SEOULTECH
Seoul National University of Science & Technology

Assignment 1

Date :

Code	ITM 524		Title	Management Science	
-	-	Questions		Weighting	5%
Student's Number	21102052		Student's Name	Lee Jeonghyun - 이정훈	

1. (20pts) Use the graphical method to find the optimal solution.

Maximize $Z = 2x_1 + x_2$ $x_2 = Z - 2x_1$

(subject to)

$x_2 \le 10$

$2x_1 + 5x_2 \le 60$ $x_2 \le (60-2x_1)\frac{1}{5}$

$x_1 + x_2 \le 18$ $x_2 \le 18 - x_1$

$3x_1 + x_2 \le 44$ $x_2 \le 44 - 3x_1$

$x_1, x_2 \ge 0$

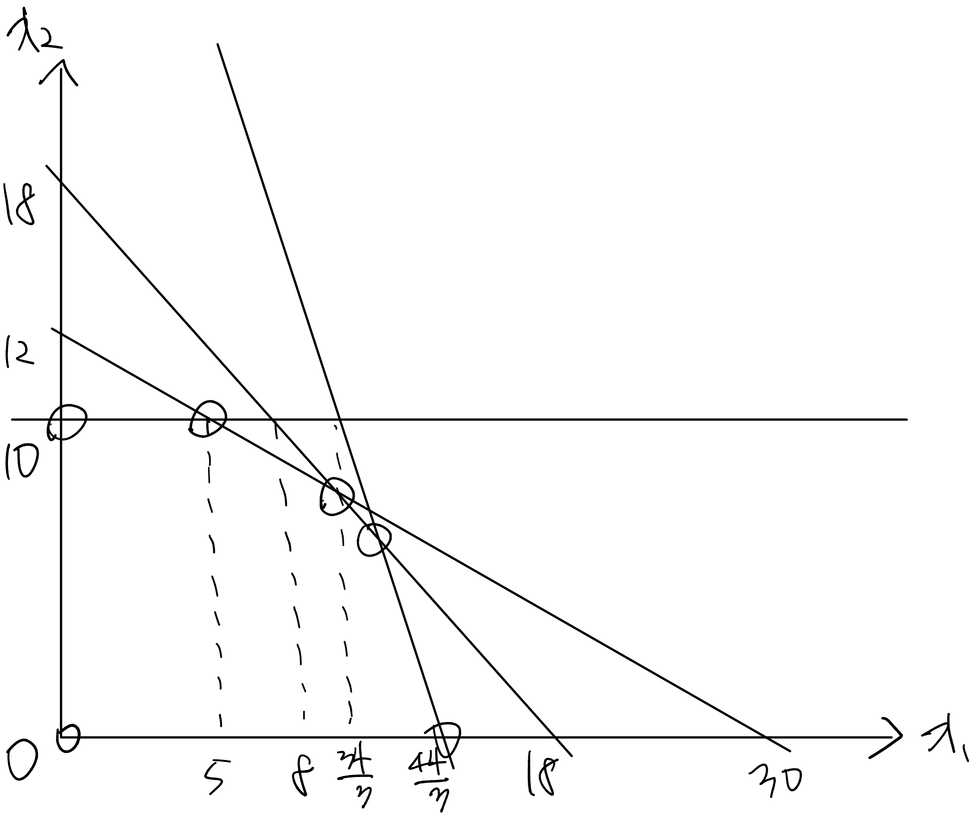
$3x_1 = 30$

$44 - 3x_1 = 18 - x_1 \Rightarrow (13, 5)$

$5 = Z - 2 \times 13, \quad Z = 31$

$60 - 2x_1 = 90 - 5x_1 \Rightarrow (10, 8)$

$Z = 20 + 8 = 28$



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2. (20pts) A financial services firm plans to launch two offerings: high-risk coverage and mortgage loans. The expected profit per unit is \$5 for high-risk coverage and \$2 for mortgage loans.
Management would like to set sales targets for the two offerings to maximize total expected profit. The required work-hours and available hours by department are:

Department	Work-Hours per Unit		Hours Available
	High Risk	Mortgage	
Underwriting	3	2	2400
Administration	0	1	800
Claims	2	0	1200

(a) Formulate a linear programming model for this problem.
(b) Solve this model by the graphical method.

a) $Z = 5x_1 + 2x_2$, x_1 for coverage, x_2 for loan. Z for profit.

