Introduction to Model Building

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Contents

Introduction to Modeling

- Operations Research (management science) is a scientific approach to decision making that seeks to best design and operate a system, usually under conditions requiring the allocation of scarce resources.

- A system is an organization of interdependent components that work together to accomplish the goal of the system.

Introduction to Modeling

- The term operations research was coined during World War II when leaders asked scientists and engineers to analyze several military problems.

- The scientific approach to decision making requires the use of one or more mathematical models. A mathematical model is a mathematical representation of the actual situation that may be used to make better decisions or clarify the situation.

Introduction to Modeling

- A modeling example
  - Eli Daisy produces the drug Wozac in huge batches by heating a chemical mixture in a pressurized container. Each time a batch is produced, a different amount of Wozac is produced. The amount produced is the process *yield*.
  - Daisy is interested in understanding the factors that influence the yield of Wozac production process.
  - The solution on subsequent slides describes a model building process for this situation.

Introduction to Modeling

- A modeling example
  - Daisy is interested in determining the factors that influence the process yield. This would be referred to as a descriptive model since it describes the behavior of the actual yield as a function of various factors.

  - Daisy might determine the following factors influence yield:
    - ✓ Container volume in liters (V)
    - ✓ Container pressure in milliliters (P)
    - ✓ Container pressure in degrees centigrade (T)
    - ✓ Chemical composition of the processed mixture

Introduction to Modeling

- A modeling example
  - Letting A, B, and C be the percentage of the mixture made up of chemical A, B, and C, then Daisy might find, for example, that
  - $Yield = 300 + 0.8 \cdot V + 0.01 \cdot P + 0.06 \cdot T + 0.001 \cdot T \cdot P - 0.01 \cdot T^2 - 0.001 \cdot P^2 + 11.7 \cdot A + 9.4 \cdot B + 16.4 \cdot C + 19 \cdot A \cdot B + 11.4 \cdot A \cdot C - 9.6 \cdot B \cdot C$

Introduction to Modeling

- A modeling example
  - To determine this relationship, the yield of the process would have to be measured for many different combinations of the previously listed factors. Knowledge of this equation would enable Daisy to describe the yield of the production process once volume, pressure, temperature, and chemical composition were known.

Introduction to Modeling

- Prescriptive or Optimization Models
  - Prescriptive models “prescribes” behavior for an organization that will enable it to best meet its goals. Components of this model include:
    - ✓ Objective function(s)
    - ✓ Decision variables
    - ✓ Constraints
  - An optimization model seeks to find values of the decision variables that optimize (maximize or minimize) an objective function among the set of all values for the decision variables that satisfy the given constraints.

Introduction to Modeling

- A modeling example – the objective function
  - The Daisy example seeks maximize the yield for the production process. In most models, there will be function we wish to maximize or minimize. This function is called the model's objective function. To maximize the process yield we need to find the values of V, P, T, A, B, and C that make the yield equation (below) as large as possible.

  - $Yield = 300 + 0.8 \cdot V + 0.01 \cdot P + 0.06 \cdot T + 0.001 \cdot T \cdot P - 0.01 \cdot T^2 - 0.001 \cdot P^2 + 11.7 \cdot A + 9.4 \cdot B + 16.4 \cdot C + 19 \cdot A \cdot B + 11.4 \cdot A \cdot C - 9.6 \cdot B \cdot C$

Introduction to Modeling

- In many situations, an organization may have more than one objective.
- For example, in assigning students to the two high schools in Bloomington, Indiana, the Monroe Count School Board stated that the assignment of students involve the following objectives:
  - Equalize the number of students at the two high schools
  - Minimize the average distance students travel to school
  - Have a diverse students body at both high schools

Introduction to Modeling

- The decision variables
  - Variables whose values are under our control and influence system performance are called decision variables. In the Daisy example, V, P, T, A, B, and C are decision variables.

- Constraints
  - In most situations, only certain values of the decision variables are possible. For example, certain volume, pressure, and temperature conditions might be unsafe. Also, A, B, and C must be non-negative numbers that sum to one. These restrictions on the decision variable values are called constraints.

Introduction to Modeling

- The modeling example
  - Suppose the Daisy example has the following constraints:
    - ✓ Volume must be between 1 and 5 liters
    - ✓ Pressure must be between 200 and 400 milliliters
    - ✓ Temperature must be between 100 and 200 degrees centigrade
    - ✓ Mixture must be made up entirely of A, B, and C.
    - ✓ For the drug to perform properly, only half the mixture at most can be product A.

