Lecture 4. Newsvendor Model (2)

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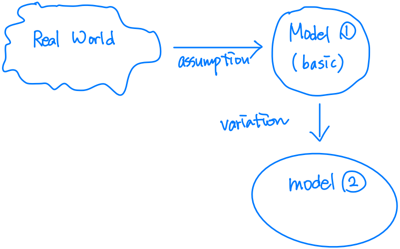
Making Newsvendor Model advanced

1. **Various situation with different cost setting.**
2. **(S,s) policy. (nonzero fixed ordering cost)**



Various situation with different cost setting.

- Assumptions in basic newsvendor model
  - "positive" salvage value
  - understock cost of lost sale opportunity.
- Variations when the above assumption is broken (in reality)
  - Unused stocks (overstock) may cost you in different ways
  - Unit cost for overstock $(= c_o)$ should be evaluated accordingly.
  - disposal cost
  - holding cost
  - reputation loss
  - compensation to customer

overstock: disposal cost, holding cost

understock: reputation loss, compensation to customer

blank

$F(y) = \dfrac{C_u}{C_u + C_o}$ $\begin{cases} C_u : \text{cost for understock} \\ C_o : \text{cost for overstock.} \end{cases}$

LHS)

$F(y) = F_D(y) = \mathbb{P}(D \leq y) = \mathbb{P}(\text{Demand} \leq \text{stock})$ ... $\begin{cases} \text{probability of overstock} \\ \text{probability that you are prepared enough to cover all demands} \\ \text{fill rate (준비율)} \end{cases}$

RHS)

	C_o	C_u	$\frac{C_u}{C_u + C_o} = F(y) = \text{fill rate}$
Department store	similar	higher	higher
Outlet	similar	lower	lower

Exercise 1

- David selling bananas
  - David buys fruits and vegetables wholesale and retails them at David's Produce on La Vista Road. One of the difficult decisions is the amount of bananas to buy.
  - Let us make some simplifying assumptions, and assume that David purchases bananas once a week at 30 cents per pound and retails them at 60 cents per pound during the week.
  - Bananas that are more than a week old are too ripe to sell and David will pay workers to take them away. It costs 5 cent to get rid of each pound of unsold bananas.
  - Suppose that the weekly demand for bananas is uniformly distributed between 500 and 1500 pounds.

$D \sim U(500, 1500)$

1. Identify c_o, c_u.
2. How many pound of bananas should David order each week?
3. What is the optimal expected weekly profit?

Solution

1. $c_o = 30 + 5$ and $c_u = 60 - 30$
2. $F(y) = \frac{y-500}{1500-500} = \frac{30}{35+30}$ gives $y = 962$
3. $60\mathbb{E}[min(962, D)] + (-5)\mathbb{E}[max(962 - D, 0)] - 30 \cdot 962$

- Now assume that the demand for bananas is exponentially distributed with mean 1000.
    1. Identify c_o, c_u.
    2. How many pound of bananas should David order each week?
    3. If David order 800 pounds this week, how much profit he should expect this week?

Solution

1. Same as before
2. $F(y) = 1 - exp(-\frac{y}{1000}) = \frac{30}{35+30}$ gives $y = 619$
3. $60\mathbb{E}[min(800, D)] + (-5)\mathbb{E}[max(800 - D, 0)] - 30 \cdot 800$

blank

2. (S, s) policy. (nonzero fixed ordering cost)

- Assumptions in basic newsvendor model
  - No fixed cost for ordering.
  - No existing inventory.

fixed order cost & variable order cost

quantity discount (많이 사면 할인)

- Variations when the above assumption is broken
  - Fixed cost
    - You may need to travel to pick up material from wholesaler.
    - Fixed shipping cost.
    - Every time you place an order, it occurs f dollars.
  - Inventory
    - You may have some inventory for sale

Example

- Solvent management
  - Next month's production at a manufacturing company will use a certain solvent for part of its production process. You need to prepare solvent in prior and the demand of solvent is random.
  - Assume that there is an ordering cost of \$1,500 incurred whenever an order for solvent is placed and the solvent costs \$50 per liter. Due to short product life cycle, unused solvent cannot be used in following months.
  - There will be a \$15 disposal charge for each liter of solvent left over at the end of the month. If there is a shortage of solvent, the production process is seriously disrupted at a cost of \$100 per liter short.
  - Assume that the initial inventory level is x, where $x = 0, 500, 700,$ and 800 liters.

