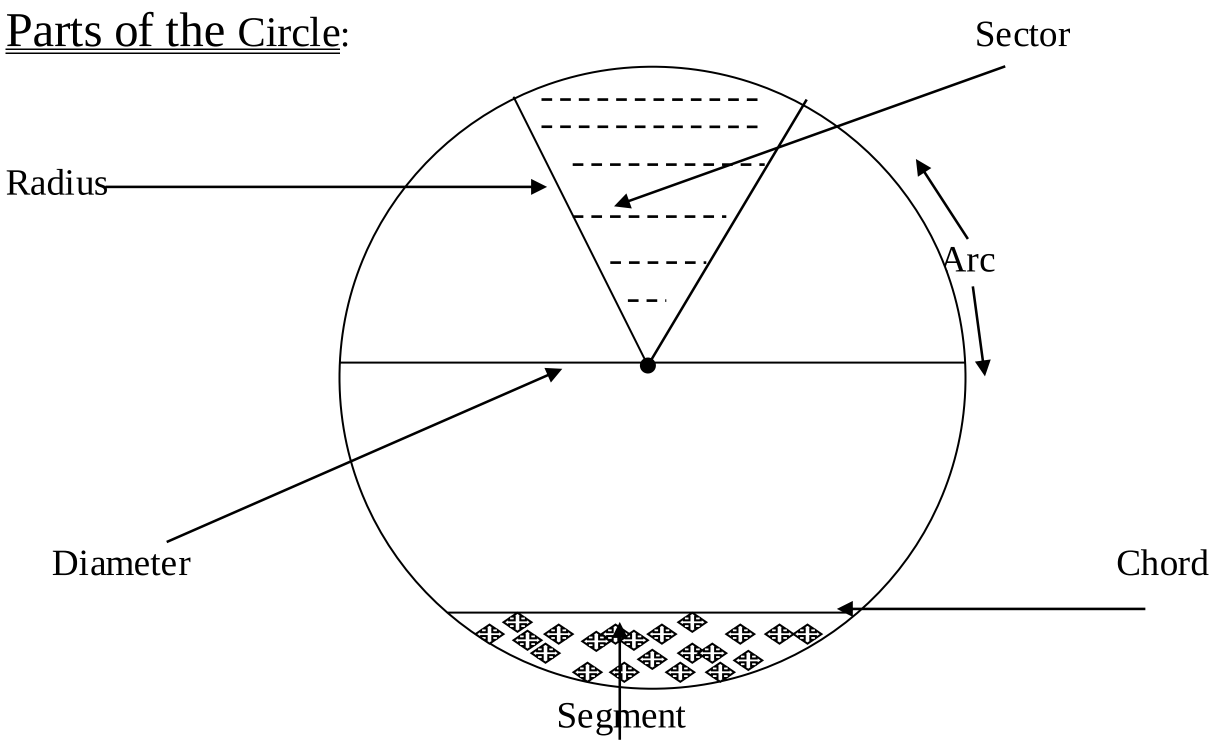
CHAPTER THIRTEEN

THE CIRCLE

Parts of the Circle:

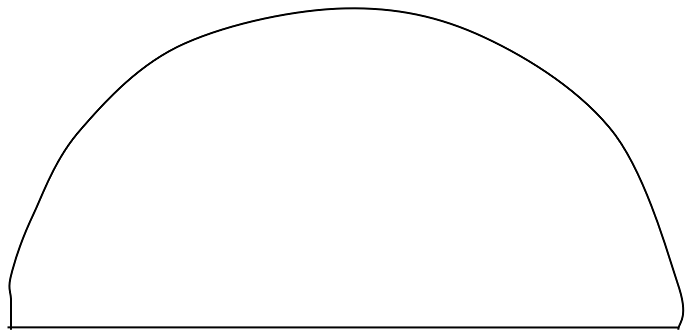


1) The Circumference: This is the distance around the circle.
2) Chord:This is a straight line which joins two points on the circumference.
3) The diameter: This is a special chord which passes through the centre of the circle.
4) The radius: This is a line drawn from the centre, to a point on the circumference.
5) Arc: This refers to a portion of the circumference.
6) The segment: This is the region between a chord and an arc.
7) The sector: This refers to the region between two radii.

