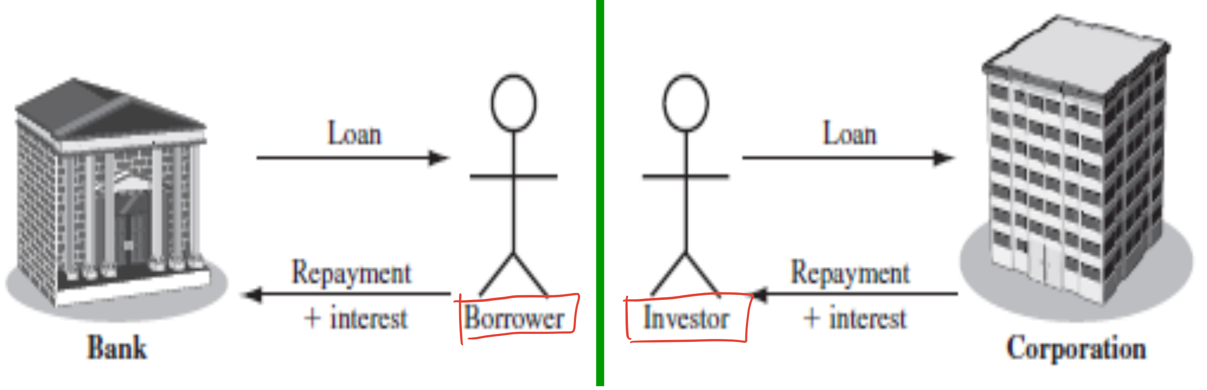
- Interest earned over a period of time is expressed as a percentage of the original amount (principal)

$$\text{Rate of return (\%)} = \frac{\text{interest accrued per time unit}}{\text{original amount}} \times 100\%$$

- Borrower's perspective – interest rate paid
- Lender's or investor's perspective – rate of return earned

Interest paid

Interest earned



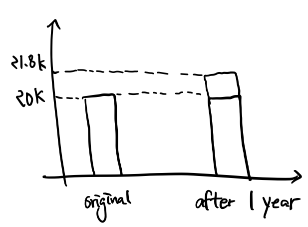
Interest rate

Rate of return

Ex 1.4) Inc $\xleftarrow{\$20k}$ Bank $\left\langle \begin{matrix} 1 \text{ year} \\ 9\% \end{matrix} \right\rangle$ Inc $\xrightarrow[\text{principal + interest}]{}$ Bank

a) interest : $20k \times 9\% = 1.8k$ / total : $21.8k = \underset{(\text{principal})}{20k} + \underset{(\text{interest})}{1.8k}$

b)



Commonly used Symbols

t = time, usually in periods such as years or months

P = value or amount of money at a time t designated as present or time 0

F = value or amount of money at some future time, such as at $t = n$ periods in the future

A = series of consecutive, equal, "end-of-period" amounts of money
연이은, serial
annual value

n = number of interest periods; years, months

i = interest rate or rate of return per time period; percent per year or month

Ex 1.7)

$P = 5000$

$A = 1000$ per year for 5 years

$F = ?$ at end of year 6

$i = 6\%$ per year

$n = 5$ years for A series and 6 for F value

Ex 1.8)

a) $P = ?$

$F = P + 5000$

$i = 6\%$

$n = 1$

b) $(P \times 1.06) = P + 5000$

$0.06P = 5000$

$P \cong 83{,}333.3\dot{3}$

Cash Flows: Terms

- **Cash Inflows –** Revenues (R), receipts, incomes, savings generated by projects and activities that flow in . Plus (+) sign used
- **Cash Outflows –** Disbursements 지출 (D), costs, expenses, taxes caused by projects and activities that flow out . Minus (−) sign used
- **Net Cash Flow** 순현금흐름 **(NCF) for each time period:**

  NCF = cash inflow − cash outflow **= R – D**

- End-of-period assumption :

  Funds flow at the end of a given interest period

  all cash flow occur at the end-of-period
  (when the other conditions are not mentioned)

Cash Flows: Estimating

✓ Point estimate – A single-value estimate of a cash flow element of an alternative ~ deterministic

Cash inflow: Income = $150,000 per month

✓ Range estimate – Min. and Max. values that estimate the cash flow ~ probablistic

Cash outflow: Cost is between $2.5 M and $3.2 M

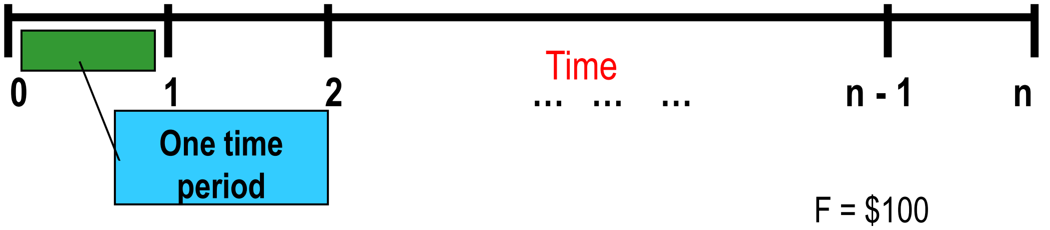
Point estimates are commonly used; however, range estimates with probabilities attached provide a better understanding of variability of economic parameters used to make decisions

Cash Flow Diagrams

A typical cash flow diagram **might look like**

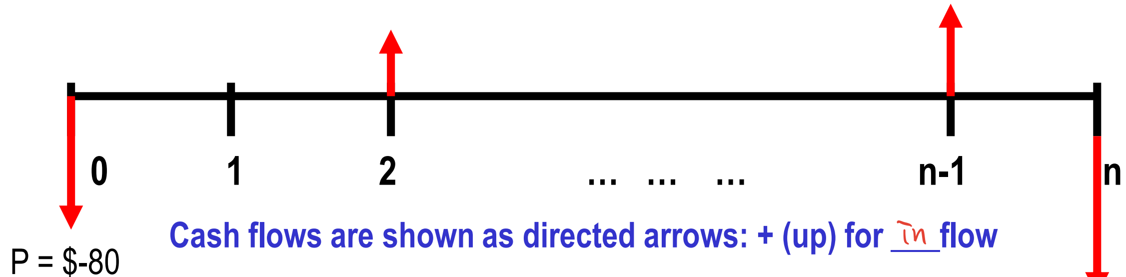
1. Draw a time line

Always assume end-of-period *cash flows*



F = $100

2. Show the cash flows and discount(interest) rate



Cash flows are shown as directed arrows: + (up) for in flow

- (down) for out flow

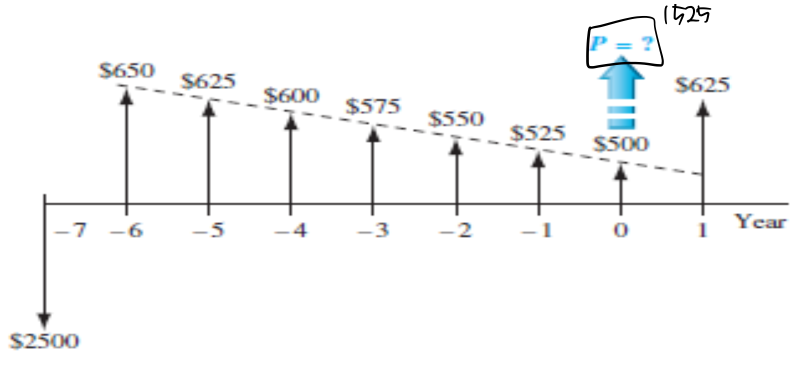
i=10 %

Ex 1.11)

Cash Flow Diagram Example

Plot observed cash flows over last 8 years and estimated sale next year for $150. Show present worth (P) arrow at present time, t = 0

End of Year	Income	Cost	Net Cash Flow
−7	$ 0	$2500	$−2500
−6	750	100	650
−5	750	125	625
−4	750	150	600
−3	750	175	575
−2	750	200	550
−1	750	225	525
0	750	250	500
1	750 + 150	275	625



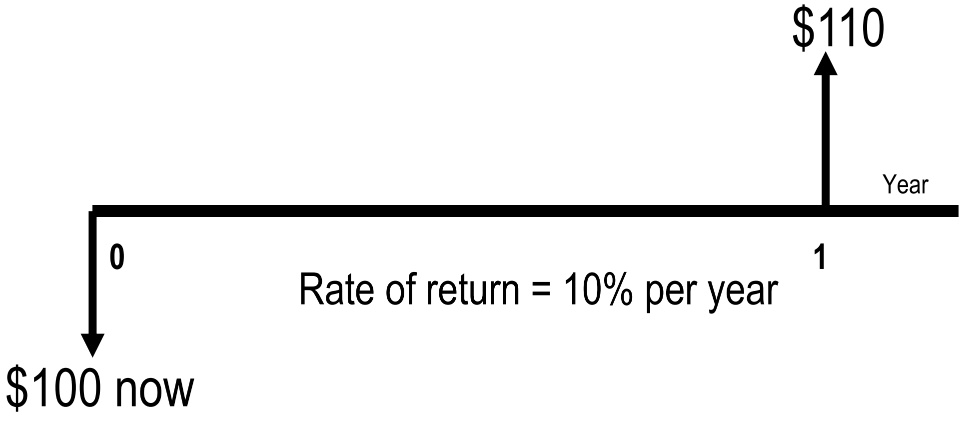
Economic Equivalence

Definition: Combination of interest rate (rate of return) and time value of money to determine different amounts of money at different points in time that are economically equivalent

How it works: Use rate *i* and time *t* in upcoming relations to move money (values of P, F and A) between time points *t* = 0, 1, …, n to make them equivalent (not equal) at the rate *i*

Example of Equivalence

Different sums of money at different times may be equal in economic value **at a given rate**



$100 now is economically equivalent to $110 one year from now, if the $100 is invested at a rate of 10% per year.

Simple and Compound Interest

❑ Simple Interest 단리

Interest is calculated using principal.

Interest = (principal)(number of periods)(interest rate)

$I = Pni$

Example: $100,000 lent for 3 years at simple i = 10% per year. What is repayment after 3 years?

Interest = 100,000(3)(0.10) = $30,000

Total due = 100,000 + 30,000 = $130,000

Simple and Compound Interest

복리

- Compound Interest

  Interest is based on principal plus all accrued interest

  That is, interest compound over time

Interest = (principal + all accrued interest) (interest rate)

Interest for time period *t* is

$$I_t = \left(P + \sum_{j=1}^{j=t-1} I_j\right)(i)$$

Compound Interest Example

Example: **$100,000 lent for 3 years at i = 10% per year compounded. What is repayment after 3 years?**

Interest, year 1:	$I_1 = 100{,}000(0.10) = \$10{,}000$
Total due, year 1:	$T_1 = 100{,}000 + 10{,}000 = \$110{,}000$
Interest, year 2:	$I_2 = 110{,}000(0.10) = \$11{,}000$
Total due, year 2:	$T_2 = 110{,}000 + 11{,}000 = \$121{,}000$
Interest, year 3:	$I_3 = 121{,}000(0.10) = \$12{,}100$
Total due, year 3:	$T_3 = 121{,}000 + 12{,}100 =$ **$133,100**

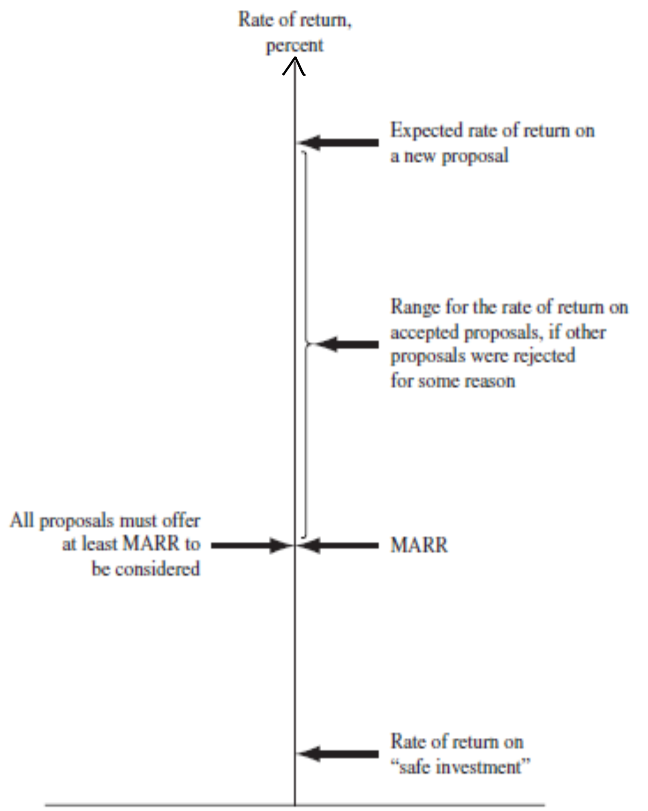
Compounded: $133,100 Simple: $130,000

cf)
TABLE 1-1

Minimum Attractive Rate of Return

최소요구수익률

- MARR is a reasonable rate of return (percent) established for evaluating and selecting alternatives
- An investment is justified economically if it is expected to return at least the MARR
- Also termed hurdle rate, benchmark rate and *cutoff rate*



Potential alternatives (MARR 이상) 중 RoR이 가장 높은 alternative select!

MARR Characteristics

- MARR is established by the financial managers of the firm
- MARR is fundamentally connected to the cost of capital (financing cost)
- Both types of capital financing are used to determine the weighted average cost of capital **(WACC)** and the MARR
- MARR usually considers the risk inherent 내재하는 to a project

Types of Financing

- Equity financing (EF) – Funds either from retained earnings, new stock issues, or owner's infusion of money.

자기자본조달 투입

- Debt financing (DF) – Borrowed funds from outside sources – loans, bonds, mortgages, venture capital pools, etc. Interest is paid to the lender on these funds

채권 담보대출 대출기관

For an economically justified project

ROR ≥ MARR > WACC

$= EF\% \times EF\ rate + DF\% \times DF\ rate$

Opportunity Cost 기회비용

- **Definition: Largest rate of return of all projects not accepted (forgone) due to some reasons**
- **If no MARR is set, the ROR of the project not undertaken establishes the opportunity cost (the de facto MARR)**

※ **"the _loss_ of potential gain from _other_ alternatives when one alternative is chosen" – New Oxford American Dictionary**

Introduction to Spreadsheet Functions

Excel financial functions

Present Value, P: = PV(i%,n,A,F)

Future Value, F: = FV(i%,n,A,P)

Equal, periodic value, A: = PMT(i%,n,P,F)

Number of periods, n: = NPER(i%,A,P,F)

Compound interest rate, i: = RATE(n,A,P,F)

Compound interest rate, i: = IRR(first_cell:last_cell)

Present value, any series, P: = NPV(i%,second_cell:last_cell) + first_cell

---

Example: Estimates are P = $5,000 n = 5 years i = 5% per year

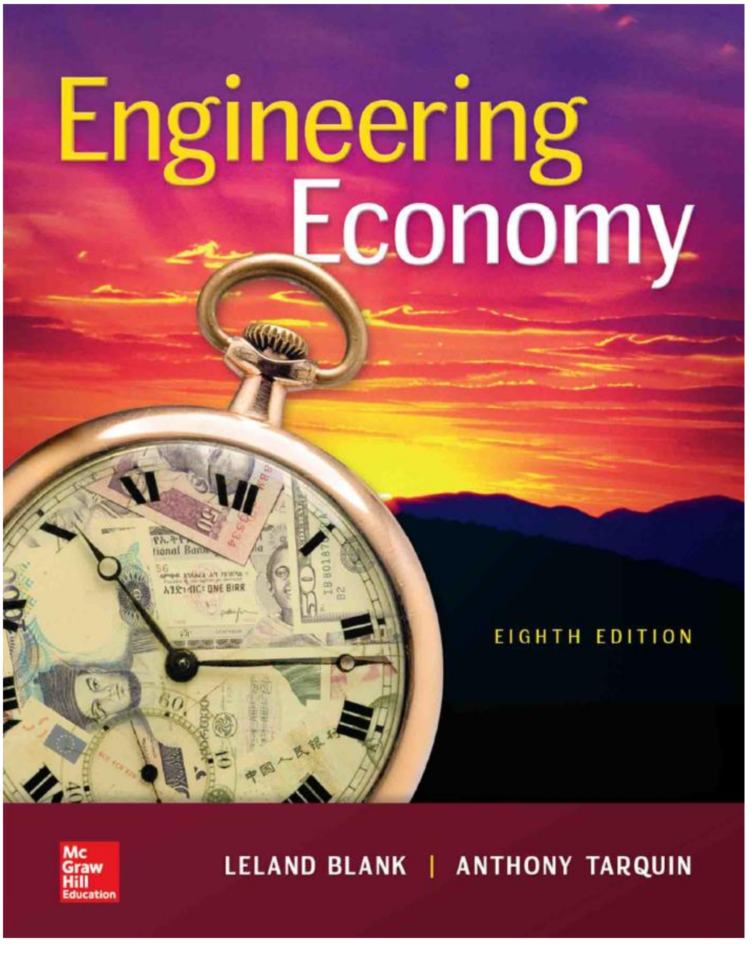
Find A in $ per year

Function and display: = PMT(5%, 5, 5000) displays A = $1,154.87

HOMEWORK

1. Please solve every Examples in your textbook. You do not have to submit your works.

2. Please upload following “PROBLEMS” solution file on “Assignment” menu in e-Class.
   ① 1.18
   ② 1.25
   ③ 1.33
   ④ 1.40
   ⑤ 1.45
   ⑥ 1.61



Chapter 2

계수

Factors: How Time and Interest Affect Money

Lecture slides to accompany

Engineering Economy

8th edition

Leland Blank

Anthony Tarquin

F : Future Value
P : Present Value
A : Annual Value

LEARNING OUTCOMES

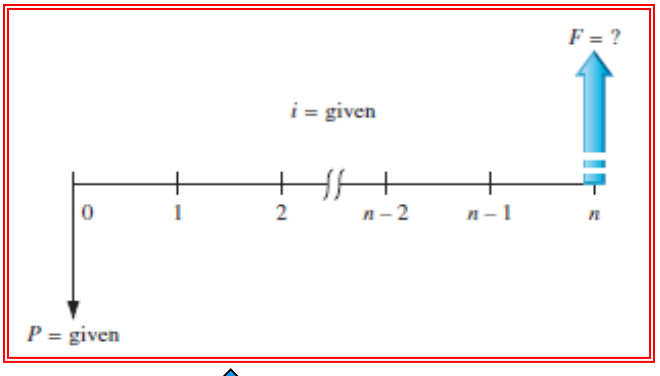
1. **F/P and P/F Factors**
2. **P/A and A/P Factors**
3. **F/A and A/F Factors**
4. **Factor Values**
5. **Arithmetic Gradient** 산수 변화도, 증감률
6. **Geometric Gradient** 기하
7. **Find i or n**

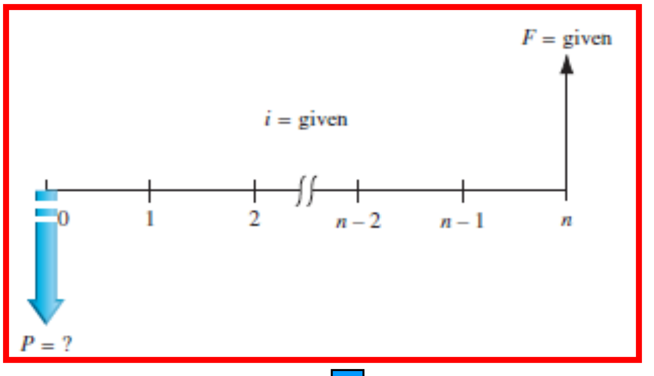
cf) PE, the steel Plant Case
- build ⇒ 200m.
- Annual profit ⇒ 30m.
- plan for 5 years

Single Payment Factors (F/P and P/F)

Single payment factors involve only P and F.

Cash flow diagrams are as follows:





Formulas are as follows:

$F = P(1+i)^{n}$

$P = F[(1+i)^{-n}]$

Terms in parentheses or brackets are called *factors.* Values are in tables for i and n values

Factors are represented in standard factor notation *such as (F/P,i,n)*,

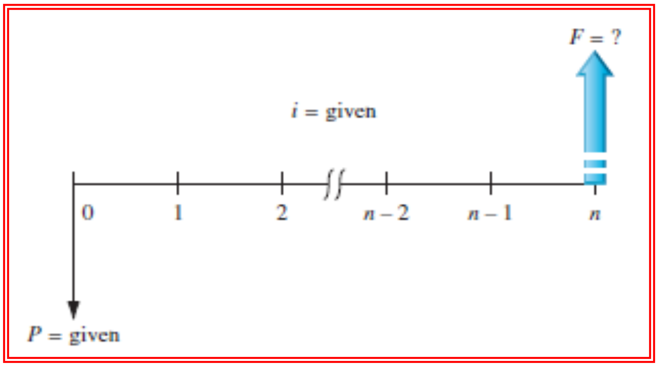
where letter to left of slash is what is sought; letter to right represents what is given

want to know

(F/P, i, n)

given

Single Payment Factors (F/P and P/F)



$$F_1 = P + Pi = P(1+i)^1$$

$$F_2 = F_1 + F_1 i = F_1(1+i)^1 = P(1+i)^2$$

.
.
.

$$F_n = F_{n-1} + F_{n-1} i = F_{n-1}(1+i)^1 = P(1+i)^n$$

Equation

$$F = P(1+i)^n$$

$$= P(F/P, i\%, n)$$

$$P = F\left[\frac{1}{(1+i)^n}\right]$$

$$= F(P/F, i\%, n)$$

Standard Factor Notation

- P = F(P/F,i,n) : Given F, i, and n, Find P
- F = P(F/P,i,n) : Given P, i, and n, Find F
- A = F(A/F,i,n) : Given F, i, and n, Find A
- A = P(A/P,i,n) : Given P, i, and n, Find A

분자 분모

As if they are numerator and denominator

- A = F(A/F,i,n) = P(F/P,i,n) x (A/F,i,n) = P(A/P,i,n)

$$= P\left(\frac{F}{P}\right) \times \left(\frac{A}{F}\right) = P(A/P, i, n)$$

Compound Interest Factor Tables

Please consult the tables in p. 595 ~ p. 623

F/P and P/F for Spreadsheets

Future value F is calculated using FV function:

= FV(i%, n,, P **)**

Present value P is calculated using PV function:

= PV(i%, n,, F **)**

Note the use of double commas in each function

Introduction to Spreadsheet Functions

Excel financial functions

수정 (5개만!)

Present Value, P: = PV (i%,n,A,F)

Future Value, F: = FV (i%,n,A,P)

주기적인

Equal, periodic value, A: annual value = PMT (i%,n,P,F) periodic amount

Number of periods, n: = NPER (i%,A,P,F)

Compound interest rate, i: = RATE (n,A,P,F)

Compound interest rate, i: = IRR (first_cell:last_cell)

Present value, any series, P: = NPV (i%,second_cell:last_cell) + first cell

handle any kind of cash flow → powerful

net present value

Assume Bank Transactions

⇒ ⊕, ⊖ signs

Example: Estimates are P = $5000 n = 5 years i = 5% per year

Find A in $ per year

Function and display: = PMT (5%, 5, -5000) displays A = $1154.87

you will deposit

(예금한 상황)

you can withdraw this amount of money each year. (5 years total)

Example: Finding Future Value

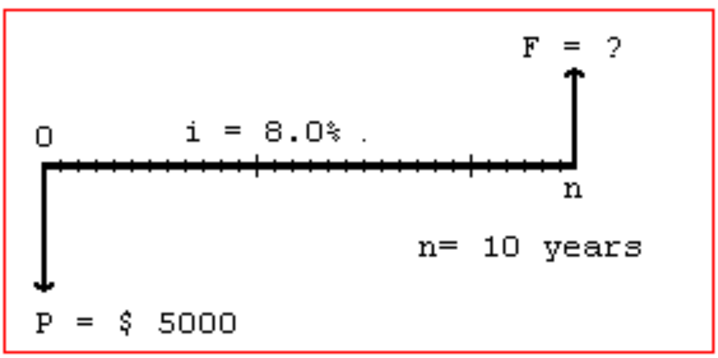
A person deposits \$5000 into an account which pays interest at a rate of 8% per year. The amount in the account after 10 years is closest to:

P=5000
i= 8
n=10

(A) \$2,792 (B) \$9,000 (C) \$10,795 (D) \$12,165

FV(8,10,,-5000)

The cash flow diagram is:



Solution:

F = P(F/P,i,n)

= 5000(F/P, 8%, 10)

= 5000(2.1589)

= \$10,794.50

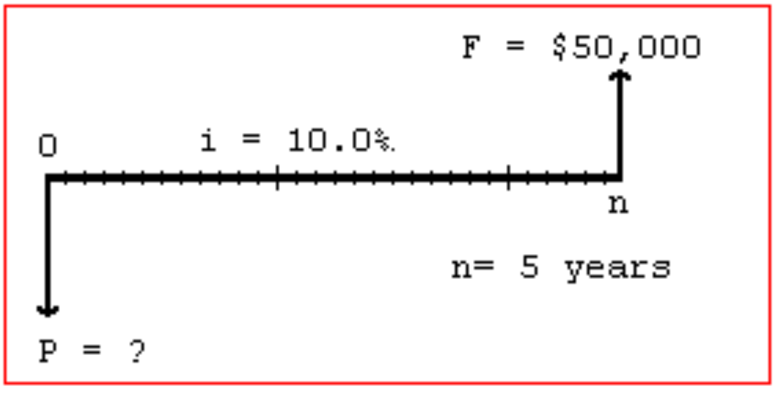
Answer is (C)

Example: Finding Present Value

A small company wants to make a single deposit now so it will have enough money to purchase a backhoe costing $50,000 five years from now. If the account will earn interest of 10% per year, the amount that must be deposited now is nearest to:

(A) $10,000 (B) $ 31,050 (C) $ 33,250 (D) $319,160

The cash flow diagram is:



Solution:

P = F(P/F,i,n)

= 50,000(P/F,10%,5)

= 50,000(0.6209)

= $31,045

Answer is (B)

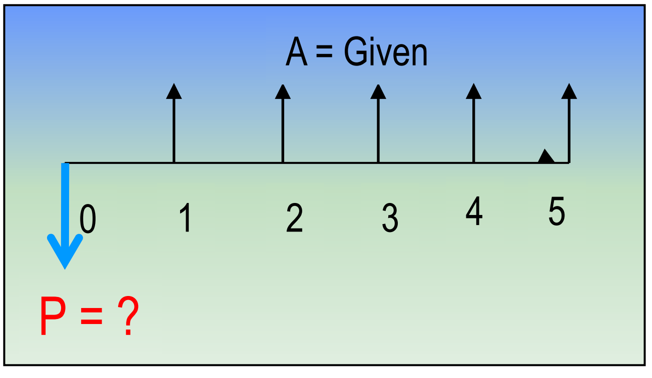
Uniform Series Involving P/A and A/P

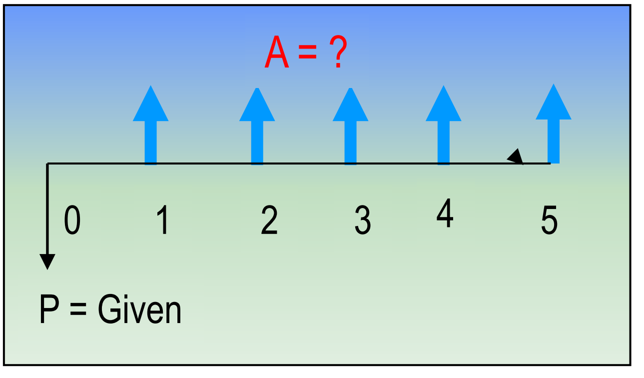
The uniform series factors that involve **P and A** are derived as follows:

(1) Cash flow occurs in consecutive interest periods

(2) Cash flow amount is same in each interest period

The cash flow diagrams are:





P = A(P/A,i,n) ⟵ Standard Factor Notation ⟶ A = P(A/P,i,n)

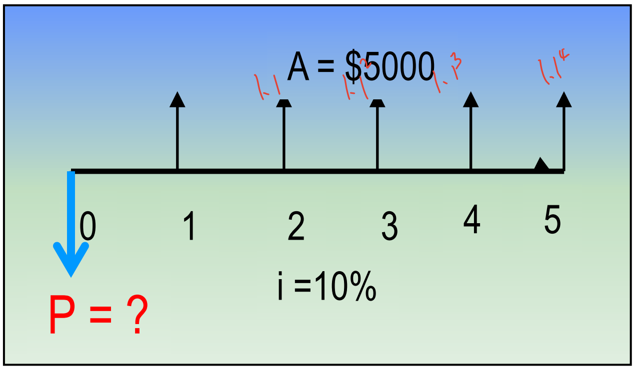
Note: P is one period ahead of first A value

Example: Uniform Series Involving P/A

A chemical engineer believes that by modifying the structure of a certain water treatment polymer, his company would earn an extra $5000 per year. At an interest rate of 10% per year, how much could the company afford to spend now to just break even over a 5 year project period?

(A) $11,170 **(B) 13,640** **(C) $15,300** **(D) $18,950**

The cash flow diagram is as follows:



Solution:

P = 5000(P/A, 10%, 5)
= 5000(3.7908)
= $18,954

Answer is (D)

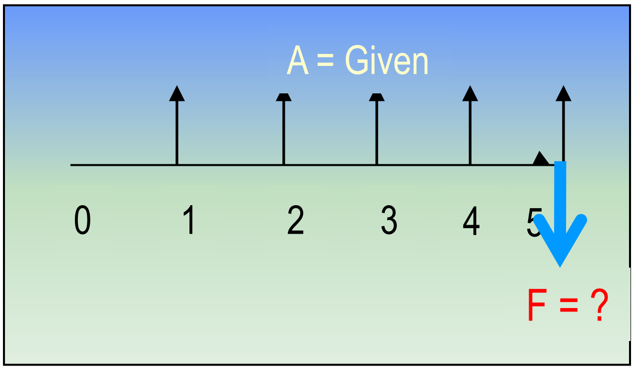
Uniform Series Involving F/A and A/F

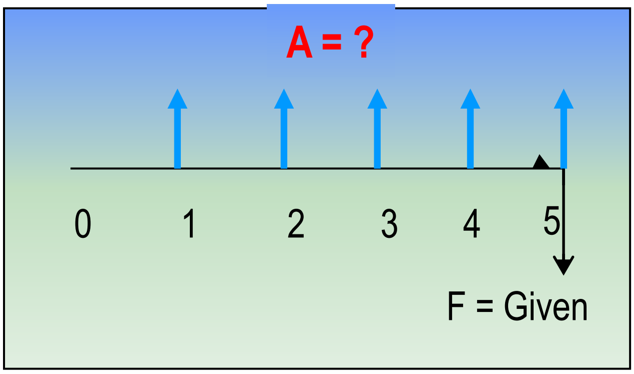
The uniform series factors that involve **F and A** are derived as follows:

(1) Cash flow occurs in ***consecutive*** interest periods

(2) Last cash flow occurs in ***same*** period as F

Cash flow diagrams are:





F = A(F/A,i,n) ⟸ Standard Factor Notation ⟹ A = F(A/F,i,n)

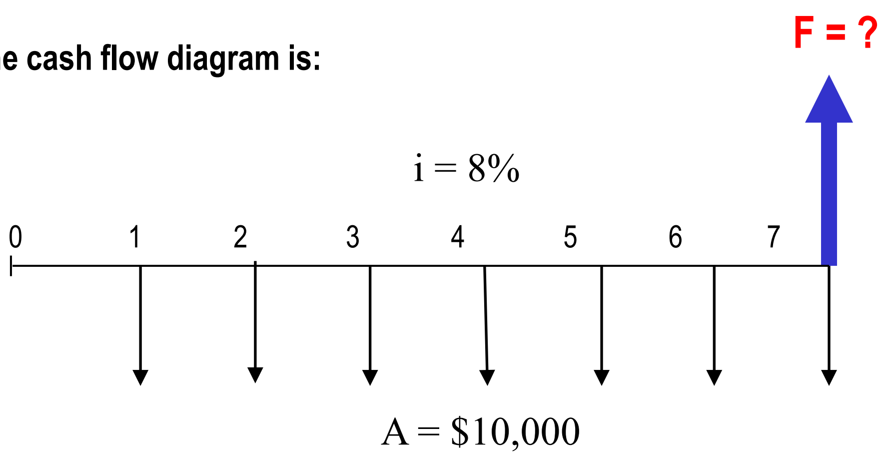
Note: F takes place in the _same_ period as last A

Example: Uniform Series Involving F/A

An industrial engineer made a modification to a chip manufacturing process that will save her company $10,000 per year. At an interest rate of 8% per year, how much will the savings amount to in 7 years?

