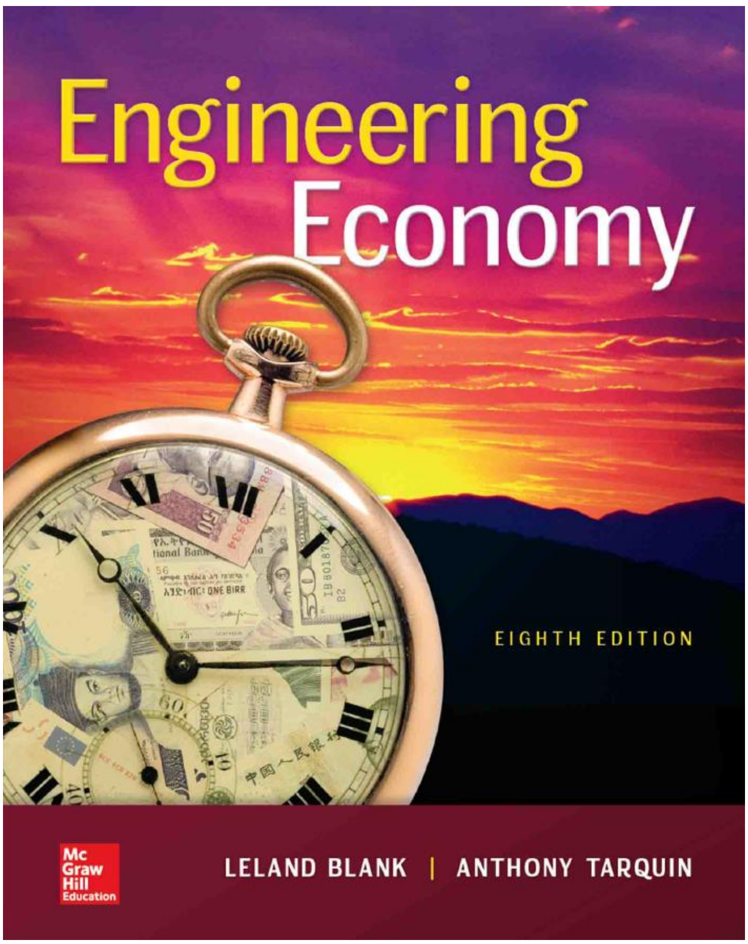

Chapter 2

계수

Factors: How Time and Interest Affect Money

Lecture slides to accompany

Engineering Economy

8th edition

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Mc Graw Hill Higher Education

F : Future Value
P : Present Value
A : Annual Value

LEARNING OUTCOMES

1. F/P and P/F Factors
2. P/A and A/P Factors
3. F/A and A/F Factors
4. Factor Values
5. Arithmetic Gradient (산수 변화도, 증감률)
6. Geometric Gradient (기하)
7. Find i or n

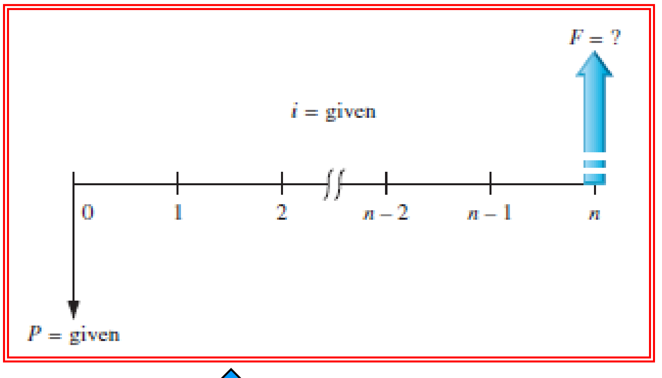
cf) PE, the steel Plant Case

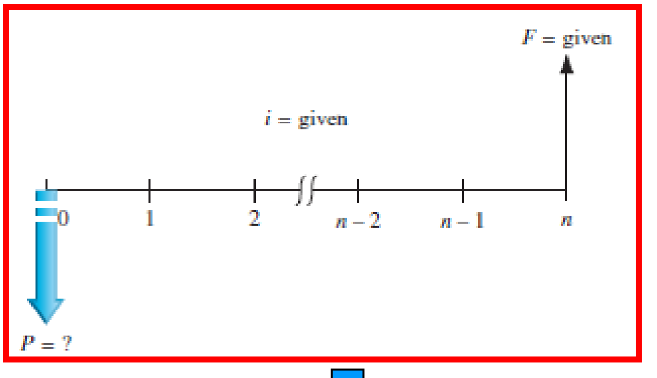
build ⇒ 200m.
Annual profit ⇒ 30m.
plan for 5 years

Single Payment Factors (F/P and P/F)

Single payment factors involve only **P** and **F**.