Note:

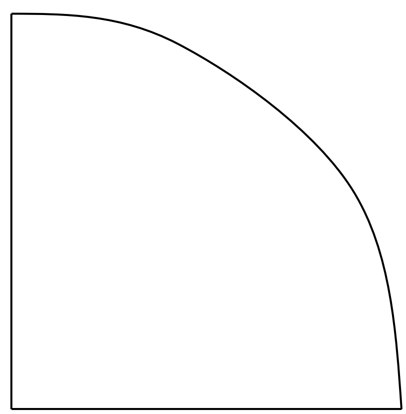
i. For any circle, the radius × 2 = the diameter i.e. twice the radius gives us the diameter.

ii. Half a circle is referred to as a semi circle.



iii. A quadrant refers to one quarter of the area of a circle.

i.e.



iv. For a circle, C$= 2\pi r$, where C= the circumference, r = the radius and $\pi = 3.14 \; or \; 3.142 \; or \; \frac{22}{7}$.

Q1) A circle has a radius of 14cm. Determine the distance round it.

<u>Solutions</u>

C $= 2\pi r$ => C=2×3.14×14=88cm.

Q2) A city is circular in shape and its diameter is 30km. Determine the distance covered by a man, who walked twice round this city.

<u>Solution</u>

The distance covered by a man who did walk round the city once = the circumference.

D = 30km, =>r =$\frac{30}{2}$ = 15km.

C= $2\pi r$, => C=2×3.14×15= 94km.

Distance covered by walking round the city twice= 2×94=188km.

Q3) A racing bike is travelling round a circular track whose radius is 40km, at a speed of 20km/h. Determine the time it will take to travel

a) once round the track.
b) thrice round the track.

<u>Solution</u>

(a) The distance covered by travelling once round the track = the circumference = $2\pi r$ =2×3.14×40 =251km.

a) the speed of racing bike = 20km/h

∴ If 20km = 1hour

then 251km =$\frac{251}{20}$× 1 = 12.6hrs.

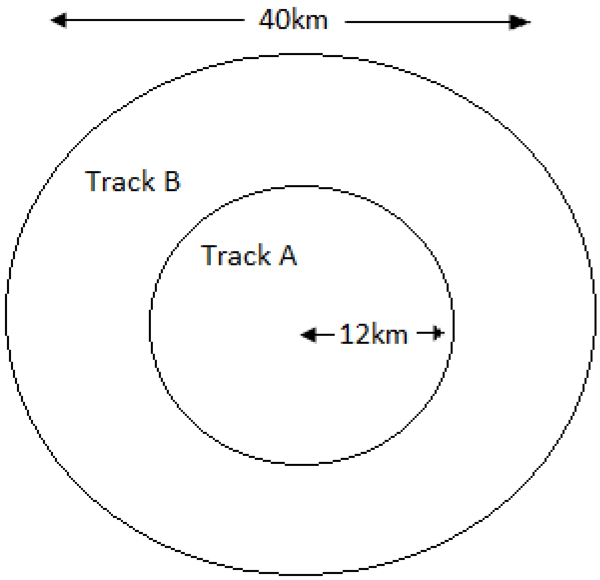
b) Distance travelled by travelling thrice round this track = 3×251= 753km.

Speed of bike = 20km/h

If 20km = 1 hour

Then 753km = $\frac{753}{20}$× 1 = 38hrs.

Q4)



Two cyclists, Addo and John are supposed to travel round two different circular tracks. Addo is to travel in track A at a speed of 40km/h and John is to travel in track B at a speed of 60km/h.

a) Determine which of these men will be the first to complete his journey.
b) Express the distance travelled by Addo as a fraction of the distance travelled by John.

<u>Solution</u>

The distance travelled by Addo = the circumference of track A= $2\pi r$ =2×3.14×12=75km.

Speed of Addo = 40km/h.

If 40km=1 hour

$=> 75km = \frac{75}{40} \times 1 = 1.9.$

$\therefore$ Time taken by Addo to move round his track = 1.9 hrs.

Distance travelled by John = the circumference of track

B = $2\pi r$ =2×3.14×20 =126km.

