Chapter Three

Linear and exponential equations

- In a simplified manner, a linear equation can be said to be an equation, in which none of the letters or numbers has been raised to any power.

- On the other hand in an exponential equation, one of the letters or numbers has been raised to a power or an exponent.

Linear equation:

- A linear equation such as $2x + 1 = -x + 1$ can also be written as $+2x + 1 = -x + 1$.

- The equation $4x + 3 = 2x - 1$ can also be written as $+4x + 3 = +2x - 1$.

- Lastly the equation $5 + 2x = 8 - x$ can be written as $+5 + 2x = +8 - x$.

- In short if there is no negative sign in front of a letter, then the sign in front of it is taken to be the positive sign.

- The sign before a number or a letter is also part of that letter or number.

- For example consider the equation $2x - 5 = -4x + x + 2$, the negative sign before the 5 is part of the 5, the positive sign before the $2x$ forms part of the $2x$, the – sign before the $4x$ also forms part of it, and the positive sign before the 2 forms part of it.

- When a number or a letter crosses the equal to sign, then the initial sign before that number or letter must be changed into the opposite sign.

- For example if the sign before the letter or number is the positive sign, then it changes into the negative sign, after crossing the equal to sign.

Q1. Given that $x + 1 = 6$, find x.

Soln.

$Since\ x + 1 = 6 => x = 6 - 1 = 5, => x = 5.$

Q2. If $2x - 4 = 6,\ find\ x.$

Soln.

$Since\ 2x - 4 = 6 => 2x = 6 + 4, => 2x = 10.$

Divide through using 2 $=> \frac{2x}{2} = \frac{10}{2}, => x = 5.$

Q3. Given that $5x = x - 8$, calculate the value of *x*.

Soln.

$From\ 5x = x - 8 => 5x - x = -8, \rightarrow 4x = -8.$

Dividing through using 4 $=> \frac{4x}{4} = \frac{-8}{4}, => x = -2.$

Q4. Determine the value of n, given that $6n + 2 = 3n + 14$.

Soln.

$$Since\ 6n + 2 = 3n + 14 => 6n + 2 - 3n = 14. From\ 6n - 3n + 2 = 14, then\ 6n - 3n = 14 - 2, => 3n = 12 \Longrightarrow n = 4.$$

Q5. If $4x + 5 = 17 + 2x,\ find\ x.$

Soln.

$$4x + 5 = 17 + 2x \Longrightarrow 4x + 5 - 2x = 17, \Longrightarrow 4x - 2x = 17 - 5 \Longrightarrow 2x = 12, \Longrightarrow \frac{2x}{2} = \frac{12}{2} \Longrightarrow x = 6.$$

Q6. If $6x - 1 = -2x + 15$, determine the value of x.

Soln.

$$Since\ 6x - 1 = -2x + 15 => 6x - 1 + 2x = 15, \Longrightarrow 6x + 2x - 1 = 15 \Longrightarrow 6x + 2x = 15 + 1, \Longrightarrow 8x = 16 \Longrightarrow \frac{8x}{8} = \frac{16}{8} \Longrightarrow x = 2.$$

Q7. If $4y + 10 = 2 + 3y, find\ y.$

Soln.

$$From\ 4y + 10 = 2 + 3y => 4y + 10 - 3y = 2, \Longrightarrow 4y - 3y + 10 = 2, \Longrightarrow 4y - 3y = 2 - 10, \Longrightarrow y = -8.$$

Q8. Solve for n, given that $2n + 4 - 6n = -8 + 2n$

Soln.

$$Since\ 2n + 4 - 6n = -8 + 2n => 2n - 6n = -8 + 2n, => -4n + 4 = -8 + 2n, -4n + 4 - 2n = -8, => -4n - 2n + 4 = -8, -6n + 4 = -8, => -6n = -8 - 4, => -6n = -12$$

Divide through using - 6 $=> \frac{-6n}{-6} = \frac{-12}{-6} => n = 2.$

Linear equations associated with cross multiplication:

- Certain linear equations may be given in the disguised form and will really only show themselves up, only after the application of cross multiplication.

- In multiplication, when there is the positive or the negative sign between a number and a letter, or between two letters, we must put them into the bracket.

- Example (1)
  Multiply a – 2 by 4.
  Soln.
  $(a - 2) \times 4 = 4(a - 2) = 4a - 8.$
  Example (2)
  Multiply x + 4 by 5.
  Soln.
  $(x + 4) \times 5 = 5(x + 4) = 5x + 20.$

Q1. If $\frac{x}{6} = 2,\ find\ x.$

Soln.

$Since\ \frac{x}{6} = 2 => \frac{x}{6} = \frac{2}{1}$

Cross multiply => $x \times 1 = 2 \times 6 => x = 12.$

Q2. Given that $\frac{2}{3} = \frac{x}{9}$, calculate the value of x.

Soln.

$\frac{2}{3} = \frac{x}{9}, and\ by\ cross\ multiplying => 2 \times 9 = 3 \times x, therefore\ since\ 18 = 3x, =$

$> 3x = 18 => x = {}^{18}/_{3} => x = 6.$