SEOULTECH
Seoul National University of Science & Technology

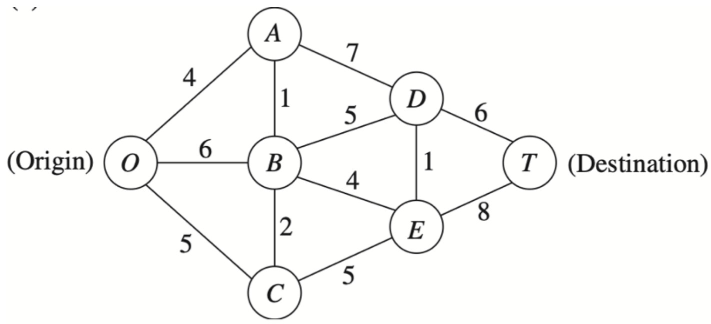
Assignment 3

Date :

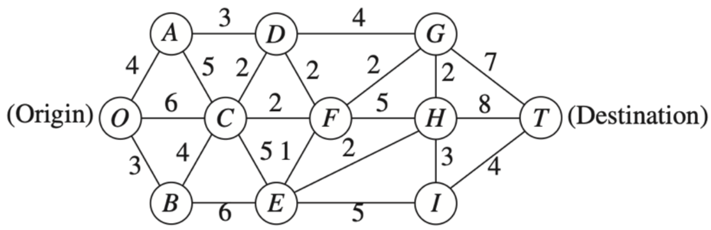
Code	**ITM 524**		**Title**	**Management Science**	
-	**-**	**Questions**		**Weighting**	**5%**
Student's Number			**Student's Name**		

1. (20pts) For each network shown below, find the shortest path. Assume the numbers on the arcs are the actual distances between the nodes, and repot both path and its total distance.

   (a) First network

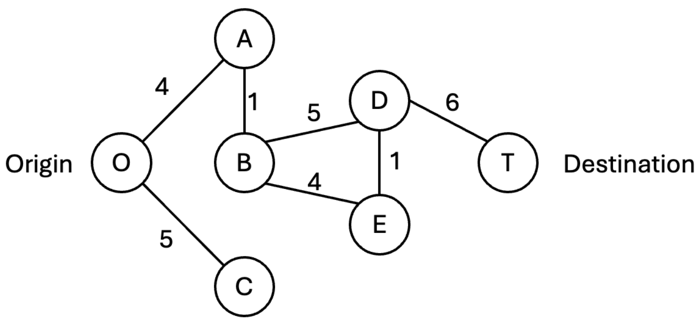


   (b) Second network



(sol)

   (a) Length of the shortest path is 16



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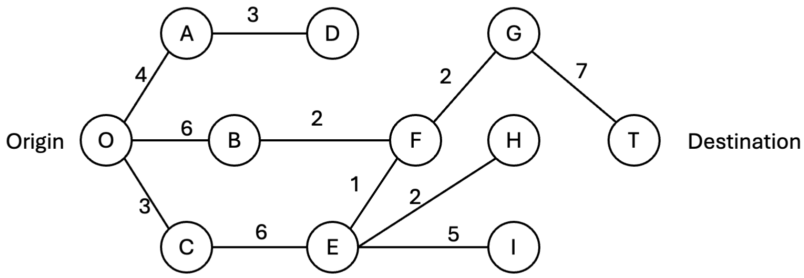
(b) The length of the shortest path is 17



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2. (20pts) A university plans to connect eight campus buildings in the same area with fiber-optic lines so that every building can reach every other (directly or indirectly). The distance (miles) between each pair of buildings is:

		Distance between pairs of buildings							
		1	2	3	4	5	6	7	8
Bldg	1	–	1.3	2.1	0.9	0.7	1.8	2.0	1.5
	2	1.3	–	0.9	1.8	1.2	2.6	2.3	1.1
	3	2.1	0.9	–	2.6	1.7	2.5	1.9	1.0
	4	0.9	1.8	2.6	–	0.7	1.6	1.5	0.9
	5	0.7	1.2	1.7	0.7	–	0.9	1.1	0.8
	6	1.8	2.6	2.5	1.6	0.9	–	0.6	1.0
	7	2.0	2.3	1.9	1.5	1.1	0.6	–	0.5
	8	1.5	1.1	1.0	0.9	0.8	1.0	0.5	–

Management wants to decide which building pairs should be directly linked so that all buildings are connected while the total length of fiber is minimized.

