CHAPTER SEVEN
MOTION

Intrduction:

- This is recognised as a change in the position of a body or of a system.
- The study of bodies in motion is called dynamics, and a body which is not in motion is said to be static.

Types of motion:

- There are different types of motion and some examples are:

(1) Rectilinear motion: This is motion in a straight line.

(2) Circular motion: This refers to the motion of an object in a circle eg, the whirling of a stone in a circle.

(3) Rational motion or spin: This is the motion of a body which spins on its axis, e.g the spin of the earth.

(4) Random motion: This is the motion of a body, in which the direction of movement is not specific and can change at any time.

- An example is the movement of gas particles within a container.

Rectilinear motion:

Speed:

- This is the rate of change of distance with time, and it is the term used to describe an object which does not move in a straight line.
- It describes how fast an object is moving.
- It is a scalar quantity, since it has only magnitude but no direction.

Average speed:

- When a body moves at different speeds in the course of a journey, the ratio of the total distance travelled to the total time taken is called the average speed.

Instantaneous speed:

- This is the measure of the speed of a body at a specific moment.
- Speed = $\frac{Distance}{Time}$

(Q1) A car covers a distance of 20Km within a time interval of 2hours. Calculate its speed or its average speed.

Soln:

Distance = 20Km, time = 2hrs.

Speed = $\frac{Distance}{Time}$ = $\frac{20}{2} = \frac{10km}{h} = 10km/h$

(Q2) A man travels a distance of 50m within 10 seconds. Find the average speed.

Soln:

Distance = 50m, and time = 10seconds.

Speed = $\frac{Distance}{Time}$ = $\frac{50}{10} = 5m/s.$

(Q3) A lorry covers a distance of 100km in 120 minutes. Find its average speed.

Soln:

Distance = 100km. Time = 120 minutes = $\frac{120}{60} = 2hrs.$

Speed = $\frac{Distance}{Time}$ = $\frac{100}{2} = 50km/h.$

(Q4) A cyclist covers a distance of 50, 000m within 4 hours. Find his speed.

N/B:

- Convert the metres into kilometre by dividing by 1000.
- If the distance is in kilometres, then the time must be in hours, and the speed will be in km/h.
- If the distance is in metres, then time should be in seconds and speed will be in m/s.
  Soln:
  Distance = 50,000m = $\frac{50{,}000}{1000} = 50km$
  Speed = $\frac{Distance}{Time}$ = $\frac{50}{4} = \frac{12.5}{h}km/h.$

Displacement:

- This is distance moved in a specific direction, or the distance moved in a straight line.

- It is measured in metres and it is a vector quantity.

Velocity:

- This is defined as speed in a specified direction, or it is the rate of change of displacement.
- The term velocity is used when an object moves in a constant direction, or along a straight line.

- Velocity = $\frac{Distance\ travelled\ in\ straihgt\ line}{Time\ taken}$

Uniform velocity:

- This occurs when a body travels in a straight line, and moves equal distances within equal time intervals.
- If the motion of a body moving in a straight line, is such that the body travels equal interval in equal time interval, no matter how small the time interval, then the body is said to be moving with a uniform velocity.

(Q5) The distance moved in a straight line by an aeroplane is 20km. If the time interval is 10hours, determine its velocity.

Soln:

Distance travelled in straight line = 200km.

Time = 10 hours.

Velocity = $\frac{200}{10} = 20km/h.$

Acceleration:

- This is the rate of change of velocity with time.

Uniform acceleration:

- If the motion of a body moving in a straight line is such that its velocity increases by equal amount in equal time interval, no matter how small the interval, the body is said to have a uniform acceleration.

(Q6) The velocity of a car increased from 10m/s to20m/s in 5 seconds. Calculate its acceleration (or average acceleration).

Soln:

Acceleration = $\frac{Increase\ in\ velocity}{Time\ taken}$

= $\frac{(20-10)}{5} = \frac{10}{5} = 2m/s^2$.

Deceleration or retardation:

- This is the rate of decrease of velocity with time, which is also referred to as negative acceleration.
-When the velocity of moving body is decreasing, it is said to be undergoing retardation.

- Retardation = $\frac{Final\ velocity - Initial\ Velocity}{Time}$

(Q7) The velocity of a car decreased from 30m/s to 19m/s in 3 seconds.

(a) Calculate its retardation.
(b) Determine its velocity after 2 seconds.

Soln:

(a) Final velocity = V = 18m/s.
Initial velocity = U = 30m/s.
Time = t = 3seconds
Retardation = $\frac{V-U}{t} = \frac{18-30}{3}$
= $\frac{-12}{3} = -4m/s^2$.

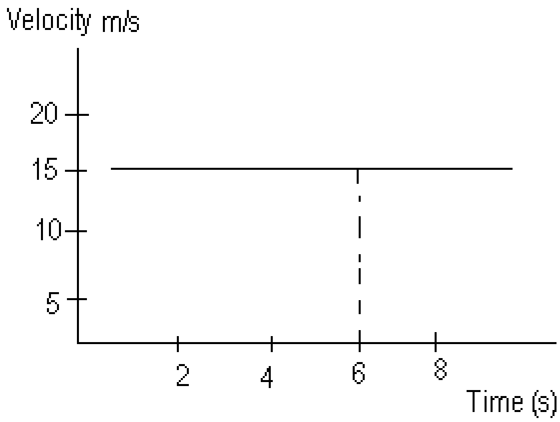
N/B: The negative sign implies that it is retardation.

(b) If the retardation is uniform, then 2 seconds later, the velocity will be reduced by $4 \times 2 = 8$ m/s^2.

The velocity will therefore be (18 – 8) = 10m/s^2.

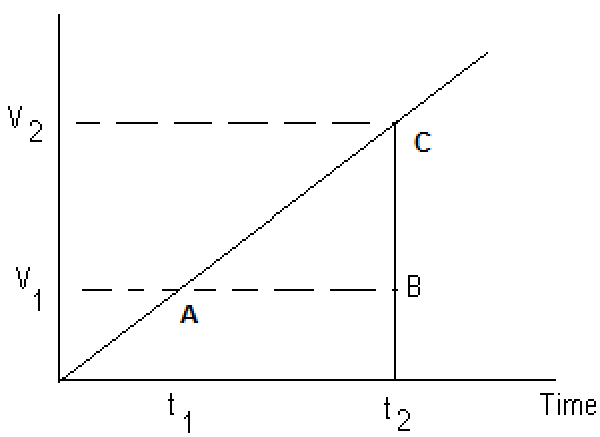
Graphs of motion:

(1) Velocity – time graphs:



- The graph is that for a body which is moving with a constant or a uniform velocity.
- The distance covered by the body in 6 seconds = velocity × time = 15 × 6 = 90m.
- When a body moves with a constant velocity, the graph is a straight line, which is parallel to the horizontal axis.
- Because the gradient of the line is zero, the acceleration is also zero.

(b)



- This graph is that for a body which starts or takes off from rest.
- Since the velocity increases uniformly, then the body is moving within uniform acceleration.
- With respect to velocity – time graph, the gradient of the line or the line graph, gives us the acceleration or the retardation of the body or the object.

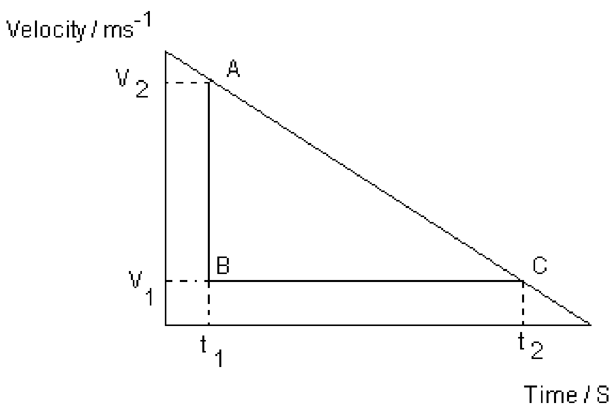
- If the gradient is positive, then the object is undergoing acceleration, but if it is negative, then the body is undergoing retardation.
- In the case under consideration.