(A) $45,300 **(B) $68,500** **(C) $89,228** **(D) $151,500**

The cash flow diagram is:



Solution:

$$F = 10{,}000(F/A, 8\%, 7)$$
$$= 10{,}000(8.9228)$$
$$= \$89{,}228$$

Answer is (C)

Factor Values for Untabulated i or n

3 ways to find factor values for untabulated i or n values

- **Use formula**
- **Use spreadsheet function with corresponding P, F, or A value set to (1)**
- **Linearly interpolate in interest tables (See FIG 2-10)**

Formula or spreadsheet function is fast and accurate
Interpolation is only approximate

표에 없는

Example: Untabulated i

Determine the value for (F/P, 8.3%,10)

Formula: $F = (1 + 0.083)^{10} = 2.2197$ ⟸ **OK**

Spreadsheet: = FV(8.3%,10,,1) = 2.2197 ⟸ **OK**

보간법

Interpolation: 8% ------ 2.1589
8.3% ------ x
9% ------ 2.3674

x = 2.1589 + [(8.3 - 8.0)/(9.0 - 8.0)][2.3674 – 2.1589]
= 2.2215 ⟸ **(Too high)**

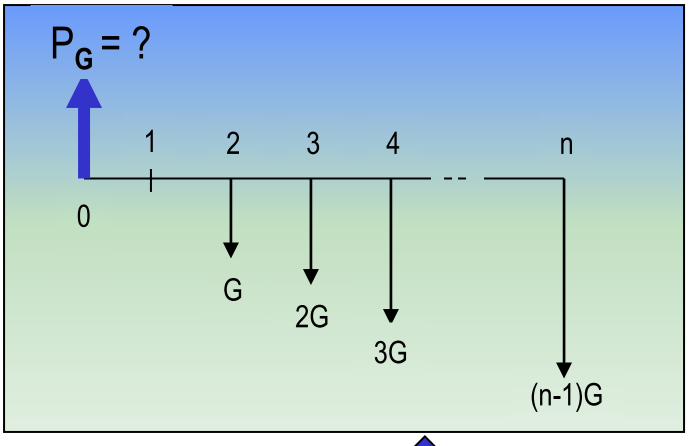
Absolute Error = 2.2215 – 2.2197 = 0.0018

Ex 2.2)
Ex 2.4)
Ex 2.6)

Arithmetic Gradients 등차급수

Arithmetic gradients ***change*** by the <u>Same</u> ***amount*** each period

The cash flow diagram for the P_G of an arithmetic gradient is:



Standard factor notation is

$P_G = G(P/G,i,n)$

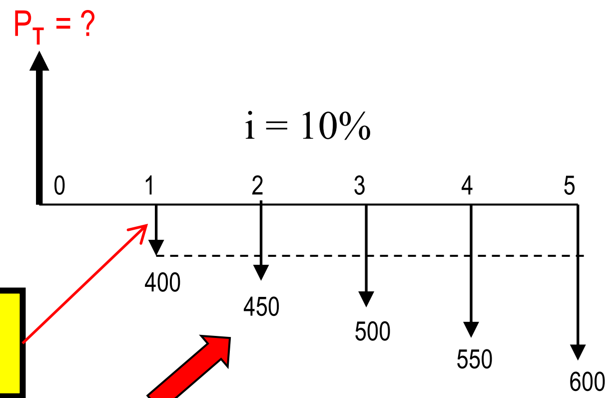
G starts between periods 1 and 2

(not between 0 and 1)

This is because cash flow in year 1 is usually not equal to G and is handled separately as a *base amount* (shown on next slide)

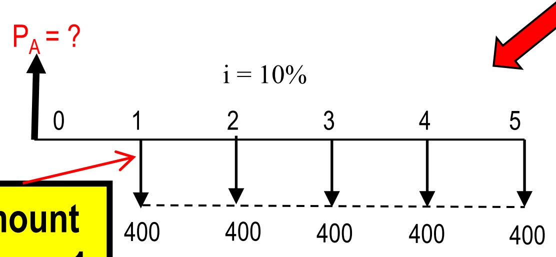
Note that P_G is located <u>Two Periods</u> **Ahead of the first change that is equal to G**

Typical Arithmetic Gradient Cash Flow



Amount in year 1 is base amount

This diagram = this *base amount* plus this gradient



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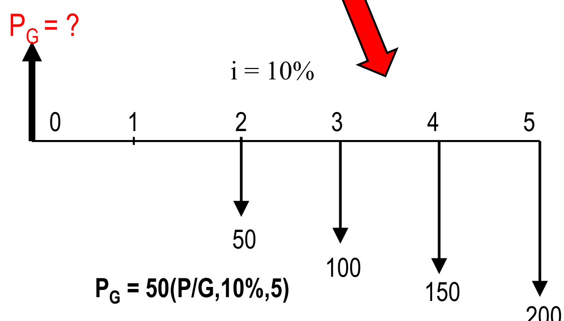


Amount in year 1 is base amount

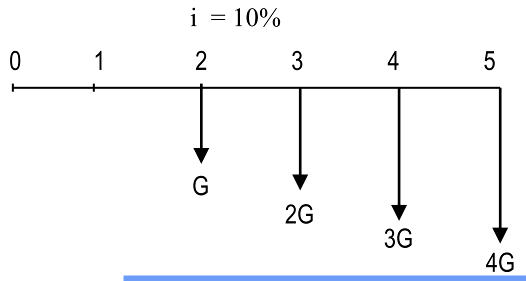
$P_A = 400(P/A,10\%,5)$

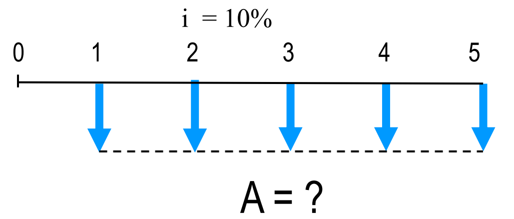
$P_G = 50(P/G,10\%,5)$

$P_T = P_A + P_G = 400(\underline{P/A}\ ,10\%,5) + 50(\underline{P/G}\ ,10\%,5)$

Converting Arithmetic Gradient to A

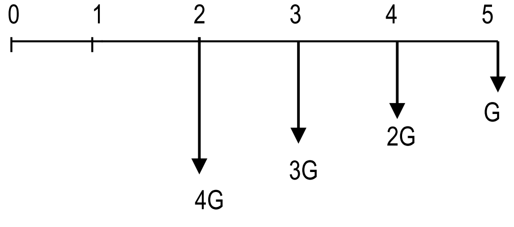
Arithmetic gradient can be converted into equivalent A value using G(A/G,i,n)





General equation when base amount is involved is

A = base amount + G(A/G,i,n)



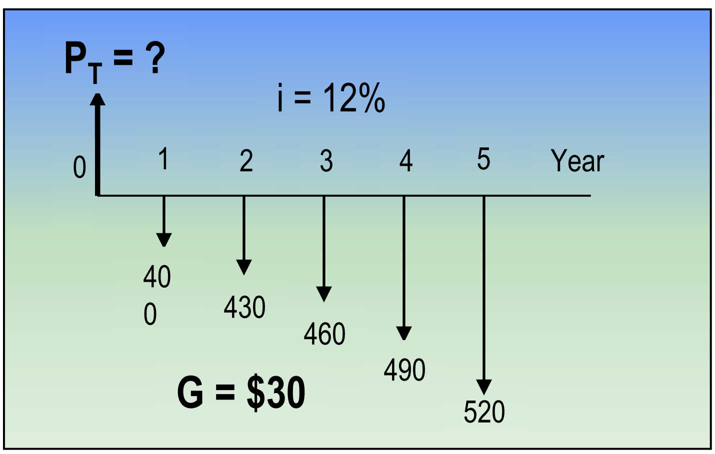
For decreasing gradients, change plus sign to minus

A = base amount - G(A/G,i,n)

Example: Arithmetic Gradient

The present worth of \$400 in year 1 and amounts increasing by \$30 per year through year 5 at an interest rate of 12% per year is closest to:

(A) \$1532 (B) \$1,634 (C) \$1,744 (D) \$1,829



Solution:

P_T = 400(P/A,12%,5) + 30(P/G,12%,5)
= 400(3.6048) + 30(6.3970)
= \$1,633.83

Answer is (B)

The cash flow could also be converted into an **A** value as follows:

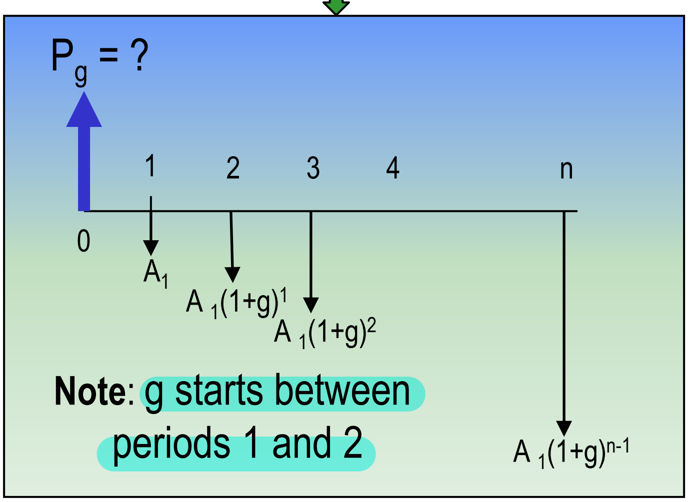
A = 400 + 30(A/G,12%,5)
= 400 + 30(1.7746)
= \$453.24

year 1에서 생각해야 함

Geometric Gradients 등비급수

Geometric gradients **change by the** ~~same percentage~~ **each period**

Cash flow diagram for present worth of geometric gradient



There are ***no tables*** for geometric factors

Use following equation for $g \neq i$:

$$P_g = A_1\{1- [(1+g)/(1+i)]^n\}/(i-g)$$

where: A_1 *= cash flow in period 1*

g *= rate of increase*

If $g = i$, $P_g = A_1n/(1+i)$

Note: If g is negative, change signs in front of both g values

$$P_g = A_1\left\{1-\left(\frac{1+g}{1+i}\right)^n\right\} \times \frac{1}{(i-g)} \quad (g>0)$$

$$= A_1\left\{1-\left(\frac{1-g}{1+i}\right)^n\right\} \times \frac{1}{(i+g)} \quad (g<0)$$

$$A_1\left\{1-\left(\frac{1+|g|}{i+|g|}\right)^n\right\} \times \frac{1}{i-|g|}$$

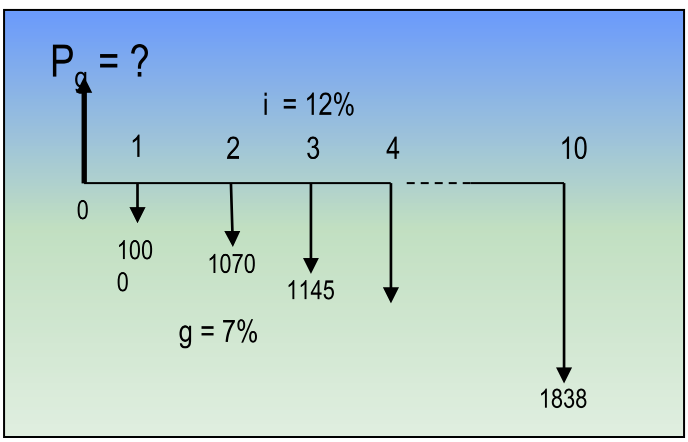
A=1000

G= 0.07 i = 0.12 n=10

Example: Geometric Gradient

Find the present worth of $1,000 in year 1 and amounts increasing by 7% per year through year 10. Use an interest rate of 12% per year.

(a) $5,670 (b) $7,333 (c) $12,670 (d) $13,550



Solution:

$P_g = 1000[1-(1+0.07/1+0.12)^{10}]/(0.12-0.07)$

$= \$7,333$

Answer is (B)

To find A, multiply P_g by (A/P,12%,10)

Unknown Interest Rate i

Unknown interest rate problems involve solving for i, given n and 2 other values (P, F, or A)

(Usually requires a trial and error solution or interpolation in interest tables)

Procedure: Set up equation with all symbols involved and solve for i

A contractor purchased equipment for $60,000 which provided income of $16,000 per year for 10 years. The annual rate of return of the investment was closest to:

(a) 15% (b) 18% (c) 20% (d) 23%

Solution: Can use either the P/A or A/P factor. Using A/P:

$$60{,}000(A/P,i\%,10) = 16{,}000$$
$$(A/P,i\%,10) = 0.26667$$

From A/P column at n = 10 in the interest tables, i is between 22 % and 24 % Answer is (d)

Unknown Recovery Period n

Unknown recovery period problems involve solving for n, given i and 2 other values (P, F, or A)

(Like interest rate problems, they usually require a trial & error solution or interpolation in interest tables)

Procedure: Set up equation with all symbols involved and solve for n

A contractor purchased equipment for \$60,000 that provided income of \$8,000 per year. At an interest rate of 10% per year, the length of time required to recover the investment was closest to:

(a) 10 years **(b) 12 years** **(c) 15 years** **(d) 18 years**

Solution: Can use either the P/A or A/P factor. Using A/P:

$$60{,}000(A/P,10\%,n) = 8{,}000$$

$$(A/P,10\%,n) = 0.13333$$

From A/P column in i = 10% interest tables, n is between 14 and 15 years **Answer is (c)**

Summary of Important Points

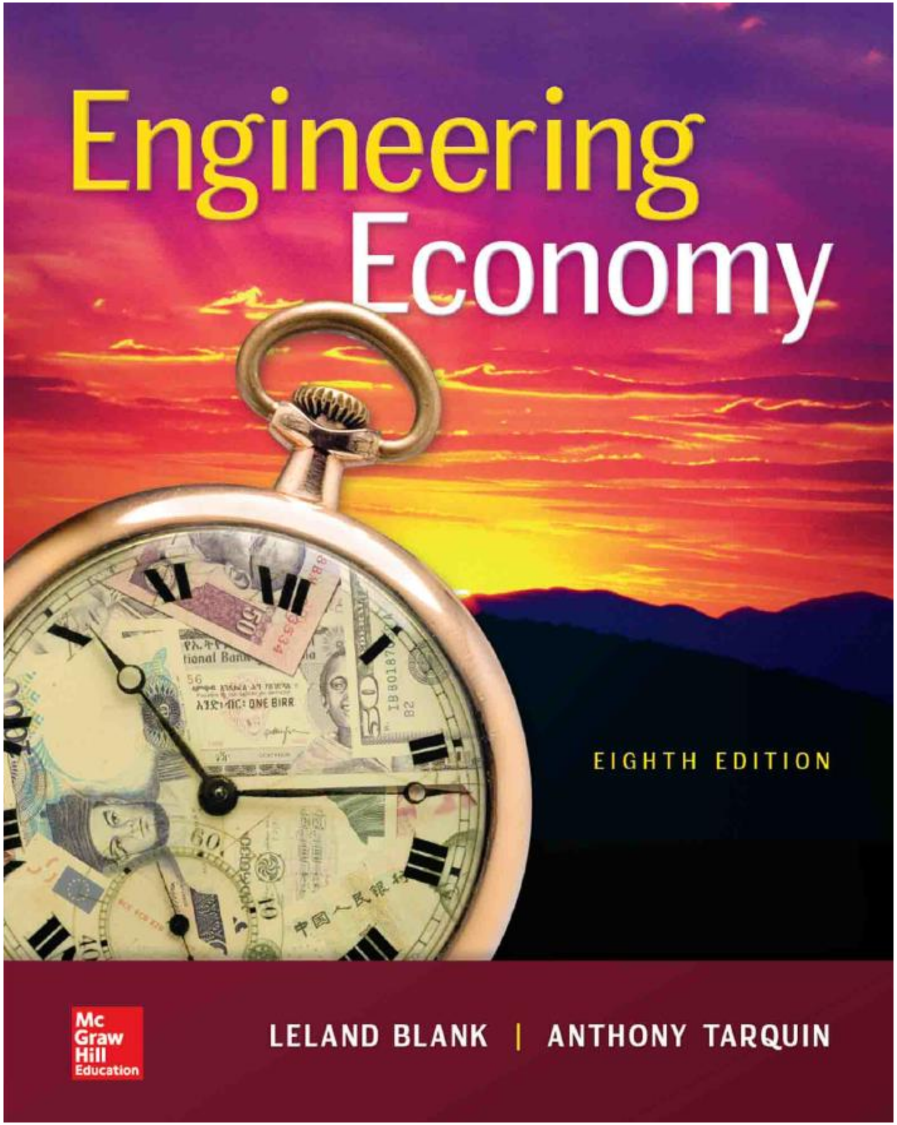
- In P/A and A/P factors, P is one period *ahead* of first A
- In F/A and A/F factors, F is in same period *as last* A
- To find untabulated factor values, best way is to use formula or spreadsheet
- For arithmetic gradients, gradient G starts between *periods* 1 & 2
- Arithmetic gradients have 2 parts, base *amount* (year 1) and gradient *amount*
- For geometric gradients, gradient g starts been *periods 1 and 2*
- In geometric gradient formula, A_1 is amount in *period* 1
- To find unknown i or n, *set up equation involving all terms* and solve for i or n

HOMEWORK

1. Please solve every Examples in your textbook. You do not have to submit your works.

2. Please upload following “PROBLEMS” solution file on “Assignment” menu in e-Class.

   ① 2.18
   ② 2.22
   ③ 2.36
   ④ 2.45
   ⑤ 2.53
   ⑥ 2.60
   ⑦ 2.65
   ⑧ 2.79



Chapter 3

Combining Factors and Spreadsheet Functions

Lecture slides to accompany

Engineering Economy

8th edition

Leland Blank

Anthony Tarquin

McGraw Hill Higher Education

LEARNING OUTCOMES

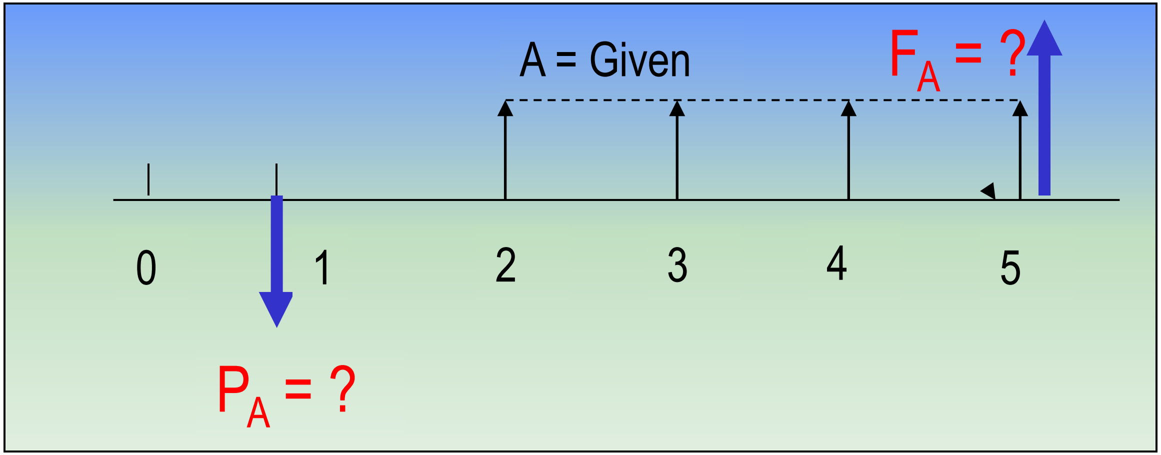
1. **Shifted uniform series** = A
2. **Shifted series and single cash flows**
3. **Shifted gradients**

Shifted Uniform Series

A shifted uniform series starts at a time *other than period 1*

The cash flow diagram below is an example of a shifted series

Series starts in period 2, not period 1



Shifted series usually require the use of *multiple factors*

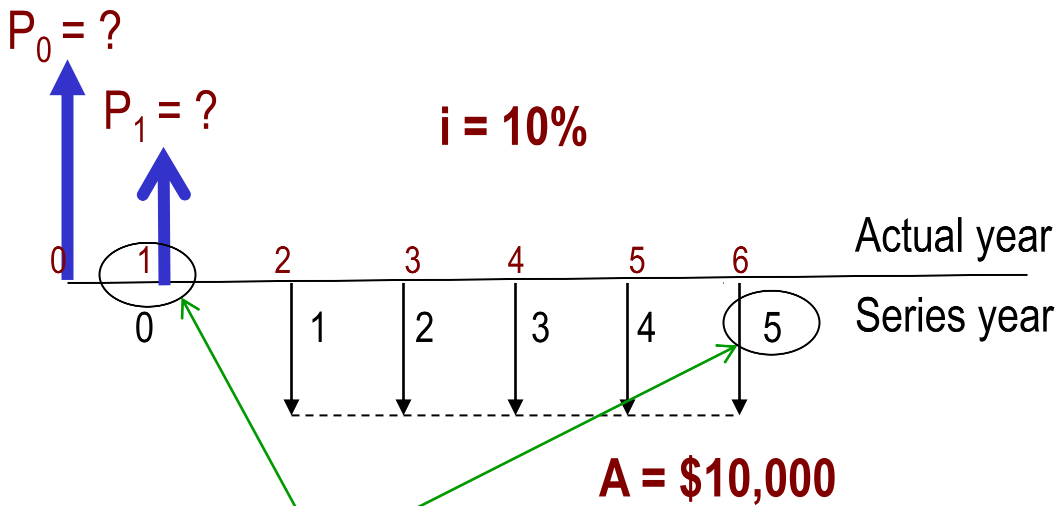
Remember: **When using P/A or A/P factor, P_A is always** one year ahead **of first A**

When using F/A or A/F factor, F_A is in same year ***as last A***

Example Using P/A Factor: Shifted Uniform Series

The present worth of the cash flow shown below at i = 10% is:

(a) \$25,304 (b) \$29,562 (c) \$34,462 (d) \$37,908



Solution: (1) Use P/A factor with n = 5 (for 5 arrows) to get P_1 **in year 1**

(2) Use P/F factor with n = 1 to move P_1 back for P_0 **in year 0**

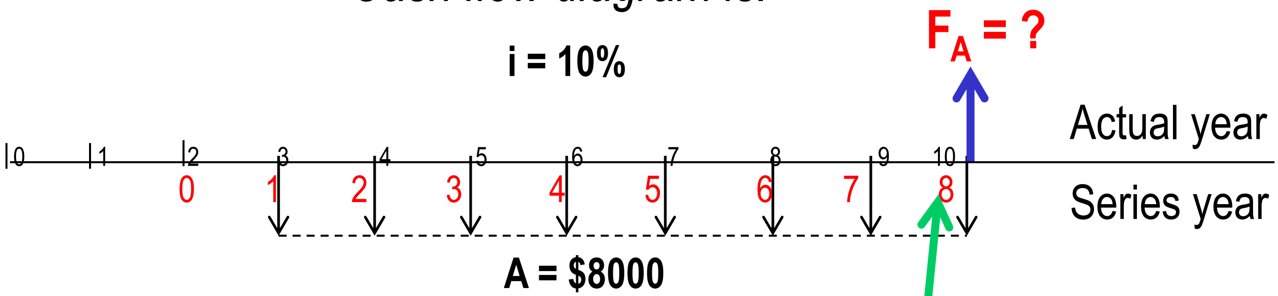
$$P_0 = P_1(P/F,10\%,1) = A(P/A,10\%,5)(P/F,10\%,1) = 10{,}000(3.7908)(0.9091) = \$34{,}462$$

Answer is C

Example Using F/A Factor: Shifted Uniform Series

How much money would be available in year 10 if $8000 is deposited each year in years 3 through 10 at an interest rate of 10% per year?

Cash flow diagram is:



Solution: Re-number diagram to determine n = 8 (number of arrows)

$$F_A = 8000(F/A,10\%,8)$$
$$= 8000(11.4359)$$
$$= 91,487$$

Shifted Series and Random Single Amounts

For cash flows that include *uniform series* **and** *randomly placed single amounts*:

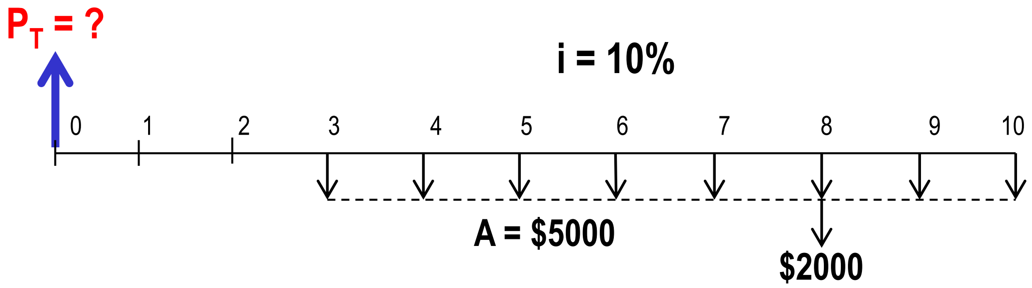
- ***Uniform series procedures*** **are applied to the** ***series amounts***
- ***Single amount formulas*** **are applied to the** ***one-time cash flows***

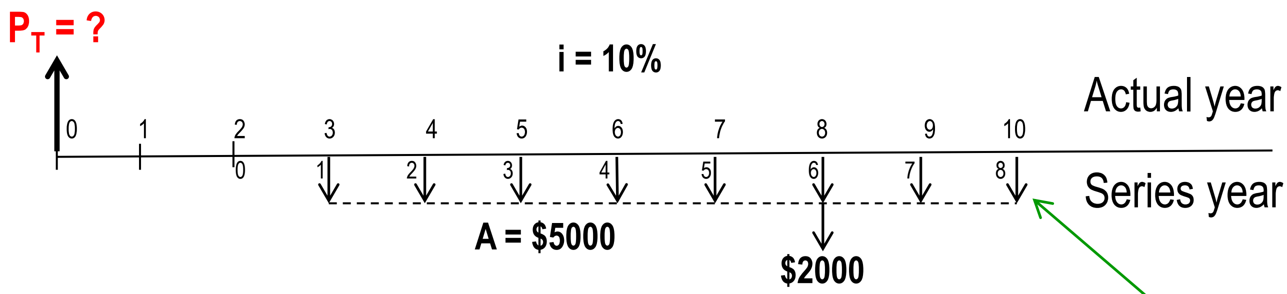
The resulting values are then combined **per the problem statement**

The following slides illustrate the procedure

Example: Series and Random Single Amounts

Find the present worth in year 0 for the cash flows shown using an interest rate of 10% per year.

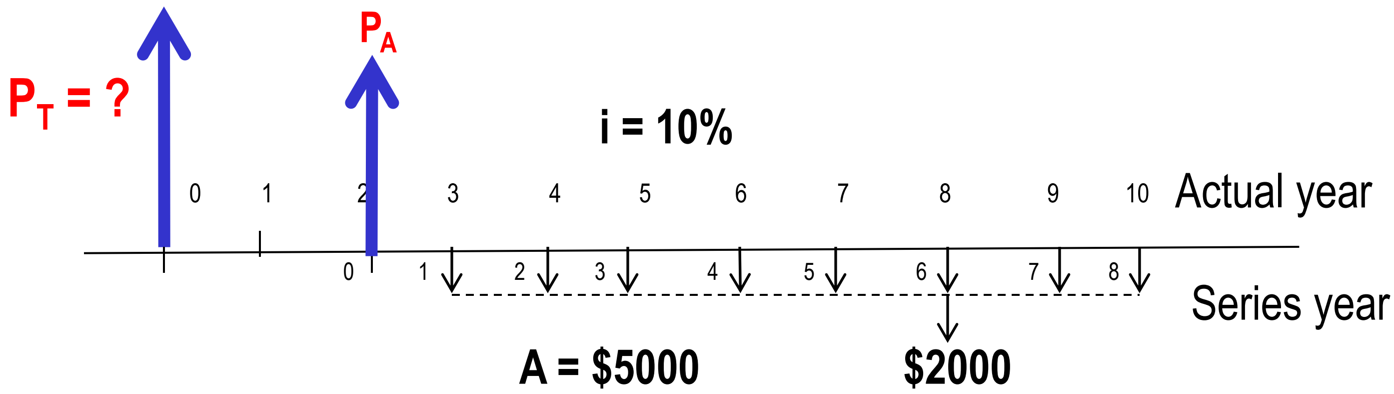




Solution:

First, re-number cash flow diagram to get n for uniform series: n = 8

Example: Series and Random Single Amounts



Use P/A to get P_A in year 2: $P_A = 5000(P/A,10\%,8) = 5000(5.3349) = \$26,675$

Move P_A back to year 0 using P/F: $P_0 = 26,675(P/F,10\%,2) = 26,675(0.8264) = \$22,044$

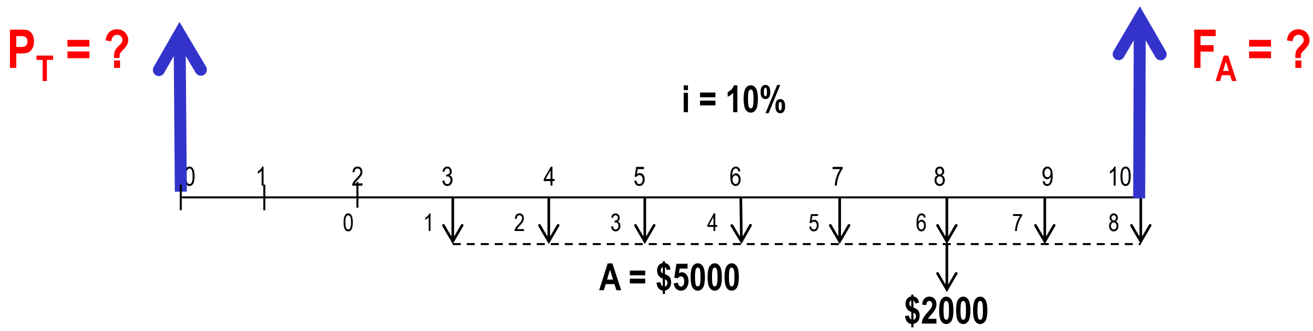
Move $2000 single amount back to year 0: $P_{2000} = 2000(P/F,10\%,8) = 2000(0.4665) = \$933$

Now, add P_0 and P_{2000} to get P_T: P_T = 22,044 + 933 = 22,977

Example Worked a Different Way

(Using F/A instead of P/A for uniform series)

The same re-numbered diagram from the previous slide is used



Solution: Use F/A to get F_A in actual year 10: $F_A = 5000(F/A,10\%,8) = 5000(11.4359) = \$57,180$

Move F_A back to year 0 using P/F: $P_0 = 57,180(P/F,10\%,10) = 57,180(0.3855) = \$22,043$

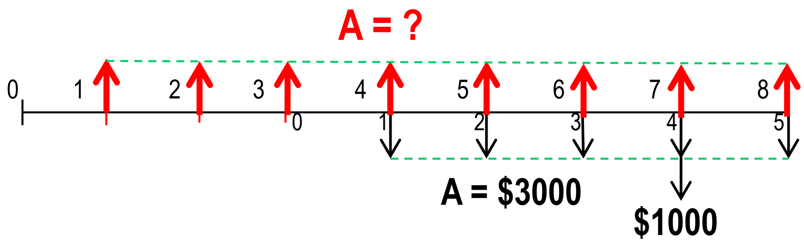
Move $2000 single amount back to year 0: $P_{2000} = 2000(P/F,10\%,8) = 2000(0.4665) = \$933$

Now, add two P values to get P_T: $P_T = 22,043 + 933 = \$22,976$ **Same as before**

As shown, there are usually multiple way to work equivalency problems

Example: Series and Random Amounts

Convert the cash flows shown below (black arrows) into an equivalent annual worth A in years 1 through 8 (red arrows) at i = 10% per year.



i = 10%

Approaches:

1. **Convert all cash flows into P in year 0** and use A/P with n = 8
2. **Find F in year 8** and use A/F with n = 8

Solution:

Solve for F: **F = 3000(F/A,10%,5) + 1000(F/P,10%,1)**
= 3000(6.1051) + 1000(1.1000)
= 19,415

Find A: **A = 19,415(A/F,10%,8)**
= 19,415(0.08744)
= 1,698

Shifted Arithmetic Gradients

Shifted gradient begins at a time other than between periods 1 and 2

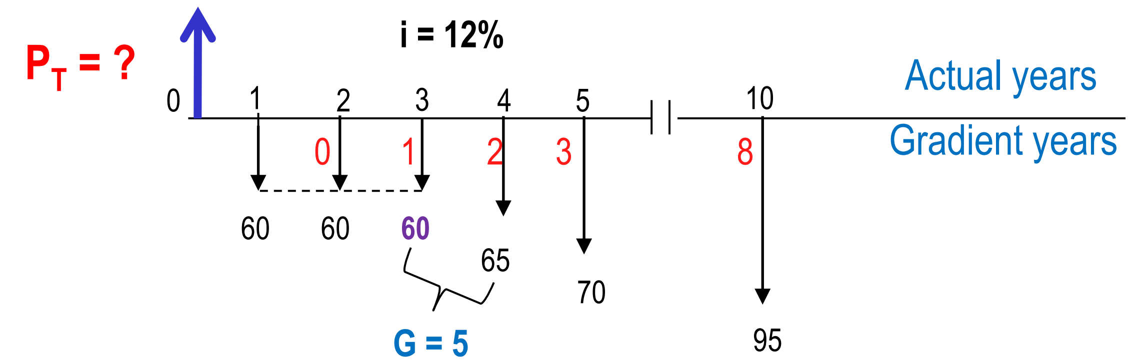
Present worth P_G is located 2 periods before gradient starts

Must use multiple factors to find P_T in actual year 0

To find equivalent A series, find P_T at actual time 0 and apply (A/P, i, n)

Example: Shifted Arithmetic Gradient

John Deere expects the cost of a tractor part to increase by $5 per year beginning 4 years from now. If the cost in years 1-3 is $60, determine the *present worth in year 0* of the cost through year 10 at an interest rate of 12% per year.



