#1. Show that the set of the following vectors are linearly indepdent. [15pt]

$$(2,3,0),\ (0,2,-1),\ (4,8,-1)$$

If the vectors are independent, $a_1=a_2=a_3$ should be satisfied

$a_1(2,3,0)+a_2(0,2,-1)+a_3(4,8,-1)=(0,0,0)$

$2a_1+0a_2+4a_3=0$ ① ... $a_1=-2a_3$ ④

$3a_1+2a_2+8a_3=0$ ② ... ②→ $3(-2a_3)+2(-a_3)+8a_3=0$.

$0a_1-a_2-a_3=0$ ③ ... $a_2=-a_3$ ⑤

$a_1=-2a_3$

$a_2=-a_3$

($a_3=0$, $a_1=a_2=a_3=0$. vectors are independent.)

$a_1=a_2=a_3$

→ $-2a_3=-a_3=+a_3$

Only $a_3=0$ can validate the equation

So, $a_3=0$.

$a_1=a_2=a_3=0$

the vectors above are independent

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#2. Complete the following theorem. [20pt]

For a $n \times n$ matrix A, the followings are all equivalents.

- (invertibility) The matrix A is invertible.
- (determinant) A has non-zero determinant
- (solution of $A\mathbf{x} = \mathbf{b}$) unique solution
- (singularity) non singular matrix A
- (column vectors) column vectors are independent

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#3. Prove the following statement. **[15pt]**

- For a 2×2 matrix A, if its column vectors are independent, then its row vectors are independent.

$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ $\quad a_{11} \cdot a_{22} - a_{12} a_{21} \neq 0$

· column vectors $(a_{11}, a_{21}), (a_{12}, a_{22})$
are independent

$a(a_{11}, a_{21}) + b(a_{12}, a_{22}) = (0,0)$

$a \cdot a_{11} + b a_{12} = 0$

$a \cdot a_{21} + b a_{22} = 0$

$a = b = 0$

row vectors
are independent $(a_{11}, a_{12}), (a_{21}, a_{22})$

$c(a_{11}, a_{12}) + d(a_{21}, a_{22}) = (0,0)$ is this true? or your assumption? or your claim?

$c a_{11} + d a_{21} = 0$

$c a_{12} + d a_{22} = 0$

if (c & $d \neq 0$)

Seem to apply 귀류법 (proof by contradiction).

$c a_{12} + d\left(a_{21} \cdot \frac{a_{12}}{a_{11}}\right) = 0$ (why?)

$c a_{12} + d a_{22} = 0$

$a_{21} \cdot \frac{a_{12}}{a_{11}} = a_{22}$ ← what if $a_{11} = 0$?

$a_{21} \cdot a_{12} = a_{11} \cdot a_{22}$ (False) $\because$ $a_{11} a_{22} \neq a_{12} a_{21}$

non-zero determinant.

there fore, $c = d = 0$.

row vectors are independent.

#4. Write the matrix formular for the following system of linear equation. Find the inverse of the coefficient matrix. Find the solution to the system of linear equation in vector form. [15pt]

$$\begin{aligned} 2x + 3y &= 13 \\ 4x + 2y &= 14 \end{aligned}$$

$\begin{bmatrix} 2 & 3 \\ 4 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 13 \\ 14 \end{bmatrix}$

$Ax = b$

$A^{-1} = -\frac{1}{8} \begin{bmatrix} 2 & -3 \\ -4 & 2 \end{bmatrix}$

$AA^{-1} = I = -\frac{1}{8} \begin{bmatrix} 2 & -3 \\ -4 & 2 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ 4 & 2 \end{bmatrix}$

$= -\frac{1}{8} \begin{bmatrix} -8 & 0 \\ 0 & -8 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\begin{bmatrix} x \\ y \end{bmatrix} = -\frac{1}{8} \begin{bmatrix} 2 & -3 \\ -4 & 2 \end{bmatrix} \begin{bmatrix} 13 \\ 14 \end{bmatrix}$

$= -\frac{1}{8} \begin{bmatrix} \cancel{26} - 42 \\ -52 + 28 \end{bmatrix} = -\frac{1}{8} \begin{bmatrix} -16 \\ -24 \end{bmatrix} = \begin{bmatrix} +2 \\ +3 \end{bmatrix}$

$x = +2, \quad y = +3$

$2(+2) + 3(3) = 13$

$2(4) + 2(3) = 14.$

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#5. Suppose that $\{\mathbf{x}, \mathbf{y}, \mathbf{z}\}$ is a basis of 3-dimensional vector space. Carefully show that $\{\mathbf{x}, \mathbf{x}-\mathbf{y}, \mathbf{x}+\mathbf{y}-\mathbf{z}\}$ is a basis of 3-dimensional vector space as well. [15pt]

x, y, z should be 3-demensional vector.

addition between vector should be conducted with same demension.
also, the result of addition is still the same demension.

So, x-y and x+y-z are 3-demensional vectors

therefore, {x, x-y, x+y-z} is basis of 3-dimensional vector space.

cf) n-dimensional vector space

→ n-dimension vectors in the set.

The number of the element in the set is n.

X

0