Definition 1

Semigroup $(S, \circ)$ - if $\forall a, b, c \in S \hookrightarrow a \circ (b \circ c) = (a \circ b) \circ c$	associativity
Monoid $(M, \circ)$ - semigroup and $\exists e : \forall m \in M \hookrightarrow m \circ e = e \circ m = m$	identity
Group $(G, \circ)$ - monoid and $\forall g \in G\ \exists g^{-1} : g \circ g^{-1} = g^{-1} \circ g = e$	inverse

Definition 2

Vector space $(V, F, +, \cdot)$ - $\forall u, v, w \in V,\ \forall a, b \in F$:

1.	$+ : V \times V \to V$	addition
2.	$\cdot : F \times V \to V$	multiplication
3.	$(u + v) + w = u + (v + w)$	associativity
4.	$\exists 0 : v + 0 = 0 + v = v$	identity
5.	$\exists -v : v + (-v) = (-v) + v = 0$	inverse
6.	$u + v = v + u$	commutativity
7.	$a \cdot (u + v) = a \cdot u + a \cdot v$	left-distributivity
8.	$(a + b) \cdot v = a \cdot v + b \cdot v$	right-distributivity
9.	$a \cdot (b \cdot v) = (a \cdot b) \cdot v$	associativity
10.	$1 \cdot v = v$	identity