efficiently use resource!

* time, memory, bandwidth of Network

Data Structures

Performance

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Contents

Data Structures . Performance

Motivation

Example

$$sum = \sum_{i=1}^{n} i$$

Algorithm A	Algorithm B	Algorithm C

`sum = 0`
for `i = 1` *to* `n`
    `sum = sum + i`
	`sum = 0`
	for `i = 1` *to* `n`
	`{`
	    for `j = 1` *to* `i`
	        `sum = sum + 1`
	`}`
		`sum = n * (n + 1) / 2`

Most Efficient

what is criteria?
∴ we should define.

- Lower complexity is better

- Usually the "best" solution to a problem balances various criteria such as time, space, generality, programming effort, and so on.

- Time complexity:
  - Time requirement
  - the time it takes to execute

- Space complexity:
  - Space requirements
  - the memory it needs to execute

Sum from 1 to n

- How much time to add 1 … n

- Algorithm A

```java
long n = 10000;

// Algorithm A
long t0 = System.currentTimeMillis();
long sum = 0;
for(long i = 1; i <= n ; i++)
    sum = sum+i;
long t1 = System.currentTimeMillis();
System.out.println(sum+" : "+(t1-t0));
```

- Algorithm B

```java
//Algorithm B
t0 = System.currentTimeMillis();
sum = 0;
for(long i = 1; i <= n ; i++)
    for(long j = 1; j<=i; j++)
        sum = sum+1;
t1 = System.currentTimeMillis();
System.out.println(sum+" : "+(t1-t0));
```

- Algorithm C

```java
// Algorithm C
t0 = System.currentTimeMillis();
sum = n*(n+1)/2;
t1 = System.currentTimeMillis();
System.out.println(sum+" : "+(t1-t0));
```

- Result

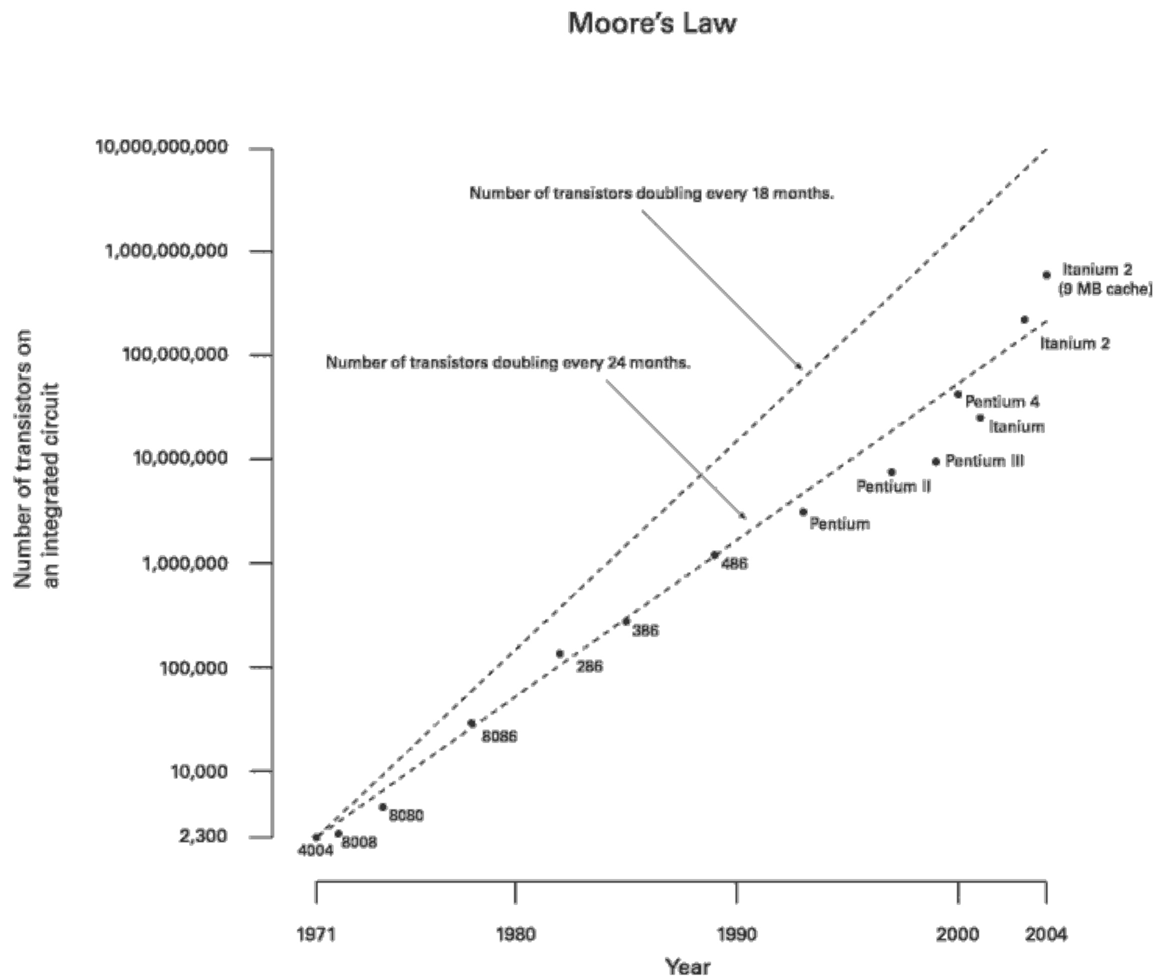
```
Problems  @ Javadoc  Declaration  Console
<terminated> AlgorithmABC [Java Application] C:₩Program Files₩Java₩
50005000 : 0
50005000 : 95
50005000 : 0
```