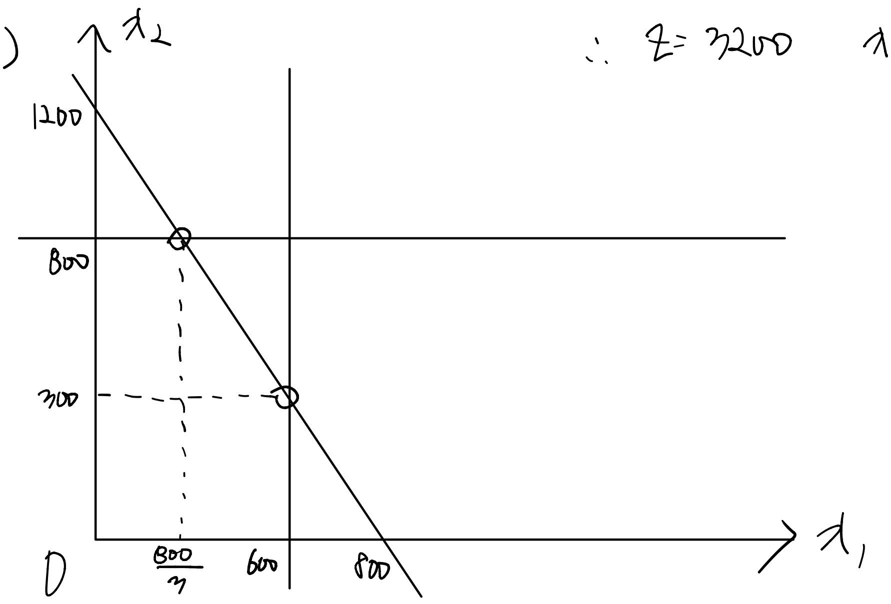
$3x_1 + 2x_2 \leq 2400 \Rightarrow x_2 \leq 1200 - \frac{3}{2}x_1$

$x_2 \leq 800$

$2x_1 \leq 1200$

$5 \times \frac{800}{3} + 2 \times 800 \fallingdotseq 2933 < 5 \times 600 + 2 \times 300 = 3200$

b)



$\therefore Z = 3200 \quad x_1 = 600 \quad x_2 = 300$

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3. (20pts) Apply the simplex algorithm, showing each pivot step, to obtain the optimal solution.

$$\text{Maximize } Z = 3x_1 + 5x_2 + 6x_3$$

(subject to)

$$2x_1 + x_2 + x_3 \leq 4$$
$$x_1 + 2x_2 + x_3 \leq 4$$
$$x_1 + x_2 + 2x_3 \leq 4$$
$$x_1 + x_2 + x_3 \leq 3$$
$$x_1, x_2, x_3 \geq 0$$

$Z - 3x_1 - 5x_2 - 6x_3 = 0$
$2x_1 + x_2 + x_3 + s_1 = 4$
$x_1 + 2x_2 + x_3 + s_2 = 4$
$x_1 + x_2 + 2x_3 + s_3 = 4$
$x_1 + x_2 + x_3 + s_4 = 4$

	Z	x_1	x_2	$\boxed{x_3}$	s_1	s_2	s_3	s_4	RHS	MRT
Z	1	-3	-5	-6					0	
s_1		2	1	1	1				4	4/1
s_2		1	2	1		1			4	4/1
$\boxed{s_3}$		1	1	2			1		4	4/2
s_4		1	1	1				1	3	3/1

pivot

	Z	x_1	$\boxed{x_2}$	x_3	s_1	s_2	s_3	s_4	RHS	
Z	1	0	-2	0			3		12	
s_1		3/2	1/2	0	1		-1/2		2	4
$\boxed{s_2}$		1/2	3/2	0		1	-1/2		2	4/3
x_3		1/2	1/2	1			1/2		2	4
s_4		1/2	1/2	0			-1/2	1	1	2

pivot

	Z	x_1	x_2	x_3	s_1	s_2	s_3	s_4	RHS
Z	1	2/3	0	0		$\frac{4}{3}$	$\frac{2}{3}$		$12+\frac{8}{3}$
s_1		$\frac{3}{2}-\frac{1}{6}$	0		1	$-\frac{1}{3}$	$-\frac{1}{2}+\frac{1}{6}$		$2-2/3$
x_2		1/3	1	0		$\frac{2}{3}$	$-\frac{1}{3}$		$\frac{4}{3}$
x_3		$1/2-\frac{1}{6}$	0	1		$-\frac{1}{3}$	$\frac{1}{2}+\frac{1}{6}$		$2-\frac{2}{3}$
s_4		$1/2-\frac{1}{6}$	0	0		$-\frac{1}{3}$	$-1/2+\frac{1}{6}$	1	$1-\frac{2}{3}$

coef of Z-row $x_1, x_2, x_3 \geq 0$.