Solution: **First find P_2 for G = $5 *and* base amount ($60) in actual year 2**

$$P_2 = 60(P/A,12\%,8) + 5(P/G,12\%,8) = \$370.41$$

Next, move P_2 back to year 0

$$P_0 = P_2(P/F,12\%,2) = \$295.29$$

Next, find P_A for the $60 amounts of years 1 and 2

$$P_A = 60(P/A,12\%,2) = \$101.41$$

Finally, add P_0 and P_A to get P_T in year 0

$$P_T = P_0 + P_A = \underline{396.7}$$

Shifted Geometric Gradients

Shifted gradient begins at a time other than between periods 1 and 2

Equation yields P_g for *all* cash flows (base amount A_1 is included)

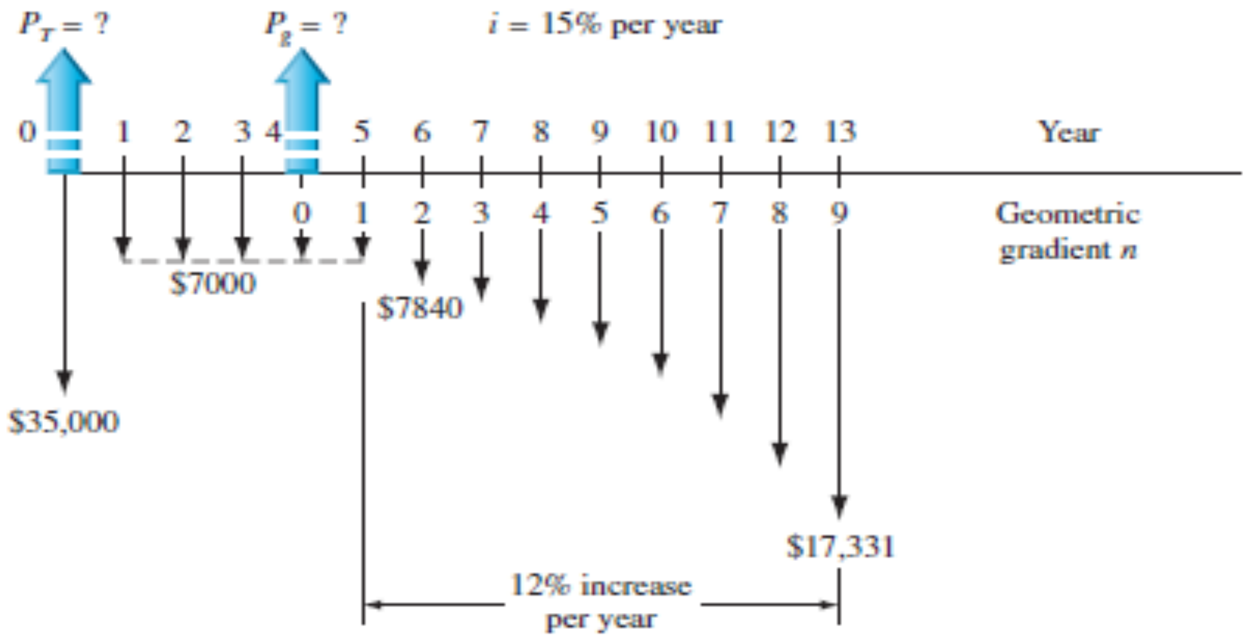
Equation ($i \neq g$): $P_g = A_1\{1 - [(1+g)/(1+i)]^n/(i-g)\}$ ($g > 0$)

For negative gradient, change signs on both g values

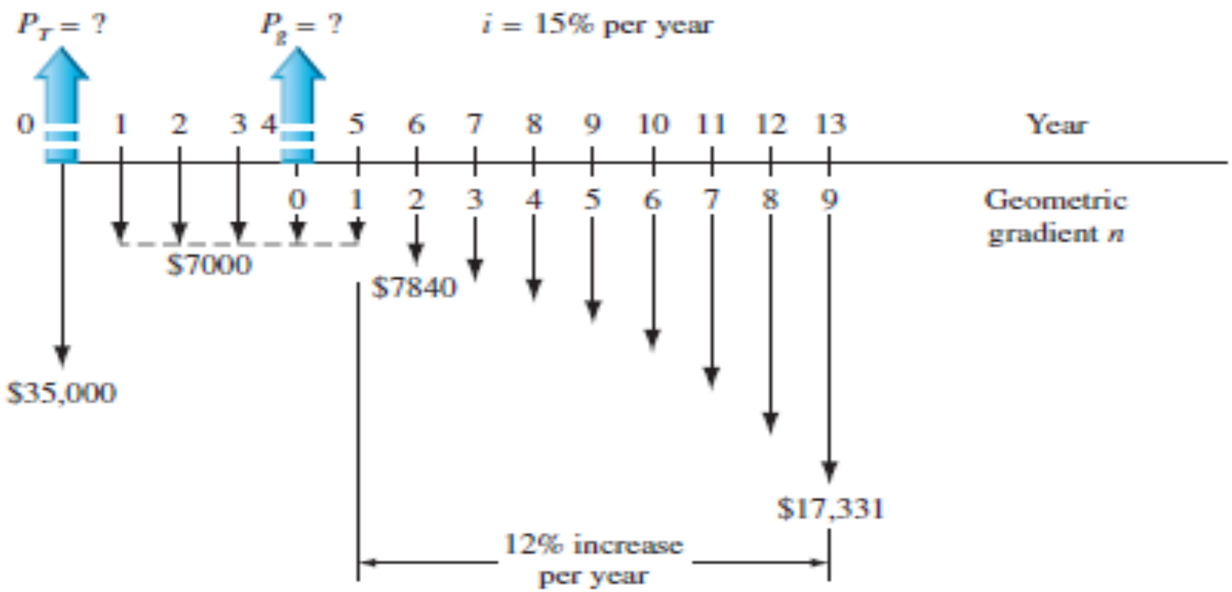
There are no tables for geometric gradient factors

Example: Shifted Geometric Gradient

Weirton Steel signed a 5-year contract to purchase water treatment chemicals from a local distributor for \$7000 per year. When the contract ends, the cost of the chemicals is expected to increase by 12% per year for the next 8 years. If an initial investment in storage tanks is \$35,000, determine the equivalent present worth in year 0 of all of the cash flows at i = 15% per year.



Example: Shifted Geometric Gradient



Gradient starts between actual years 5 and 6; these are gradient years 1 and 2. P_g is located in gradient year 0, which is actual year 4

$P_g = 7000\{1-[(1+0.12)/(1+0.15)]^9/(0.15-0.12)\} = \$49{,}401$

Move P_g and other cash flows to year 0 to calculate P_T

P_T = **35,000 + 7000(P/A,15%,4) + 49,401(P/F,15%,4) =** $62,230

Negative Shifted Gradients

For negative arithmetic gradients, change sign on G term from + to -

General equation for determining P: **P = present worth of base amount - P_G**

Changed from + to -

For negative geometric gradients, change signs on both g values

Changed from + to -

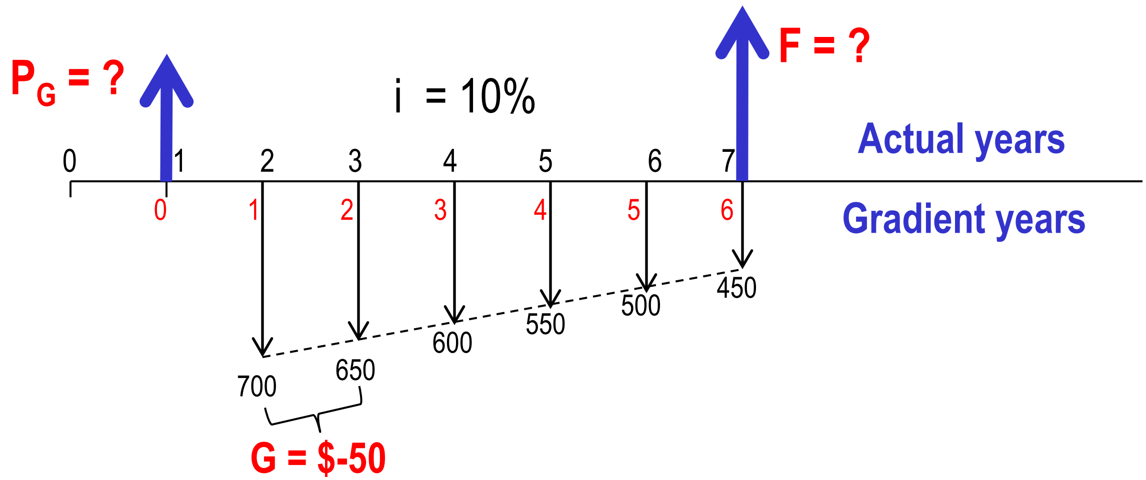
$$P_g = A_1\{1-[(1-g)/(1+i)]^n/(i+g)\}$$

Changed from - to +

All other procedures are the same as for positive gradients

Example: Negative Shifted Arithmetic Gradient

For the cash flows shown, find the future worth in year 7 at i = 10% per year



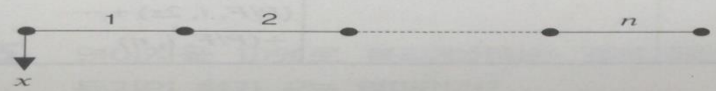
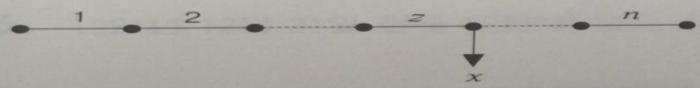
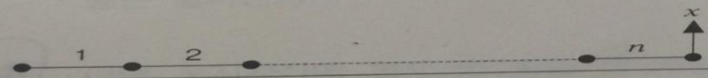
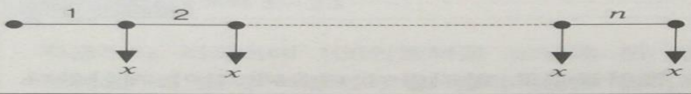
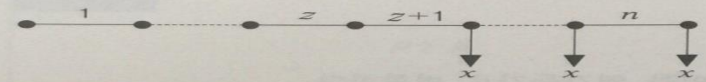
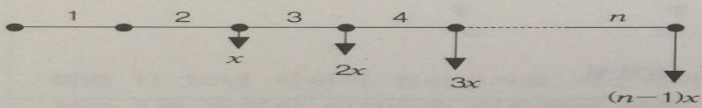
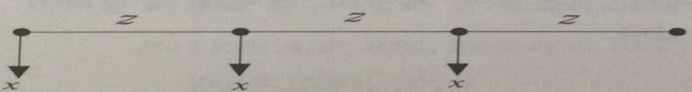
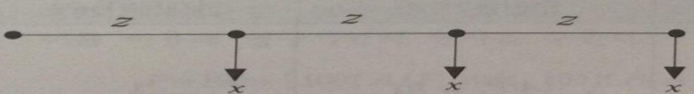
Solution: Gradient G first occurs between actual years 2 and 3; these are gradient years 1 and 2

P_G is located in gradient year 0 (actual year 1); base amount of $700 is in gradient years 1-6

$$P_G = 700(P/A,10\%,6) – 50(P/G,10\%,6) = 700(4.3553) – 50(9.6842) = \underline{\quad 2565 \quad}$$

$$F = P_G(F/P,10\%,6) = 2565(1.7716) = \underline{\quad 4544 \quad}$$

그림 3-7 선택된 현금흐름 X를 연간등가와 현재등가로 변환시 곱하는 인자

현금흐름의 형태	사례	연간등가(A)	현재등가(P)
1. 초기비용 	건물	$(A/P,\ i,\ n)$	1
2. 중간발생비용 	건물의 증축	$(P/F,\ i,\ z)\times(A/P,\ i,\ n)$	$(P/F,\ i,\ z)$
3. 잔존가치 	건물의 재판매	$(A/F,\ i,\ n)$	$(P/F,\ i,\ n)$
4. 연간등가 	재산세, 보험료	1	$(P/A,\ i,\ n)$
5. z+1년에서 시작하는 연간등가 	증가된 노무비	$(F/A, i, n-z)\times(A/F, i, n)$ 또는, $[(P/A, i, n)-(P/A, i, z)]\cdot(A/P, i, n)$	$(P/A,\ i,\ n-z)\times(P/F,\ i,\ z)$ 또는, $(P/A,\ i,\ n)-(P/A,\ i,\ z)$
6. 산술적 증가 	유지비	$(A/G,\ i,\ n)$	$(P/G,\ i,\ n)$
7. 주기적비용(0년부터 각 z년마다 발생) 	대수리비	$(A/P,\ i,\ z)$	$(A/P, i, z)\times(P/A, i, n)$ 또는 $1+(P/F,\ i,\ z)+\cdots+(P/F, i, n-z)$
8. 주기적비용(z년부터 각 z년마다 발생) 	대수리비	$(A/F,\ i,\ z)$	$(A/F, i, z)\times(P/A, i, n)$ 또는 $(P/F, i, z)+(P/F, i, 2z)+\cdots+(P/F, i, n)$

Summary of Important Points

NPV

P for shifted uniform series is one period ahead of first A;
n is equal to number of A values

F for shifted uniform series is in same year as last A;
n is equal to number of A values

For gradients, *first change* equal to G or g occurs between gradient years 1 and 2

For negative arithmetic gradients, change sign on G from + to -

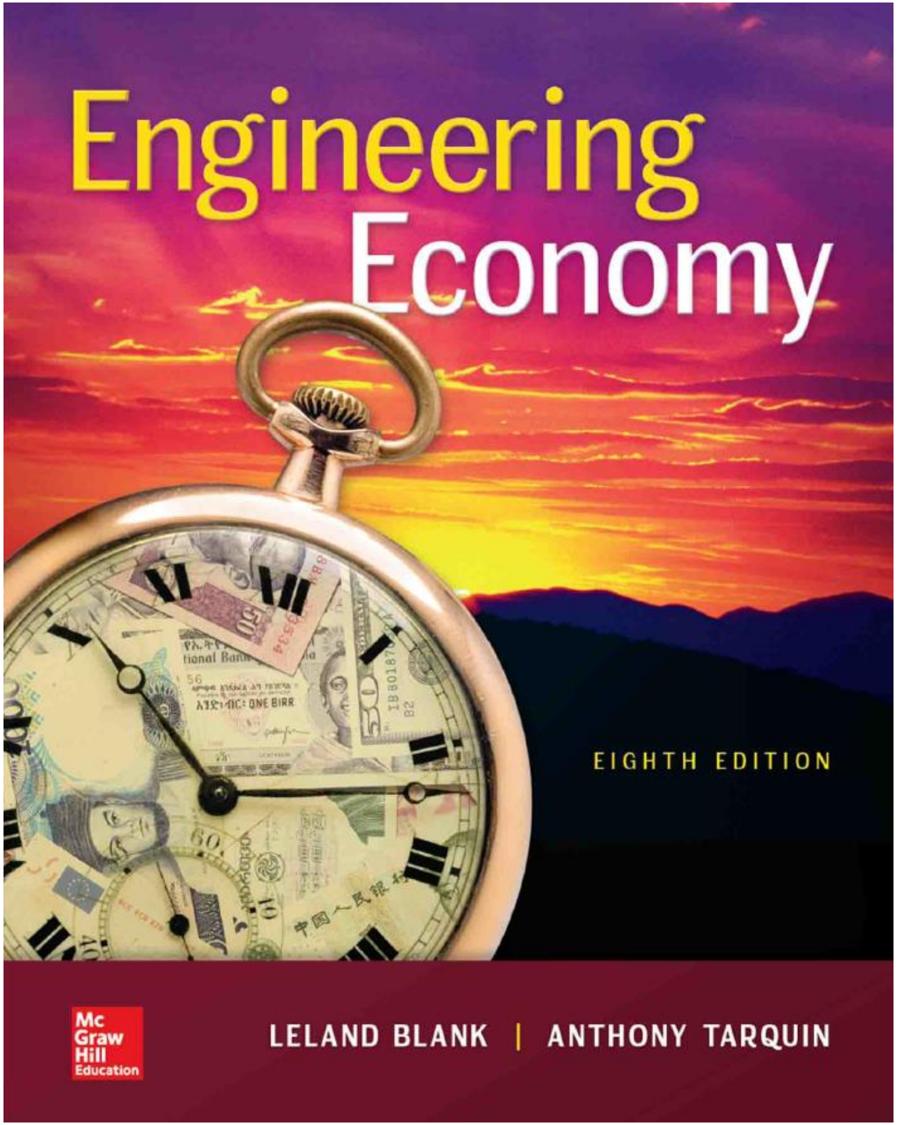
For negative geomatric gradients, change sign on g from + to -

HOMEWORK

1. Please solve every Examples in your textbook. You do not have to submit your works.

2. Please upload following "PROBLEMS" solution file on "Assignment" menu in e-Class. → Do with Excel function instead of equations.

   ① 3.10
   ② 3.12
   ③ 3.28
   ④ 3.33
   ⑤ 3.42
   ⑥ 3.60
   ⑦ 3.75
   ⑧ 3.77



Chapter 4

명목 상의

Nominal and Effective Interest Rates

Lecture slides to accompany

Engineering Economy

8th edition

Leland Blank

Anthony Tarquin

McGraw Hill Higher Education

LEARNING OUTCOMES

1. Understand interest rate statements → Nominal or Effective
2. Use formula for effective interest rates
3. Determine interest rate for any time period
4. Determine payment period (PP) and compounding period (CP) for equivalence calculations
5. Make calculations for single cash flows
6. Make calculations for series and gradient cash flows with PP ≥ CP
7. Perform equivalence calculations when PP < CP
8. Use interest rate formula for continuous compounding
9. Make calculations for varying interest rates

Interest Rate Statements

The terms ‘ nominal ’ and ‘ effective ’ enter into consideration when the interest period is *less than one year.*

New time-based definitions to understand and remember

Interest period (t) – period of time over which interest is expressed. For example, 1% *per month.*

Compounding period (CP) – Shortest time unit over which interest is charged or earned. For example,10% per year compounded monthly.

Compounding frequency (m) – Number of times compounding occurs within the interest period t. For example, at i = 10% per year, compounded monthly, interest would be compounded 12 times during the one year interest period.

Understanding Interest Rate Terminology

A nominal interest rate (r) is obtained by multiplying an interest rate that is expressed over a short time period by the number of compounding periods in a longer time period: That is:

r = interest rate per period x number of compounding periods → simple interest rate

Example: If i = 1% per month, nominal rate per year is
r = (1)(12) = 12% per year)

Effective (actual) interest rates (i) take compounding into account (effective rates can be obtained from nominal rates via a formula to be discussed later).

"APR ≤ APY" → Nominal < Effective

IMPORTANT: Nominal interest rates are essentially simple interest rates. Therefore, they can never be used in interest formulas.

Effective rates must always be used hereafter in all interest formulas.

APR(Annual Percentage Rate) vs. APY(Annual Percentage Yield)

APR → Nominal rate … credit cards, loan, mortgages

APY → effective rate … RoR of investment, CD, saving account

Advantages of nominal rate : represent different effective rate by 1 nominal rate

specify CP + Nominal rate → describe all effective rate simply

TABLE 4–2 Effective Annual Interest Rates Using Equation [4.3] Nominal

$r = \boxed{18\%}$ per year, compounded CP-ly

actual time value of money

CP Compounding Period, CP	m Times Compounded per Year, *m*	i Rate per Compound Period, *i*%		Distribution of *i* over the Year of Compounding Periods	Effective Annual Rate, $i_a = (1 + i)^m - 1$
Year	1 × CP = 1 YEAR	18	18/1	18% (period 1)	$(1.18)^1 - 1 = 18\%$
6 months	2 × CP = 1 YEAR	9	18/2	9%, 9% (periods 1, 2)	$(1.09)^2 - 1 = 18.81\%$
Quarter	4 × CP = 1 YEAR	4.5	18/4	4.5%, 4.5%, 4.5%, 4.5% (periods 1, 2, 3, 4)	$(1.045)^4 - 1 = 19.252\%$
Month	12 × CP = 1 YEAR	1.5	18/12	1.5% in each (periods 1 2 3 4 5 6 7 8 9 10 11 12)	$(1.015)^{12} - 1 = 19.562\%$
Week	52 × CP = 1 YEAR	0.34615	18/52	0.34615% in each (periods 1 2 3 … 24 26 28 … 50 52)	$(1.0034615)^{52} - 1 = 19.684\%$

More About Interest Rate Terminology

There are 3 general ways to express interest rates as shown below

Sample Interest Rate Statements		**Comment**
(1)	i = 2% per month i = 12% per year $(1+(12/2=6)\%)^2-1$	When no compounding period is given, rate is effective
(2)	i = 12% per year, comp'd semiannually i = 12% per year, comp'd monthly $(1+(12/12=1)\%)^{12}-1$	When compounding period is given and it is *not the same* as interest period, it is nominal
(3)	i = effective 9.4%/year, comp'd semiannually i = effective 4% per quarter, comp'd monthly	When compounding period is given and rate is *specified as effective*, rate *is* effective over stated period

EXAMPLE 4.2 The Credit Card Offer Case

As described in the introduction to this case, Dave has been offered what is described as a credit card deal that should not be refused—at least that is what the Chase Bank offer letter implies. The balance transfer APR interest rate of 14.24% is an annual rate, with no compounding period mentioned. Therefore, it follows the format of the third entry in Table 4–1, that is, interest rate stated, no CP stated. Therefore, we should conclude that the CP is 1 year, the same as the annual interest period of the APR. However, as Dave and we all know, credit card payments are required monthly.

(*a*) First, determine the effective interest rates for *compounding periods of 1 year and 1 month* so Dave knows some effective rates he might be paying when he transfers the $1000 balance from his current card.

(*b*) Second, assume that immediately after he accepts the card and completes the $1000 transfer, Dave gets a bill that is due 1 month later. What is the amount of the total balance he owes?

Now, Dave looks a little closer at the fine print of the "pricing information" sheet and discovers a small-print statement that Chase Bank uses the daily balance method (including new transactions) to determine the balance used to calculate the interest due at payment time.

(*c*) We will reserve the implication of this new finding until later, but for now help Dave by determining the *effective daily interest rate* that may be used to calculate interest due at the end of 1 month, provided the CP is 1 day.

Solution

(*a*) The interest period is 1 year. Apply Equation [4.2] for both CP values of 1 year ($m = 1$ compounding period per year) and 1 month ($m = 12$ compounding periods per year).

CP of year: Effective rate per year $= 14.24/1 = 14.24\%$

CP of month: Effective rate per month $= 14.24/12 = 1.187\%$

(*b*) The interest will be at the monthly effective rate, plus the balance transfer fee of 3%.

$$\begin{aligned}\text{Amount owed after 1 month} &= 1000 + 1000(0.01187) + 0.03(1000)\\ &= 1000 + 11.87 + 30\\ &= \$1041.87\end{aligned}$$

Including the $30 fee, this represents an interest rate of $(41.87/1000)(100\%) = 4.187\%$ for only the 1-month period.

(*c*) Again apply Equation [4.2], now with $m = 365$ compounding periods per year.

CP of day: Effective rate per day $= 14.24/365 = 0.039\%$

Effective Annual Interest Rates

Effective rates are converted into effective annual rates via the equation:

$$i_a = \frac{\text{interest}}{\text{principal}} = \frac{P(1+i)^m - P}{P} - 1 = (1+i)^m - 1$$

where i_a = effective annual interest rate

i = effective rate for one compounding period

m = number times interest is compounded per year

Example: For a nominal interest rate of 12% per year, determine the nominal and effective rates per year for (a) quarterly, and (b) monthly compounding

Solution:

(a) Nominal r / year = 12% per year

Nominal r / quarter = 12/4 = 3.0% per quarter = effective i per quarter

Effective i / year = $(1 + 0.03)^4 - 1$ = 12.55% per year

(b) Nominal r / year = 12% per year

Nominal r /month = 12/12 = 1.0% per month

Effective i / year = $(1 + 0.01)^{12} - 1$ = 12.68% per year

Effective Interest Rates

Nominal rates can be converted into effective rates for any time period via the following equation:

$$i = \left(1 + \frac{r}{m}\right)^m - 1$$

where i = effective interest rate for any time period

r = nominal rate for same time period as i

m = no. times interest is comp'd in period specified for i

Spreadsheet function is = EFFECT(r%,m) where r = nominal rate per period specified for i

Example: For an interest rate of 1.2% per month, determine the nominal and effective rates (a) per quarter, and (b) per year

Solution:

1 quater = 3 months

4 quaters = 1 year

(a) Nominal r / quarter = (1.2)(3) = 3.6% per quarter

Effective i / quarter = $(1 + 0.036/3)^3 - 1$ = 3.64 % per quarter

(b) Nominal i /year = (1.2)(12) = 14.4% per year

Effective i / year = $(1 + 0.144 / 12)^{12} - 1$ = 15.39 % per year

EXAMPLE 4.4 The Credit Card Offer Case

In our Progressive Example, Dave is planning to accept the offer for a Chase Bank credit card that carries an APR (nominal rate) of 14.24% per year, or 1.187% per month. He will transfer a balance of $1000 and plans to pay it and the transfer fee of $30, due at the end of the first month. Let's assume that Dave makes the transfer, and only days later his employer has a 1-year assignment for him in the country of Cameroon in West Africa. Dave accepts the employment offer, and in his hurried, excited departure, he forgets to send the credit card service company a change of address. Since he is now out of mail touch, he does not pay his monthly balance due, which we calculated in Example 4.2 to be $1041.87.

(*a*) If this situation continues for a total of 12 months, determine the total due after 12 months and the effective annual rate of interest Dave has accumulated. Remember, the fine print on the card's interest and fee information states a penalty APR of 29.99% per year after one late payment of the minimum payment amount, plus a late payment fee of $39 per occurrence.

(*b*) If there were no penalty APR and no late-payment fee, what effective annual interest rate would be charged for this year? Compare this rate with the answer in part (*a*).

EXAMPLE 4.5

Tesla Motors manufactures high-performance battery electric vehicles. An engineer is on a Tesla committee to evaluate bids for new-generation coordinate-measuring machinery to be directly linked to the automated manufacturing of high-precision vehicle components. Three bids include the interest rates that vendors will charge on unpaid balances. To get a clear understanding of finance costs, Tesla management asked the engineer to determine the effective semiannual and annual interest rates for each bid. The bids are as follows:

Bid 1: 9% per year, compounded quarterly

Bid 2: 3% per quarter, compounded quarterly

Bid 3: 8.8% per year, compounded monthly

(*a*) Determine the effective rate for each bid on the basis of semiannual periods.
(*b*) What are the effective annual rates? These are to be a part of the final bid selection.
(*c*) Which bid has the lowest effective annual rate?

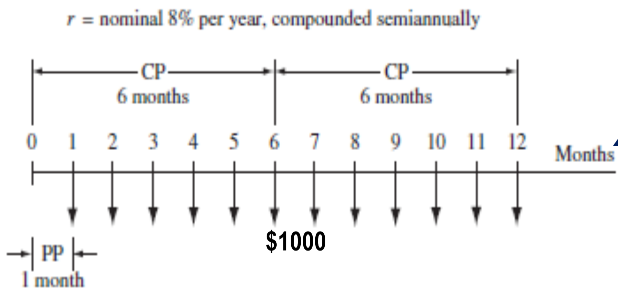
Equivalence Relations: PP and CP

New definition: **Payment Period (PP)** – Length of time between cash flows

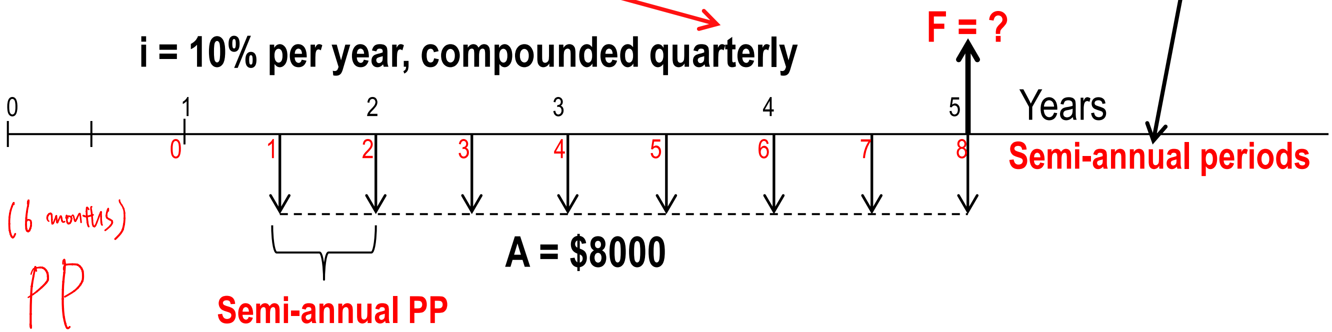
In the diagram below, the compounding period (CP) is semiannual and the payment period (PP) is monthly

(6 months) (a month)

CP > PP



Similarly, for the diagram below, the CP is quarterly and the payment period (PP) is semiannual



(3 months) (6 months)

CP < PP

Single Amounts with PP > CP

normal case

For problems involving single amounts, the payment period (PP) is usually longer than the compounding period (CP). For these problems, there are an infinite number of i and n combinations that can be used, with only two restrictions:

(1) The i must be an effective **interest rate, and**

(2) The time units on n must be the same **as those of i**

(i.e., if i is a rate per quarter, then n is the number of quarters between P and F)

There are two equally correct ways to determine i and n

Method 1: Determine effective interest rate over the compounding period CP, and set n equal to the number of compounding periods between P and F

Method 2: Determine the effective interest rate for any time period t, and set n equal to the total number of those **same time periods**.

Example: Single Amounts with PP ≥ CP

How much money will be in an account in 5 years if \$10,000 is deposited now at an interest rate of 1% per month? Use three different interest rates: (a) monthly, (b) quarterly , and (c) yearly.

CP

PP

(a) For monthly rate, 1% is effective [n = (5 years) × (12 CP per year) = 60]

F = 10,000(F/P,1%,60) = \$18,167

months
effective i per month

i and n must *always* have same time units

(b) For a quarterly rate, effective i/quarter = $(1 + 0.03/3)^3 - 1$ = 3.03%

1% per month, 3% per quarter

1 quarter = 3 months

F = 10,000(F/P,3.03%,20) = \$18,167

20 quarter = 5 years
4 quarter = 1 year

quarters
effective i per quarter

i and n must *always* have same time units

(c) For an annual rate, effective i/year = $(1 + 0.12/12)^{12} - 1$ = 12.683%

1% per month, 12% per year

1 year = 12 months

F = 10,000(F/P,12.683%,5) = \$18,167

years
effective i per year

i and n must *always* have same time units

Series with PP ≥ CP

For series cash flows, *first step* is to determine *relationship* between PP and CP

Determine if PP ≥ CP, or if PP < CP

When PP ≥ CP, the *only* procedure (2 steps) that can be used is as follows:

(1) First, find effective i per PP

Example: if PP is in quarters, *must* find effective *i per* quarter

(2) Second, determine n, the number of A values involved

Example: quarterly payments for 6 years yields n = 4 × 6 = 24

Note: Procedure when PP < CP is discussed later

Example: Series with PP ≥ CP

How much money will be accumulated in 10 years from a deposit of $500 every 6 months if the interest rate is 1% per month?

PP

CP

Solution: First, find relationship between PP and CP

PP = ***six months***, CP = ***one month***; Therefore, **PP > CP**

Since PP > CP, find effective i per PP of six months

Step 1. i **/6 months =** $(1 + 0.06/6)^6 - 1 = 6.15\%$

1% x6
= 6% per semiannual $\boxed{\text{6 months}}$

Next, determine n (number of 6-month periods)

Step 2: $n = 10(2) = 20$ **six month periods**

1 year = 2 six months
10 year = 20 six months

Finally, set up equation and solve for F

F = 500(F/A,6.15%,20) = $18,692 (by factor or spreadsheet)

EXAMPLE 4.10

The Scott and White Health Plan (SWHP) has purchased a robotized prescription fulfillment system for faster and more accurate delivery to patients with stable, pill-form medication for chronic health problems, such as diabetes, thyroid, and high blood pressure. Assume this high-volume system costs \$3 million to install and an estimated \$200,000 per year for all materials, operating, personnel, and maintenance costs. The expected life is 10 years. An SWHP biomedical engineer wants to estimate the total revenue requirement for each 6-month period PP that is necessary to recover the investment, interest, and annual costs. Find this semiannual *A* value both by hand and by spreadsheet, if capital funds are evaluated at 8% per year, using two different compounding periods:

CP

Rate 1. 8% per year, compounded *semiannually*.