Cash flow diagrams are as follows:





Formulas are as follows:

$F = P(1+i)^n$

$P = F[(1+i)^{-n}]$

Terms in parentheses or brackets are called *factors.* Values are in tables for i and n values

Factors are represented in standard factor notation *such as* ***(F/P,i,n)****,*

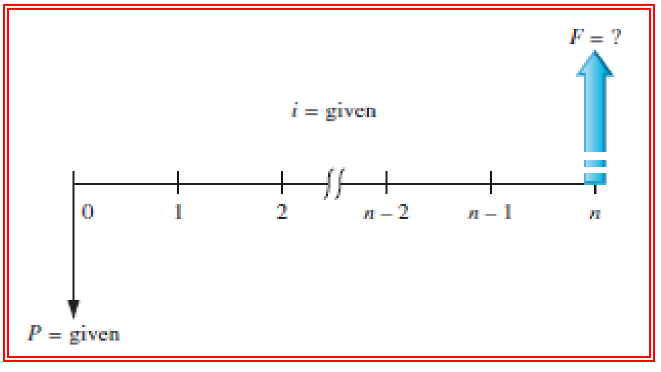
where letter to left of slash is what is sought; letter to right represents what is given

want to know

$(F/\boxed{P, i, n})$

given

Single Payment Factors (F/P and P/F)



$$F_1 = P + Pi = P(1+i)^1$$

$$F_2 = F_1 + F_1 i = F_1(1+i)^1 = P(1+i)^2$$

.

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$$F_n = F_{n-1} + F_{n-1} i = F_{n-1}(1+i)^1 = P(1+i)^n$$

Equation

$$F = P(1+i)^n$$

$$= P(F/P, i\%, n)$$

$$P = F[\frac{1}{(1+i)^n}]$$

$$= F(P/F, i\%, n)$$

Standard Factor Notation

- P = F(P/F,i,n) : Given F, i, and n, Find P
- F = P(F/P,i,n) : Given P, i, and n, Find F
- A = F(A/F,i,n) : Given F, i, and n, Find A
- A = P(A/P,i,n) : Given P, i, and n, Find A

분자 분모
As if they are numerator and denominator

- A = F(A/F,i,n) = P(F/P,i,n) x (A/F,i,n) = P(A/P,i,n)

$$= P\left(\frac{F}{P}\right) \times \left(\frac{A}{F}\right) = P(A/P,\ i,\ n)$$

Compound Interest Factor Tables

Please consult the tables in p. 595 ~ p. 623

F/P and P/F for Spreadsheets

Future value F is calculated using FV function:

= FV(i%, n,, P)

Present value P is calculated using PV function:

= PV(i%, n,, F)

Note the use of double commas in each function

Introduction to Spreadsheet Functions

Excel financial functions

수정 (5개만!)

Present Value, P: = PV (i%,n,A,F)

Future Value, F: = FV (i%,n,A,P)

주기적인 / annual value

Equal, periodic value, A: = PMT (i%,n,P,F) periodic amount

Number of periods, n: = NPER (i%,A,P,F)

Compound interest rate, i: = RATE (n,A,P,F)

Compound interest rate, i: = IRR (first_cell:last_cell)

Assume Bank Transactions ⇒ ⊕, ⊖ signs

Present value, any series, P: = NPV (i%,second_cell:last_cell) + first cell

handle any kind of cash flow → powerful

net present value

Example: Estimates are P = $5000 n = 5 years i = 5% per year

Find A in $ per year

Function and display: = PMT (5%, 5, -5000) displays A = $1154.87

you will deposit (예금한 상황)

you can withdraw this amount of money each year. (5 years total)

