Assignment 2

Date :

Code	**ITM 524**		**Title**	**Management Science**	
-	**-**	**Questions**		**Weighting**	**5%**
Student's Number	21102052		**Student's Name**	Lee Jeong-yun 이정윤	

1. (20pts) The company operates two factories that produce electric scooters, which are then shipped to three regional warehouses. The manufacturing costs are the same at both factories, but the shipping costs (per scooter) vary depending on the factory-warehouse route, as shown below:

		Warehouse		
		1	2	3
Factory	A	800	700	400
	B	600	800	500

A total of 60 scooters are produced and shipped each week. Each factory has a maximum weekly capacity of 50 units, so production can be split between them in various ways. At the same time, each warehouse must receive exactly 20 scooters per week.

Management wants to decide how many scooters should be produced at each factory and how shipments should be allocated to the warehouses in order to minimize the overall shipping cost.

Formulate this problem as a transportation problem by constructing the appropriate parameter table.

factory \ warehouse	1	2	3	4(D)	supply
A	800	700	400	0	50
B	600	800	500	0	50
demand	20	20	20	40	100

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2. (20pts) The distribution company needs to ship goods from three warehouses to three retail stores. The per-unit shipping costs (in dollars) are given in the following table, along with the available supply at each warehouse and the demand at each store:

		Store			Supply
		1	2	3	
Warehouse	1	15	9	13	7
	2	11	M	17	5
	3	9	11	9	3
Demand		7	3	5	

(a) Use Vogel's approximation method to identify the first basic variable chosen for an initial basic feasible solution.

(b) Use the northwest corner rule to construct the complete initial basic feasible solution.

a)

	1	2	3	s	Δ
1	15	9	13	7	4
2	11	M	17	5	6
3	9	11	9	3	0
d	7	3	5	$\lambda_{21}=5$	
Δ	2	2	4		

	1	2	3	s	Δ
1	15	9	13	7	4
2					
3	9	11	9	3	0
d	2	3	5	$\lambda_{31}=2$	
Δ	6	2	4		

	1	2	3	s	Δ
1		9	13	7	4
2					
3		11	9	1	2
d		3	5	$\lambda_{33}=1$	
Δ		2	4		

	1	2	3	s	Δ
1		9	13	7	4
2					
3					
d		3	4	$\lambda_{13}=4$	
Δ		9	13	$\lambda_{12}=3$	

⇒

	1	2	3	s
1		3	4	7
2	5			5
3	2		1	3
	7	3	5	

b)

	1	2	3	s_i
1	15	9	13	7
2	11	M	17	5
3	9	11	9	3
d_j	7	3	5	

	1	2	3	s_i
1	7			0
2	0	3	2	0
3			3	0
d_j	0	0	0	

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3. (20pts) A company serves four stores from four manufacturing sites. The per-shipment shipping cost (in dollars) from each site to each store is:

		Unit shipping cost to each store				
		1	2	3	4	S
Site	1	700	800	500	200	10
	2	200	900	100	400	20
	3	400	500	300	100	20
	4	200	100	400	300	10
	d	20	10	10	20	

Sites 1-4 can send 10, 20, 20, and 10 shipments per month, respectively. Stores 1-4 must receive 20, 10, 10, and 20 shipments per month, respectively.

The distribution manager must decide how many shipments to route from each site to each store so as to minimize the total shipping cost.

(a) Formulate this problem as a transportation problem by constructing appropriate parameter table (supplies, demands, and unit costs).

(b) Use the northwest corner rule to construct an initial basic feasible solution.

(c) Starting from the solution in (b), apply the transportation simplex method step by step to reach an optimal solution.

a)

	1	2	3	4	s_i
1	700	800	500	200	10
2	200	900	100	400	20
3	400	500	300	100	20
4	200	100	400	300	10
d_j	20	10	10	20	

b)

	1	2	3	4	s_i
1	10				0
2	10	10			0
3		0	10	10	0 0
4				10	
d_j	0	0	0	0	

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3. (20pts) A company serves four stores from four manufacturing sites. The per-shipment shipping cost (in dollars) from each site to each store is:

		Unit shipping cost to each store			
		1	2	3	4
Site	1	700	800	500	200
	2	200	900	100	400
	3	400	500	300	100
	4	200	100	400	300

Sites 1-4 can send 10, 20, 20, and 10 shipments per month, respectively. Stores 1-4 must receive 20, 10, 10, and 20 shipments per month, respectively.

