ITM426, Quiz 3, 2021 Fall

Solution and Grading

- ITM 426 Engineering Mathematics 2021 F
- Nov 26, 2021
- Duration: 90 minutes
- Weights: 25% or 30% depending on other quiz scores
- 5 Questions

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- Write legibly.
- In on-line exam, start every problem in a new page.
- Justification is necessary unless stated otherwise.
- Partial points are given only sparingly for the most problems because you are expected to 1) carry out proper sanity check and 2) correct your mistake by doing so.

1	24
2	20
3	16
4	20
5	20
Total	100

#1. Mark True or False. No justification is necessary. `[Each 6pt]`

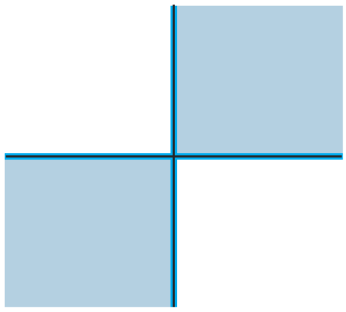
- A is a $m \times n$ matrix. Its null space is in $\mathbb{R}^m$.
- A is a $m \times n$ matrix. If the equation $A\mathbf{x} = \mathbf{b}$ is consistent, then $Col\,A$ is $\mathbb{R}^m$
- A is $n \times n$ matrix. If A is diagonalizable, then A is invertible.
- A is $n \times n$ matrix. If A has n eigenvectors, then A is diagonalizable.

Difficulty: Medium
Amount of work: 24 %
Suggested answer:

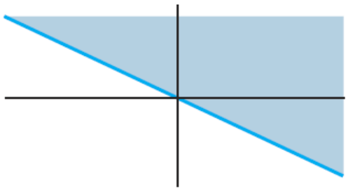
- False. See Theorem 2 in Chapter 4.
- False. The equation $A\mathbf{x} = \mathbf{b}$ must be consistent for every $\mathbf{b}$. See #7 in the table on page 206.
- False. Invertibility depends on 0 not being an eigenvalue. (See the Invertible Matrix Theorem.) A diagonalizable matrix may or may not have 0 as an eigenvalue. See Examples 3 and 5 in Chapter 5 for both possibilities.
- False. The n eigenvectors must be linearly independent. See the Diagonalization Theorem.

#2. Following figures display sets in $\mathbb{R}^2$. Assume the sets include the bounding lines. In each case, give a specific reason why the set H is not a subspace of $\mathbb{R}^2$. `[Each 10pt]`

(a)

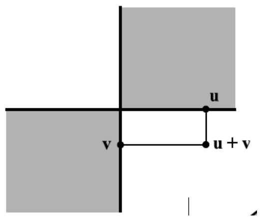


(b)



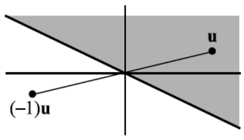
Difficulty: Easy
Amount of work: 20 %
Suggested answer:
(a)

The set is closed under scalar multiples but not sums. For example, the sum of the vectors **u** and **v** shown here is not in *H*.



(b)

No. The set is closed under sums, but not under multiplication by a negative scalar.



#3. Compute the determinants of the following matrix [16pt]

$$A = \begin{bmatrix} 3 & 0 & 0 & 0 \\ 7 & -2 & 0 & 0 \\ 2 & 6 & 3 & 0 \\ 3 & -8 & 4 & -3 \end{bmatrix}$$

Difficulty: Easy
Amount of work: 16 %
Suggested answer:

First use a row replacement to create zeros in the fourth column, and then expand down the fourth column: $\begin{vmatrix} -1 & 2 & 3 & 0 \\ 3 & 4 & 3 & 0 \\ 11 & 4 & 6 & 6 \\ 4 & 2 & 4 & 3 \end{vmatrix} = \begin{vmatrix} -1 & 2 & 3 & 0 \\ 3 & 4 & 3 & 0 \\ 3 & 0 & -2 & 0 \\ 4 & 2 & 4 & 3 \end{vmatrix} = 3\begin{vmatrix} -1 & 2 & 3 \\ 3 & 4 & 3 \\ 3 & 0 & -2 \end{vmatrix}$

