Rc.

#1. Show that the set of the following vectors are linearly indepdent. [15pt]

$$(2,3,0),\ (0,2,-1),\ (4,8,-1)$$

If the vectors are independent, $a_1=a_2=a_3$ should be satisfied

$a_1(2,3,0)+a_2(0,2,-1)+a_3(4,8,-1)=(0,0,0)$

$2a_1+0a_2+4a_3=0$ ① … $a_1=-2a_3$ ④

$3a_1+2a_2+8a_3=0$ ② … ②$\to 3(-2a_3)+2(-a_3)+8a_3=0.$

$0a_1-a_2-a_3=0$ ③ … $a_2=-a_3$ ⑤

$a_1=-2a_3$

$a_2=-a_3$

($a_3=0$, $a_1=a_2=a_3=0$.

vectors are independent.)

$a_1=a_2=a_3$

$\to -2a_3=-a_3=+a_3$

Only $a_3=0$ can validate the equation

So, $a_3=0$.

$a_1=a_2=a_3=0$

the vectors above are Independent

15

#1. Show that the set of the following vectors are linearly dependent.`[15pt]`

$$(2,3,0),\ (0,2,-1),\ (4,8,-1)$$

If k vectors $v_1, \dots, v_k$ are linearly independent with real numbers $a_1, \dots, a_k$,
the sol'n of $a_1v_1 + \dots + a_kv_k = 0$ is only $a_1 = a_2 = \dots = a_k = 0$

not linearly independent means linearly dependent.

$$a_1\begin{pmatrix}2\\3\\0\end{pmatrix} + a_2\begin{pmatrix}0\\2\\-1\end{pmatrix} + a_3\begin{pmatrix}4\\8\\-1\end{pmatrix} = \begin{pmatrix}0\\0\\0\end{pmatrix}$$

if the sol'n is $a_1 = a_2 = a_3 = 0$, the vectors are linearly independent,
otherwise the vectors are linearly dependent.

$2a_1 + 4a_3 = 0$ ... ①
$3a_1 + 2a_2 + 8a_3 = 0$ ... ②
$-a_2 - a_3 = 0$ ... ③

$a_1 = -2a_3$ (∵ ①)
$a_2 = -a_3$ (∵ ③)

② $= 3(-2a_3) + 2(-a_3) + 8a_3 = 0 \times a_3 = 0$ ... ④

a_3 has infinite solution to satisfy ④
a_1 & a_2 is decided by a_3. then the vectors are not linearly independent.

if $a_3 = 1$, $a_1 = -2$ and $a_2 = -1$

$2 \times (-2) + 4 \times 1 = 0$
$3 \times (-2) + 2 \times (-1) + 8(1) = 0$
$-(-1) - 1 = 0$

a_3 has other solution except 0.

Re

#3. Prove the following statement. [15pt]

- For a 2×2 matrix A, if its column vectors are independent, then its row vectors are independent.

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \qquad a_{11} \cdot a_{22} - a_{12} a_{21} \neq 0$$

· column vectors $(a_{11}, a_{21}), (a_{12}, a_{22})$
are independent

$$a(a_{11}, a_{21}) + b(a_{12}, a_{22}) = (0,0)$$
$$a \cdot a_{11} + b a_{12} = 0$$
$$a \cdot a_{21} + b a_{22} = 0$$
$$a = b = 0$$

row vectors $(a_{11}, a_{12}), (a_{21}, a_{22})$
are independent

$$c(a_{11}, a_{12}) + d(a_{21} + a_{22}) = (0,0)$$

is this true?
or your assumption?
or your claim?

$$c a_{11} + d a_{21} = 0$$
$$c a_{12} + d a_{22} = 0$$

if (c & $d \neq 0$)

Seem to apply
귀류법
(proof by contradiction).

$$c a_{12} + d \, a_{21} \cdot \frac{a_{12}}{a_{11}} = 0$$

(why?)

$$c a_{12} + d \, a_{22} = 0$$

$$a_{21} \cdot \frac{a_{12}}{a_{11}} = a_{22}$$

what if $a_{11} = 0$?

$$a_{21} a_{12} = a_{11} a_{22} \quad \text{(false)} \quad \because \quad a_{11} a_{22} \neq a_{12} a_{21}$$

non-zero determinant.

there fore, $c = d = 0$.
row vectors are independent.