Example: Finding Future Value

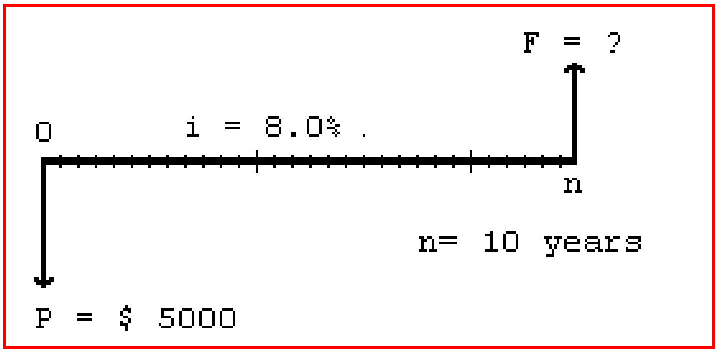
A person deposits \$5000 into an account which pays interest at a rate of 8% per year. The amount in the account after 10 years is closest to:

P=5000
i= 8
n=10

(A) \$2,792 (B) \$9,000 (C) \$10,795 (D) \$12,165

FV(8,10,,-5000)

The cash flow diagram is:



Solution:

F = P(F/P,i,n)

= 5000(F/P, 8%, 10**)**

= 5000(2.1589)

= \$10,794.50

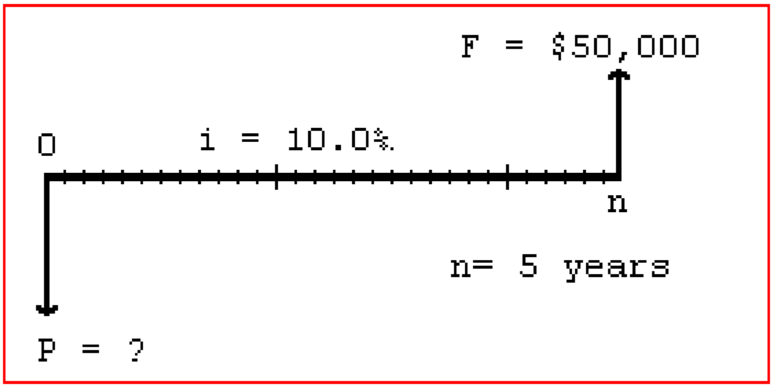
Answer is (C **)**

Example: Finding Present Value

A small company wants to make a single deposit now so it will have enough money to purchase a backhoe costing $50,000 five years from now. If the account will earn interest of 10% per year, the amount that must be deposited now is nearest to:

(A) $10,000 (B) $ 31,050 (C) $ 33,250 (D) $319,160

The cash flow diagram is:



Solution:

P = F(P/F,i,n)

= 50,000(P/F,10%,5)

= 50,000(0.6209)

= $31,045

Answer is (B)

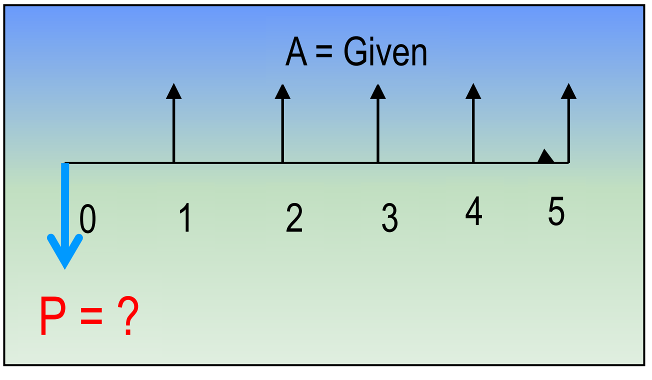
Uniform Series Involving P/A and A/P

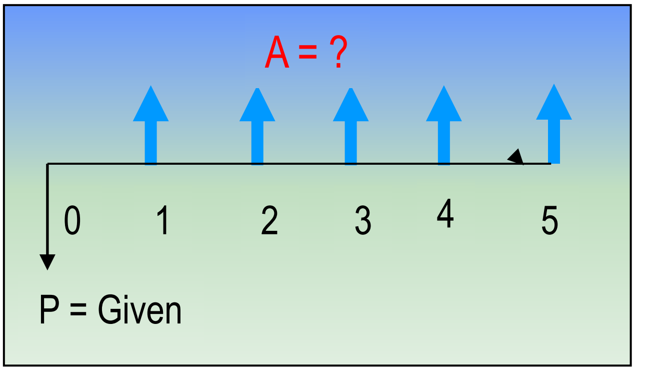
The uniform series factors that involve **P and A** are derived as follows:

