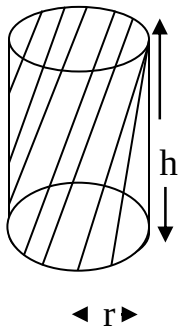
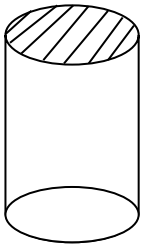
CHAPTER FOURTEEN

CYLINDERS AND CONES

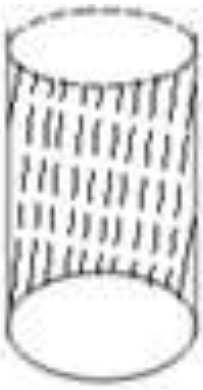
The cylinder:



- The above figure is known as a cylinder.
- The height of this cylinder is h and its radius is r.
- The shaded portion is called the total surface area of the cylinder, also referred to as the area of the cylinder.
- The area of a cylinder is made up of three parts and these are:
  1. The top circular flat surface area, which is also referred to as the top surface area, and which is indicated in the next diagram, by means of shading:

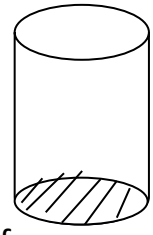


- The flat top circular surface area $= \pi r^2$, since it is circular in shape where r = the radius.
  2. The curved surface area, which is indicated by means of shading, in the next figure:



- The curved surface area = 2 πrh, where h = the height.

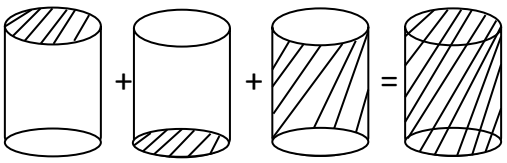
The bottom circular surface area, which is indicated in the next diagram by means of shading:



- The bottom surface area = $= \pi r^2$, since it is also circular in shape.

The area of a cylinder:

The total surface area of a cylinder is therefore had by adding together all these three surface areas,

i.e 

$$\pi r^2 + \pi r^2 + 2\pi rh = 2\pi r^2 + 2\pi rh = 2\pi r(r + h).$$

Q1. The height of a cylinder is 5cm and its radius is 2cm. Calculate

a. its flat top circular area.

b) its flat bottom circular area.

c) its curved surface area.

d)its total surface area. [Take $\pi = 3.14$].

Soln.

h = 5cm, r = 2cm and $\pi = 3.14$.

a. The flat top surface area = $\pi r^2 = 3.14 \times 2^2 = 3.14 \times 4 = 12.56cm^2$
b. The flat bottom surface area = $\pi r^2 = 3.14 \times 2^2 = 3.14 \times 4 = 12.56cm^2$
c. The curved surface area = $2\pi rh = 2 \times 3.14 \times 2 \times 5 = 62.8cm^2$
d. The total surface area = top surface area + bottom surface area + curved surface area = $12.56cm^2 + 12.56cm^2 + 62.8cm^2 = 87.9cm^2$.

N/B: Also the total surface area = $2\ \pi r(r + h) = 2 \times 3.14 \times 2(2 + 5) = 12.56(7)$ = 87.9cm^2.

Q2. A cylinder has a height of 40m and a diameter of 12m. Determine

a. its bottom circular area .
b. its curved surface area.
c. Its total surface area.
   [Take $\pi = 3.142$]

   Soln:

   Since d =12m$\Longrightarrow r = \frac{12}{2} = 6m$.

   Also $\pi = 3.142\ and\ h = 40m$.

a. The bottom circular surface area = $\pi r^2 = 3.142 \times 6^2 = 3.142 \times 36 = 113m^2$
b. The curved surface area = $2\pi rh = 2 \times 3.142 \times 6 \times 40 = 1508m^2$
c. The total surface area = $2\pi r(r + h) = 2 \times 3.142 \times 6(6 + 40) = 1734$m^2.

