Cache Memories and Virtual Memories

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Most parts are based on slides written by Brayant and O'Hallaon, CMU
(http://csapp.cs.cmu.edu/3e/instructors.html)

Contents

Previous Lecture

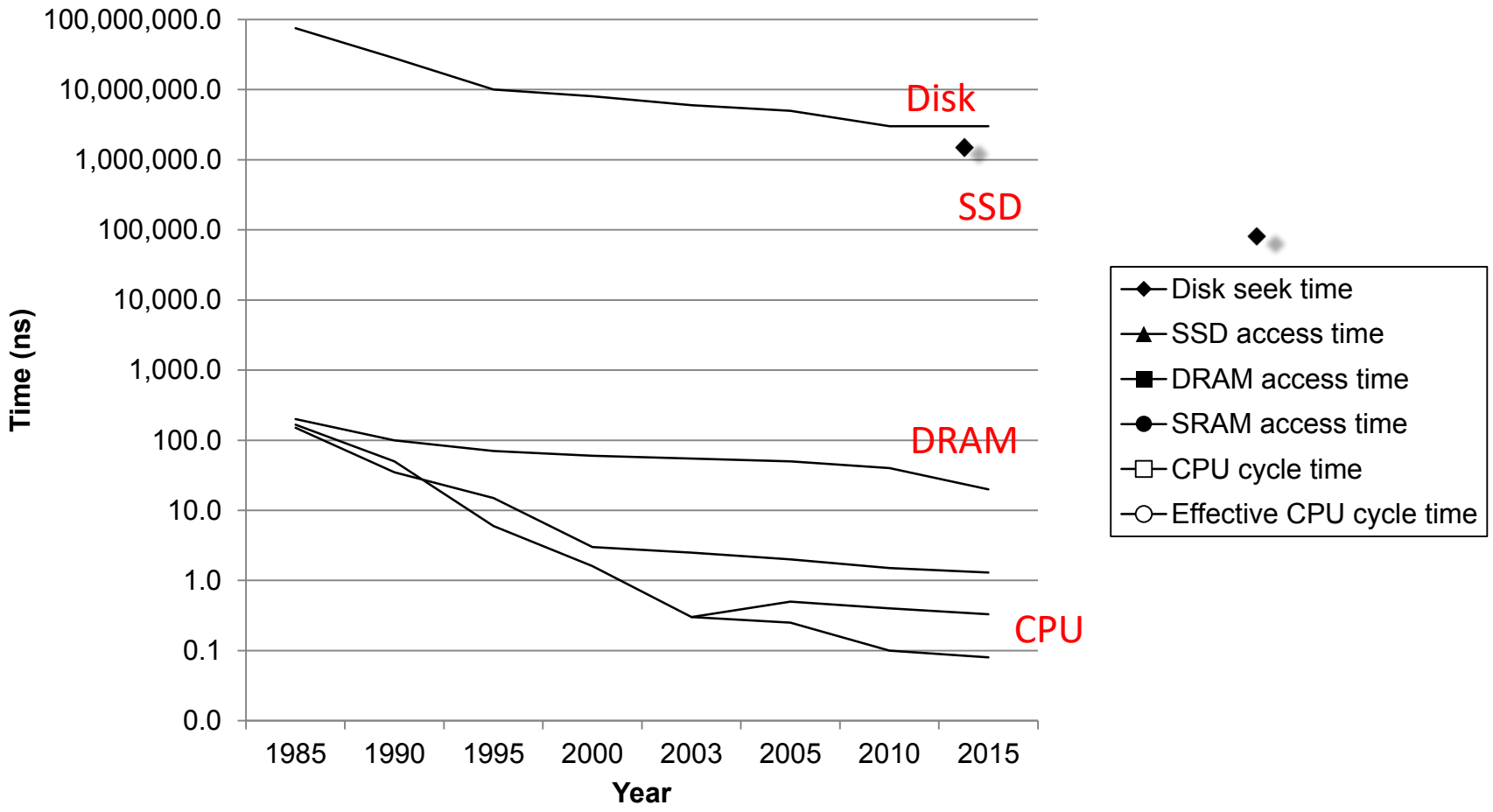
- Memory hierarchy

Today’s Lecture

- Cache memories
- Virtual memory

The CPU-Memory Gap

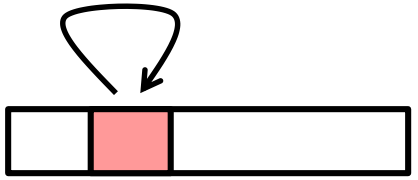
The gap widens between DRAM, disk, and CPU speeds.



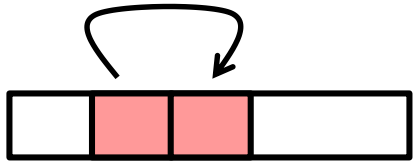
Locality

- **Principle of Locality:** **Programs use data and instructions with addresses near or equal to those they have used recently**

- **Temporal locality:**
  - Recently referenced items are likely to be referenced again in the near future



- **Spatial locality:**
  - Items with nearby addresses tend to be referenced close together



Caches

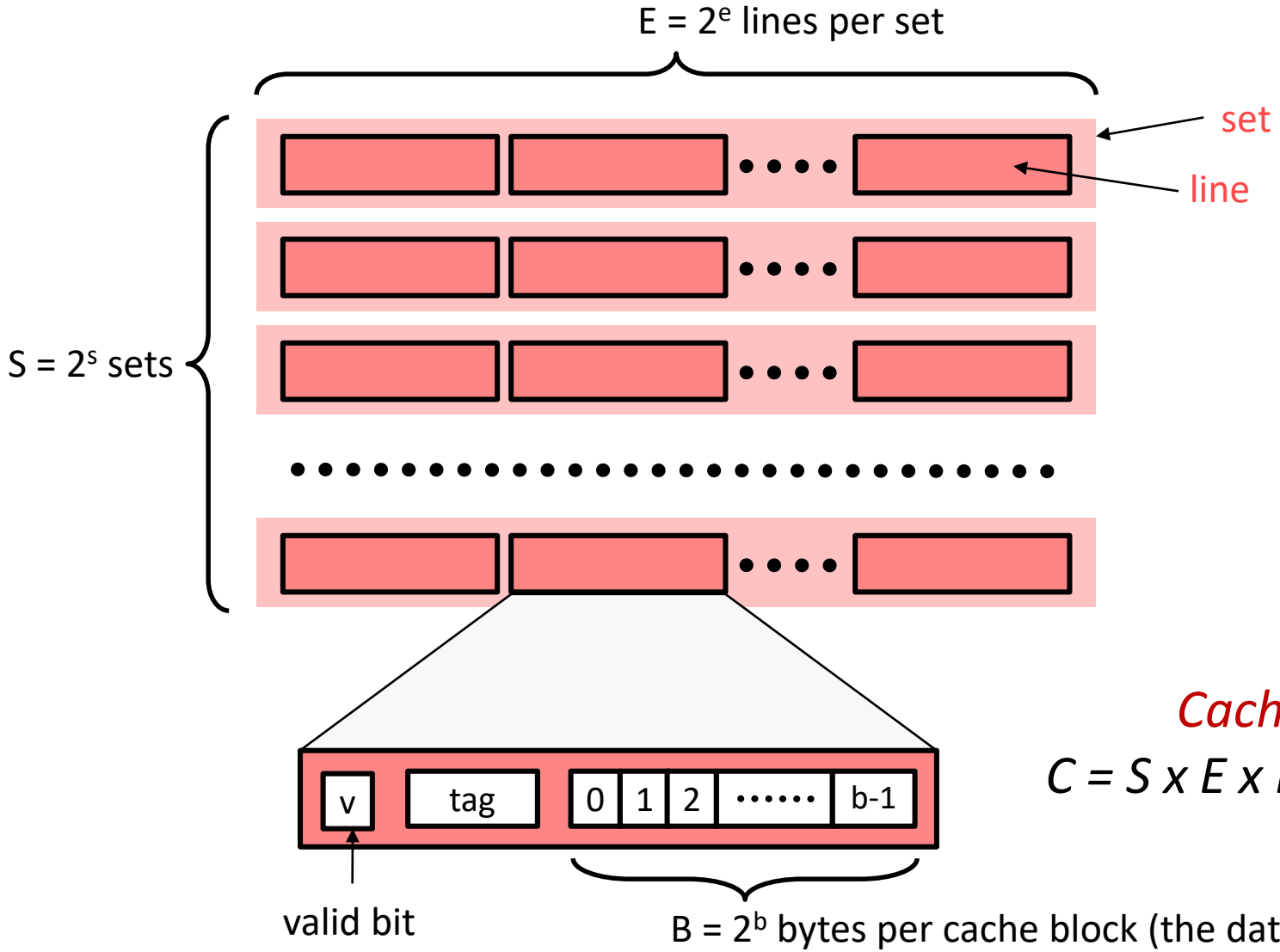
- ***Cache:*** **A smaller and faster storage device as a staging area for a subset of the entire data**

- **Fundamental idea of a memory hierarchy:**
  - For each *k*, device at level *k* serves as a cache for the device at level *k*+1

- **Why do memory hierarchies work?**
  - Because of locality, programs access the data at level *k* more often than the data at level *k*+1.
  - Thus, the storage at level *k*+1 can be slower.

- ***Big Idea:*** **The memory hierarchy creates a large pool of storage that costs the cheap storage near the bot tom, but that serves data at the rate of the fast storage near the top.**

General Cache Organization (S, E, B)



Cache size:

C = S x E x B data bytes

Let’s think about those numbers

- **Huge difference between a hit and a miss**
  - Could be 100x, if just L1 and main memory

- **Would you believe 99% hits is twice as good as 97%?**
  - Consider:
    cache hit time of 1 cycle
    miss penalty of 100 cycles

  - Q. Average access time?

    97% hits: 1 cycle + 0.03 * 100 cycles = **4 cycles**

    99% hits: 1 cycle + 0.01 * 100 cycles = **2 cycles**

- **This is why “miss rate” is used instead of “hit rate”**

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Memory Mountain Test Function

```
long data[MAXELEMS];  /* Global array to traverse */

/* test - Iterate over first "elems" elements of
 *      array “data” with stride of "stride",
 *      using 4x4 loop unrolling.
 */
int test(int elems, int stride) {
    long i, sx2=stride*2, sx3=stride*3, sx4=stride*4;
    long acc0 = 0, acc1 = 0, acc2 = 0, acc3 = 0;
    long length = elems, limit = length - sx4;

    /* Combine 4 elements at a time */
    for (i = 0; i < limit; i += sx4) {
        acc0 = acc0 + data[i];
        acc1 = acc1 + data[i+stride];
        acc2 = acc2 + data[i+sx2];
        acc3 = acc3 + data[i+sx3];
    }

    /* Finish any remaining elements */
    for (; i < length; i++) {
        acc0 = acc0 + data[i];
    }
    return ((acc0 + acc1) + (acc2 + acc3));
}
```

mountain/mountain.c

Call `test()` with many combinations of `elems` and `stride`.

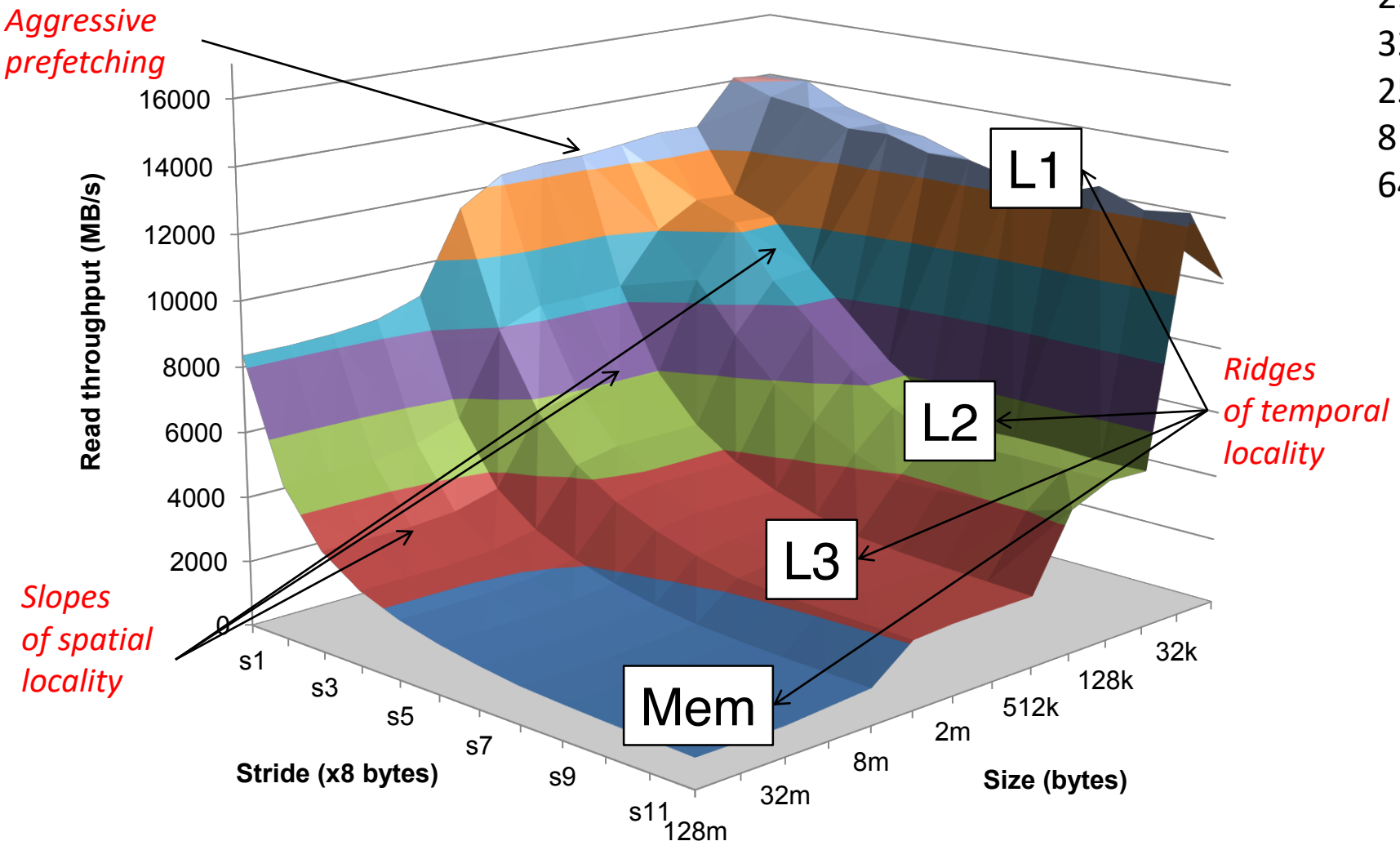
For each `elems` and `stride`:

1. Call `test()` once to warm up the caches.

2. Call `test()` again and measure the read throughput (MB/s)

The Memory Mountain

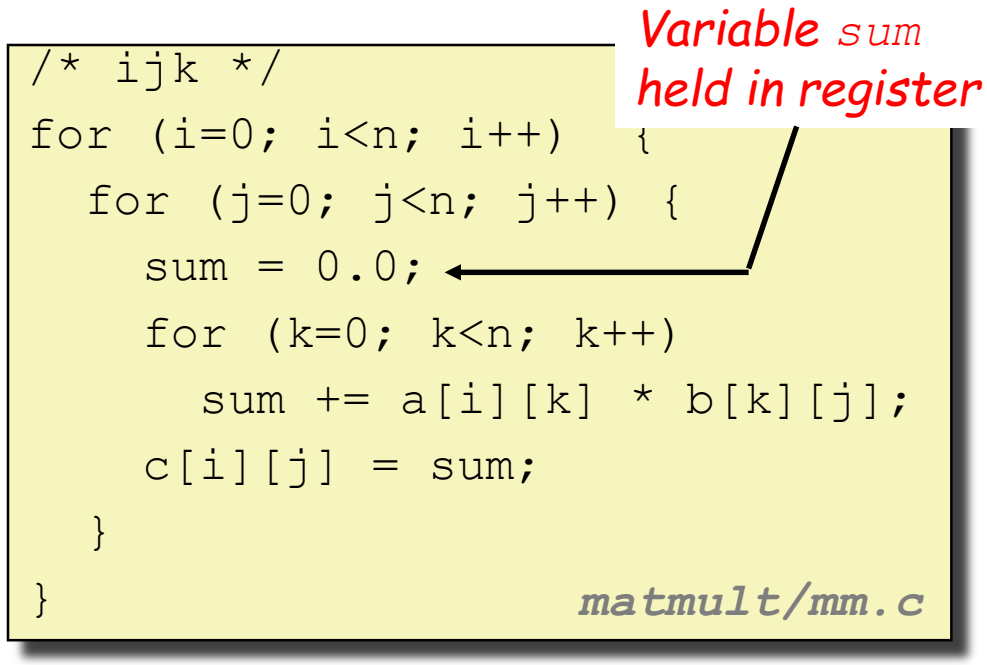
Core i7 Haswell
2.1 GHz
32 KB L1 d-cache
256 KB L2 cache
8 MB L3 cache
64 B block size



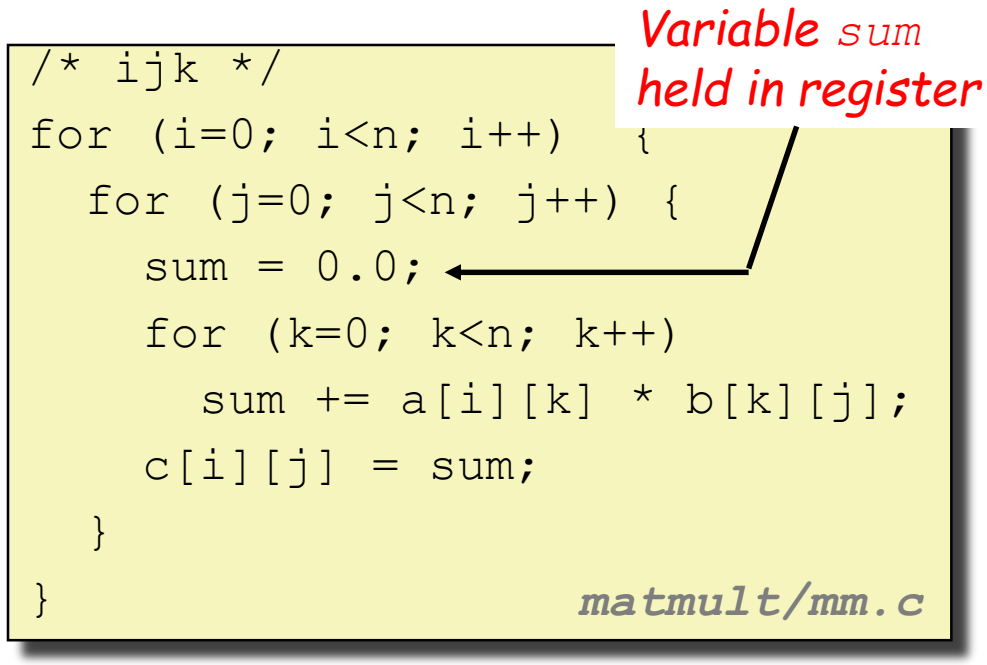
Today

- Cache organization and operation
- Performance impact of caches
  - The memory mountain
  - Rearranging loops to improve spatial locality
  - Using blocking to improve temporal locality

