CHAPTER SEVEN

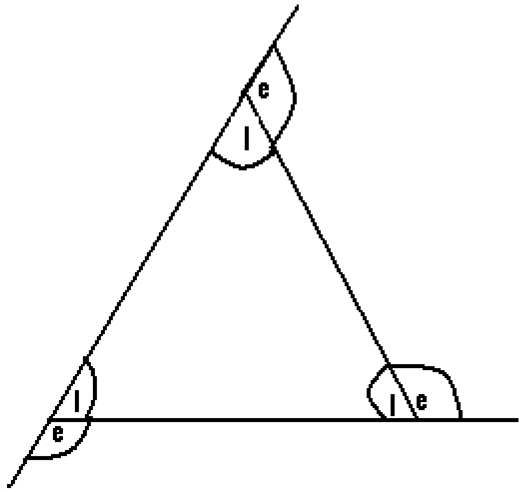
Polygons

Definition:

A polygon is a plane figure which is bounded by straight lines.

Polygons	
Number of sides	Name
3	triangle
4	quadrilateral
5	pentagon
6	hexagon
7	heptagon
8	octagon
9	nonagon
10	decagon

- A polygon has both interior as well as exterior angles.
- The interior angles of a polygon are those angles which lie within the polygon.
- The exterior angles of a polygon lie outside the polygon.



I = interior angle.

e = exterior angle.

N/B: For any polygon, the sum of the exterior angles = 360^0.

Q1. Calculate the value of each exterior angle of a regular decagon.

Soln.

Decagon has 10 sides and as such 10 exterior angles.

But the sum of the exterior angles of any polygon = 360^0.

$\Rightarrow$ 10 exterior angles = 360^0.

$\therefore$ 1 exterior angle = $\frac{1}{10} \times 360 = 36^0$.

$\Rightarrow$ each exterior angle of a decagon = 36^0.

Q2. Find the exterior angle of a regular pentagon.

Soln.

Pentagon has 5 sides, and as such 5 exterior angles. But the sum of the exterior angles of a polygon = 360^0

$\Rightarrow$ 5 exterior angles = 360

$\Rightarrow$ 1 exterior angle = $\frac{1}{5} \times 360$

= 72^0.

$\therefore$ Each exterior angle of the regular pentagon =72^0. For any polygon, the sum of the exterior angle and the exterior angle at any of its vertices = 180^0.

Determination of the interior angle of a regular polygon:

- We must first determine the value of the exterior angle.

- Using the fact that at any vertex, exterior angle + interior angle = 180^0.

$\Rightarrow$ interior angle = 180^0 - exterior angle.

Q1. Calculate the interior angles of a regular decagon.

Soln.

Decagon has 10 exterior angles

$\Rightarrow$ 10 exterior angles = 360^0.

$\therefore$ 1 exterior angle = $\frac{1}{10} \times 360$

= 36^0.

But at any vertex, exterior angle + interior angle = 180^0.

$\Rightarrow$ 36^0 + interior angle = 180^0.

Interior angle =$180^0 - 36^0 = 144^0$.

The interior angle of the decagon = 144^0.

Q2. Find the value of each Interior angle of a triangle.

Soln.

A triangle has 3 sides and as such 3 exterior angles.

$\Rightarrow$ 3 exterior angles = 360^0

$\therefore$ 1 exterior angle = $\frac{1}{3} \times 360 = 120^0$.

But at any vertex, interior angle + exterior angle = 180^0

$\Rightarrow$ Interior angle + 120^0 = 180^0

$\therefore$ Interior angle = 60^0

Determination of the sum or the total interior angles of a polygon:

For any polygon, the sum of the interior angles = the number of sides of the polygon $\times$ the value of one interior angle.

Q1. Calculate the sum of the interior angles of a regular decagon.

Soln.

Decagon has 10 exterior angles

$\Rightarrow$ 10 exterior angles = 360^0

$\therefore$ 1 exterior angle = $\frac{1}{10} \times 360^0$

= 36^0.

But at any vertex, interior angle + exterior angle = 180^0

$\Rightarrow$ Interior angle + 36^0 = 180^0

$\Rightarrow$ Interior angle = 180 – 36

$\Rightarrow$ Interior angle = 144^0.

But the sum of the interior angles of a decagon = interior angle $\times$ the number of sides.