$\therefore Z = \frac{44}{3}$ done.

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4. (20pts) Consider the following problem.

Maximize $Z = x_1 - 7x_2 + 3x_3$

(subject to)

$2x_1 + x_2 - x_3 \leq 4$

$4x_1 - 3x_2 \leq 2$

$-3x_1 + 2x_2 + x_3 \leq 3$

$x_1, x_2, x_3 \geq 0$

Max Z

$Z - x_1 + 7x_2 - 3x_3 = 0$

$2x_1 + x_2 - x_3 + s_1 = 4$

$4x_1 - 3x_2 + s_2 = 2$

$-3x_1 + 2x_2 + x_3 + s_3 = 3$

Apply the simplex algorithm and show each pivot step to obtain the optimal solution.

	x_1	x_2	x_3	s_1	s_2	s_3	RHS	
Z	-1	7	-3				0	
s_1	2	1	-1	1			4	
s_2	4	-3			1		2	
s_3	-3	2	1			1	3	3/1

	x_1	x_2	x_3	s_1	s_2	s_3	RHS	
Z	-10	13	0			3	9	
s_1	-1	3	0	1		1	7	
s_2	4	-3			1		2	2/4
x_3	-3	2	1			1	3	

	x_1	x_2	x_3	s_1	s_2	s_3	RHS
Z	0	1/2			5/2	3	14
s_1	0	9/4		1	1/4	1	15/2
x_1	1	-3/4			1/4		1/2
x_3	0	-1/4	1	0	3/4	1	9/2

$Z = 1/2 - 7 \times 0 + 3 \times 9/2 = 14.$

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5. (20pts) Consider the following problem.

$$\text{Maximize } Z = -5x_1 + 5x_2 + 13x_3$$

(subject to)

$$-x_1 + x_2 + 3x_3 \leq 20$$
$$12x_1 + 4x_2 + 10x_3 \leq 90$$
$$x_1, x_2, x_3 \geq 0$$

Let s_1 and s_2 be the slack variables for the respective constraints. After applying the simplex method, the final system can be written as:

$$Z + 2x_3 + 5s_1 = 100$$
1) $$-x_1 + x_2 + 3x_3 + s_1 = 20$$
2) $$16x_1 - 2x_3 - 4s_1 + s_2 = 10$$

BV	Z	X1	X2	X3	S1	S2	RHS
Z	1	0	0	2	5	0	100
X2	0	-1	1	3	1	0	20
S2	0	16	0	-2	-4	1	10

Now, perform sensitivity analysis by considering each of the following nine modifications independently. For each change, update the above equations (i.e., the tableau form) and rewrite them as needed to evaluate the current basic solution. Then, test this solution for feasibility and for optimality (You don't need to carry out a full re-optimization when it is no longer optimal. Just indicate that re-optimization would be required.).

(a) Change the RHS of constraint 1 to $b_1 = 30$.
(b) Change the RHS of constraint 2 to $b_2 = 70$.
(c) Change the RHSs to $b_1 = 10, b_2 = 100$.
(d) Change the coefficient of x_3 in the objective function to $c_3 = 8$.
(e) Change the coefficients of x_1 to $c_1 = -2, a_{11} = 0, a_{21} = 5$.
(f) Change the coefficients of x_2 to $c_2 = 6, a_{12} = 2, a_{22} = 5$.
(g) Introduce a new variable x_4 with coefficients $c_4 = 10, a_{14} = 3, a_{24} = 5$.
(h) Introduce a new constraint $2x_1 + 3x_2 + 5x_3 \leq 50$. (Denote its slack variable by s_3)
(i) Change constraints 2 to $10x_1 + 5x_2 + 10x_3 \leq 100$.