$C_o: 50-(-15)=65$

$C_u: 100-50=50$

Problem

$D \sim U(300, 900)$ $x = 0, 500, 700, 800$

1. What is the optimal ordering quantity for each case when the demand is governed by the continuous uniform distribution varying between three hundred and nine hundred liters?

2. What is the optimal ordering policy for arbitrary initial inventory level x?
   i) if $x < 437$, order up to 560.
   ii) if $x \geq 437$, order nothing.

3. Assume optimal quantity will be ordered. What is the total expected cost when the initial inventory $x = 0$? What is the total expected cost when the initial inventory $x = 700$?

Discussion

What are the differences from basic model?

- We learned basic newsvendor model for the case when there is no “fixed” ordering cost but only “variable” ordering cost.
- And your optimal preparation for random demand was obtained by matching $F(y)$ to the critical fraction. $\left(\frac{C_u}{C_u+C_o}\right)$
- However, when there exists fixed ordering cost, you need to consider more.
- For example, suppose that y is 500 and you have 499 units currently available. If there is no fixed cost, you should order 1 unit to prepare 500 units available for sale.
- However, when fixed cost is large, then you might want to just give up some potential sale opportunity and stay with 499 level inventory.

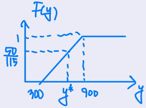
Opt. decision when nonzero inventory and nonzero fixed cost

$F(y) = \frac{C_u}{C_u + C_o}$

- Still y provides "the order up to quantity", which means if you are going to order then you should order up to y. (If you are paying fixed cost, then why not order up to optimal?)
- Therefore, at any level of your inventory, your decision is either
  - i: order up to y $\Rightarrow$ order $(y - x)$, y: newsvendor sol. x: current inventory
  - ii: order nothing.
- Without having the newsvendor optimal equality (or inequality for discrete distribution), you don't know whether you need to order 0 unit or 1 unit or 2 unit,...? Can't consider all cases!
- Basic Newsvendor solution narrows down your action set to two.
  - At some level of inventory, i) is optimal choice.
  - At some level, ii) can be optimal choice.

Solution to Problem 1

- The example asked you to find which action is optimal when you have different inventory level $x = 0, 500, 700, 800$
- You need to evaluate total expected costs for two possible actions (order || no-order) and make decision based on the comparison.
- Solution to 1.
  - What is y? optimal order quantity w/o fixed cost. $F(y) = \frac{C_u}{C_u + C_o} = \frac{50}{65+50} \Rightarrow D \sim U(300, 900)$
  - What are the two possible actions at inventory x?
    - order up to 560 (order $560 - x$)
    - order nothing (order 0)
  - Formula for total cost?



$y^* = 300 + 600 \times \frac{50}{115} \approx 560.$

Total cost = order cost + shortage cost + disposal cost

Total cost = order cost + shortage cost + disposal cost

- Solution to Problem 1. (continued) $D \sim U(300, 900)$
  - What would you do when $x = 0$?

i) $\mathbb{E}[\text{Total cost} \mid \text{if order } 560 \text{ at } x=0] = (1500 + 50 \times 560) + 100\,\mathbb{E}[(D-560)^+] + 15\,\mathbb{E}[(560-D)^+]$

$= 29500 + 100 \times 96 + 15 \times 56 = 39940.$

ii) $\mathbb{E}[\text{Total cost} \mid \text{if order } 0 \text{ at } x=0] = (0 + 50 \cdot 0) + 100 \cdot \mathbb{E}[(D-0)^+] + 15 \cdot \mathbb{E}[(0-D)^+]$

$= 100 \times 600 = 60000$

At $x=0$, we should order 560.

