CHAPTER TWO

ALGEBRAIC FRACTIONS

Multiplication of algebraic fractions:

- One must go through these steps when multiplying algebraic fraction
- Factors which are common to the numerator and the denominator must be cancelled.
- Multiply the numerators and after that do a similar thing to the denominators.

Q1) Simplify the following:

a) $\frac{12ab}{6a} \times \frac{10xy}{5x}$

<p align="center">Soln.</p>

$.\frac{12ab}{6a} \times \frac{10xy}{5x} = \frac{2ab}{a^1} \times \frac{2xy}{x^1} = 2 \times a^1 \times a^{-1}$

$\times b \times 2x^1 \times x^{-1} \times y = 2b \times 2y = 4by.$

Q2. $\frac{6a^4b}{2a} \times \frac{8x^6y}{4x^2}$

<p align="center">Soln.</p>

$$\frac{6a^4b}{2a} \times \frac{8x^6y}{4x^2} = \frac{3a^4b}{a^1} \times \frac{2x^6y}{x^2} = 3 \times a^4 \times a^{-1} \times b \times 2 \times x^6 \times x^{-2}y = 3a^3b \times 2x^4y$$
$$= 6a^3bx^4y.$$

Q3. $\frac{4x^2y^6}{2x^5y^2} \times \frac{9a^2}{3a^6}$

<p align="center">Soln.</p>

$$\frac{4x^2y^6}{2x^5y^2} \times \frac{9a^2}{3a^6} = \frac{2x^2y^6}{x^5y^2} \times \frac{3a^2}{a^6} = 2 \times x^2 \times x^{-5} \times y^6 \times y^{-2} \times 3a^2 \times a^{-6}$$
$$= 2x^{-3}y^4 \times 3a^{-4} = 6x^{-3}y^4a^{-4}$$

Q4. $\frac{3x^{-2}y^{-4}}{2x^4y-2} \times \frac{14v^2t^{-3}}{7v^{-5}}$

<center>Soln.</center>

$$\frac{3x^{-2}y^{-4}}{2x^4y-2} \times \frac{14v^2t^{-3}}{7v^{-5}} = \frac{3}{2} \times x^{-2} \times x^{-4} \times y^{-4} \times y^2 \times 2 \times v^2 \times v^5 \times t^{-3}$$

$$= \frac{3}{2}x^{-6}y^{-2} \times 2v^7t^{-3} = \frac{3}{2} \times 2 \times x^{-6}y^{-2}v^7t^{-3} = 3x^{-6}y^{-2}v^7t^{-3}$$

Q5. $\frac{15xy^{-6}}{25yx^2} \times \frac{2a^4b}{3b^3}$

<center>Soln.</center>

$$\frac{15xy^{-6}}{25yx^2} \times \frac{2a^4b}{3b^3} = \frac{3}{5} \times x^1 \times x^{-2} \times y^{-6} \times y^{-1} \times \frac{2}{3} \times a^4 \times b^1 \times b^{-3}$$

$$= \frac{3}{5}x^{-1}y^{-7} \times \frac{2}{3}a^4b^{-2} = \frac{3}{5} \times \frac{2}{3}x^{-1}y^{-7}a^4b^{-2} = \frac{3}{5}x^{-1}y^{-7}a^4b^{-2}$$

Q6. $\frac{6ab^2}{4x^{-4}y} \times \frac{12x^2y^2}{2a^{-2}b}$

<center>Soln.</center>

$$\frac{6ab^2}{4x^{-4}y} \times \frac{12x^2y^2}{2a^{-2}b} = \frac{6ab^2}{2a^{-2}b} \times \frac{12x^2y^2}{4x^{-4}y}$$

$$= 3 \times a^1 \times a^2 \times b^2 \times b^{-1} \times 3 \times x^2 \times x^4 \times y^2 \times y^{-1} = 3a^3b^1 \times 3x^6y^1 = 9a^3bx^6y$$

Q7. $\frac{2xy}{ab} \times \frac{4a^2}{y^2} \times \frac{a^3b^2}{x}$

N/B: Since there are no common factors to be cancelled out, multiply all the numerators together, as well as well the denominators together.

The numerator refers to the top letters and numbers, while the denominators refer to the numbers and letters which are down

<center>Soln.</center>

Multiplying the numerators => $2xy \times 4a^2 \times a^3b^2 = 8xya^5b^2$

Multiplying the denominator $\Rightarrow ab \times y^2 \times x = aby^2x, \Rightarrow \frac{2xy}{ab} \times \frac{4a^2}{y^2} \times \frac{a^3b^2}{x} =$

$\frac{8xya^5b^2}{aby^2x} = 8 \times x \times x^{-1} \times a^5 \times a^{-1} \times y \times y^{-2} \times b^2 \times b^{-1} = 8a^4y^{-1}b$

Q8. $\frac{6x^2y}{2r^2s} \times \frac{4r^3}{y} \times \frac{12x^2y}{4y}$

<center>Soln.</center>

First cancel out the common factors i.e $\frac{6x^2y}{2r^2s} \times \frac{4r^3}{y} \times \frac{12x^2y}{4y} = \frac{3x^2y}{r^2s} \times \frac{4r^3}{y} \times \frac{3x^2y}{y}$

Multiply the numerators together $\Rightarrow 3x^2y \times 4r^3 \times 3x^2y = 36x^4y^2r^3$

Multiply the denominators together $\Rightarrow r^2s \times y \times y = r^2sy^2 \Rightarrow \frac{3x^2y}{r^2s} \times \frac{4r^3}{y} \times \frac{3x^2y}{y} =$

$\frac{36x^4y^2r^3}{r^2sy^2} = 36 \times x^4 \times y^2 \times y^{-2} \times r^3 \times r^{-2} \times s^{-1}$

$= 36x^4rs^{-1}$

Q9. $\frac{3x^2y}{2ab} \times \frac{3a^2b}{2x^{-2}y^4} \times \frac{3xy}{4a^2b^2}$

N/B: There are no common factors to be cancelled out.