	Z	x_1	x_2	x_3	s_1	s_2	RHS
Z	1	5	-5	-13	0	0	0
s_1	0	-1	1	3	1	0	20
s_2	0	12	4	10	0	1	90

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BV	Z	x_1	x_2	x_3	s_1	s_2	RHS
Z	1	0	0	2	5	0	100
x_2	0	-1	1	3	1	0	20
s_2	0	16	0	-2	-4	1	10

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BV	Z	X	X_S	RHS	
Z	1	-c	0	0	$+C_B B^{-1}(\ )$
X_S	0	A	I	b	$\times B^{-1}$

$\Rightarrow$

BV	Z	X	X_S	RHS
Z	1	$C_B B^{-1}A - C$	$C_B B^{-1}$	$C_B B^{-1} b$
X_B	0	$B^{-1}A$	B^{-1}	$B^{-1}b$

(a) $b = \begin{bmatrix} 20 \\ 90 \end{bmatrix} \Rightarrow \bar{b} = \begin{bmatrix} 30 \\ 90 \end{bmatrix}$

$B^{-1}\bar{b} = \begin{bmatrix} 1 & 0 \\ -4 & 1 \end{bmatrix}\begin{bmatrix} 30 \\ 90 \end{bmatrix} = \begin{bmatrix} 30 \\ -30 \end{bmatrix}$

$C_B B^{-1}\bar{b} = \begin{bmatrix} 5 & 0 \end{bmatrix}\begin{bmatrix} 30 \\ 90 \end{bmatrix} = 150$

BV	Z	x_1	x_2	x_3	s_1	s_2	RHS	
Z	1	0	0	2	5	0	150	
x_2	0	-1	1	3	1	0	30	
s_2	0	16	0	-2	-4	1	-30	< 0 ∴ infeasible

(b) $b = \begin{bmatrix} 20 \\ 90 \end{bmatrix} \Rightarrow \bar{b} = \begin{bmatrix} 20 \\ 70 \end{bmatrix}$

$B^{-1}\bar{b} = \begin{bmatrix} 1 & 0 \\ -4 & 1 \end{bmatrix}\begin{bmatrix} 20 \\ 70 \end{bmatrix} = \begin{bmatrix} 20 \\ -10 \end{bmatrix}$

$C_B B^{-1}\bar{b} = \begin{bmatrix} 5 & 0 \end{bmatrix}\begin{bmatrix} 20 \\ 70 \end{bmatrix} = 100$

BV	Z	x_1	x_2	x_3	s_1	s_2	RHS	
Z	1	0	0	2	5	0	100	
x_2	0	-1	1	3	1	0	20	
s_2	0	16	0	-2	-4	1	-10	< 0 ∴ infeasible

(c) $b = \begin{bmatrix} 20 \\ 90 \end{bmatrix} \Rightarrow \bar{b} = \begin{bmatrix} 10 \\ 100 \end{bmatrix}$

$B^{-1}\bar{b} = \begin{bmatrix} 1 & 0 \\ -4 & 1 \end{bmatrix}\begin{bmatrix} 10 \\ 100 \end{bmatrix} = \begin{bmatrix} 10 \\ 60 \end{bmatrix}$

$C_B B^{-1}\bar{b} = \begin{bmatrix} 5 & 0 \end{bmatrix}\begin{bmatrix} 10 \\ 100 \end{bmatrix} = 50$

BV	Z	x_1	x_2	x_3	s_1	s_2	RHS	
Z	1	0	0	2	5	0	50	
x_2	0	-1	1	3	1	0	10	≥ 0
s_2	0	16	0	-2	-4	1	60	≥ 0

Z-row ≥ 0 ∴ optimal.

* allowance range of b_i

if b_1 : $b = \begin{bmatrix} 20 \\ 90 \end{bmatrix} \Rightarrow \bar{b} = \begin{bmatrix} 20+\theta_1 \\ 90+0 \end{bmatrix}$, $B^{-1}\bar{b} = \begin{bmatrix} 1 & 0 \\ -4 & 1 \end{bmatrix}\left(\begin{bmatrix} 20 \\ 90 \end{bmatrix} + \begin{bmatrix} \theta_1 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 20 \\ 10 \end{bmatrix} + \begin{bmatrix} \theta_1 \\ -4\theta_1 \end{bmatrix} = \begin{bmatrix} 20+\theta_1 \\ 10-4\theta_1 \end{bmatrix} \geq 0$. $\theta_1 \geq -20$, $\theta_1 \leq 5/2$ $\quad -20 \leq \theta_1 \leq \frac{5}{2}$

if b_2 : $b = \begin{bmatrix} 20 \\ 90 \end{bmatrix} \Rightarrow \bar{b} = \begin{bmatrix} 20+0 \\ 90+\theta_2 \end{bmatrix}$, $B^{-1}\bar{b} = \begin{bmatrix} 1 & 0 \\ -4 & 1 \end{bmatrix}\left(\begin{bmatrix} 20 \\ 90 \end{bmatrix} + \begin{bmatrix} 0 \\ \theta_2 \end{bmatrix}\right) = \begin{bmatrix} 20 \\ 10 \end{bmatrix} + \begin{bmatrix} 0 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} 20 \\ 10+\theta_2 \end{bmatrix} \geq 0$. $10+\theta_2 \geq 0$. $\theta_2 \geq -10$

for feasibility check.