Q3. A water storage tank is to be constructed using aluminum. If it is to have a diameter of 40m and a height of 120m, determine the amount of aluminum that will be needed to construct

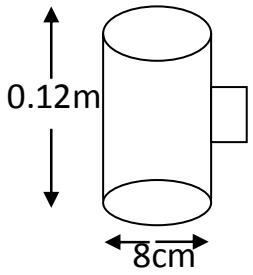
a. its curved surface area .
b. the whole tank. [Take $\pi \; or \; pie = 3.14$].

Soln.

Since d = 40m $\Longrightarrow r = \frac{40}{2} = 20m$. Also pie = 3.14 and h = 120m.

a. The amount of aluminum which is needed to construct the curved surface area = the curved surface area = $2\pi rh = 2 \times 3.14 \times 20 \times 120 = 15072m^2$
b. The amount of aluminum needed to construct the whole tank = the total surface area = $2\pi r(r + h) = 2 \times 3.14 \times 20(20 + 120)$
$= 126(140) = 17640m^2$

Q4.



The given figure is that of a drinking cup, which is to be constructed using plastic. If it is to be 0.12m long and have a diameter of 8cm, determine the quantity of plastic needed for its construction. [Take $\pi = 3.142$].

N/B:

- A drinking cup has no top surface area $\Longrightarrow plastic$ will only be needed to construct the curved surface area and the bottom surface area.
- Also since the height is given in metres and the diameter in centimetres, the metres must be converted into centimetres.

Soln.

h = 0.12m = 0.12 x 100 = 12cm.

D = 8cm $\Longrightarrow r = 4cm$.

The amount of plastic needed to construct the curve surface area $= 2\pi rh = 2 \times 3.142 \times 4 \times 12 = 302cm^2$.

The amount of plastic needed to construct the bottom surface area = bottom surface area = $\pi r^2 = 3.142 \times 4^2 = 3.142 \times 16 = 50cm^2$.

The quantity of plastic needed to construct the cup = amount of plastic needed to construct the curved portion + the amount of plastic needed to construct the bottom surface = 302 + 50 = $352cm^2$.

Q5. The curved surface area of a cylinder of height 80cm is $2880cm^2$. Calculate

i. Its total surface area .
ii. Its circular top surface area. [Take $\pi = 3.14$]

Soln.

The curved surface area = $2\pi rh$, and since the curved surface area of the cylinder is given as $2880cm^2 \Longrightarrow 2\pi rh = 2880, \Longrightarrow 2 \times 3.14 \times r \times 80 = 2880,$

$$\Longrightarrow 502r = 2880, \Longrightarrow r = \frac{2880}{502} \Longrightarrow r = 5.7cm.$$

i. The total surface area $= 2\pi r(r + h) = 2 \times 3.14 \times 5.7(5.7 + 80) = 36(85.7) = 3085cm^2$
ii. The top circular surface area = $\pi r^2 = 3.14 \times 5.7^2 = 102cm^2$.

N/B: Since in the question the heights as well as the curved surface areas were given, we must first determine the radius.

- In the next question, the curved surface area is given as well as the radius. We must therefore first determine the height.

Q6. The curved surface area of a cylinder whose radius is 5cm is $628cm^2$.Determine its total surface area.

Soln.

r = 5cm and h = ?

Since the curved surface area = $628cm^2$, then $2\pi rh = 628 \Longrightarrow 2 \times 3.14 \times 5 \times h = 628, \Longrightarrow 31.4h = 628 \Longrightarrow h = \frac{628}{3.14} = 20.$

Total surface area = $2\pi r(r+h) = 2 \times 3.14 \times 5(5+20) = 31.4(25) = 785cm^2$

Q7. A cylinder has a top surface area of 12.56cm^2 and a height of 0.8m. Calculate

a. its curved surface area.
b. its total surface area.
   [Take $\pi = 3.142$]

Soln.

Top surface area = 12.56cm^2, h = 0.8m = 0.8m x 100 = 80cm.