$i = (1+0.04)-1, \quad n=20$

Rate 2. 8% per year, compounded *monthly*.

CP

$i = \left(1+0.04/6\right)^{6}-1, \quad n=20$

Series with PP < CP

Two policies: (1) interperiod cash flows earn *no interest* (most common)
(2) interperiod cash flows earn *compound interest*

For policy (1), *positive cash flows* are moved *to* beginning of the interest period in which they occur and *negative cash flows are moved to the* end *of the interest period*

For policy (2), cash flows are *not moved* and equivalent P, F, and A values are determined using the *effective interest rate per payment period*

Example: Series with PP < CP

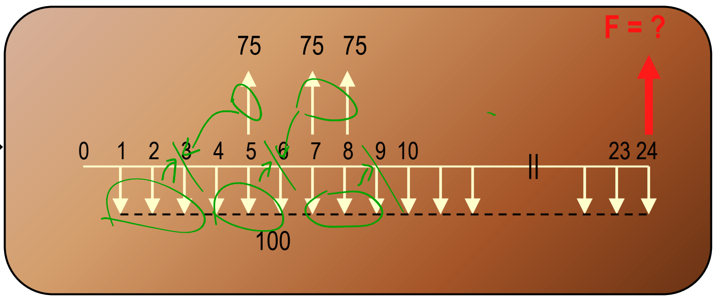
PP

A person deposits $100 per month into a savings account for 2 years. If $75 is withdrawn in months 5, 7 and 8 (in addition to the deposits), construct the cash flow diagram to determine how much will be in the account after 2 years at i = 6% per year, compounded quarterly. Assume there is no interperiod interest.

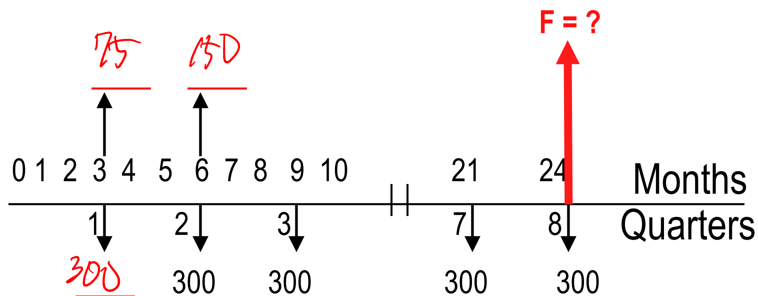
CP

Solution: Since PP < CP with no interperiod interest, the cash flow diagram must be *changed using quarters as the time periods*

from this



to this



Continuous Compounding

When the interest period is infinitely small, interest is *compounded continuously*. Therefore, PP > CP and m increases.

Take limit as m → ∞ to find the effective interest rate equation

$$i = e^r - 1$$

$$i = (1 + r / m)^m - 1$$

where i = effective interest rate for any time period

r = nominal rate for same time period as i

m = no. times interest is comp'd in period specified for i

$$\lim_{m\to\infty} i = \lim_{m\to\infty} \left\{\left(1+\frac{r}{m}\right)^{\frac{m}{r}}\right\}^{r} - 1 = e^{r} - 1$$

Continuous Compounding

When the interest period is infinitely small, interest is *compounded continuously.* Therefore, PP > CP and m increases.

Take limit as m → ∞ to find the effective interest rate equation

$$i = \underline{\;e^r\;} - 1$$

Example: If a person deposits $500 into an account every 3 months at an interest rate of 6% per year, compounded continuously, how much will be in the account at the end of 5 years?

6% per year

3 month

Solution: Payment Period: PP = 3 months

Nominal rate per *three months*: r = 6%/4 = 1.50%

Effective rate per 3 months: $i = e^{0.015} - 1 = 1.51\%$

5 years

F = 500(F/A,1.51%,20) = $11,573

r= 1.5

n=20.4

$i = (e^{1.5\%} - 1)$

Varying Rates

When interest rates vary over time, use the interest rates associated with their respective time periods to find P

Example: Find the present worth of $2500 deposits in years 1 through 8 if the interest rate is 7% per year for the first five years and 10% per year thereafter.

Solution: $P = 2{,}500(P/A,7\%,5) + \{2{,}500(P/A,10\%,3)\}(P/F,7\%,5)$

$= \mathbf{\$14{,}683}$

An equivalent annual worth value can be obtained by replacing each cash flow amount with 'A' and setting the equation equal to the calculated P value

$14{,}683 = A(P/A,7\%,5) + A(P/A,10\%,3)(P/F,7\%,5)$

$A =$ **$2500 per year**

Summary of Important Points

Must understand: interest period, compounding period, compounding frequency, and payment period

Always use effective *rates* in interest formulas

$$i = (1 + r / m)^m - 1$$

Interest rates are stated different ways; must know how to get effective rates

For single amounts, make sure units on i and n are the same

Important Points (cont'd)

For uniform series with PP ≥ CP, find effective i over PP

For uniform series with PP < CP and no interperiod interest, move cash flows to match compounding period

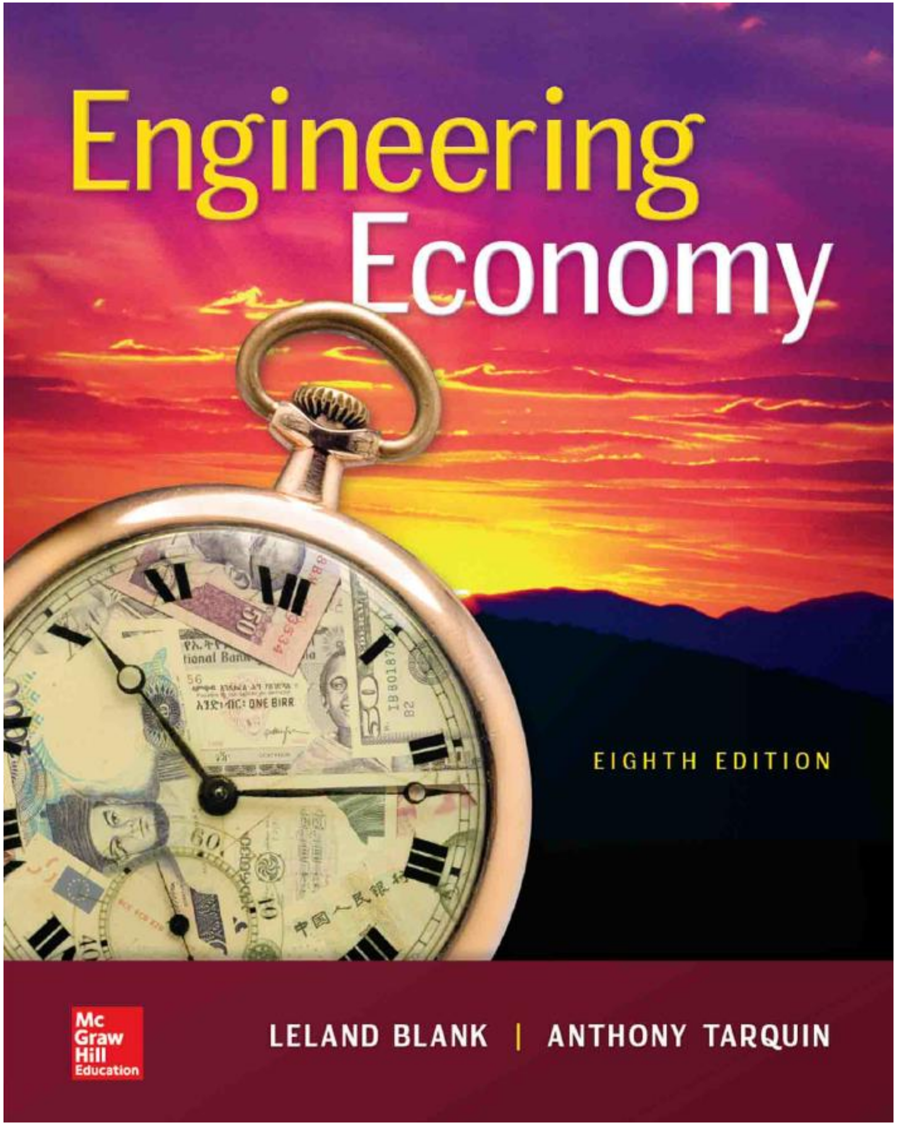
For continuous compounding, use i = $\underline{\;e^r\;}$ – 1 to get effective rate

For varying rates, use stated i values for respective time periods

HOMEWORK

1. Please solve every Examples in your textbook. You do not have to submit your works.

2. Please upload following "PROBLEMS" solution file on "Assignment" menu in e-Class.
   ① 4.14
   ② 4.44
   ③ 4.49
   ④ 4.56
   ⑤ 4.61
   ⑥ 4.74
   ⑦ 4.81



Chapter 5

Present Worth Analysis

Lecture slides to accompany

Engineering Economy

8th edition

Leland Blank

Anthony Tarquin

McGraw Hill Higher Education

LEARNING OUTCOMES

1. **Formulate Alternatives**
2. **PW of equal-life alternatives**
3. **PW of different-life alternatives**
4. **Future Worth analysis**
5. **Capitalized Cost analysis**

Formulating Alternatives

Two types of economic proposals

- **Mutually Exclusive (ME) Alternatives:** Only ***one*** **can be selected; Compete against each other**
- **Independent Projects:** More than ***one*** **can be selected; Compete only against DN**

Do Nothing (DN) – An ME alternative or independent project to maintain the current approach; no new costs, revenues or savings

also can be alternacte or project !

Formulating Alternatives

Two types of cash flow estimates

- **Revenue:** Alternatives include estimates of costs (cash outflows) and revenues (cash inflows)

- **Cost:** Alternatives include only costs; revenues and savings assumed equal for all alternatives

PW Analysis of Alternatives

- Convert all cash flows to PW using MARR
- Precede *costs* by minus(-) sign; **receipts** by plus (+) sign

---

EVALUATION

- For one project, if PW $\geq$ 0, it is justified
- For mutually exclusive alternatives, select **one** with ***numerically*** largest ***PW***
- For independent projects, select all with ***PW ≥ 0***

$i = MARR = 10\%$

110

100

$PW = -100 + \frac{110}{(1+0.1)} = 0$

$\boxed{PW=0}$ → No ~~loss & gain~~ → earned at least minimum profit

$i=10\%$

100

100

⇒ $PW<0$

→ cost

$-10M$ vs $-15M$

⇒ $\boxed{-10M\ A}$ ∵ $-10 > -15$

Selection of Alternatives by PW

For the alternatives shown below, which should be selected if they are (a) mutually exclusive; (b) independent?

Project ID	Present Worth
A	$30,000
B	$12,500
C	$-4,000
D	$ 2,000

Solution: (a) **Select numerically largest PW; alternative** A

(b) **Select all with PW ≥ 0; projects** A, B, D

Ex 5.2

Example: PW Evaluation of Equal-Life ME Alts.

Alternative X has a first cost of \$20,000, an operating cost of \$9,000 per year, and a \$5,000 salvage value after 5 years. Alternative Y will cost \$35,000 with an operating cost of \$4,000 per year and a salvage value of \$7,000 after 5 years. At an MARR of 12% per year, which should be selected?

Solution: Find PW at MARR and select numerically largest PW value

$$PW_X = -20{,}000 - 9000(P/A,12\%,5) + 5000(P/F,12\%,5)$$
$$= -\$49{,}606$$

$$PW_Y = -35{,}000 - 4000(P/A,12\%,5) + 7000(P/F,12\%,5)$$
$$= -\$45{,}447$$

Select alternative Y

PW of Different-Life Alternatives

Must compare alternatives for equal **_service_**
(i.e., alternatives must **end at the** same **time**)

Two ways to compare equal service:

최소공배수

- **Least common multiple (LCM) of lives**
- **Specified study period**

(The LCM procedure is used unless otherwise specified)

Assumptions of LCM approach

- Service provided is needed over the LCM or more years

- Selected alternative can be repeated over each life cycle of LCM in exactly the same manner

- Cash flow estimates are the same for each life cycle

Example: Different-Life Alternatives

Compare the machines below using present worth analysis at i = 10% per year

	Machine A	Machine B
First cost, $	20,000	30,000
Annual cost, $/year	9000	7000
Salvage value, $	4000	6000
Life, years	3	6

Solution: **LCM = 6 years; repurchase A after 3 years**

$$PW_A = -20{,}000 - 9000(P/A,10\%,6) - 16{,}000(P/F,10\%,3) + 4000(P/F,10\%,6)$$
$$= \$-68{,}961$$

-4000 + 20000 in year 3

$$PW_B = -30{,}000 - 7000(P/A,10\%,6) + 6000(P/F,10\%,6)$$
$$= \$-57{,}100$$

Select alternative B

PW Evaluation Using a Study Period

- Once a study period is specified, all cash flows after this time are ignored
- Salvage value is the estimated market **value** at the end of study period

Short study periods are often defined by management when business goals are short-term

Study periods are commonly used in equipment replacement analysis

Example: Study Period PW Evaluation

Compare the alternatives below using present worth analysis at i = 10% per year and a 3-year study period

	Machine A	Machine B
First cost, $	-20,000	-30,000
Annual cost, $/year	-9,000	-7,000
Salvage/market value, $	4,000	6,000 (after 6 years)
		10,000 (after 3 years)
Life, years	3	6

Solution: **Study period = 3 years; disregard all estimates after 3 years**

$$PW_A = -20{,}000 - 9000(P/A,10\%,3) + 4000(P/F,10\%,3) = \$-39{,}376$$

$$PW_B = -30{,}000 - 7000(P/A,10\%,3) + 10{,}000(P/F,10\%,3) = \$-39{,}895$$

Marginally, select A; different selection than for LCM = 6 years

Ex 5.4

Future Worth Analysis

FW exactly like PW analysis, except calculate FW

Must compare alternatives for equal ***service***
(i.e. alternatives must ***end*** at the same time)

Two ways to compare equal service:

- **Least common multiple (LCM) of lives**
- **Specified study period**

(The LCM procedure is used unless otherwise specified)

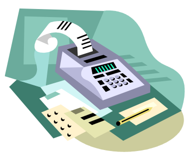
FW of Different-Life Alternatives

Compare the machines below using future worth analysis at i = 10% per year

	Machine A	Machine B
First cost, $	-20,000	-30,000
Annual cost, $/year	-9000	-7000
Salvage value, $	4000	6000
Life, years	3	6

Solution: **LCM = 6 years; repurchase A after 3 years**

$$FW_A = -20{,}000(F/P,10\%,6) - 9000(F/A,10\%,6) \underbrace{-\ 16{,}000}_{= +4000 - 20000}(F/P,10\%,3) + 4000$$
$$= \$-122{,}168$$



$$FW_B = -30{,}000(F/P,10\%.6) - 7000(F/A,10\%,6) + 6000$$
$$= \$-101{,}157$$

Select B (**Note: PW and FW methods will** always **result in** same ***selection***)

Ex. 5.5

Capitalized Cost (CC) Analysis

CC refers to the present worth of a project with a very long life, that is, PW as n becomes infinite

Basic equation is: $CC = P = \frac{A}{i}$ ※ $\boxed{P \times i = A}$

"A" essentially represents the interest on a perpetual investment
↳ (오래 동안) 끊임없는, 영원한

For example, in order to be able to withdraw $50,000 per year forever at i = 10% per year, the amount of capital required is 50,000/0.10 = $500,000
= CC for A=50,000

For infinite *life* alternatives, convert all cash flows into an A value over one *life cycle* and then divide by i

Example: Capitalized Cost

Compare the machines shown below on the basis of their capitalized cost. Use i = 10% per year

	Machine 1	Machine 2
First cost,$	-20,000	-100,000
Annual cost,$/year	-9000	-7000
Salvage value, $	4000	-----
Life, years	3	∞

Solution: Convert machine 1 cash flows into A and then divide by i

$$A_1 = -20{,}000(A/P,10\%,3) - 9000 + 4000(A/F,10\%,3) = \$-15{,}834$$

$$CC_1 = -15{,}834 / 0.10 = \$-158{,}340$$

$$CC_2 = -100{,}000 - 7000/ 0.10 = \$-170{,}000$$

Select machine 1

Ex 5.8

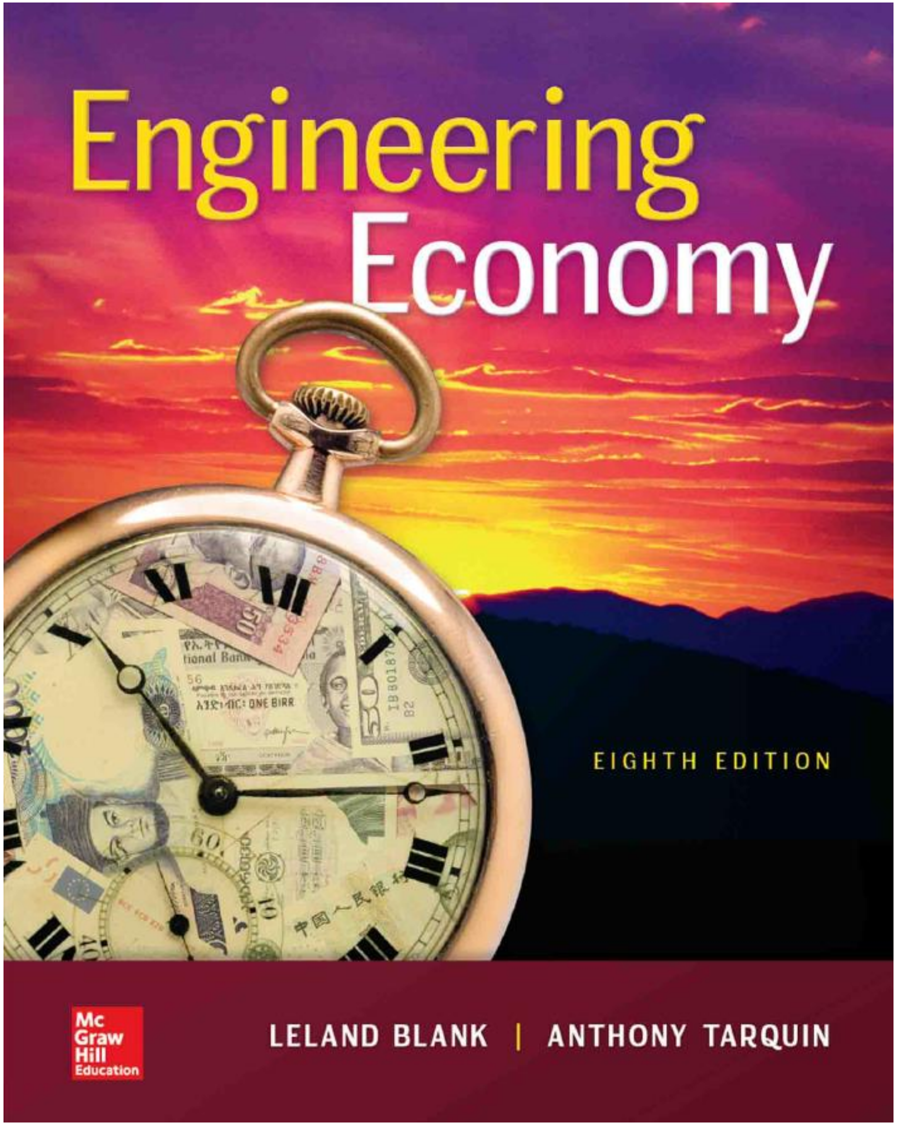
Summary of Important Points

- PW method converts all cash flows to ***present value at*** MARR
- Alternatives can be ***mutually*** exclusive or independent
- *Cash flow estimates can be for* revenue *or* cost *alternatives*
- PW comparison must always be made for equal ***service***
- Equal service is achieved by using LCM or study period
- Capitalized cost is PW of project with infinite ***life***;
  CC = P = A/i

HOMEWORK

1. Please solve every Examples in your textbook. You do not have to submit your works.

2. Please upload following "PROBLEMS" solution file on "Assignment" menu in e-Class.
   ① 5.16
   ② 5.26
   ③ 5.31
   ④ 5.47
   ⑤ 5.51



Chapter 6

Annual Worth Analysis

Lecture slides to accompany

Engineering Economy

8th edition

Leland Blank

Anthony Tarquin

Higher Education

LEARNING OUTCOMES

1. **Advantages of AW** → Don't consider life-time.
2. **Capital Recovery and AW values**
3. **AW analysis**
4. **Perpetual life** infinite
5. **Life-Cycle Cost analysis** (LCC)

Advantages of AW Analysis

AW calculated for only one life cycle

- AW = PW(A/P, i, n) = FW(A/F, i, n)
- It is not necessary to use the LCM of lives to satisfy the equal-Service requirement
- All cash flows *will be* same in every life cycle

Alternatives usually have the following cash flow estimates

(±) P ↔ (∓) S

- **Initial investment, P** – First cost of an asset
- **Salvage value, S** – Estimated value of asset at end of useful life
- **Annual amount, A** – Cash flows associated with asset, such as annual operation cost (AOC), etc.

Relationship between AW, PW and FW

$$AW = PW(A/P,i\%,n) = FW(A/F,i\%,n)$$

amount per year

n is years for equal-service comparison (value of LCM or specified study period)

Same time unit
→ don't care about lifetime

Not needed to get AW

Calculation of Annual Worth

AW for one life cycle is the ___same for all___ **life cycles!!**

An asset has a first cost of \$20,000, an annual operating cost of \$8000 and a salvage value of \$5000 after 3 years. Calculate the AW for one and two life cycles at i = 10%

$$AW_{one} = -20{,}000(A/P,10\%,3) - 8000 + 5000(A/F,10\%,3)$$

$$= \$\text{-}14{,}532$$

$$AW_{two} = -20{,}000(A/P,10\%,6) - 8000 \boxed{-\,15{,}000}(P/F,10\%,3)(A/P,10\%,6) + 5000(A/F,10\%,6)$$

$$= \$\text{-}14{,}532$$

$$= \underbrace{+5000}_{\text{salvage}} + \underbrace{(-20000)}_{\text{second purchase}}$$

Capital Recovery and AW

자본회수

Capital recovery (CR) is the equivalent annual amount that an asset, process, or system must earn each year to just recover the first cost and a stated rate of return over its expected life. Salvage value is considered when calculating CR.

$$CR = -P(A/P,i\%,n) + S(A/F,i\%,n)$$

Use previous example: (note: AOC not included in CR)

CR = -20,000(A/P,10%,3) + 5000(A/F,10%,3) = $ – 6532 per year

Now **AW = CR + A**

AW = – 6532 – 8000 = $ – 14,532

Selection Guidelines for AW Analysis

One alternative: If AW $\geq$ 0, the requested MARR is met or exceeded and the alternative is economically justified.

Two or more alternatives: Select the alternative with the AW that is **numerically largest,** that is, less negative or more positive. This indicates a lower AW of cost for cost alternatives or a larger AW of net cash flows for revenue alternatives.

Ex 6.3

ME Alternative Evaluation by AW

Not necessary to use LCM for different life alternatives

A company is considering two machines. Machine X has a first cost of \$30,000, AOC of \$18,000, and S of \$7000 after 4 years.
Machine Y will cost \$50,000 with an AOC of \$16,000 and S of \$9000 after 6 years.
Which machine should the company select at an interest rate of 12% per year?

Solution: $AW_X = -30{,}000(A/P,12\%,4) - 18{,}000 + 7{,}000(A/F,12\%,4)$
$= \$-26{,}412$

$AW_Y = -50{,}000(A/P,12\%,6) - 16{,}000 + 9{,}000(A/F,12\%,6)$
$= \$-27{,}052$

Select Machine X; it has the numerically larger AW value

AW of Permanent Investment

Use A = Pi for AW of infinite life alternatives
Find AW over one *life cycle* for *finite* life alternatives

Compare the alternatives below using AW and i = 10% per year

	C	D
First Cost, $	-50,000	-250,000
Annual operating cost, $/year	-20,000	-9,000
Salvage value, $	5,000	75,000
Life, years	5	∞

→ meaningless in infinite lifetime.

Solution: Find AW of C over 5 years and AW of D using relation A = P*i*

$$AW_C = -50{,}000(A/P,10\%,5) - 20{,}000 + 5{,}000(A/F,10\%,5)$$
$$= \$-32{,}371$$

$$AW_D = Pi + AOC = -250{,}000(0.10) - 9{,}000$$
$$= \$-34{,}000$$

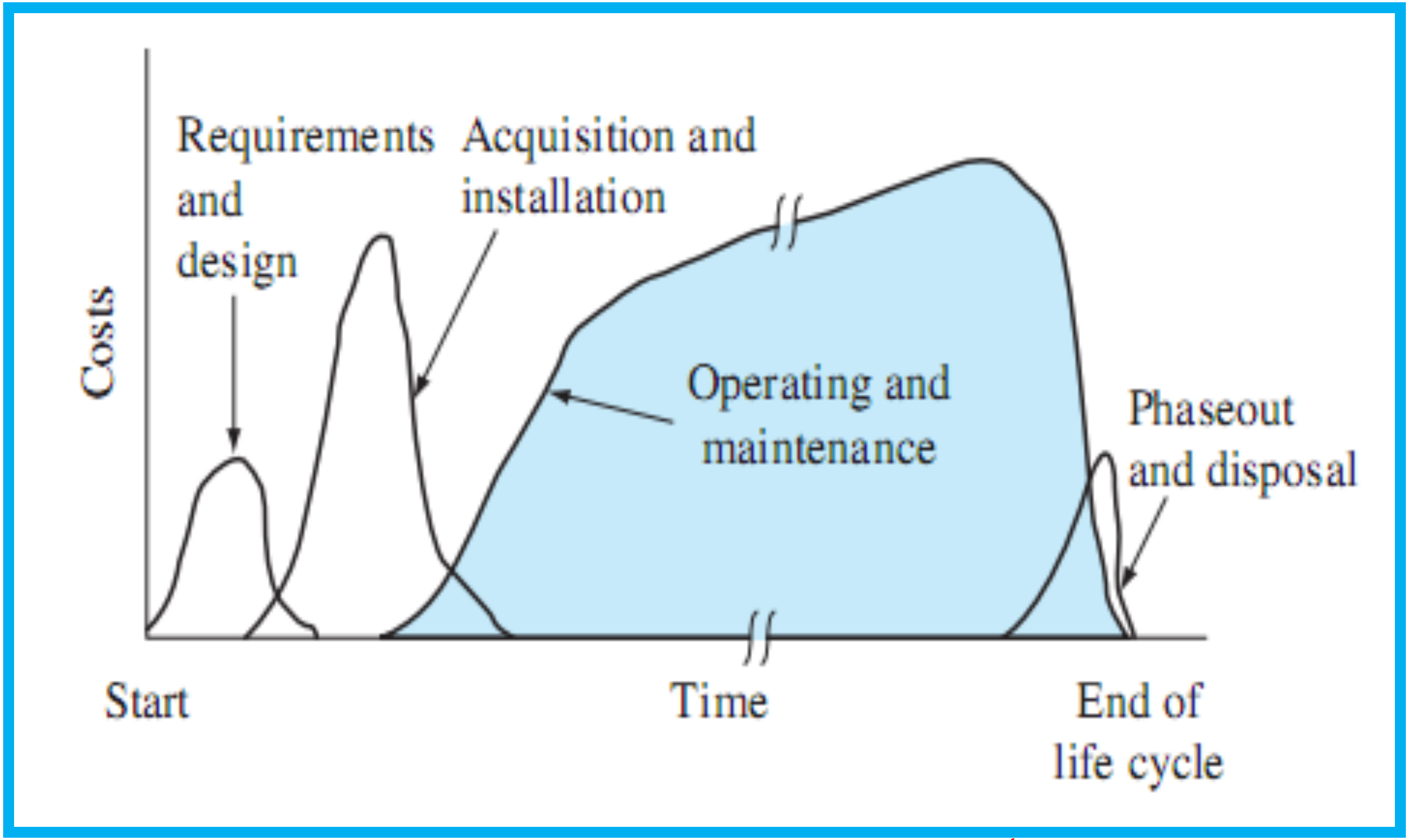
$P \times i + AOC$

Select alternative C

$(\because C > D)$

Ex 6.6

Typical Life-Cycle Cost Distribution by Phase



consider all these cost for fair comparison.

Life-Cycle Cost Analysis

LCC analysis includes all costs for entire life span, from concept to disposal

Best when large percentage of costs are M&O

Includes phases of acquisition, operation, & phaseout

- ✓ **Apply the AW method for LCC analysis of 1 or more cost alternatives**
- ✓ **Use PW analysis if there are revenues and other benefits considered**

choose anything!

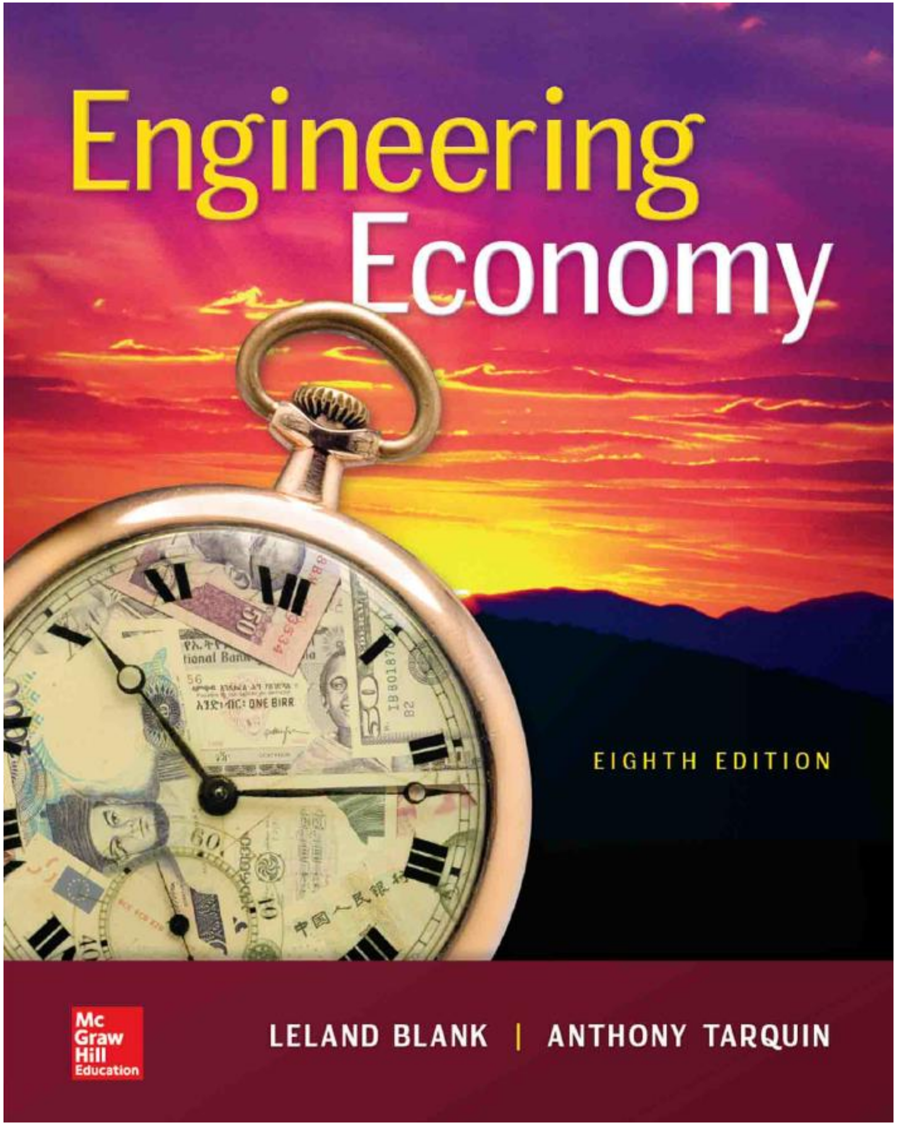
Summary of Important Points

- AW method converts all cash flows to annual *value at* MARR
- Alternatives can be *mutually exclusive, independent, revenue, or cost*
- AW comparison is *only* one *life cycle of each alternative*
- For infinite life alternatives, annualize *initial cost as A =* $P \times i$
- Life-cycle cost analysis includes all *costs* over a project's life span

HOMEWORK

1. Please solve every Examples in your textbook. You do not have to submit your works.

2. Please upload following “PROBLEMS” solution file on “Assignment” menu in e-Class.
   ① 6.7
   ② 6.16
   ③ 6.31
   ④ 6.36
   ⑤ 6.49



Chapter 7

Rate of Return One Project

Lecture slides to accompany

Engineering Economy

8th edition

Leland Blank

Anthony Tarquin

LEARNING OUTCOMES

1. Understand meaning of ROR
2. Calculate ROR for cash flow series
3. Understand difficulties of ROR
4. Determine multiple ROR values
5. Calculate External ROR (EROR)
6. Calculate *r* and *i* for bonds 채권

Ex 7-1

Interpretation of ROR

Rate paid on unrecovered ***balance*** of borrowed money such that final payment brings balance to exactly zero with interest considered

ROR equation can be written in terms of PW, AW, or FW

Use trial and error solution by *factor* or spreadsheet

ROR Calculation and Project Evaluation

- To determine ROR, find the *i** value in the relation

$$PW = \underline{0} \quad \text{or} \quad AW = \underline{0} \quad \text{or} \quad FW = \underline{0}$$

- Alternatively, a relation like the following finds i*

$$PW_{outflow} = PW_{inflow}$$

- For evaluation, a project is economically viable if

$$i^* \geq \underline{\text{MARR}}$$

Finding ROR by Spreadsheet Function

Using the RATE function

= RATE(n,A,P,F)

P = $-200,000   A = $-15,000

n = 12   F = $435,000

Function is

= RATE(12,-15000,-200000,435000)

Display is i* = 1.9%

Using the IRR function

= IRR(first_cell, last_cell)

	A	B
1	Year	CF,$
2	0	-200,000
3	1	-15,000
4	2	-15,000
5	3	-15,000
6	4	-15,000
7	5	-15,000
8	6	-15,000
9	7	-15,000
10	8	-15,000
11	9	-15,000
12	10	-15,000
13	11	-15,000
14	12	435,000
15	**IRR function**	**1.9%**

= IRR(B2:B14)

ROR Calculation Using PW, FW or AW Relation

ROR is the unique i* rate at which a PW, FW, or AW relation equals exactly 0 zero

Example: An investment of $20,000 in new equipment will generate income of $7000 per year for 3 years, at which time the machine can be sold for an estimated $8000. If the company's MARR is 15% per year, should it buy the machine?

