SEOULTECH
Seoul National University of Science & Technology

Assignment 1

Date :

Code	ITM 524		Title	Management Science	
-	-	Questions		Weighting	5%
Student's Number			Student's Name		

1. (20pts) Use the graphical method to find the optimal solution.

Maximize $Z = 2x_1 + x_2$

(subject to)

$$x_2 \leq 10$$
$$2x_1 + 5x_2 \leq 60$$
$$x_1 + x_2 \leq 18$$
$$3x_1 + x_2 \leq 44$$
$$x_1, x_2 \geq 0$$

(Sol) Optimal solution is $(x_1^*, x_2^*) = (13, 5)$ and $Z^* = 31$.

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2. (20pts) A financial services firm plans to launch two offerings: high-risk coverage and mortgage loans. The expected profit per unit is $5 for high-risk coverage and $2 for mortgage loans.
   Management would like to set sales targets for the two offerings to maximize total expected profit. The required work-hours and available hours by department are:

Department	Work-Hours per Unit		Hours Available
	High Risk	Mortgage	
Underwriting	3	2	2400
Administration	0	1	800
Claims	2	0	1200

   (a) Formulate a linear programming model for this problem.
   (b) Solve this model by the graphical method.

(sol)

(a) Maximize $z = 5x_1 + 2x_2$
(subject to)

$$3x_1 + 2x_2 \leq 2400$$
$$x_2 \leq 800$$
$$2x_1 \leq 1200$$
$$x_1, x_2 \geq 0$$

(b) Optimal solution is $(x_1^*, x_2^*) = (600, 300)$ and $Z^* = 3600$.

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3. (20pts) Apply the simplex algorithm, showing each pivot step, to obtain the optimal solution.

$$\text{Maximize } Z = 3x_1 + 5x_2 + 6x_3$$

(subject to)

$$2x_1 + x_2 + x_3 \le 4$$
$$x_1 + 2x_2 + x_3 \le 4$$
$$x_1 + x_2 + 2x_3 \le 4$$
$$x_1 + x_2 + x_3 \le 3$$
$$x_1, x_2, x_3 \ge 0$$

(sol) Optimal solution is $(x_1^*, x_2^*, x_3^*) = \left(0, \frac{4}{3}, \frac{4}{3}\right)$ and $Z^* = \frac{44}{3}$.

BV	# Eq.	Coefficient								RHS
		Z	X1	X2	X3	S1	S2	S3	S4	
Z	0	1	-3	-5	-6	0	0	0	0	0
S1	1	0	2	1	1	1	0	0	0	4
S2	2	0	1	2	1	0	1	0	0	4
S3	3	0	1	1	2	0	0	1	0	4
S4	4	0	1	1	1	0	0	0	1	3

BV	# Eq.	Coefficient								RHS
		Z	X1	X2	X3	S1	S2	S3	S4	
Z	0	1	0	-2	0	0	0	3	0	12
S1	1	0	1.5	0.5	0	1	0	-0.5	0	2
S2	2	0	0.5	1.5	0	0	1	-0.5	0	2
X3	3	0	0.5	0.5	1	0	0	0.5	0	2
S4	4	0	0.5	0.5	0	0	0	-0.5	1	1

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BV	# Eq.	Coefficient								RHS
		Z	X1	X2	X3	S1	S2	S3	S4	
Z	0	1	0.667	0	0	0	1.333	2.333	0	14.667
S1	1	0	1.333	0	0	1	-0.333	-0.333	0	1.333
X2	2	0	0.333	1	0	0	0.667	-0.333	0	1.333
X3	3	0	0.333	0	1	0	-0.333	0.667	0	1.333
S4	4	0	0.333	0	0	0	-0.333	-0.333	1	0.333

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4. (20pts) Consider the following problem.

$$\text{Maximize } Z = x_1 - 7x_2 + 3x_3$$

(subject to)

$$2x_1 + x_2 - x_3 \leq 4$$
$$4x_1 - 3x_2 \leq 2$$
$$-3x_1 + 2x_2 + x_3 \leq 3$$
$$x_1, x_2, x_3 \geq 0$$

Apply the simplex algorithm and show each pivot step to obtain the optimal solution.

(sol)

Optimal solution is $(x_1^*, x_2^*, x_3^*) = (0.5, 0, 4.5)$ and $Z^* = 14$.

BV	# Eq.	Coefficient							RHS
		Z	X1	X2	X3	S1	S2	S3	
Z	0	1	-1	7	-3	0	0	0	0
S1	1	0	2	1	-1	1	0	0	4
S2	2	0	4	-3	0	0	1	0	2
S3	3	0	-3	2	1	0	0	1	3

BV	# Eq.	Coefficient							RHS
		Z	X1	X2	X3	S1	S2	S3	
Z	0	1	-10	13	0	0	0	3	9
S1	1	0	-1	3	0	1	0	1	7
S2	2	0	4	-3	0	0	1	0	2
X3	3	0	-3	2	1	0	0	1	3