  Acceleration = $\frac{Change\ in\ velocity}{Time}$

= $\frac{BC}{AB} = \frac{V_2 - V_1}{t_2 - t_1}$

- In this particular case, the gradient is positive and as such the object or body is undergoing acceleration.

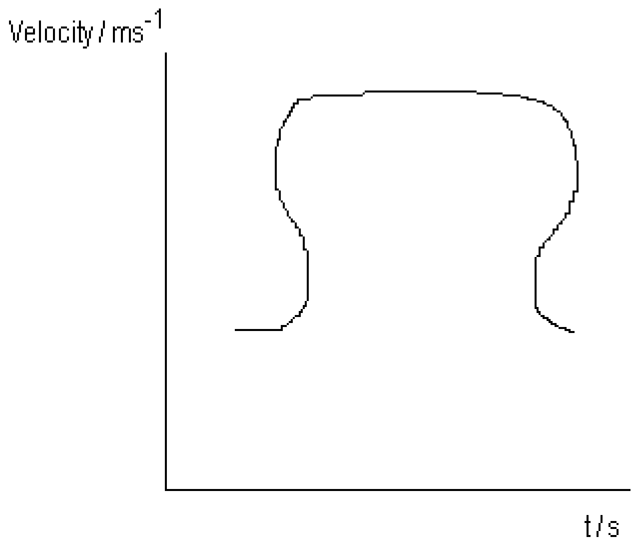
(c)

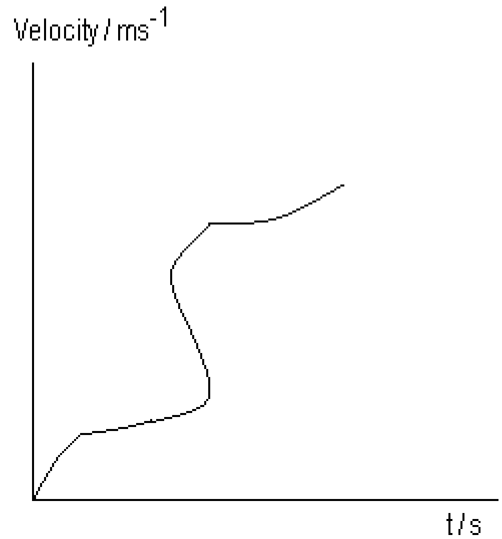


- Gradient = $\frac{AB}{BC} = \frac{V_2 - V_1}{t_2 - t_1}$

  = $\frac{Change\ in\ velocity}{Time}$
- The case just given is retardation since the gradient is negative.

(d)

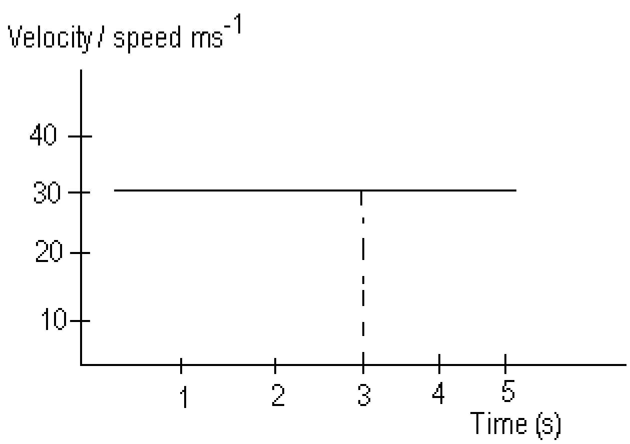




- The two given cases are the velocity – time graphs of a body, moving with non uniform acceleration.

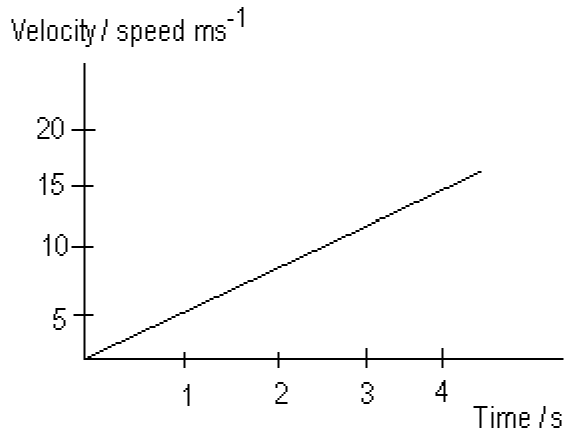
Summary:

(1)



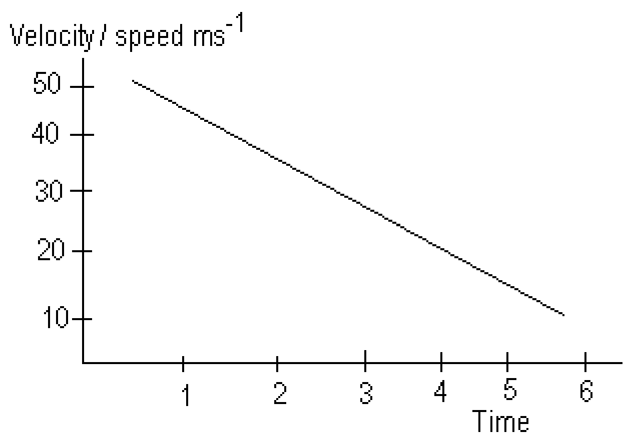
The velocity – time graph of a body moving with a constant velocity, and as such zero acceleration.

(2)



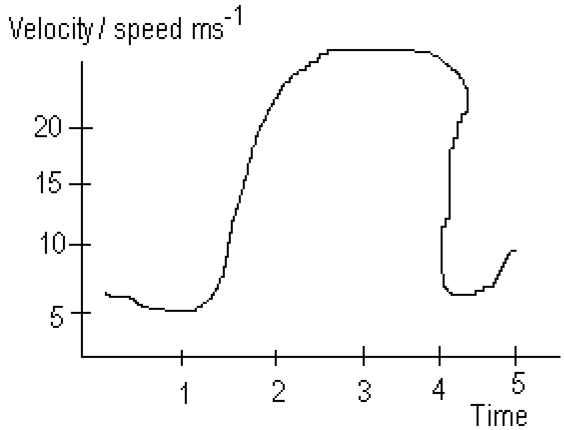
The velocity - time graph of a body moving with constant or uniform acceleration.

(3)



The velocity – time graph of a body moving with constant retardation.

(4)

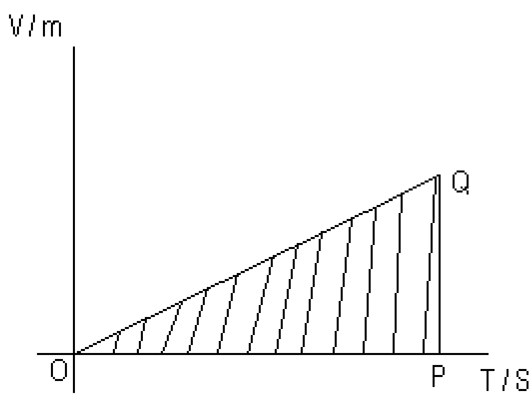


The velocity – time graph of a body moving with a non uniform acceleration

The distance travelled:

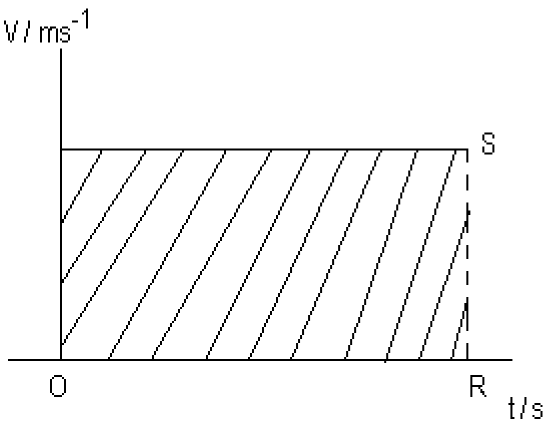
- The distance covered by an object can be obtained from its velocity – time graph.
- In this case, the area under the graph gives us the distance travelled by the object.