  - What would you do when $x = 500$?

i) $\mathbb{E}[\text{Total cost} \mid \text{if order } 60 \text{ at } x=500] = (1500 + 50 \times 60) + 100\,\mathbb{E}[(D-560)^+] + 15\,\mathbb{E}[(560-D)^+]$

$= 4500 + 100 \times 96 + 15 \times 56 = 14940$

ii) $\mathbb{E}[\text{Total cost} \mid \text{if order } 0 \text{ at } x=500] = (0 + 50 \cdot 0) + 100 \cdot \mathbb{E}[(D-500)^+] + 15 \cdot \mathbb{E}[(500-D)^+]$

$= 100 \times 133 + 15 \times 27 = 13795$

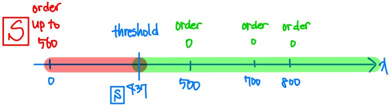
At $x=500$, we should not order.

- Solution to Problem 1. (continued)
  - What would you do when $x = 700$? No order.

why?

i) already have enough inventory... ($\lambda > 560$)

ii) not make an order at ($\lambda = 500$) ... ($\lambda > 500$)



  - What would you do when $x = 800$? No order

why?

i) "

ii) "

problem 2.

find optimal action identified for all value of x.

① all contingent value of x
② readily available solution
} strategy
policy

- Problem 2. Opt. ordering policy for arbitrary initial inventory level x?
- Discussion
  - You tend to go ahead and make an order when inventory level is low, despite of the fixed cost
  - On the other hand, you tend not to order when inventory level is high.
  - Then there is the critical "threshold" where you should
    - order up to y if inventory level below "the threshold"
    - order nothing if above.
  - At the threshold, the two actions should result the same total expected cost.

- (S, s) policy?
  - Definition: (S, s) policy is the inventory policy where you **make an order up to point S if and only if your inventory level is below s.**
  - What is S and s?
    - S: order up to quantity obtained from basic newsvendor
    - s: threshold of inventory level on which two actions the same expected result
  - Once S and s are found, what's the advantage?
    - readily available solution.
    - quick decision making.
    -

optimal action

blank

find value of threshold, S

$D \sim U(300, 900)$

Total Cost = order cost + disruption cost + disposal cost

$\mathbb{E}[\text{Total cost} \mid \text{if order } 560-S \text{ at } x=S] = \mathbb{E}[\text{Total Cost} \mid \text{if order } 0 \text{ at } x=S]$

$\text{LHS} = (1500 + 50(560-S)) + 100 \cdot \mathbb{E}[(D-560)^+] + 15\,\mathbb{E}[(560-D)^+]$

$= 1500 + 50 \cdot 560 - 50 \cdot S + 100 \cdot 96 + 15 \cdot 56 = 39940 - 50S$

$\text{RHS} = 0 + 100 \cdot \mathbb{E}[(D-S)^+] + 15 \cdot \mathbb{E}[(S-D)^+]$

$\mathbb{E}[(D-S)^+] = \int_{300}^{900} (y-S)^+ \frac{1}{600}\,dy = \int_{300}^{S} (y-S)^+ \frac{1}{600}\,dy + \int_{S}^{900} (y-S)^+ \frac{1}{600}\,dy = \frac{1}{1200}(900-S)^2$

$\mathbb{E}[(S-D)^+] = \int_{300}^{900} (S-y)^+ \frac{1}{600}\,dy = \frac{1}{1200}(S-300)^2$

LHS = RHS

$39940 - 50S = 100 \cdot \frac{1}{1200}(900-S)^2 + 15 \cdot \frac{1}{1200}(S-300)^2$

$\Rightarrow 115S^2 - 129000S + 34422000 = 0.$

$S = 684$ or $\underline{437}$ $(S < 560)$

Solution to problem 2.

Solution to problem 3.

$\lambda = 0$, order up to 560.

$\mathbb{E}[\text{Total Cost}] = 39940$.

