SEOULTECH
Seoul National University of Science & Technology

Assignment 4

Date :

Code	**ITM 524**		**Title**	**Management Science**	
-	**-**	**Questions**		**Weighting**	**5%**
Student's Number	21102072		**Student's Name**	Lee Jeongyun 이정윤	

1. (30pts) Solve the following 0-1 integer programming problem using a branch-and-bound algorithm. At each step, report your balancing choice, the LP-relaxation bound at the node, and the current best feasible 0-1 solution (and its objective value).

$$\text{Maximize } Z = 8x_1 + 11x_2 + 6x_3$$
(subject to)
$$5x_1 + 7x_2 + 4x_3 \leq 14$$
$$x_j \text{ is binary for } j = 1, 2, 3$$

(Note: You must follow the procedure of branch and bound method)
Refer attachment for LP relaxation solutions

$x_1=1$, $x_2=1$, $x_3=0.5$ $\Rightarrow$ $5\times1+7\times1+4\times0.5 \leq 14$ $\Rightarrow$ $Z_{LP} = 8\times1+11\times1+6\times0.5 = 22$, $Z^* = -\infty$

$Z_{LP} = 22$, ($x_3 = 0.5$)

① (?,?,0)
$x_3=0$, $5x_1+7x_2 \leq 14$. $x_1=1$, $x_2=1$ $\Rightarrow$ $5+7 \leq 14$.
$(1,1,0) \Rightarrow 8+11+0 = 19 \Rightarrow Z^* = 19$
fathomed. f3

② (?,?,1)
$x_3=1$, $5x_1+7x_2 \leq 10$. $x_1=1$, $x_2=\frac{5}{7}$ $\Rightarrow$ $5+5 \leq 10$
$(1, 5/7, 1) \Rightarrow 8+11\times\frac{5}{7}+6 = 21.86 > Z^*=19$

④ (?,1,1)
$x_2=1$ $5x_1 \leq 3$ $x_1=\frac{3}{5}$ $\Rightarrow$ $3 \leq 3$
$(\frac{3}{5},1,1) \Rightarrow 8\times\frac{3}{5}+11+6 = 21.8 > Z^*=19$

③ (?,0,1)
$x_2=0$. $5x_1 \leq 10$. $x_1=1$ $\Rightarrow$ $5 \leq 10$
$(1,0,1) \Rightarrow 8+6 = 14 < Z^*=19$ fathomed
f2

⑤ (0,1,1)
$x_1=0$, $7+4 \leq 11$
$(0,1,1) \Rightarrow 11+6 = 17 < Z^*=19$
fathomed f1

⑥ (1,1,1)
$x_1=1$ $5+7+4 > 14$
infeasible fathomed
f2

$\therefore Z^* = 19$ at $(1,1,0)$

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Subproblems	LP relaxation – optimal solution	Z_{LP}
Original	(1,1,0.5)	22
x1=0	(0,1,1)	17
x1=1	(1,1,0.5)	22
x2=0	(1,0,1)	14
x2=1	(1,1,0.5)	22
x3=0	(1,1,0)	19
x3=1	(1,5/7,1)	21.86
x1=0,x2=0	(0,0,1)	6
x1=0,x2=1	(0,1,1)	17
x1=0,x3=0	(0,1,0)	11
x1=0,x3=1	(0,1,1)	17
x2=0,x3=0	(1,0,0)	8
x2=0,x3=1	(1,0,1)	14
x1=1,x2=0	(1,0,1)	14
x1=1,x2=1	(1,1,0.5)	22
x1=1,x3=0	(1,1,0)	19
x1=1,x3=1	(1,5/7,1)	21.86
x2=1,x3=0	(1,1,0)	19
x2=1,x3=1	(0.6,1,1)	21.8
(0,0,0)	(0,0,0)	0
(0,0,1)	(0,0,1)	6
(0,1,0)	(0,1,0)	11
(0,1,1)	(0,1,1)	17
(1,0,0)	(1,0,0)	8
(1,0,1)	(1,0,1)	14
(1,1,0)	(1,1,0)	19
(1,1,1)	Infeasible	-

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2. (30pts) From the inequality below, which arises in a pure 0-1 integer programming problem, derives as many valid cutting planes as possible (use minimum cover).

$$3x_1 + 5x_2 + 4x_3 + 8x_4 \leq 10$$

$\{x_1, x_4\}$ $3+8 > 10 \cdots x_1+x_4 \leq 1$ $\{x_1, x_2\}$ $\{x_1, x_3\}$ $\{x_2, x_3\}$ ≤ 10.

$8 \leq 10,\ 3 \leq 10$

$\{x_2, x_4\}$ $5+8 > 10 \cdots x_2+x_4 \leq 1$

$8 \leq 10,\ 5 \leq 10$

$\{x_3, x_4\}$ $4+8 > 10 \cdots x_3+x_4 \leq 1$

$8 \leq 10,\ 4 \leq 10$

$\{x_1, x_2, x_3\}$ $3+5+4 > 10 \cdots x_1+x_2+x_3 \leq 2$.

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3. (40pts) A student has 7 days to prepare for final exam in four courses and wants to allocate this time as effectively as possible. She must spend at least 1 day on each course, and prefers to focus on a single course per day. Therefore, the allowable study-time choices per course are 1, 2, 3, or 4 days. She decides to use dynamic programming to choose the allocation that maximizes total grade points. The estimated grade points for each course under the alternative day allocations are

Study days	Estimated Grade Points			
	Course			
	C 1	C 2	C 3	C 4
1	1	5	4	4
2	3	6	6	4
3	6	8	7	5
4	8	8	9	8

Use dynamic programming to determine how many days to assign to each course (subject to 7 day total and the 1-4 days per course limits) so as to maximize the sum of grade points.

$s_1 = 7 \quad s_2 = 7 - x_1 \quad s_3 = s_2 - x_2 \quad s_4 = s_3 - x_3$

s_4	1	2	3	4
$f_4^*(s_4)$	4	4	5	8

$x_3 \backslash s_3$	2	3	4	5
1	$\boxed{4+4=8}$	4+4=8	4+5=9	4+8=12
2		$\boxed{6+4=10}$	6+4=10	6+5=11
3			$\boxed{7+4=11}$	7+4=11
4				$\boxed{9+4=13}$

$x_2 \backslash s_2$	3	4	5	6
1	$\boxed{5+8=13}$	$\boxed{5+10=15}$	5+11=16	$\boxed{5+13=18}$
2		6+8=14	6+10=16	6+11=17
3			8+8=16	8+10=18
4				8+8=16

$x_1 \backslash s_1$	7
1	1+18=19
2	3+16=19
3	6+15=21
4	8+13=21

C1 C2 C3 C4

sol. { 3-1-2-1 ; 4-1-1-1 }

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C_1 C_2 C_3 C_4