Matrix Multiplication Example

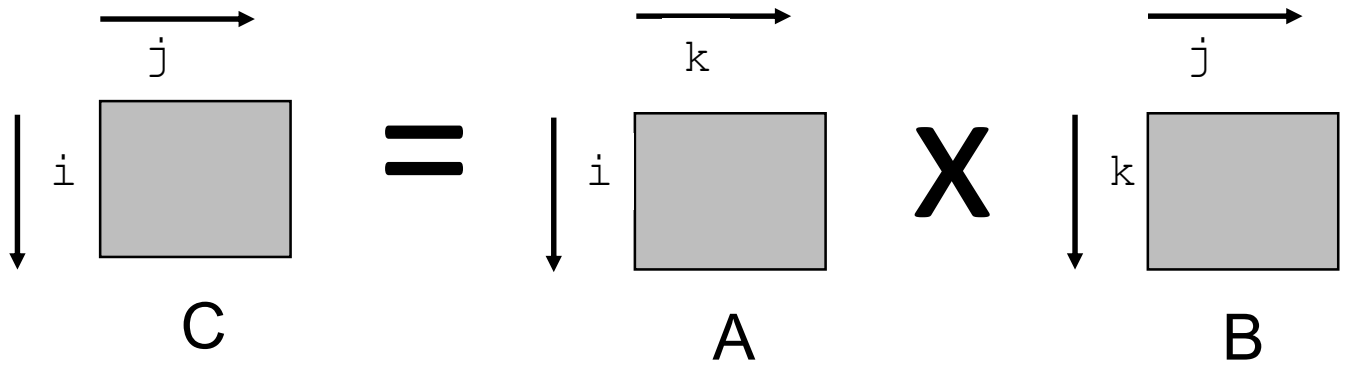


Practice: Matrix Multiplication Example



Miss Rate Analysis for Matrix Multiply

- **Assume:**
  - Block size = 32B (big enough for four doubles)
  - Matrix dimension (N) is very large
    - Approximate 1/N as 0.0
  - Cache is not even big enough to hold multiple rows
- **Analysis Method:**
  - Look at access pattern of inner loop



Layout of C Arrays in Memory (review)

- **C arrays allocated in row-major order**

- **Stepping through columns in one row:**
  - `for (i = 0; i < N; i++)`
    `sum += a[0][i];`
  - accesses successive elements
  - if block size (B) > sizeof(a_{ij}) bytes, exploit spatial locality
    - miss rate = sizeof(a_{ij}) / B

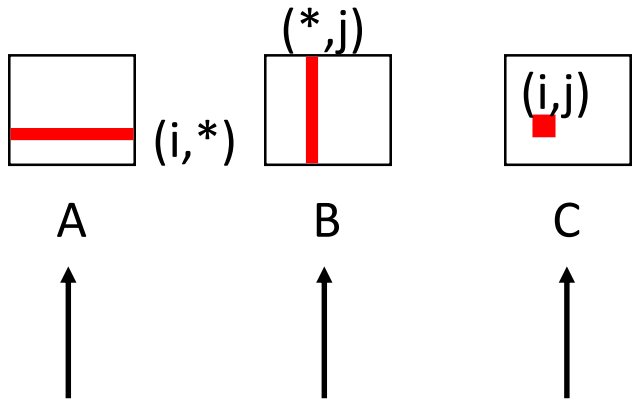
- **Stepping through rows in one column:**
  - `for (i = 0; i < n; i++)`
    `sum += a[i][0];`
  - accesses distant elements
  - no spatial locality!
    - miss rate = 1 (i.e. 100%)

Matrix Multiplication (ijk)

```
/* ijk */
for (i=0; i<n; i++)  {
  for (j=0; j<n; j++) {
    sum = 0.0;
    for (k=0; k<n; k++)
      sum += a[i][k] * b[k][j];
    c[i][j] = sum;
  }
}
```

matmult/mm.c

Inner loop:



Misses per inner loop iteration:

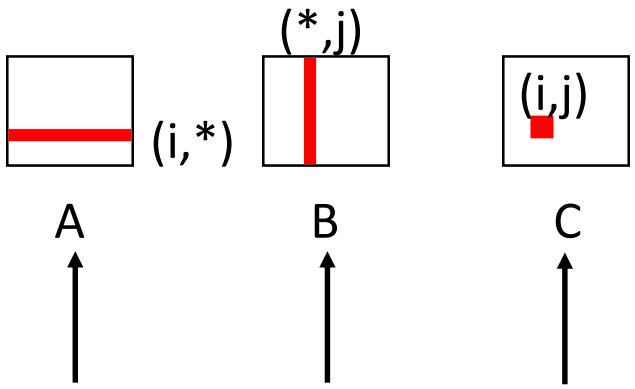
A	B	C
0.25	1.0	0.0

Matrix Multiplication (jik)

```
/* jik */
for (j=0; j<n; j++) {
  for (i=0; i<n; i++) {
    sum = 0.0;
    for (k=0; k<n; k++)
      sum += a[i][k] * b[k][j];
    c[i][j] = sum
  }
}
```

matmult/mm.c

Inner loop:



Misses per inner loop iteration:

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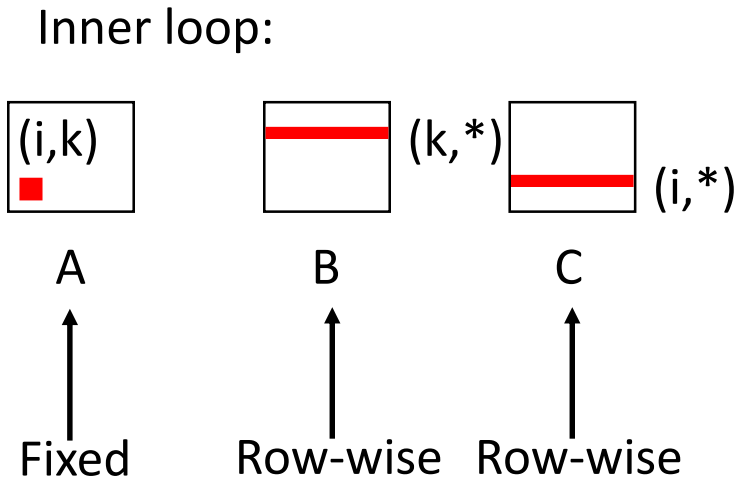
0.25	1.0	0.0

Matrix Multiplication (kij)

```
/* kij */
for (k=0; k<n; k++) {
  for (i=0; i<n; i++) {
    r = a[i][k];
    for (j=0; j<n; j++)
      c[i][j] += r * b[k][j];
  }
}
```

matmult/mm.c



Misses per inner loop iteration:

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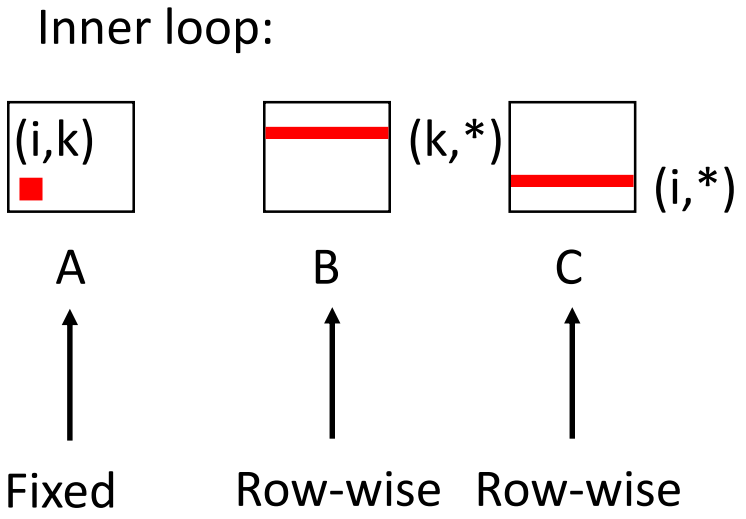
Matrix Multiplication (ikj)

```
/* ikj */
for (i=0; i<n; i++) {
  for (k=0; k<n; k++) {
    r = a[i][k];
    for (j=0; j<n; j++)
      c[i][j] += r * b[k][j];
  }
}
```

matmult/mm.c



Misses per inner loop iteration:

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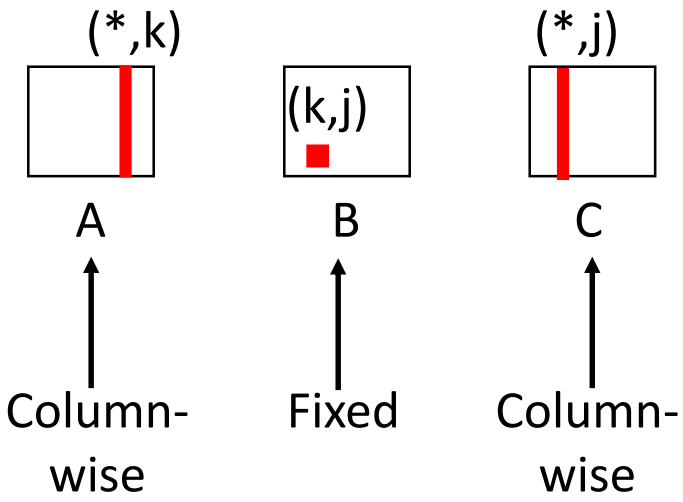
Matrix Multiplication (jki)

```
/* jki */
for (j=0; j<n; j++) {
  for (k=0; k<n; k++) {
    r = b[k][j];
    for (i=0; i<n; i++)
      c[i][j] += a[i][k] * r;
  }
}
```

matmult/mm.c

Inner loop:



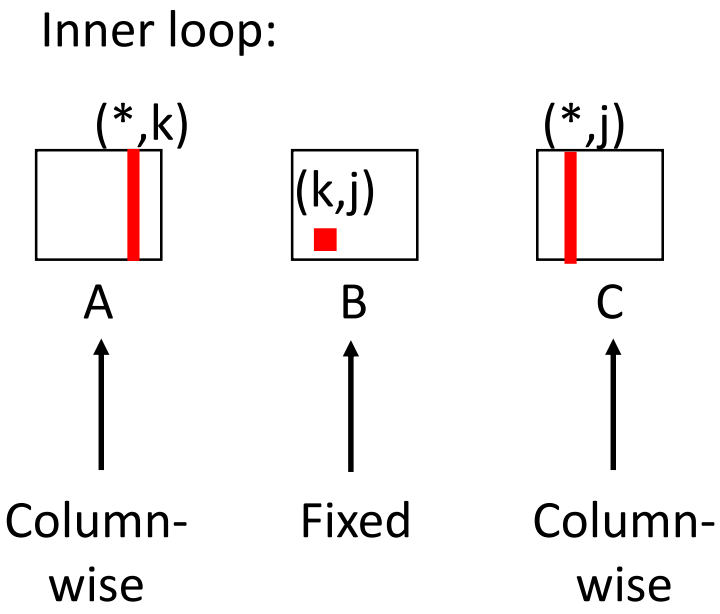
Misses per inner loop iteration:

A	B	C
1.0	0.0	1.0

Matrix Multiplication (kji)

```
/* kji */
for (k=0; k<n; k++) {
  for (j=0; j<n; j++) {
    r = b[k][j];
    for (i=0; i<n; i++)
      c[i][j] += a[i][k] * r;
  }
}
```

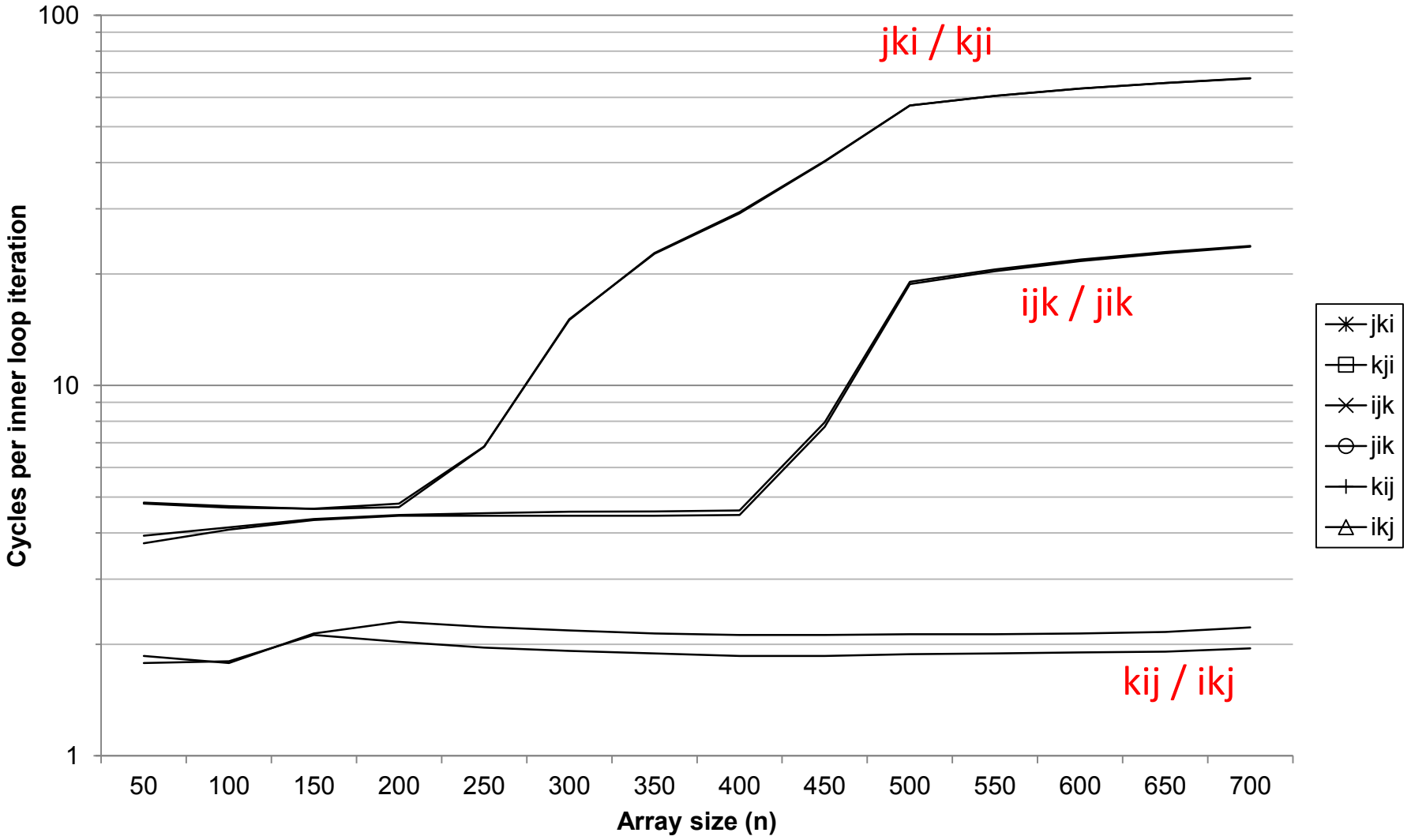
matmult/mm.c



Misses per inner loop iteration:

A	B	C
1.0	0.0	1.0

Core i7 Matrix Multiply Performance

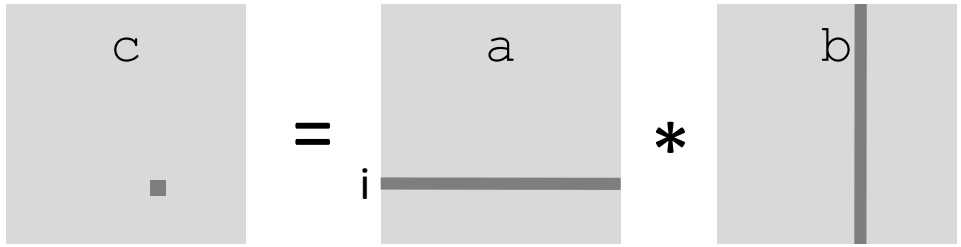


Today

- Cache organization and operation
- Performance impact of caches
  - The memory mountain
  - Rearranging loops to improve spatial locality
  - Using blocking to improve temporal locality

Example: Matrix Multiplication

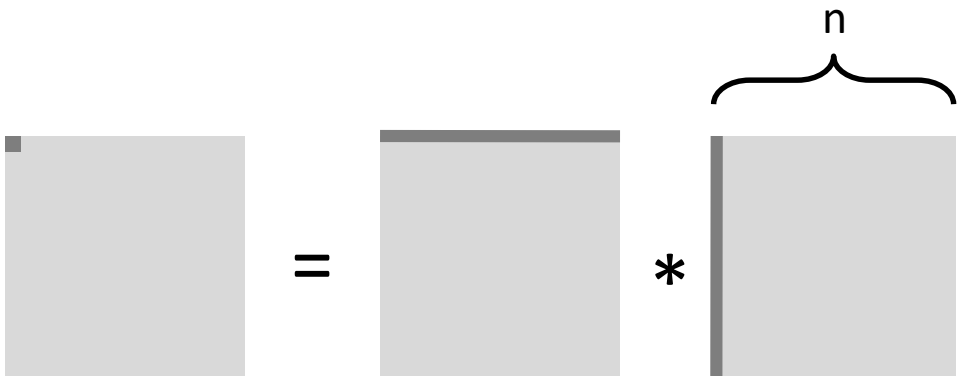
```
c = (double *) calloc(sizeof(double), n*n);

/* Multiply n x n matrices a and b  */
void mmm(double *a, double *b, double *c, int n) {
    int i, j, k;
    for (i = 0; i < n; i++)
        for (j = 0; j < n; j++)
            for (k = 0; k < n; k++)
                c[i*n + j] += a[i*n + k] * b[k*n + j];
}
```



Cache Miss Analysis

- **Assume:**
  - Matrix elements are doubles
  - Cache block = 8 doubles
  - Cache size C << n (much smaller than n)