Factors

- Factors that determine running time of a program
  - problem size
  - basic algorithm / actual processing
  - memory access speed
  - CPU/processor speed
  - the of processors?
  - compiler/linker optimization?

Moore's law

upgrade hardware.



process of measuring complexity => analysis of algorithm.

focusing on time complexity!

time > space

Data Structures . Performance

Measuring an Algorithm Efficiency

Terminology

- Analysis of algorithms
  - the process of measuring the complexity of algorithms
- Problem size
  - the number of items that an algorithm processes
- Basic operation
  - the most significant contributor to its total time requirement
  - the most frequent operation is not necessarily the basic operation such as assignments, control loop
  - Simplified analysis can be based on : number of arithmetic operations performed, Number of comparisons made, Number of times through a critical loop, Number of array elements accessed, etc
- directly proportional
  - the time requirement increases by some factor
- growth-rate function: T(n)
  - how an algorithm's
  - The number of basic operations for n

Constant time

- T(n)=3

```
// Algorithm C
t0 = System.currentTimeMillis();
sum = n*(n+1)/2;
t1 = System.currentTimeMillis();
System.out.println(sum+" : "+(t1-t0));
```

3 operations

constant algorithm

Linear time

```
long n = 10000;

// Algorithm A
long t0 = System.currentTimeMillis();
long sum = 0;
for(long i = 1; i <= n ; i++)
    sum = sum+i;
long t1 = System.currentTimeMillis();
System.out.println(sum+" : "+(t1-t0));
```

#(n+1) assignments

#(n) additions

loop control

#(n+1) assignments #(n) addition

#(n+1) comparison

- The number of basic operations

n	0	1	2	3	4	5
T(n)	0	1	2	3	4	5

- T(n) = n

= → not basic operation

linear algorithm

Quadratic time

```
//Algorithm B
t0 = System.currentTimeMillis();
sum = 0;
for(long i = 1; i <= n ; i++)
     for(long j = 1; j<=i; j++)
          sum = sum+1;   constant
t1 = System.currentTimeMillis();
System.out.println(sum+" : "+(t1-t0));
```

