# CHAPTER FIVE

## DENSITY, ARCHEMEDES PRINCIPLE AND FLOATING BODIES

Density:

- The density of a sibstance is defined as its mass per unit volume.

- Density = $\frac{Mass}{Volume}$ $or$ $e = \frac{m}{v}$ ,

where $e$ = density, m = mass and v = volume.

**Importance of density measurement:**

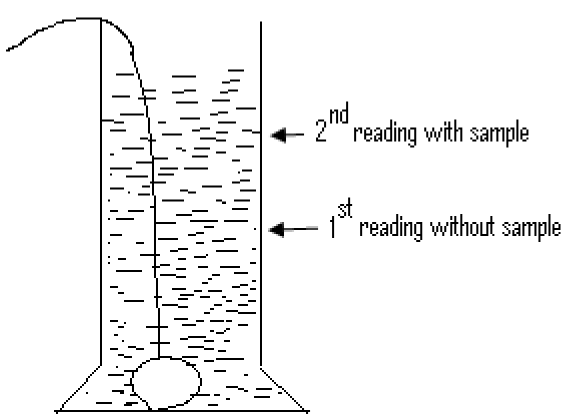
(1) It can be used to determine whether a substance or an item is pure or not.
(2) Architects and engineers make us of the densities of materials, when they design structures such as bridges and flyovers.

**Simple measurement of density:**

**(1) Liquid:**

- To measure the density of a liquid, a convenient volume of the liquid is run off into a clean, dry, previously weighed beaker, using eihter a pipettte or burette.
- The beaker and the liquid are then weighed and the mass of the liquid found by subtraction.
- The volume of the liquid delivered by the pipette or the burette is noted.
- We finally divide the mass by the volume to get the density.

**(2) Solid:**



- If the substance has a regular shape e.g cylinder, sphere or a rectangular bar, its volume can be found by calculation.

- Its mass can then be found through weighing.-
- The mass is then divided by the volume to get the density.
- If the solid has an irregular shape, such as that of a stone, then its vol;ume is found in the following manner.
- Water is first placed inside a measuring cylinder and its volume is noted.
- The stone tied to a thread is completely immersed into the water in the cylinder.- The new volume of the water is also noted, and the difference between the two volumes gives us the volume of the solid.
- The mass of the solid is found through weighing and we divide the mass by the volume to get the density.

## Relative density:

- The relative density of a substance is the ratio of the mass of any volume of it, to the mass of an equal volume of water.

- Relative density = $\frac{mass\ of\ any\ volume\ of\ the\ substances}{mass\ of\ equal\ volume\ of\ water}$

- Because the mass of a body is proportional to its weight

=> relative density = $\frac{weight\ of\ any\ volume\ of\ the\ substances}{weight\ of\ an\ equal\ volume\ of\ water}$

- This explains why relative density is also called specific gravty, since gravity implies weight.
- Relative density has no units but density is expressed in $kg/m^3$ or $g/cm^3$.

## Measuring the relative density of a liquid:

- To ensure getting identical equal volumes of water and a liquid, we make use of the density bottle.
- This botttle is first weighed empty, and later weighed when it is filled with the given liquid.
- The liquid is then removed from the density bottle, and after rinsing or cleaning the bottle, it is filled with water and weighed again.
- To get the mass of the liquid, the mass of the empty bottle is subtracted from the mass of the bottle filled with the liquid.- To get the mass of the water, we

subtract the mass of the empty bottle from the mass of the bottle filled with water.