Introduction to Modeling

- The modeling example
  - Mathematically these constraints can be expressed as:
    - ✓ $V \leq 5$ and $V \geq 1$
    - ✓ $P \leq 400$ and $P \geq 200$
    - ✓ $T \leq 200$ and $T \geq 100$
    - ✓ $A \geq 0, \mathrm{B} \geq 0, \mathrm{C} \geq 0$
    - ✓ $A + B + C = 1.0$
    - ✓ $A \leq 0.5$

Introduction to Modeling

- The modeling example
  - The complete Daisy optimization model
    - ✓ Letting z represents the value of the objective function (the yield), the entire optimization model may be written as:
      - Maximize $z = 300 + 0.8 \cdot V + 0.01 \cdot P + 0.06 \cdot T + 0.001 \cdot T \cdot P - 0.01 \cdot T^2 - 0.001 \cdot P^2 + 11.7 \cdot A + 9.4 \cdot B + 16.4 \cdot C + 19 \cdot A \cdot B + 11.4 \cdot A \cdot C - 9.6 \cdot B \cdot C$
      - Subject to
        - $V \leq 5, \;\; V \geq 1, \;\; P \leq 400, \;\; P \geq 200, \;\; T \leq 200, \;\; T \geq 100$
        - $A \geq 0, \;\; B \geq 0, \;\; C \geq 0, \;\; A + B + C = 1.0, \;\; A \leq 0.5$

Introduction to Modeling

- The modeling example
  - Any specification of the decision variables that satisfies all the model's constraints is said to be in the feasible region. For example, V=2, P=300, T=150, A=0.4, B=0.3, C=0.3 is in the feasible region.

  - An optimal solution to an optimization model is any point in the feasible region that optimizes (in this case, maximizes) the objective function.

Introduction to Modeling

- Static and Dynamic Models
  - A static model is one in which the decision variables do not involve sequences of decisions over multiple rounds.
  - A dynamic model is a model in which the decision variables do involve sequences of decisions over multiple periods.
  - In a static model, we solve a one-shot problem whose solutions are prescribe optimal values of the decision variables at all points in time. The Daisy problem is an example of a static model.

Introduction to Modeling

- Static and Dynamic Models
  - For a dynamic model, consider a company (SailCo) that must determine how to minimize the cost of meeting (on-time) the demand for sail boats it produces during the next year. SailCo must determine the number of sail boats to produce during each of the next four quarters. SailCo's decisions must be made over multiple periods and thus posses a dynamic model.

Introduction to Modeling

- Linear and non-linear models
  - Suppose that whenever decision variables appear in the objective function and in the constraints of an optimization model, the decision variables are always multiplied by constants and then added together. Such a model is a linear model.
  - The Daisy example is a nonlinear model. While the decision variables in the constraints are linear, the objective function is nonlinear since the objective terms are non-linear:
    - ✓ $0.001 \cdot T \cdot P, \quad -0.01 \cdot T^2, \quad -0.001 \cdot P^2, \quad 19 \cdot A \cdot B, \quad 11.4 \cdot A \cdot C, \quad -9.6 \cdot B \cdot C$
  - In general, nonlinear models are much harder to solve.

Introduction to Modeling

- Integer and non-integer models
  - If one or more the decision variables must be integer, then we say that an optimization model is an integer model. If all the decision variables are free to assume fractional values, then an optimization model is a non-integer model.
  - The Daisy example is a non-integer example since volume, pressure, temperature, and percentage compositions are all decision variables which may assume fractional values.
  - If decision variables in a model represent the number of workers starting during each shift, then we clearly have an integer model.
  - Integer models are much harder to solve then non-integer models.

Introduction to Modeling

- Deterministic and Stochastic Models
  - Suppose that, for any value of the decision variables, the value of the objective function and whether the constraints are satisfied or not is known with certainty. We then have a deterministic model. If this is not the case, then we have a stochastic model.

  - If we view the Daisy example as a deterministic model, then we are making the assumption that for given values of V, P, T, A, B, and C, the process yield will always be the same. Since this is unlikely, the objective function can be viewed as the average yield of the process given decision variables values.

The Seven-step Model-building Process

- Operations research used to solve an organization's problem follows a seven-step model building procedure:

  - 1. Formulate the problem
    - ✓ Define the problem
    - ✓ Specify objectives
    - ✓ Determine parts of the organization to be studied

  - 2. Observe the system
    - ✓ Determine parameters affecting the problem
    - ✓ Collect data to estimate values of the parameters

The Seven-step Model-building Process

- Operations research used to solve an organization’s problem follows a seven-step model building procedure:
  - 3. Formulate a mathematical model of the problem

  - 4. Verify the model and use the model for prediction
    - ✓ Does the model yield results for values of decision variables not used to develop the model?
    - ✓ What eventualities might cause the model to become invalid?

  - 5. Select a suitable alternative
    - ✓ Given a model and a set of alternative solutions, determine which solution best meets the organizations objectives.

The Seven-step Model-building Process

- Operations research used to solve an organization's problem follows a seven-step model building procedure:
  - 6. Present the results and conclusions of the study to the organization
    - ✓ Present the results to the decision makers
    - ✓ If necessary, prepare several alternative solutions and permit the organization to choose the one that best meets their need
    - ✓ Any non-approval of the study's recommendations may have stemmed from an incorrect problem definition or failure to involve the decision makers from the start of the project. In such a case, return to step 1, 2, or 3.

The Seven-step Model-building Process

- Operations research used to solve an organization's problem follows a seven-step model building procedure:
  - 7. Implement and evaluate recommendations
    - ✓ Upon acceptance of the study by the organization, the analyst:
      - Assists in implementing the recommendations
      - Monitors and dynamically updates the system as the environment and parameters change to ensure that recommendations enable the organization to meet its goals.