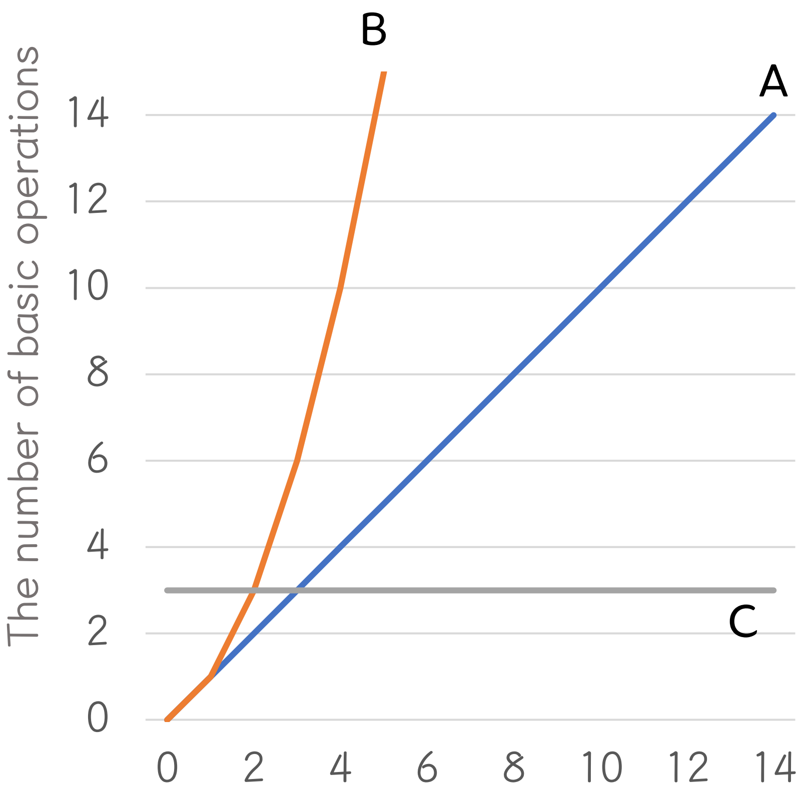
- The number of basic operations

n	0	1	2	3	4	5
i	0	1	1 2	1 2 3	1 2 3 4	1 2 3 4 5
j	0	1	1 12	1 12 123	1 12 123 1234	... 12345
T(n)	0	1	3	6	10	15

- $T(n) = \sum_{i=1}^{n} i = \frac{n(n+1)}{2} = \frac{n^2}{2} + \frac{n}{2}$

Necessary time

	A	B	C
Additions	n	n(n+1)/2	1
Multiplications			1
Divisions			1
Total	n	$\frac{n^2}{2}+\frac{n}{2}$	3



Growth-rate function

- Dominant term: the term the one dominating as n gets bigger

다항 시간

Polynomial-time algorithm

solvable
tractable

fast

f(n) T(n)	n=10^3	n=10^5	n=10^6	
$\log_2(n)$	10^{-5} sec	1.7 * 10^{-5} sec	2 * 10^{-5} sec	Logarithmic algorithm
n	10^{-3} sec	0.1 sec	1 sec	Linear algorithm
$n*\log_2(n)$	0.01 sec	1.7 sec	20 sec	
n^2	1 sec	3 hr	12 days	Quadratic algorithm
n^3	17 min	32 yr	317 centuries	
2^n	10^{285} centuries	10^{10000} years	10^{100000} years	Exponential algorithm

