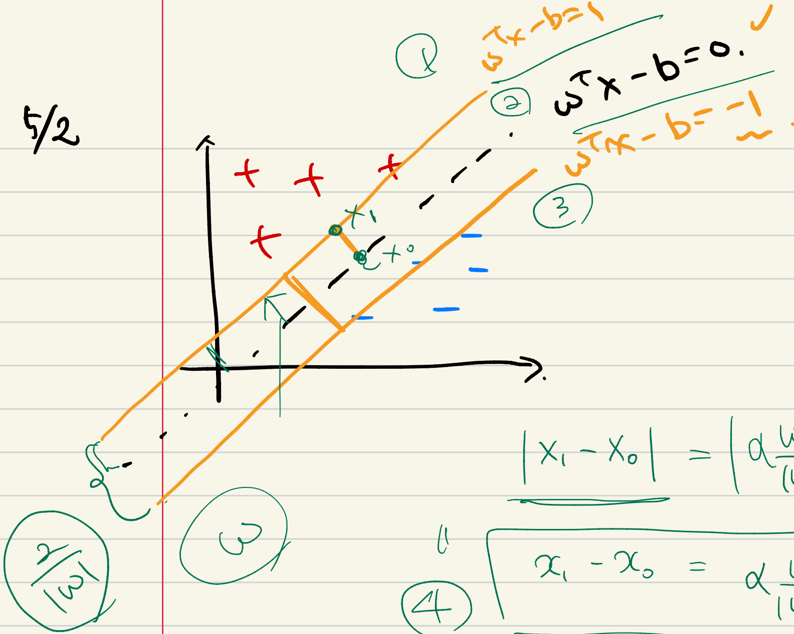
$5/2$

$①$.



$\alpha > 0$

$$|x_1 - x_0| = \left|\alpha \frac{w}{|w|}\right| = \frac{|\alpha||w|}{|w|} = |\alpha|$$

$$= \alpha.$$

④ $$\boxed{x_1 - x_0 = \alpha \frac{w}{|w|}}$$

$$w^Tw = |w|^2$$

① $w^Tx_1 - b = 1$

② $w^Tx_0 - b = 0$

③ $w^T(x_1 - x_0) = 1$ ① − ②

by ④

$$w^T \alpha \frac{w}{|w|} = 1$$

$$\alpha \frac{w^Tw}{|w|} = 1$$

$$\alpha \frac{|w|^2}{|w|} = 1$$

$$\alpha = \frac{1}{|w|}$$

□

5/2 최적화 (Numerical Optimization). // Convex optimization. 볼록최적화.

$$\min_{x \in \mathbb{R}^d} f(x)$$

decision variable

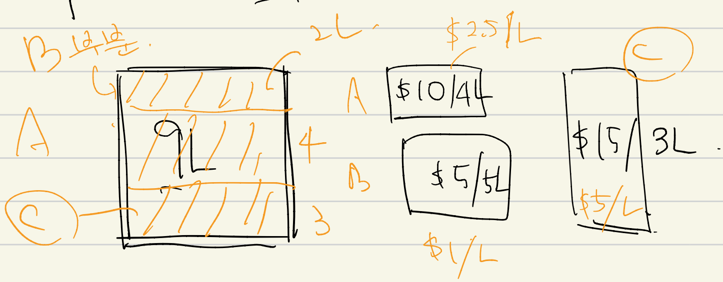
· Continuous optimization

$$\min_{x \in \mathbb{Z}^d} f(x)$$

· Discrete optimization.

Continuous relaxation

Knapsack np-hard



$$\min_{x \in \{0,1\}^3} p^T x$$

$$\boxed{\text{s.t. } v^T x \le 9}$$

constraint

· Mixed Integer Programming.

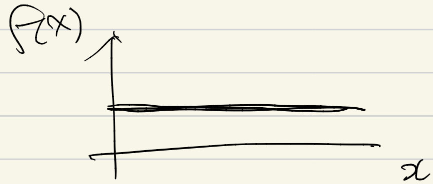
5/2

$\min_{x \in \mathbb{R}^d} f(x)$ → unconstrained opt.