Solution:: The ROR equation, based on a PW relation, is:

$$0 = -20{,}000 + 7000(P/A,i^*,3) + 8000(P/F,i^*,3)$$

Solve for i* by trial and error or spreadsheet: i* = 18.2% per year

Since $i^* > \text{MARR} = 15\%$, *the company should buy the machine*

Special Considerations for ROR

- May get multiple ***i* values*** (discussed later)

- Incremental ***analysis*** necessary for multiple alternative evaluations (discussed later)

Multiple ROR Values

숫자부호 변화

Multiple i* values may exist when there is more than one sign change in net cash flow (CF) series.
Such CF series are called non-conventional

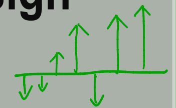
Two tests for multiple i* values:

Descarte's rule of signs: total number of real i* values is ≤ the number of sign changes in *net cash flow series.*

Norstrom's criterion: if the cumulative *cash flow* starts off negatively and has only one *sign change*, there is one positive root .

Multiple ROR Values

Multiple i* values may exist when there is more than one sign change in net cash flow (CF) series.
Such CF series are called non-conventional



Three stricter conditions for multiple i* values:

1. Starts off Negatively
2. *Has only* one sign change (Not Cumulative)
3. $\sum$ Positive *cash flow* $\geq$ $\sum$ Negative *Cash Flow*

⇒ 1 rate of return by RoR methods.

Non-convention
↳ PW FW AW methods!

Plot of PW for CF Series with Multiple ROR Values

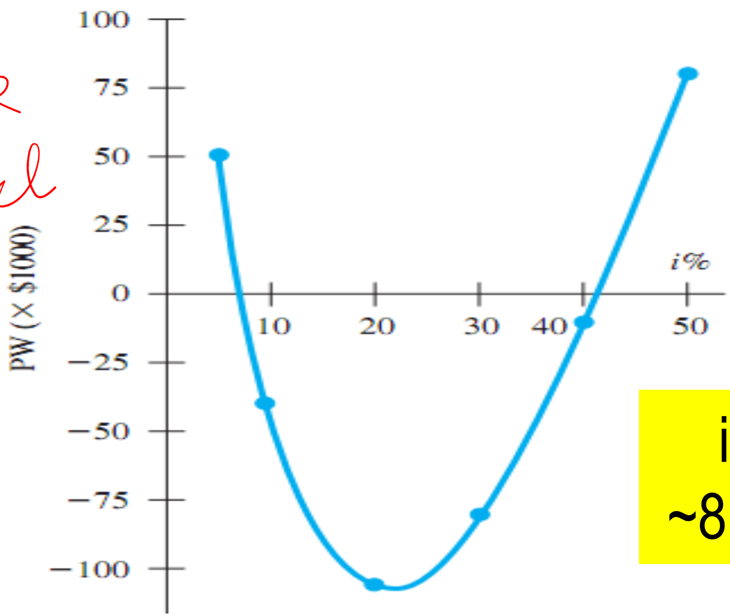
Year	Cash Flow ($1000)	Sequence Number	Cumulative Cash Flow ($1000)
0	+2000	S_0	+2000
1	−500	S_1	+1500
2	−8100	S_2	−6600
3	+6800	S_3	+200

] + → −
] − → +

i%	5	10	20	30	40	50
PW ($1000)	+51.44	−39.55	−106.13	−82.01	−11.83	+81.85

2 sign change
∴ 2 RoR
by Norstrom's criteria
⇓
Do not use RoR method

Use IRR In Excel



→ Non-conventional

i* values at ~8% and ~41%

Example: Multiple i* Values

Determine the maximum number of i* values for the cash flow shown below

Year	Expense	Income	**Net cash flow**	**Cumulative CF**
0	-12,000	-	-12,000	-12,000
1	-5,000	+ 3,000	-2,000	-14,000
2	-6,000	+9,000	+3,000	-11,000
3	-7,000	+15,000	+8,000	-3,000
4	-8,000	+16,000	+8,000	+5,000
5	-9,000	+8,000	-1,000	+4,000

Solution:

The sign on the net cash flow changes twice, indicating two possible i* values

The cumulative cash flow begins negatively with one *sign change*

Therefore, there is only one i* value (i* = 8.7%) > 0

Use PW, FW, AW & 3 strict condition.

Not true / recommend to not use | Decartes Norstrom

Removing Multiple i* Values

Two new interest rates to consider:

- ***Investment rate i_i*** **– rate at which extra funds are** invested ***external*** **to the project**

- ***Borrowing rate i_b*** **– rate at which funds are** borrowed ***from an external source*** **to provide funds to the project**

Two approaches to determine External ROR (EROR)

(1) Modified ROR (MIRR)

(2) Return on Invested Capital (ROIC)

Modified ROR Approach (MIRR)

Four step Procedure:

- Determine PW in *year 0* of all negative CF at i_b
- Determine FW in *year n* of all positive CF at i_i
- Calculate EROR = i' by FW = PW(F/P,i',n)
- If i' ≥ MARR, project is economically justified

Example: EROR Using MIRR Method

For the NCF shown below, find the EROR by the MIRR method if MARR = 9%, i_b = 8.5%, and i_i = 12%

Year	0	1	2	3
NCF	+2000	-500	-8100	+6800

Solution:

$$PW_0 = -500(P/F,8.5\%,1) - 8100(P/F,8.5\%,2)$$
$$= \$-7342$$

$$FW_3 = 2000(F/P,12\%,3) + 6800$$
$$= \$9610$$

$$PW_0(F/P,i',3) + FW_3 = 0$$
$$-7342(1 + i')^3 + 9610 = 0$$

i' = 0.939 (9.39%)

Since i' > MARR of 9%, project is justified

Return on Invested Capital Approach

- Measure of how effectively project uses funds that ***remain internal to project***
- ROIC rate, i", is determined using ***net-investment procedure***

Three step Procedure

(1) Develop series of FW relations for each year t using:

$$F_t = F_{t-1}(1 + k) + NCF_t$$

where: k = i_i if $F_{t-1} > 0$ and k = i" if $F_{t-1} < 0$

(2) Set future worth relation for last year n equal to 0 (i.e., $F_n = 0$); solve for i"

(3) If i" ≥ MARR, *project is* justified ; otherwise, reject

ROIC Example

For the NCF shown below, find the EROR by the ROIC method if MARR = 9% and i_i = 12%

Year	0	1	2	3
NCF	+2000	-500	-8100	+6800

Solution:

Year 0: $F_0 = \$+2000$ — **$F_0 > 0$; invest in year 1 at $i_i = 12\%$**

Year 1: $F_1 = 2000(1.12) - 500 = \$+1740$ — **$F_1 > 0$; invest in year 2 at $i_i = 12\%$**

Year 2: $F_2 = 1740(1.12) - 8100 = \$-6151$ — **$F_2 < 0$; use i'' for year 3**

Year 3: $F_3 = -6151(1 + i'') + 6800$ — **Set $F_3 = 0$ and solve for i''**

$$-6151(1 + i'') + 6800 = 0$$
$$i'' = 10.55\%$$

Since i'' > MARR of 9%, project is justified

Important Points to Remember

About the computation of an EROR value

- EROR values are dependent upon the selected investment and/or borrowing rates
- Commonly, multiple i* rates, i' from MIRR and i'' from ROIC have different values

About the method used to decide

- For a definitive economic decision, set the MARR value and *use the PW or AW method* to determine economic viability of the project

U.S. Saving Bonds

액면가 수익률 (nominal)

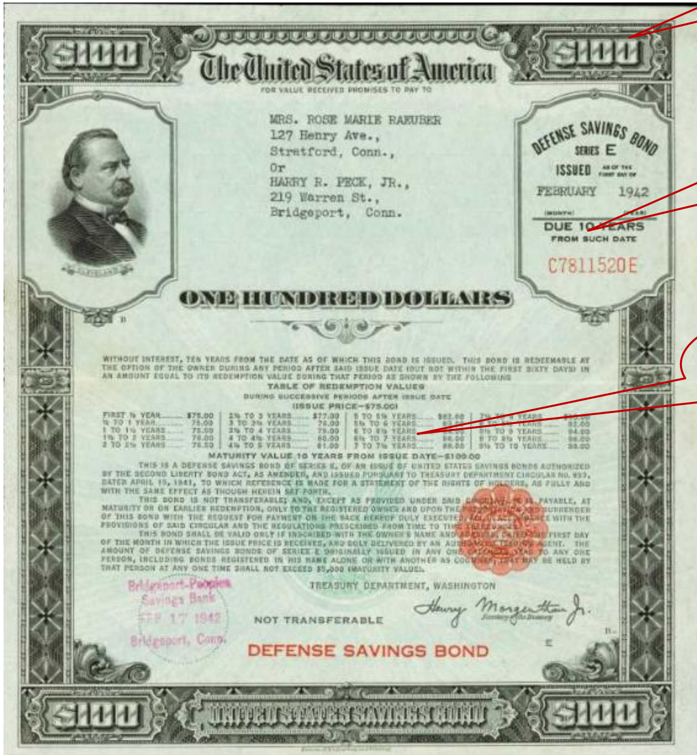
Bond is ***IOU*** with face value **(V)**, coupon rate **(b)**, no. of payment periods/year **(c)**, dividend **(I)**, and maturity date **(n).**

만기

V

n

b, c, I



ROR of Bond Investment

Bond is ***IOU*** with <u>face</u> value **(V)**, <u>coupon</u> rate **(b)**, no. of payment periods/year **(c)**, <u>Dividend</u> **(I)**, and <u>maturity</u> date **(n)**. Amount paid for the bond is **P.** 채권가격

$$I = Vb/c$$

General equation for i*: $0 = -P + I(P/A,i^*,n\times c) + V(P/F,i^*,n\times c)$

V b c=4 P

A $10,000 bond with 6% interest payable quarterly is purchased for $8000. If the bond matures in 5 years, what is the ROR (a) per quarter, (b) per year?

n

Solution: **(a)** $I = 10{,}000(0.06)/4 = \$150$ per quarter

ROR equation is: $0 = -8000 + 150(P/A,i^*,\overset{=4\times 5}{\boxed{20}}) + 10{,}000(P/F,i^*,20)$

By trial and error or spreadsheet: $i^* = 2.8\%$ per quarter

(b) Nominal i^* per year $= 2.8(4) = 11.2\%$ per year

Effective i^* per year $= (1 + \underbrace{0.028}_{=11.2\%/4})^4 - 1 = 11.7\%$ per year

Summary of Important Points

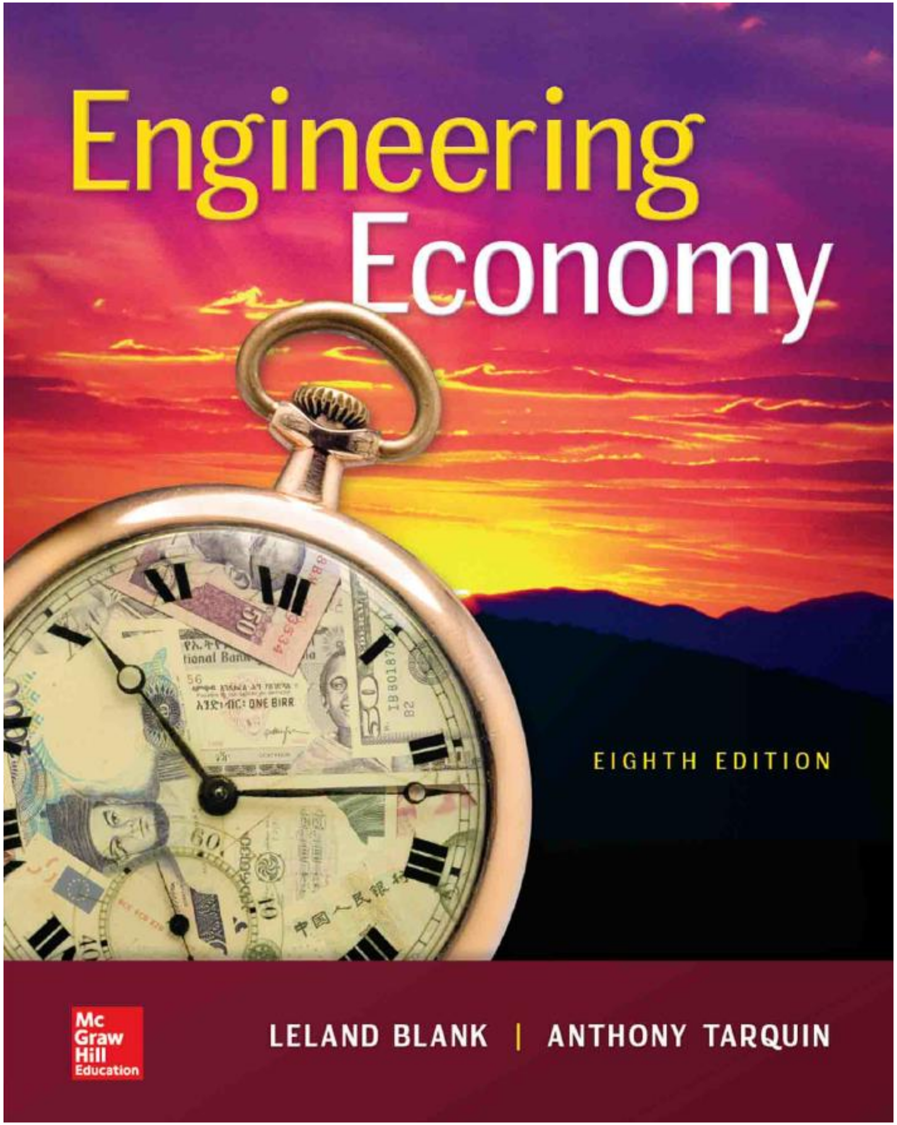
- ROR equations can be written in terms of PW, FW, or AW and usually require *trial and error solution*
- i* assumes *reinvestment* of positive cash flows *at i* rate*
- More than 1 sign change in NCF may cause *multiple i* values*
- Descarte's rule of signs and Norstrom's criterion (3 stricter conditions) useful when multiple *i* values* are suspected
- Not recommended
  EROR can be calculated using MIRR or ROIC approach. Assumptions about investment and borrowing rates is required.
- General ROR equation for bonds is

$$0 = - P + I(P/A,i^*,n_xc) + V(P/F,i^*,n_xc)$$

HOMEWORK

1. Please solve every Examples in your textbook. You do not have to submit your works.

2. Please upload following “PROBLEMS” solution file on “Assignment” menu in e-Class.
   ① 7.12
   ② 7.22
   ③ 7.52
   ④ 7.71



Chapter 8

Rate of Return Multiple Alternatives

Lecture slides to accompany

Engineering Economy

8th edition

Leland Blank

Anthony Tarquin

McGraw Hill Higher Education

LEARNING OUTCOMES

1. **Why incremental analysis is required in ROR**
2. **Incremental cash flow (CF) calculation**
3. **Interpretation of ROR on incremental CF**
4. **Select alternative by ROR based on PW relation**
5. **Select alternative by ROR based on AW relation**
6. **Select best from several alternatives using ROR method**

Why Incremental Analysis is Necessary

- Selecting the alternative with highest ROR may not yield highest return on ***available capital***
- Must consider weighted ***average*** of total capital available
- Capital ***not*** invested in a project is assumed to ***earn at*** MARR

Example: Assume $90,000 is available for investment and MARR = 16% per year. If alternative A would earn 35% per year on investment of $50,000, and B would earn 29% per year on investment of $85,000, the weighted averages are:

(90000-50000)

Overall ROR_A = [50,000(0.35) + 40,000(0.16)]/90,000 = 26.6%

Overall ROR_B = [85,000(0.29) + 5,000(0.16)]/90,000 = 28.3%

(90000-85000)

Which investment is better, economically?

Total Capital : 200

MARR	10%				
Year	0	1		PW	IRR
Alt. 1 ①	Smaller investment -100	150		36.36	50.0%
Alt. 2 ②	larger investment -200	270		45.45	35.0%
Marr	-100	110		0.00	10.0%
② -① Alt2-Alt1	-100	120			
Alt. 1	-200	260			30%

IRR → ① is larger ∴ select ① ?

larger RoR doesn't guarantee a better solution

PW analysis → ② is larger ∴ select ② !

Rest of money doesn't be invested will give you MARR

if select ① ∴

-100 + 150 (50%)

-100 + 110 (10%)

-200 + 260 (30%)

MARR	22%			
Year	0	1	PW	IRR
Alt. 1	-100	150	22.95	50.0%
Alt. 2	-200	270	21.31	35.0%
Marr	-100	122	0.00	22.0%
Alt2-Alt1	-100	120		
Alt. 1	-200	272		36%

MARR increase 10% → 22%

Consistent with PW analysis

When perform RoR analysis,

need to perform incremental analysis.

(* when $\underbrace{\text{multiple alternatives}}_{\text{mutual exclusive.}}$ exists.)

if 'independant' → select alternatives larger than MARR.

Why Incremental Analysis is Necessary

If selection basis is higher ROR:

Select alternative A (wrong answer)

If selection basis is higher overall ROR:

Select alternative B

Incremental RoR > Marr

→ larger initial investment

Conclusion: Must use an incremental ROR analysis to make a consistently correct selection

Unlike PW, AW, and FW values, if not analyzed correctly, ROR values can lead to an incorrect alternative selection. This is called the ranking inconsistency problem (discussed later)

higher RoR $\not\Rightarrow$ Better alternative

not always!

Calculation of Incremental CF

Incremental cash flow = cash flow$_B$ – cash flow$_A$

where larger *initial investment* is Alternative B

Example: Either of the cost alternatives shown below can be used in a grinding process. Tabulate the incremental cash flows.

|-60000| > |-40000|

	A	B	B - A
First cost, $	-40,000	- 60,000 → larger initial investment !	-20,000
Annual cost, $/year	-25,000	-19,000	+6000
Salvage value, $	8,000	10,000	+2000

"incremental cash flow vs MARR → compare!"

The incremental CF is shown in the (B-A) column

*The ROR on the extra **$20,000** investment in B determines which alternative to select (as discussed later)*

Interpretation of ROR on Extra Investment

Based on concept that any avoidable *investment* that does not yield at least the MARR should not be made.

Once a lower-cost alternative *has been economically* justified, the ROR on the extra *investment* (*i.e., additional amount of money* associated with a higher first-cost alternative) must also yield a ROR < MARR (because the extra investment *is* avoidable by selecting the economically-justified lower-cost alternative).

for mutually exclusive !

This incremental ROR is identified as Δi^*

For independant projects, select all that have ROR ≥ MARR

(No incremental analysis is necessary)

ROR Evaluation for Two ME Alternatives

(1) Order alternatives by increasing *initial investment cost*

(2) *Develop incremental CF series* using LCM of years

(3) Draw incremental *cash flow diagram*, if needed

(4) Count sign changes to see if *multiple Δi* values exist*

(5) Set up PW, AW, or FW = 0 relation and *find* Δi^*_{B-A}

Note: Incremental ROR analysis requires equal service comparison. The LCM of lives must be used in the relation

(6) If Δi^*_{B-A} < MARR, *select* A; otherwise, select B.

If multiple Δi* values exist, ***find*** $\boxed{\textbf{EROR}}$ using either MIRR or ROIC approach. (Not Recommended!)

different Lifetime ⇒ PW or AW analysis is better.

Example: Incremental ROR Evaluation

Either of the cost alternatives shown below can be used in a chemical refining process. If the company's MARR is 15% per year, determine which should be selected on the basis of ROR analysis?

	A		B	B−A!
First cost ,$	|-40,000|	<	|-60,000|	−20,000
Annual cost, $/year	-25,000		-19,000	+6,000
Salvage value, $	8,000		10,000	+2,000
Life, years	5		5	

Initial observations: ME, cost alternatives with equal life estimates and no multiple ROR values indicated

Example: ROR Evaluation of Two Alternatives

Solution, using procedure:

	A	B	B - A
First cost , $	-40,000	-60,000	-20,000
Annual cost, $/year	-25,000	-19,000	+6000
Salvage value, $	8,000	10,000	+2000
Life, years	5	5	

Order by first ***cost*** **and** ***find*** incremental ***cash flow*** **B - A**

Write ROR equation **(in terms of PW, AW, or FW) on incremental CF**

$$0 = -20{,}000 + 6000(P/A,\Delta i^*,5) + 2000(P/F,\Delta i^*,5)$$

Solve **for Δi* and** ***compare*** **to MARR**

$$\Delta i^*_{B\text{-}A} = 17.2\% > \text{MARR of } 15\%$$

ROR on $20,000 extra investment is acceptable: **Select** B

ROR Analysis – Multiple Alternatives

Six-Step Procedure for Mutually (Exclusive) Alternatives

(1) Order alternatives from smallest *to largest initial investment*

(2) For revenue alts, calculate **i*** (vs. DN) and eliminate *all with i* < MARR*; remaining alternative with lowest cost is ***defender***. **F**or cost alternatives, go to step (3)

(3) Determine incremental CF between ***defender*** and ***next lowest-cost*** alternative (known as the ***challenger***). Set up ROR relation

(4) Calculate Δ**i*** on incremental CF between *two alternatives from step (3)*

(5) If **Δi* ≥ MARR,** *eliminate* defender and *challenger becomes new defender* against next alternative on list

(6) Repeat steps (3) through (5) *until only* one *alternative* remains. ***Select it***.

For Independent Projects

Compare each alternative vs. DN and select ***all with ROR ≥ MARR***

Example: ROR for Multiple Alternatives

The five mutually exclusive alternatives shown below are under consideration for improving visitor safety and access to additional areas of a national park. If all alternatives are considered to last indefinitely, determine which should be selected on the basis of a rate of return analysis using an $\boxed{\text{interest rate of 10\%.}}$

	A	B	C	D	E
First cost, $ millions	-20	-40	-35	-90	-70
Annual M&O cost, $ millions	-2	-1.5	-1.9	-1.1	-1.3

Solution: **Rank on the basis of initial cost: A < C < B < E < D; calculate CC values** $\rightarrow \frac{A}{i} = P$

C−A **C vs. A:** $0 = -15 + 0.1/\Delta i^*$ **Δi^* = 6.7%** (eliminate C)

B−A **B vs. A:** $0 = -20 + 0.5/\Delta i^*$ **Δi^* = 25%** (eliminate A)

E−B **E vs. B:** $0 = -30 + 0.2/\Delta i^*$ **Δi^* = 6.7%** (eliminate E)

D−B **D vs. B:** $0 = -50 + 0.4/\Delta i^*$ **Δi^* = 8%** (eliminate D)

Select alternative B

Summary of Important Points

- Must consider incremental **cash flows** for mutually exclusive alternatives

Incremental cash flow = cash flow$_B$ – cash flow$_A$

where alternative with larger initial investment is **Alternative B**

- Eliminate B if incremental ROR $\Delta i^* <$ ***MARR****; otherwise,* **eliminate** A

- For multiple mutually exclusive alternatives, compare two at a time and eliminate alternatives until ***only*** one ***remains***

- For independent alternatives, compare each against **DN** and ***select*** all ***that have ROR ≥ MARR***

HOMEWORK

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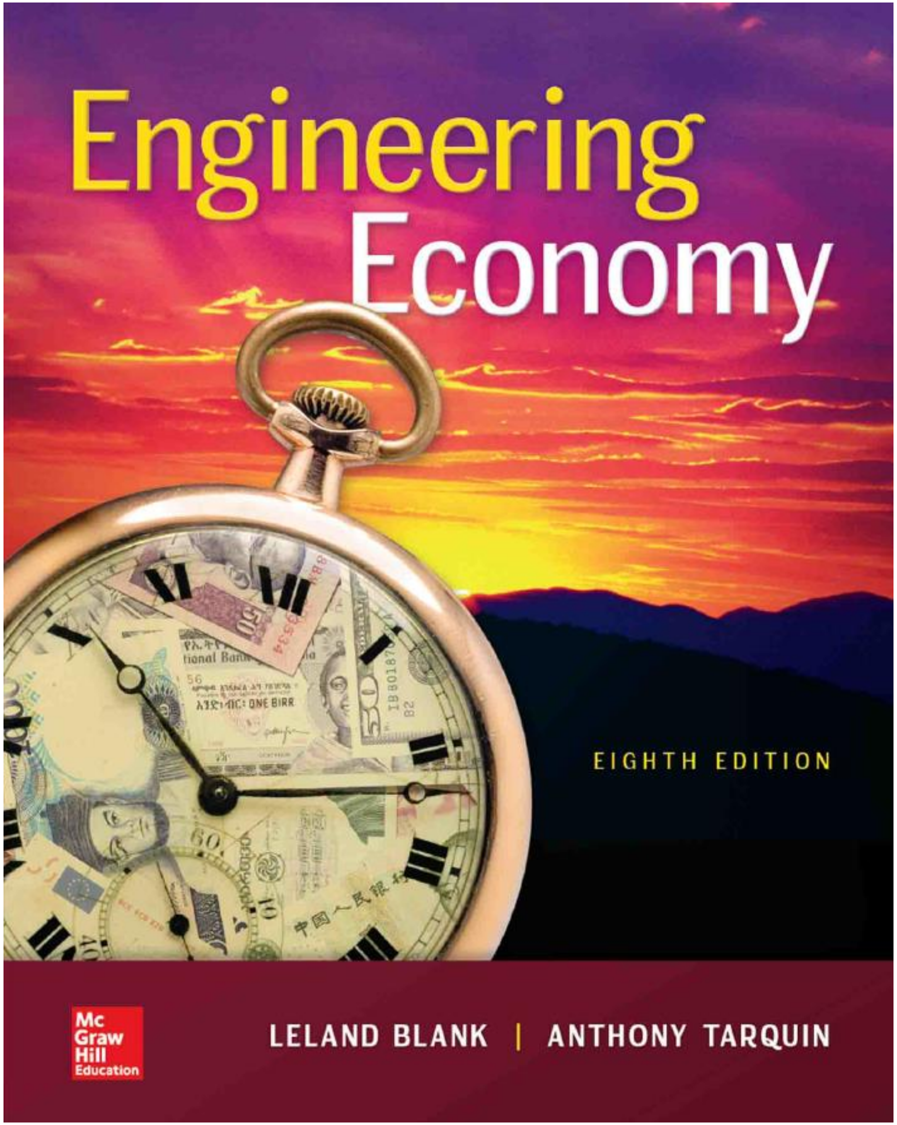


Chapter 9

Benefit/Cost Analysis

mainly for public project

Lecture slides to accompany

Engineering Economy

8th edition

Leland Blank

Anthony Tarquin

Mc Graw Hill Higher Education

LEARNING OUTCOMES

1. Explain difference in public vs. private sector projects
2. Calculate B/C ratio for single project
3. Select better of two alternatives using B/C method
4. Select best of multiple alternatives using B/C method
5. Use cost-effectiveness analysis (CEA) to evaluate service sector projects
6. Describe how ethical compromises may enter public sector projects

Differences: Public vs. Private Projects

Characteristic	Public	Private
Size of Investment	Large	Small, medium, large
Life	Longer (30 – 50+ years)	Shorter (2 – 25 years)
Annual CF	No profit	Profit-driven
Funding	Taxes, fees, bonds, etc.	Stocks, bonds, loans, etc.
Interest rate	Lower	Higher
Selection criteria	Multiple criteria	Primarily ROR
Environment of evaluation	Politically inclined	Economic

Types of Contracts

Contractors does not share project risk

- **Fixed price** - lump-sum payment ... 고정금액 (정액) 계약
- **Cost reimbursable** - Cost plus, as negotiated ... 실비정산식 계약

Contractor shares in project risk

- **Public-private partnerships (PPP)**, such as:
  - **Design-build projects** - Contractor responsible from design stage to operations stage
  - **Design-build-operate-maintain-finance (DBOMF) projects** - Turnkey project with contractor managing **financing** (manage cash flow); government obtains **funding** for project
  - BTO(Build-Transfer-Operate): does not guarantee proper profit by the government → 수익형민자사업
  - BTL(Build-Transfer-Lease): guarantee proper profit by the government → 임대형민자사업

Cash Flow Classifications and B/C Relations

Must identify each cash flow as either benefit, disbenefit, or cost

Benefit (B) -- Advantages to the *public*

Disbenefit (D) -- Disadvantages to the *public*

Cost (C) -- Expenditures by the *government*

Preferred

Conventional B/C ratio = (B–D) / C

M&O cost

Modified B/C ratio = [(B–D) – $\boxed{C}$] / Initial Investment → separate cost into two parts

Profitability Index = $NCF_{i \neq 0}$ / Initial Investment

net cash flow

Note 1: All terms must be expressed in ***same units***, i.e., PW, AW, or FW

Note 2: *Do not use* minus *sign* ahead of $\boxed{\text{costs}}$ → using absolute value

Decision Guidelines for B/C and PI

Benefit/cost analysis

If B/C ≥ 1.0, project **is** economically justified at discount rate applied

If B/C < 1.0, project **is** not economically acceptable

Profitability index analysis of revenue projects

If PI ≥ 1.0, project **is** economically justified at discount rate applied

If PI < 1.0, project **is** not economically acceptable

B/C Analysis – Single Project

Conventional B/C ratio = $\dfrac{B - D}{C}$

Modified B/C ratio = $\dfrac{B - D - M\&O}{C}$

If B/C ≥ 1.0,
accept project;
otherwise, reject

분모

Denominator is
initial investment

PI = $\dfrac{\text{PW of NCF}_{t \neq 0}}{\text{PW of initial investment}}$

If PI ≥ 1.0,
accept project;
otherwise, reject

B/C Analysis – Single Project

conventional

$$\frac{B}{I + M}$$

benefit

initial maintenance

OR

modified

$$\frac{B - M}{I}$$

?

consistency !

	Annual value				IRR	$\frac{B}{I+M}$		$\boxed{\frac{B-M}{I}}$
	I	B	M	PV				
a	8,024	23,856	7,880	7,952	15%	1.50		1.99
b	8,024	16,500	524	7,952	15%	1.93		1.99
c	8,024	17,500	1,524	7,952	15%	1.83		1.99
d	8,024	18,500	2,524	7,952	15%	1.75		1.99

Example: B/C Analysis – Single Project

A flood control project will have a first cost of $1.4 million with an annual maintenance cost of $40,000 and a 10 year life. Reduced flood damage is expected to amount to $175,000 per year. Lost income to farmers is estimated to be $25,000 per year. At an interest rate of 6% per year, should the project be undertaken?