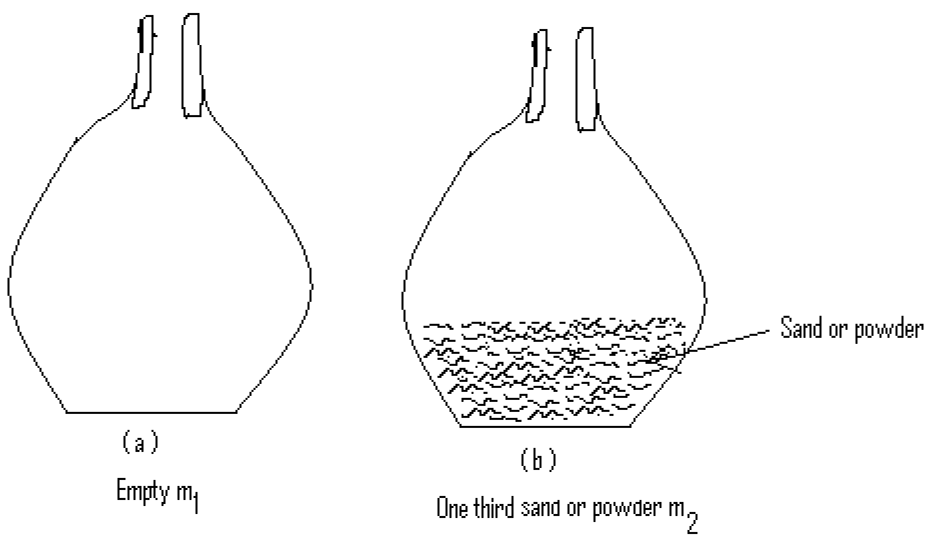
- To get the relative density, we divide the mass of the liquid by the mass of the water.
- The precautions taken are:

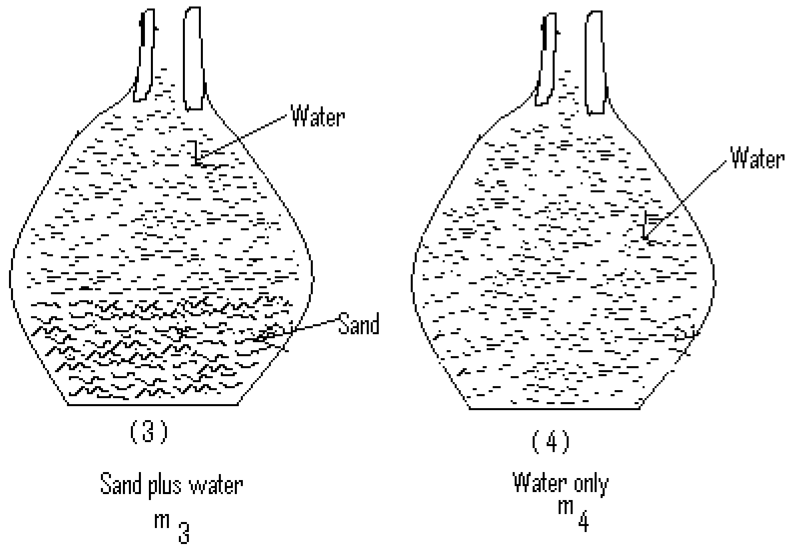
  (I) The outside of the bottle should be wiped dry before weighing.

  (I) The bottle should not be held in warm hands, otherwise some of the liquid may be lost through expansion.

## **Measuring the relative density of a powder:**

- In this case, the powder must not be soluble in water, and the same method can be used to determine the relative density of other items which are insoluble in water, and are in the powdered or granular form such as sand.
- The density bottle is used in this case.






Relative density of sand (powder) = $\frac{mass\ of\ any\ volume\ of\ sand\ (powder)}{mass\ of\ an\ equal\ volume\ of\ water}$

= $\frac{mass\ of\ sand\ in\ (2)}{mass\ of\ water\ in\ (4) - mass\ of\ water\ in (3)}$

=$\frac{m_2 - m_1}{(m_4 - m_1) - (m_3 - m_2)}$

(Q1) A piece of a material has a volume of 15cm$^3$ and a mass of 27g. Calculate its density in g/cm$^3$.

Soln:

Mass = m = 27g

Volume = 15cm$^3$

Density $e$ = $\frac{m}{v} = \frac{27}{15}$

= 1.8gcm$^{-3}$

N/B: The density of water = 1g/cm$^3$.

(Q2) Calculate the mass of air in a room of floor dimension 10m $\times$ $12m$ and height 4m. (Density of air = 1.26kg/m$^3$).

Soln:

Volume of the room = L $\times$ B $\times$ $H$ = 12 $\times$ 10 $\times$ 4 = 480m$^3$.

Mass = ?

Density = $(e) = \frac{mass}{volume}$

=> mass = density × volume

=> mass = 1.26 × 480 = 605kg.

(Q3) An empty 60litre petrol tank weighs 10kgf. Determine its weight when it is filled with a fuel of relative density 0.72.