(1) Cash flow occurs in consecutive interest periods

(2) Cash flow amount is same in each interest period

The cash flow diagrams are:





P = A(P/A,i,n) ⟵ Standard Factor Notation ⟶ A = P(A/P,i,n)

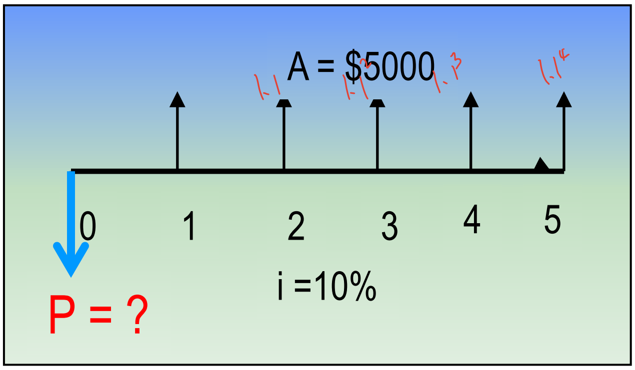
Note: P is one period ahead of first A value

Example: Uniform Series Involving P/A

A chemical engineer believes that by modifying the structure of a certain water treatment polymer, his company would earn an extra \$5000 per year. At an interest rate of 10% per year, how much could the company afford to spend now to just break even over a 5 year project period?

(A) \$11,170 **(B) 13,640** **(C) \$15,300** **(D) \$18,950**

The cash flow diagram is as follows:



Solution:

$P = 5000(\underline{P/A,\ 10\%,\ 5})$

$= 5000(3.7908)$

$= \$18,954$

Answer is ($\underline{D}$)

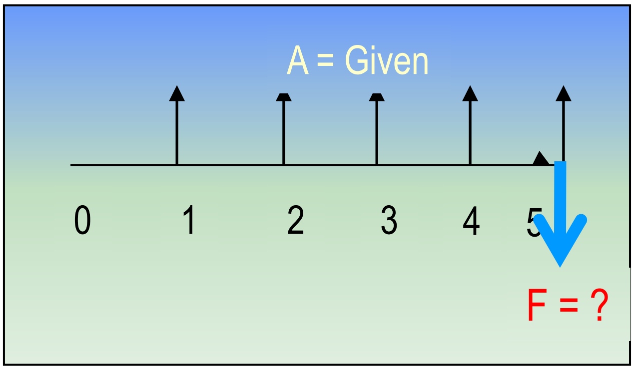
Uniform Series Involving F/A and A/F

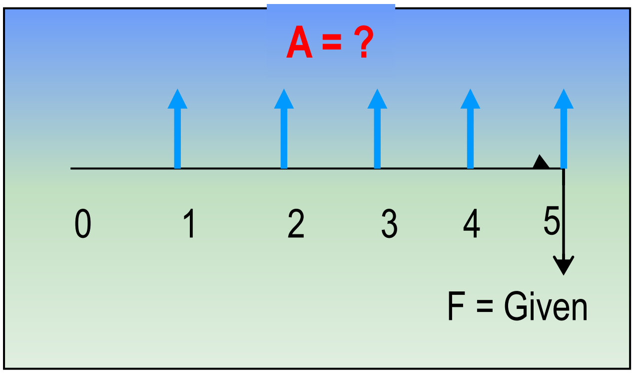
The uniform series factors that involve **F and A** are derived as follows:

(1) Cash flow occurs in ***consecutive*** interest periods

(2) Last cash flow occurs in ***same*** period as F

Cash flow diagrams are:





F = A(F/A,i,n) ⟵ Standard Factor Notation ⟶ A = F(A/F,i,n)