Solution: Express all values in AW terms and find conventional B/C ratio

AW

B = $175,000

D = $25,000

C = 1,400,000(A/P,6%,10) + $40,000 = $230,218

B/C = (175,000 – 25,000)/230,218
= 0.65 < 1.0

Do not build project

Ex 9.3

Defender, Challenger and Do Nothing Alternatives

When selecting from two or more ME alternatives, there is a:

- ✓ **Defender –** in-place system or currently selected alternative
- ✓ **Challenger –** Alternative challenging the defender
- ✓ **Do-nothing option –** Status quo system

General approach for incremental B/C analysis of two ME alternatives:

- Lower total cost alternative is first compared to Do-nothing ***(DN)***
- If B/C for the lower cost alternative is < 1.0, the DN option is compared to B/C of the higher-cost alternative
- If both alternatives lose out to DN option, DN prevails, unless overriding needs requires selection of one of the alternatives

Alternative Selection Using Incremental B/C Analysis – Two or More ME Alternatives

Procedure similar to RoR analysis for multiple alternatives

(1) Determine *equivalent total cost* for each alternative
(2) *Order alternatives by increasing total cost*
(3) Identify *B and D for each alternative*
(4) *Calculate B/C for each alternative and eliminate all with B/C < 1.0*
(5) *Determine incremental costs and benefits for first two alternatives*
(6) *Calculate ΔB/C; if >1.0, higher cost alternative becomes defender*
(7) *Repeat steps 5 and 6 until only one alternative remains*

Example: Incremental B/C Analysis

Compare two alternatives using i = 10% and B/C ratio

Alternative	X	Y
First cost, $	320,000	540,000
M&O costs, $/year	45,000	35,000
Benefits, $/year	110,000	150,000
Disbenefits, $/year	20,000	45,000
Life, years	10	20

Solution: **First, calculate equivalent total cost**

AW of $costs_X$ = 320,000(A/P,10%,10) + 45,000 = $97,080

AW of $costs_Y$ = 540,000(A/P,10%,20) + 35,000 = $98,428

Order of analysis is X, then Y

X vs. DN: (B-D)/C = (110,000 – 20,000) / 97,080 = **0.93** **Eliminate** X

Y vs. DN: (150,000 – 45,000) / 98,428 = **1.07** **Eliminate** DN

conventional B/C

Select Y

Example: ΔB/C Analysis; Selection Required

Must select one of two alternatives using i = 10% and ΔB/C ratio

Alternative	X	Y
First cost, $	320,000	540,000
M&O costs, $/year	45,000	35,000
Benefits, $/year	110,000	150,000
Disbenefits, $/year	20,000	45,000
Life, years	10	20

Solution: **Must select X or Y; DN not an option, *compare Y to X***

AW of costs_X = $97,080   AW of costs_Y = $98,428

Incremental values: ΔB = 150,000 – 110,000 = $40,000

ΔD = 45,000 – 20,000 = $25,000

ΔC = 98,428 – 97,080 = $1,348

Y vs. X: (ΔB - ΔD) / ΔC = (40,000 – 25,000) / 1,348 = **11.1**   **Eliminate X**

Select Y

B/C Analysis of Independent Projects

- Independent projects comparison does **not require** incremental **analysis**
- Compare each alternative's overall B/C with **DN option**

- **No budget limit:** **Accept** all alternatives with **B/C ≥ 1.0**
- **Budget limit specified:** capital budgeting problem; selection follows different procedure (Knapsack Problem)

Cost Effectiveness Analysis

Service sector projects primarily involve intangible, **not physical facilities**; examples include health care, security programs, credit card services, etc.

Cost-effectiveness analysis (CEA) combines monetary cost estimates with ***non-***monetary benefit estimates to calculate the

Cost-effectiveness ratio (CER)

$$\text{CER} = \frac{\text{Equivalent total costs}}{\text{Total effectiveness measure}}$$

$$= C/E$$

CER Analysis for Independent Projects

Procedure is as follows:

(1) Determine equivalent total cost **C**, total effectiveness measure **E** and **CER**

(2) **Order projects by** smallest **to largest CER**

(3) Determine cumulative cost of projects and compare to budget limit ***b***

(4) Fund all projects such that ***b* is not exceeded**

Example: The effectiveness measure ***E*** is the number of graduates from adult training programs. For the CERs shown, determine which ***independent*** programs should be selected; b = $500,000.

Program	CER, $/graduate	Program Cost, $
A	1203	305,000 ②
B	752	98,000 ①
C	2010	126,000
D	1830	365,000

+ 403,000 < 500,000

Example: CER for Independent Projects

First, rank programs according to increasing *CER:*

Program	CER, $/graduate	Program Cost, $	Cumulative Cost, $
B	752	98,000	98,000
A	1203	305,000	403,000
D	1830	365,000	768,000
C	2010	126,000	894,000

Next, select programs until budget is not exceeded

Select programs B and A at total cost of $403,000

Note: To expend the entire $500,000, accept as many additional individuals as possible from D at the per-student rate

Ethical Considerations

Engineers are routinely involved in two areas where ethics may be compromised:

Public policy making – **Development of strategy**, e.g., water system management (supply/demand strategy; ground vs. surface sources)

Public planning - **Development of projects**, e.g., water operations (distribution, rates, sales to outlying areas)

Engineers must maintain integrity and impartiality and *always* adhere to Code of Ethics

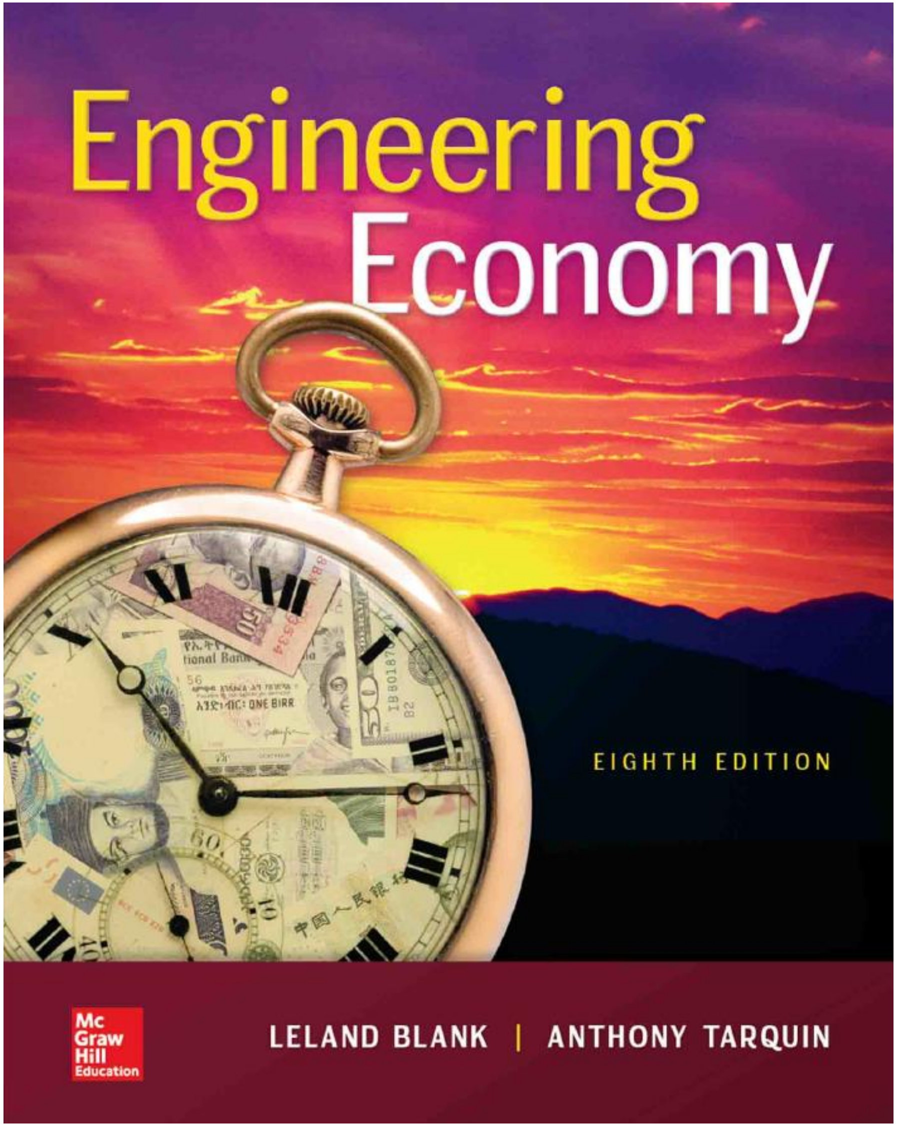
Summary of Important Points

- B/C method used in ***public sector*** project evaluation
- Can use PW, AW, or FW for incremental B/C analysis, but must ***be consistent*** with units for B,C, and D estimates
- For multiple mutually exclusive alternatives, compare two at a time and eliminate alternatives until only one ***remains***
- For independent alternatives with no budget limit, compare each against **DN** and select ***all alternatives that have B/C*** $\geq$ ***1.0***
- **CEA analysis** for service sector projects combines cost and non-monetary ***measures***
- Ethical dilemmas are ***especially prevalent*** in public sector projects

HOMEWORK

1. Please solve every Examples in your textbook. You do not have to submit your works.

2. Please upload following “PROBLEMS” solution file on “Assignment” menu in e-Class.
   ① 9.16
   ② 9.26
   ③ 9.35
   ④ 9.44



Chapter 14

Effects of Inflation

Lecture slides to accompany

Engineering Economy

8th edition

Leland Blank

Anthony Tarquin

Higher Education

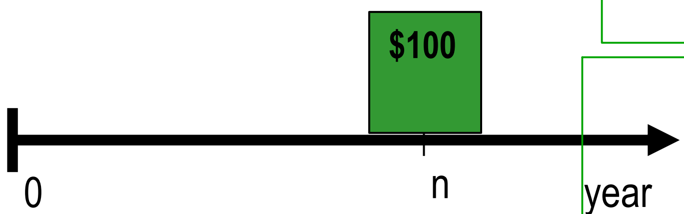
LEARNING OUTCOMES

1. Understand inflation/deflation
2. Calculate PW of cash flows with inflation
3. Calculate FW with inflation considered
4. Calculate AW with inflation considered

Understanding Inflation

Actual \$ vs. Constant \$

- **Actual \$** : expressed in the amount of __\$ bills__ that you are expected to have at year n then current dollar unit
- **Constant \$** : expressed in the amount of __purchasing power__ as of year __0__ that you are expected to have at year n



$100(Actual)

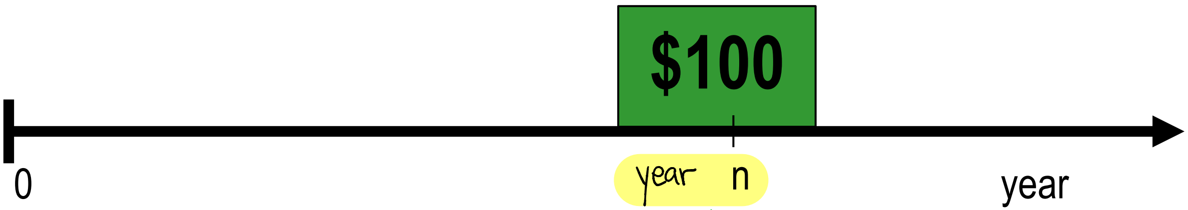
$100(Constant)

+

Constant $

Could be purchased with  at year 0

$100 bill



$100 + a

Convert

+

Conversion
from Constant $ to Actual $

(Inflation, f = 5%)

Period	NCF **Constant $**	Conversion Factor	NCF in **Actual $**
0	-$250.00	(1+0.05)^0	-$250.00
1	$100.00	(1+0.05)^1	$105.00
2	$100.00	(1+0.05)^2	$110.25
3	$100.00	(1+0.05)^3	$115.76
4	$100.00	(1+0.05)^4	$121.55
5	$100.00	(1+0.05)^5	$127.63

Understanding Inflation

Inflation: Increase in amount of money needed to purchase same amount of goods or services. Inflation results in a decrease *in purchasing power*, *i.e., one unit of money buys* less *goods or services*

Two ways to work problems *when considering inflation: (Fig. 14-1)*

(1) Convert to **constant value** (CV) dollars, then use real rate *i.*

If *f* = inflation rate (% per year), the equation is:

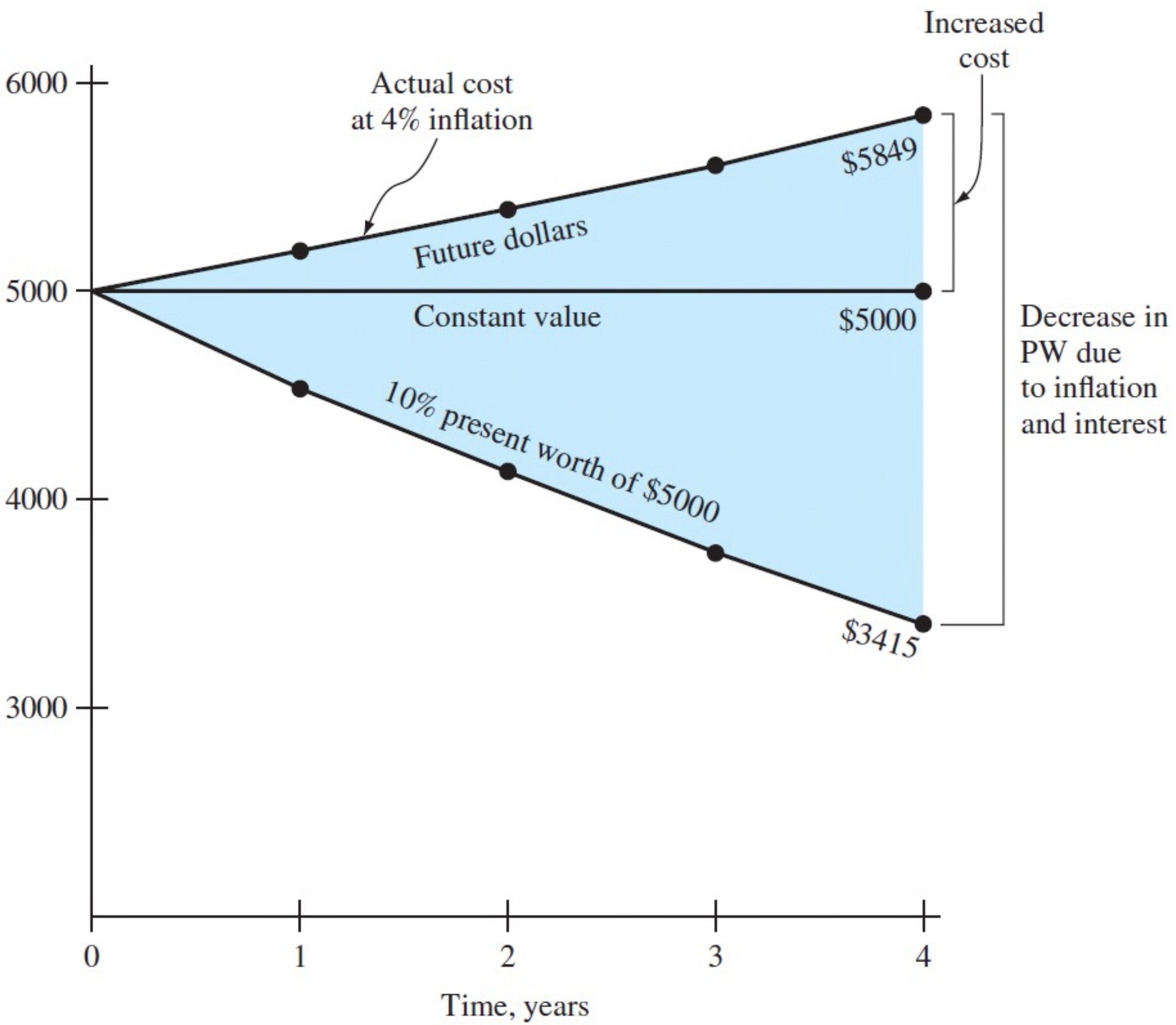
$$\text{Constant \$} = \frac{\text{Actual \$}}{(1+f)^n}, \qquad \therefore \text{PV} = \frac{\text{Constant \$}}{(1+i)^n} = \frac{\text{Actual \$}}{(1+f)^n \times (1+i)^n}$$

(2) Leave money amounts ***as is*** and use ***interest rate adjusted for inflation***, $i_f = i + f + (i)(f)$,

$$\therefore \text{PV} = = \frac{\text{Actual \$}}{(1+i+f+i\cdot f)^n} = \frac{\text{Actual \$}}{(1+i_f)^n}$$

Understanding Inflation

Figure 14–1
Comparison of constant-value dollars, future dollars, and their present worth values.



Example: Constant Value Dollars

How much would be *required today* to purchase an item that increased in cost by exactly the inflation rate? The cost 30 years ago was $1,000 and inflation has consistently averaged 4% per year.

Solution: Solve for future dollars

$$\textbf{Future dollars} = \text{constant value dollars}(1 + f)^n$$
$$= 1{,}000(1 + 0.04)^{30}$$
$$= \mathbf{\$3{,}243}$$

Note: This calculation only accounts for the decreasing *purchasing power of the currency*. It does ***not*** take into account the ***time value of money*** (to be discussed)

Deflation: **Opposite of inflation; purchasing power of money is greater in future than at present; however, money, credit, jobs are 'tighter'**

Three Different Rates

- Real or inflation-free rate i – Rate at which interest is earned when *effects of inflation are* removed ; i represents the real increase in purchasing power

- Market or inflation-adjusted rate i_f – Rate that *takes* inflation *into account.* Commonly stated rate everyday

- Inflation rate f – Rate of *change in value of currency*

Relation between three rates is derived using the relation

$$P = F\frac{1}{(1+i_f)^n} = F(P/F,i_f,n)$$

Market rate is: $i_f = i + f + (i)(f)$

Example: Market vs. Real Rate

Money in a medium-risk investment makes a guaranteed **8%** i_f per year. Inflation rate has averaged **5.5%** per year. What is the real rate of return on the investment? f

i

$i_f = i + 0.055 + 0.055i = 0.08$

Solution: Solve for the real rate *i* in relation for i_f

$i = \frac{0.08 - 0.055}{1.055} = 0.024$

$= 2.4\%$

$$i_f = i + f + (i)(f)$$

$$i = \frac{i_f - f}{1 + f}$$

$$= \frac{0.08 - 0.055}{1 + 0.055}$$

$$= 0.024$$

Investment pays only 2.4 % per year in real terms vs. the stated 8%

PW Calculations with Inflation

Two ways to account for inflation in PW calculations

(1) Convert cash flow into *constant-value* (CV) *dollars* and use regular i

where: CV = future dollars/$(1 + f)^n$ = then-current dollars/$(1 + f)^n$

(actual dollar)

f = inflation rate

(**Note:** *Calculations up to now have assumed constant-value dollars*)

(2) Express cash flow in *future* (then-current) *dollars* and use inflated interest rate where $i_f = i + f + (i)(f)$

(**Note:** *Inflated interest rate is the market interest rate*)

Example: PW with Inflation

A honing machine will have a cost of $25,000 (future cost) six years from now. Find the PW of the machine, if the real interest rate is 10% per year and the inflation rate is 5% per year using (a) constant-value dollars, and (b) future dollars.

Solution: (a Determine constant-value dollars and ***use i*** in PW equation

$$\mathbf{CV} = 25{,}000 / (1 + \underbrace{0.05}_{f})^6 = \$18{,}655$$

$$\mathbf{PW} = 18{,}655(P/F,\underbrace{10\%}_{i},6)$$

$$\mathbf{= \$10{,}530}$$

(b) Leave as future (then-current) dollars and ***use i_f*** in PW equation

$$i_f = 0.10 + 0.05 + (0.10)(0.05) = 15.5\%$$

$$\mathbf{PW} = 25{,}000(P/F,\underbrace{15.5\%}_{i_f},6)$$

$$\mathbf{= \$10{,}530}$$

Example: PW with Inflation

A 15-year \$50,000 bond that has a dividend rate of 10% per year, payable semiannually, is currently for sale. If the expected rate of return of the purchaser is 8% per year, compounded semiannually, and if the inflation rate is expected to be 2.5% each 6-month period, what is the bond worth now (*a*) without an adjustment for inflation, and (*b*) when inflation is considered? Show both hand and spreadsheet solutions.

Solution by Hand

(*a*) *Without inflation adjustment*: The semiannual dividend is $I = [(50{,}000)(0.10)]/2 = \$2500$.
At a nominal 4% per 6 months for 30 periods,

$(8/2)\%$

$$\text{PW} = 2500(P/A{,}4\%{,}30) + 50{,}000(P/F{,}4\%{,}30) = \$58{,}645$$

(*b*) *With inflation*: Use the inflated rate i_f.

$$i_f = \overset{i}{0.04} + \overset{f}{0.025} + (\overset{i}{0.04})(\overset{f}{0.025}) = 0.066 \text{ per semiannual period}$$

$$\begin{aligned}\text{PW} &= 2500(P/A{,}6.6\%{,}30) + 50{,}000(P/F{,}6.6\%{,}30)\\ &= 2500(12.9244) + 50{,}000(0.1470)\\ &= \$39{,}660\end{aligned}$$

	A	B	C
1		Function	PW value, \$
2	(a) PW without inflation	`= PV(4%,30,-2500,-50,000)	58,646
3			
4	(b) PW with inflation adjustment	`= PV(6.6%,30,-2500,-50,000)	39,660
5			
6			

FW Calculations with Inflation

FW values can have *four different* interpretations

(1) The *actual amount accumulated*

✓ Use i_f in FW equation → **FW = PW(F/P, i_f, n)**

(2) The *purchasing power* in terms of CV dollars *of the future amount*

✓ Use i_f in FW equation and divide by $(1+f)^n$ or **use real *i***

where real $i = (i_f - f)/(1 + f)$ → **FW = PW(F/P,i,n)**

(3) The *number of future dollars required to have the same purchasing power* as a dollar today with no time value of money considered

✓ Use *f* instead of *i* in F/P factor → **FW = PW(F/P,f,n)**

(4) The amount required to *maintain the purchasing power of the present sum **and** earn a stated real rate of return*

✓ Use i_f in FW equation → **FW = PW(F/P, i_f, n)**

Example: FW with Inflation

$i_f = 0.08 + 0.05 + 0.004 = 0.134 = 13.4\ \%$

An engineer invests $15,000 in a savings account that pays interest at a real 8% per year. i If the inflation rate is 5% per year, f determine (a) the amount of money that will be accumulated in 10 years, (b) the purchasing power of the accumulated amount (in terms of today's dollars), (c) the number of future dollars that will have the same purchasing power as the $15,000 today, and (d) the amount to maintain purchasing power and earn a real 8% per year return.

Solution:

(a) The *amount accumulated* is a function of the *market interest rate*, i_f

$$i_f = 0.08 + 0.05 + (0.08)(0.05) = 13.4\%$$

i_f

Amount Accumulated = 15,000(F/P,13.4%,10)

= **$52,750**

Example: FW with Inflation (cont'd)

(b) To find the *purchasing power* of the accumulated amount *deflate* the inflated dollars

use i

Purchasing power = 15,000(F/P,13.4%,10) / $(1 + 0.05)^{10}$ = 15000 (F/P, 8%, 10)
= **$32,384**

(c) The number of future dollars required to purchase goods that cost $15,000 now is the inflated cost of the goods

use f

Number of future dollars = 15,000(F/P,5%,10)
= **$24,434**

(d) In order to maintain purchasing power *and* earn a real return, money must grow by the inflation rate ***and*** the interest rate, or i_f = 13.4%, as in part (a)

use i_f

FW = 15,000(F/P,13.4%,10)
= **$52,750**

Capital Recovery with Inflation

↳ investment - salvage value.

The A/P and A/F factors require the use of i_f when inflation is considered

AW → PMT → if

If a small company invests $150,000 in a new production line machine, how much must it receive each year to recover the investment in 5 years? The real interest rate is 10% and the inflation rate is 4% per year.

i f

Solution: Capital recovery (CR) is the AW value

$$i_f = 0.10 + 0.04 + (0.10)(0.04) = 14.4\%$$

CR = AW = 150,000(A/P,14.4%,5)

= $44,115 per year

Summary of Important Points

- **Inflation occurs because** value **_*of currency has changed*_**
- **Inflation** reduces **purchasing power; one unit buys less goods or services**
- **Two ways to account for inflation in economic analyses:**
  (1)Convert all cash flows into constant ***-value dollars and use i***
  (2)Leave cash flows as inflated dollars and use i_f
- During deflation, purchasing power of money is ***greater*** in future than at present
- **Future worth values can have** ***four different interpretations,*** requiring different interest rates to find FW
- ***Use*** i_f **in calculations involving A/P or A/F when inflation is considered**

HOMEWORK

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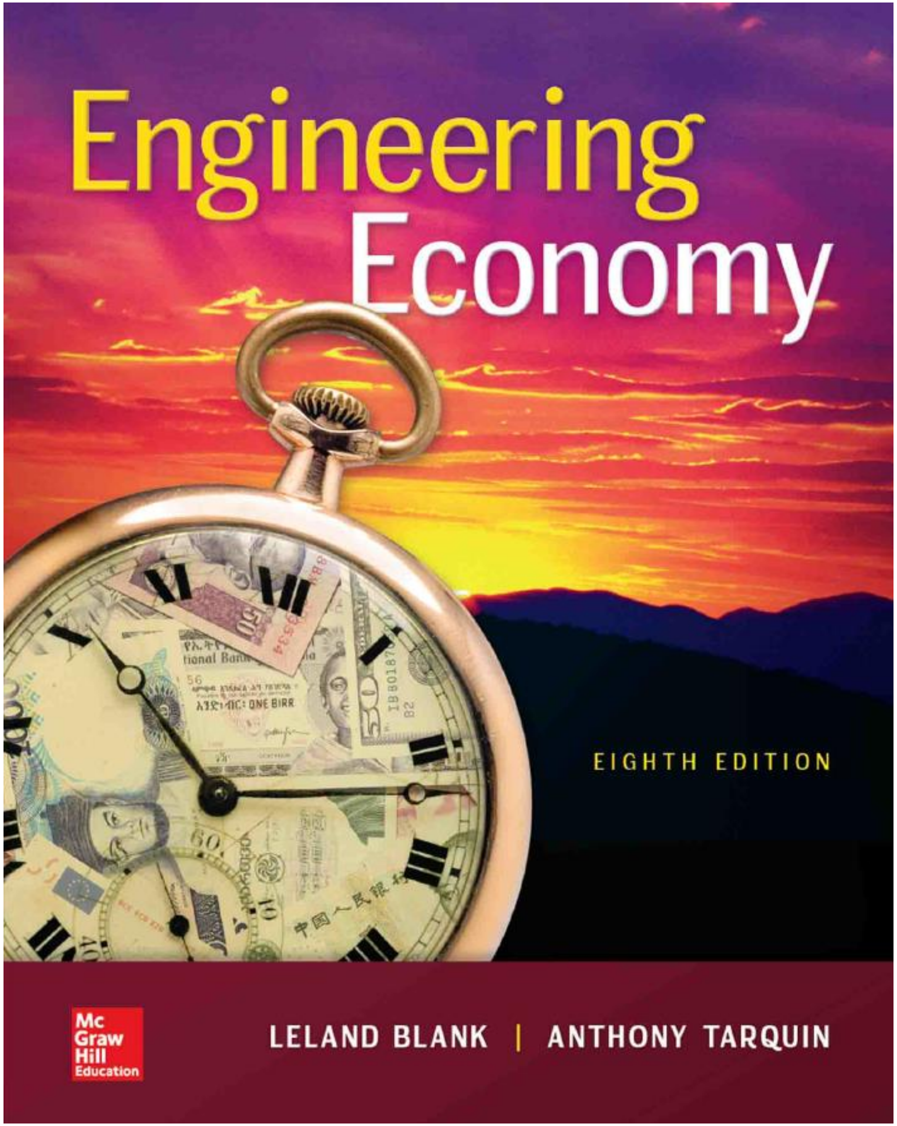
   ① 14.29
   ② 14.41
   ③ 14.56
   ④ 14.67



Chapter 16
Depreciation Methods

Lecture slides to accompany

Engineering Economy

8th edition

Leland Blank

Anthony Tarquin

Mc Graw Hill Higher Education

LEARNING OUTCOMES

1. Understand basic terms of asset depreciation
2. Apply straight line method of depreciation 정액법
3. Apply DB 정률법 and DDB methods of depreciation
4. Apply MACRS method of depreciation
5. Select asset recovery period for MACRS
6. Explain depletion 고갈 and apply cost depletion & percentage depletion methods

Depreciation Terminology

value decreasing ≠ pay money

장부

Definition: ***Book (noncash) method*** to represent **decrease in value** of a tangible asset over time

Two types: **book depreciation and tax depreciation**

Book depreciation: used for *internal accounting* to track value of assets

Tax depreciation: used to determine *taxes due* based on tax laws

In USA only, **tax depreciation** *must be calculated* ***using MACRS***; **book depreciation** *can be calculated* ***using any method***

Depreciation Terminology

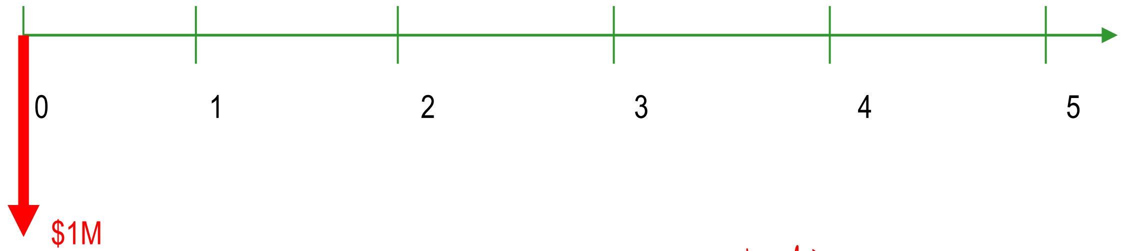
Assets		Liabilities & Owner's Equity	
Cash	$21,000	Invested Capital	$50,000
Accounts Receivable	$10,000	Retained Earning	$16,000
Tools	$35,000		
Total	$66,000	Total	$66,000

Assets		Liabilities & Owner's Equity	
Cash	$21,000	Invested Capital	$50,000
Accounts Receivable	$10,000	Retained Earning	$9,000
Tools(Tangible Asset)			($16,000 - $7,000)
	$28,000		
	($35,000 - $7,000)		
Total	$59,000	Total	$59,000

Concept

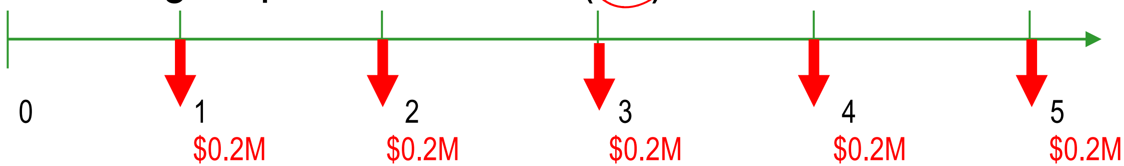
Assume A Co. purchase $1M machine that has 5 years of life with SL(or DB) depreciation.

- Actual Cash Flows



Straight line

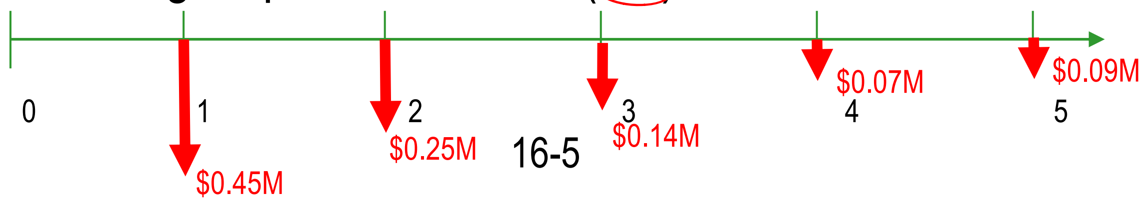
- Accounting Depreciation Cost (SL)



OR

Declining Balance

- Accounting Depreciation Cost (DB)

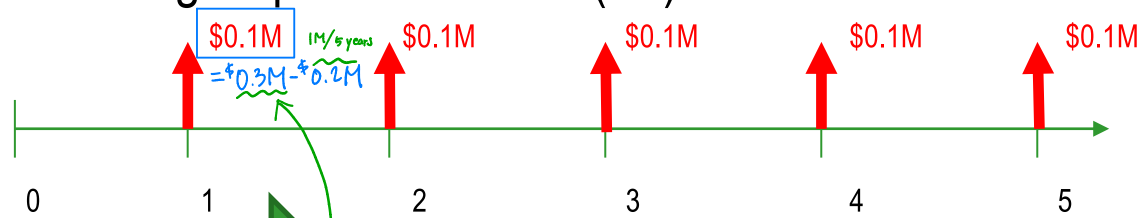


Concept (Discussed later)

Assume A Co. purchase $1M machine that has 5 years of life with SL depreciation. Assuming $\boxed{\text{NOPAT}}$ is $0.1M, compare Accounting analyses and Actual cash flows with i=10%.