Soln:

Volume = 60 litres.

Relative density = 0.72.

Relative density = $\frac{mass}{volume}$

=> 0.72 = $\frac{mass}{volume}$

=> 0.72 = $\frac{mass}{60}$

=> mass = 0.72 × 60 = 43.2kg,

=> the mass of fuel = 43.2kg.

But since the mass of the tank is 10kgf => the mass of the tank when filled with fuel = 43.2 + 10 = 53.2kg.

(Q4) A bottle full of water weighs 45gf. When it is full of mercury, it weighs 360gf. If the empty bottle weighs 20.0gf, calculate the density of mercury.

Soln:

Weight of bottle filled with water = 45gf.

Weight of empty bottle = 20gf,

=> weight of water = 45 – 20 = 25g (i.e. weight of water within the density bottle).

Density = $\frac{mass}{volume}$

Since the density of water = 1g/cm$^3$ => 1 = $\frac{25}{v}$

=> V = $\frac{25}{1} = 25cm^3$

=> the volume of the bottle = 25cm$^3$.

Mass of bottle filled with mercury = 360gf.

Since the mass of the bottle = 20gf => the mass of mercury = 360 – 20 = 340gf.

Volume of the bottle = 25cm$^3$

Density = $\frac{mass}{volume}$ => density of mercury = $\frac{340}{25}$ = 13.6gcm$^{-3}$.

(Q5) A density bottle weighs 18gf when empty, 44gf when full of water and 40gf when full of a second liquid. Determine the density of this liquid.

Soln:

Weight of empty bottle = 18gf.

Weight of bottle filled with water = 44gf,

=> weight of water = 44 – 18 = 26g.

But density = $\frac{mass}{volume}$, and since the density of water = 1g/ cm$^3$

=> 1 = $\frac{26}{v} => v$ =26cm$^2$ => the volume of the bottle = 26cm$^3$.
Since weight of the bottle filled with the second liquid = 40g, => weight of the second liquid = 40 – 18 = 22g.

Volume of bottle = 26cm$^3$

Density of this liquid = $\frac{mass}{volume}$ = $\frac{22}{26}$

= 0.9gcm$^{-3}$.

## Archimedes principle and floating bodies:

**Upthrust:** When a body is placed in a liquid, an upward force (i.e. a force which acts in the upward direction) acts on it.This upward acting force is called the upthrust.

## Apparent loss in weight:

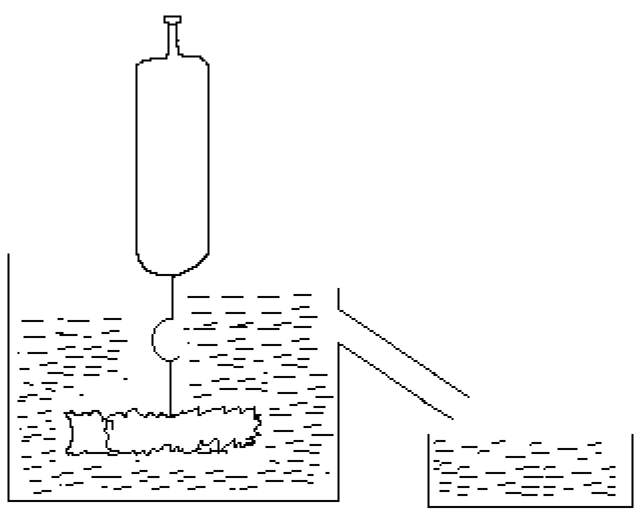
- A simple but striking experiment to illustrate the upthrust exerted by a liquid, can be shown by tying a length of cotton to a brick.
- Any attempt made to lift the brick by the cotton fails through the breakage of the cotton.
- But if the brick is immersed in water, it may be lifted quite easily.
- This is due to the fact that because of the upthrust exerted on the brick by the water, it appears to weigh less in water, than in air.
- In short, there is a decrease in the weight of a body when it is placed in a liquid such as water.

- Stone boulders immersed in water have an upthrust acting on them, which causes decreases in their weights.
- This explains why these boulders can so easily be moved by flood waters.