$\min_{x} f(x)$ → constrained opt.

s.t. $h(x) \le 0$.

(국부) 최적해 전역 최적해.



∘ Local Optimum.

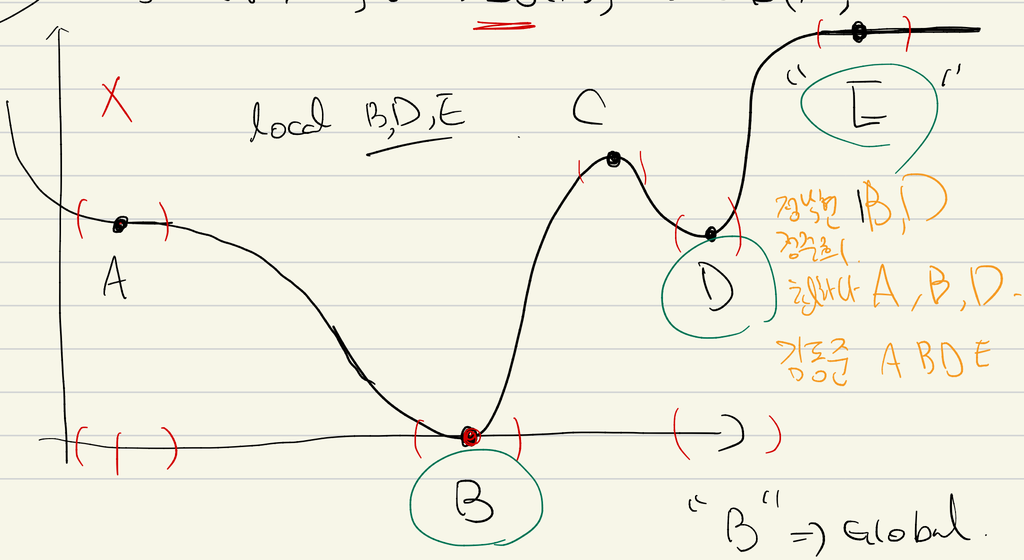
∘ Global Optimum $f(x^*) \le f(x)$, $\forall x \in X$