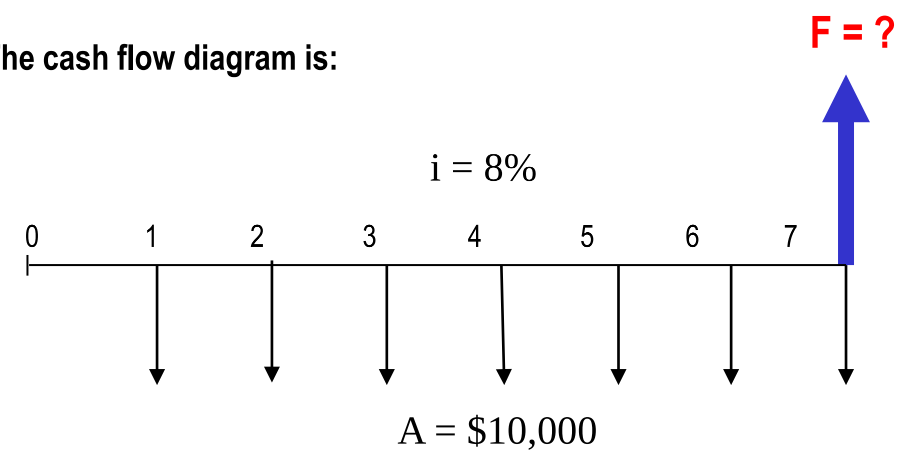
Note: F takes place in the _same_ period as last A

Example: Uniform Series Involving F/A

An industrial engineer made a modification to a chip manufacturing process that will save her company $10,000 per year. At an interest rate of 8% per year, how much will the savings amount to in 7 years?

(A) $45,300 (B) $68,500 (C) $89,228 (D) $151,500

The cash flow diagram is:



Solution:

$F = 10{,}000(F/A, 8\%, 7)$

$= 10{,}000(8.9228)$

$= \$89{,}228$

Answer is (C)

Factor Values for Untabulated i or n

3 ways to find factor values for untabulated i or n values

- **Use formula**
- **Use spreadsheet function with corresponding P, F, or A value set to (1)**
- **Linearly interpolate in interest tables (See FIG 2-10)**

Formula or spreadsheet function is fast and accurate
Interpolation is only approximate

표에 없는

Example: Untabulated i

Determine the value for (F/P, 8.3%,10)

Formula: $F = (1 + 0.083)^{10} = 2.2197$ ⟵ **OK**

Spreadsheet: = FV(8.3%,10,,1) = 2.2197 ⟵ **OK**

보간법

Interpolation:

8%	------	2.1589
8.3%	------	x
9%	------	2.3674

x = 2.1589 + [(8.3 - 8.0)/(9.0 - 8.0)][2.3674 – 2.1589]
= 2.2215 ⟵ **(Too high)**

Absolute Error = 2.2215 – 2.2197 = 0.0018

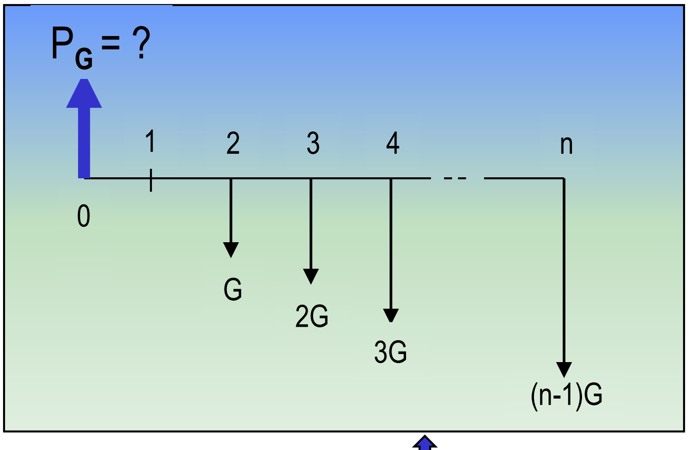
Ex 2.2)
Ex 2.4)
Ex 2.6)

Arithmetic Gradients

등차급수

Arithmetic gradients ***change*** by the Same ***amount*** each period

The cash flow diagram for the P_G of an arithmetic gradient is:



Standard factor notation is

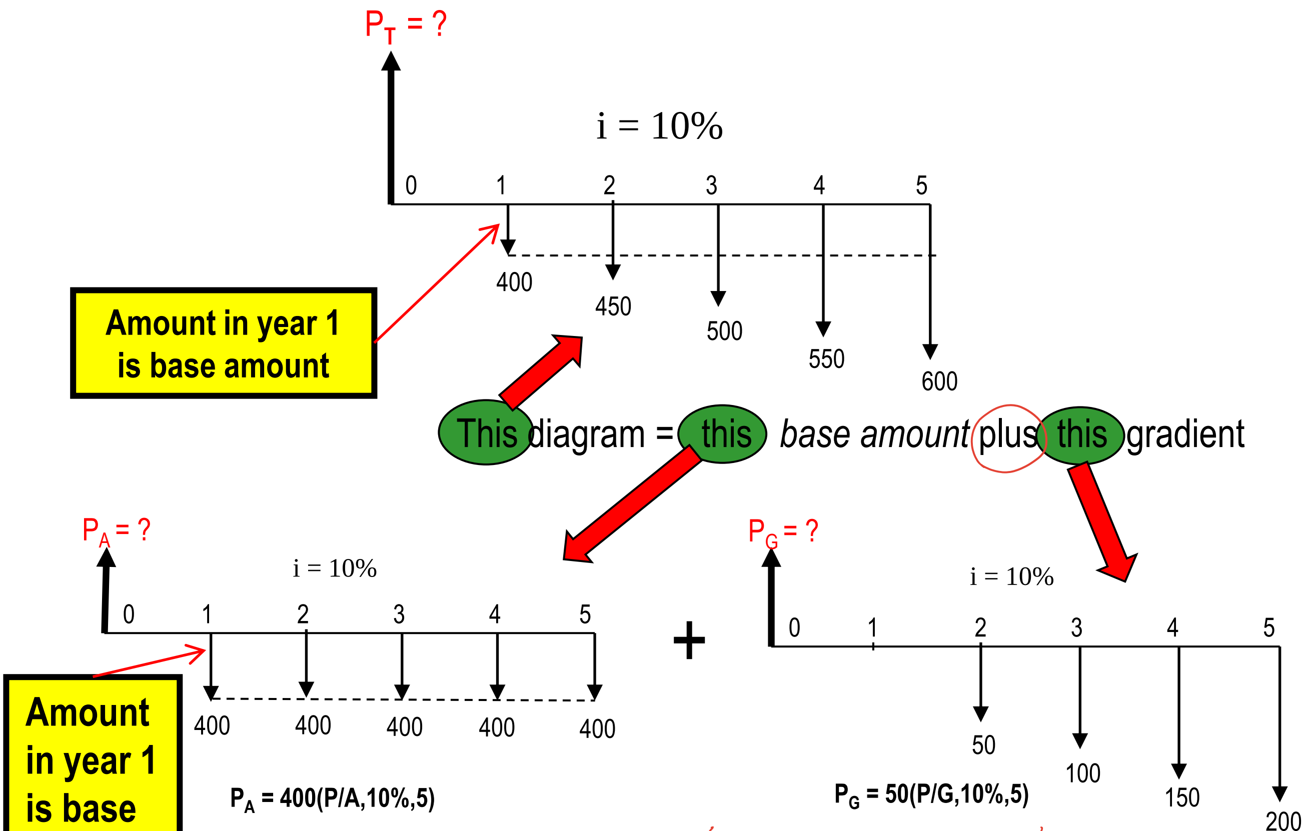
$P_G = G(P/G,i,n)$

G starts between periods 1 and 2
(not between 0 and 1)

This is because cash flow in year 1 is usually not equal to G and is handled separately as a *base amount* (shown on next slide)

Note that P_G is located Two Periods **Ahead of the first change that is equal to G**