- **First iteration:**
  - n/8 + n = 9n/8 misses



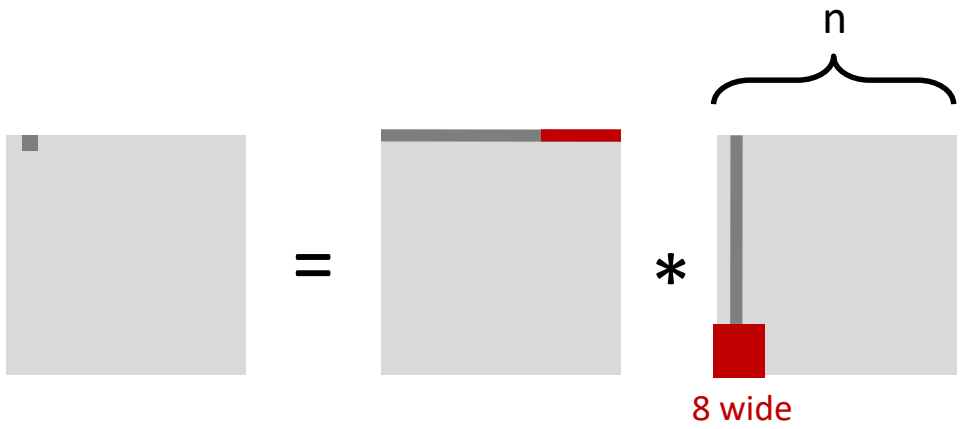
  - Afterwards in cache:
    (schematic)

Cache Miss Analysis

- **Assume:**
  - Matrix elements are doubles
  - Cache block = 8 doubles
  - Cache size C << n (much smaller than n)

- **Second iteration:**
  - Again:

    n/8 + n = 9n/8 misses

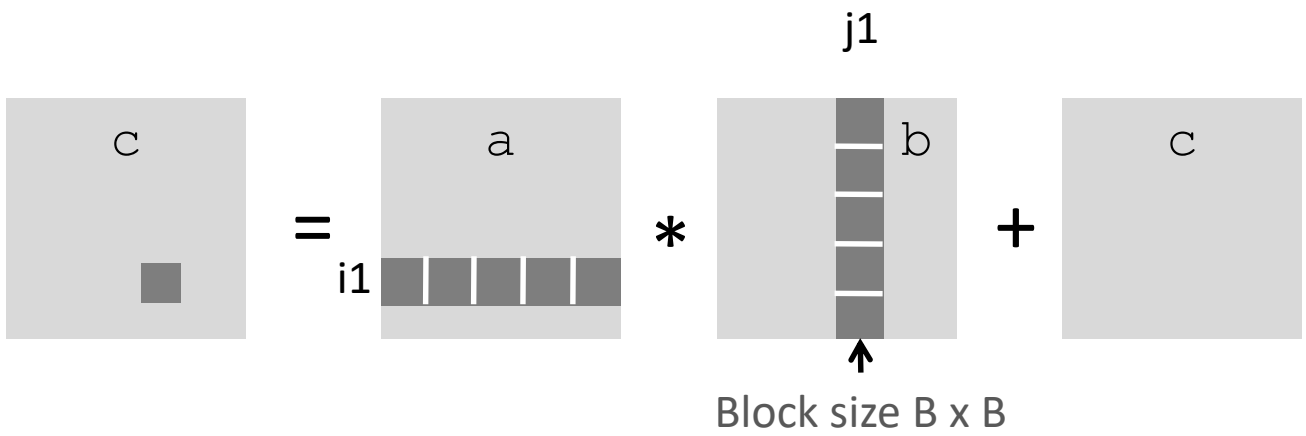


- **Total misses:**
  - $9n/8 * n^2 = (9/8) * n^3$

Blocked Matrix Multiplication

```
c = (double *) calloc(sizeof(double), n*n);

/* Multiply n x n matrices a and b  */
void mmm(double *a, double *b, double *c, int n) {
    int i, j, k;
    for (i = 0; i < n; i+=B)
        for (j = 0; j < n; j+=B)
            for (k = 0; k < n; k+=B)
                /* B x B mini matrix multiplications */
                for (i1 = i; i1 < i+B; i++)
                    for (j1 = j; j1 < j+B; j++)
                        for (k1 = k; k1 < k+B; k++)
                            c[i1*n+j1] += a[i1*n + k1]*b[k1*n + j1];
}
```

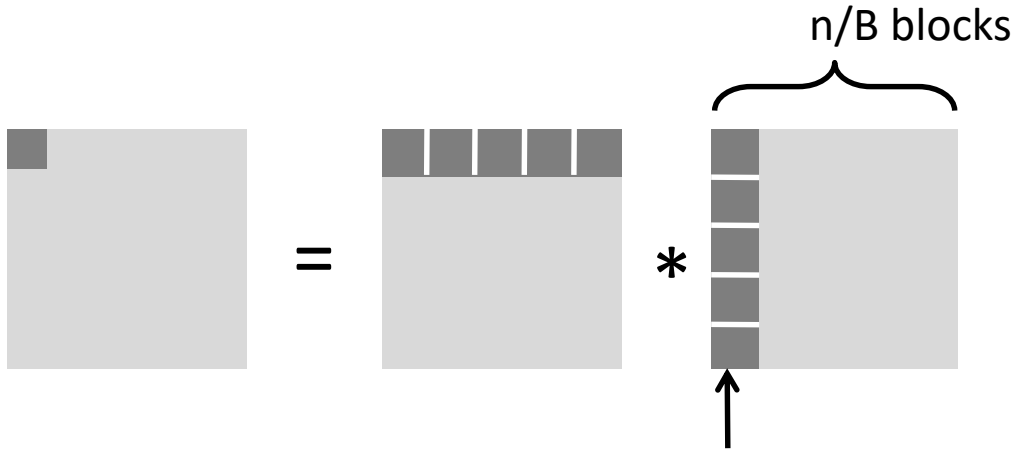
matmult/bmm.c



Cache Miss Analysis

- **Assume:**
  - Cache block = 8 doubles
  - Cache size C << n (much smaller than n)
  - Three blocks ■ fit into cache: $3B^2 < C$

- **First (block) iteration:**
  - $B^2/8$ misses for each block
  - $2n/B * B^2/8 = nB/4$
    (omitting matrix c)

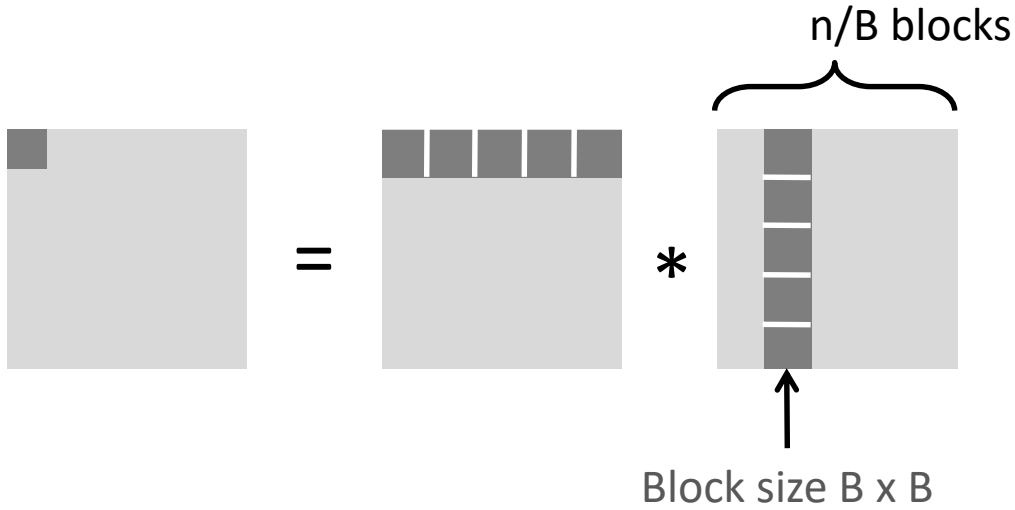


  - Afterwards in cache
    (schematic)

Cache Miss Analysis

- **Assume:**
  - Cache block = 8 doubles
  - Cache size C << n (much smaller than n)
  - Three blocks ■ fit into cache: $3B^2 < C$

- **Second (block) iteration:**
  - Same as first iteration
  - $2n/B * B^2/8 = nB/4$



- **Total misses:**
  - $nB/4 * (n/B)^2 = n^3/(4B)$

Blocking Summary

- **No blocking: (9/8) * n^3**
- **Blocking: 1/(4B) * n^3**
- **Suggest largest possible block size B, but limit $3B^2 < C$!**
- **Reason for dramatic difference:**
  - Matrix multiplication has inherent temporal locality:
    - Input data: $3n^2$, computation $2n^3$
    - Every array elements used O(n) times!
  - But program has to be written properly

Cache Summary

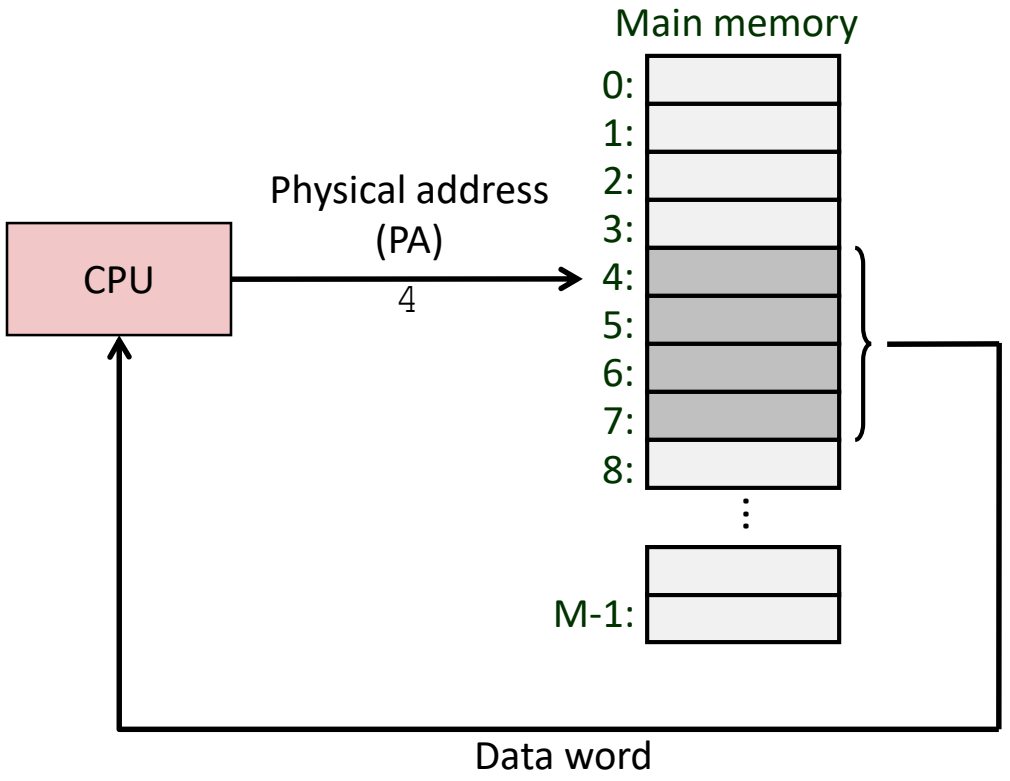
- **Cache memories can have significant performance impact**

- **You can write your programs to exploit this!**
  - Focus on the inner loops, where bulk of computations and memory accesses occur.
  - Try to maximize spatial locality by reading data objects with sequentially with stride 1.
  - Try to maximize temporal locality by using a data object as often as possible once it’s read from memory.

Virtual Memory

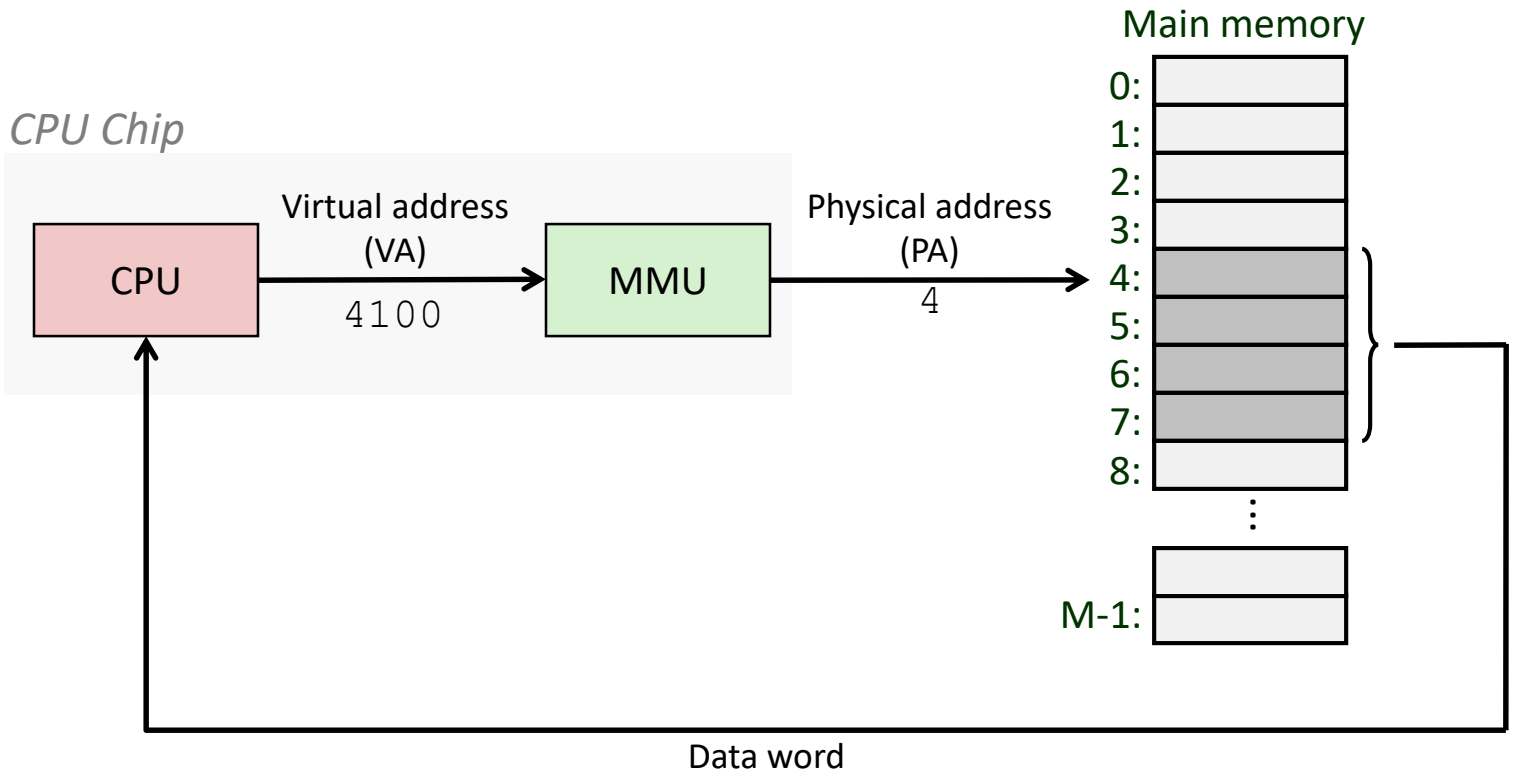
- **Address spaces**
- VM as a tool for caching
- VM as a tool for memory management
- VM as a tool for memory protection
- Address translation

A System Using Physical Addressing



- Used in “simple” systems like embedded microcontrollers in devices like cars, elevators, and digital pict ure frames

A System Using Virtual Addressing



- Used in all modern servers, laptops, and smart phones
- One of the great ideas in computer science

Address Spaces

- **Linear address space:** Ordered set of contiguous non-negative integer addresses:

  {0, 1, 2, 3 … }

- **Virtual address space:** Set of $N = 2^n$ virtual addresses

  {0, 1, 2, 3, …, N-1}

- **Physical address space:** Set of $M = 2^m$ physical addresses

  {0, 1, 2, 3, …, M-1}

Why Virtual Memory (VM)?

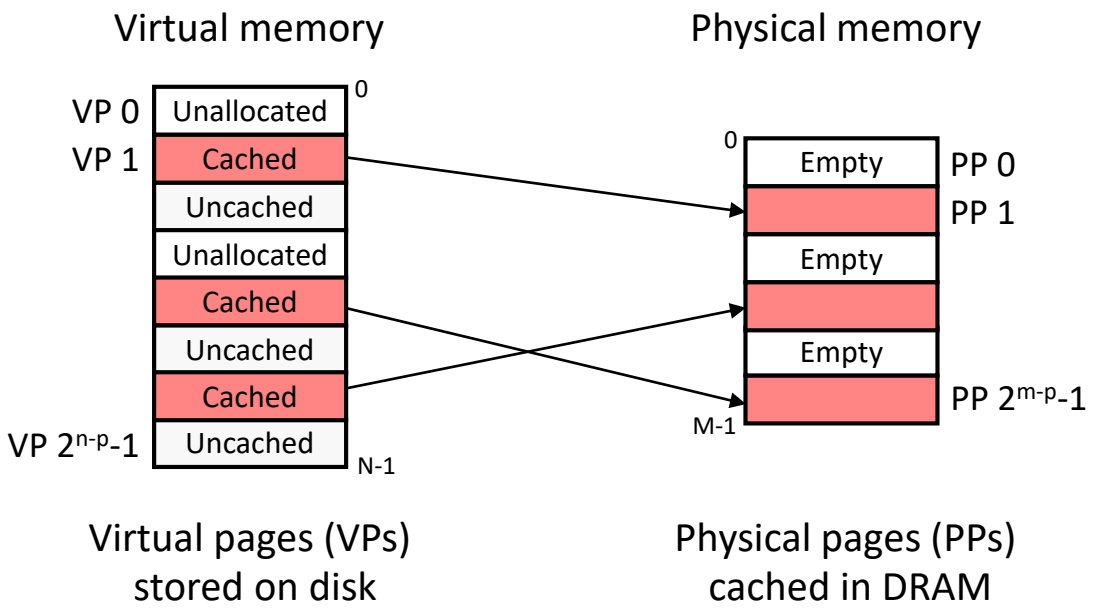
- **Uses main memory efficiently**
  - Use DRAM as a cache for parts of a virtual address space
- **Simplifies memory management**
  - Each process gets the same uniform linear address space
- **Isolates address spaces**
  - One process can’t interfere with another’s memory
  - User program cannot access privileged kernel information and code

Virtual Memory

- Address spaces
- **VM as a tool for caching**
- VM as a tool for memory management
- VM as a tool for memory protection
- Address translation

VM as a Tool for Caching

- **Conceptually, *virtual memory* is an array of N contiguous bytes stored on disk.**
- **The contents of the array on disk are cached in *physical memory* (*DRAM cache*)**
  - These cache blocks are called *pages* (size is $P = 2^p$ bytes)