BV	# Eq.	Coefficient							RHS
		Z	X1	X2	X3	S1	S2	S3	
Z	0	1	0	5.5	0	0	2.5	3	14

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S1	1	0	0	2.25	0	1	0.25	1	7.5
X1	2	0	1	-0.75	0	0	0.25	0	0.5
X3	3	0	0	-0.25	1	0	0.75	1	4.5

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5. (20pts) Consider the following problem.

$$\text{Maximize } Z = -5x_1 + 5x_2 + 13x_3$$

(subject to)

$$-x_1 + x_2 + 3x_3 \le 20$$
$$12x_1 + 4x_2 + 10x_3 \le 90$$
$$x_1, x_2, x_3 \ge 0$$

Let s_1 and s_2 be the slack variables for the respective constraints. After applying the simplex method, the final system can be written as:

$$Z + 2x_3 + 5x_4 = 100$$
$$-x_1 + x_2 + 3x_3 + s_1 = 20$$
$$16x_1 - 2x_3 - 4s_1 + s_2 = 10$$

Now, perform sensitivity analysis by considering each of the following nine modifications independently. For each change, update the above equations (i.e., the tableau form) and rewrite them as needed to evaluate the current basic solution. Then, test this solution for feasibility and for optimality (You don't need to carry out a full re-optimization when it is no longer optimal. Just indicate that re-optimization would be required.).

(a) Change the RHS of constraint 1 to $b_1 = 30$.
(b) Change the RHS of constraint 2 to $b_2 = 70$.
(c) Change the RHSs to $b_1 = 10, b_2 = 100$.
(d) Change the coefficient of x_3 in the objective function to $c_3 = 8$.
(e) Change the coefficients of x_1 to $c_1 = -2, a_{11} = 0, a_{21} = 5$.
(f) Change the coefficients of x_2 to $c_2 = 6, a_{12} = 2, a_{22} = 5$.
(g) Introduce a new variable x_4 with coefficients $c_4 = 10, a_{14} = 3, a_{24} = 5$.
(h) Introduce a new constraint $2x_1 + 3x_2 + 5x_3 \le 50$. (Denote its slack variable by s_3)
(i) Change constraints 2 to $10x_1 + 5x_2 + 10x_3 \le 100$.

(sol)

(a) $\Delta b_1 = 10, \Delta b_2 = 0 \rightarrow \Delta Z^* = (5\ 0)\begin{pmatrix}10\\0\end{pmatrix} = 50$

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$$\Delta b_1^* = (1\ 0)\begin{pmatrix}10\\0\end{pmatrix} = 10$$

$$\Delta b_2^* = (-4\ 1)\begin{pmatrix}10\\0\end{pmatrix} = -40$$

New tableau:

BV	# Eq.	Z	X1	X2	X3	S1	S2	RHS
Z	0	1	0	0	2	5	0	150
X2	1	0	-1	1	3	1	0	30
S2	2	0	16	0	-2	-4	1	-30

The current basic solution is infeasible, so it is not optimal (require re-optimization).

(b) $\Delta b_1 = 0, \Delta b_2 = -20 \rightarrow \Delta Z^* = (5\ 0)\begin{pmatrix}0\\-20\end{pmatrix} = 0$

$$\Delta b_1^* = (1\ 0)\begin{pmatrix}0\\-20\end{pmatrix} = 0$$

$$\Delta b_2^* = (-4\ 1)\begin{pmatrix}0\\-20\end{pmatrix} = -20$$

New tableau:

BV	# Eq.	Z	X1	X2	X3	S1	S2	RHS
Z	0	1	0	0	2	5	0	100
X2	1	0	-1	1	3	1	0	20
S2	2	0	16	0	-2	-4	1	-10

The current basic solution is infeasible and not optimal (require re-optimization).

(c) $\Delta b_1 = -10, \Delta b_2 = 10 \rightarrow \Delta Z^* = (5\ 0)\begin{pmatrix}-10\\10\end{pmatrix} = -50$

$$\Delta b_1^* = (1\ 0)\begin{pmatrix}-10\\10\end{pmatrix} = -10$$

$$\Delta b_2^* = (-4\ 1)\begin{pmatrix}-10\\10\end{pmatrix} = 50$$

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New tableau:

BV	# Eq.	Z	X1	X2	X3	S1	S2	RHS
Z	0	1	0	0	2	5	0	50
X2	1	0	-1	1	3	1	0	10
S2	2	0	16	0	-2	-4	1	60

The current basic solution is feasible and optimal.

(d) $\Delta c_3 = -5 \rightarrow \Delta(z_3^* - c_3) = 5$

New tableau:

BV	# Eq.	Z	X1	X2	X3	S1	S2	RHS
Z	0	1	0	0	7	5	0	100
X2	1	0	-1	1	3	1	0	20
S2	2	0	16	0	-2	-4	1	10

The current basic solution is feasible and optimal.