x^* is said to be a local minimum

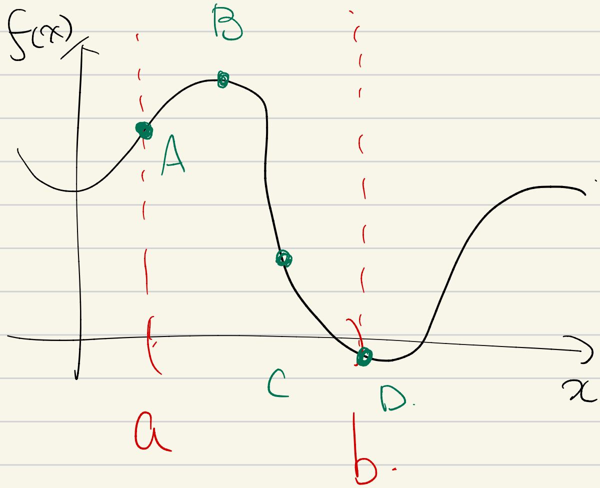
$\nabla f(x) = 0$

FONC

if $\exists r > 0$, $f(x^*) \le f(x)$, $\forall x \in B(x^*, r)$



5/3

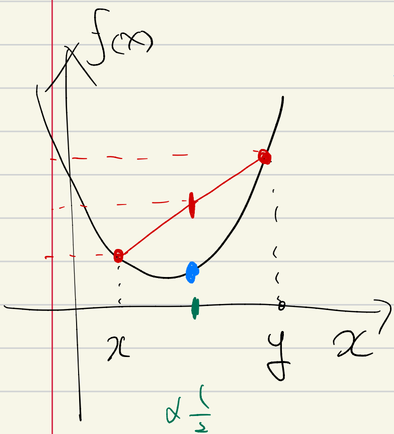


$$\min_x f(x)$$

$$\text{s.t. } a \le x \le b.$$

5/2 Convex optimization

if x^* is a local minimum
then x^* is a global minimum. $\square$



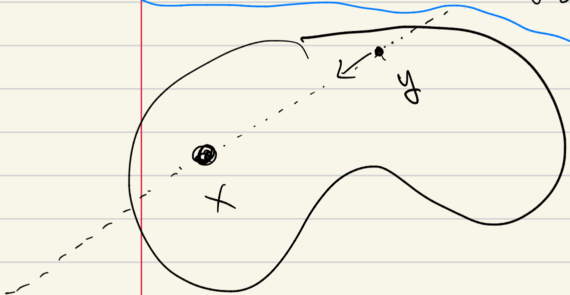
$\star$ $\boxed{f(\alpha x+(1-\alpha)y)} \le \alpha f(x) + (1-\alpha) f(y)$

$\forall \alpha$ $\quad 0<\alpha<1$

$\forall x, y \in X$

$\star$ Convex set.

S is cvx if $\forall x, y \in S$, $\alpha x+(1-\alpha)y \in S$.
$\forall \alpha \quad 0<\alpha<1$



$\alpha(x-y) + y$

$\alpha x - \alpha y + y$,

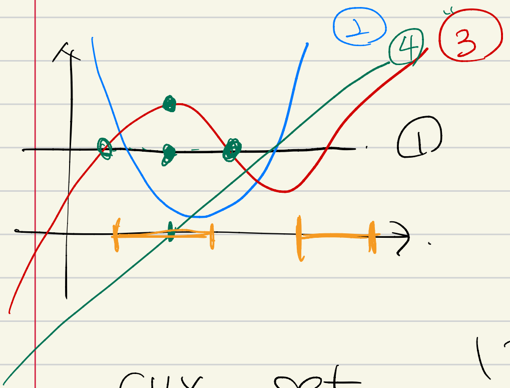
$\boxed{\alpha x + (1-\alpha) y}$

cvx, combination

$\boxed{\sum_i \alpha_i x_i, \ \sum \alpha_i = 1}$ $\quad \alpha$ affine, combination
unconstrained.

cvx comb. $\boxed{1 \ge \alpha_i \ge 0.}$

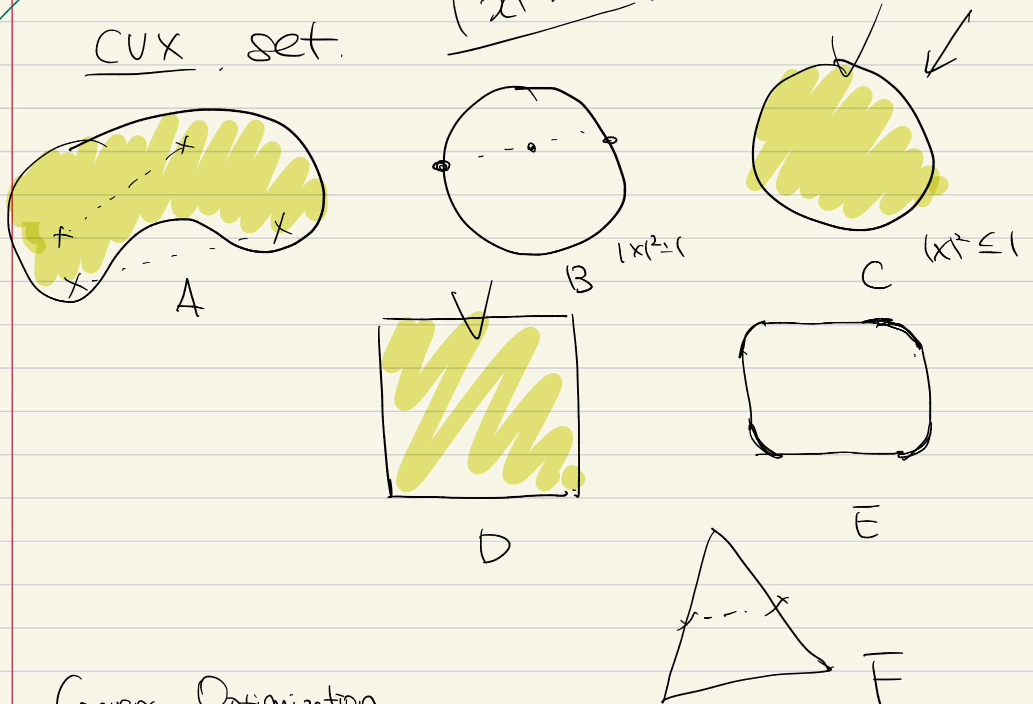
5/2 $\quad f(\alpha x + (1-\alpha)y) \leq \alpha f(x) + (1-\alpha) f(y)$ $\quad \geq$ $\quad$ 4