$\lambda = 700$, not making order

$\mathbb{E}[\text{Total Cost} \mid \lambda = 700, \text{ no order}]$

$= 0 + 100 \cdot \mathbb{E}[(D-700)^+] + 15 \cdot \mathbb{E}[(700-D)^+]$

$= 100 \times 73 + 15 \times 133$

$= 9295$

Summary

Basic newsvendor model $\rightarrow$ (S, s) policy

$\exists$ fixed ordering cost

$\exists$ inventory

Q1. (solution). What would you do at particular inventory level x.

$\mathbb{E}[\text{Total Cost} \mid \text{order } S-x \text{ from } x \text{ to get } S]$ vs. $\mathbb{E}[\text{Total Cost} \mid \text{order } 0 \text{ from } x \text{ to get } x]$

Q2. (policy) How to find the threshold value "small s"

$\mathbb{E}[\text{Total Cost} \mid \text{order } S-s \text{ from } s \text{ to get } S] = \mathbb{E}[\text{Total Cost} \mid \text{order } 0 \text{ from } S \text{ to get } S]$

Solve for small s

blank

Exercise 2

- Repeat the problem 1.,2.,3. where there is an ordering cost of $10000.

(Solution)

$y \approx 560$ stays same.

i) Same conclusion for $x = 700$ or $x = 800$. (Optimal order quantity is zero because initial inventory level is higher than "big S")

ii) When $x = 0$, compare the following two expected costs.

$$\begin{aligned}
&\mathbb{E}[\text{Cost}|\text{if order } 560 \text{ at } x = 0] \\
&= (10000 + 560 * 50) + 100\mathbb{E}[(D - 560)^+] + 15\mathbb{E}[(560 - D)^+] \\
&= 8500 + \text{RHS}^1 \text{of (1)} = 8500 + 39940 = 48440 \\
&\mathbb{E}[\text{Cost}|\text{if order } 0 \text{ at } x = 0] \\
&= 60000 \ (\text{Same as lecture note})
\end{aligned}$$

Thus, optimal order quantity at $x = 0$ is 560.

iii) When $x = 500$, compare following two expected costs.

$$\begin{aligned}
\mathbb{E}[\text{Cost}|\text{if order } 60 \text{ at } x = 500] &= (10000 + 60 * 50) + 100\mathbb{E}[(D - 560)^+] + 15\mathbb{E}[(560 - D)^+] \\
&= 8500 + \text{RHS of (4)} = 8500 + 14940 = 23440 \\
\mathbb{E}[\text{Cost}|\text{if order } 0 \text{ at } x = 500] &= 13795 \ (\text{Same as lecture note})
\end{aligned}$$

Thus, optimal order quantity at $x = 500$ is 0

For the “small s” of current inventory level,

$$
\begin{aligned}
& \mathbb{E}[\text{Cost}|\text{if order } 560-s \text{ at } x=s] \\
= \quad & (10000+(560-s)*50)+100\mathbb{E}[(D-560)^+]+15\mathbb{E}[(560-D)^+] \\
= \quad & 8500+\text{RHS of (6)} = 8500+39940-50s \\
& \mathbb{E}[\text{Cost}|\text{if order } 0 \text{ at } x=s] \\
= \quad & \frac{100}{1200}(s-900)^2+\frac{15}{1200}(s-300)^2 \;\; (\text{Same as lecture note})
\end{aligned}
$$

Solving for s gives $s \approx 881$ or $s \approx 241$. (SEE the next pages for solving the quadratic equation) Because s must be positive but lower than 560 (“big S”), we have $s \approx 241$. For an arbitrary inventory level x, optimal order policy is order $560-x$ if $x \leq 241$, and order 0 otherwise. Because of the increased fixed cost from original setting, you tend to become conservative when you make a decision of ordering.