(a)



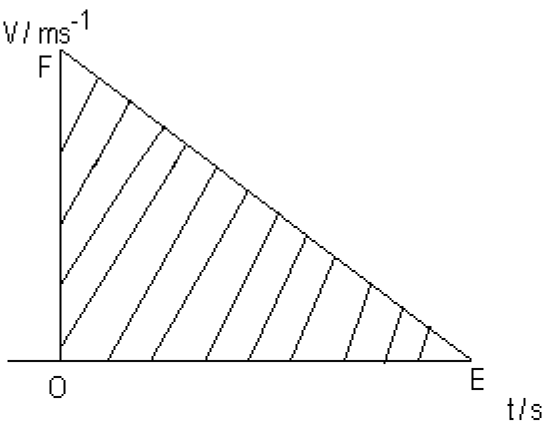
- This distance travelled = the area of the triangle = $\frac{base \times height}{2} = \frac{OP \times PQ}{2}$

(b)



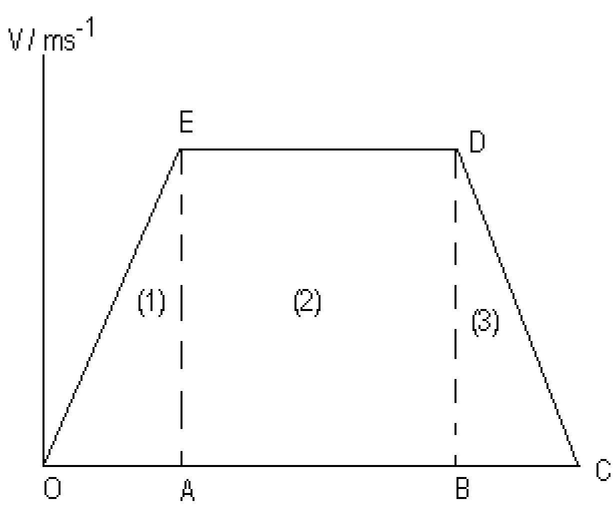
- The distance travelled = area under the rectangle = length × breadth = OR × RS.

(c)



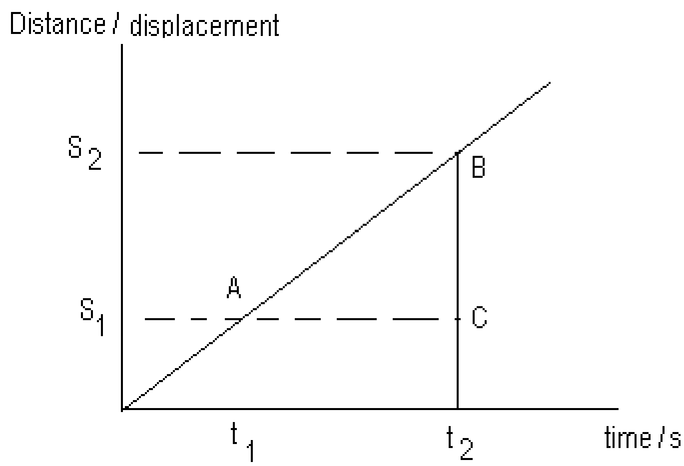
For the above graph of retardation, the distance covered = the area under the triangle = $\frac{b \times h}{2} = \frac{OE \times OF}{2}$

(d)



- For this velocity – time graph, the distance travelled = Area of figure (1) + Area Of figure (2) + Area of figure (3) or distance travelled = the area under the graph or trapezium OEDC = $\frac{1}{2}(OC + DE) \times AE$.

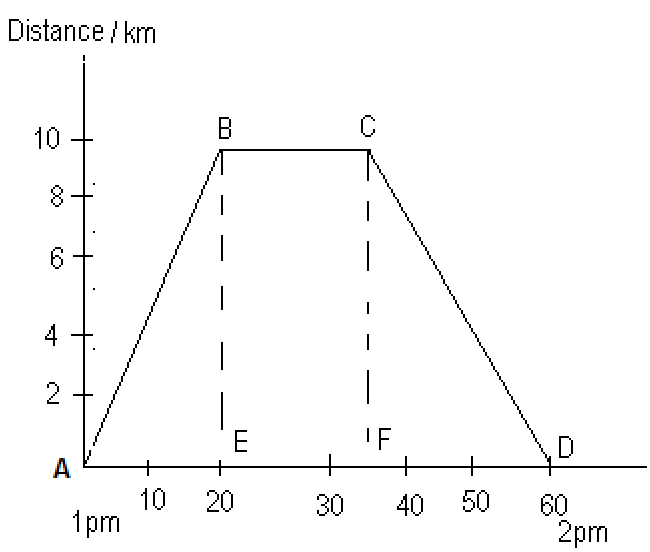
The distance – time graph:



- The gradient = the speed or the velocity.
- The speed or velocity = $\frac{BC}{AC} = \frac{S_2 - S_1}{t_2 - t_1}$

(Q8) A man starts a journey from his home at 1 pm and travels at a uniform speed. After 20 minutes, he had covered a distance of 10km and the then rests for 15 minutes. He then returns to his home arriving at 2. 00 pm. Calculate the velocities at the various stages of the journey, using a displacement – time graph.

Soln:



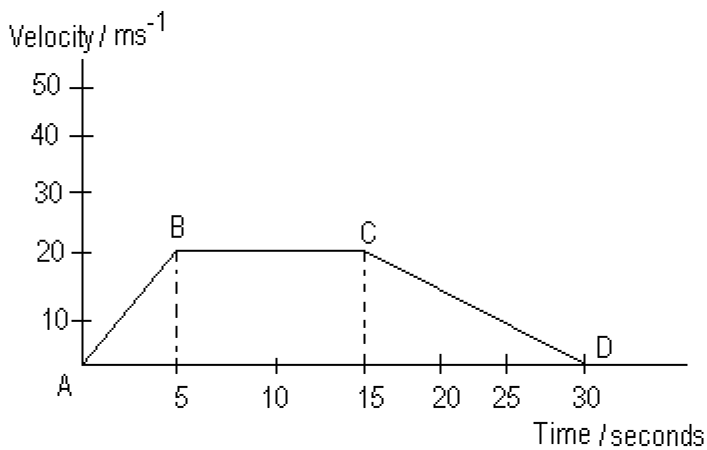
Velocity for the outward journey = $\frac{displacement\ BE}{Time\ AE}$

= $\frac{10}{20} = 0.5km/m$

= 30 km/h.

The velocity for the return journey = $\frac{CF}{FD} = \frac{10}{25} = 0.4km/\min = 24km/h.$

(Q9) A motorist started from rest and acquired a velocity of 20m/s in 5 seconds. He kept this constant for 10 seconds and finally drove to rest in 15 seconds. Represent this on a velocity – time graph.



N/B:

The acceleration = the gradient of line AB, and the retardation = the gradient of line CD.

(Q10) A cyclist started from rest and accelerated for 5 seconds at 6m/s^2. He then kept this speed constant for 10 seconds. This speed was then reduced to 15m/s within 5 seconds and after driving at this speed for 10 seconds, he finally drove to rest in 5 seconds.

(a) Represent this on a velocity time graph.
(b) Using your graph determine the total distance travelled.