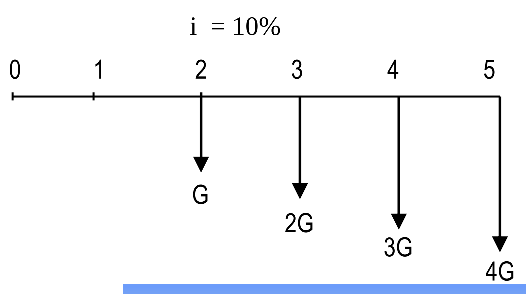
Typical Arithmetic Gradient Cash Flow

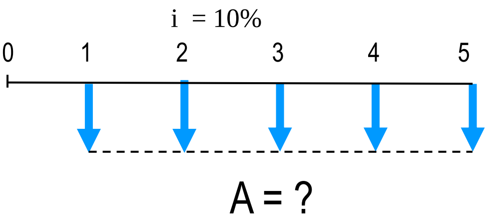


$P_T = P_A + P_G = 400(P/A\ ,10\%,5) + 50(P/G\ ,10\%,5)$

Converting Arithmetic Gradient to A

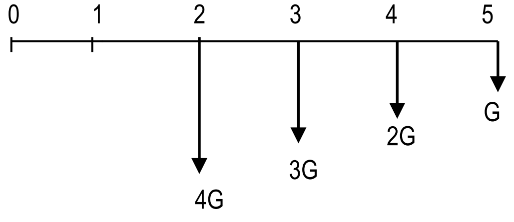
Arithmetic gradient can be converted into equivalent A value using G(A/G,i,n)





General equation when base amount is involved is

A = base amount + G(A/G,i,n)



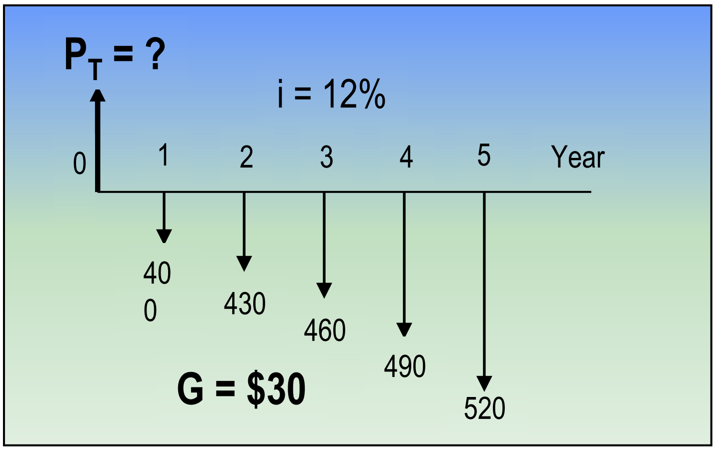
For decreasing gradients, change plus sign to minus

A = base amount - G(A/G,i,n)

Example: Arithmetic Gradient

The present worth of \$400 in year 1 and amounts increasing by \$30 per year through year 5 at an interest rate of 12% per year is closest to:

(A) \$1532 (B) \$1,634 (C) \$1,744 (D) \$1,829



year 1에서 생각해야 함

Solution:

P_T = 400(P/A,12%,5) + 30(P/G,12%,5)

= 400(3.6048) + 30(6.3970)

= \$1,633.83

Answer is (B)

The cash flow could also be converted into an **A** value as follows:

A = 400 + 30(A/G,12%,5)

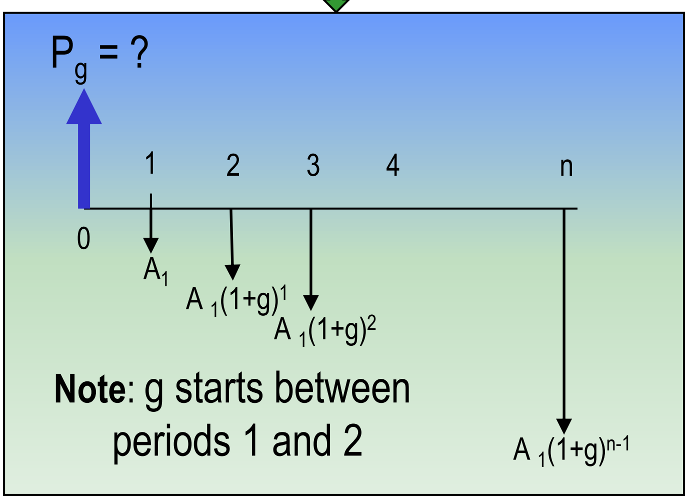
= 400 + 30(1.7746)