[**Supplememt** - review previous homework and solution if you have trouble understanding following]

$$\mathbb{E}[(D-560)^+] = \int_{560}^{900} \frac{1}{600}(x-560)dx = \frac{1}{600}\cdot\frac{1}{2}\cdot 340\cdot 340 \approx 96 \quad (1)$$

$$\mathbb{E}[(560-D)^+] = \int_{300}^{560} \frac{1}{600}(560-x)dx = \frac{1}{600}\cdot\frac{1}{2}\cdot 260\cdot 260 \approx 56 \quad (2)$$

$$\mathbb{E}[(D-500)^+] = \int_{500}^{900} \frac{1}{600}(x-500)dx = \frac{1}{600}\cdot\frac{1}{2}\cdot 400\cdot 400 \approx 133 \quad (3)$$

$$\mathbb{E}[(500-D)^+] = \int_{300}^{500} \frac{1}{600}(500-x)dx = \frac{1}{600}\cdot\frac{1}{2}\cdot 200\cdot 200 \approx 33 \quad (4)$$

$$\mathbb{E}[(D-700)^+] = \int_{700}^{900} \frac{1}{600}(x-700)dx = \frac{1}{600}\cdot\frac{1}{2}\cdot 200\cdot 200 \approx 33 \quad (5)$$

$$\mathbb{E}[(700-D)^+] = \int_{300}^{700} \frac{1}{600}(700-x)dx = \frac{1}{600}\cdot\frac{1}{2}\cdot 400\cdot 400 \approx 133 \quad (6)$$

$$\mathbb{E}[(D-s)^+] = \int_{s}^{900} \frac{1}{600}(x-s)dx = \frac{1}{600}\cdot\frac{1}{2}\cdot (900-s)\cdot(900-s) \approx \frac{1}{1200}(s-900)^2 \quad (7)$$

$$\mathbb{E}[(s-D)^+] = \int_{300}^{s} \frac{1}{600}(s-x)dx = \frac{1}{600}\cdot\frac{1}{2}\cdot (s-900)\cdot(s-900) \approx \frac{1}{1200}(s-300)^2 \quad (8)$$

[**Supplememt** - RHS in solution 2, Same as lecture note.]

i) When $x = 0$, compare following two expected costs.

$$\begin{aligned}
&\mathbb{E}[\text{Cost}|\text{if order } 560 \text{ at } x = 0] \\
&= (1500 + 560 * 50) + 100\mathbb{E}[(D - 560)^+] + 15\mathbb{E}[(560 - D)^+] \\
&= 29500 + 100 \cdot 96 + 15 \cdot 56 = 39940 \qquad (9) \\
&\mathbb{E}[\text{Cost}|\text{if order } 0 \text{ at } x = 0] \\
&= 0 + 100\mathbb{E}[(D - 0)^+] + 15\mathbb{E}[(0 - D)^+] \\
&= 0 + 100\mathbb{E}D + 0 \\
&= 100 \cdot 600 = 600000 \qquad (10)
\end{aligned}$$

ii) When $x = 500$, compare following two expected costs.

$$\begin{aligned}
&\mathbb{E}[\text{Cost}|\text{if order } 60 \text{ at } x = 500] \\
&= (1500 + 60 * 50) + 100\mathbb{E}[(D - 560)^+] + 15\mathbb{E}[(560 - D)^+] \\
&= 4500 + 100 \cdot 96 + 15 \cdot 56 = 14940 \qquad (11) \\
&\mathbb{E}[\text{Cost}|\text{if order } 0 \text{ at } x = 500] \\
&= 0 + 100\mathbb{E}[(D - 500)^+] + 15\mathbb{E}[(500 - D)^+] \\
&= 0 + 100 \cdot 133 + 15 \cdot 33 = 13795 \qquad (12)
\end{aligned}$$

iii) When $x = s$, compare following two expected costs.