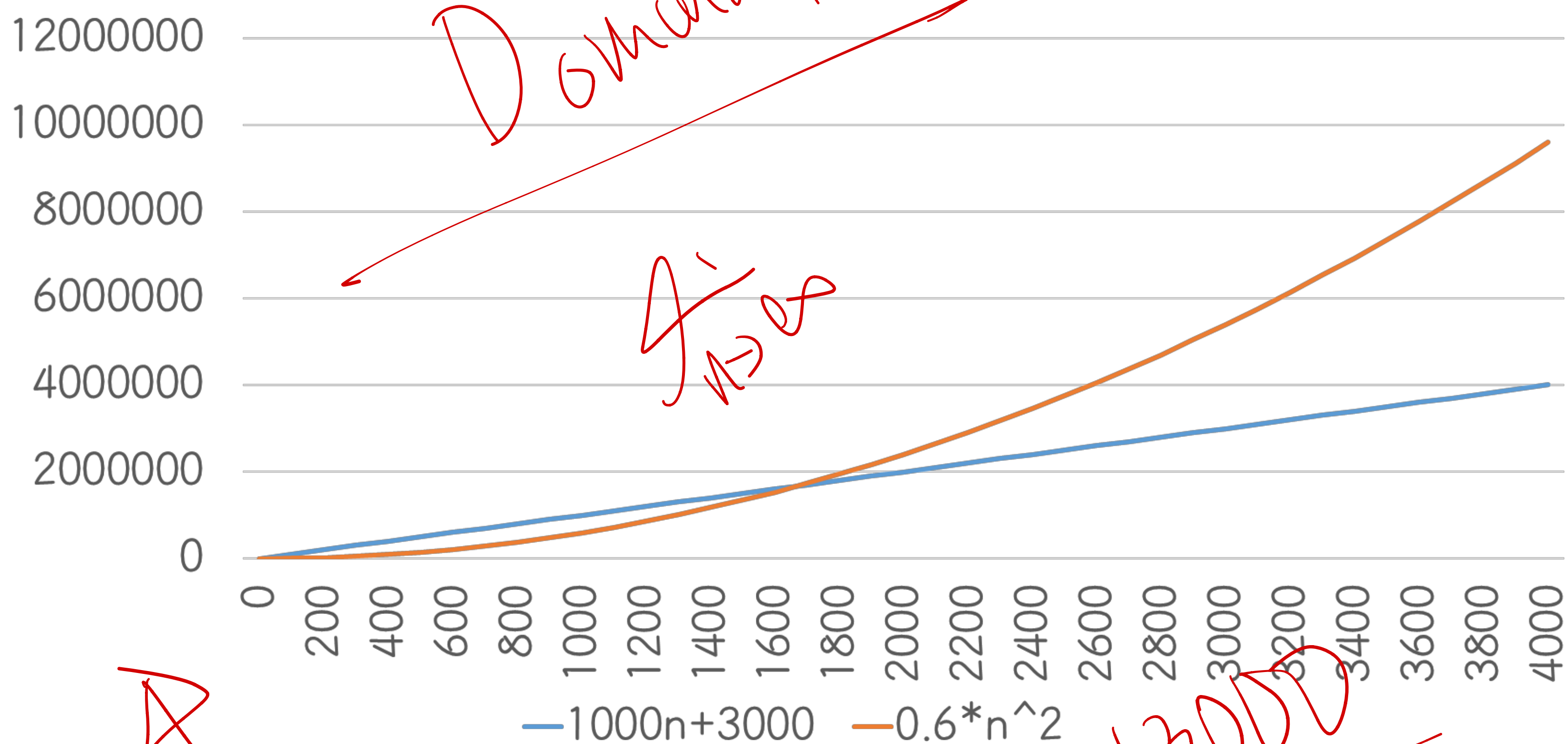
n!