$\therefore$ Sum of interior angles of the decagon = $144^0 \times 10 = 1440^0$.

Q2. Find the sum of the interior angles of a regular octagon.

Soln.

Octagon has eight sides and as such eight exterior angles.

$\Rightarrow$ 8 exterior angles = 360^0

$\therefore$ 1 exterior angle = $\frac{1}{8} \times \ 360^0 \ = 45^0$.

But at any vertex, exterior angle + interior angle = 180^0

$\therefore 45^0$ + interior angle = 180^0

$\Rightarrow$ Interior angle = 180 – 45 = 135^0.

But the sum of interior angle = the number of sides of the polygon $\times$ interior angle = 8 $\times$ 135^0= 1080^0.

Q3.The interior angles of a regular triangle are marked $20^0 + \ 2x^0, 10^0 + 5x^0$and $40^0 + 4x^0$. Find the actual values of each of these angles.

N/B: First calculate the sum of the interior angles of the triangle.

<u>Soln.</u>

Triangle has 3 exterior angles

$\Rightarrow$ 3 exterior angles = 360^0

$\therefore$ 1 exterior angle = $\frac{1}{3} \times 360^0$

= 120^0.

But at any vertex, exterior angle + interior angle = 180^0

$\Rightarrow$ 120^0 + interior angle = 180^0

$\Rightarrow$ Interior angle = 180^0 - 120^0= 60^0.

But the sum of the interior angles of the triangle = the number of sides $\times$ interior angle = 3 $\times$ 60 = 180^0.

But the interior angles of the triangle are given as $20^0 + \ 2x^0, 10^0 + 5x^0$and $40^0 + 4x^0$.The sum of these interior angles = $20^0 + \ 2x^0$+$10^0 + 5x^0$+ $40^0 + 4x^0$

$= 20^0 + 10^0 + 40^0 + 2x^0 + 5x^0 + 4x^0 = 70^0 + 11x.$

But the sum of the interior angles of the polygon or triangle = 180^0

$\Rightarrow 70 + 11x = 180^0$

$\Rightarrow 11x = \ 180^0 - \ 70 = 110^0$

$\Rightarrow x = \frac{110}{11} = 10^0.$

$\therefore$ The angle marked $20^0 + 2x = 20 + 2(10) = \ 20^0 + 20^0 = 40^0$.

The angle marked $10^0 + 5x^0 = 10^0 + 50(10) = 10 + 50^0 = 60^0$.

Lastly, the angle marked $40^0 + 4x^0 = 40 + 4(10) = 40 + 40 = 80^0$.

Q4. The angles of a pentagon are marked $x^0, (x^0 + 20^0), (x^0 + 25^0), 2x^0 \ and \ (2x^0 + 5)$.

(a) Find the value of x.
(b) Determine the value of each of those angles.
Soln.

Pentagon has 5 exterior angles.

5 exterior angles = 360^0

$\therefore$ 1 exterior angle = $\frac{1}{5} \times 360$

$= 72^0.$

But at any vertex, exterior angle + interior angle = 180^0

$\Rightarrow 72^0 + interior \ angle = \ 180^0$

$\Rightarrow$ interior angle = $180 - 72 = 108^0$.

Sum of the interior angles of the pentagon = number of sides $\times$ interior angle

$= 5 \times 108 = 540^0.$

The given angles which are x^0, $x + 20^0$, $x + 25^0, 2x$ and $2x + 5^0$ are the interior angles of the pentagon.

Sum of these interior angles = $x^0 + \ x + 20^0 + \ x + 25^0 + \ 2x + \ 2x + 5^0$

= 7x + 50.

Since the sum of the interior angles of the pentagon has been calculated to be equal to $540^0 \Rightarrow 7x + 50 = 540^0 \Rightarrow 7x = 540 - 50 \Rightarrow 7x = 490$

$\Rightarrow x = \frac{490}{7} = 70$, $\therefore$ x = 70^0.

The value of the angle marked $x^0 = 70^0$.

The value of the one marked x + $20^0 = 70 + 20 = 90^0$.

The angle marked x + 25 = 70 + 25 = 95^0.

The angle marked 2x = 2 $\times$ $70 = 140^0$.

Lastly, the angle marked 2x + 5 = 2(70) + 5 = 140 + 5 = 145^0