$$\begin{aligned}
&\mathbb{E}[\text{Cost}|\text{if order } 560 - s \text{ at } x = s] \\
&= (1500 + (560 - s) * 50) + 100\mathbb{E}[(D - 560)^+] + 15\mathbb{E}[(560 - D)^+] \\
&= (1500 + 50 \cdot 560 - 50s) + 100 \cdot 96 + 15 \cdot 56 = 39940 - 50s
\end{aligned} \tag{13}$$

$$\begin{aligned}
&\mathbb{E}[\text{Cost}|\text{if order } 0 \text{ at } x = s] \\
&= 0 + 100\mathbb{E}[(D - s)^+] + 15\mathbb{E}[(s - D)^+] \\
&= \frac{100}{1200}(s - 900)^2 + \frac{15}{1200}(s - 300)^2
\end{aligned} \tag{14}$$

[**Supplememt** - solving the quadratic equation] We start with

$$100(s-900)^2 + 15(s-300)^2 = 1200(39940 - 50s)$$

Be moving all terms to (LHS) yields

$$115s^2 - (100\cdot 2\cdot 900 - 15\cdot 2\cdot 300)s + (100\cdot 900^2 + 15\cdot 300^2) + (1200\cdot 50)s - 1200\cdot 39940 = 0 \quad (15)$$

Reduces it into

$$115s^2 - 129000s + 34422000 = 0 \quad (16)$$

Then, using the formula for quadratic solution...

$$s = \frac{\frac{129000}{2} \pm \sqrt{(\frac{129000}{2})^2 - 115\cdot 34422000}}{115} \approx \frac{64500 \pm 14203}{115} = 684 \text{ or } 437 \quad (17)$$

[**Supplememt** - solving the quadratic equation] We start with

$$100(s-900)^2 + 15(s-300)^2 = 1200(48440 - 50s)$$

Reduces into

$$115s^2 - 129000s + 24422000 = 0 \tag{18}$$

Then, using the formula for quadratic solution…

$$s \approx \frac{64500 \pm 36766}{115} = 881 \text{ or } 241 \tag{19}$$

Exercise 3

- Repeat the problem 1.,2.,3, when the demand is following: $\mathbb{P}(D = 300) = .2, \mathbb{P}(D = 500) = .4, \mathbb{P}(D = 700) = .3$, and $\mathbb{P}(D = 900) = .1$. (Fixed order cost is still $1500)

(Solution)

(a) If you are going to order, then you should order up to point smallest y such that $F(y) \geq \frac{50}{65+50}$. Thus, we have $y = 500$ (This is the "big S").

i) When $x = 500, 700, 800$, the optimal order quantity is zero because initial inventory level is greater than or equal to "big S".

ii) When $x = 0$, compare following two expected costs. (The very same procedure as lecture note)

$$
\begin{aligned}
&\mathbb{E}\big[\text{Cost}\big|\text{if order } 500 \text{ at } x = 0\big] \\
&= (1500 + 500 * 50) + 100\mathbb{E}[(D - 500)^+] + 15\mathbb{E}[(500 - D)^+] \\
&= 26500 + 100 \cdot 100 + 15 \cdot 40 = 37100 \\
&\mathbb{E}\big[\text{Cost}\big|\text{if order } 0 \text{ at } x = 0\big] \\
&= 100\mathbb{E}[(D - 0)^+] + 15\mathbb{E}[(0 - D)^+] \\
&= 100\mathbb{E}[D] = 100 \cdot 560 = 56000
\end{aligned}
$$

Thus, optimal order quantity at $x = 0$ is 500.

(b) We need to find the small s (≤ 500) such that i) ordering up to "big S"$(= 500)$ and ii) not ordering both result in same total expected costs.

$$\begin{aligned}
&\mathbb{E}[\text{Cost}|\text{if order } 500 - s \text{ at } x = s] \\
&= (1500 + (500 - s) * 50) + 100\mathbb{E}[(D - 500)^+] + 15\mathbb{E}[(500 - D)^+] \\
&= 37100 - 50s \qquad (1) \\
&\mathbb{E}[\text{Cost}|\text{if order } 0 \text{ at } x = s] \\
&= 100\mathbb{E}[(D - s)^+] + 15\mathbb{E}[(s - D)^+] \\
&= 100 \sum p(x)(x - s)^+ + 15 \sum p(x)(s - x)^+ \qquad (2)
\end{aligned}$$

We need to solve the following equation (3) by letting RHS of (1) and (2) equal.

$$37100 - 50s = 100 \sum p(x)(x - s)^+ + 15 \sum p(x)(s - x)^+ \qquad (3)$$

Solving for s in (3) requires more than simple thought. Because the demand x is varying from 300 to 900, it's not straightforward to evaluate the expectations. There are at least two following methods to get around this issue.