The distribution manager must decide how many shipments to route from each site to each store so as to minimize the total shipping cost.

(a) Formulate this problem as a transportation problem by constructing appropriate parameter table (supplies, demands, and unit costs).

(b) Use the northwest corner rule to construct an initial basic feasible solution.

(c) Starting from the solution in (b), apply the transportation simplex method step by step to reach an optimal solution.

c)

	1	2	3	4	S_i	u_i
1	700) $-\theta$ 10	800) 400	500) -700	200) -800 θ	10	900
2	200) θ 10	900) $-\theta$ 10	100) -600	400) -100	20	400
3	400) 600	500) θ 0	300) 10	100) $-\theta$ 10	20	0
4	200) -200	100) -600	400) -100	300) 10	10	200
d_j	20	10	10	20		
v_j	-200	500	300	100		

λ_{14}	θ	10
λ_{11}	$10-\theta \geq 0$	0
λ_{21}	$10+\theta$	20
λ_{22}	$10-\theta \geq 0$	0
λ_{32}	θ	10
λ_{34}	$10-\theta \geq 0$	0

$\theta=10$ leaving λ_{11}

	1	2	3	4	S_i	u_i
1	700) 800	800) 200	500) 100	200) 10	10	100
2	200) 20	$-\theta$ 900) 0	θ 100) -600	400) -100	20	400
3	400) 600	θ 500) 10	$-\theta$ 300) 10	100) 0	20	0
4	200) 200	100) -600	400) -100	300) 10	10	200
d_j	20	10	10	20		
v_j	-200	500	300	100		

λ_{23}	θ	0
λ_{22}	$0-\theta \geq 0$	0
λ_{32}	$10+\theta$	10
λ_{33}	$10-\theta \geq 0$	10

$\theta=0$, leaving λ_{22}

	1	2	3	4	S_i	u_i
1	700) 200	800) 200	500) 100	200) 10	10	100
2	200) 20	900) 600	100) 0	400) 300	20	-200
3	400) 0	$-\theta$ 500) 10	300) 10	100) θ 0	20	0
4	200) -400	θ 100) -600	400) -100	300) $-\theta$ 10	10	200
d_j	20	10	10	20		
v_j	400	500	300	100		

λ_{42}	θ	10
λ_{44}	$10-\theta \geq 0$	0
λ_{34}	$0+\theta$	10
λ_{32}	$10-\theta \geq 0$	0

$\theta=10$. leaving λ_{44}

	1	2	3	4	S_i	u_i
1	700) 200	800) 200	500) 100	200) 10	10	100
2	200) 20	900) 600	100) 0	400) 500	20	-200
3	400) 0	500) 0	300) 10	100) 10	20	0
4	200) 200	100) 10	400) 500	300) 600	10	-400
d_j	20	10	10	20		
v_j	400	500	300	100		

all non negative
$\Rightarrow$ optimal

$200 \times 10 + 200 \times 20 + 300 \times 10 + 100 \times 10 + 100 \times 10 = 11000$

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4. (20pts) A logistics company needs to send four truck drivers on four different delivery routes (labeled 1-4). Each driver must be assigned to exactly one route. Because of differences in driver experience and route conditions, the cost of completing each route varies by driver, as shown below.

		Route			
		1	2	3	4
Driver	1	500	400	600	700
	2	600	600	700	500
	3	700	500	700	600
	4	500	400	600	600

The objective is to assign the four drivers to the four routes in order to minimize the total cost.

(a) Explain how this problem fits the assignment problem framework.

(b) Reformulate this situation as an equivalent transportation problem with supplies, demands, and unit costs.

(c) Use the model in (b), apply the northwest corner rule to obtain an initial basic feasible solution.