N/B:

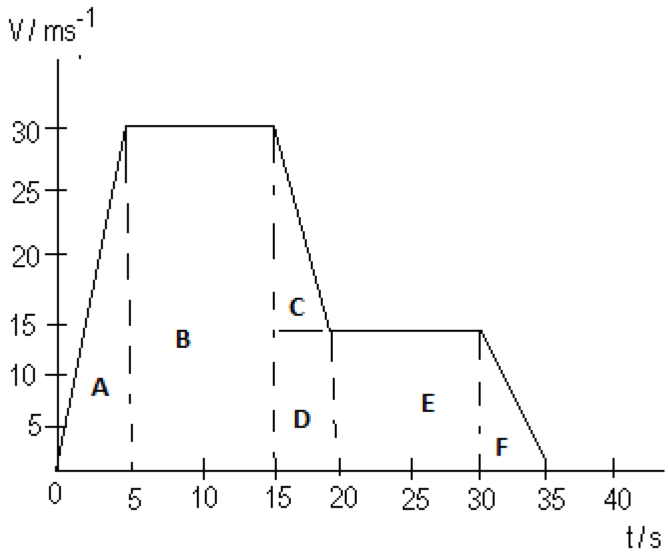
- The cyclist started from rest (i.e. when the speed = 0) and accelerated at 6m/s^2 for 5 seconds.

=>The increase in speed acquired by the cyclist in the first 5 seconds = 6 $\times$ 5 = 30m/s.

Then speed of the cyclist = 0 + 30 = 30m/s.

=> Within the first 5 seconds, the speed of the cyclist will increase to 30m/s.

(a)



(c) The total distance travelled = area of figure A + area of figure B + area of figure D + area of figure E + area of figure F + area of figure C.

Area of figure A = $\frac{b \times h}{2}$

= $\frac{5\times30}{2} = 75\text{m}^2$

Area of figure B = L $\times$ B

= 10 $\times$ 30 = 300m^2

Area of figure C = $\frac{b \times h}{2}$

= $\frac{5\times15}{2} = 37.5\text{m}^2$

Area of figure D = L $\times$ B =15 $\times$ 5 = 75m^2

Area of figure E = L $\times$ B

= 15$\times$ 10 = 150m^2

Area of figure F = $\frac{b \times h}{2}$

= $\frac{5\times15}{2} = 37.5\text{m}^2$

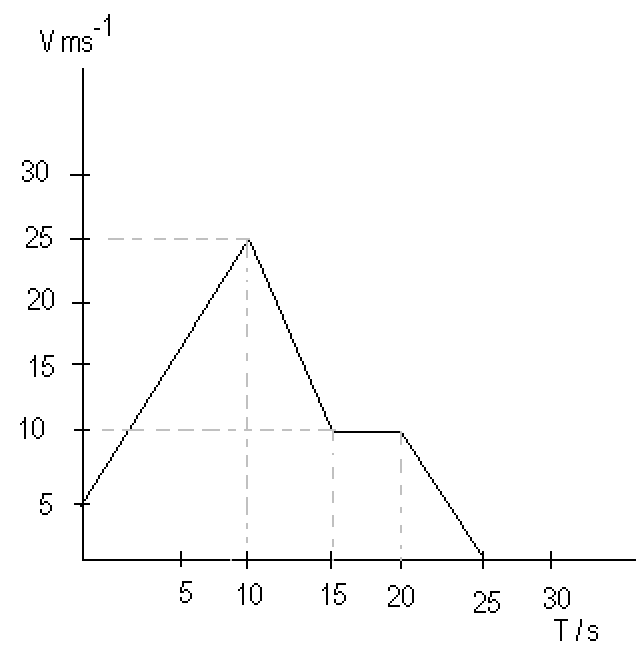
Total distance travelled = 75 + 300 + 37.5 + 150 + 37.5 = 601m.

(Q11) A cyclist who was moving at an initial velocity of 5m/s, accelerated at 2m/s for 10 seconds. He then decelerated at 3m/s for the next 5 seconds and rode at this speed for 5 seconds. He finally rode to rest in 5 seconds. Represent this on a velocity-time graph.

N/B:

- The initial velocity of the cyclist = 5m/s.
- Since he accelerated at 2m/s for 10 seconds, => the speed increase = 2 $\times$ 10 = 20m/s.
  - ⇨ The speed acquired by the cyclist within the first 10 minutes = the initial speed + the increase in speed = 5 + 20 = 25m/s.
- From this velocity, he decelerated for the next 5 seconds at 3m/s => the decrease in velocity within this time period = 3 $\times$ 5 = 15m/s.
  - ⇨ The velocity of the cyclist at the end of this 3 seconds = 25 – 15 = 10m/s.

Soln:

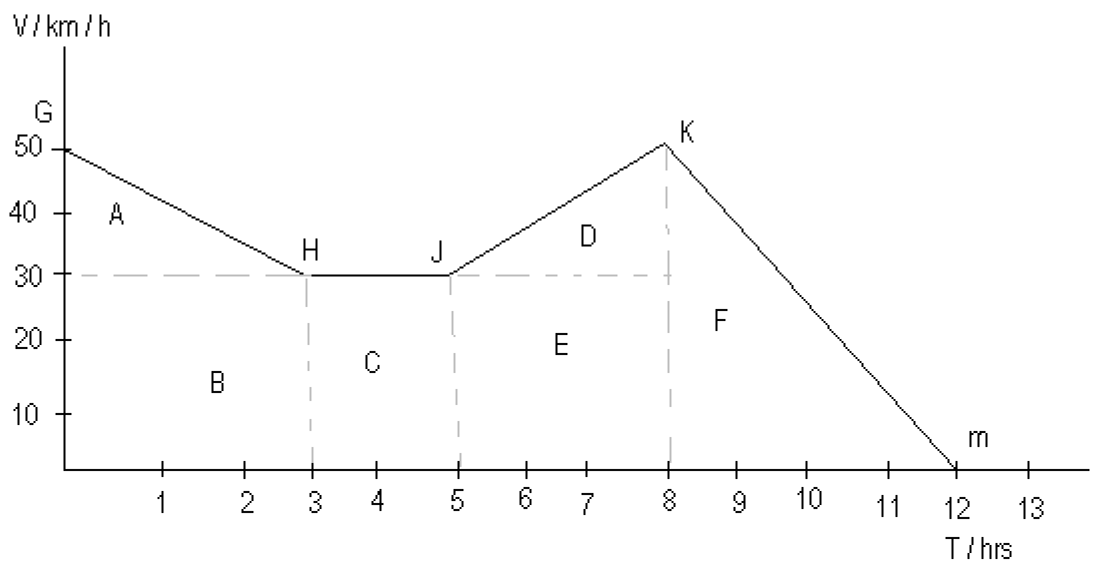


(Q12) A motorist riding at a speed of 50km/h, subjected his vehicle to retardation such that in 3 hours time, his velocity has decreased to 30km/h. He travelled at this speed for 2 hours, before increasing it to 50km/h within 3 hours. He finally drove to rest in 4 hours time.

(a) Represent this on a velocity-time graph.
(b) Using your graph, determine
 (i) his retardation during the first part of his journey in km/h.
 (ii) his retardation during the last stage of journey.
 (iii) his total distance travelled in km/h.

Soln:

(a)



(b) (i) The retardation in km/h during the first part of the journey = the gradient of the line GH = $\frac{50-30}{3-0} = \frac{20}{3} = 6.6.$

= -6.6 since from observation, the line has a negative slope. Retardation = 6.6km/h.

(ii) The retardation during the last stage of the journey = the gradient of line km = $\frac{50-0}{12-8} = 12.5 = -12.5.$

=> Retardation = 12.5km/h.

(iii) The total distance travelled in km/h = the area under the graph.

Area of figure A = $\frac{b \times h}{2} = \frac{3 \times 20}{2}$ = 30km^2.

Area of figure B = L $\times$ B

= 30×3 = 90km^2.

Area of figure C = L $\times$ B

$= 30 \times 2$ = 60km^2.

Area of figure D = $\frac{b \times h}{2} = \frac{3 \times 20}{2}$ = 30km^2.

Area of figure E = L $\times$ B

= 30×3 = 90km^2.

Area of figure F = $\frac{b \times h}{2} = \frac{4 \times 50}{2}$

= 100km^2.

Distance travelled = 30 + 90 + 60 + 30 + 90 + 100 = 400km.

(Q13) A train starts a journey from station A with an acceleration of 0.2m/s^2, and attains the maximum speed in 1.5 minutes. After continuing for 4 minutes, it is uniformly retarded for 60 seconds before coming to rest in station B. Find by drawing a suitable graph

(i) the maximum speed in km/h.
(ii) the distance between A and B in kilometres.