= \$453.24

Geometric Gradients 등비급수

Geometric gradients **change by the** same percentage **each period**

Cash flow diagram for present worth of geometric gradient



There are ***no tables*** for geometric factors

Use following equation for $g \neq i$:

$$P_g = A_1\{1- [(1+g)/(1+i)]^n\}/(i-g)$$

where: A_1 *= cash flow in period 1*

g *= rate of increase*

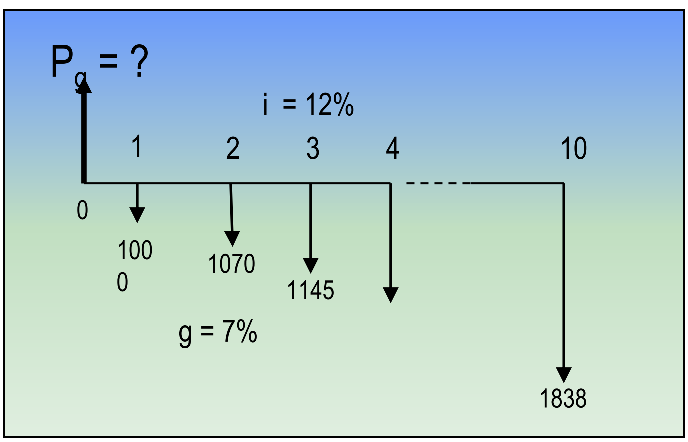
If $g = i$, $P_g = A_1 n/(1+i)$

Note: If g is negative, change signs in front of both g values

Example: Geometric Gradient

Find the present worth of $1,000 in year 1 and amounts increasing by 7% per year through year 10. Use an interest rate of 12% per year.

(a) $5,670 **(b) $7,333** **(c) $12,670** **(d) $13,550**



Solution:

$P_g = 1000[1-(1+0.07/1+0.12)^{10}]/(0.12-0.07)$

$= \$7,333$

Answer is (B)

To find A, multiply P_g by (A/P,12%,10)

Unknown Interest Rate i

Unknown interest rate problems involve solving for i, given n and 2 other values (P, F, or A)

(Usually requires a trial and error solution or interpolation in interest tables)

Procedure: Set up equation with all symbols involved and solve for i

A contractor purchased equipment for $60,000 which provided income of $16,000 per year for 10 years. The annual rate of return of the investment was closest to:

(a) 15% (b) 18% (c) 20% (d) 23%

Solution: Can use either the P/A or A/P factor. Using A/P:

$$60{,}000(A/P,i\%,10) = 16{,}000$$

$$(A/P,i\%,10) = 0.26667$$

From A/P column at n = 10 in the interest tables, i is between 22 % and 24 %   Answer is (d)

Unknown Recovery Period n

Unknown recovery period problems involve solving for n, given i and 2 other values (P, F, or A)

(Like interest rate problems, they usually require a trial & error solution or interpolation in interest tables)

Procedure: Set up equation with all symbols involved and solve for n

A contractor purchased equipment for $60,000 that provided income of $8,000 per year. At an interest rate of 10% per year, the length of time required to recover the investment was closest to:

(a) 10 years (b) 12 years (c) 15 years (d) 18 years

Solution: Can use either the P/A or A/P factor. Using A/P:

$$60{,}000(A/P,10\%,n) = 8{,}000$$

$$(A/P,10\%,n) = 0.13333$$

From A/P column in i = 10% interest tables, n is between 14 and 15 years **Answer is (C)**

Summary of Important Points

- In P/A and A/P factors, P is one period *ahead* of first A
- In F/A and A/F factors, F is in same period *as last* A
- To find untabulated factor values, best way is to use fomula or spreadsheet
- For arithmetic gradients, gradient G starts between *periods* 1 & 2
- Arithmetic gradients have 2 parts, base *amount* (year 1) and gradient *amount*
- For geometric gradients, gradient g starts been *periods 1 and 2*
- In geometric gradient formula, A_1 is amount in *period* 1
- To find unknown i or n, *set up equation involving all terms* and solve for i or n

HOMEWORK

1. Please solve every Examples in your textbook. You do not have to submit your works.

2. Please upload following “PROBLEMS” solution file on “Assignment” menu in e-Class.
   ① 2.18
   ② 2.22
   ③ 2.36
   ④ 2.45
   ⑤ 2.53
   ⑥ 2.60
   ⑦ 2.65
   ⑧ 2.79