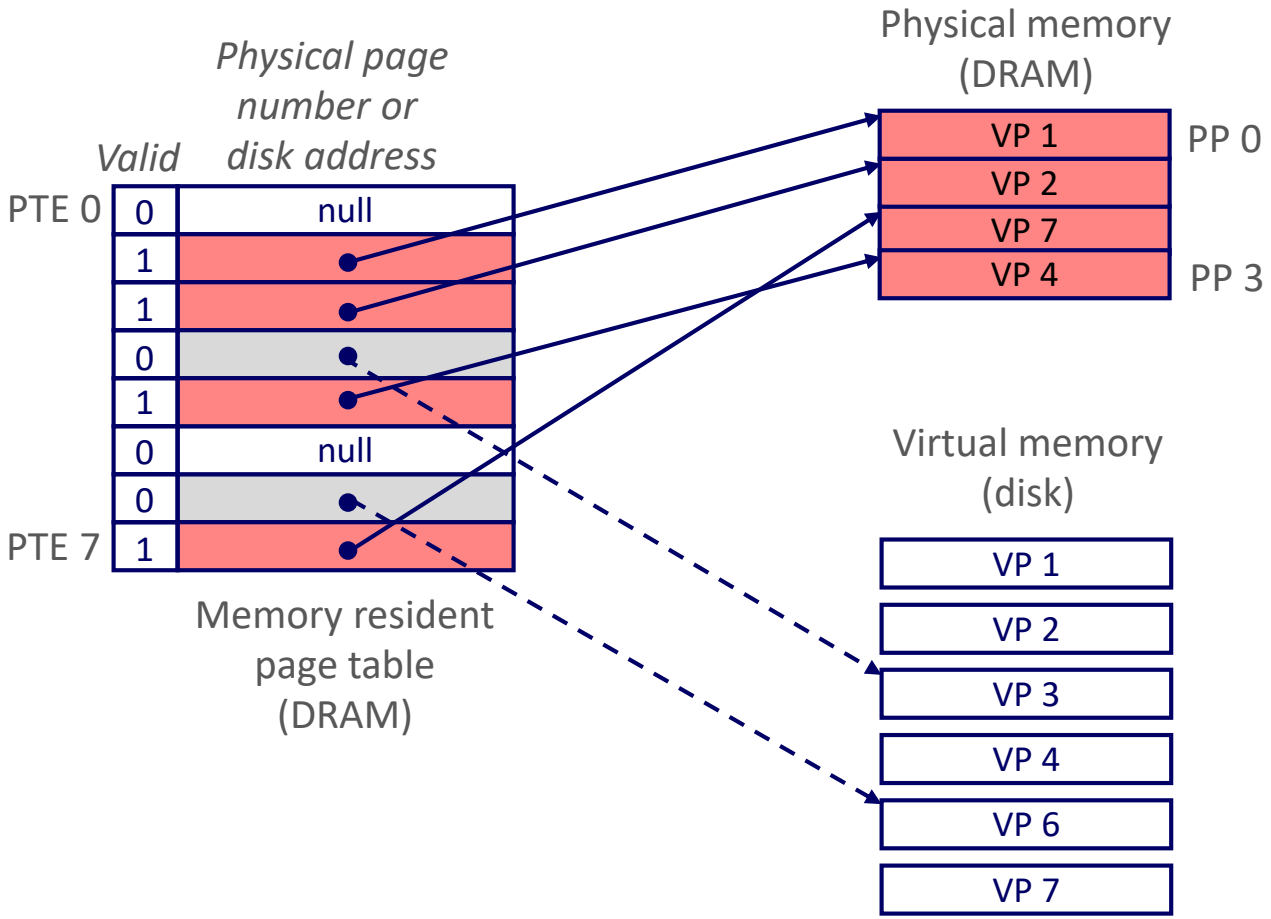
DRAM Cache Organization

- **DRAM cache organization driven by the enormous miss penalty**
  - DRAM is about ***10x*** slower than SRAM
  - Disk is about ***10,000x*** slower than DRAM

- **Consequences**
  - Large page (block) size: typically 4 KB, sometimes 4 MB
  - Fully associative
    - Any VP can be placed in any PP
  - Highly sophisticated, expensive replacement algorithms
  - Write-back rather than write-through

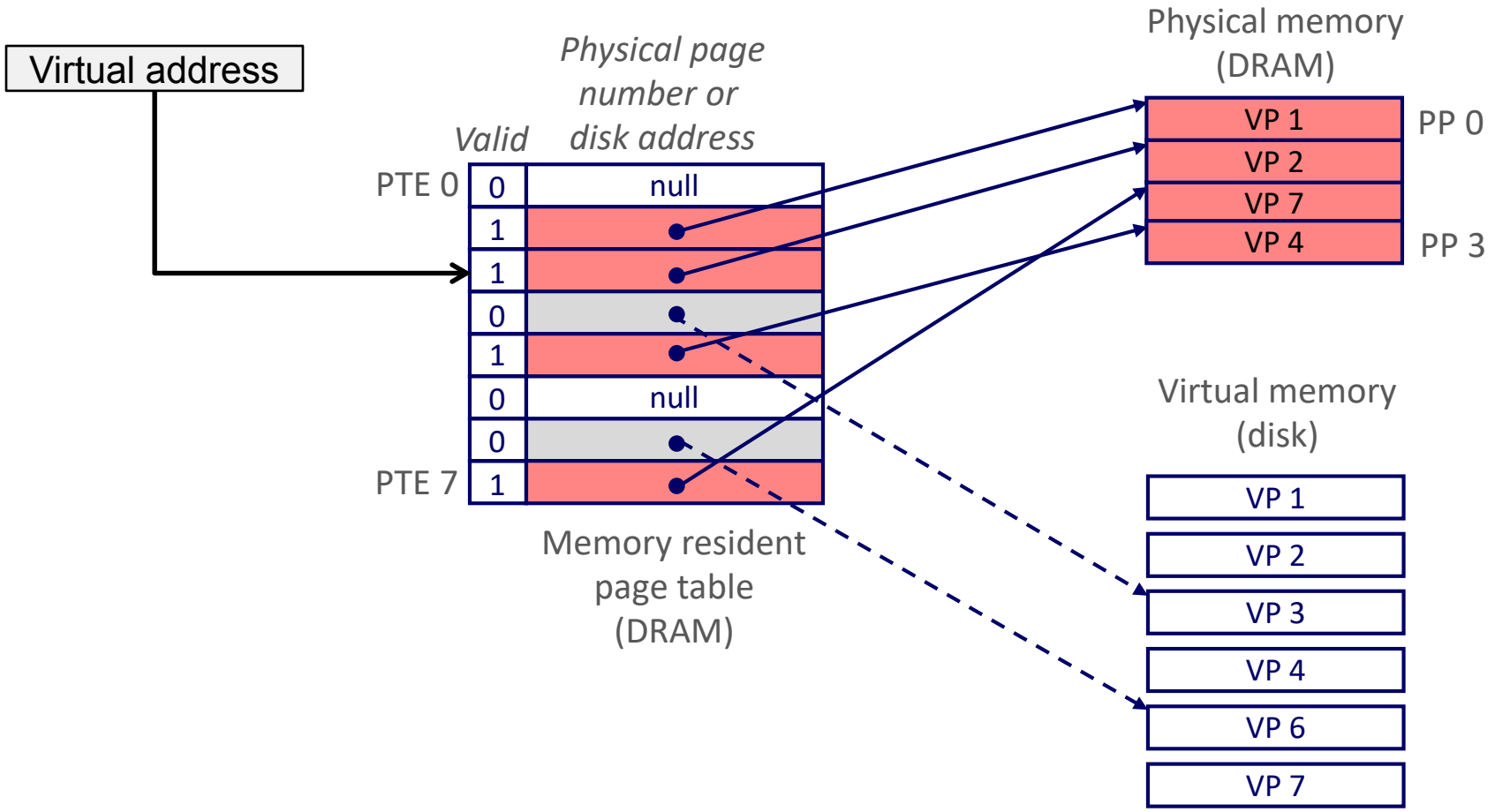
Enabling Data Structure: Page Table

- **A *page table* is an array of page table entries (PTEs) that maps virtual pages to physical pages.**
  - Per-process kernel data structure in DRAM



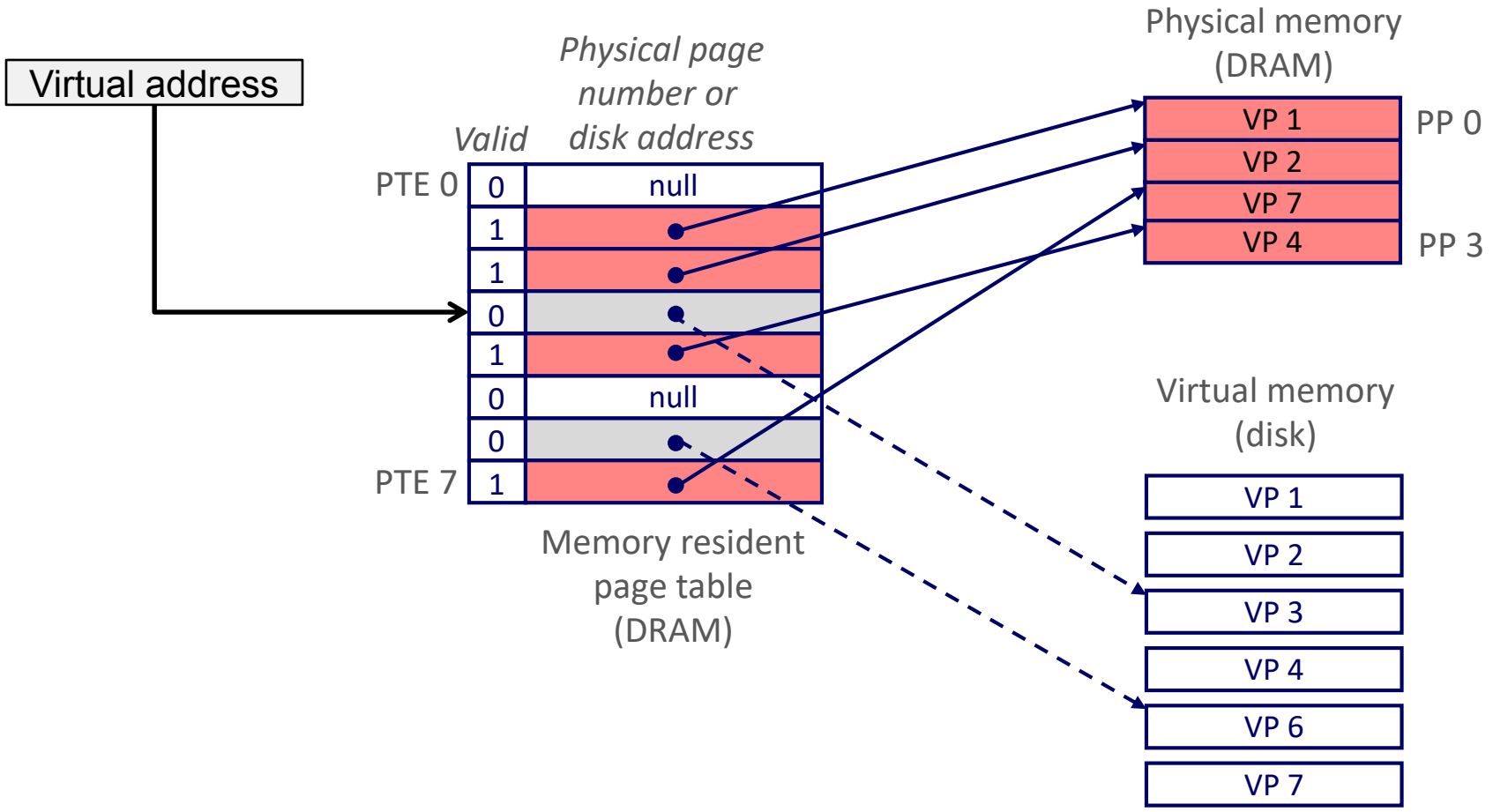
Page Hit

- ***Page hit:*** **reference to VM word that is in physical memory (DRAM cache hit)**



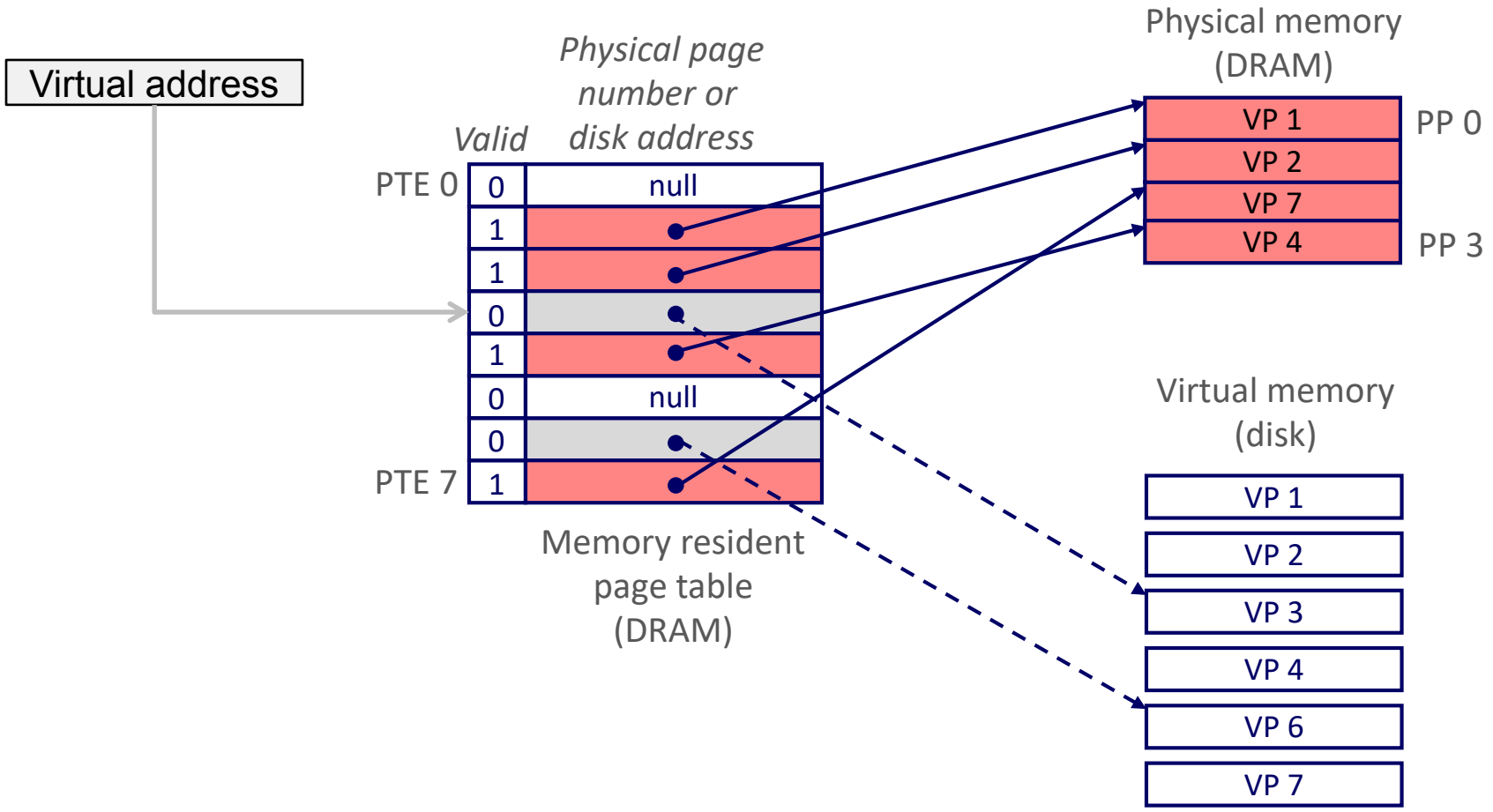
Page Fault

- ***Page fault:*** **reference to VM word that is not in physical memory (DRAM cache miss)**



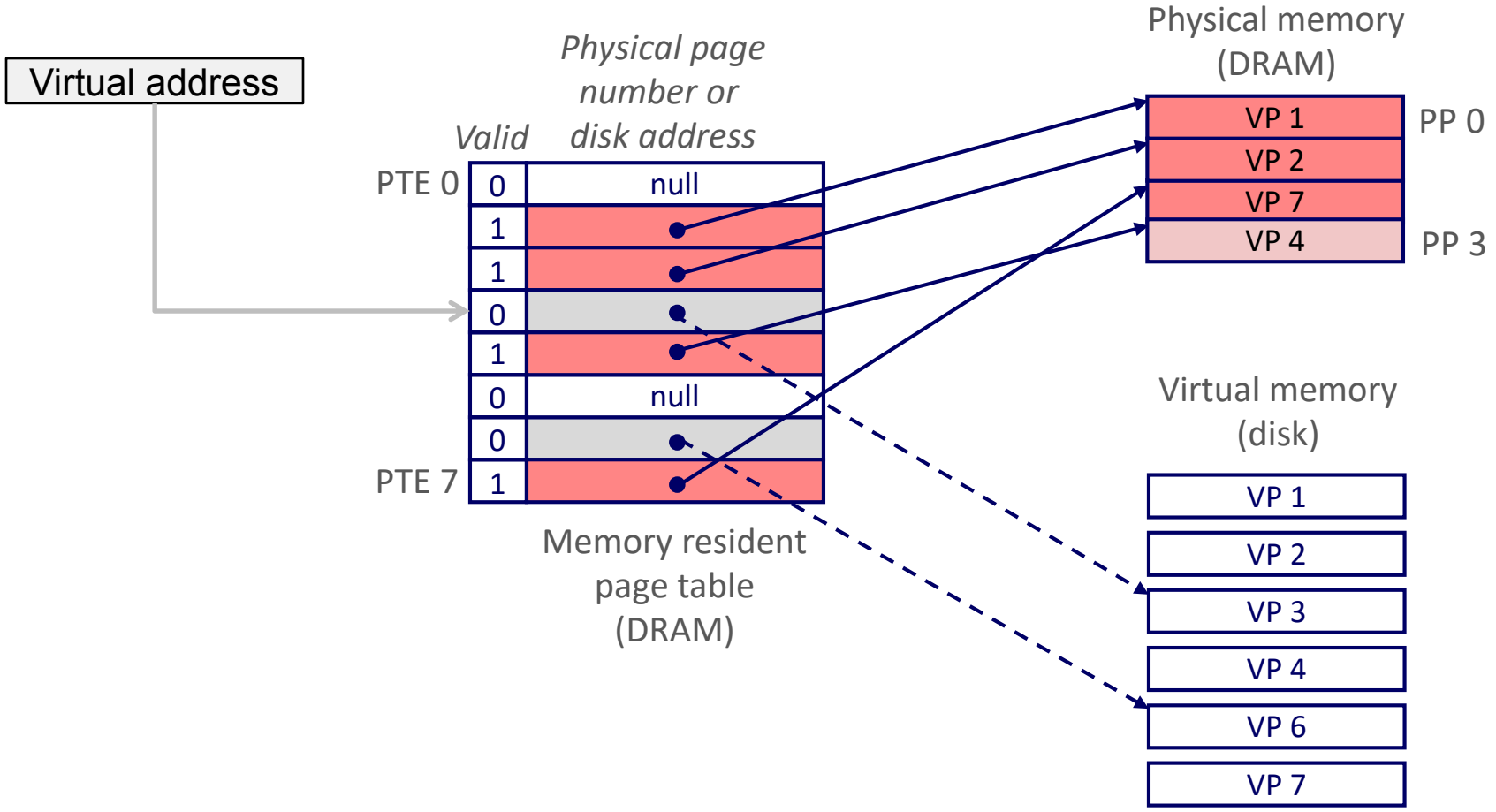
Handling Page Fault

- Page miss causes page fault (an exception)



