ITM426, Final, 2019 Fall

Code	ITM 426
Title	Engineering Math.
Time for Exam	2.5 hours
Questions	9
Weighting	40 %

- Name: ______________________
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- Closed book, closed notes, no calculator.
- Only writing instrments are allowed on the desk.
- Absolutely no phone on the desk.
- Do not remove the original staple.
- You may detach the last three sheets of this packet.
- Write legibly.

"Exams are formidable even to the best prepared, for the greatest fool may ask more than the wisest man can answer". - Charles Caleb Colton

1	5
2	5
3	11
4	5
5	5
6	5
7	8
8	10
9	8
Total	62

#1. For each of the following statements, write either True or False. No justification is necessary.[`each 2.5pts`]

- (a) A is diagonalizable if and only if A has n eigenvalues, counting multiplicities.

- (b) An $n \times n$ matrix that is orthogonally diagonalizable must be symmetric.

#2. Short answers are expected. No justification is necessary. [`each 2.5pts`]

- (a) If the null space of an 8×5 matrix A is 2-dimensional, what is the dimension of the row space of A?

- (b) Find a unit vector in the direction of a vector $[-6, 4, 3]^t$.

#3. The matrix A is row equivalent to B.

$$A = \begin{bmatrix} 2 & -3 & 6 & 2 & 5 \\ -2 & 3 & -3 & -3 & -4 \\ 4 & -6 & 9 & 5 & 9 \\ -2 & 3 & 3 & -4 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 2 & -3 & 6 & 2 & 5 \\ 0 & 0 & 3 & -1 & 1 \\ 0 & 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(a) Without calculations, list $rank\, A$ and $dim\, Nul\, A$. [2pts]

(b) Find bases for $Col\, A$. [3pts]

(c) Find bases for $Row\, A$. [3pts]

(d) Find bases for $Nul\, A$. [3pts]

#4. Find the characteristic polynomial and the eigenvalues of the matrices. [each `2.5pts`]

(a)

$$\begin{bmatrix} 3 & -2 \\ 1 & -1 \end{bmatrix}$$

(b)

$$\begin{bmatrix} 2 & 1 \\ -1 & 4 \end{bmatrix}$$

#5. Diagonalize the following matrix. Eigenvalues are $\lambda = 5, 4$. Be careful with multiplicity. [`5pts`]

$$\begin{bmatrix} 4 & 0 & -2 \\ 2 & 5 & 4 \\ 0 & 0 & 5 \end{bmatrix}$$

#6. Set up a weighted normal equation where the weight vector is given as $\mathbf{w} = [.1\ .2\ .3\ .4]^t$, and A and $\mathbf{y}$ are given as follows. [5pts]

$$A = \begin{bmatrix} 1 & -1 \\ 1 & 4 \\ 1 & -1 \\ 1 & 4 \end{bmatrix}, \ \mathbf{y} = \begin{bmatrix} -1 \\ 6 \\ 5 \\ 7 \end{bmatrix}$$

#7. From some experiment, pairs of (x, y) are observed as $(3, 5), (-1, 1), (1, 0)$.

(a) Identify the linear regression line of $y = \alpha + \beta x$ using the least-squares method. [`5pts`]

(b) Draw the points and the regression line that you obtained in 2D plane. (If you failed to obtain regression line from (a), then still draw points and regression line at your imagination. And discuss the properties of the regression line in relation to the data points.) [`3pts`]

#8. Consider the following matrix A.

$$A = \begin{bmatrix} 2 & 9 \\ 9 & 42 \end{bmatrix}$$

(a) Show that the matrix A is positive definite. [5pts]

(b) Since A is symmatric and positive definite, we know decomposition such that $A = LDL^t$ and $A = LL^t$ are possible. Find L that satisfies $A = LL^t$. (Cholesky decomposition). [5pts]

#9. Consider a sample covariance matrix S as following. Its eigenvalues are equal to 9, 6, and 3.

$$S = \begin{bmatrix} 5 & 2 & 0 \\ 2 & 6 & 2 \\ 0 & 2 & 7 \end{bmatrix}$$

(a) Find the principal components of the data. [`5pts`]

(b) How many principal components are needed to explain more than 80% of variance? [`3pts`]

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