	z	x_1	x_2	x_3	s_1	s_2	RHS
z	1	5	-5	-13	0	0	0
s_1	0	-1	1	3	1	0	20
s_2	0	12	4	10	0	1	90

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BV	z	x_1	x_2	x_3	s_1	s_2	RHS
z	1	0	0	2	5	0	100
x_2	0	-1	1	3	1	0	20
s_2	0	16	0	-2	-4	1	10

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BV	Z	X	X_S	RHS	
Z	(1	$-C$	0	0)	$+C_B B^{-1}(\ )$
X_S	(0	A	I	b)	$\times B^{-1}$

$\Rightarrow$

BV	Z	X	X_S	RHS
Z	1	$C_B B^{-1} A - C$	$C_B B^{-1}$	$C_B B^{-1} b$
X_B	0	$B^{-1} A$	B^{-1}	$B^{-1} b$

(d) $C_3 = 13 \Rightarrow \bar{C}_3 = 8$. x_3 is NBV.

$C_B B^{-1} A_3 - \bar{C}_3 = [5\ 0]\begin{bmatrix} 3 \\ 10 \end{bmatrix} - 8$
$= 15 - 8 = 7$

BV	z	x_1	x_2	x_3	s_1	s_2	RHS	
z	1	0	0	7	5	0	100	
x_2	0	-1	1	3	1	0	20	≥ 0
s_2	0	16	0	-2	-4	1	10	≥ 0

z-row ≥ 0
∴ optimal

(e) $C_1 = -5 \Rightarrow \bar{C}_1 = -2$ x_1 is NBV

$A_1 = \begin{bmatrix} -1 \\ 12 \end{bmatrix} \Rightarrow \bar{A}_1 = \begin{bmatrix} 0 \\ 5 \end{bmatrix}$

$B^{-1}\bar{A}_1 = \begin{bmatrix} 1 & 0 \\ -4 & 1 \end{bmatrix}\begin{bmatrix} 0 \\ 5 \end{bmatrix} = \begin{bmatrix} 0 \\ 5 \end{bmatrix}$

$C_B B^{-1} \bar{A}_1 - \bar{C}_1 = [5\ 0]\begin{bmatrix} 0 \\ 5 \end{bmatrix} - (-2) = 2$

BV	z	x_1	x_2	x_3	s_1	s_2	RHS	
z	1	2	0	2	5	0	100	
x_2	0	0	1	3	1	0	20	≥ 0
s_2	0	5	0	-2	-4	1	10	≥ 0

z-row ≥ 0
∴ optimal

* allowable range of NBV C_j

$\left.\begin{array}{l} \bar{C}_j = C_j + \theta \\ C_B B^{-1} A_j - \bar{C}_j \geq 0. \end{array}\right\}$ for optimality check

(f) $C_2 = 5 \Rightarrow \bar{C}_2 = 6$. x_2 is BV.

$A_2 = \begin{bmatrix} 1 \\ 4 \end{bmatrix} \Rightarrow \bar{A}_2 = \begin{bmatrix} 2 \\ 5 \end{bmatrix}$

$B^{-1}\bar{A}_2 = \begin{bmatrix} 1 & 0 \\ -4 & 1 \end{bmatrix}\begin{bmatrix} 2 \\ 5 \end{bmatrix} = \begin{bmatrix} 2 \\ -3 \end{bmatrix}$

$C_B B^{-1} \bar{A}_2 = [5\ 0]\begin{bmatrix} 2 \\ 5 \end{bmatrix} = 10$

$C_B B^{-1} \bar{A}_2 - \bar{C}_2 = 10 - 6 = 4$

BV	z	x_1	x_2	x_3	s_1	s_2	RHS
z	1	0	4	2	5	0	100
x_2	0	-1	2	3	1	0	20
s_2	0	16	-3	-2	-4	1	10

BV	z	x_1	x_2	x_3	s_1	s_2	RHS	
z	1	2	0	-4	3	0	60	
x_2	0	$-1/2$	1	$3/2$	$1/2$	0	10	≥ 0
s_2	0	$29/2$	0	$5/2$	$-5/2$	1	40	≥ 0.

z-row < 0
∴ re-optimize!