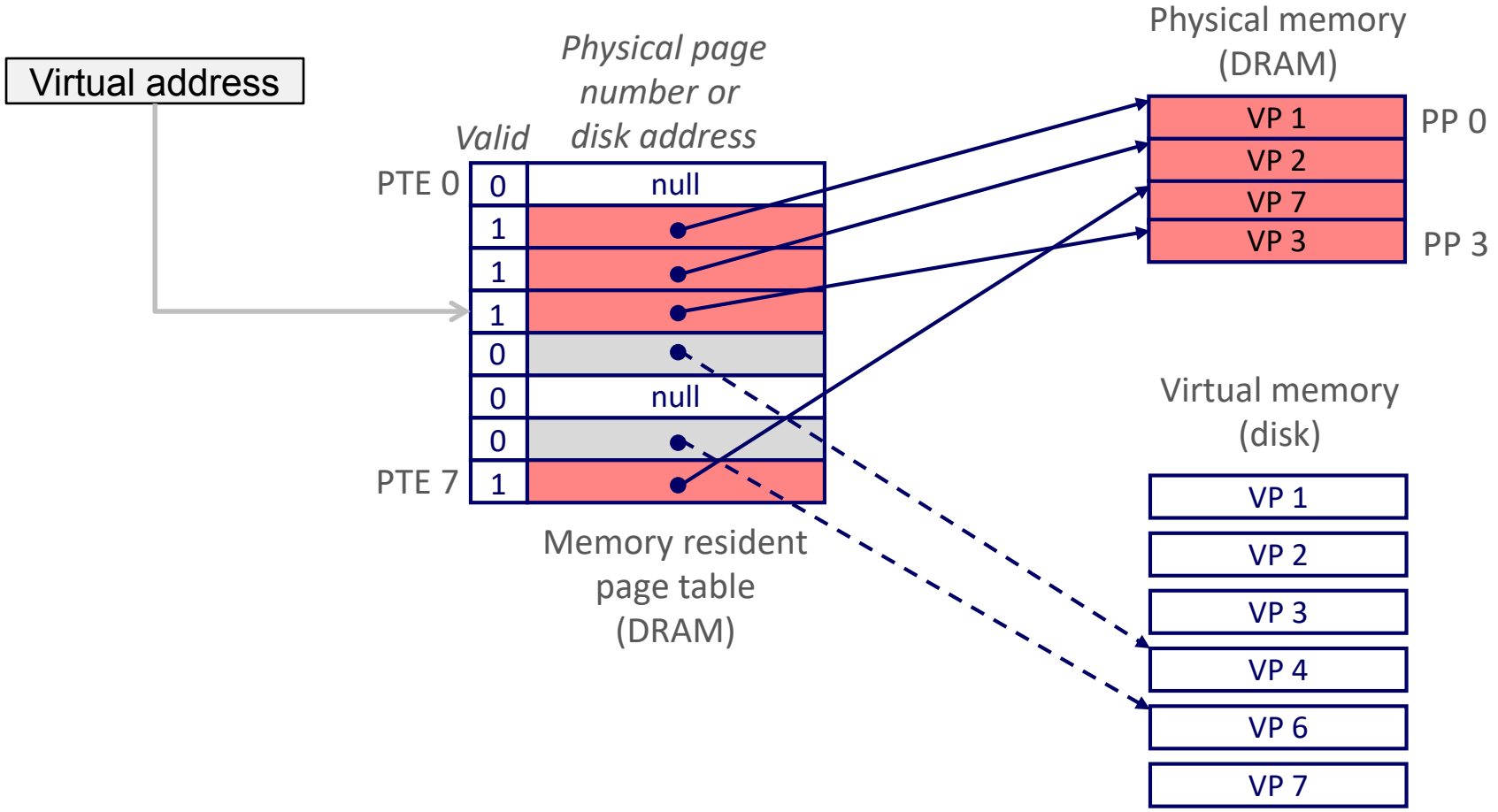
Handling Page Fault

- Page miss causes page fault (an exception)
- Page fault handler selects a victim to be evicted (here VP 4)



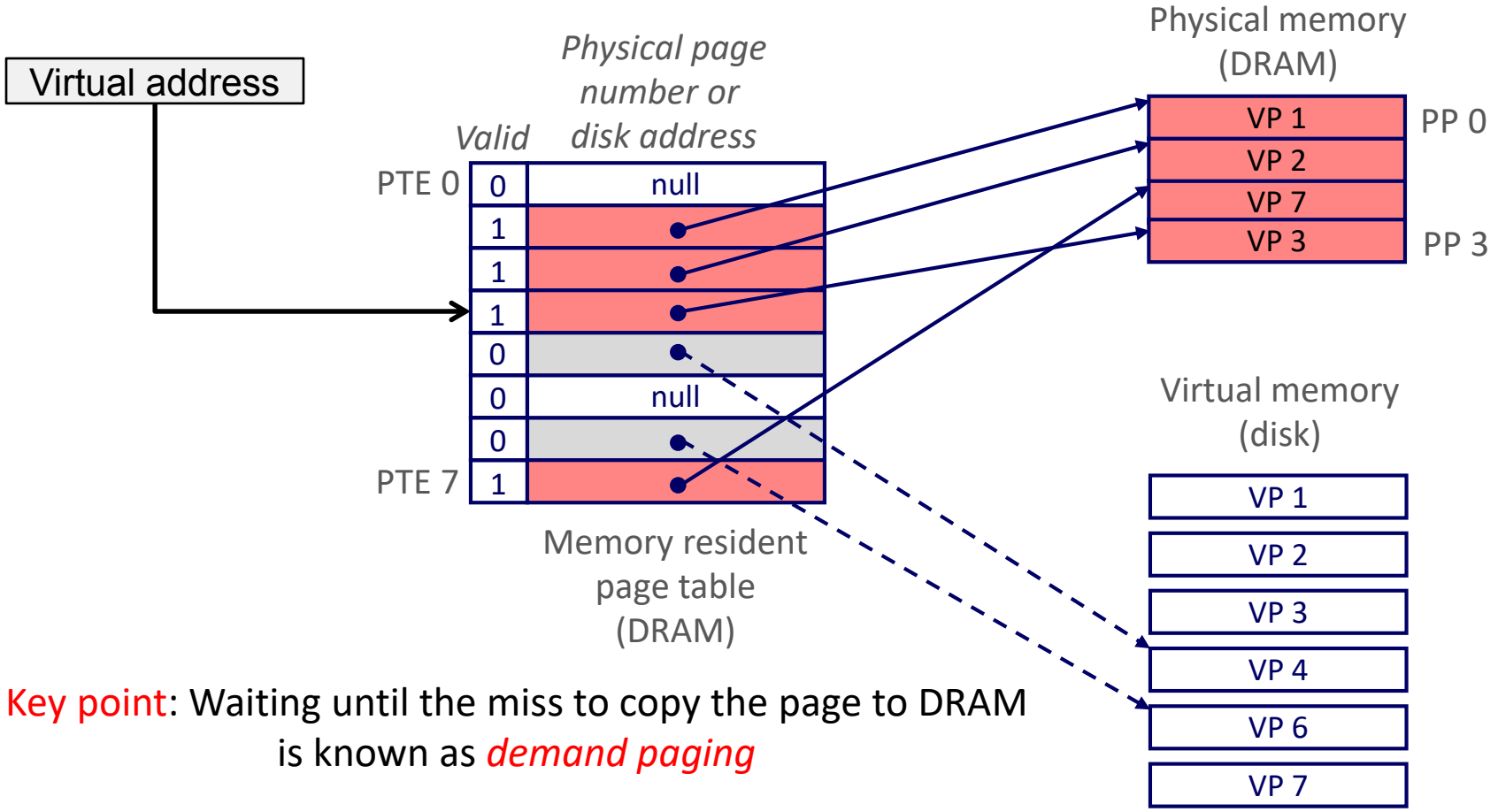
Handling Page Fault

- Page miss causes page fault (an exception)
- Page fault handler selects a victim to be evicted (here VP 4)



Handling Page Fault

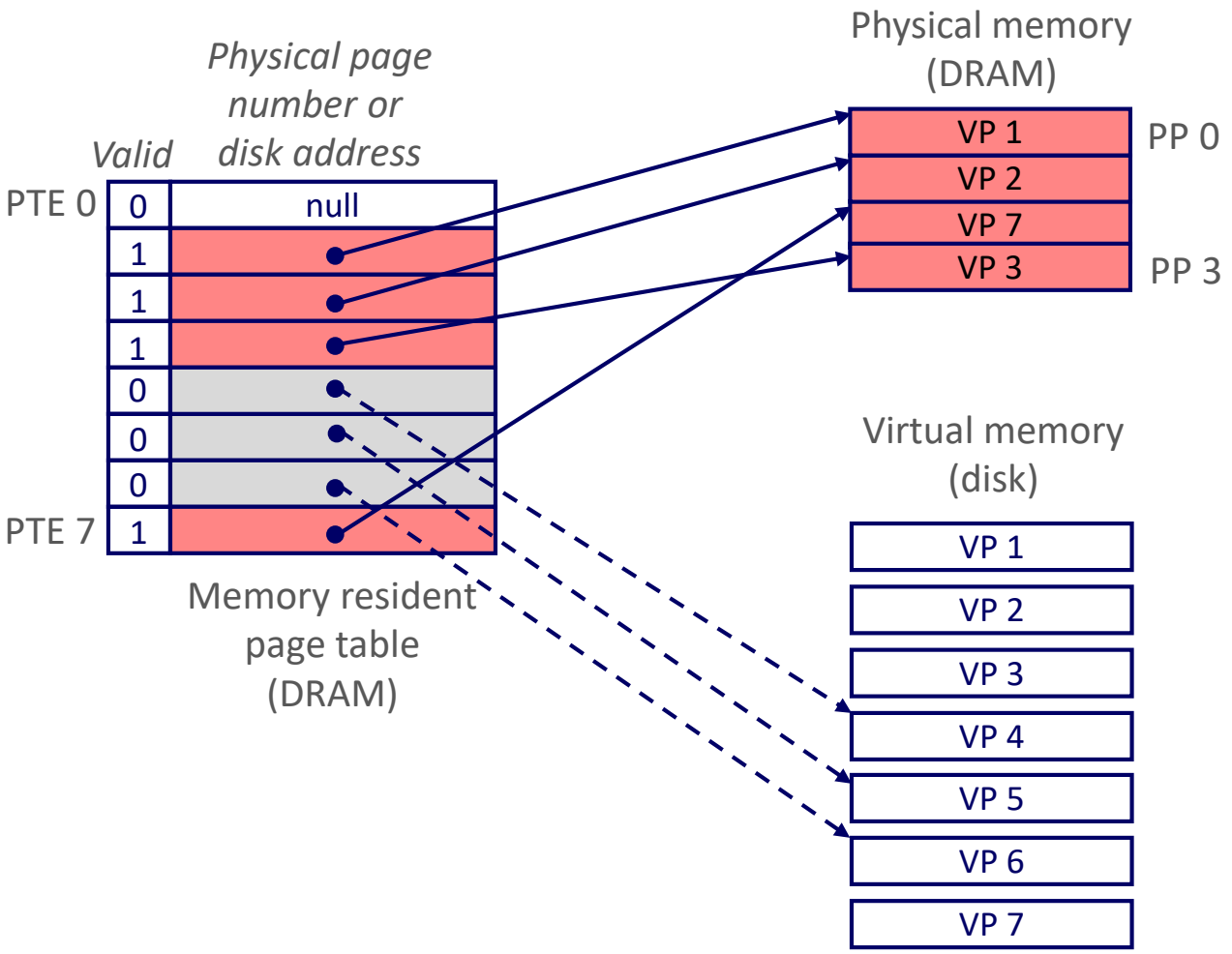
- Page miss causes page fault (an exception)
- Page fault handler selects a victim to be evicted (here VP 4)
- Offending instruction is restarted: page hit!



Key point: Waiting until the miss to copy the page to DRAM is known as *demand paging*

Allocating Pages

- **Allocating a new page (VP 5) of virtual memory.**



Locality to the Rescue Again!

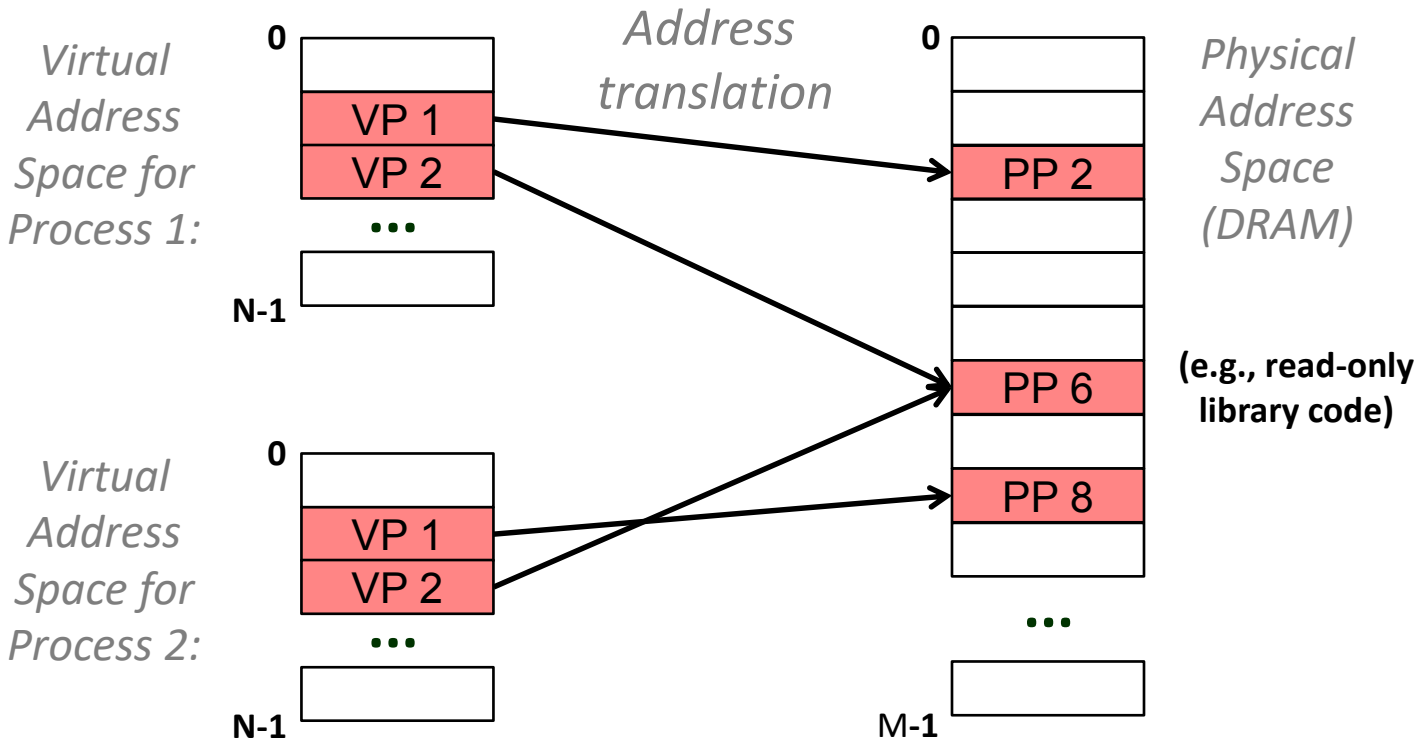
- **Virtual memory seems terribly inefficient, but it works because of locality.**
- **At any point in time, programs tend to access a set of active virtual pages called the *working set***
- **If (working set size < main memory size)**
  - Good performance for one process after compulsory misses
- **If (SUM(working set sizes) > main memory size)**
  - *Thrashing:* Performance meltdown where pages are swapped (copied) in and out continuously

Today

- Address spaces
- VM as a tool for caching
- **VM as a tool for memory management**
- VM as a tool for memory protection
- Address translation

VM as a Tool for Memory Management

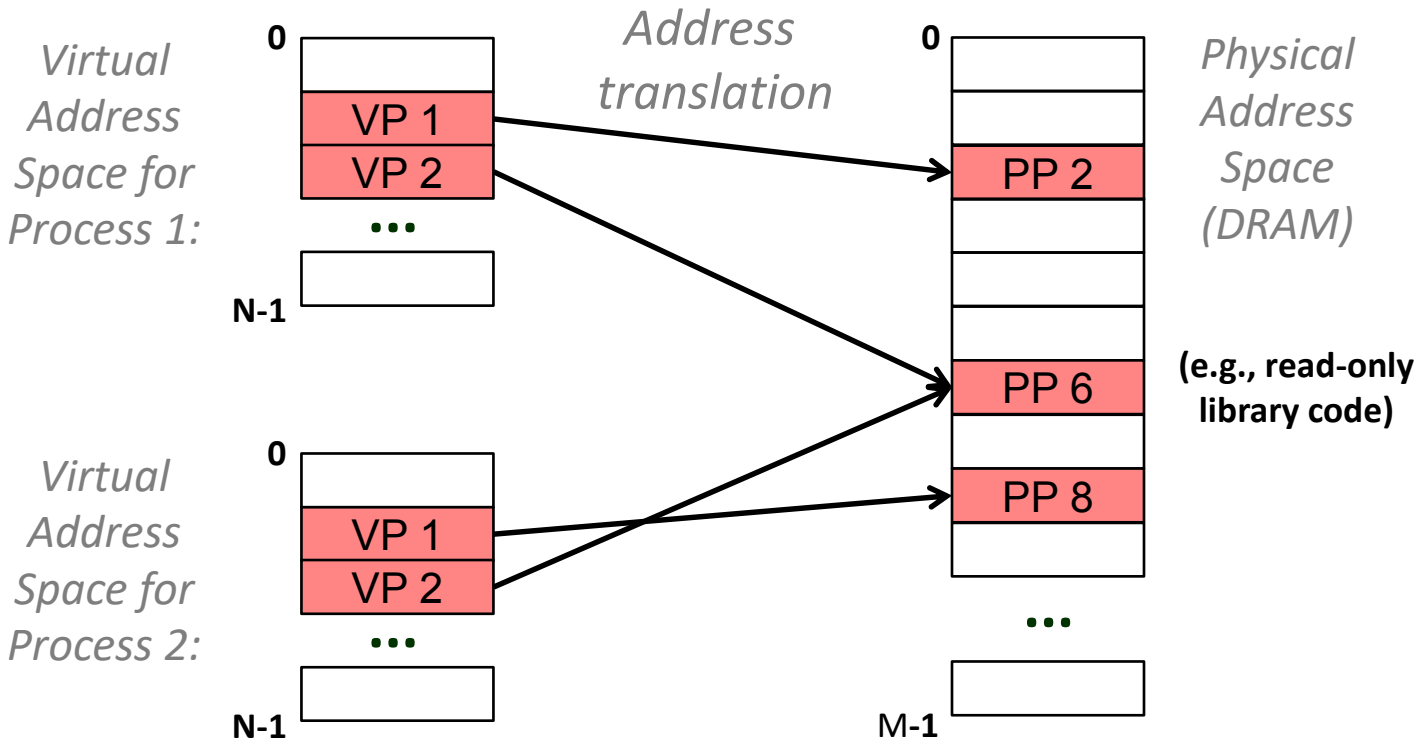
- **Key idea: each process has its own virtual address space**
  - It can view memory as a simple linear array
  - Mapping function scatters addresses through physical memory