## Archimedes principle:

Experiment to measure the upthrust of a liquid was first carried out by Archimedes, who came out with a discovery called the Archimedes principle. This principle states that when a body is wholly or partially immersed in a fluid, it experiences an upthrust which is equal to the weight of the fluid displaced by that body. The word fluid may either be referring to either a liquid or a gas.

## To verify Archimede principle for a body in liquid:



- An eureka or displacement can is placed on a bench, with a beaker under its spout.
- Water is then poured into the can until it rans from the spout.
- When the water has ceased dropping, the beaker is removed and replaced by another which has previously been dried and weighed.
- Any suitable solid body such as a piece of metal or stone, is suspended by a thread from the hook of a spring – balance, and the weight of the body in air is measured.
- The body, still attached to the balance is carefully lowered into the displacement can, and when it is completely immersed, its new weight is noted.

- The displaced water is caught in a weighed beaker, and when no more water drips from the spout, the beaker and the water is weighed.
- The result should be set down as follow:
  Weight of body in air =
  Weight of body in water =
  Weight of empty beaker =
  Weight of water plus displaced water =

- Apparent loss in weight of body = the weight of the water displaced.
- The apparent loss of weight of the body, or the upthrust on it, should be equal to the weight of water displaced.
- This verifies Archimedes principle in the case of water.
- A similar result will be obtained, if any other liquid is used.

**<u>To measure the relative density of a solid by using Archimedes principle:</u>**

- The relative density of a substance = $\frac{mass\ of\ any\ volume\ of\ the\ substance}{mass\ of\ an\ equal\ volume\ of\ water}$

- Archimedes principle gives us a simple but accurate method for finding the relative density of a solid.
- If we take a sample of the solid and weigh it first in air, and then in water, the apparent loss in weight obtained by subtraction, is the volume of water displaced by that sample.

- The relative density of a substance = $\frac{weight\ of\ a\ sample\ of\ substance}{apparent\ loss\ in\ weight\ of\ sample\ in\ H_2O}$

- For example, this method can be used to determine the relative density of a piece of brass.

- The weight of the brass is first determined in air.-
- The brass is then completely immersed in a beaker which contains water.
- The results are recorded as follows:

Weight of brass in air = ………………… gf.
Weight of water = ………………

Apparent loss in weight of brass =

Relative density of brass = $\frac{Weight\ of\ brass\ in\ air}{apparent\ loss\ in\ weight\ in\ H_2O}$

**Determination of the relative density of a liquid, using Archimedes principle:**

- A sinker (i.e. any convenient solid body such as a piece of metal or a glass stopper) is first weighed in air, and then weighed in a liquid such as white spirit.
- The apparent loss of weight of the sinker in the spirit is then noticed.
- The sinker is then immersed in water, and the apparent loss of weight in water is also determined.
- The relative density of the spirit = $\frac{apparent\ loss\ of\ weight\ of\ the\ sinker\ in\ the\ spirit}{apparent\ loss\ of\ weight\ of\ the\ sinker\ in\ water}$

**Reasons why a body may sink or float in water:**

- If the density of a material or a body is greater that that of water, and the material or the body is placed in water, then it will sink, but if the density of the material or the body is less than that of water, then it will float.
- For this reason, a piece of solid steel will sink in water since its density is greater than that of water, but a ship made of steel will not sink in water due to the fact that its density is less than that of water.
- The ship has a hollow portion which contains air, which causes its density to be lower than that of water.

- Also a cork held below the surface of water rises to the surface when released, because its density is less than that of water.

## The submarine:

- The submarine has ballast tanks which can be filled or emptied when the need arises, and by filling these tanks with water, there will be an increase in the mass of the submarine.
- This causes its density to increase and become greater than that of water, causing the submarine to sink below the surface of the water.
- In order for it to float or rise, the water is rejected or pumped out from these ballast tanks using compressed air.
- This causes a decrease in the mass of the submarine, causing its density to decrease and become less than that of water.
- The submarine therefore rises to the surface of the water and floats.
- The buoyance of a submarine depends therefore on the quantity of water in its ballast tanks.

## Ballons:

- Gases also have densities, just like solids or liquids.
- When a gas is put into a ballon and the density of the gas within the ballon is less than that of air, then the ballon will rise in the air.

- On the other hand, if the density of the gas within the ballon is greater than that of air, then the ballon will not rise.
- Since hydrogen has a lower density than air, then a ballon filledwith hydrogen gas will rise in the air.