(d) Starting from (c), apply the transportation simplex method to obtain the optimal set of driver-route assignments.

a) each driver must be assigned only single route. 4 drivers & 4 routes.

b)

	1	2	3	4	S
1	500	400	600	700	1
2	600	600	700	500	1
3	700	500	700	600	1
4	500	400	600	600	1
d	1	1	1	1	

c)

	1	2	3	4	S
1	1				0
2	0	1			0
3		0	1		0
4			0	1	0
d	0	0	0	0	

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4. (20pts) A logistics company needs to send four truck drivers on four different delivery routes (labeled 1-4). Each driver must be assigned to exactly one route. Because of differences in driver experience and route conditions, the cost of completing each route varies by driver, as shown below.

		Route			
		1	2	3	4
Driver	1	500	400	600	700
	2	600	600	700	500
	3	700	500	700	600
	4	500	400	600	600

The objective is to assign the four drivers to the four routes in order to minimize the total cost.

(a) Explain how this problem fits the assignment problem framework.

(b) Reformulate this situation as an equivalent transportation problem with supplies, demands, and unit costs.

(c) Use the model in (b), apply the northwest corner rule to obtain an initial basic feasible solution.

(d) Starting from (c), apply the transportation simplex method to obtain the optimal set of driver-route assignments.

d)

	1	2	3	4	s	u_i
1	(500) 1	(400) -100	(600) -100	(700) 0	1	0
2	(600) 0	−θ (600) 1	(700) -100	θ (500) -300	1	100
3	(700) 200	θ (500) 0	−θ (700) 1	(600) -100	1	0
4	(500) 100	(400) 0	θ (600) 0	−θ (600) 1	1	-100
d	1	1	1	1		
v_j	500	500	700	700		

x_{24}	$+\theta$	1
x_{44}	$1-\theta \geq 0$	0
x_{43}	$0+\theta$	1
x_{33}	$1-\theta \geq 0$	0
x_{32}	$0+\theta$	1
x_{22}	$1-\theta \geq 0$	0

$\theta=1$, leaving x_{44}

	1	2	3	4	s	u_i
1	−θ (500) 1	θ (400) -100	(600) 100	(700) 300	1	0
2	θ (600) 0	−θ (600) 0	(700) -100	(500) 1	1	100
3	(700) 200	(500) 1	(700) 0	(600) 200	1	0
4	(500) 100	(400) 0	(600) 1	(600) 300	1	-100
d	1	1	1	1		
v_j	500	500	700	400		

x_{12}	θ	0
x_{11}	$1-\theta \geq 0$	1
x_{21}	$0+\theta$	0
x_{22}	$0-\theta \geq 0$	0

$\theta=0$, leaving x_{22}

	1	2	3	4	s	u_i
1	(500) 1	(400) 0	(600) 0	(700) 300	1	-100
2	(600) 0	(600) 100	(700) 0	(500) 1	1	0
3	(700) 100	(500) 1	(700) 0	(600) 100	1	0
4	(500) 0	(400) 0	(600) 1	(600) 200	1	-100
d	1	1	1	1		
v_j	600	500	700	500		

all non-negative
⇒ optimal

$500\times1 + 500\times1 + 500\times1 + 600\times1$
$= 2100.$

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5. (20pts) Use the Hungarian algorithm to find the minimum-cost assignment for the following cost table. Show the row and column reductions, indicate the zero coverings and augmentations as needed, and report the final assignment and the total cost.

		Task			
		1	2	3	4
Assignee	A	4	$\boxed{1}$	0	1
	B	$\boxed{1}$	3	4	0
	C	3	2	$\boxed{1}$	3
	D	2	2	3	$\boxed{0}$

after row reduction

	1	2	3	4
A	4	1	0	1
B	1	3	4	0
C	2	1	0	2
D	2	2	3	0

2 lines

after column reduction

	1	2	3	4
A	3	$\boxed{0}$	0	1
B	$\boxed{0}$	2	4	0
C	1	0	$\boxed{0}$	2
D	1	1	3	$\boxed{0}$

4 lines

total cost = 1+1+1+0=3