VM as a Tool for Memory Management

- **Simplifying memory allocation**

- **Sharing code and data among processes**
  - Map virtual pages to the same physical page (here: PP 6)



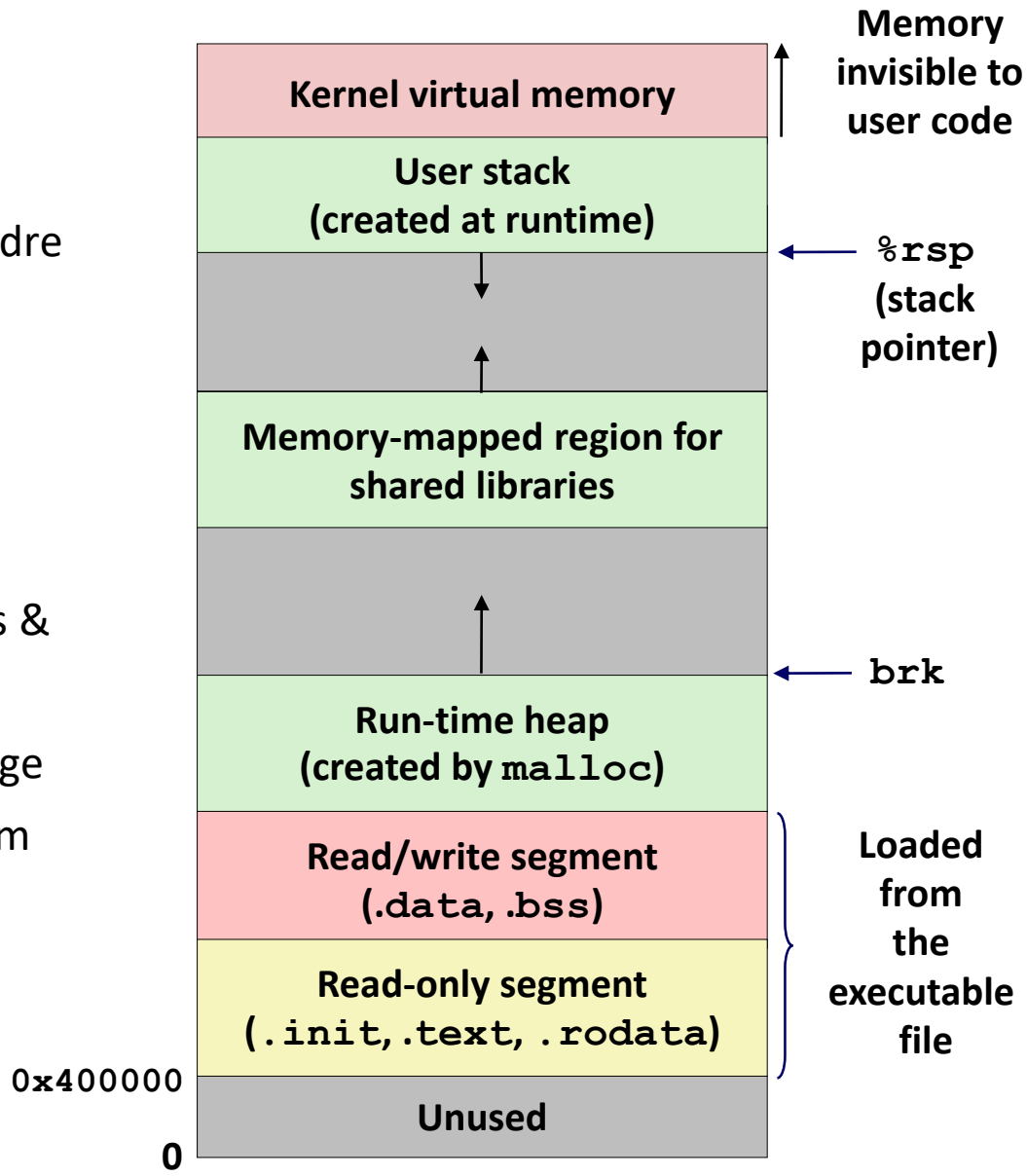
Simplifying Linking and Loading

Linking

- Each program has similar virtual address space
- Code, data, and heap always start at the same addre sses.

Loading

- Allocates virtual pages for .text and .data sections & creates PTEs marked as invalid
- The `.text` and `.data` sections are copied, page by page, on demand by the virtual memory system

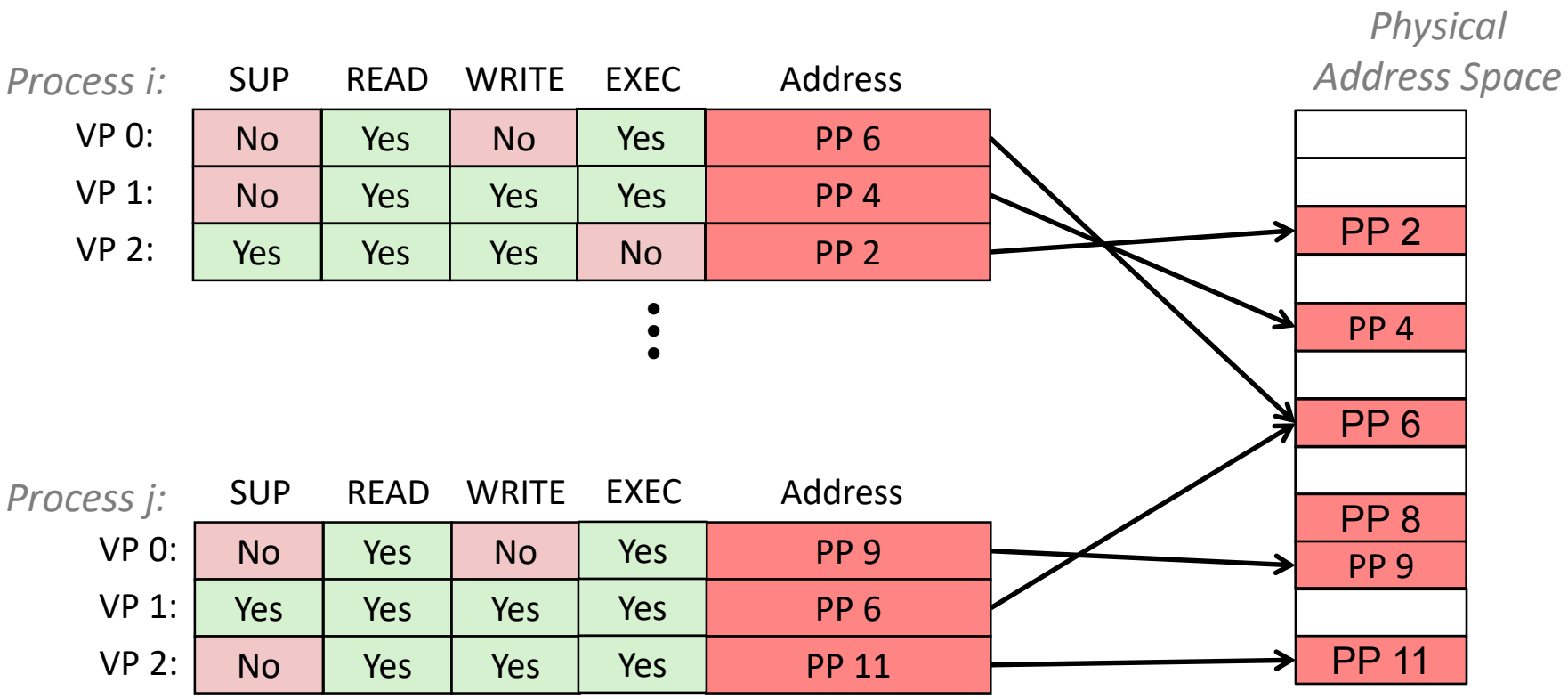


Today

- Address spaces
- VM as a tool for caching
- VM as a tool for memory management
- **VM as a tool for memory protection**
- Address translation

VM as a Tool for Memory Protection

- Extend PTEs with permission bits
- MMU checks these bits on each access



Today

- Address spaces
- VM as a tool for caching
- VM as a tool for memory management
- VM as a tool for memory protection
- **Address translation**

VM Address Translation

- **Virtual Address Space**
  - *V = {0, 1, …, N–1}*
- **Physical Address Space**
  - *P = {0, 1, …, M–1}*
- **Address Translation**
  - ***MAP: V → P U {∅}***
  - For virtual address ***a***:
    - ***MAP(a) = a'*** if data at virtual address ***a*** is at physical address ***a'*** in ***P***
    - ***MAP(a) = ∅*** if data at virtual address ***a*** is not in physical memory

Summary of Address Translation Symbols

Basic Parameters

- **$N = 2^n$** : Number of addresses in virtual address space
- **$M = 2^m$** : Number of addresses in physical address space
- **$P = 2^p$** : Page size (bytes)

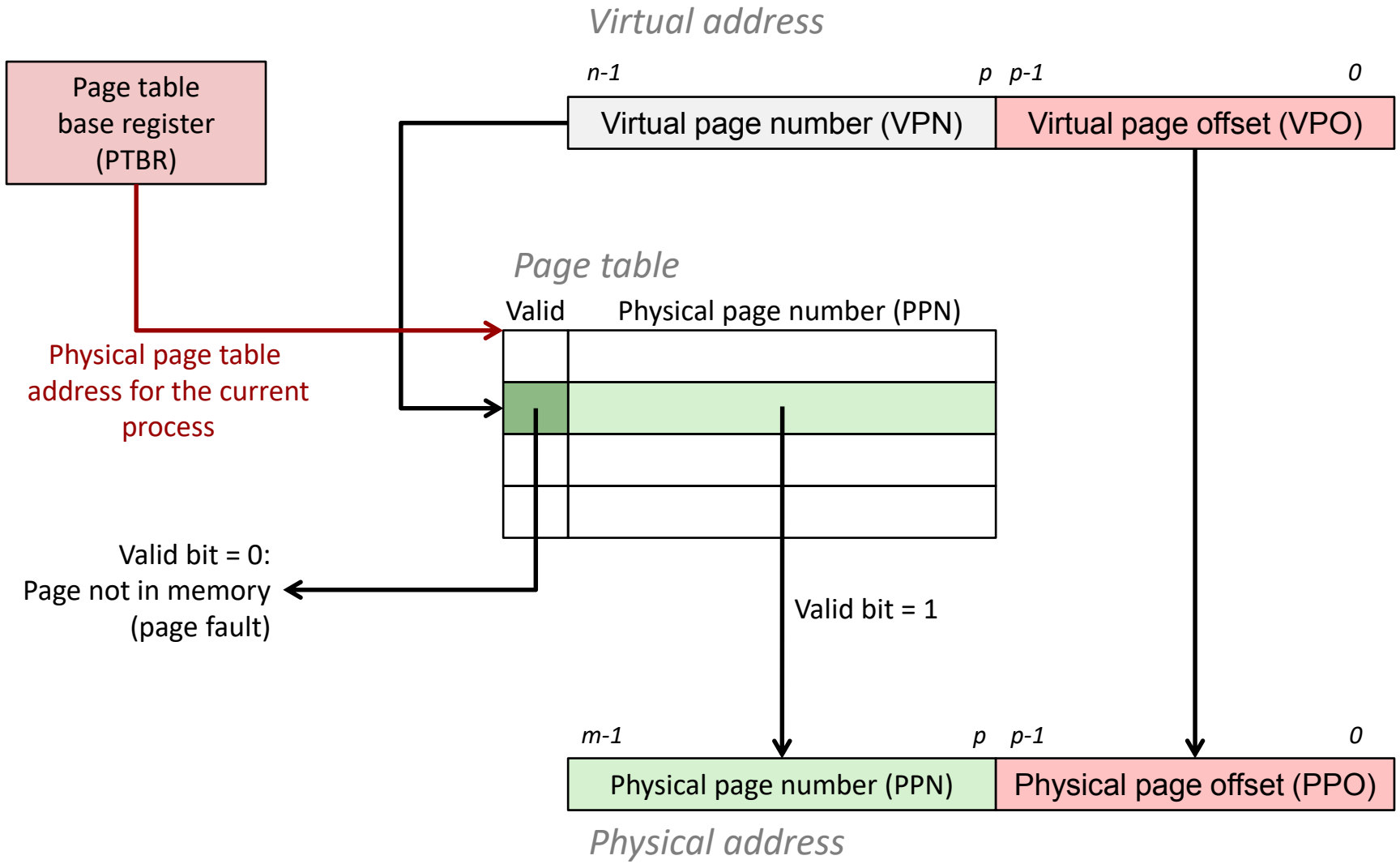
Components of the virtual address (VA)

- **TLBI**: TLB index
- **TLBT**: TLB tag
- **VPO**: Virtual page offset
- **VPN**: Virtual page number

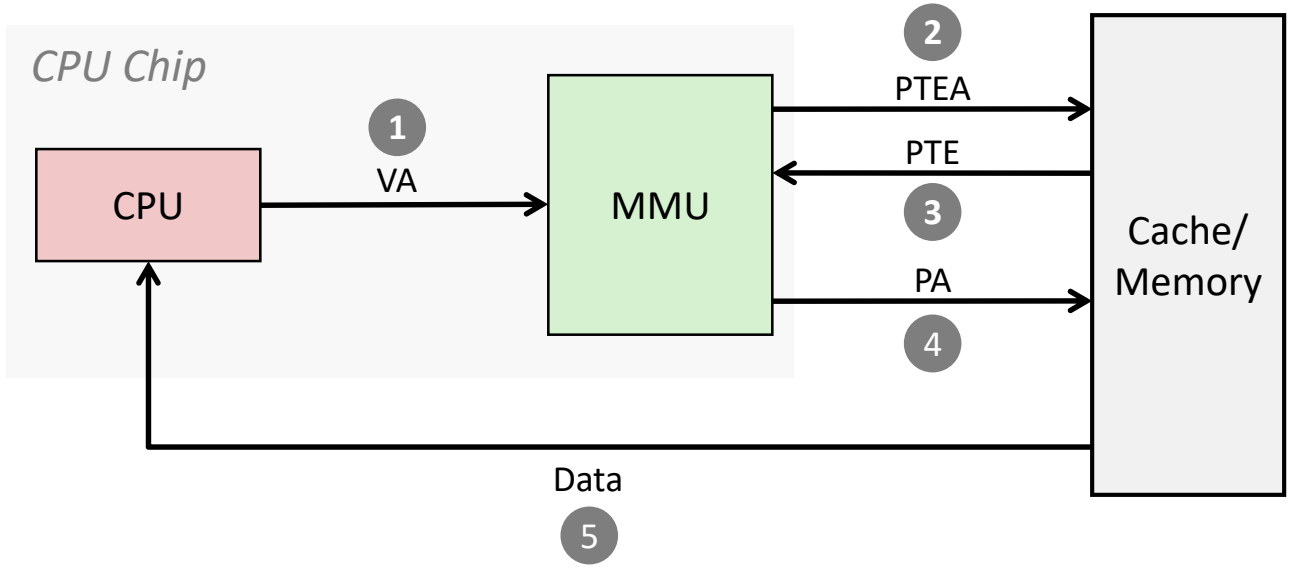
Components of the physical address (PA)

- **PPO**: Physical page offset (same as VPO)
- **PPN:** Physical page number

Address Translation With a Page Table



Address Translation: Page Hit



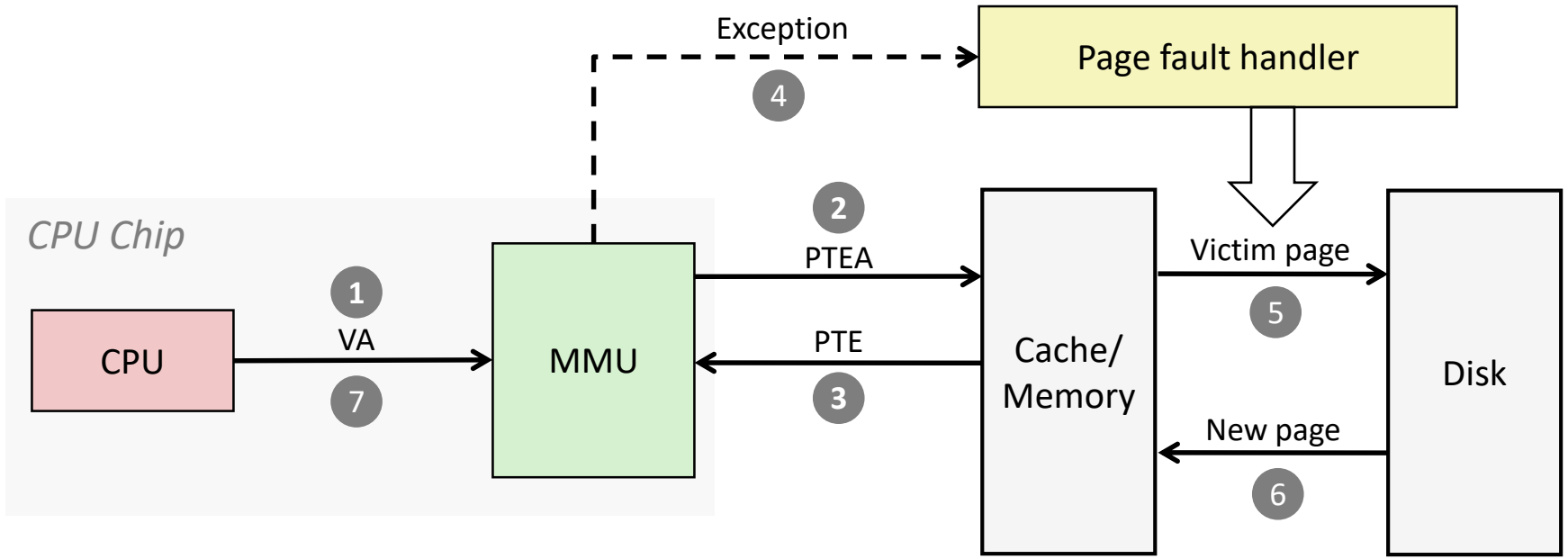
1) Processor sends virtual address to MMU