Speed of John = 60km/h.

If 60km = 1 hour

$=> 126km = \frac{126}{60} \times 1 = 2.1$

John will complete his journey in 2.1 hours..

Addo will finish first

b)Distance travelled by Addo = 75km and distance travelled by John = 126km

Distance travelled by Addo as a fraction of that travelled by John = $\frac{75}{126} = \frac{25}{42}$

Q5) A city which is circular in shape has a length of 420km. Determine the distance walked by Mr. Abu, if he walked from the centre of this city to a point on the city's boundary.

Solution

Length of the city = the circumference = 420km.

Distance travelled by Mr. Abu = the radius =?

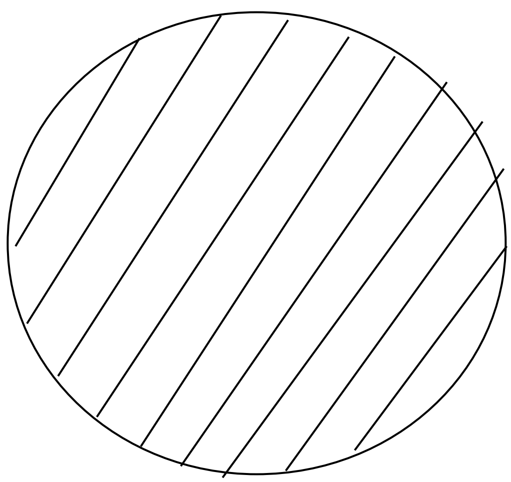
But since C=$2\pi r$ => 420 =2×3.14×r,

=>420 = 6.28r => r = $\frac{420}{6.28}$,

=>r = 67, =>Distance walked by Mr. Abu = 67km.

The area of a circle:

The area of a circle refers to the region within the circle.



For example the shaded portion refers to the area of the given circle.

The area of a circle = πr^2, where r = radius of the circle.

1) A circle has a radius of 7cm. Determine its area.

$\left[\text{Take } \pi = \frac{22}{7}\right]$

Solution

Area = $\pi r^2 = \frac{22}{7} \times 7^2 = 154\text{cm}^2$

2) A circular plot of land has an area of 255cm^2.

Determine its diameter.

$\left[\text{Take } \pi = 3.142\right]$

Solution

Since $A = \pi r^2 \Rightarrow 255 = 3.14 \times r^2$

$\Rightarrow \frac{255}{3.14} = r^2 \Rightarrow 81 = r^2 \Rightarrow r = \sqrt{81} = 9\text{cm}.$

Diameter = 2r = 2 × 9 = 18cm..

3) A man charges ¢2 for weeding an area of 5m^2.Determine how much he will charge if he weeds a circular field of radius 8m. $\left[\text{Take } \pi = 3.142\right]$.

Solution

Area of circular field to be weeded = $\pi r^2 = 3.142 \times 8^2 = 201\text{m}^2$

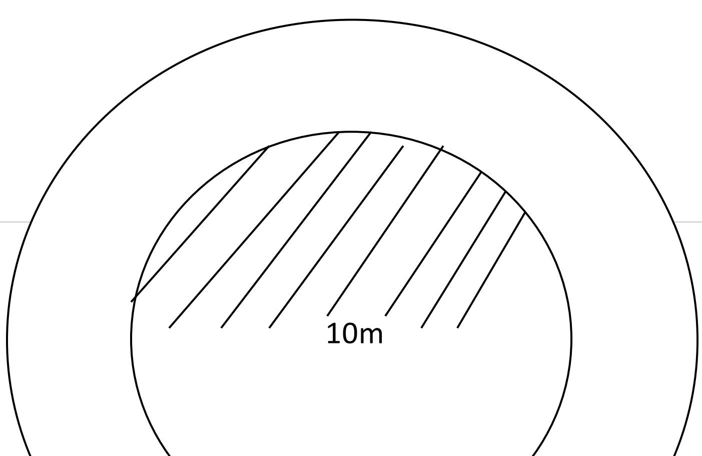
If 5m^2 = ¢2

$\Rightarrow 201\text{m}^2 = \frac{201}{5} \times 2 = 80$, $\Rightarrow$ amount charged = ¢80.