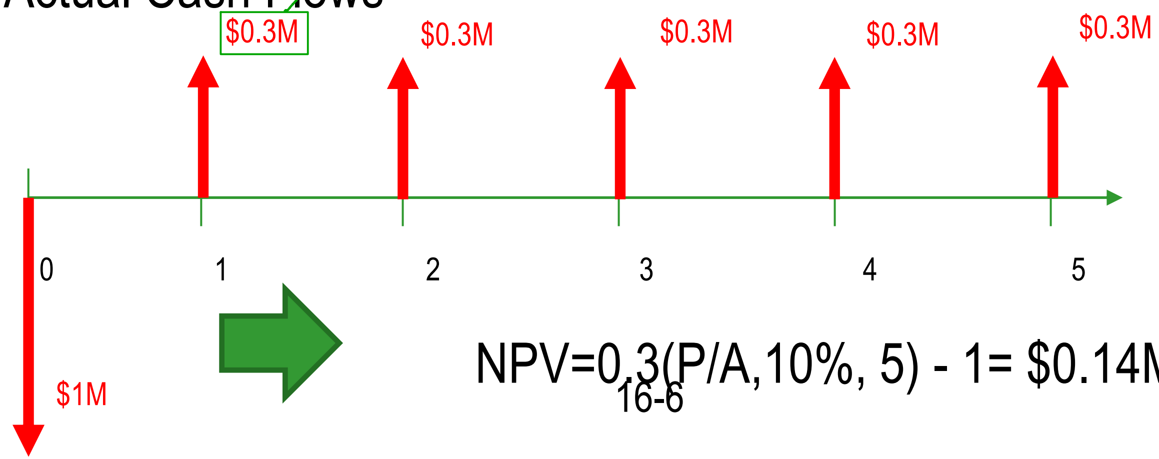
당기순이익

- Accounting Depreciation Cost (SL)



NPV=0.1(P/A,10%, 5)=$0.38M

- Actual Cash Flows



NPV=0.3(P/A,10%, 5) - 1= $0.14M

Common Depreciation Terms

No depreciation

First cost *P* or unadjusted basis *B*: Total installed cost of asset

Book value BV_t: Remaining undepreciated capital investment in year t

Recovery period *n*: Depreciable life of asset in years

Market value *MV*: Amount realizable if asset were sold on open market

Salvage value *S*: Estimated trade-in or MV at end of asset's useful life

Depreciation rate d_t: Fraction of first cost or basis removed each year t

---

Personal property: Possessions of company used to conduct business

Real property: Real estate and all improvements **(land is not depreciable)**

부동산

Half-year convention: Assumes assets are placed in service in midyear

Straight Line Depreciation

➡ **Book value decreases** ***linearly with time***

$$\boxed{D_t = \frac{B - S}{n}}$$

Where: **D_t** = annual depreciation charge
t = year
B = first cost or unadjusted basis
S = salvage value
n = recovery period

$$\boxed{BV_t = B - tD_t}$$

Where: **BV_t** = book value after t years

SL depreciation rate is **constant** for each year: $d = d_t = 1/n$

Example: SL Depreciation

An argon gas processor has a first cost of \$20,000 with a \$5,000 salvage value after 5 years. Find (a) D_3 and (b) BV_3 for year three. (c) Plot book value vs. time.

Solution: (a) $D_3 = (B - S)/n$

$= (20{,}000 - 5{,}000)/5$

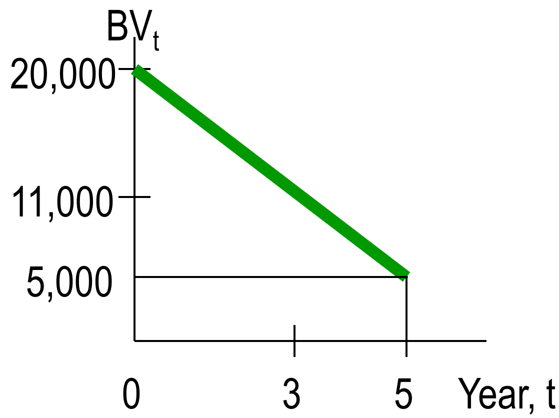
$= \$3{,}000$

(b) $BV_3 = B - tD_t$

$= 20{,}000 - 3(3{,}000)$

$= \$11{,}000$

(c) Plot BV vs. time



Declining Balance (DB) and Double Declining Balance (DDB) Depreciation

➡ **Determined by multiplying BV at beginning of year *by fixed percentage d***

$d \le 2/n$

- **Max rate for d is twice straight line ra**
- **Cannot depreciate below salvage val**

$BV_0 = B$
$BV_1 = BV_0 - BV_0 \cdot d = B(1 - d)$
$BV_2 = BV_1 - BV_1 \cdot d = B(1 - d)^2$
:
$BV_t = BV_{t-1} - BV_{t-1} \cdot d = B(1 - d)^t$

Book value for year t is given by:

$$\boxed{BV_t = B(1 - d)^t}$$

Depreciation for year t is obtained by either relation:

$$\boxed{D_t = dBV_{t-1} = \boxed{d}B(1 - d)^{t-1}}$$

Where: D_t = depreciation for year t
d = uniform depreciation rate (2/n for DDB)
B = first cost or unadjusted basis
BV_{t-1} = book value at end of previous year

Declining Balance (DB) Depreciation

➡ **Determined by multiplying B at beginning of year *by fixed percentage d where Max. d = 2/n***

Book value for year t is given by:

$$B_n = S = B(1 - d)^n$$

$$\therefore d = 1 - \sqrt[n]{\frac{S}{B}} \quad \leq \frac{2}{n}$$

Where: B_n = Book value for year n

d = uniform depreciation rate

B = first cost or unadjusted basis

Example: Double Declining Balance

A depreciable construction truck has a first cost of \$20,000 with a \$4,000 salvage value after 5 years. Find the (a) depreciation, and (b) book value after 3 years using DDB depreciation.

Solution:

(a) $d = 2/n = 2/5 = 0.4$

$$D_3 = dB(1 - d)^{t-1}$$
$$= 0.4(20{,}000)(1 - 0.40)^{3-1}$$
$$= \$2880$$

(b) $$BV_3 = B(1 - d)^t$$
$$= 20{,}000(1 - 0.4)^3$$
$$= \$4320$$

Spreadsheet Functions for Depreciation

Straight line function: SLN(B,S,n)

Declining balance function: DB(B,S,n,t)

default value : 2.

Double declining balance function: DDB(B,S,n,t,$\boxed{d}$)

$= \frac{2}{n}$

$1 - \sqrt[n]{\frac{S}{B}}$

Note: It is better to use the DDB(d$= n * depreciation\ rate$) function for DB and DDB depreciation. DDB function checks for BV < S and is more accurate than the DB function(3-digits only).

not accurate

EXAMPLE 16.3

Freeport-McMoRan Copper and Gold has purchased a new ore grading unit for \$80,000. The unit has an anticipated life of 10 years and a salvage value of \$10,000. Use the DB and DDB methods to compare the schedule of depreciation and book values for each year. Solve by hand and by spreadsheet.

	A	B	C	D	E
1					
2	First cost, B	\$ 80,000			
3	Salvage, S	\$ 10,000			
4	Rec'vry pd, n	10			
5	Depr. rate, d	0.1877476			
6		Declining balance (DB), \$		Double declining (DDB), \$	
7	Year, t	D(t)	DB book value	D(t)	DDB book value
8	0		80,000		80,000
9	1	15,020	64,980	16,000	64,000
10	2	12,200	52,780	12,800	51,200
11	3	9,909	42,871	10,240	40,960
12	4	8,049	34,822	8,192	32,768
13	5	6,538	28,284	6,554	26,214
14	6	5,310	22,974	5,243	20,972
15	7	4,313	18,661	4,194	16,777
16	8	3,503	15,157	3,355	13,422
17	9	2,846	12,311	2,684	10,737
18	10	2,311	10,000	737	10,000

= 1−(B3/B2)^0.1

= DDB(B\$2,B\$3,B\$4,\$A18 10*\$B\$5)

= DDB(B\$2,B\$3,B\$4,\$A18)

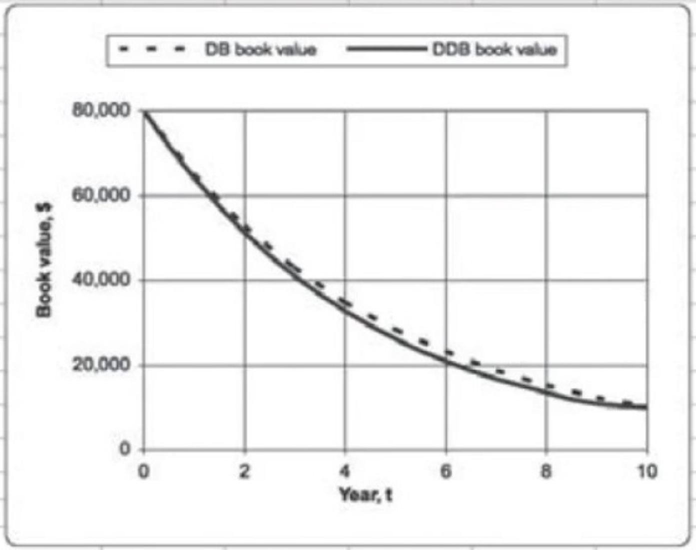


Figure 16–3

Annual depreciation and book value using DB and DDB methods, Example 16.3.

EXAMPLE 16.3

were used in column B (Figure 16–3), the fixed rate applied would be 0.188. The resulting D_t and BV_t values for years 8, 9, and 10 would be as follows:

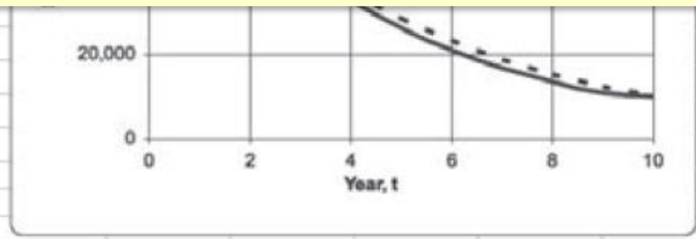
round-off error

t	D_t, \$	BV_t, \$
8	3,501	15,120
9	2,842	12,277
10	2,308	9,969

≠ 10,000

Also noteworthy is the fact that the DB function uses the implied rate without a check to halt the book value at the estimated salvage value. Thus, BV_{10} will go slightly below $S = \$10{,}000$, as shown above. However, the DDB function uses a relation different from that of the DB function to determine annual depreciation—one that correctly stops depreciating at the estimated salvage value, as shown in Figure 16–3, cells E17–E18.

15	7	4,313	18,661	4,194	16,777
16	8	3,503	15,157	3,355	13,422
17	9	2,846	12,311	2,684	10,737
18	10	2,311	10,000	737	10,000
19					
20					
21					
22					
23					
24					



= DDB(B$2,B$3,B$4,$A18,10*B5)

= DDB(B$2,B$3,B$4,$A18)

Figure 16–3

Annual depreciation and book value using DB and DDB methods, Example 16.3.

MACRS Depreciation

Required method to use for tax depreciation in **USA only**

Originally developed to offer accelerated depreciation for economic growth

$$D_t = d_t B$$

Where: D_t = depreciation charge for year t
B = first cost or unadjusted basis
d_t = depreciation rate for year t (decimal)

Get value for d_t from IRS table for MACRS rates

$$BV_t = B - \sum_{j=1}^{j=t} D_j$$

Where: D_j = depreciation in year j
$\sum D_j$ = all depreciation through year t

MACRS Depreciation

- Always depreciates to **zero**; no salvage value considered
- Incorporates **switching from DDB to SL** depreciation
- ***Standardized recovery periods*** (n) are tabulated
- MACRS recovery time is always **n+1 years**; **half-year convention** assumes purchase in midyear
- No special spreadsheet function; can arrange **VDB function** to display MACRS depreciation each year

Example: MACRS Depreciation

A finishing machine has a first cost of \$20,000 with a \$5,000 salvage value after 5 years. Using MACRS, find (a) D and (b) BV for year 3.

Solution: (a) From table, $d_3 = 19.20$

$$D_3 = 20{,}000(0.1920)$$
$$= \$3{,}840$$

(b) $BV_3 = 20{,}000 - 20{,}000(\overset{d_1}{0.20} + \overset{d_2}{0.32} + \overset{d_3}{0.1920})$

$$= \$5{,}760$$

Note: Salvage value **S = \$5,000** is **not** used by MACRS and $BV_{\boxed{6}} = 0$

$= 5+1$ years

EXAMPLE 16.4

Chevron Phillips Chemical Company in Baytown, Texas, acquired new equipment for its polyethylene processing line. This chemical is a resin used in plastic pipe, retail bags, blow molding, and injection molding. The equipment has an unadjusted basis of $B = \$400{,}000$, a life of only 3 years, and a salvage value of 5% of B. The chief engineer asked the finance director to provide an analysis of the difference between (1) the DDB method, which is the internal book depreciation and book value method used at the plant, and (2) the required MACRS tax depreciation and its book value. He is especially curious about the differences after 2 years of service for this short-lived, but expensive asset. Use hand and spreadsheet solutions to do the following:

(*a*) Determine which method offers the larger total depreciation after 2 years.
(*b*) Determine the book value for each method after 2 years and at the end of the recovery period.

cf) Chapter 16 Depreciation Example.xlsx

TABLE 16–3 Comparing MACRS and DDB Depreciation, Example 16.4

		MACRS		DDB	
Year	**Rate**	**Tax Depreciation, $**	**Book Value, $**	**Book Depreciation, $**	**Book Value, $**
0			400,000		400,000
1	0.3333	133,320	266,680	D_1 266,667	133,333
2	0.4445	177,800	88,880	D_2 88,889	44,444
3	0.1481	59,240	29,640	D_3 24,444	20,000
4	0.0741	29,640	0		

$$\begin{cases} d = 2/3 \\ D_t = dB(1-d)^{t-1} \end{cases}$$

MACRS Recovery Period

Recovery period (n) is function of ***property class***

Two systems for determining recovery period:

- **general depreciation system (GDS)** – **fastest write-off allowed**
- **alternative depreciation system (ADS)** – **longer recovery; uses SL**

IRS publication 946 gives n values for an asset. For example:

Asset description	MACRS n value GDS	MACRS n value ADS range
Special manufacturing devices, racehorses, tractors	3	3 - 5
Computers, oil drilling equipment, autos, trucks, buses	5	6 - 9.5
Office furniture, railroad car, **property not in another class**	7	10 – 15
✓ ✓ ✓ ✓	✓	✓
Nonresidential real property (not land itself)	39	40

Depletion Methods

Depletion: **book (noncash) method to represent decreasing value of *natural resources***

Two methods: ***cost*** depletion (CD) and ***percentage*** depletion (PD)

for cost for revenue

Cost depletion: **Based on level of activity to remove a natural resource**

- Calculation: Multiply factor CD_t by amount of resource removed

  Where: $\mathbf{CD_t}$ **= first cost / resource capacity**

- Total depletion can not exceed first cost of the resource

Percentage depletion: **Based on gross income (GI) from resource**

- Calculation: Multiply GI by standardized rate (%) from table
- Annual depletion can not exceed 50% of company's taxable income (TI)

Example: Cost and Percentage Depletion

A mine purchased for $3.5 million has a total expected yield of one million ounces of silver. Determine the depletion charge in year 4 when 300,000 ounces are mined and sold for $30 per ounce using (a) cost depletion, and (b) percentage depletion. (c) Which is larger for year 4?

Solution: Let depletion amounts equal CDA_4 and PDA_4

(a) Factor, CD_4 = 3,500,000/ 1,000,000 = \$3.50 per ounce

CDA_4 = 3.50(300,000) = **\$1,050,000**

from the table

(b) Percentage depletion rate for silver mines is 0.15

PDA_4 = (0.15)(300,000)(30) = **\$1,350,000**

(c) Claim **percentage depletion** amount, provided it is ≤ 50% of TI

Summary of Important Points

MACRS SL DB DDB

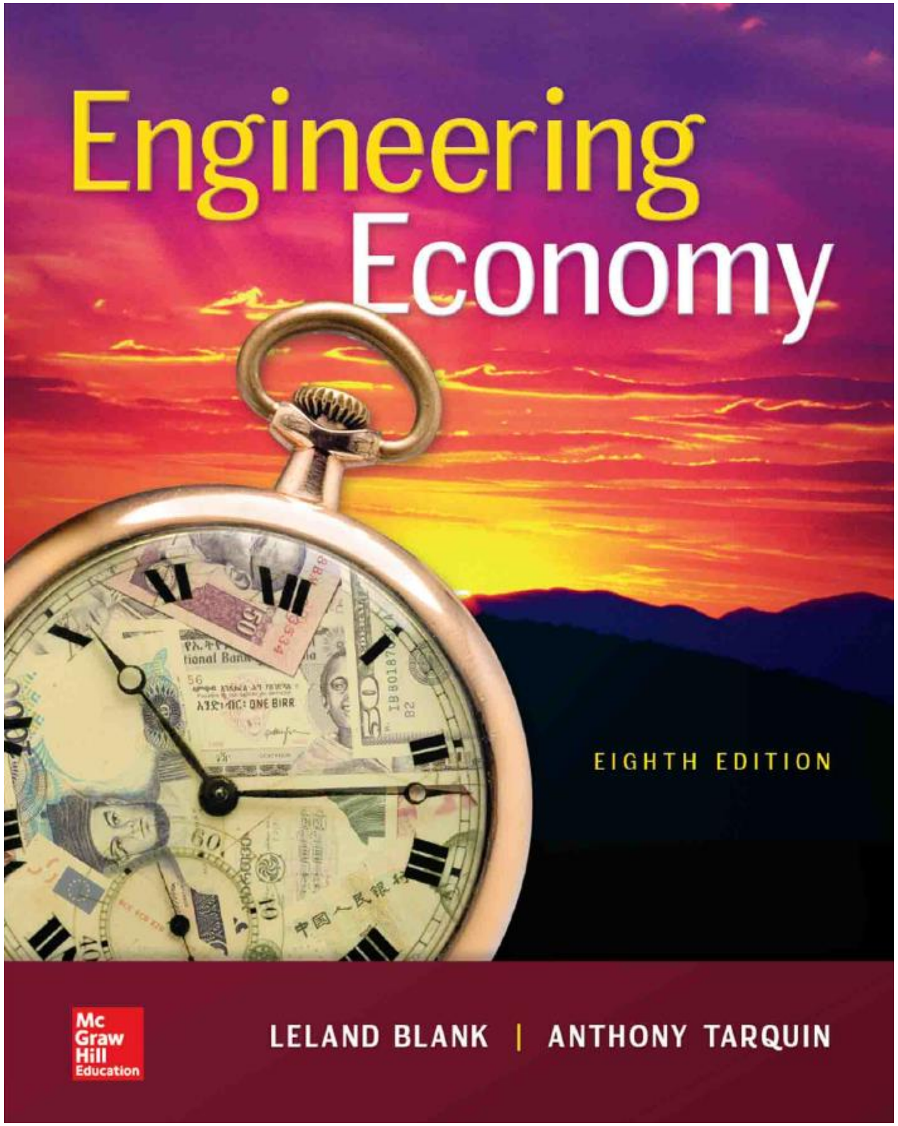
- **Two types** for depreciation: **tax and book**
- Classical methods are **straight line** and **declining balance**
- In USA only, **MACRS** method is ***required*** for ***tax depreciation***
- Determine MACRS recovery period using either **GDS** or **ADS**
- Switching between methods is allowed; MACRS switches automatically from DDB to SL to maximize write-off
- **Depletion** (instead of depreciation) used for ***natural resources***

Two methods *of depletion:* ***cost*** *(amount resource removed* $\times$ CD_t *factor) and* ***percentage*** *(gross income* $\times$ *tabulated %)*

HOMEWORK

1. Please solve every Examples in your textbook. You do not have to submit your works.

2. No Exercise Homework.



Chapter 17

After-tax Economic Analysis

Lecture slides to accompany

Engineering Economy

8th edition

Leland Blank

Anthony Tarquin

McGraw Hill Higher Education

LEARNING OBJECTIVES

1. Terminology and rates; marginal tax tables
2. Determining cash flows before taxes(CFBT) and after taxes (CFAT)
3. Effects of depreciation on taxes
4. Depreciation recapture and capital gains
5. Performing an after-tax analysis
6. Performing after-tax replacement studies
7. Economic value-added analysis
8. Tax structures outside the United States
9. Understanding value-added tax (VAT)

Tax Terms and Relations - Corporations

- Gross Income ***GI*** or operating revenue ***R*** -- Total income for the tax year realized from all revenue producing sources

매출원가

- Operating expenses ***OE*** -- All annual operating costs (AOC) and maintenance & operating (M&O) costs incurred in transacting business; these are tax deductible; **depreciation not included** here

세금공제

- Income Taxes and tax rate ***T*** -- Taxes due annually are based on taxable income *TI* and tax rates, which are commonly **graduated** (or **progressive**) by TI level.

누진세

$$\text{Taxes} = \text{tax rate} \times \boxed{\text{taxable income}}$$
$$= T \times \boxed{(GI - OE - \boxed{D})}$$

depreciation cost : tax deductible.

당기순이익

- Net operating profit after taxes ***NOPAT*** – Money remaining as a result of capital invested during the year; amount left after taxes are paid.

$$\text{NOPAT} = \text{taxable income} - \text{taxes} = TI - T \times (TI)$$
$$= TI \times (1 - T)$$

Income Tax Terms and Relations (Corporations)

Income taxes are real cash flow payments to governments levied against income and profits. The (noncash) allowance of asset depreciation is used in income tax computations.

Two fundamental relations: NOI and TI

Net operating income = gross revenue – operating expenses

영업이익

$$NOI = GI - OE$$

(only actual cash involved)

NOI is also called **EBIT(DA)** (earnings before interest, taxes and depreciation)

Taxable income = gross revenue – operating expenses – depreciation

$$TI = GI - OE - D$$

(involves noncash item)

Note: **All terms and relations are calculated for each year t, but the subscript is often omitted for simplicity**

Profit and Loss Statements	
Gross Income (GI)	3,000,000
Operating Expense (OE)	2,000,000
Net Operating Income (NOI) = GI-OE	1,000,000 = 3,000,000-2,000,000
Depreciation (D)	150,000
Taxable Income (TI) = GI-OE-D	850,000 = 1,000,000-150,000
Tax = TI x tax rate.	200,000
Net Operating Profit After Tax (NOPAT) TI x (1-Tax rate) = TI-Tax	650,000 = 850,000-200,000

cash

non cash

not actual spending

Actual amount of cash in pocket = 650,000 + $\underbrace{150,000}_{\text{non cash}}$

Average and Effective Tax Rates

Marginal tax rates change as TI increases. Calculate an **average tax rate** using:

$$\text{Average tax rate} = \frac{\text{total taxes paid}}{\text{taxable income}} = \boxed{\frac{\text{taxes}}{\text{TI}}}$$

To approximate a **single-figure tax rate** that combines local (e.g., state) and federal rates calculate the **effective tax rate T_e**

$$T_e = \text{local rates} + (1 - \text{local rates}) \times \text{federal rate}$$

Then, $$\textbf{Taxes} = T_e \times TI$$

See Example 17.1

Rate of Federal Taxes (누진세)

TABLE 17–1 U.S. Corporate Income Tax Rate Schedule

If Taxable Income ($) Is:			
Over	**But Not Over**	**Tax Is**	**Of the Amount Over**
0	50,000	15%	0
50,000	75,000	7,500 + 25%	50,000
75,000	100,000	13,750 + 34%	75,000
100,000	335,000	22,250 + 39%	100,000
335,000	10,000,000	113,900 + 34%	335,000
10,000,000	15,000,000	3,400,000 + 35%	10,000,000
15,000,000	18,333,333	5,150,000 + 38%	15,000,000
18,333,333	—	35%	0

50,000 × 15%

e.g., TI = 60,000
Taxes = 7500 + (60000 − 50,000) × 25%
= 10,000

7500 + 25000 × 25%
=

EX 17.1

EXAMPLE 17.1

REI (Recreational Equipment Incorporated) sells outdoor equipment and sporting goods through retail outlets, the Internet, and catalogs. Assume that for 1 year REI has the following financial results in the state of Kentucky, which has a flat tax rate of 6% on corporate taxable income.

Total revenue R or GI	\$19.9 million
Operating expenses OE	\$8.6 million
Depreciation and other allowed deductions D	\$1.8 million

(*a*) Determine the state taxes and federal taxes due using Table 17–1 rates.
(*b*) Find the average federal tax rate paid for the year.
(*c*) Determine a single-value tax rate useful in economic evaluations using the average federal tax rate determined in part (*b*).
(*d*) Estimate federal and state taxes using the single-value rate, and compare their total with the total in part (*a*).

Solution

(*a*) Calculate TI by Equation [17.2] and use Table 17–1 rates for federal taxes due.

$$\begin{aligned}
\text{Kentucky state TI} &= \text{GI} - \text{OE} - \text{D} = 19.9 \text{ million} - 8.6 \text{ million} - 1.8 \text{ million} \\
&= \$9.5 \text{ million} \\
\text{Kentucky state taxes} &= 0.06(\text{TI}) = 0.06(9{,}500{,}000) = \$570{,}000 \\
\text{Federal TI} &= \text{GI} - \text{OE} - \text{D} - \text{state taxes} = 9{,}500{,}000 - 570{,}000 \\
&= \$8{,}930{,}000 \\
\text{Federal taxes} &= 113{,}900 + 0.34(8{,}930{,}000 - 335{,}000) = \$3{,}036{,}200 \\
\text{Total federal and state taxes} &= 3{,}036{,}200 + 570{,}000 = \boxed{\$3{,}606{,}200} \qquad [17.8]
\end{aligned}$$

(*b*) From Equation [17.5], the average tax rate paid is approximately 32% of TI.

$$\text{Average federal tax rate} = 3{,}036{,}200/9{,}500{,}000 = \boxed{0.3196}$$

(*c*) By Equation [17.6], T_e is slightly over 36% per year for combined state and federal taxes.

$$T_e = 0.06 + (1 - 0.06)(0.3196) = \boxed{0.3604} \quad (36.04\%)$$

(*d*) Use the effective tax rate and TI = \$9.5 million from part (*a*) in Equation [17.7] to approximate total taxes.

$T_e \times TI$

$$\text{Taxes} = 0.3604(9{,}500{,}000) = \boxed{\$3{,}423{,}800}$$

Compared to Equation [17.8], this approximation is $\boxed{\$182{,}400}$ low, a $\boxed{5.06\%}$ underestimate.

difference difference

Cash Flow After Taxes (CFAT)

- NCF is cash inflows – cash outflows. Now, **consider taxes and deductions**, such as **depreciation**
- Cash Flow Before Taxes (CFBT)

CFBT = gross income – expenses – initial investment + salvage value

$$= GI - OE - P + S$$

- Cash Flow After Taxes (CFAT)

CFAT = CFBT – taxes

$$= GI - OE - P + S - \underbrace{(GI - OE - D)}_{\text{A negative TI value is considered as tax savings for the project}}(T_e)$$

A **negative TI** value is considered as **tax savings** for the project

The **negative** tax will **offset** taxes for the same year in other income-producing areas of the corporation
OR Carry-forward and carry-back rules

See Example 17.2

TABLE 17–2 Suggested Column Headings for Calculation of CFAT

Year	Gross Income GI	Operating Expenses OE	Investment and Salvage *P* and *S*	CFBT	Depreciation *D*	Taxable Income TI	Taxes	CFAT
	(1)	(2)	(3)	(4) = (1) - (2) + (3)	(5)	(6) = (1) - (2) − (5)	(7) = T_e(6)	(8) = (4) − (7)

EXAMPLE 17.2

Wilson Security has received a contract to provide additional security for corporate and government personnel along the international border between two countries in South America. Wilson plans to purchase listening and detection equipment for use in the 6-year contract. The equipment is expected to cost $550,000 and have a resale value of $150,000 after 6 years. Based on the incentive clause in the contract, Wilson estimates that the equipment will increase contract revenue by $200,000 per year and require an additional M&O expense of $90,000 per year. MACRS depreciation allows recovery in 5 years, and the effective corporate tax rate is 35% per year. Tabulate and plot the CFBT and CFAT series.

감가상각 재평가

Depreciation Recapture (DR) and Capital Gain (CG)

DR, also called **ordinary gain**, in year t occurs when an asset is **sold for <u>more than</u> its BV_t** Book Value at year t

$$DR = \text{selling price} - \text{book value} = SP - BV_t$$

CG occurs when an asset is **sold for <u>more than</u> its unadjusted basis *B*** (or first cost *P*)

$$CG = \text{selling price} - \text{basis} = SP - B$$

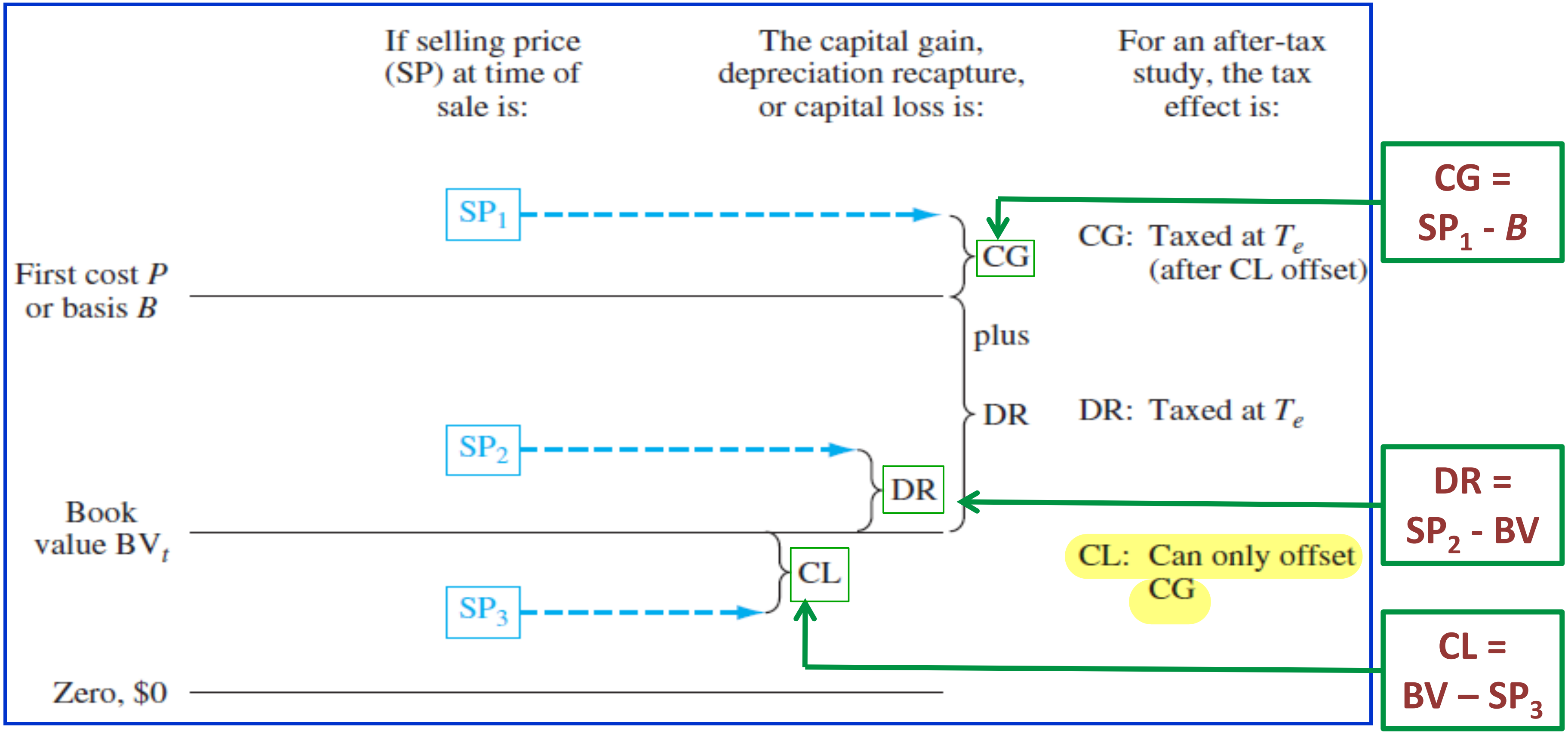
구매가격 (원가?)