Now use a row replacement to create zeros in the second column, and then expand down the second column: $3\begin{vmatrix} -1 & 2 & 3 \\ 3 & 4 & 3 \\ 3 & 0 & -2 \end{vmatrix} = 3\begin{vmatrix} -1 & 2 & 3 \\ 5 & 0 & -3 \\ 3 & 0 & -2 \end{vmatrix} = 3(-2)\begin{vmatrix} 5 & -3 \\ 3 & -2 \end{vmatrix} = 3(-2)(-1) = 6$

#4. Assume that A is row equivalent to B. Find bases for $Nul\,A$ and $Col\,A$.[Each 10pt]

$$A = \begin{bmatrix} -2 & 4 & -2 & -4 \\ 2 & -6 & -3 & 1 \\ -3 & 8 & 2 & -3 \end{bmatrix}, B = \begin{bmatrix} 1 & 0 & 6 & 5 \\ 0 & 2 & 5 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Difficulty: Easy
Amount of work: 20 %
Suggested answer:

Since B is a row echelon form of A, we see that the first and second columns of A are its pivot columns. Thus a basis for Col A is $\left\{ \begin{bmatrix} -2 \\ 2 \\ -3 \end{bmatrix}, \begin{bmatrix} 4 \\ -6 \\ 8 \end{bmatrix} \right\}$.

To find a basis for Nul A, we find the general solution of $A\mathbf{x} = \mathbf{0}$ in terms of the free variables: $x_1 = -6x_3 - 5x_4$, $x_2 = (-5/2)x_3 - (3/2)x_4$, with x_3 and x_4 free. So

$$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = x_3 \begin{bmatrix} -6 \\ -5/2 \\ 1 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} -5 \\ -3/2 \\ 0 \\ 1 \end{bmatrix}, \text{ and a basis for Nul } A \text{ is } \left\{ \begin{bmatrix} -6 \\ -5/2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -5 \\ -3/2 \\ 0 \\ 1 \end{bmatrix} \right\}.$$

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#5. The eigenvalues are 2 and 8. Diagonalize the matrix. [20pt]

$$A = \begin{bmatrix} 4 & 2 & 2 \\ 2 & 4 & 2 \\ 2 & 2 & 4 \end{bmatrix}$$

Difficulty: Medium
Amount of work: 20 %
Suggested answer:

The eigenvalues of A are given to be 2 and 8.

<u>For $\lambda = 8$</u>: $A - 8I = \begin{bmatrix} -4 & 2 & 2 \\ 2 & -4 & 2 \\ 2 & 2 & -4 \end{bmatrix}$, and row reducing $\begin{bmatrix} A-8I & \mathbf{0} \end{bmatrix}$ yields $\begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$. The general solution is $x_3 \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$, and a basis vector for the eigenspace is $\mathbf{v}_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$.

<u>For $\lambda = 2$</u>: $A - 2I = \begin{bmatrix} 2 & 2 & 2 \\ 2 & 2 & 2 \\ 2 & 2 & 2 \end{bmatrix}$, and row reducing $\begin{bmatrix} A-2I & \mathbf{0} \end{bmatrix}$ yields $\begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$. The general solution is $x_2 \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$, and a basis for the eigenspace is $\{\mathbf{v}_2, \mathbf{v}_3\} = \left\{ \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} \right\}$.

From $\mathbf{v}_1, \mathbf{v}_2$ and $\mathbf{v}_3$ construct $P = \begin{bmatrix} \mathbf{v}_1 & \mathbf{v}_2 & \mathbf{v}_3 \end{bmatrix} = \begin{bmatrix} 1 & -1 & -1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$. Then set $D = \begin{bmatrix} 8 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$, where the eigenvalues in D correspond to $\mathbf{v}_1, \mathbf{v}_2$ and $\mathbf{v}_3$ respectively.

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Write your name before detaching this page. Your Name: ______________________