ITM426, Quiz 2, 2020 Fall

Solution and Grading

- ITM 426 Engineering Mathematics 2020 F
- Oct 23, 2020
- Duration: 90 minutes
- 5 Questions
- Weighting of 25 % or 30 % depending on other quiz scores

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- Write legibly.
- In on-line exam, start every problem in a new page.
- Justification is necessary unless stated otherwise.
- Partial points are given only sparingly for the most problems because you are expected to 1) carry out proper sanity check and 2) correct your mistake by doing so.

1	15
2	20
3	15
4	10
5	15
Total	75

#1. Find the solution set of the following system of linear equations by describing in parametric vector form. Also, give a geometric description of the solution set. [15pt]

$$\begin{aligned} x_1 + 3x_2 + x_3 &= 1 \\ -4x_1 - 9x_2 + 2x_3 &= -1 \\ -3x_2 - 6x_3 = -3 \end{aligned}$$

Difficulty: Easy
Amount of work: 20 %
Suggested answer:

Row reduce the augmented matrix for the system:

$$\begin{bmatrix} 1 & 3 & 1 & 1 \\ -4 & -9 & 2 & -1 \\ 0 & -3 & -6 & -3 \end{bmatrix} \sim \begin{bmatrix} 1 & 3 & 1 & 1 \\ 0 & 3 & 6 & 3 \\ 0 & -3 & -6 & -3 \end{bmatrix} \sim \begin{bmatrix} 1 & 3 & 1 & 1 \\ 0 & 3 & 6 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 3 & 1 & 1 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & -5 & -2 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}. \quad \begin{aligned} x_1 \quad - 5x_3 &= -2 \\ x_2 + 2x_3 &= 1 \\ 0 &= 0 \end{aligned}.$$

Thus $x_1 = -2 + 5x_3$, $x_2 = 1 - 2x_3$, and x_3 is free. In parametric vector form,

$$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -2 + 5x_3 \\ 1 - 2x_3 \\ x_3 \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 5x_3 \\ -2x_3 \\ x_3 \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} 5 \\ -2 \\ 1 \end{bmatrix}$$

The solution set is the line through $\begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix}$, parallel to the line that is the solution set of the homogeneous system in Exercise 5.

#2. Find the inverse of these matrices.[20pt]

(a)

$$\begin{bmatrix} -2 & -7 & -9 \\ 2 & 5 & 6 \\ 1 & 3 & 4 \end{bmatrix}$$

Difficulty: Medium
Amount of work: 10 %
Suggested answer:

$$\begin{bmatrix} 2 & 1 & 3 \\ -2 & 1 & -6 \\ 1 & -1 & 4 \end{bmatrix}$$

(b)

$$\begin{bmatrix} 1 & & & \\ 1 & 1 & & \\ 1 & 1 & 1 & \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

Difficulty: Easy
Amount of work: 10 %
Suggested answer:

$$\begin{bmatrix} 1 & & & \\ -1 & 1 & & \\ & -1 & 1 & \\ & & -1 & 1 \end{bmatrix}$$

#3. Find an LU factorization of the following matrix. `[15pt]`

$$\begin{bmatrix}2&-4&4&-2\\6&-9&7&-3\\-1&-4&8&0\end{bmatrix}$$

Difficulty: Easy
Amount of work: 20 %
Suggested answer:

$$A=\begin{bmatrix}2&-4&4&-2\\6&-9&7&-3\\-1&-4&8&0\end{bmatrix}\sim\begin{bmatrix}2&-4&4&-2\\0&3&-5&3\\0&-6&10&-1\end{bmatrix}\sim\begin{bmatrix}2&-4&4&-2\\0&3&-5&3\\0&0&0&5\end{bmatrix}=U$$

$$\begin{bmatrix}2\\6\\-1\end{bmatrix}\quad\begin{bmatrix}3\\-6\end{bmatrix}\quad\begin{bmatrix}5\end{bmatrix}$$

$$\div 2\qquad \div 3\qquad \div 5$$

$$\begin{bmatrix}1&&\\3&1&\\-1/2&-2&1\end{bmatrix},\quad L=\begin{bmatrix}1&0&0\\3&1&0\\-1/2&-2&1\end{bmatrix}$$

#4. If the following statement is True, then provide a mathematical proof or explain it further. If False, then provide a counter-example. `[10pt]`

For a $m \times n$ matrix A, if $A\mathbf{x} = \mathbf{0}$ has a nontrivial solution, then the number of solution to $A\mathbf{x} = \mathbf{b}$ is infinite.

Difficulty: Medium
Amount of work: 15 %
Suggested answer:
False. $\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ is a possible counter-example.

#5. After a linear transformation, SNUTech's amblem has been transformed. Specifically, the emblem is horizontally reversed and its horizontal length is doubled. Suggest a standard matrix for this linear transformation and explain it. [15pt]

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	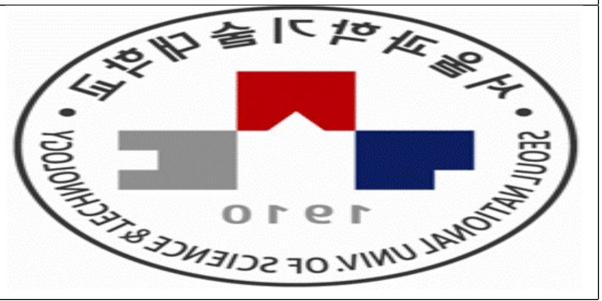

Difficulty: Hard
Amount of work: 25 %
Suggested answer:
Doubing in x-axis can be achieved by $\begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$, and reflection through y-axis can be achieved by $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$. Doing these consecutively means multiplication of these matrices, which leads to a standard matrix of $\begin{bmatrix} -2 & 0 \\ 0 & 1 \end{bmatrix}$

Write your name before detaching this page. Your Name: ______________________

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