CL occurs when an asset is **sold for <u>less than</u> its current BV_t**

capital loss

$$CL = \text{book value} - \text{selling price} = BV_t - SP$$

Effects of DR, CG and CL on TI and Taxes



Update of TI relation: $TI = GI - OE - D + DR + \text{net } CG - \text{net } CL$

After-Tax Evaluation

- ✓ **Use CFAT values to calculate PW, AW, FW, ROR, B/C or other measure of worth using after-tax MARR**
- ✓ Same guidelines as before-tax; e.g., using PW at after-tax MARR:
  - **One project**: PW ≥ 0, project is viable
  - **Two or more alternatives**: select one ME alternative with best (numerically largest) PW value
- ✓ For costs-only CFAT values, use + sign for OE, D, and other savings and use same guidelines
- ✓ Remember: **equal-service requirement** for PW-based analysis
- ✓ ROR analysis is same as before taxes, except use CFAT values:
  - **One project**: if i* ≥ after-tax MARR, project is viable
  - **Two alternatives**: select ME alternative with Δi* ≥ after-tax MARR for incremental CFAT series

Approximating After-Tax ROR Value

To adjust a before-tax ROR without details of after-tax analysis, an **approximating** relation is:

After-tax ROR ≈ before-tax ROR × $(1 - T_e)$

Example: P = \$-50,000 GI – OE = \$20,000/year

n = 5 years D = \$10,000/year $T_e = 0.40$

Estimate after-tax ROR from before-tax ROR analysis

Solution: Set up **before-tax PW** relation and solve for i*

$$0 = -50{,}000 + 20{,}000(P/A, i^*\%, 5)$$

i* = 28.65%

After-tax ROR ≈ 28.65% × (1 – 0.40) = 17.19%

(Note: Actual after-tax analysis results in i* = 18.03%)

Example: After-Tax Analysis

Asset: B = $90,000 S = 0 n = 5 years

Per year: R = $65,000 OE = $18,500 D = $18,000

Effective tax rate: $T_e = 0.184$

Find ROR (a) before-taxes, (b) after-taxes actual and (c) approximation

	A	B	C	D	E	F	G	H	I
1		Revenue,	Operating	Basis, B and		Depreciation,	Taxable	Taxes	
2	Year	R	Expenses, OE	Salvage, S	CFBT	D	Income, TI	at T_e = 0.184	CFAT
3	0			90,000	-90,000				-90,000
4	1	65,000	18,500		46,500	18,000	28,500	5,244	41,256
5	2	65,000	18,500		46,500	18,000	28,500	5,244	41,256
6	3	65,000	18,500		46,500	18,000	28,500	5,244	41,256
7	4	65,000	18,500		46,500	18,000	28,500	5,244	41,256
8	5	65,000	18,500	0	46,500	18,000	28,500	5,244	41,256
9									
10			(a) Before-tax ROR:		43%		(b) After-tax ROR:		36%

Solution: (a) Using IRR function, i* = 43% (b) Using IRR function, i* = 36%

(c) By approximation: **after-tax ROR = 43% × (1 – 0.1840) = 35%**

EXAMPLE 17.5

Biotech, a medical imaging and modeling company, must purchase a bone cell analysis system for use by a team of bioengineers and mechanical engineers studying bone density in athletes. This particular part of a 3-year contract with the NBA will provide additional gross income of $100,000 per year. The effective tax rate is 35%. Estimates for two alternatives are summarized below.

	Analyzer 1	**Analyzer 2**
Basis *B*, $	150,000	225,000
Operating expenses, $ per year	30,000	10,000
MACRS recovery, years	5	5

Answer the following questions, solving by hand and spreadsheet.

(*a*) The Biotech president, who is very tax conscious, wishes to use a criterion of minimizing total taxes incurred over the 3 years of the contract. Which analyzer should be purchased?

(*b*) Assume that 3 years have now passed, and the company is about to sell the analyzer. Using the same total tax criterion, did either analyzer have an advantage? Assume the selling price is $130,000 for analyzer 1, or $225,000 for analyzer 2.

TABLE 17–4 Comparison of Total Taxes for Two Alternatives, Example 17.5*a*

Year	Gross Income GI, $	Operating Expenses OE, $	Basis *B*, $	MACRS Depreciation *D*, $	Book Value BV, $	Taxable Income TI, $	Taxes at 0.35TI, $
				Analyzer 1			
0			150,000		150,000		
1	100,000	30,000		30,000	120,000	40,000	14,000
2	100,000	30,000		48,000	72,000	22,000	7,700
3	100,000	30,000		28,800	43,200	41,200	14,420
							36,120
				Analyzer 2			
0			225,000		225,000		
1	100,000	10,000		45,000	180,000	45,000	15,750
2	100,000	10,000		72,000	108,000	18,000	6,300
3	100,000	10,000		43,200	64,800	46,800	16,380
							38,430

	A	B	C	D	E	F	G	H
1								
2								
3	Year	GI	OE	B and S	D	BV	TI	Taxes
4				**Analyzer 1**				
5	0			-150,000		150,000		
6	1	100,000	-30,000		30,000	120,000	40,000	14,000
7	2	100,000	-30,000		48,000	72,000	22,000	7,700
8	3	100,000	-30,000		28,800	43,200	41,200	14,420
9	Total							36,120
10	**Revised 3**	**100,000**	**-30,000**	**130,000**	**28,800**	**43,200**	**128,000**	**44,800**
11	**New Total**							**66,500**
12								
13				**Analyzer 2**				
14	0			-225,000		225,000		
15	1	100,000	-10,000		45,000	180,000	45,000	15,750
16	2	100,000	-10,000		72,000	108,000	18,000	6,300
17	3	100,000	-10,000		43,200	64,800	46,800	16,380
18	Total							38,430
19	**Revised 3**	**100,000**	**-10,000**	**225,000**	**43,200**	**64,800**	**207,000**	**72,450**
20	**New Total**							**94,500**

= B6+C6−E6

= B10+C10−E10+(D10−F10)

Last term calculates the depreciation recapture

Total tax advantage of analyzer 1 over 2 without sale and with sale in year 3

Figure 17–4

Impact of depreciation recapture on total taxes, Example 17.5.

EXAMPLE 17.9

In Example 17.5 an after-tax analysis of two bone cell analyzers was initiated due to a new 3-year NBA contract. The criterion used to select analyzer 1 was the total taxes for the 3 years. The complete solution is in Table 17–4 (hand) and Figure 17–4 (spreadsheet).

Continue the spreadsheet analysis by performing an after-tax ROR evaluation, assuming the analyzers are sold after 3 years for the amounts estimated in Example 17.5: $130,000 for analyzer 1 and $225,000 for analyzer 2. The after-tax MARR is 10% per year.

	A	B	C	D	E	F	G	H	I	J	K
1	Year	GI	OE	B and S	D	BV	TI	Taxes	CFAT		
2					Analyzer 1						
3	0			-150,000		150,000			-150,000		
4	1	100,000	-30,000		30,000	120,000	40,000	14,000	56,000		
5	2	100,000	-30,000		48,000	72,000	22,000	7,700	62,300		
6	3	100,000	-30,000	130,000	28,800	43,200	128,000	44,800	155,200		
7	i*							IRR→	30.2%	Overall i*	
8	PW at 10%								$69,001		
9										Incremental	$A_2 - A_1$
10					Analyzer 2					CFAT	
11	0			-225,000		225,000			-225,000	-75,000	
12	1	100,000	-10,000		45,000	180,000	45,000	15,750	74,250	18,250	
13	2	100,000	-10,000		72,000	108,000	18,000	6,300	83,700	21,400	
14	3	100,000	-10,000	225,000	43,200	64,800	207,000	72,450	242,550	87,350	
15	i*							IRR→	27.9%		
16	PW at 10%								$93,905		MARR after tax
17	Incr i*									23.6%	> 10%
18											
19											
20											
21											
22											

= B14+C14−E14+(D14−F14)

This term calculates DR = $160,200

CFAT calculation
= B14+C14+D14−H14

Incremental i*
= IRR(J11:J14)

Figure 17–7
Incremental ROR analysis of CFAT with depreciation recapture, Example 17.9.

PW → Analyzer 2
RoR → 〃

After-Tax Replacement Analysis

- Consider depreciation recapture (DR) or capital gain (CG), if challenger is selected over defender
- Can include capital loss, if trade occurs at very low (‘sacrifice’) trade-in for defender
- An after-tax analysis can reverse the selection compared to before-tax analysis, but more likely it will provide information about differences in PW, AW or ROR value when taxes are included
- Apply same procedure as before-tax replacement evaluation once CFAT series is estimated

EXAMPLE 17.10

Midcontinent Power Authority purchased emission control equipment 3 years ago for $600,000. Management has discovered that it is technologically and legally outdated now. New equipment has been identified. If a market value of $400,000 is offered as the trade-in for the current equipment, perform a replacement study using (*a*) a before-tax MARR of 10% per year and (*b*) a 7% per year after-tax MARR. Assume an effective tax rate of 34%. As a simplifying assumption, use classical straight line depreciation with $S = 0$ for both alternatives.

	Defender	Challenger
Market value, $	400,000	
First cost, $		−1,000,000
Annual cost, $/year	−100,000	−15,000
Recovery period, years	8 (originally)	5

Select Defender

Defender Age	Year	OE	P or S	CFBT	Depre.C.	TI GI+OE−D	Tax TI × 34%	CFAT CFBT−TAX	
Defender									
3	0								
4	1	- 100,000		- 100,000	$600,000 / 8 years $\boxed{75,000}$	- 175,000	- 59,500	- 40,500	
5	2	- 100,000		- 100,000	75,000	- 175,000	- 59,500	- 40,500	
6	3	- 100,000		- 100,000	75,000	- 175,000	- 59,500	- 40,500	
7	4	- 100,000		- 100,000	75,000	- 175,000	- 59,500	- 40,500	
8	5	- 100,000		- 100,000	75,000	- 175,000	- 59,500	- 40,500	
AW at 10%				- 100,000				- 40,500	AW at 7%
Challenger									
	0		$\boxed{-600,000}$	- 600,000		$\boxed{25,000}$	8,500	- 608,500	
	1	- 15,000		- 15,000	$1M / 5 years $\boxed{200,000}$	- 215,000	- 73,100	58,100	
	2	- 15,000		- 15,000	200,000	- 215,000	- 73,100	58,100	
	3	- 15,000		- 15,000	200,000	- 215,000	- 73,100	58,100	
	4	- 15,000		- 15,000	200,000	- 215,000	- 73,100	58,100	
	5	- 15,000		- 15,000	200,000	- 215,000	- 73,100	58,100	
AW at 10%				- 173,278				- 102,421	AW at 7%

Defender is sold.

Defender market value.

400,000 − 1,000,000

challenger first cost.

Selling Price : 400,−

DR : 25,−

Book Value : 375,−

DR

34%

* defender first cost

year

depreciation

(600,000) − 3×(75,000)

Example 17.10: Before- and After-Tax Replacement Study

	Defender	Challenger
Market value, $	400,000	
First cost, $		−1,000,000
Annual cost, $/year	−100,000	−15,000
Recovery period, years	8 (originally)	5

Purchased 3 years ago
Before-tax MARR = 10%
After-tax MARR = 7%
T_e = 34%
SL depreciation; S = 0

SOLUTION

		Before Taxes			After Taxes			
Defender Age	Year	Expenses OE, $	*P* and *S*, $	CFBT, $	Depreciation *D*, $	Taxable Income TI, $	Taxes* at 0.34TI, $	CFAT, $
				Defender				
3	0		−400,000	−400,000				−400,000
4	1	−100,000		−100,000	75,000	−175,000	−59,500	−40,500
5	2	−100,000		−100,000	75,000	−175,000	−59,500	−40,500
6	3	−100,000		−100,000	75,000	−175,000	−59,500	−40,500
7	4	−100,000		−100,000	75,000	−175,000	−59,500	−40,500
8	5	−100,000	0	−100,000	75,000	−175,000	−59,500	−40,500
AW at 10%				−205,520	AW at 7%			−138,056
				Challenger				
	0		−1,000,000	−1,000,000		+25,000†	8,500	−1,008,500
	1	−15,000		−15,000	200,000	−215,000	−73,100	+58,100
	2	−15,000		−15,000	200,000	−215,000	−73,100	+58,100
	3	−15,000		−15,000	200,000	−215,000	−73,100	+58,100
	4	−15,000		−15,000	200,000	−215,000	−73,100	+58,100
	5	−15,000	0	−15,000	200,000	−215,000‡	−73,100	+58,100
AW at 10%				−278,800	AW at 7%			−187,863

* Minus sign indicates a tax savings for the year.
† Depreciation recapture on defender trade-in.
‡ Assumes challenger's salvage actually realized is S = 0; no tax.

Select defender both ways

Economic Value Added (EVA)™ Analysis

부가가치

- **Definition:** The **economic worth added** by a product or service from the perspective of the consumer, owner or investor

- In other words, it is the contribution of a capital investment to the net worth of a corporation **after taxes**

Example: **The average consumer is willing to pay significantly more for potatoes processed and served at a fast-food restaurant as fries (chips) than as raw potatoes in the skin from a supermarket.**

- Value-added analysis is performed in a different way than CFAT analysis, however…

- Selection of the better economic alternative is the same for EVA and CFAT analysis, because it is always correct that …

AW of EVA estimates = AW of CFAT estimates

or PW　　　　or PW

™ At this time, the term EVA is a registered trademark of Stern Stewart & Co.

EVA Analysis: Procedure

DIFFERENCE BETWEEN CFAT AND EVA APPROACHES

- CFAT estimates (describes) how actual **cash will flow**
- EVA estimates **extra worth** that an alternative adds
- EVA is a measure of worth that **mingles actual cash flows and noncash flows**

합치다

PROCEDURE FOR EVA ANALYSIS

Each year *t* determine the following for each alternative:

$$\begin{aligned} EVA_t &= \mathbf{NOPAT_t} - \text{cost of invested capital} \\ &= \mathbf{NOPAT_t} - (\text{after-tax MARR})(BV_{t-1}) \\ &= \mathbf{TI_t \times (1 - \mathit{T_e})} - \boxed{i} \times (BV_{t-1}) \end{aligned}$$

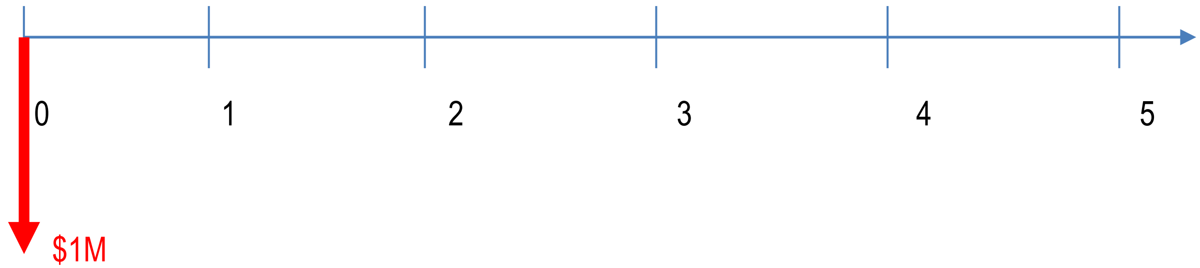
Selection: Choose alternative with better AW of EVA series

Remember: Since AW of EVA series will always = AW of CFAT series, the same alternative is selected by either method

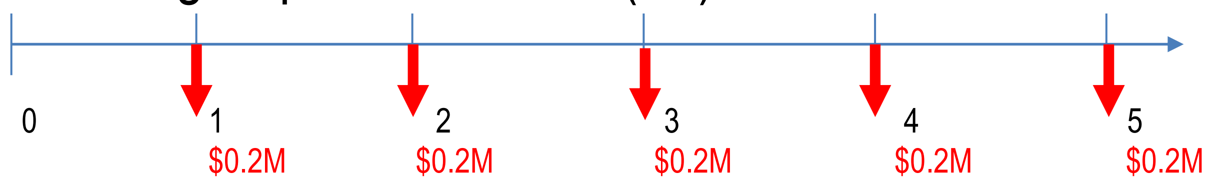
Concept

Assume A Co. purchase $1M machine that has 5 years of life with SL(or DB) depreciation.

- Actual Cash Flows

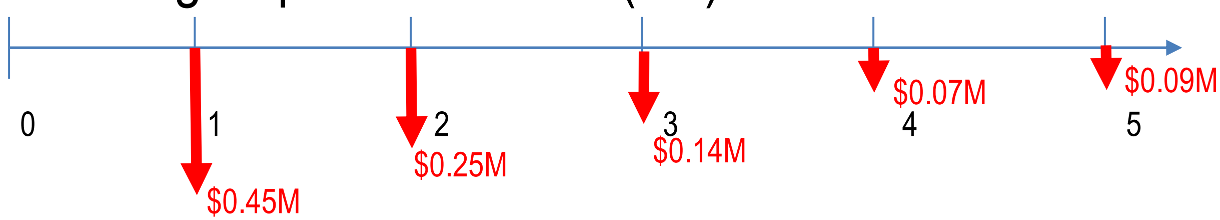


- Accounting Depreciation Cost (SL)



OR

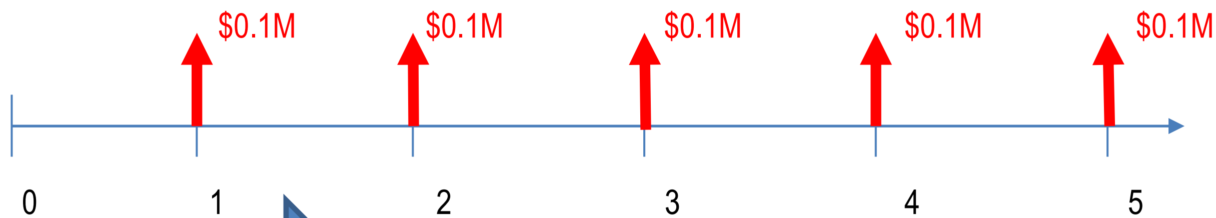
- Accounting Depreciation Cost (DB)



Concept

Assume A Co. purchase $1M machine that has 5 years of life with SL depreciation. Assuming NOPAT is $0.1M, compare Accounting analyses and Actual cash flows with i=10%.

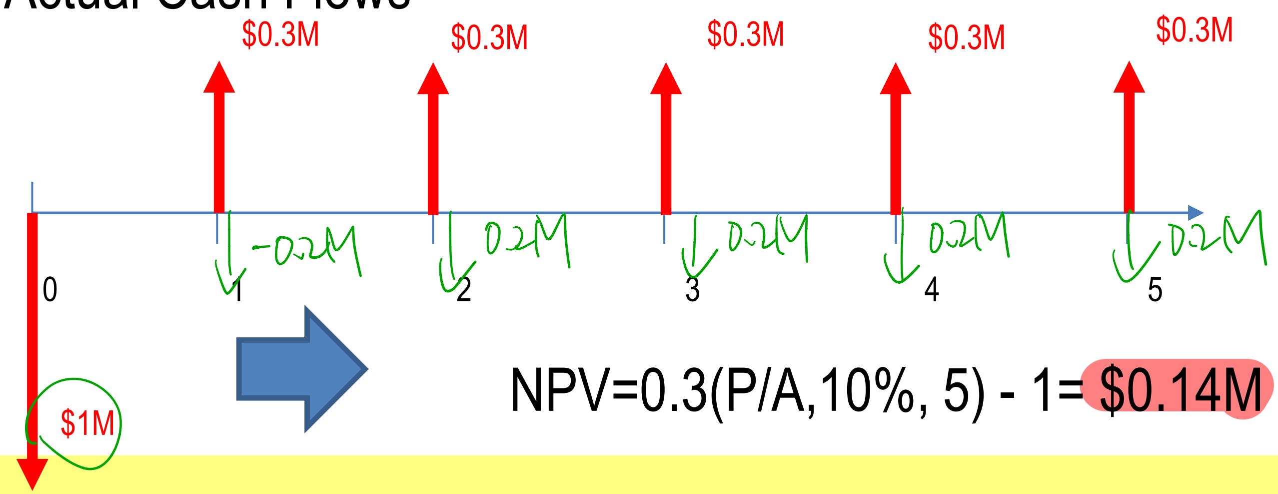
- Accounting Depreciation Cost (SL)



NPV=0.1(P/A,10%, 5)=$0.38M

use this

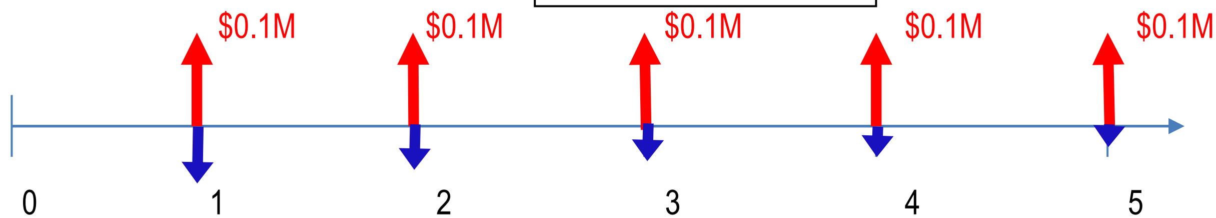
- Actual Cash Flows



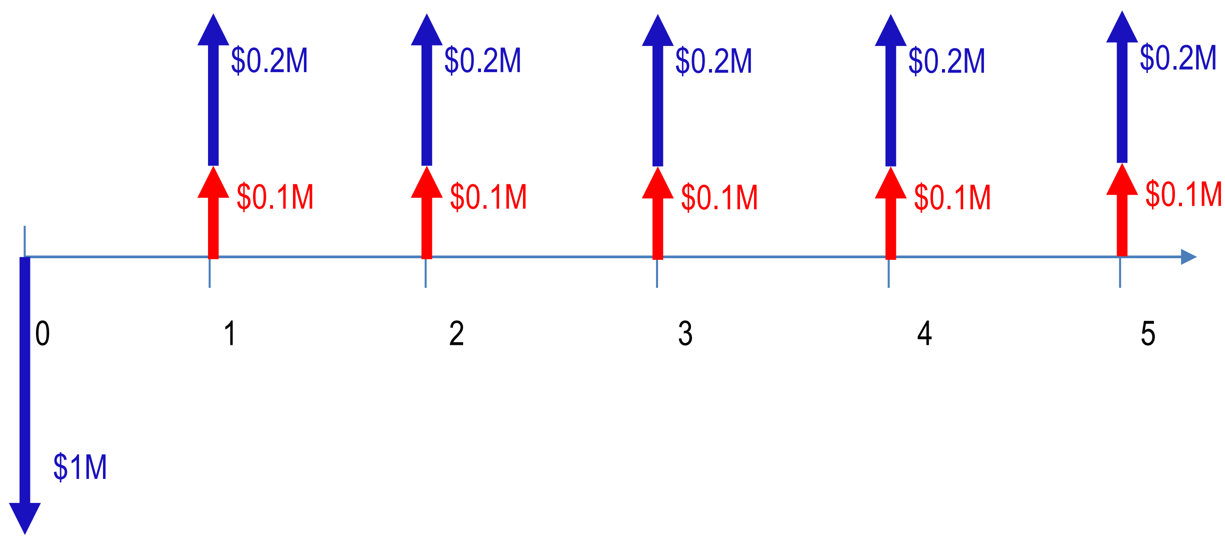
NPV=0.3(P/A,10%, 5) - 1= $0.14M

How to Make Them Equal?

- EVA=NOPAT – (i)x(BV_{t-1}) **PW = 0.14M**



- Actual Cash Flows **PW = 0.14M**



EXAMPLE 17.12

Biotechnics Engineering has developed two mutually exclusive plans for investing in new capital equipment with the expectation of increased revenue from its medical diagnostic services to cancer patients. The estimates are summarized below. (*a*) Use classical straight line depreciation, an after-tax MARR of 12%, and an effective tax rate of 40% to perform two annual worth after-tax analyses: EVA and CFAT. (*b*) Explain the fundamental difference between

	A	B	C	D	E	F	G	H	I	J	K
1						**PLAN A**					
2									EVA analysis		CFAT analysis
3			Investment P	SL	Book value	Taxable			Cost of		
4	Year	GI - OE	(Basis B)	Depreciation	BV	income, TI	Taxes	NOPAT	inv. capital	EVA	CFAT
5	0		-500,000		500,000						-500,000
6	1	170,000		125,000	375,000	45,000	18,000	27,000	-60,000	-33,000	152,000
7	2	170,000		125,000	250,000	45,000	18,000	27,000	-45,000	-18,000	152,000
8	3	170,000		125,000	125,000	45,000	18,000	27,000	-30,000	-3,000	152,000
9	4	170,000		125,000	0	45,000	18,000	27,000	-15,000	12,000	152,000
10	**AW values**									**-$12,617**	**-$12,617**
11						**PLAN B**					
12									EVA analysis		CFAT analysis
13			Investment P	SL	Book value	Taxable			Cost of		
14	Year	GI - OE	(Basis B)	Depreciation	BV	income, TI	Taxes	NOPAT	inv. capital	EVA	CFAT
15	0		-1,200,000		1,200,000						-1,200,000
16	1	600,000		300,000	900,000	300,000	120,000	180,000	-144,000	36,000	480,000
17	2	500,000		300,000	600,000	200,000	80,000	120,000	-108,000	12,000	420,000
18	3	400,000		300,000	300,000	100,000	40,000	60,000	-72,000	-12,000	360,000
19	4	300,000		300,000	0	0	0	0	-36,000	-36,000	300,000
20	**AW values**									**$3,388**	**$3,388**
21											
22	Functions for Plan B, year 3			`= -C15/4	`= E17-D18	`= B18-D18	`= F18*0.4	`= F18-G18	`=-0.12*E17	`= H18+I18	`= B18+C18-G18

Figure 17–9

Comparison of two plans using EVA and CFAT analyses, Example 17.12.

International Tax Structures

- **Tax related questions for internationally located projects concentrate on items such as:**
  - Depreciation methods approved by host country
  - Capital investment allowances
  - Business expense deductibility
  - Corporate tax rates
  - Indirect tax rates – Value-Added Tax / Goods and Service Tax (VAT/GST)

- **Rules and laws vary considerably from country to country**

Summary of International Corporate Tax Rates

Tax Rate Levied on TI, %	For These Countries
≥ 40	**United States, Japan**
35 to < 40	Pakistan, Sri Lanka
32 to < 35	France, India, South Africa
28 to < 32	Australia, United Kingdom, Canada, New Zealand, Spain, Germany, Mexico ?
24 to < 28	China, Indonesia, **South Korea**, ~~Israel~~
20 to < 24 ?	Russia, Turkey, Saudi Arabia
< 20	Singapore, Hong Kong, Taiwan, Chile, Ireland, Iceland, Hungary

Source: KPMG *Corporate and Indirect Tax Rate Survey, 2010*

Value-Added Tax (VAT)

VAT is an **indirect tax placed on goods and services**, not on people and corporations like an income tax. The VAT is charged sequentially throughout the process of manufacturing a good or providing a service. The VAT is also called **Goods and Service Tax (GST).**

VAT CHARACTERISTICS

- A percent, e.g., 10%, of current value, of **unfinished** goods or service (G/S) is charged to the purchaser and sent to taxing entity by manufacturer or provider
- VAT charged to buyer at purchase time whether buyer is an end user or intermediate business
- As next transfer occurs, VAT previously paid on unfinished G/S is **subtracted from VAT currently due**

SALES TAX CHARACTERISTICS

- Charged only **once** at final product sale to the **end user or consumer**
- Selling merchant sends tax to taxing entity
- Businesses do **not** pay sales tax on raw materials or unfinished goods or service
- Businesses **do pay** sales tax on items for which they are the end user

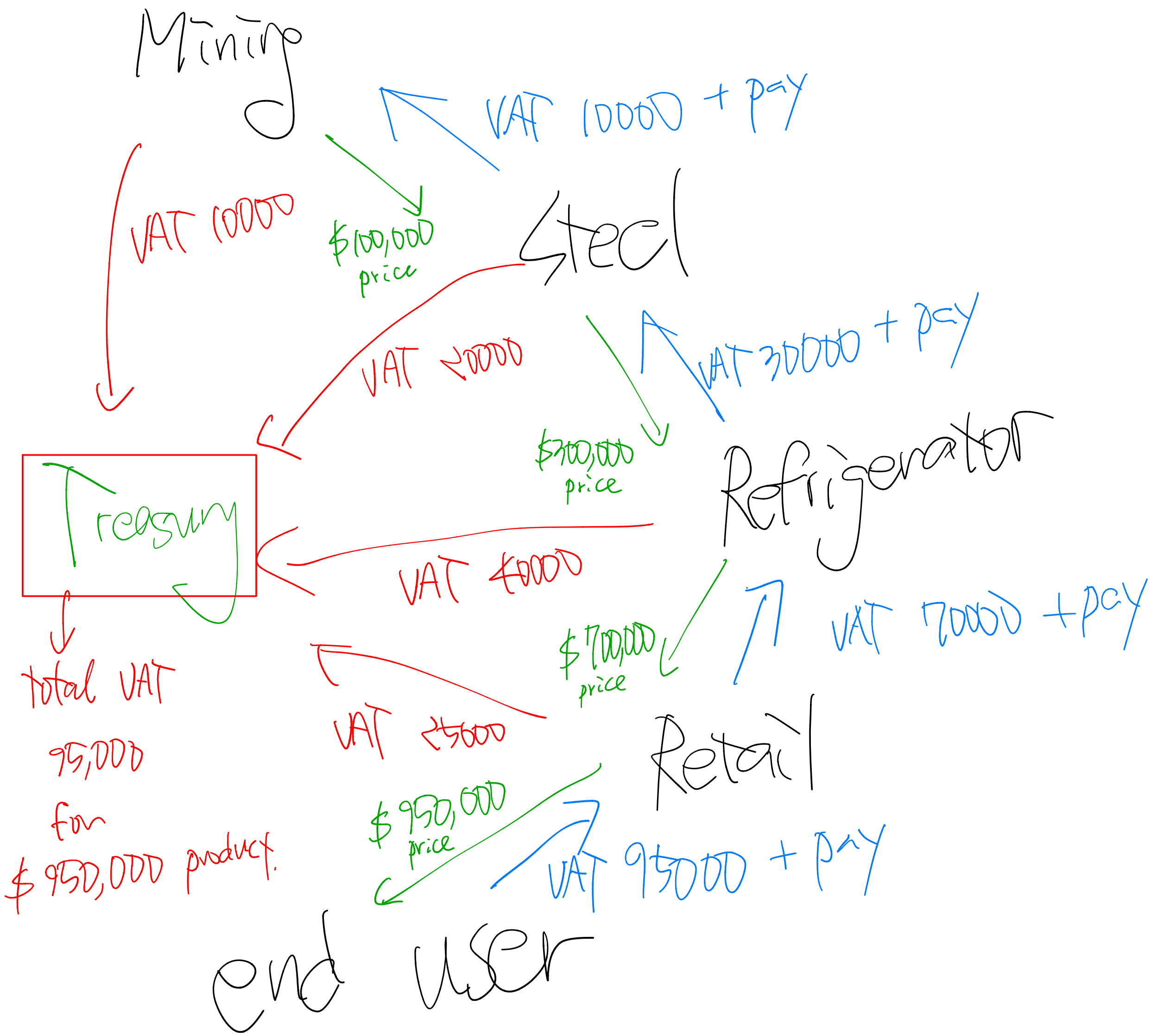
Example: How a 10% VAT Could Work

1. **Mining company** sells $100,000 of iron ore to **Steel company** and charges Steel company 10% VAT, or $10,000. **Mining company** sends **$10,000** to US Treasury.
2. **Steel company** sells steel for $300,000 to **Refrigerator company** and charges Refrigerator company 10% VAT, or $30,000. **Steel company** sends $30,000 – 10,000 = **$20,000** to US Treasury.
3. **Refrigerator company** sells refrigerators to **Retail company** for $700,000 and charges Retailer 10% VAT, or $70,000. **Refrigerator company** sends $70,000 – 30,000 = **$40,000** to US Treasury.
4. Finally, **Retailer** sells refrigerators to **end users/consumers** - for $950,000 and collects 10% VAT, or $95,000, from consumers. **Retailer** sends $95,000 – 70,000 = **$25,000** to US Treasury.

Conclusion: US Treasury received $25,000 + 40,000 + 20,000 + 10,000 = $95,000, which is 10% of final sales price of $950,000

Less Evasion of Taxes than Sales Tax

회피


Mining
VAT 10000 + pay
VAT 10000
$100,000 price
Steel
VAT 20000
VAT 30000 + pay
$300,000 price
Refrigerator
Treasury
VAT 40000
$700,000 price
VAT 70000 + pay
total VAT
95,000
for
$950,000 product.
VAT 25000
Retail
$950,000 price
VAT 95000 + pay
end user

Summary of Important Points

- For a corporation's taxable income (TI), operating expenses and asset depreciation are **deductible items**
- Income tax rates for corporations and individuals are **graduated** by increasing TI levels
- CFAT indirectly includes **(noncash) depreciation** through the TI computation
- Depreciation recapture (DR) occurs when an asset is **sold for more than the book value**; DR is taxed as regular income in all after-tax evaluations
- After-tax analysis uses **CFAT values** and the **same guidelines** for alternative selection as before-tax analysis
- EVA estimates **extra worth** that an alternative adds to net worth after taxes; it **mingles actual cash flows and noncash flows**
- A **VAT system** collects taxes progressively on **unfinished goods and services;** different than a sales tax system where only end users pay

HOMEWORK

1. Please solve every Examples in your textbook. You do not have to submit your works.

2. Please upload following "PROBLEMS" solution file on "Assignment" menu in e-Class.
   ① 17.28
   ② 17.42
   ③ 17.52
   ④ 17.56(00)
   ⑤ 17.59(XX)
   ⑥ 17.66
   ⑦ 17.67