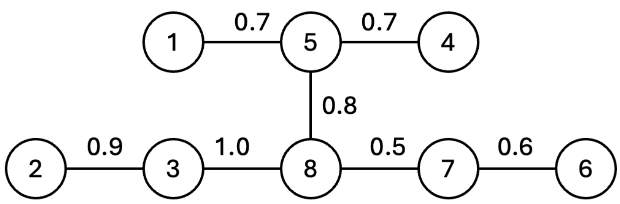
(a) Explain why this problem can be modeled as a minimum spanning tree problem on a network.

(b) Solve the problem (identify the links to build and the minimum total length).

(sol)

(a) The nodes represent buildings and the branches represent the lines.

(b) Length is 5.2



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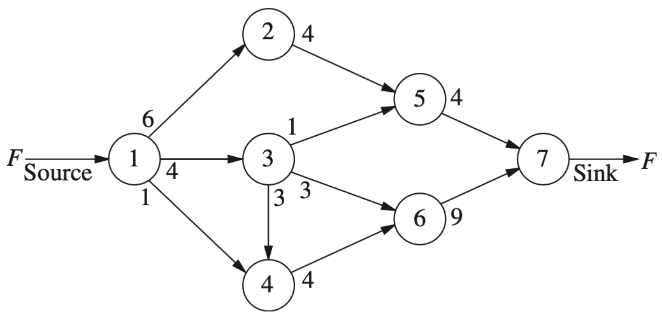
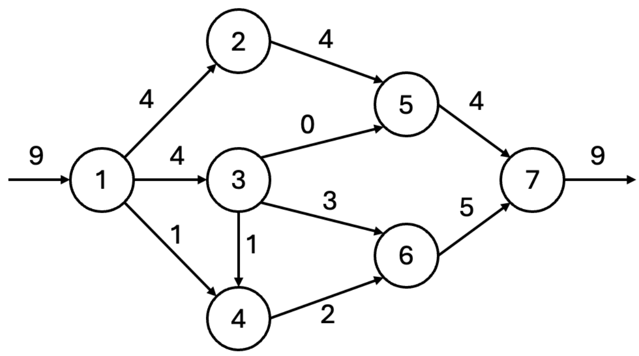
3. (20pts) For the network shown below, apply the augmenting path algorithm to determine the maximum flow from the source to the sink. The capacity of each arc (i,j) is given by the number closest to node i along the arc.
   Report the sequence of augmenting paths used, the final flow on each arc, and the value of the maximum flow.



(sol)

Maximum flow is 9.



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4. (20pts) A company will produce the same product at two factories and then ship it to two warehouses.

Factory 1 has a dedicated rail link to Warehouse 1 only (no upper limit).

Factory 2 has a dedicated rail link to Warehouse 2 only (no upper limit).

In addition, third-party truckers can move up to 50 units from each factory to a distribution center (DC), and from the DC up to 50 units to each warehouse.

To / From	Unit shipping cost			Output
	Distribution center	Warehouse 1	Warehouse 2	
Factory 1	3	7	–	80
Factory 2	4	–	9	70
Distribution center		2	4	
Allocation (demand)		60	90	

Task. Formulate a minimum-cost flow network for this problem:

- Identify the nodes and their supplies/demands (Factories 1-2 supply 80 and 70. Warehouses 1-2 demands 60 and 90; DC is a transshipment node).
- List the arcs with their per-unit costs and capacitites (factory → DC arcs have capacity 50 each; DC→warehouse arcs have capacity 50 each; rail arcs F1→W1 and F2→W2 have no upper bound).

Represent the model either by a diagram or by an explicit arc list in the form ($i \rightarrow j$: cost, capacity).

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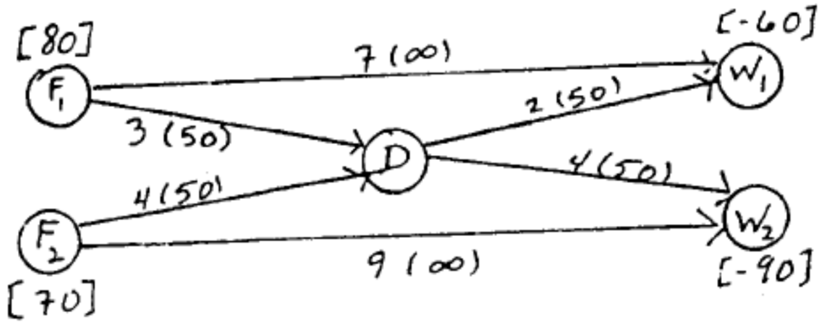