2-3) MMU fetches PTE from page table in memory

4) MMU sends physical address to cache/memory

5) Cache/memory sends data word to processor

Address Translation: Page Fault



1) Processor sends virtual address to MMU

2-3) MMU fetches PTE from page table in memory

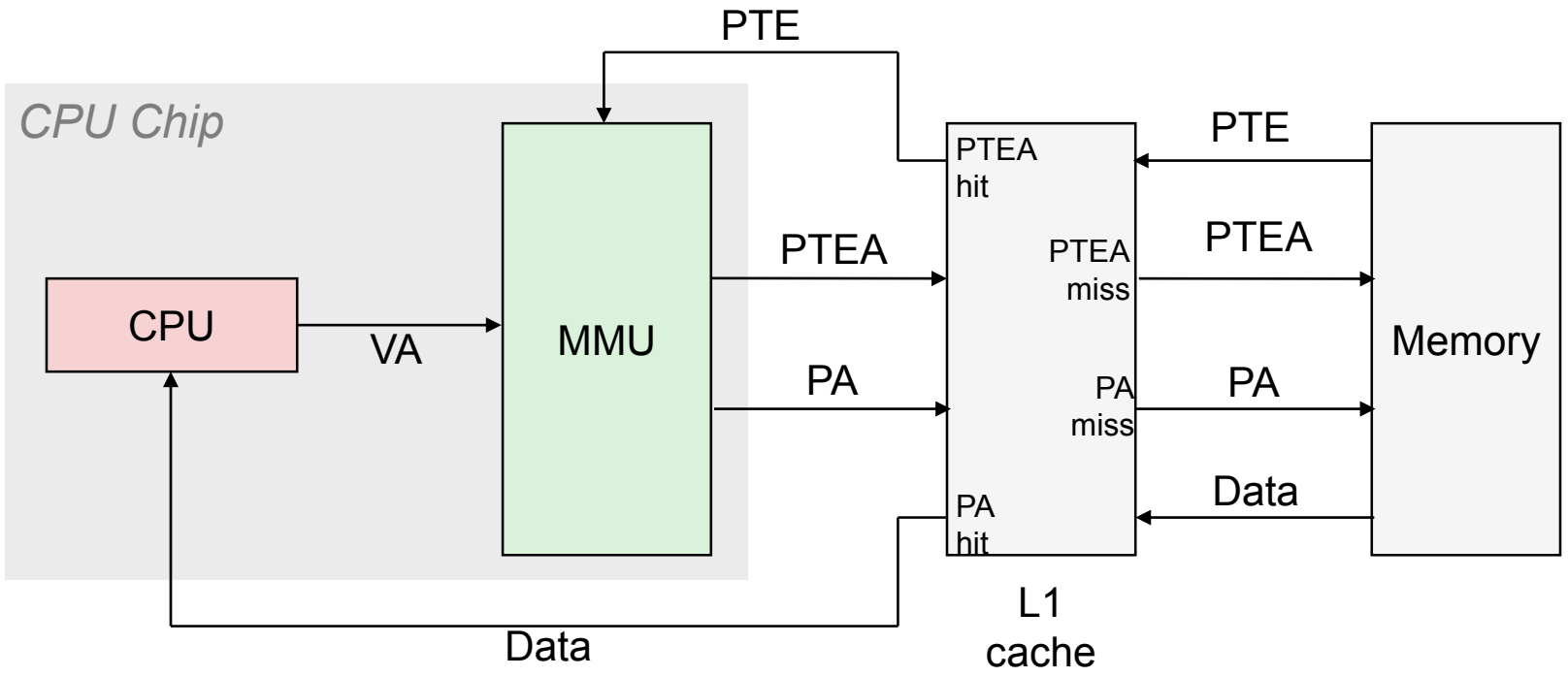
4) Valid bit is zero, so MMU triggers page fault exception

5) Handler identifies victim (and, if dirty, pages it out to disk)

6) Handler pages in new page and updates PTE in memory

7) Handler returns to original process, restarting faulting instruction

Integrating VM and Cache



VA: virtual address, PA: physical address, PTE: page table entry, PTEA = PTE address

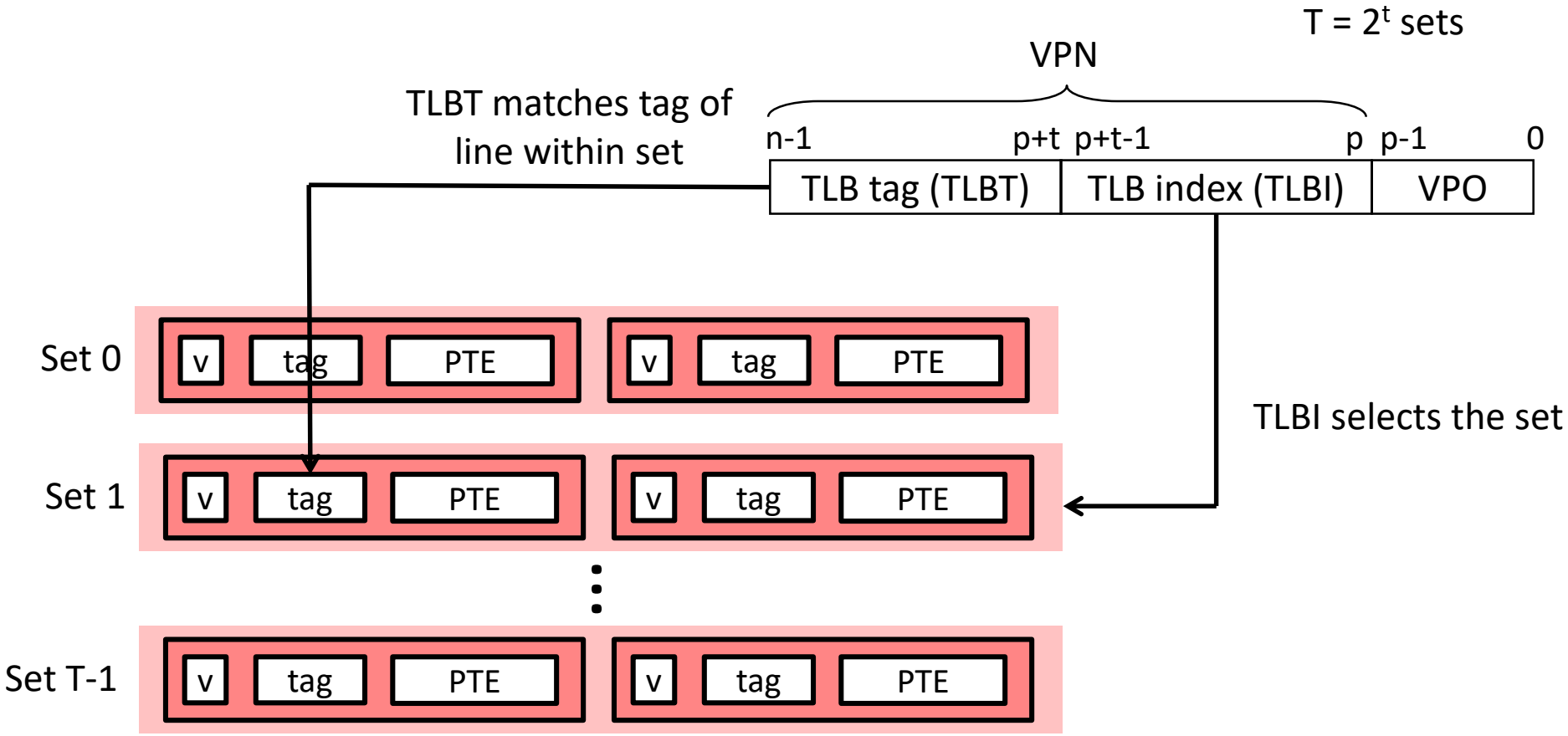
Speeding up Translation with a TLB

- **Page table entries (PTEs) are cached in L1 like any other memory word**
  - PTEs may be evicted by other data references
  - PTE hit still requires a small L1 delay

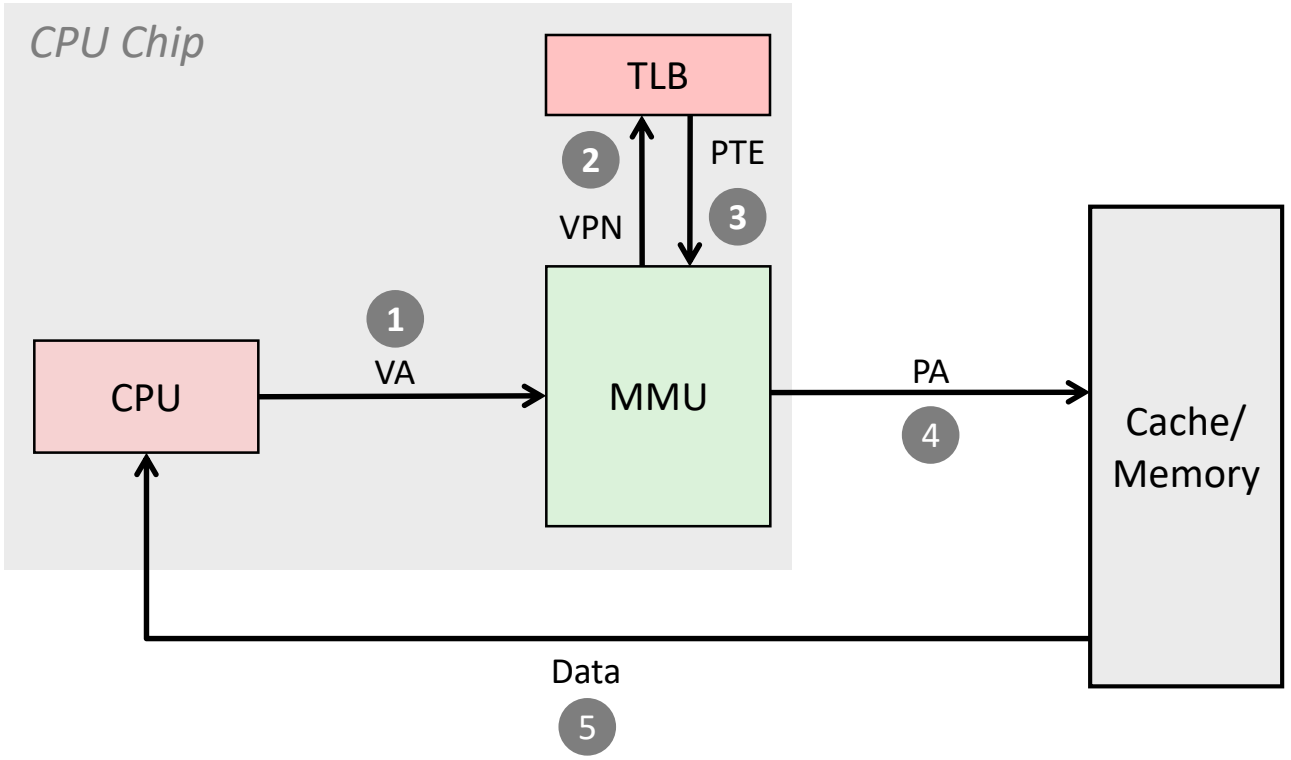
- **Solution: *Translation Lookaside Buffer* (TLB)**
  - Small set-associative hardware cache in MMU
  - Maps virtual page numbers to physical page numbers
  - Contains complete page table entries for small number of pages

Accessing the TLB

- **MMU uses the VPN portion of the virtual address to access the TLB:**

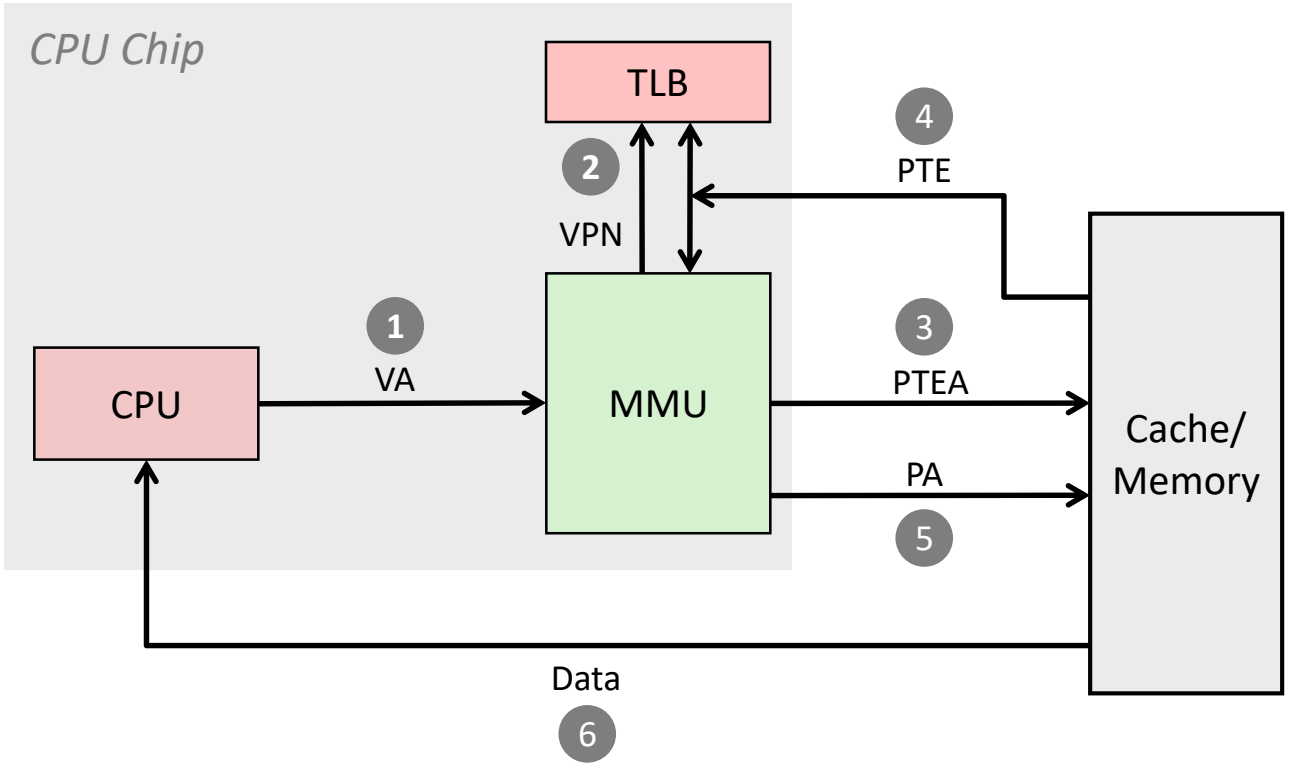


TLB Hit



A TLB hit eliminates a memory access

TLB Miss

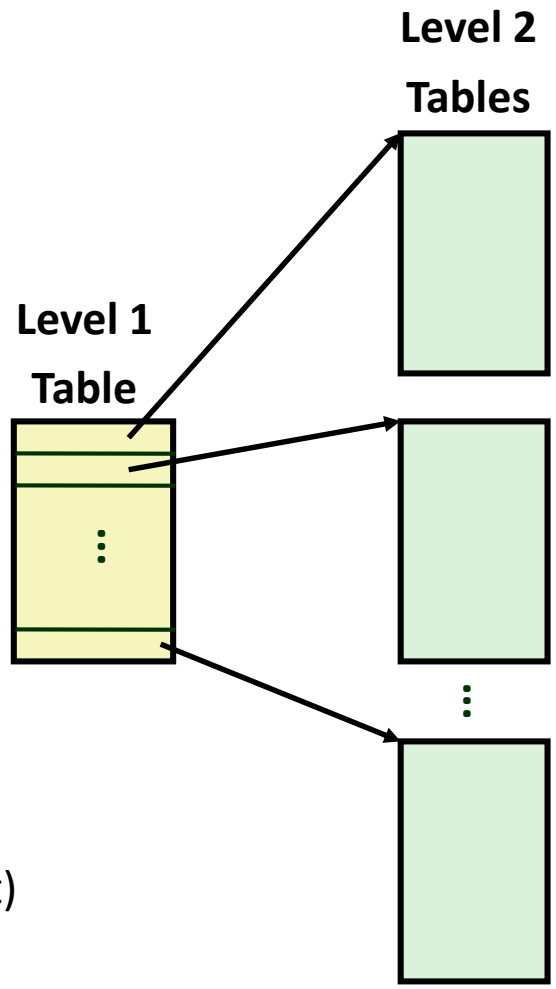


A TLB miss incurs an additional memory access (the PTE)

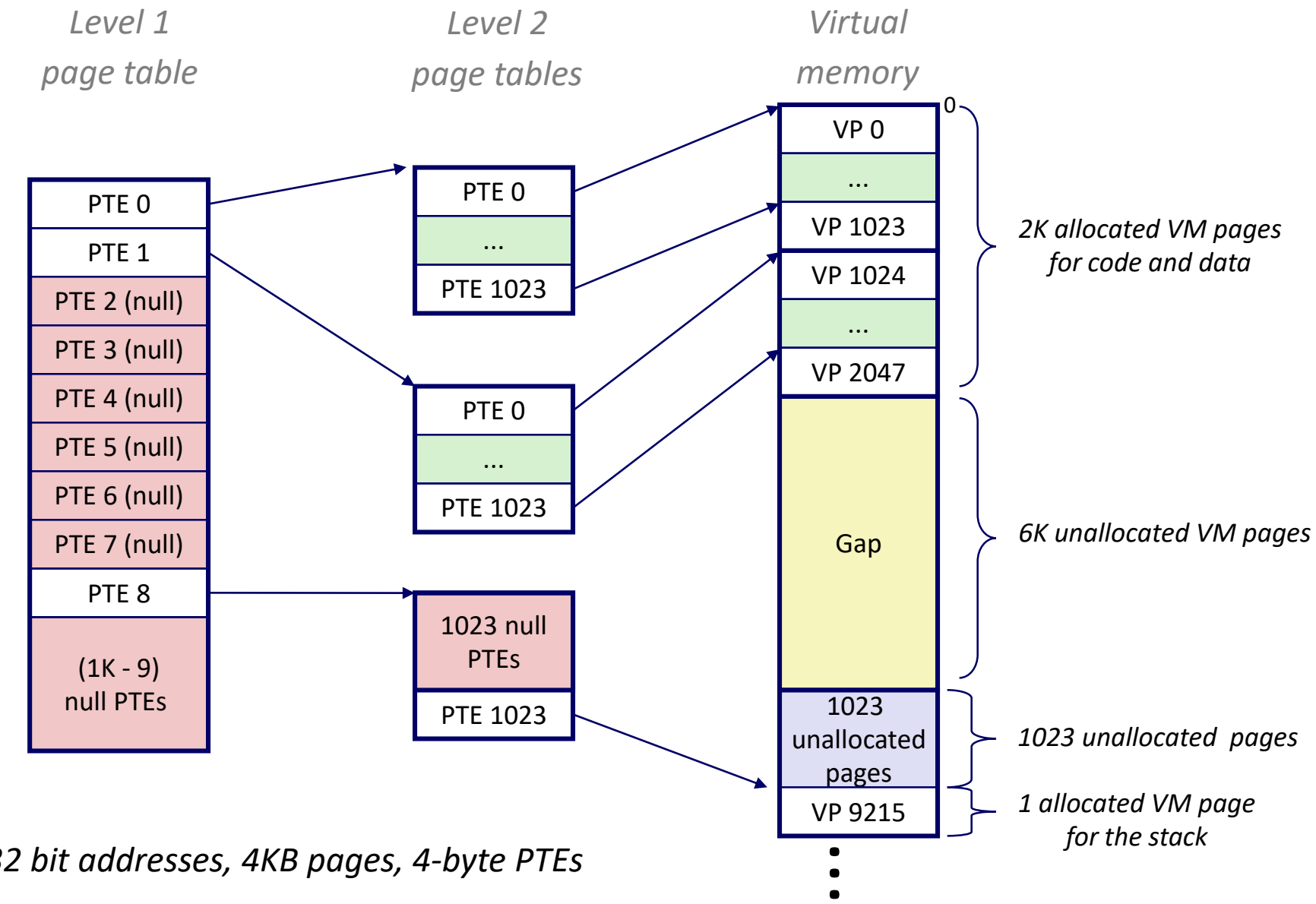
Fortunately, TLB misses are rare. Why?