N/B: - Since the train started from rest, then its initial velocity = U = 0.

Its final velocity = the maximum speed = V = ?.

Time = t = 1.5 minutes = 90 seconds .

Acceleration = a = 0.2m/s^2.

But since V = U + at

=> V = 0 + (0.2) (90) = 0 + 18 = 18,

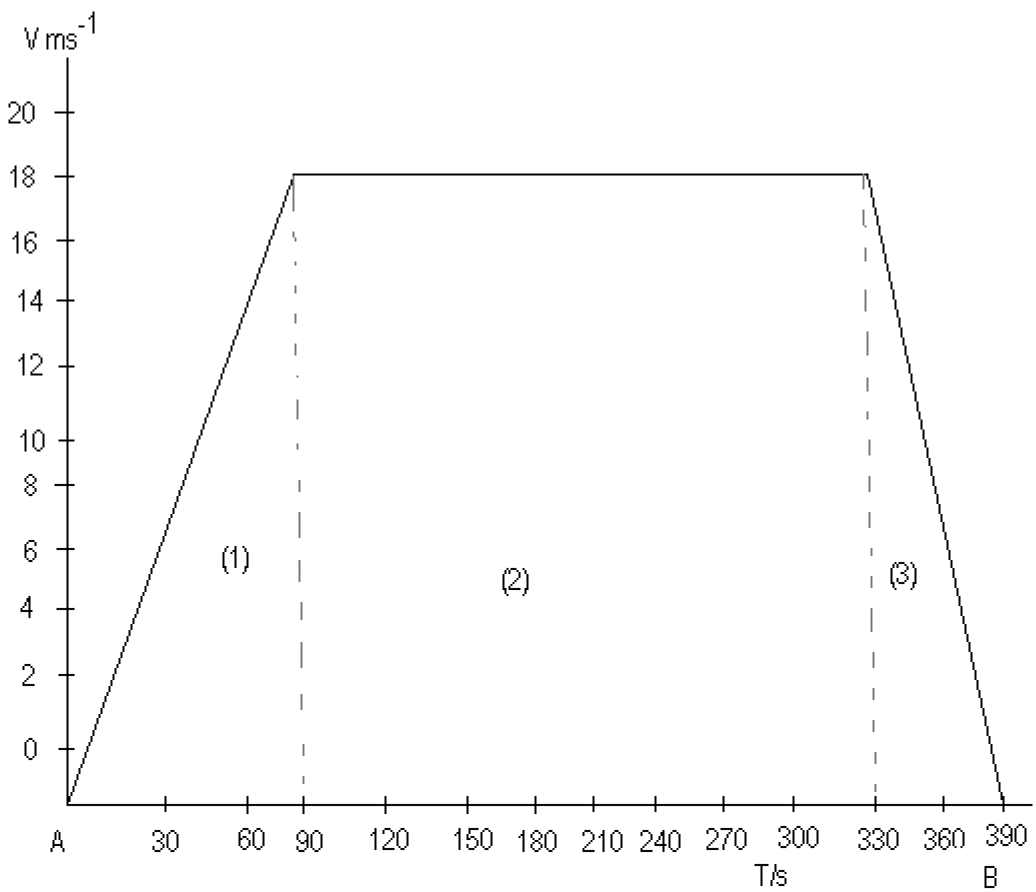
=> V = 18 => the train attained a maximum speed of 18m/s in 90 seconds.

It then travelled at this speed for a time period of 4 minutes or 240seconds. This implies that, from 90 seconds to 330 seconds i.e. (90 seconds + 240 seconds), the train travelled at a constant speed of 18m/s.

- Finally, within a time interval of 60 second, it was brought to rest.

(b) As already stated, the train attained a maximum speed of 18m/s in 90 seconds. But 18m = 0.018km. Speed of train = 0.018km/s. If 1 second = 0.018km, then 60 seconds

$= \frac{60}{1} \times 0.018 = 1.08km/h.$



The distance between A and B = the area under the graph = area of figure (1) + area of figure (2) + area of figure (3).

Area of figure (1) = $\frac{b \times h}{2} = \frac{90 \times 18}{2}$ = 810m^2.

Area of figure (2) = $L \times B$

= 240 × 18 = 4320m^2

Area of figure (3) = $\frac{b \times h}{2}$ = $\frac{60 \times 18}{2}$ = 540m^2

Total distance travelled = 810 + 4320 + 540 = 5670m = 5.7km.

(Q14) The speed – time relationship for a motor car on a short test is given as below

Speed (km/h)	0	32.5	65	65	45.5	26	6.5	0
Time/s	0	3	6	12	15	18	21	22

Plot the speed – time graph and use it to find the

(i) acceleration in m/s^2.
(ii) distance covered in km.

N/B: Since the speed is given in km/h and the time in seconds, convert the speed in km/h into m/s.

For example: consider the second speed of 32.5km/h

32.5km/h = 32.5km/1h.

32.5km = 32.5 $\times 1000 = 32500m$.

1 hour = 60 × 60 = 3600 seconds.

=> 32.5km/h = 32500m/3600 seconds.

Now 3600 seconds = 32500m

⇨ 1 second = $\frac{1}{3600} \times 32500$
= 9m

A speed of 32.5km/h = a speed of 9m/s.

Consider the speed of 65km/h.

65km = 65 × 1000 = 65000m.

1 hour = 60 × 60 = 3600seconds

Now if 3600 seconds = 65ooom, then 1 second = $\frac{1}{3600} \times 65000$

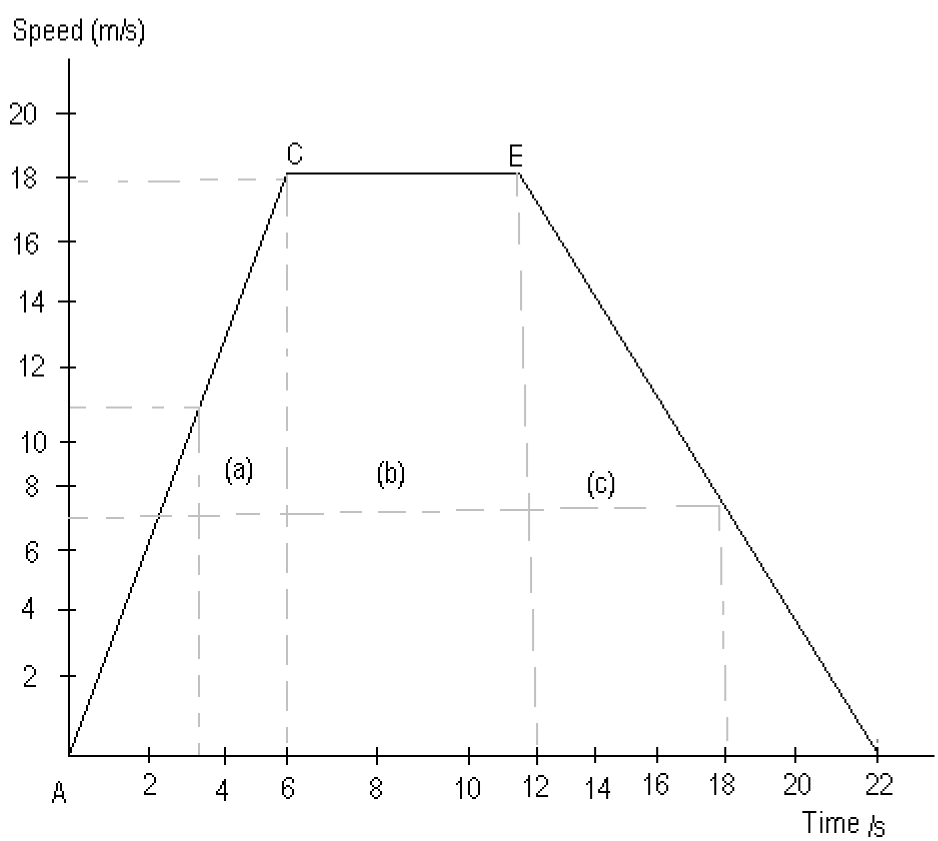
= 18m

=> A speed of 65km/h = 18m/s.