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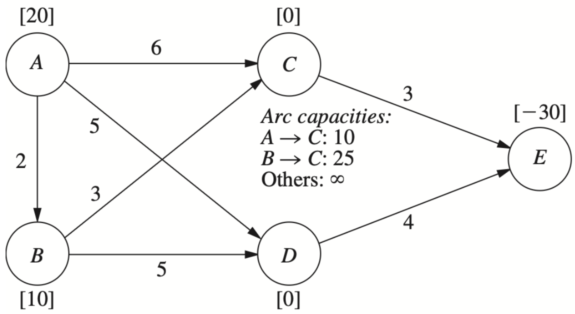
5. (20pts) Consider the minimum cost flow network shown below.
   - Each node displays its net supply/demands value b_i.
   - Each arc is labeled with its unit cost c_{ij}.
   - The capacity u_{ij} is specified between nodes C and D.



(a) Construct an initial basic feasible solution using the spanning tree whose basic arcs are $A \rightarrow B, C \rightarrow E, D \rightarrow E$, and $C \rightarrow A$ (a reverse arc). Among the nonbasic arcs, $C \rightarrow B$ is also a reverse arc.
Draw the resulting network (including b_i, c_{ij}, and u_{ij}) in the same format as the one provided (except use dashed lines to draw the nonbasic arcs). For each basic arc, write the flow value in parentheses next to the arc.

(b) Use the optimality test to confirm that this initial basic feasible solution is optimal and that multiple optimal solutions exist. Perform one iteration of the network simplex method to obtain the another optimal basic feasible solution, and then use these results to identify the other optimal solutions that are not basic feasible solutions.

(c) Now, consider the following basic feasible solution.

Basic Arc	Flow	Nonbasic Arc
$A \rightarrow D$	20	$A \rightarrow B$
$B \rightarrow C$	10	$A \rightarrow C$
$C \rightarrow E$	10	$B \rightarrow D$
$D \rightarrow E$	20	

Starting from this basis, carry out one iteration of the network simplex method.

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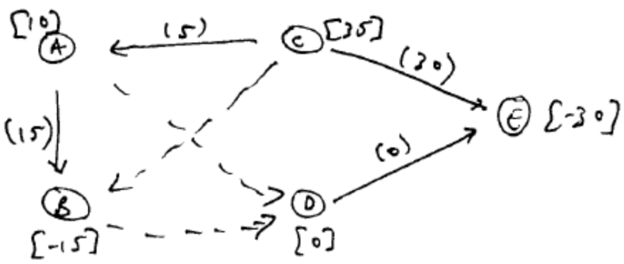
Identify the entering basic arc, the leaving basic arc, and the next basic feasible solution. Do not proceed beyond this iteration.

(sol)

(a)

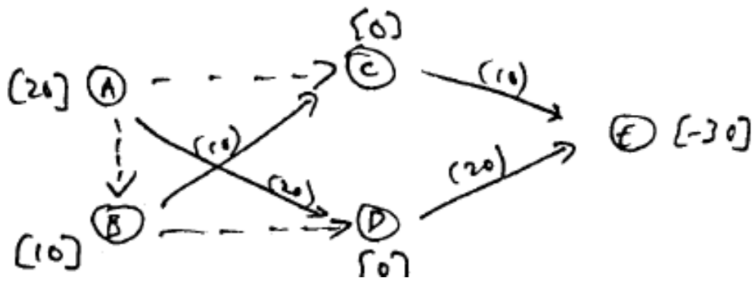


(b) Compute Δ for nonbasic variables

$$\Delta_{BD} = 5 + 4 - 3 + (-6) + 2 = 2$$
$$\Delta_{AD} = 5 + 4 - 3 + (-6) = 0$$
$$\Delta_{CB} = (-3) - 2 - (-6) = 1$$

All of them are nonnegative, so this solution is optimal. Since $\Delta_{AD} = 0$, multiple optimal solutions exist.

(c) Start with



Network simplex: $\Delta_{AC} = 0, \Delta_{AB} = -1, \Delta_{BD} = 3$.

$\theta = 15$ and BC is leaving arc (reverses). The next solution is optimal.