The CITGO Petroleum Problem

- Klingman et al. (1987) applied a variety management science techniques to CITGO Petroleum. Their work saved the company an estimated $70 per year. CITGO is an oil-refining and marketing company that was purchased by Southland Corporation (7-11 Stores).
- The following slides focus on two aspects of the CITGO's team's work:
  - A mathematical model to optimize the operation of CITGO's refineries.
  - A mathematical model – supply, distribution and marketing (SDM) – used to develop an 11-week supply, distribution and marketing plan for the entire business.

The CITGO Petroleum Problem

- Optimizing Refinery Operations
  - Step 1. (Formulate the Problem)
    Klingman et al. wanted to minimize the cost of CITGO’s refineries.

  - Step 2. (Observe the System)
    The Lake Charles, Louisiana, refinery was closely observed in an attempt to estimate key relationships such as:
    - ✓ How the cost of producing each of CTIGO’s products (motor fuel, no. 2 fuel oil, turbine fuel, naphtha, and several blended motor fuels) depends upon the inputs used to produce each product.
    - ✓ The amount of energy needed to produce each product (requiring the installation of a new metering system).

The CITGO Petroleum Problem

- Optimizing Refinery Operations
  - Step 2 (Continued)
    - ✓ The yield with each input-output combination. For example, if 1 gallon of crude oil would yield 0.52 gallons of motor fuel, then the yield would be 52%.
    - ✓ To reduce maintenance costs, data were collected on parts inventories and equipment breakdowns. Obtaining accurate data required the installation of a new data based-management system and integrated maintenance information system. Additionally, a process control system was also installed to accurately monitor the inputs and resources used to manufacture each product.

The CITGO Petroleum Problem

- Optimizing Refinery Operations
  - Step 3. (Formulate a Mathematical Model of the Problem)
    Using linear programming (LP), a model was developed to optimize refinery operations.

    - ✓ The model:
      - Determines the cost-minimizing method for mixing blending together inputs to obtain desired results.
      - Contains constraints that ensure that inputs are blended to produce desired results.
      - Includes constraints that ensure inputs are blended so that each output is of the desired quality.
      - Other model constraints ensure plant capacities are not exceeded and allow for inventory for each end product.

The CITGO Petroleum Problem

- Optimizing Refinery Operations
  - Step 4. (Verify the Model and Use the Model for Prediction)
    To validate the model, inputs and outputs from the refinery were collected for one month. Given actual inputs used at the refinery during that month, actual outputs were compared those predicted by the model. After extensive changes, the model's predicted outputs were close to actual outputs.

The CITGO Petroleum Problem

- Optimizing Refinery Operations
  - Step 5. (Select Suitable Alternative Solutions)
    Running the LP yielded a daily strategy for running the refinery. For instance, the model might say, produce 400,000 gallons of turbine fuel using 300,000 gallons of crude 1 and 200,000 of crude 2.

The CITGO Petroleum Problem

- Optimizing Refinery Operations
  - Step 6. (Present the Results and Conclusions) and Step 7. (Implement and Evaluate Recommendations)

    Once the data base and process control were in place, the model was used to guide day-to-day operations. CITGO estimated the overall benefits of the refinery system exceeded $50 million annually.

The CITGO Petroleum Problem

- The Supply Distribution Marketing (SDM) System
  - Step 1. (Formulate the Problem)
    CITGO wanted a mathematical model that could be used to make supply, distribution, and marketing decisions such as
    - ✓ Where should the crude oil be purchased?
    - ✓ Where should products be sold?
    - ✓ What price should be charged for products?
    - ✓ How much of each product should be held in inventory?

The goal was to maximize profitability associated with these decisions.

The CITGO Petroleum Problem

- The Supply Distribution Marketing (SDM) System
  - Step 2. (Observe the System)
    A database that kept tack of sales, inventory, trades, and exchanges of all refined goods was installed. Also, regression analysis was used to develop forecasts of for wholesale prices and wholesale demand for each CITGO product.

  - Step 3. (Formulate a Mathematical Model of the Problem) and Step 5. (Select Suitable Alternative Solutions)
    A minimum-cost network flow model (MCNFM) is used to determine an 11-week supply, marketing and distribution strategy. The model makes all decisions discussed in Step 1. A typical model run involved 3,000 equations and 15,000 decision variables required only 30 seconds on an IBM 4381.

The CITGO Petroleum Problem

- The Supply Distribution Marketing (SDM) System
  - Step 4. (Verify the Model and Use the Model for Prediction)
    The forecasting models are continuously evaluated to ensure that they continue to give accurate forecasts.

  - Step 6. (Present the Results and Conclusions) and Step 6. (Implement and Evaluate Recommendations)
    Implementing the SDM required several organizational changes. A new vice-president was appointed to coordinate the operation of the SDM and refinery LP model. The product supply and product scheduling departments were combined to improve communications and information flow.

General Comment on Model Building

- Model building is a prime example of so called
  - Interdisciplinary work that spans various engineering, scientific (primarily mathematics), and business fields.
  - It is a very productive and an exciting area to work in.

Thank you