Soln:

By means of this conversion, the given table becomes as shown next.:

Speed (m/s)	0.0	9.0	18	18	12.6	7.2	1.8	0.0
Time/s	0	3	6	12	15	18	21	22



The acceleration = the slope of line AC = $\frac{18-0}{6-0} = 3ms^{-2}$

(b) The total distance covered = the area under the graph = 353.4m = $\frac{353.4}{1000} = 0.3534km$.

N/B: Kilometres goes with hours and metres goes with seconds.

(Q15) A body which starts from rest and slides down an inclined air – track covers the following distances x in time t.

x/cm	0	12.8	20.0	28.8	39.2	51.2	64.8
t/s	0	0.8	1.0	1.2	1.4	1.6	1.8

Show by the graphical method that the body moves with uniform acceleration, and find its value in m/s^2.

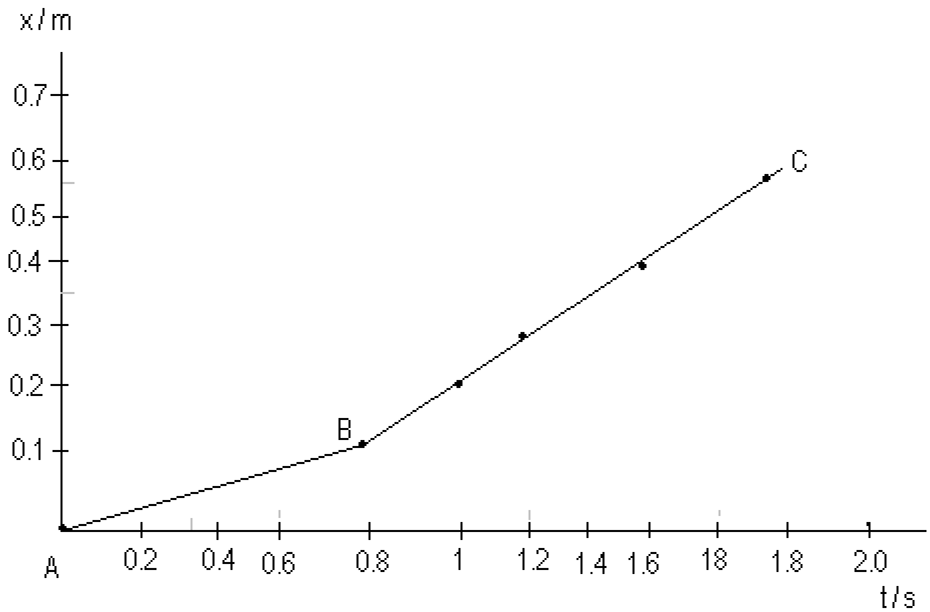
N/B: Convert the values of x given in centimetres into metres, by dividing by 100. By means of this conversion, the given table becomes as shown next.

Soln:

x/m	0	0.13	0.2	0.3	0.4	0.6
t/s	0	0.8	1.0	1.2	1.6	1.8

N/B: - Some of the values have been approximated.

We next plot a graph of x against t.

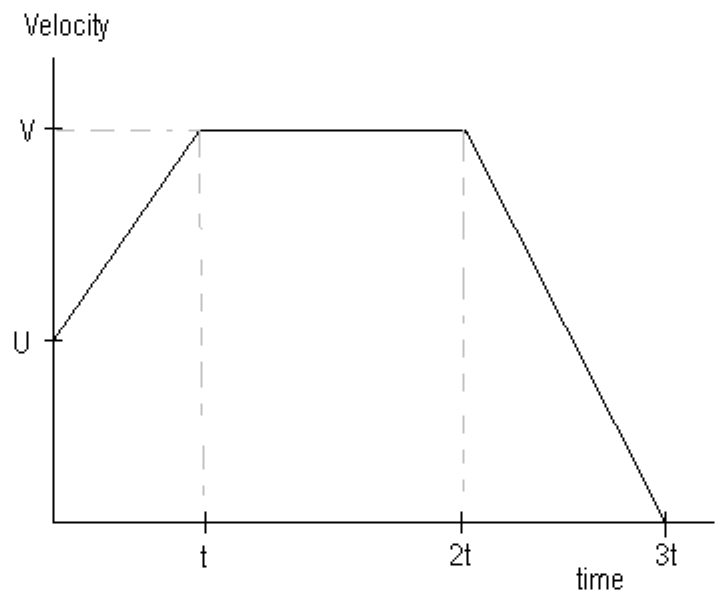


Since the line had after plotting distance against time is almost a straight line, then the body moves with almost a uniform velocity.

- The acceleration = the gradient of line BC.

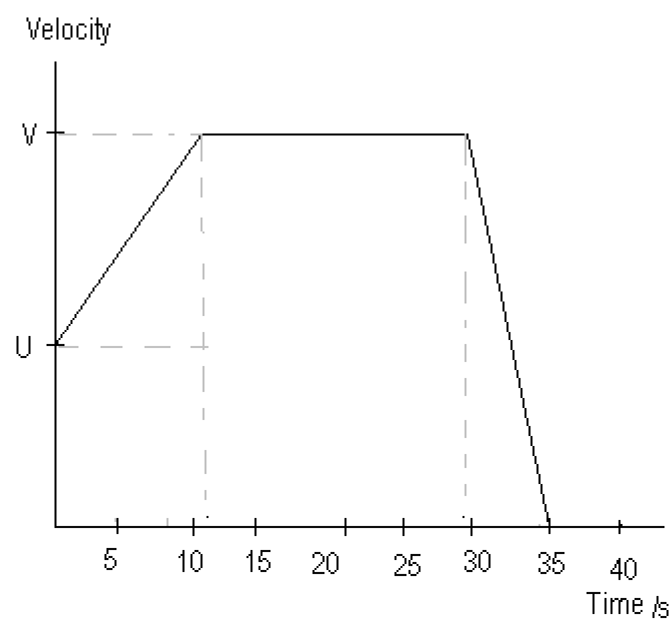
(Q16) A motorist starts from rest with an initial velocity U, and accelerated uniformly for time t until the velocity becomes V. It continues with this velocity for a further time t, and then decelerated uniformly to rest within time t. Represent this on an appropriate graph.

Soln:



(Q17) The initial velocity of a lorry was Um/s, but this was increased to Vm/s in 10 seconds. It then travelled at this speed for the next 20 seconds, before coming to rest in 5 seconds time. Represent this on a velocity – time graph.

Soln:

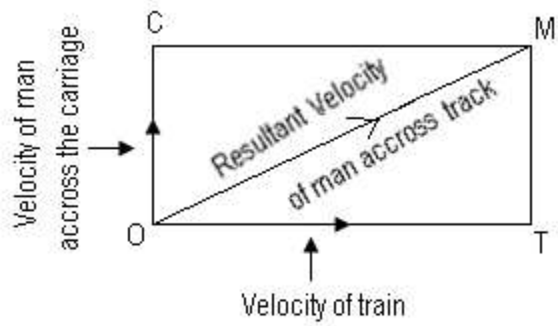


Relative Velocity:

- The velocity of a body A relative to a body B, is obtained by combining the velocity of A with the velocity of B, which has been reversed.

- If the velocities of A and B are in the same straight line, then the relative velocity becomes a matter of simple addition or subtraction.
- For example, if a car A which is travelling at 50km/h is moving in the same direction as another car B, which is travelling at 60km/h, then the relative velocity of B to A = 60 – 50 = 10km/h.
- If however, the cars are travelling in opposite directions, then the relative velocity of B to A = 60 – (-50) = 60 + 50 = 110km/h.
- But if the velocities of A and B are not in the same straight line, then the parallelogram of velocities must be applied.