<center>Soln.</center>

Multiply the numerators $\Rightarrow 3x^2y \times 3a^2b \times 3xy = 27x^3y^2ba^2$

Multiply the denominators $\Rightarrow 2ab \times 2x^{-2}y^4 \times 4a^2b^2 = 16a^3b^3y^4x^{-2} \Rightarrow \frac{3x^2y}{2ab} \times$

$\frac{3a^2b}{2x^{-2}y^4} \times \frac{3xy}{4a^2b^2} = \frac{27x^3y^2ba^2}{16a^3b^3y^4x^{-2}} = \frac{27}{16} \times x^3 \times x^2 \times y^6 \times y^{-4} \times b \times b^{-3} \times a^2 \times a^{-3} \times$

$y^{-4} \times y^2 = \frac{27}{16}x^5y^{-2}b^{-2}a^{-1}$

Q10. $\frac{24x^2y}{8rv^2} \times \frac{y^2}{2x^4y^2} \times \frac{6v^4}{x^{-3}}$

N/B: use the 8 to cancel the 24 and use the 2 to cancel the 6

<center>Soln.</center>

$\frac{24x^2y}{8rv^2} \times \frac{y^2}{2x^4y^2} \times \frac{6v^4}{x^{-3}} = \frac{3x^2y}{rv^2} \times \frac{y^2}{x^4y^2} \times \frac{3v^4}{x^{-3}}$

Multiply the numerators $\Rightarrow 3x^2y \times y^2 \times 3v^4 = 9x^2y^3v^4$

Multiply the denominators $\Rightarrow rv^2 \times x^4y^2 \times x^{-3} = rv^2xy^2$

$$\frac{3x^2y}{rv^2} \times \frac{y^2}{x^4y^2} \times \frac{3v^4}{x^{-3}} = \frac{9x^2y^3v^4}{rv^2xy^2} = 9 \times x^2 \times x^{-1} \times y^3 \times y^{-2} \times v^4 \times v^{-2} \times r^{-1}$$
$$= 9xyv^2r^{-1}$$

N/B: Certain questions may demand the application of factorization, for its simplification.

Q11. Simplify $\frac{6x^2+2xy}{5m} \times \frac{20m^2}{3x+y}$

<div align="center">Soln.</div>

$$\frac{6x^2+2xy}{5m} \times \frac{20m^2}{3x+y} = \frac{2x(3x+y)}{5m} \times \frac{20m^2}{3x+y} = \frac{2x(3x+y)}{3x+y} \times \frac{20m^2}{5m} = 2x \times \frac{4m^2}{m}$$
$$= 2x \times 4m^2 \times m^{-1} = 8x \times m = 8xm$$

Q12. Simplify $\frac{9y^2-6y}{3r} \times \frac{9r^3}{3y-2}$

<div align="center">Soln.</div>

$$\frac{9y^2-6y}{3r} \times \frac{9r^3}{3y-2} = \frac{3y(3y-2)}{3r} \times \frac{9r^3}{3y-2} = \frac{3y(3y-2)}{3y-2} \times \frac{9r^3}{3r} = 3y \times \frac{3r^3}{r}$$
$$= 3y \times 3r^3 \times r^{-1} = 3y \times 3r^2 = 9yr^2$$

Division of algebraic fractions:

N/B: i. $\frac{a}{b} \div \frac{x}{y} = \frac{a}{b} \times \frac{y}{x} = \frac{ay}{bx}$

ii. $\frac{a}{b} \div \frac{3}{5} = \frac{a}{b} \times \frac{5}{3} = \frac{5a}{3b}$

From these given examples, it becomes obvious that in the division of algebraic fractions, the division sign must be changed into the multiplication sign, while the fraction on the right is turned upside down.

Q1. Solve the following:

a) $\frac{6x^2y}{2r^4s} \div \frac{x^3}{r^2s^2}$

<div align="center">Soln.</div>

$$\frac{6x^2y}{2r^4s} \div \frac{x^3}{r^2s^2} = \frac{6x^2y}{2r^4s} \times \frac{r^2s^2}{x^3} = \frac{3x^2y}{r^4s} \times \frac{r^2s^2}{x^3}$$

$$= 3 \times x^2 \times x^{-3} \times y \times r^2 \times r^{-4} \times s^2 \times s^{-1} = 3x^{-1}r^{-2}sy$$

b) $\frac{4a^3b}{2xy^2} \div \frac{ab}{3x^2y^{-2}}$

<div align="center">Soln.</div>

$$\frac{4a^3b}{2xy^2} \div \frac{ab}{3x^2y^{-2}} = \frac{4a^3b}{2xy^2} \times \frac{3x^2y^{-2}}{ab} = \frac{2a^3b}{ab} \times \frac{3x^2y^{-2}}{xy^2}$$

$$= 2 \times a^3 \times a^{-1} \times b \times b^{-1} \times 3x^2 \times x^{-1} \times y^{-2} \times y^{-2} = 2a^2b^0 \times 3x^1y^{-4} = 6a^2xy^{-4}$$