## Floating bodies:

- When a piece of wood or any other material whose density is less than that of water is placed in water, it sinks until the weight of the displaced water is just equal to the weight of the object which has been immersed into the the water.
- The object will then float.
- Even though the hydrogen ballon will ascend in still air, it can be made to float if the quantity of gas it contains can be adjusted, so that average density becomes equal to that of the surrounding air.
- At this stage, the weight of the displaced air will be equal to the weight of the ballon.
- These two given examples illustrate the law of floatation, which states that a floating body displaces its own weight of fluid in which it floats.

## Floating bodies in liquids of different densities:

- Accordind to the priciple of floatation, a body placed in a liquid sinks and only floats when the weight of the liquid it displaces, is just equal to the weight of that body.
- But the less dense (i.e. the lower the density) a liquid is, the less its weight.

- For this reason, if a body is floating in a liquid and the density of the liquid decreases, then more liquid must be displaced in order for the weight of the displaced liquid to be equal to that of the body.
- In order to displace more liquid, the body must sink further into the liquid.
- In short, when a body moves from a liquid of high density into another of low density, it sinks in order to displace more liquid whose weight will balance that of the body.
- On the other hand, if a body moves from a liquid of low density into any of high density, then it will rise since a lesser amount of water must be displaced in order to balance its weight.

## Application of relative density:

- Such application can be seen in the operation of boats, ships, the submarine and the hydrometer.

## Boats and ships:

- Inorder to be capable of floating in water, a boat or ship must sink to a certain level, so as to displace its own weight of water.
- If a ship has been loaded fully to the correct level in waters of high density and then it moves into waters of low density, then it will sink deeper.
- This is due to the fact that according to the principle of floatation, it must displace more of the low density liquid in order to balance or equalize its weight.

- The new level to which the ship may sink, may be so dangerous that a storm may cause it to sink.
- For this reason, insurance companies have lines called primsoll lines marked on ships, so that safe levels may be kept to.
- Loading of the ship may be done to different levels.
- In cold winter waters, loading is done to a particular level.
- It will be done to another level in the warm waters of the tropics, and also to a different level in fresh waters.

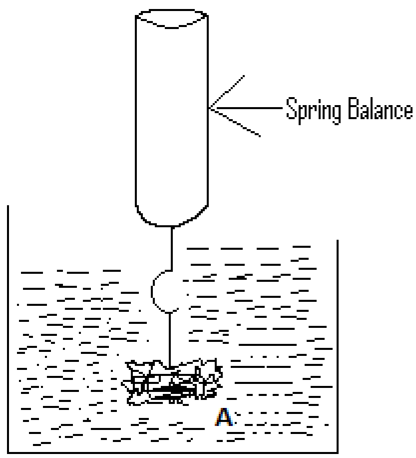
(Q1) (a) State Archimedes principle.

(b) Describe and explained an experiment to measure the relative density of a liquid such as paraffin oil, based on this principle.

Soln:

(a) Archimedes principle states that the upthrust on an object immersed in a fluid, is equal to the weight of the fluid displaced.

(b)




- The apparatus needed are a spring balance, a piece of metal and a beaker.
- The metal A is attached to a spring balance and its weight W was determined.
- It was totally immersed in water in a beaker, taking care it does not touch the sides of the beaker and the apparent weight $W_1$ was noted.
- The metal A was then totally immersed in the liquid, (such as the paraffin oil) in the same way and the apparent weight $W_2$ was noted.
- The loss in weight of metal A in the liquid = $W - W_2$, and the loss in weight of metal A in water = $W - W_1$.
- The relative density of the liquid = $\frac{upthrust}{weight\ of\ water\ displaced}$

  = $\frac{W - W_2}{\mathrm{W} - \mathrm{W}_1}$, since the volume of the metal = the volume of displaced water = the volume of the liquid.

(Q2) A piece of iron weighed in air has a weight of 177.5N. When completely immersed in water, its weight is 152.5N, and when completely immersed in alcohol, its weight is 157.5N. Determine the relative density of alcohol.

Soln:

Weight of iron in air = 177.5N.