$\pi = 3.142 \; and \; r =?$

The top surface area is given by πr^2, and since this = 12.56cm^2, then

$\pi r^2 = 12.56, \Longrightarrow r^2 = \frac{12.56}{3.142} = 4.$

Since $r^2 = 4 \Rightarrow r = \sqrt{4} = 2.$

a. Curved surface area = $2\pi rh = 2 \times 3.142 \times 2 \times 80 = 1005cm^2$
b. The total surface area = $2\pi r(r+h) = 2 \times 3.142 \times 2(2+80) = 12.56(82) = 1030cm^2$.

The volume of cylinder:

- The volume of a cylinder is the amount of gas, liquid or solid which it can contain or hold.
- The volume of a cylinder is given by $v = \pi r^2 h$, where r = the radius and h = the height.

Q1. A cylinder has a height of 80cm and a diameter of 20cm. Calculate

a. its volume
b. the volume of air it will contain when it is
   i. full ii. half full.

[Take $\pi = 3.143$]

Soln.

d = 20cm $\Rightarrow r = 10cm$.

a. Volume $= \pi r^2 h = 3.14 \times 10^2 \times 80 = 25120cm^3$.
b. i. The volume of air it will contain when it is full = $25120cm^3$.

ii. The volume of air it will contain when it is half full $= \frac{1}{2} \times 25120 = 12560cm^2$.

Q2. A cylinder is to be constructed in order to have a volume of 5540cm^3. If it is to have a radius is 20cm, calculate its height.

Soln.

$v = 5540cm^3, r = 20cm \ and \ h =?$

Since $v = \pi r^2 h$, then 5540 = 3.14$\times$ $20^2 \times h, \Rightarrow 5540 = 1256h \Rightarrow h = \frac{5540}{1256} =$ 4.4,

$\therefore$ the height = 4.4cm

Q3. A cylindrically shaped water tank, can hold 7000cm^3 of water when it is full. If it has a height of 50cm, determine its radius.

Soln.

$v = 7000cm^3, h = 50cm \ and \ r =?$

Since $v = \pi r^2 h$, then 7000 = $3.14 \times r^2 \times 50, \Rightarrow 7000 = 157r^2, \Rightarrow r^2 = \frac{7000}{157}, \Rightarrow r^2 = 44.5, \Rightarrow r = \sqrt{44.5} => r = 6.6cm$

Q4. Water for sale is stored in a cylindrically shaped tank, of height 120m and diameter 40m. If the tank is full and a bucket whose volume id 200m^3, is used to sell the water at a price of ¢2 per bucket, calculate the total amount expected if all the water was sold. [Take $\pi = 3.142$]

Soln.

D = 40m $\Rightarrow r = 20m.$

Also h = $120m \; and \; \pi = 3.142$

The amount of water the tank will contain when full = the volume of the tank = $\pi r^2 h = 3.142 \times 20^2 \times 120 = 3.142 \times 400 \times 120 = 150816m^3$.

The volume of the bucket used in selling the water = 200m^3 $\Rightarrow$ the number of buckets of water which can be had from the tank $= \frac{150816}{200} = 754$ buckets.

Since the price of water per bucket = ¢2, then the total amount had = 754 x 2 = ¢1508.

Q5. The total surface area of a closed circular cylinder of radius 3.5cm is 1320cm^2. Calculate the volume of the cylinder.

Soln

Area of the cylinder = 1320cm^2.

Radius = r = 3.5cm

Height = h = ?

We must first find the height

Area of cylinder = $2\pi r(r+h)$.

Since the area of the given cylinder = 1320cm^2, then $2\pi r(r+h) = 1320, => 2 \times 3.14 \times 3.5(3.5+h) = 1320, \Longrightarrow 77 + 22h = 1320,$

$\Longrightarrow 22h = 1320 - 77 \Longrightarrow 22h = 1243, \Longrightarrow h = \frac{1243}{22}$

$\Longrightarrow h = 56.5.$

Volume of cylinder = $\pi r^2 h = 3.14 \times 3.5^2 \times 56.5 = 2173cm^3$.