slow

Data Structures . Performance

Asymptotic Notation

Asymptotic



Dominat

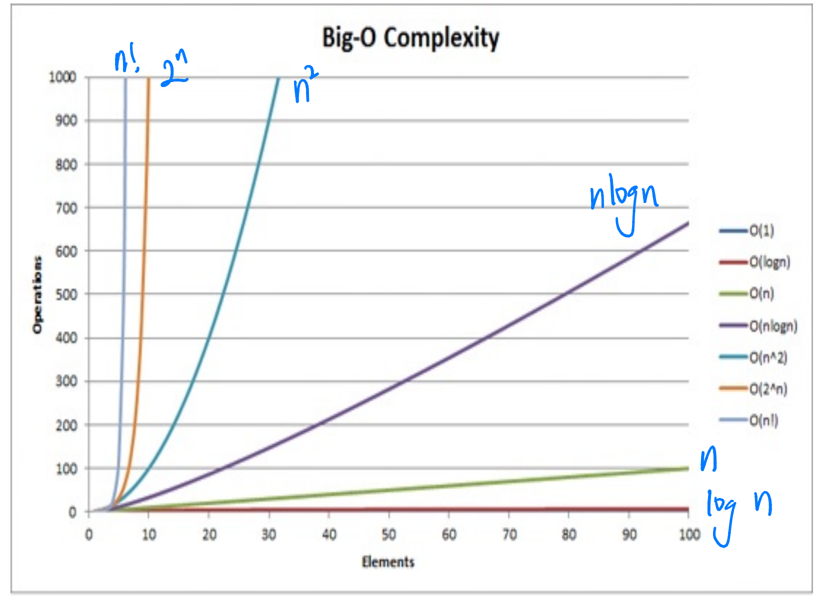
$\lim_{n \to \infty}$

$0.6n^2 + 1000n + 3000$

Khan Academy

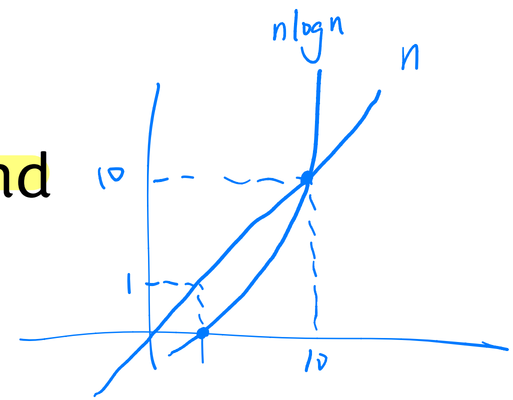
Asymptotic notations

- O
  - Big O notation
  - Asymptotic upper bound



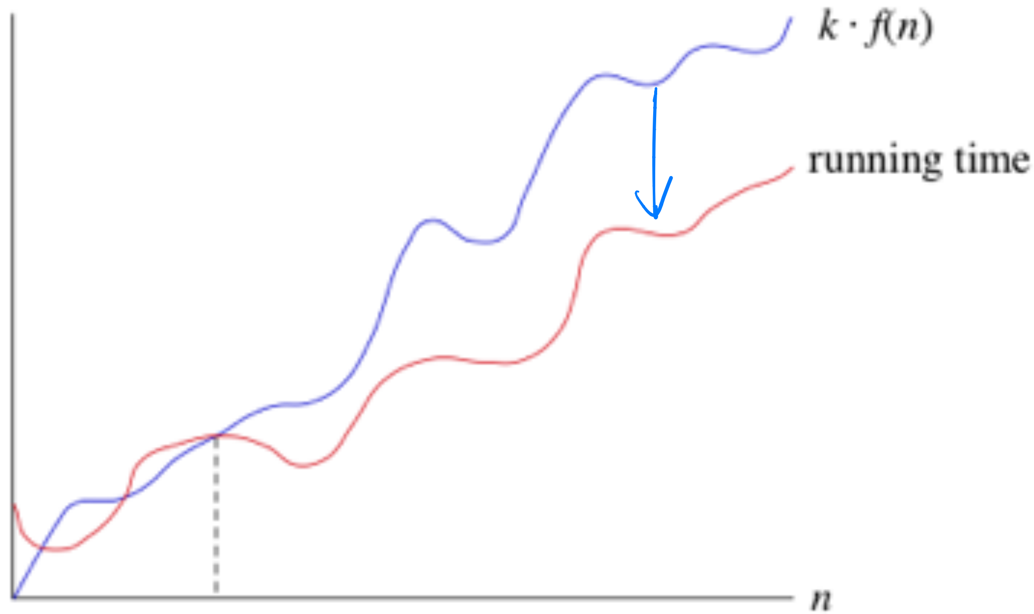
- Ω
  - Big Omega
  - Asymptotic lower bound

