ITM426, Quiz 1, 2021 Fall

Solution and Grading

- ITM 426 Engineering Mathematics 2021 F
- Sep 24, 2021
- Duration: 60 minutes
- Weights: 10% or 20% depending on other quiz scores
- 6 Questions

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- Write legibly.
- In on-line exam, start every problem in a new page.
- Justification is necessary unless stated otherwise.
- Partial points are given only sparingly for the most problems because you are expected to 1) carry out proper sanity check and 2) correct your mistake by doing so.

1	20
2	10
3	20
4	15
5	15
6	20
Total	100

#1. Investigate whether the following sets of vectors are linearly independent or dependent.`[20pt]`

- (a) $(1, 1, 0), (1, 2, 3), (0, 0, 0)$
- (b) $(2, 3, 0), (0, 2, -1), (4, 4, -1)$

Difficulty: Easy
Amount of work: 20 %
Suggested answer:
For (a), linearly dependent because $0 \cdot (1, 1, 0) + 0 \cdot (1, 2, 3) + 1 \cdot (0, 0, 0) = 0$. For (b), linearly indepdent because only solution to the equation $a_1 \cdot (2, 3, 0) + a_2 \cdot (0, 2, -1) + a_3 \cdot (4, 4, -1) = (0, 0, 0)$ is $a_1 = a_2 = a_3 = 0$.

#2. Find the 3×3 matrix $A = [a_{ij}]$ such that $a_{ij} = i - 2j - 1$.`[10pt]`

Difficulty: Easy
Amount of work: 10 %
Suggested answer:

$$\begin{bmatrix} -2 & -4 & -6 \\ -1 & -3 & -5 \\ 0 & -2 & -4 \end{bmatrix}$$

#3. Suppose we have a $n \times n$ matrix A such that $A\mathbf{x} = \mathbf{b}$ has a unique solution. Pick all of statements that are true. `[20pt]`

- (a) A has zero determinant
- (b) A has non-zero determinant
- (c) A is singular
- (d) A is non-singular
- (e) A set of column vectors in A is linearly independent
- (f) A set of column vectors in A is linearly dependent
- (g) A is invertible
- (h) A is not invertible.
- (i) A^{-1} exists.
- (j) A^{-1} does not exist.

Difficulty: Medium
Amount of work: 20 %
Suggested answer:
(b),(d),(e),(g),(i)

#4. Write the matrix formular for the following system of linear equation. Find the inverse of the coefficient matrix and find the solution to the system of linear equation in vector form. `[15pt]`

$$\begin{aligned} x - y &= 3 \\ 2x + 3y &= 7 \end{aligned}$$

Difficulty: Medium
Amount of work: 15 %
Suggested answer:
After writing $\begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 7 \end{bmatrix}$, the solution is $\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{5} \cdot \begin{bmatrix} 3 & 1 \\ -2 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 7 \end{bmatrix} = \begin{bmatrix} 16/5 \\ 1/5 \end{bmatrix}$

#5. We have a matrix $A = \begin{bmatrix} a & b & c & d \\ e & f & g & h \\ i & j & k & l \\ m & n & o & p \end{bmatrix}$ and want to multiply another matrix to the left of the matrix A to generate another matrix, A'. That is, $A' = \begin{bmatrix} 2 & & & \\ -1 & 1 & & \\ & & 3 & \\ & 2 & & 1 \end{bmatrix} \times A$.

a) Write A' `[7pt]`

$$A' = \begin{bmatrix} & & & \\ & & & \\ & & & \\ & & & \end{bmatrix}$$

b) Complete the following for the above operation by completing the underlined. `[8pt]`

Let each row vector of matrix A as $(R1), (R2), (R3)$, and $(R4)$, respectively. Then, each row vector of matrix A' can be expressed as following:

1. The first row of A' is $\underline{2 \times (R1)}$
2. The second row A' is $\underline{(R2) - (R1)}$
3. The third row of A' is ____________
4. The fourth row A' is ________

Difficulty: Medium
Amount of work: 15%
Solution:

a) $\begin{bmatrix} 2a & 2b & 2c & 2d \\ -a+e & -b+f & -c+g & -d+h \\ 3a & 3b & 3c & 3d \\ 2e+m & 2f+n & 2g+l & 2h+o \end{bmatrix}$

b) $3 \times (R3)$; $(R4) - 2(R2)$

#6. Let $\mathbf{y} = [2 \quad 4]$ and $\mathbf{u} = [6 \quad 2]$.

a) Compute the vector $\mathbf{z}$ such that $\mathbf{z} = \frac{\mathbf{y} \bullet \mathbf{u}}{\mathbf{u} \bullet \mathbf{u}} \mathbf{u}$, where $\bullet$ is the dot-product operator. `[10pt]`

b) Draw the vector $\mathbf{y}$, $\mathbf{u}$, and $\mathbf{z}$ in a two-dimensional space as precisely as possible. `[10pt]`

Difficulty: Medium
Amount of work: 20%
Solution: $\mathbf{z} = [3 \quad 1]$. Students are expected to mark the vectors in 2D grid, where $\mathbf{y}$ and $\mathbf{z}$ are overlapped.

Write your name before detaching this page. Your Name: ______________________________

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