[Method I - relying on mathematics]

This method is relying on the pure math skill by considering two different case of s.

i) For $s \leq 300$, (3) becomes

$$37100 - 50s = 100(560 - s) \tag{4}$$

Solving (4) will give $s = 378$, but then $s \leq 300$ condition gets violated. Thus, no solution in the range of $s \leq 300$.

ii) For $300 < s \leq 500$, (3) becomes

$$37100 - 50s = 100(500 - 0.8s) + 15(0.2s - 60) \tag{5}$$

and we have $s \approx 444$. (and $s = 444$ is in the range of $300 < s \leq 500$)

Therefore, $s = 444$. For an arbitrary inventory level x, optimal order policy is order $500 - x$ if $x \leq 444$, and order 0 otherwise.

[Method II - relying on the property of (S,s) policy]
Because $s \leq 500$, we should know whether s or 300 is bigger. One way to get around is by *finding which decision is optimal when current inventory is exactly 300*. (If ordering up to 500 is optimal at the inventory level of 300, then $s < 300$; If ordering nothing is optimal at the inventory level of 300, then $s > 300$; If both decisions will lead same expected costs, then $s = 300$.)
When $x = 300$, we have

$$\begin{aligned}
&\mathbb{E}[\text{Cost}|\text{if order } 500 - 300 \text{ at } x = 300] \\
&= (1500 + 200 * 50) + 100\mathbb{E}[(D - 500)^+] + 15\mathbb{E}[(500 - D)^+] \\
&= 11500 + 100 \cdot 100 + 15 \cdot 40 = 22100 \\
&\mathbb{E}[\text{Cost}|\text{if order } 0 \text{ at } x = 300] \\
&= 100\mathbb{E}[(D - 300)^+] + 15\mathbb{E}[(300 - D)^+] \\
&= 100 \cdot 260 + 15 \cdot 0 = 26000
\end{aligned}$$

Thus, when $x = 300$, optimal decision is order up to 500. Now we know that $s > 300$. Using this fact allow us to solve (3). For $300 < s \leq 500$, (3) becomes

$$37100 - 50s = 100(500 - 0.8s) + 15(0.2s - 60) \tag{6}$$

and we have $s ≈444 $. (and $s = 444$ is in the range of $300 < s \leq 500$)
Therefore, $s = 444$. For an arbitrary inventory level x, optimal order policy is order $500 - x$ if $x \leq 444$, and order 0 otherwise.

Exercise 4

- What is (S, s) policy? What is the difference between *solution* and *policy*?

(Solution)

What is (S, s) policy?

(S, s) policy is a type of inventory control policy used to manage the stock levels of products or materials. The policy is named after the two parameters used to define it: S and s.

The S parameter represents the order-up-to level, which is the maximum level of inventory that the organization wants to maintain. When an order is placed, the inventory is restocked to level S. (See L4.p14 for calculating level S).

The s parameter represents the reorder point or the minimum level of inventory that triggers a replenishment order. When the inventory level reaches s, the organization places an order to restock the inventory to level S.

What is the difference between *solution* and *policy*?

A *policy* is a set of rules or guidelines that guide decision-making and actions within an organization. On the other hand, a *solution* is a specific approach or action taken to address a particular problem or issue.

The main difference between a *policy* and a *solution* is that a *policy* is a broader, more general approach to a particular issue, while a *solution* is a specific action taken to address a particular problem. Policies provide a framework for decision-making and actions within an organization, while solutions are more flexible and can be adapted or changed depending on the situation.

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"An idea is always a generalization, and generalization is a property of thinking.
To generalize means to think. - George Wilhelm Friedrich Hegel"
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