- Θ
  - Big Theta notation
  - Asymptotic upper and the lower bound



O-notation

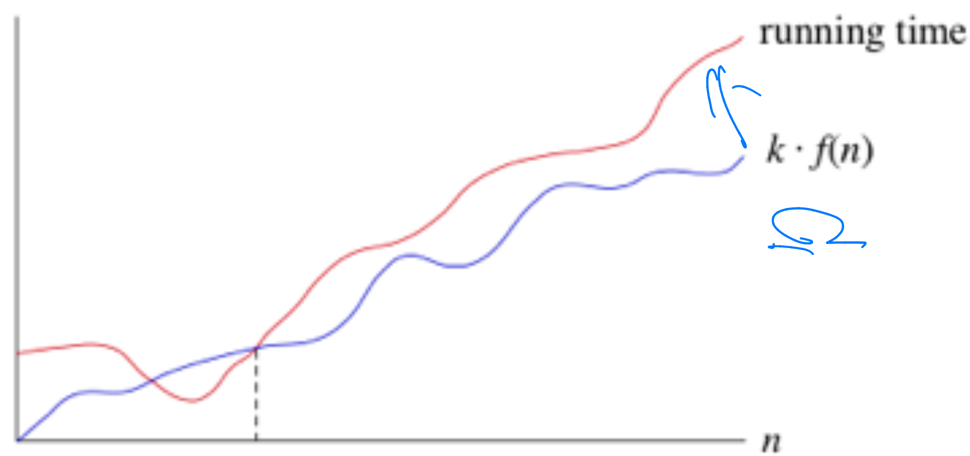
- Upper bound of the running time of an algorithm
- $f(n) \in O(n^2)$
  - n^2
  - $3n^2+2n$
  - $3n^2+n \log n$
  - $n \log n$
  - $3n$



Ω-notation

Lower

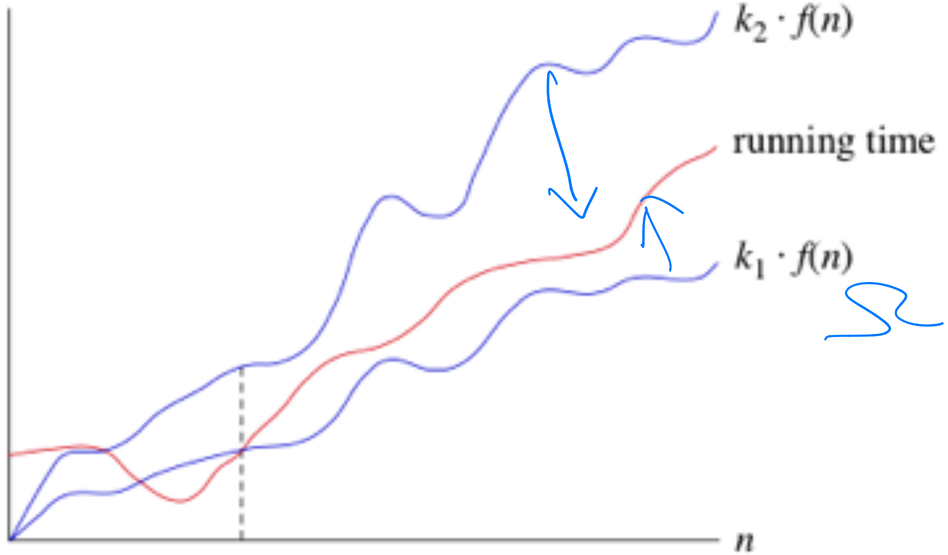
- ~~Upper~~ bound of the running time of an algorithm
- $f(n) \in \Omega(n^2)$
  - n^2
  - $3n^2+2n$
  - $3n^2+n \log n$
  - $7n^3+5n$



Θ-notation

- Tight bound of the running time of an algorithm
- $\Theta(n^2) = O(n^2) \cap \Omega(n^2)$