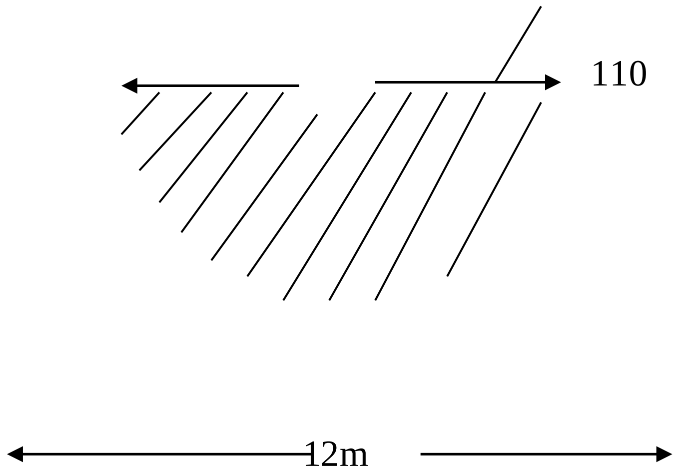
1. 4) A man owns a circular plot of land whose diameter is 12m. On a portion of this land which is circular in shape and of diameter 10m, he has planted onion. Determine

i. the fraction of the land on which the onion farm is located.
ii. the percentage of the land on which the onion farm is located
iii. the quantity of land left for future cultivation.

Solution



.



Let the shaded portion represent the onion farm. Since it diameter = 10m => radius = 5m.

The area of the onion farm = $\pi r^2 = 3.14 \times 5^2 = 78.5\text{m}^2$

Also since the diameter of the circular field = 12m => its radius = 6m.

The area of this field = $\pi r^2 = 3.14 \times 6^2 = 113\text{m}^2$

i) Fraction of the field on which the onion farm is located = $\frac{78.5}{113}$

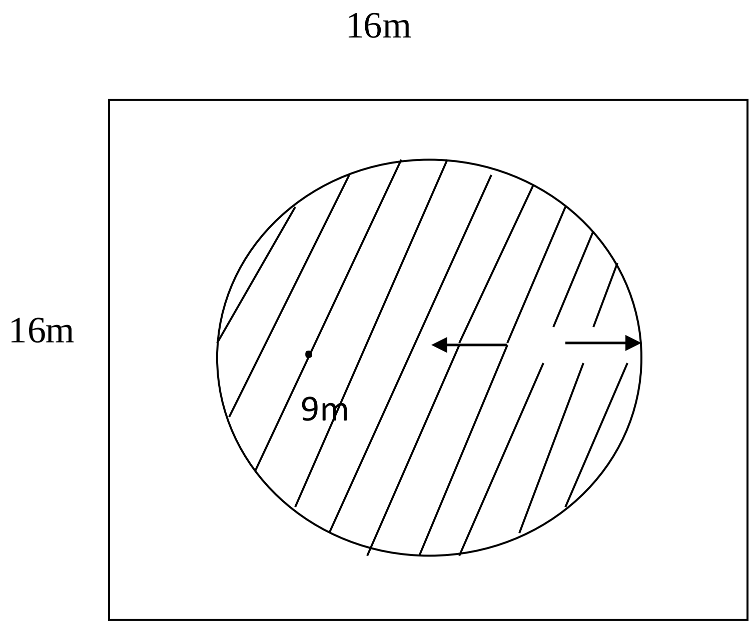
ii) The percentage of the land on which the onion farm is located = $\frac{78.5}{113} \times 100 =$ 69%.

iii) The portion of the land left for future cultivation = 113 - 78.5 = $34.5\text{m}^{2.}$

(5) An onion farm which is circular in shape and of radius 9m is situated within a plot of land, which is in the shape of a square of side 16m.

Determine the fraction of the plot on which the onion farm is situated.

Solution



The area of the square plot = $16^2 = 256m^2$

The area of the shaded portion which represents the onion farm = $\pi r^2 = 3.14 \times 9^2 = 254m^2$.

The fraction of the plot on which the onion farm is located = $\frac{254}{256} = \frac{127}{128}$