(e) $\Delta a_{11} = 1, \Delta a_{21} = -7$

$$\Delta c_1 = 3 \rightarrow \Delta(z_1^* - c_1) = -3 + (5\ 0)\begin{pmatrix} 1 \\ -7 \end{pmatrix} = 2$$

$$\Delta a_{11}^* = (1\ 0)\begin{pmatrix} 1 \\ -7 \end{pmatrix} = 1$$

$$\Delta a_{21}^* = (-4\ 1)\begin{pmatrix} 1 \\ -7 \end{pmatrix} = -11$$

New tableau:

BV	# Eq.	Z	X1	X2	X3	S1	S2	RHS
Z	0	1	2	0	2	5	0	100
X2	1	0	0	1	3	1	0	20
S2	2	0	5	0	-2	-4	1	10

The current basic solution is feasible and optimal.

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(f) $\Delta a_{12} = 1, \Delta a_{22} = 1$

$$\Delta c_2 = 1 \rightarrow \Delta(z_2^* - c_2) = -1 + (5\ 0)\begin{pmatrix}1\\1\end{pmatrix} = 4$$

$$\Delta a_{12}^* = (1\ 0)\begin{pmatrix}1\\1\end{pmatrix} = 1$$

$$\Delta a_{22}^* = (-4\ 1)\begin{pmatrix}1\\1\end{pmatrix} = -3$$

New tableau:

BV	# Eq.	Z	X1	X2	X3	S1	S2	RHS
Z	0	1	0	4	2	5	0	100
X2	1	0	-1	2	3	1	0	20
S2	2	0	16	-3	-2	-4	1	10

Proper form:

BV	# Eq.	Z	X1	X2	X3	S1	S2	RHS
Z	0	1	2	0	-4	3	0	60
X2	1	0	-0.5	1	1.5	0.5	0	10
S2	2	0	14.5	0	2.5	-2.5	1	40

The current basic solution is feasible but not optimal.

(g) $\Delta a_{14} = 3, \Delta a_{24} = 5$

$$\Delta c_4 = 10 \rightarrow \Delta(z_4^* - c_4) = -10 + (5\ 0)\begin{pmatrix}3\\5\end{pmatrix} = 5$$

$$\Delta a_{14}^* = (1\ 0)\begin{pmatrix}3\\5\end{pmatrix} = 3$$

$$\Delta a_{24}^* = (-4\ 1)\begin{pmatrix}3\\5\end{pmatrix} = -7$$

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New tableau:

BV	# Eq.	Z	X1	X2	X3	X4	S1	S2	RHS
Z	0	1	0	0	2	5	5	0	100
X2	1	0	-1	1	3	3	1	0	20
S2	2	0	16	0	-2	-7	-4	1	10

The current basic solution is feasible and optimal.

(h) New tableau and proper form:

BV	# Eq.	Z	X1	X2	X3	S1	S2	S3	RHS
Z	0	1	0	0	2	5	0	0	100
X2	1	0	-1	1	3	1	0	3	20
S2	2	0	16	0	-2	-4	1	-7	10
S3	3	0	2	3	5	0	0	1	50

BV	# Eq.	Z	X1	X2	X3	S1	S2	S3	RHS
Z	0	1	0	0	2	5	0	0	100
X2	1	0	-1	1	3	1	0	0	20
S2	2	0	16	0	-2	-4	1	0	10
S3	3	0	5	0	-4	-3	0	1	-10

The current basic solution is infeasible and not optimal (require re-optimization).

(i) $\Delta a_{11} = 0, \Delta a_{21} = -2$

$$\Delta c_1 = 0 \rightarrow \Delta(z_1^* - c_1) = 0 + (5\ 0)\begin{pmatrix} 0 \\ -2 \end{pmatrix} = 0$$

$$\Delta a_{11}^* = (1\ 0)\begin{pmatrix} 0 \\ -2 \end{pmatrix} = 0$$

$$\Delta a_{21}^* = (-4\ 1)\begin{pmatrix} 0 \\ -2 \end{pmatrix} = -2$$

$$\Delta a_{12} = 0, \Delta a_{22} = 1$$

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$$\Delta c_2 = 0 \rightarrow \Delta(z_2^* - c_2) = 0 + (5\ 0)\begin{pmatrix}0\\1\end{pmatrix} = 0$$

$$\Delta a_{12}^* = (1\ 0)\begin{pmatrix}0\\1\end{pmatrix} = 0$$

$$\Delta a_{22}^* = (-4\ 1)\begin{pmatrix}0\\1\end{pmatrix} = 1$$

$$\Delta b_1 = 0, \Delta b_2 = 10 \rightarrow \Delta Z^* = (5\ 0)\begin{pmatrix}0\\10\end{pmatrix} = 0$$

$$\Delta b_1^* = (1\ 0)\begin{pmatrix}0\\10\end{pmatrix} = 0$$

$$\Delta b_2^* = (-4\ 1)\begin{pmatrix}0\\10\end{pmatrix} = 10$$

New tableau:

BV	# Eq.	Z	X1	X2	X3	S1	S2	RHS
Z	0	1	0	0	2	5	0	100
X2	1	0	-1	1	3	1	0	20
S2	2	0	14	1	-2	-4	1	20

Proper form:

BV	# Eq.	Z	X1	X2	X3	S1	S2	RHS
Z	0	1	0	0	2	5	0	100
X2	1	0	-1	1	3	1	0	20
S2	2	0	15	0	-5	-5	1	0