* allowable range of BV C_j

$\bar{C}_j = C_j + \theta$

$C_B B^{-1} A_j - \bar{C}_j = -\theta$.

(z-row) $+ \theta \times$ (BV-row) ≥ 0.

	Z	x_1	x_2	x_3	s_1	s_2	RHS
Z	1	5	-5	-13	0	0	0
s_1	0	-1	1	3	1	0	20
s_2	0	12	4	10	0	1	90

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BV	Z	x_1	x_2	x_3	s_1	s_2	RHS
Z	1	0	0	2	5	0	100
x_2	0	-1	1	3	1	0	20
s_2	0	16	0	-2	-4	1	10

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BV	Z	X	X_S	RHS	
Z	(1	$-C$	0	0)	$+C_BB^{-1}(\ )$
X_S	(0	A	I	b)	$\times B^{-1}$

$\Rightarrow$

BV	Z	X	X_S	RHS
Z	1	$C_BB^{-1}A-C$	C_BB^{-1}	$C_BB^{-1}b$
X_B	0	$B^{-1}A$	B^{-1}	$B^{-1}b$

(g) new var x_4.

$C_4=10.\quad A_4=\begin{bmatrix}3\\5\end{bmatrix}$

$B^{-1}A_4=\begin{bmatrix}1&0\\-4&1\end{bmatrix}\begin{bmatrix}3\\5\end{bmatrix}=\begin{bmatrix}3\\-7\end{bmatrix}$

$C_BB^{-1}A_4-C_4=\begin{bmatrix}5&0\end{bmatrix}\begin{bmatrix}3\\5\end{bmatrix}-10$

$=15-10=5$

BV	Z	x_1	x_2	x_3	x_4	s_1	s_2	RHS	
Z	1	0	0	2	5	5	0	100	
x_2	0	-1	1	3	3	1	0	20	≥ 0
s_2	0	16	0	-2	-7	-4	1	10	≥ 0

Z-row ≥ 0

$\therefore$ optimal

(h) new constraint : $2x_1+3x_2+5x_3+s_3=50$.

BV	Z	x_1	x_2	x_3	s_1	s_2	s_3	RHS
Z	1	0	0	2	5	0	0	100
x_2	0	-1	1	3	1	0	0	20
s_2	0	16	0	-2	-4	1	0	10
s_4	0	2	3	5	0	0	1	50

$\Rightarrow$

BV	Z	x_1	x_2	x_3	s_1	s_2	s_3	RHS
Z	1	0	0	2	5	0	0	100
x_2	0	-1	1	3	1	0	0	20
s_2	0	16	0	-2	-4	1	0	10
s_4	0	5	0	-4	-3	0	1	$-10<0$

$\therefore$ infeasible

(i) $\bar{A}=\begin{bmatrix}-1&1&3\\10&5&10\end{bmatrix}\quad \bar{b}=\begin{bmatrix}20\\100\end{bmatrix}$

$B^{-1}\bar{b}=\begin{bmatrix}1&0\\-4&1\end{bmatrix}\begin{bmatrix}20\\100\end{bmatrix}=\begin{bmatrix}20\\20\end{bmatrix}$

$C_BB^{-1}\bar{b}=\begin{bmatrix}5&0\end{bmatrix}\begin{bmatrix}20\\100\end{bmatrix}=100$

$B^{-1}\bar{A}=\begin{bmatrix}1&0\\-4&1\end{bmatrix}\begin{bmatrix}-1&1&3\\10&5&10\end{bmatrix}=\begin{bmatrix}-1&1&3\\14&1&-2\end{bmatrix}$

$C_BB^{-1}\bar{A}-C=\begin{bmatrix}5&0\end{bmatrix}\begin{bmatrix}-1&1&3\\10&5&10\end{bmatrix}-\begin{bmatrix}-5&5&13\end{bmatrix}$

$=\begin{bmatrix}-5&5&15\end{bmatrix}-\begin{bmatrix}-5&5&13\end{bmatrix}$

$=\begin{bmatrix}0&0&2\end{bmatrix}$

BV	Z	x_1	x_2	x_3	s_1	s_2	RHS
Z	1	0	0	2	5	0	100
x_2	0	-1	1	3	1	0	20
s_2	0	14	1	-2	-4	1	20

BV	Z	x_1	x_2	x_3	s_1	s_2	RHS	
Z	1	0	0	2	5	0	100	
x_2	0	-1	1	3	1	0	20	≥ 0
s_2	0	15	0	-5	-5	0	0	≥ 0

Z-row ≥ 0. $\therefore$ optimal