The Parallelogram of Velocities:



- If a body has two component velocities which are represented by the two adjacent sides of a parallelogram, then the resultant velocity is represented by the diagonal of the parallelogram, drawn from the intersection of the two sides.

Newton`s laws of motion:

Newton`s first law of motion:

- This states that everybody or object continues in its state of rest or uniform motion in a straight line, unless acted upon by an external force.
- Any object or body trends to resist any change in its state of rest, or a change in its state of motion.

- This resistance to change in its state of rest or motion is referred to as the inertia of the body.

- There are two types of inertia and these are:

(I) Inertia of rest.

(II) Inertia of motion.

Examples of inertia of rest:

(1) A passenger in a bus is jerked backward if the bus suddenly moves forward.

- This is due to the fact that the feet of the passenger is in contact with the floor of the bus, and as such forced to move along with the bus, while the upper part of the body tends to maintain its original state of rest or its original position.
- The upper part therefore gets left behind causing the passenger to be jerked backward.

(2) – A coin resting on a cardboard placed at the mouth of a bottle or flask, falls down into the flask or bottle if the cardboard is suddenly flicked away from underneath.

- This is due to the fact that the coin in maintaining its state of rest falls, because it is no longer being supported.

Examples of inertia of motion:

(1) –A passenger moves forward when there is a sudden decrease in speed.

- This is due to the fact that as the feet moves with the floor of the bus at a slower speed, his upper part of the body continues to move at its original faster speed.

(2) – Another example is that a passenger standing in a bus is jerked backwards if the bus suddenly starts moving.

- This is due to the fact that as the feet moves along with the bus, his upper part of his body tends to maintain its original position.
- Newton`s first law is also referred to as the law of inertia.

Momentum:

- The momentum of a body is defined as the product of its mass and its velocity.
- Momentum = Mass $\times Velocity$.
- Its S.I unit is $kgms^{-1}$ and it is a vector quantity.

Newton`s second law of motion:

- This states that the rate of change of momentum of a body is proportional to the applied force, and takes place in the direction of the force.

The relationship between force, mass and acceleration:

- Newton`s second law of motion enables us to find an absolute unit force, which remains constant under all conditions.
-Suppose a force F acted on a body of mass m for a time t, and caused its velocity to change from U to V, then the momentum changed from mu to mv in time t.

-The rate of change of momentum = $\frac{mv-mu}{t}$.

- But from the 2nd law, the rate of change of momentum is proportional to the applied force => F $\propto$ $\frac{mv-mu}{t}$.

=> F$\propto$ $\frac{m(v-u)}{t}$.

But $\frac{v-u}{t} = \frac{change\ in\ velocity}{time}$ = acceleration= a.

From F$\propto$ $\frac{m(v-u)}{t}$ => F$\propto$ ma,

=> F = kma, where k is constant.
- If m = 1kg, a = 1ms^{-2} and the constant k is taken as 1, then F = 1N.

- The Newton is the S.I unit of force, and is defined as the force which acts on 1kg mass of body, to produce an acceleration of 1ms^{-2}.

- Therefore when F is in Newtons, m in kg and acceleration (a)s in ms^{-2}, then F = ma.

Newton`s third law:

This states that for every action force acting on a body, there is an equal and opposite reaction force acting. When you push on a heavy object, the object also pushes on you back. Your push is called the action force, and the push of the object is called the reaction force. These two forces are equal and act in the opposite direction.

Impulse:

- This is the product of the force and the time for which the force acts.-
- Impulse = F$\times\Delta$T, where F = force and Δt = the time duration for which the force acts.-

The greater and longer a force acts on a body, the greater the change in momentum it will produce.

(Q1) Find the impulse of a force which acts on a body of 2kg mass, to change its velocity from $10ms^{-1}$ in one direction to $25ms^{-1}$ in the opposite direction.

N/B: - Impulse = F t, and since F t = change in momentum = > impulse = change in momentum.

Soln:

Let the initial momentum`s direction be negative.

Let the final momentum`s direction be positive.

Change in momentum = impulse = $(25 \times 2) - (-10 \times 2) = 50 + 20 = 70$N/S.

(Q2) Find the impulse of a force which acts on a 2kg mass, and changes its velocity from $10ms^{-1}$ to $25ms^{-1}$ in the same direction.

Soln:

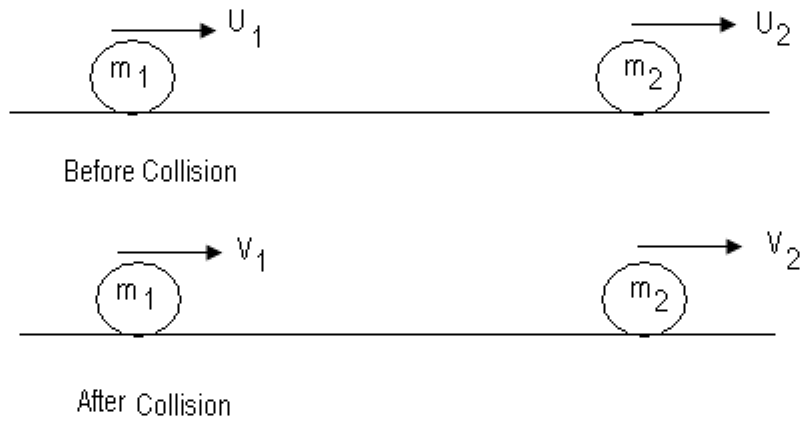
Initial momentum = 2 x 10 = 20

Final momentum = 2 x 25 = 50

Change in momentum = impulse = (2 x 25) – (2 x 10) = 50 – 20 = 30N/S.

The law of the conservation of momentum:

- This law states that momentum can neither be created nor be destroyed.
- For example when two particles in motion collide, the sum of their initial momentum before collision will be equal to the sum of their final momentum.
- The change of momentum will therefore be zero.



m_1 = mass of the first sphere.

m_2 = mass of the second sphere.

U_1 =Initial velocity of m_1.

V_1 = Final Velocity of m_1.

U_2 = Initial Velocity of m_2.

V_2 = final velocity of m_2.

- Total momentum before collision = $m_1u_1 + m_2u_2$.
- Total moment after collision = $m_1v_1 + m_2v_2$.
- From the law of conservation of momentum, the total initial momentum = the total final momentum => $m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$.

(Q2) A body of mass 5g moving with a velocity of $2ms^{-1}$ collides with another body of mass 4g moving with a velocity of $3ms^{-1}$ in the same direction. After collision, both bodies continue to move in their original directions, but that of 5g mass moves with a velocity of $1ms^{-1}$. Calculate the velocity of the other body.

Soln:

Let m_1 = 5g = 0.005kg,

$U_1 = 2ms^{-1}$, $V_1 = 1ms^{-1}$,

M_2 = 4g = 0.004kg,

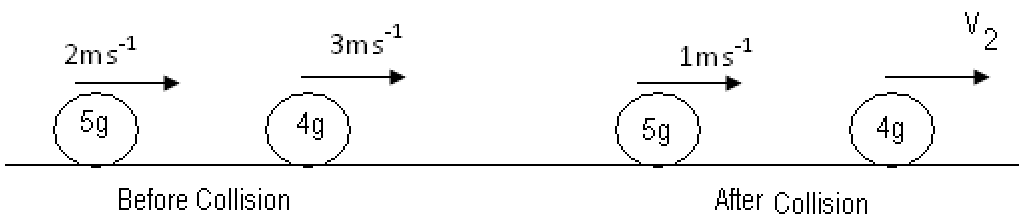
$U_2 = 3ms^{-1}$, V_2=?

Where U_1 = initial velocity of the 5g mass.

V_1 = final velocity of the 5g mass.

U_2 = Initial velocity of the 4g body.

V_2 = Final velocity of the 4g body.