1, 2, 4

$|x|^2 \leq 1$ ← convex 4

CVX set.



Convex Optimization

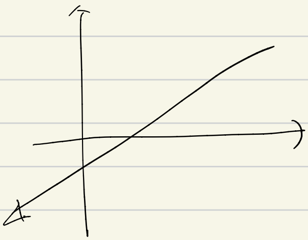
$$\min_{x \in \Theta} f(x)$$

f is cvx

Θ is cvx

feasible set.

5/2 Q1. CVX f → 항상 global optimum ?
solution



"unbounded"

No

Q2. CVX, bounded → 항상 global optimum ?

No

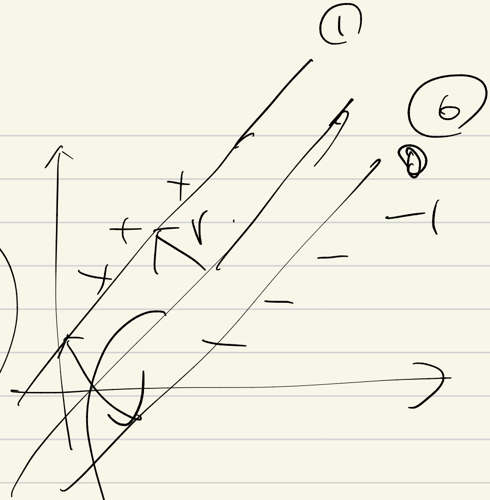
5/2

①

$$\min_{w} \|w\|^2$$

s.t. $w^T x_i - b \ge 1$ if $y_i = +1$,

$w^T x_i - b \le -1$ if $y_i = -1$,



②

$$\min_{w} \|w\|^2$$

s.t. $y_i(w^T x_i - b) \ge 1$

$y_i \in \{-1, 1\}$.

$w^T x - b = 0$

$\frac{2}{\|w\|}$

② $\|w\|$

//

③ $\|w\|^2$.

③

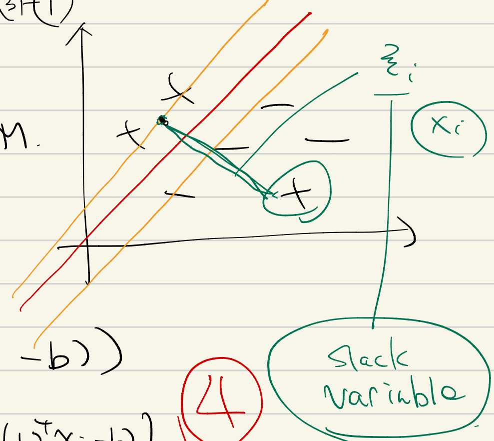
$$\min_{w} \|w\|^2 + \sum_i \xi_i$$

s.t. $y_i(w^T x_i - b) \ge 1 - \xi_i$ — ③+①

$\xi_i \ge 0$

Soft-margin SVM.

linearly non separable



③+① $\xi_i \ge 1 - y_i(w^T x_i - b)$

$$\xi_i = \max(0, 1 - y_i(w^T x_i - b))$$

④

$$\min_{w} \lambda \|w\|^2 + \sum_i \max(0, 1 - y_i(w^T x_i - b))$$