CHAPTER THREE

BINARY OPERATION

Operation rules in ordinary Algebra:

These operational rules are

1) Closure:

- A statement is opened when no limitation is placed on it, and closed when a limitation is placed on it.
- For example Kofi is a boy is an opened statement, but Kofi is a boy in the class is a closed statement.
- Also x is a number is an opened statement, but x is a number less than 10 is a closed one.

2) Commutative law:

a) If a + b = b + a, then the given operation which is + or addition is commutative.
b) If a × b = b × a, then the given operation which is × or multiplication is commutative.
c) Lastly if a Δ b = b Δ a, then the given operation which is Δ is commutative.

Q1) Using the numbers 3 and 4, determine whether or not addition is commutative.

N/B L.H.S = Left hand side and R.H.S= Right hand side.
Soln.

For addition (+) to be commutative, then 3+ 4 = 4 + 3.

Consider the L.H.S i.e 3 + 4 =7.

Consider the R.H.S i.e 4+3 =7.

Since L.H.S = R.H.S, then addition is commutative.

Q2) Using the numbers 3 and 4, determine whether or not subtraction is commutative.

Soln.

If subtraction is commutative, then 3 – 4 = 4 – 3.

Considering the L.H.S, 3 – 4 = -1.

Considering the R.H.S, 4 – 3 =1.

Since the L.H.S $\neq$ R.H.S

i.e L.H.S is not equal to the R.H.S, then subtraction is not commutative.

Q3) Using the numbers 3 and 4, determine whether or not multiplication is commutative.

Soln.

For multiplication to be commutative, then $3\times4 = 4\times3$.

L.H.S = 3×4 =12.

R.H.S = 4×3 =12.

Since L.H.S = R.H.S, then the operation which multiplication is commutative.

3) Associative Law:

a) If (a + b) + c = a + (b + c), then addition is associative.
b) If $(a \times b) \times c = a \times (b \times c)$, then multiplication is associative.
c) If (a * b) * c = a * (b * c), then the operation which is *, is associative.

Q1) Using the numbers 2, 3 and 5, determine whether or not addition is associative.

Soln.

If + (addition) is associative, then (2+3) +5 = 2+ (3+5).

L.H.S = (2+3) +5 = 5+5=10

R.H.S = 2+ (3+5) = 2+8=10

Since the L.H.S = R.H.S, then addition is associative.

Q2) Using 2, 3 and 5, determine whether or not multiplication is associative.

<u>Soln.</u>

If multiplication is associative, then $(2\times3) \times 5 = 2\times (3\times5)$.

L.H.S = $(2\times3) \times5 = 6\times5 = 30$

R.H.S = $2\times (3\times5) = 2\times15 = 30$

Since L.H.S = R.H.S => × (multiplication) is associative.

Q3) Using 2,3 and 5, determine whether or not subtraction is associative.

<u>Soln</u>

If subtraction (-) is associative, then $(5 - 3) - 2 = 5 - (3 - 2)$

L.H.S = $(5-3) -2 = 2 - 2 = 0$

R.H.S = $5 - (3 - 2) = 5 - 1= 4$

Since R.H.S ≠ L.H.S, then (–) or subtraction is not associative.

NB: $a\times (b + c)$ is the same as $a (b + c)$

4)<u>Distributive Law:</u>

- If $a (b + c)$ or $a \times (b + c) = ab + ac$, then multiplication is said to be distributive over addition (+), or multiplication is said to be distributive with respect to addition.

- Also if $a (b \Delta c) = ab \Delta ac$, then multiplication is distributive over Δ, or multiplication is distributive with respect to the operation Δ.

<u>An operation:</u>

* An operation is a symbol, with a given meaning.

* For example if a*b = 2a+b, then the symbol * becomes an operation, and a*b means that take twice of a and add it to b.

* Also given that a Δ b = $a^2 + b^2$, then the symbol Δ becomes an operation, and a Δ b means add a squared to b squared.

* Other examples of operation which we are familiar with are addition (+), subtraction (-), division (÷) and multiplication (×).

* Lastly any symbol can be used to represent an operation, provided its meaning is given.

The Identity Element:

The identity element of a given operation has no effect on that given operation.

* For example the identity element of addition is 0 (zero), since any number added to zero gives us the same number. (i.e zero has no effect on addition).

* For examples are

3 + 0 = 3

5 + 0 = 5

2 + 0 =2

* The identity element of multiplication is one, since one has no effect on multiplication.

i.e 2×1=2

4×1= 4

5×1= 5

* Therefore assume Δ to be a given operation, and if i.e = the identity element of Δ, then

2 Δ i.e = 2

4 Δ i.e = 4

1 Δ i.e = 6

Q1)

Δ	1	2	3	4
1	4	1	7	2
2	6	2	3	3
3	5	3	4	5
4	1	4	1	4

The given table is that for the operation Δ. By making a careful study of it, determine the identity element of the given operation.

Soln.

From the table

$1 \Delta 2 = 1$

$2 \Delta 2 = 2$

$3 \Delta 2 = 3$

$4 \Delta 2 = 4$

=> Any number Δ 2 = that number

=> 2 has no effect on the given operation, and as such it is the identity element.

Q2)

*	2	3	5	7
2	3	1	2	4
3	1	4	3	2

5	7	3	5	6
7	6	2	7	4

The given table is that drawn for a certain operation, which is represented by the symbol *. By careful analysis, determine the identity element for the given operation.

Soln.

A careful study indicates the following:

2 * 5 = 6

3 * 5 = 3

5 * 5 = 5

7 * 5 = 7

- This implies that 5 had no effect on the given operation. Therefore the identity element = 5

The Inverse:

The inverse of an item with respect to a given operation, acts on that particular item, to give us the identity element of that given operation.

Q1) Find the inverse of 3, with respect to the addition operation.

Soln.

Since the identity element of addition is 0, then 3 + inverse = 0

=> Inverse = 0 – 3 = -3

Q2) Find the inverse of 7 with reference to addition.

Soln.

7 + inverse = 0

=> inverse = 0 – 7 = -7

Q3) Find the inverse of 3 with respect to multiplication.

Soln.

Since the identity element of multiplication is 1, then 3 × inverse = 1

=> inverse = $\frac{1}{3}$.