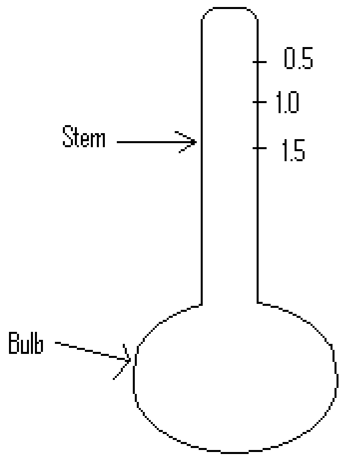
Weight of iron in water = 152.5N.

From Archimedes principle, the weight of water displaced = the apparent loss of weight in water = 177.5 – 152.5 = 25N.

- Also the weight of alcohol displaced = the apparent loss of weight in alcohol = (177.5 – 157.5) = 20N.
- Since the volume of iron = the volume of displaced water = thethe volume of displaced alcohol, then the volume of the displaced liquids are the same.
- For this reason, the relative density of alcohol = $\frac{weight\ of\ alcohol}{weight\ of\ equal\ volume\ of\ H_2O}$

$= \frac{20}{25} = 0.8.$

## The hydrometer:




- This is an instrument which is used to measure density or relative density.
- For example it can be used to measure the density of the dilute acid found in car battery, so as to determine whether the acid is charged or discharged.
- A body will displace different amount of liquids, from liquids of different densities, in order to be able to float in each of them.
- The volume or the amount of liquid displaced and as such, the level to which the body will sink in each of these liquids, depends on the density of the liquid under consideration.
- For the lower the density of the liquid, the deeper will the body sink in it in order to float and vice versa.
- The hydrometer works on this principle.
- The hydrometer consists of a weight, which is in the form of a bulb.

- Attached to this bulb is a marked stem, and the weight keeps the hydrometer erect in water.
- When placed in a particular liquid, the volume of the liquid which needs to be displaced as well as the level to which the hydrometer will sink, is determined by the density of that liquid.
- For the lower the density, the deeper will be the sinking of the hydrometer and vice versa.
- The level of sinking is then read from the markings on the stem of the hydrometer as the density or the relative density of the liquid.
- The lactometer is a special type of hydrometer which is used to measure the density of milk.

(Q1) List the chief features of the hydrometer and state the reasons for them.

Soln:

(1) <u>The narrow stem:</u>

- It gives greater stability and makes the hydrometer float upright.

(2) <u>The wide bulb:</u>

- This makes the volume of liquid displaced large, and as such provide an upthrust which can balance the weight of the floating hydrometer.

(Q2) (a) In which direction along the stem of the hydrometer do the scale graduation increases.

(b) Give a reason for your answer.

Soln:

(a) The density increases downwards from the top of the narrow stem.

- According to Archimedes principle, a body sinking in a liquid only stop sinking to float in it, when the mass of displaced water is equal to the mass of the body.
- But the lower the density of the liquid, the greater the amount of liquid which must be displaced to balance the body`s weight, and vice versa.
- The hydrometer therefore sinks further into the liquid as its density decreasess and vice versa.
- In short, the hydrometer sinks more in the liquid of lower density, and less in the liquids of higher density.
- This explains why on the graduation scale of the hydrometer, the values increases downwards.

(Q3) A simple hydrometer of mass 50g floats in a liquid of relative density 1.4. Calculate the volume of liquid displaced by the hydrometer.

Soln:

- Since if the hydrometer floats in the liquid => the weight of the liquid displaced = the upthrust on the hydrometer, then weight of the liquid displaced = Veg,

  where v = the volume of the liquid displaced,

  $e$=the density of the liquid and $g = \text{acceleration due to gravity.}$

  The upthrust on the hydrometer = mg, where m = the mass of the hydrometer.

  Since weight of displaced liquid = upthrust => V $e$g = mg => V = $^{m}/_{e}$.

  The relative density of the liquid = 1.4.

  But relative density = $\frac{Density\ of\ liquid}{Density\ of\ water}$.

  But since the density of water = 1g/$cm^3$.

  => 1.4 = $\frac{Density\ of\ the\ liquid}{1}$

=> Density of the liquid = $1 \times 1.4 = 1.4$, =>

$e$ = 1.4.

But V = $\frac{m}{e}$ = $\frac{50}{1.4}$ = 35.71$cm^3$.