#3 by 21102092 Lee Jeong-yun.

Q. Prove the following statement.

"For a 2x2 matrix A, if its column vectors are independent, then its row vectors are independent."

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$x_1 = (a, c)$ $x_2 = (b, c)$

x_1 and x_2 is linearly independent.

So, A has non-zero determinant.

Consider, $A' = \begin{bmatrix} a & c \\ b & d \end{bmatrix}$

if $y_1 = (a, b)$ and $y_2 = (c, d)$ are linearly independent, row vectors in A are linearly independent.

y_1 & y_2 are linearly independent; then $|A'| \neq 0$.

$|A| = ad - bc \neq 0$ ($\because$ x_1 & x_2 are linearly independent.)

$|A'| = ad - bc \neq 0$

A' has non-zero determinant.

So column vectors in A' are linearly independent.

$y_1 = (a, b)$ $y_2 = (c, d)$

row vectors in A

$z_1 = (a, b)$ $z_2 = (c, d)$

$y_1 = z_1$ & $y_2 = z_2$

y_1 & y_2 $\rightarrow$ linearly independent

z_1 & z_2 $\rightarrow$ linearly independent

Re.

#5. Suppose that $\{\mathbf{x}, \mathbf{y}, \mathbf{z}\}$ is a basis of 3-dimensional vector space. Carefully show that $\{\mathbf{x}, \mathbf{x}-\mathbf{y}, \mathbf{x}+\mathbf{y}-\mathbf{z}\}$ is a basis of 3-dimensional vector space as well. [15pt]

x, y, z should be 3-demensional vector.

addition between vector should be conducted with same demension.
also, the result of addition is still the same demension.
So, x-y and x+y-z are 3-demensional vectors
therefore, {x, x-y, x+y-z} is basis of 3-dimensional vector space.

cf) n-dimensional vector space

→ n-dimension vectors in the set
The number of the element in the set is n.

X

0

#5 by 21102052 Lee Jeong-yun

Q. Suppose that $\{x, y, z\}$ is a basis of 3-dimensional vector space. Carefully show that $\{x, x-y, x+y-z\}$ is a basis of 3-dimensional vector space as well

---

Since $\{x, y, z\}$ is a basis of 3D vector space, any 3D vector v can be expressed by

$$v = a_1 x + a_2 y + a_3 z \qquad (a_1, a_2, a_3 \in \mathbb{R})$$

if $\{x, x-y, x+y-z\}$ is a basis of 3D vector space, then

$$v = b_1 x + b_2 (x-y) + b_3 (x+y-z)$$

should be satisfied.

In other words, v should be express by a linear combination of x, $x-y$, $x+y-z$.

$$v = a_1 x + a_2 y + a_3 z$$
$$= (b_1 + b_2 + b_3) x + (-b_2 + b_3) y + (-b_3) z$$

$$a_1 = b_1 + b_2 + b_3$$
$$a_2 = -b_2 + b_3$$
$$a_3 = -b_3$$

then

$$b_3 = -a_3$$
$$b_2 = -a_2 - a_3$$
$$b_1 = a_1 - (-a_2 - a_3) - (-a_3) = a_1 + a_2 + 2a_3$$

v can be expressed by

$$v = (a_1 + a_2 + 2a_3) x + (-a_2 - a_3)(x-y) + (-a_3)(x+y-z)$$

Since $(a_1, a_2, a_3 \in \mathbb{R})$,

$$(a_1 + a_2 + 2a_3, -a_2 - a_3, -a_3 \in \mathbb{R})$$

Thus, above expression any 3D vector v can be expressed as a linear combination of $\{x, x-y, x+y-z\}$, i.e., $\{x, x-y, x+y-z\}$ is a basis of 3D vector space.