ITM529. Stochastic Processes

Short Quiz 5, 2025F, Weighting of 5%

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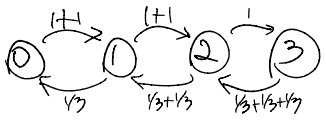
#1. A production system has three machines working in parallel. The up times of a machine is assumed to be iid, exponentially distributed with mean 3 hours. When a machine is down, its repair times are iid, exponentially distributed with mean 1 hour. There are two repairmen, working at the same speed. Using a CTMC to find the long-run fraction of time that all machines are up and running. [5pt]

up time $\sim \exp(1/3)$ repair $\sim \exp(1)$

$X(t)$ is # machine that are up at time t. $S = \{0,1,2,3\}$



cutting method	L→R	=	R→L
0→1,2,3	$2\pi_0$	=	$1/3\,\pi_1$
0,1→2,3	$2\pi_1$	=	$2/3\,\pi_2$
0,1,2→3	$1\pi_2$	=	$2/3\,\pi_3$

$\pi_1 = 6\pi_0$

$\pi_2 = 3\pi_1 = 18\pi_0$

$\pi_3 = \pi_2 = 18\pi_0$

$\pi_0 = 1/43$

$\pi = \{1/43, 6/43, 18/43, 18/43\}$

long-run frac. of time that all machines are up & running $= \pi_3 = 18/43$