Multi-Level Page Tables

- **Suppose:**
  - 4KB (2^{12}) page size, 48-bit address space, 8-byte PTE
- **Problem:**
  - Would need a 512 GB page table!
    - $2^{48} * 2^{-12} * 2^{3} = 2^{39}$ bytes
- **Common solution: Multi-level page table**
- **Example: 2-level page table**
  - Level 1 table: each PTE points to a page table (always memory resident)
  - Level 2 table: each PTE points to a page
    (paged in and out like any other data)

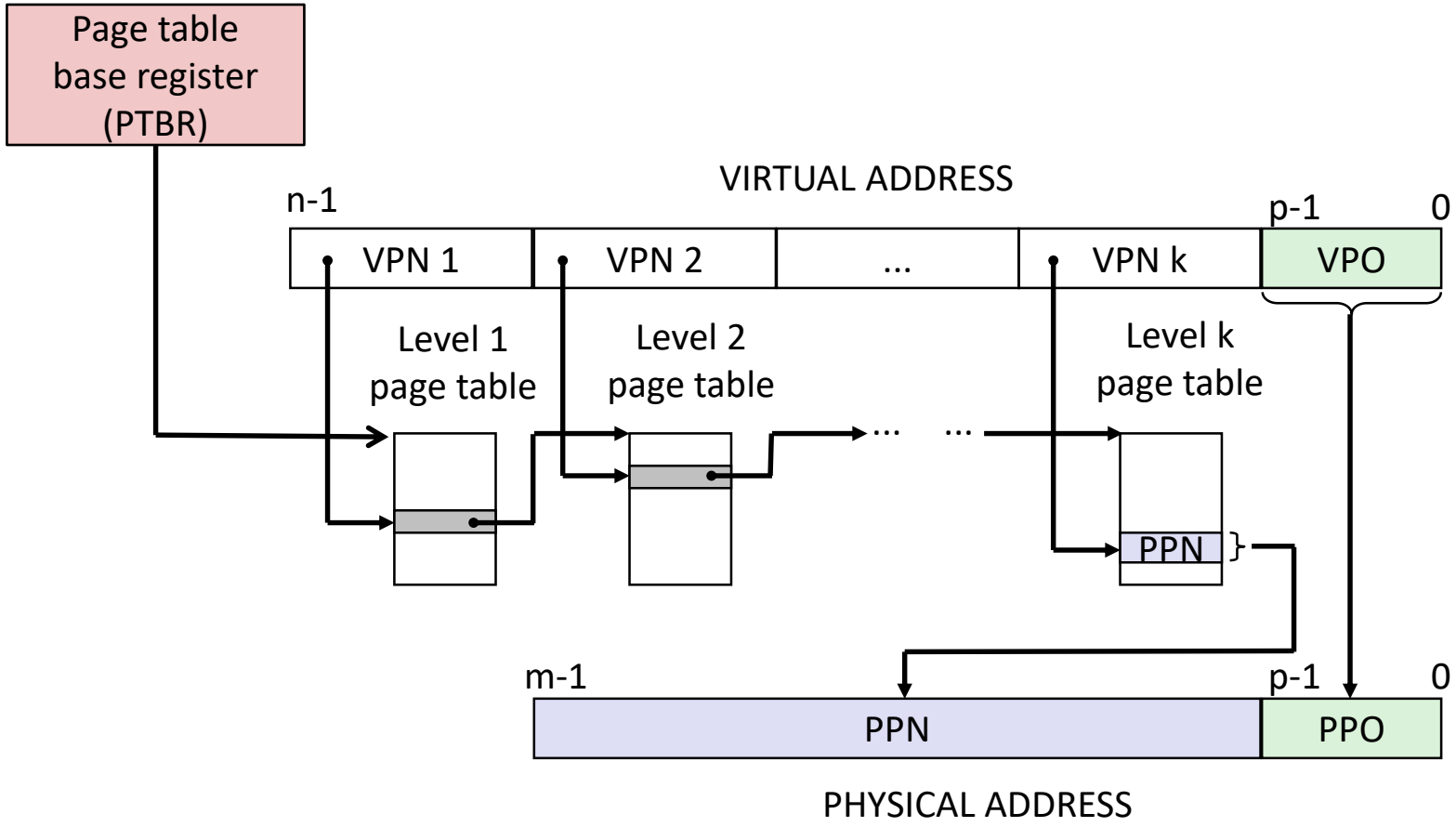


A Two-Level Page Table Hierarchy



32 bit addresses, 4KB pages, 4-byte PTEs

Translating with a k-level Page Table



Summary

Programmer's view of virtual memory

- Each process has its own private linear address space
- Cannot be corrupted by other processes

System view of virtual memory

- Uses memory efficiently by caching virtual memory pages
- Simplifies memory management and programming
- Simplifies protection by providing a convenient interpositioning point to check permissions

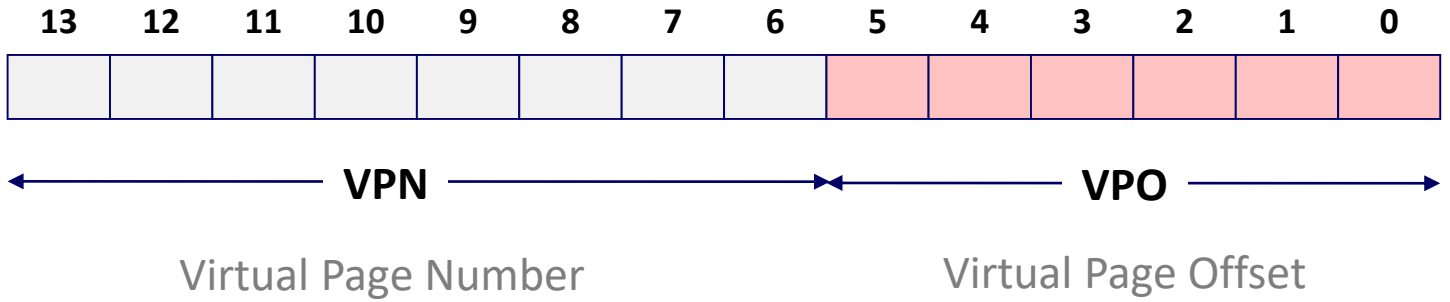
Advanced Virtual Memory

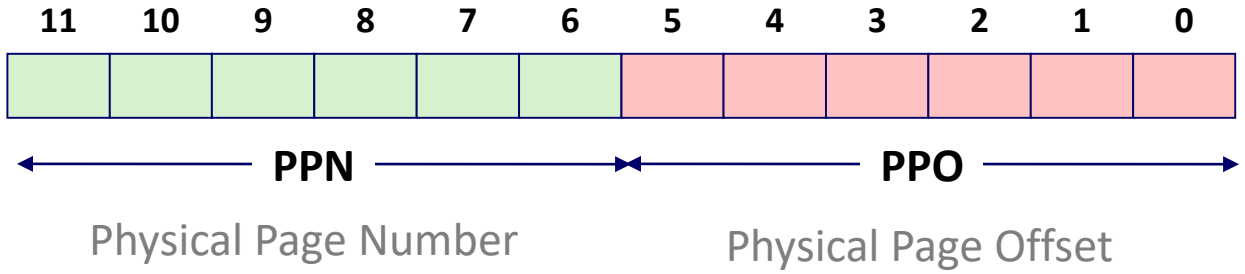
- Simple memory system example

Simple Memory System Example

Addressing

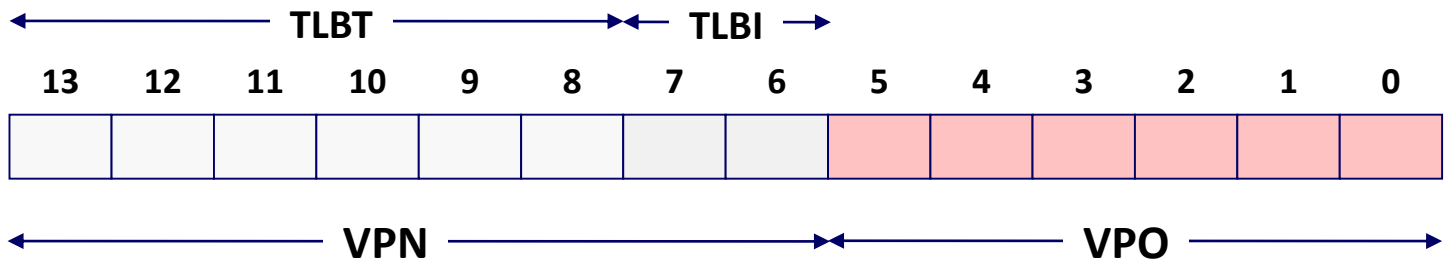
- 14-bit virtual addresses
- 12-bit physical address
- Page size = 64 bytes





1. Simple Memory System TLB

- 16 entries
- 4-way associative



Set	*Tag*	*PPN*	*Valid*	*Tag*	*PPN*	*Valid*	*Tag*	*PPN*	*Valid*	*Tag*	*PPN*	*Valid*
0	03	–	0	09	0D	1	00	–	0	07	02	1
1	03	2D	1	02	–	0	04	–	0	0A	–	0
2	02	–	0	08	–	0	06	–	0	03	–	0
3	07	–	0	03	0D	1	0A	34	1	02	–	0

2. Simple Memory System Page Table

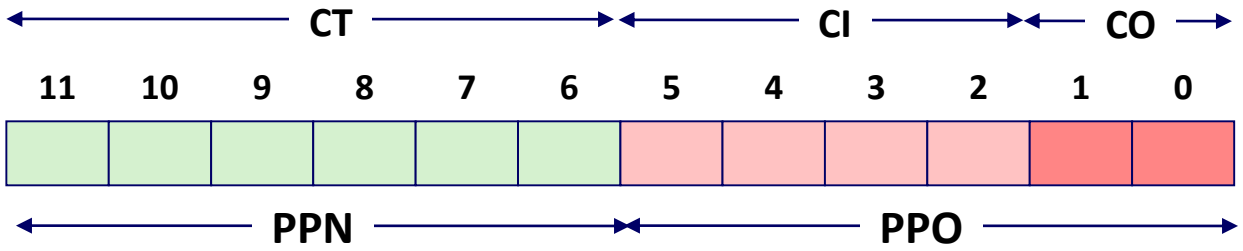
- Only show first 16 entries (out of 256)

VPN	*PPN*	*Valid*
00	28	1
01	–	0
02	33	1
03	02	1
04	–	0
05	16	1
06	–	0
07	–	0

VPN	*PPN*	*Valid*
08	13	1
09	17	1
0A	09	1
0B	–	0
0C	–	0
0D	2D	1
0E	11	1
0F	0D	1

3. Simple Memory System Cache

- 16 lines, 4-byte block size
- Physically addressed
- Direct mapped

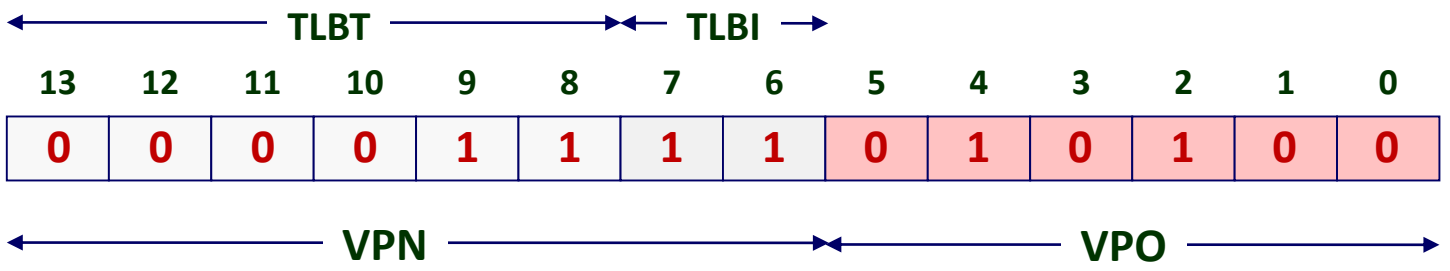


Idx	*Tag*	*Valid*	*B0*	*B1*	*B2*	*B3*
0	19	1	99	11	23	11
1	15	0	–	–	–	–
2	1B	1	00	02	04	08
3	36	0	–	–	–	–
4	32	1	43	6D	8F	09
5	0D	1	36	72	F0	1D
6	31	0	–	–	–	–
7	16	1	11	C2	DF	03

Idx	*Tag*	*Valid*	*B0*	*B1*	*B2*	*B3*
8	24	1	3A	00	51	89
9	2D	0	–	–	–	–
A	2D	1	93	15	DA	3B
B	0B	0	–	–	–	–
C	12	0	–	–	–	–
D	16	1	04	96	34	15
E	13	1	83	77	1B	D3
F	14	0	–	–	–	–

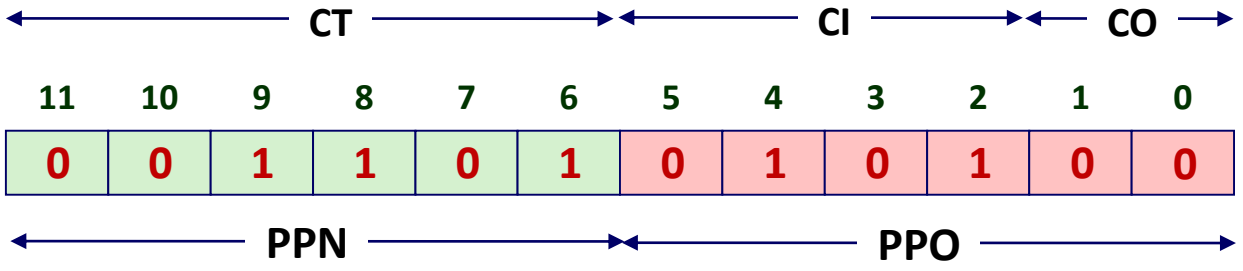
Address Translation Example #1

Virtual Address: 0x03D4



VPN 0x0F TLBI 0x3 TLBT 0x03 TLB Hit? Y Page Fault? N PPN: 0x0D

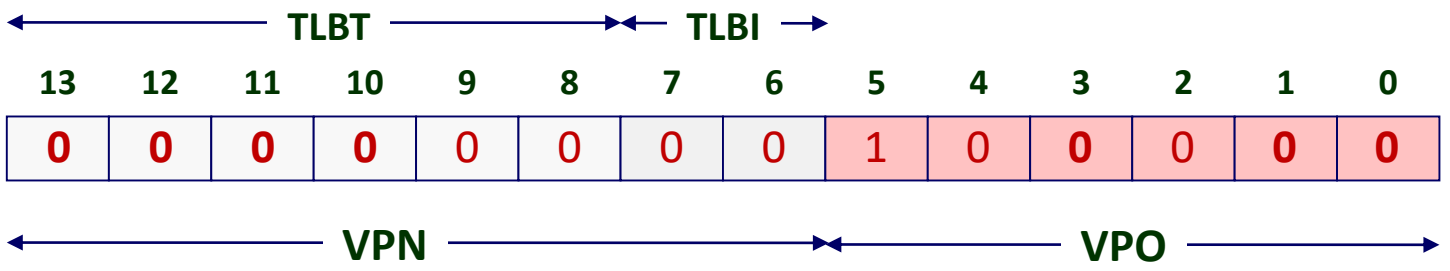
Physical Address



CO 0 CI 0x5 CT 0x0D Hit? Y Byte: 0x36

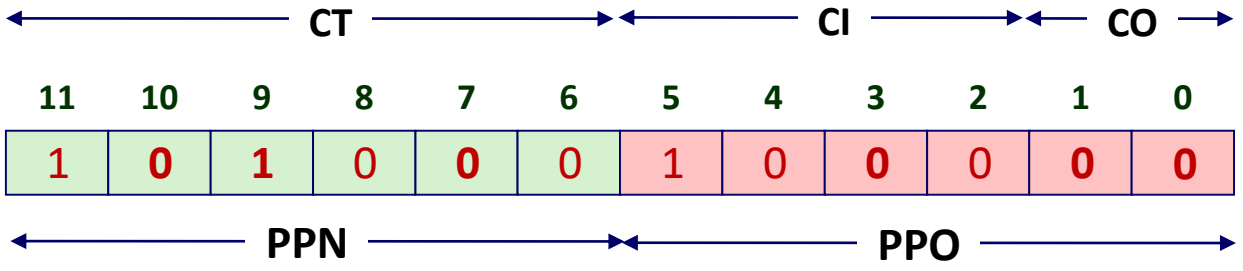
Address Translation Example #2

Virtual Address: 0x0020



VPN **0x00** TLBI 0 TLBT **0x00** TLB Hit? N Page Fault? N PPN:**0x28**

Physical Address



CO **0** CI **0x8** CT **0x28** Hit? N Byte: Mem

Final Exam

Location

- Frontier 107 (the same place for the Lecture)

Data and Time

- Date: 9 Dec. 2019
- Time: 3PM ~ 5PM (for 120 MINs)

Scope

- Lecture contents (i.e., slides + explanations) after mid-term exam
- Mainly, focus on the Lecture slide
  - You can refer to the textbook to understand the concept in the slides
  - However, the contents that are not presented in the slides are not score of the exam

Computer Systems?