Θ

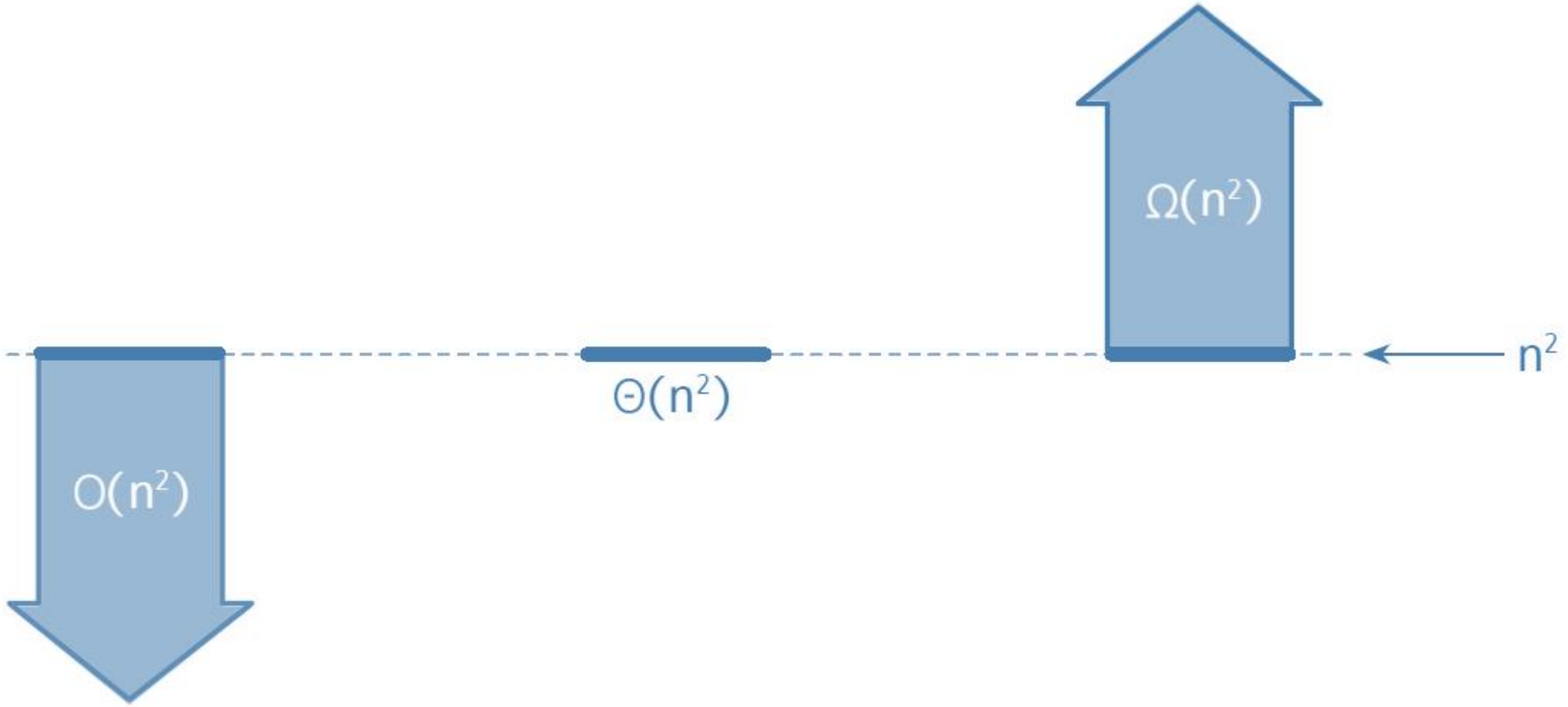


Ω

$\in \rightarrow =$

- Proper notation
  - $T(n) \in O(n^2)$
- General notation
  - $T(n) = O(n^2)$
- Wrong notation
  - $O(n^2) = T(n)$

Relation of notations




Thank you!
Questions?
Exit