From the law of conservation of momentum,

$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

=> $0.005 \times 2 + 0.004 \times 3 = 0.005 \times 1 + 0.004 \times V_2,$

=> $0.010 + 0.012 = 0.005 + 0.004V_2,$

=> $0.022 = 0.005 + 0.004V_2,$

=> $0.022 - 0.005 = 0.004V_2,$

=> $0.004V_2 = 0.017,$

=> $V_2 = \frac{0.017}{0.004} = 4.3,$

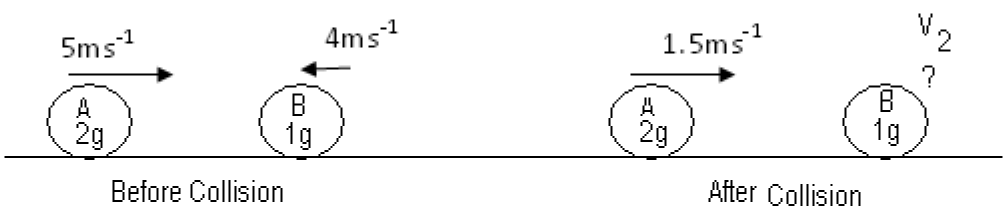
=> V_2 = 4.3m/s.

(Q3) Two spheres A and B of masses 2g and 1 g respectively move with velocities $5ms^{-1}$ and $4ms^{-1}$ respectively in opposite direction. After collision, sphere A maintained its original direction.

(a) If sphere A then acquired a new velocity of $1.5ms^{-1}$, calculate the velocity of sphere B.

(b) Determine also the direction in which it moves.

Soln:



Let all quantities in the left hand side direction be negative, and let those to the right hand side be positive.

$m_1 = 2g = 0.002kg$.

$U_1 = 5ms^{-1}$.

$m_2 = 1g = 0.001kg$

$U_2 = 4ms^{-1} = -4ms^{-1}$, since it is moving to the left hand side or the negative side direction.

$V_1 = 1.5ms^{-1}$, $V_2 = ?$

(a) From the law of conservation of momentum,

$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

=> $0.002(5) + 0.001(-4) = 0.002(1.5) + 0.001V_2$.

=> $0.0010 - 0.004 = 0.003 + 0.001V_2$

Where V_2 = the final velocity of sphere B.

=> $-0.003 = 0.003 + 0.001V_2$

=> $-0.003 - 0.003 = 0.001V_2$,

=> $-0.006 = 0.001V_2$

=> $V_2 = \frac{-0.006}{0.001} = -6ms^{-1}$.

(b) The negative sign implies that after collision, sphere B moved in the left hand side direction.

(Q4) A body of mass 8g moving with a velocity of $10ms^{-1}$ collided with a stationary body of mass 4g. After the collision, the movement of these two bodies was in the same direction as the original direction of the 8g mass. (a) If the final velocity of the 8g mass was $4ms^{-1}$, determine the final velocity of the other body.

(b) If after collision the two bodies were to have coalased or joined together, determine their common velocity.

N/B: The momentum of a stationary body i.e. a body which is not in motion is zero.

M_1 = 8g = 0.008kg.

M_2 = 4g = 0.004kg.

U_1 = 10m/s. U_2 = 0m/s.

V_1 = 4m/s and V_2 =?

Initial momentum = $m_1u_1 + m_2u_2$ = 0.008(10) + 0.004(0)

Final momentum = $m_1v_1 + m_2v_2$ = 0.008(4) + 0.004V_2

From the law of conservation of momentum, initial momentum = the final momentum,

i.e. $m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

=> 0.008(10) + 0.004(0) = 0.008(4) + 0.004V_2.

=> 0.08 + 0 = 0.0032 + 0.004V_2,

=> - 0.004V_2 = 0.0032 – 0.08

=> $V_2 = \frac{-0.0768}{-0.004} = 19\text{m/s}$

(b) If the two bodies joined together after collision, their total mass = 8 + 4 = 12g = 0.012kg.

Let their common velocity of the two joined bodies = V_2.

Then the final momentum = $0.012 \times V_2$

The initial momentum = $0.008 \times 10 + 0.004 \times 0 = 0.08$

From the principle of momentum conservation, initial momentum = final momentum.

=> $0.08 = 12V_2$ => $V_2 = \frac{0.08}{12}$

=> $V_2 = 0.0067ms^{-1}$

(Q5) Two particles M and N of masses 4g and 6g respectively collided. Before collision, M was moving with a velocity of 8m/s and N, 10m/s in opposite direction. If their momentum is preserved after collision, determine their common velocity after impact.

Soln:

Let all quantities in the direction of M, or to the right hand side be positive. Let all quantities in the direction of N, or to the left hand side be negative.

<u>Before collision:</u>

Then the momentum of M before collision = $0.004 \times 8 = 0.0032kgms^{-1}$.

The momentum of N before collision = $0.006 \times (-10)$

$= -0.06kgms^{-1}$.

=> The total momentum before collision = 0.0032 + (- 0.06)

$= 0.0032 – 0.06 = -0.057kgs^{-1}$.

<u>After collision:</u>

Since the two bodies are said to have a common velocity, => they were joined together after collision.

Let V = their common velocity after collision. Then their total momentum after collision or impact = (0.004 + 0.006) V= 0.01V.

By the principle of momentum conservation, sum of momentum before impact = sum of momentum after impact,

=> - 0.057 = 0.01V

=> $V = \frac{-0.057}{0.01} = -5.7m/s.$

Since there is a negative sign => the direction of their common velocity is to the left.

In short, after impact, the two bodies moved in the left hand side direction.

N/B: If two bodies travel towards each other => they are travelling in opposite direction.

(Q7) Two bodies of masses 80kg and 100kg travel towards each other with velocities of $30ms^{-1}$ and $20ms^{-1}$ respectively. If they joined to form one body, determine,

(a) their common velocity.

(b) the direction of this velocity.

Soln:

Let the direction of the 80kg mass be positive, and let that of the 100kg mass be negative.

<u>Before collision:</u>

M_1 = 80kg.

$U_1 = 30ms^{-1}$.

M_2 = 100kg.

$U_2 = 20ms^{-1} = -20ms^{-1}$, since the direction of the 100kg mass is to be taken as negative.

Total momentum before collision = $m_1u_1 + m_2u_2$

$= (80 \times 30) + (100 \times -20)$

= (2400) + (-2000)

= 2400 – 2000 = 400.

After collision:

If V_2 = their common velocity after impact, then their final momentum = (100 + 80) V_2

= $180V_2$.

But since the initial momentum = final momentum,

=> 400 = $180V_2$.

=> $V_2 = \frac{400}{180} = 2.2$. => V_2 = 2.2 m/s.

(b) Since V_2 has a positive value, then it has the same direction as the 80kg mass.

(Q8) A body of mass 500g moving with a velocity of 3m/s, had an head on collision with another body of mass 100g, moving with a velocity of 40m/s. if the two bodies coalase after impact, determine

(a) their common velocity.

(b) the direction of this velocity.

(c) Hint:

N/B: If head on collision took place, then the two bodies were moving towards each other, or in opposite direction.

(Q9) An arrow of mass 100g and velocity 15m/s was shot into a block of mass 400g, resting on a table. If after the collision, the two bodies joined together, find their common velocity.

Soln:

M_1 = 100g = 0.1kg.

U_1 = 15m/s.

M_2 = 400g = 0.4kg.

U_2 = 0.

Initial momentum = $m_1u_1 + m_2u_2$

= $0.1 \times 15 + 0.4 \times 0 = 1.5$.

Let their common velocity after impact = V, => the final momentum = (0.1 + 0.4) V = 0.5V.

But since initial momentum = final momentum, then 1.5 = 0.5V => V = $\frac{1.5}{0.5} = 3ms^{-1}$.

(Q10) A bullet of mass 5g is fired into a block of mass 1kg, resting on a horizontal frictionless floor.

(a) What was the initial speed of the bullet, if it gets embedded inside the block and the velocity of the block after impact was $40ms^{-1}$.

(b) What will be the velocity of the block after impact, if the block was initially moving at $5ms^{-1}$in opposite direction to that of the bullet.

Soln:

(a)

Initial momentum before collision:

M_1 = mass of the bullet = 5g = 0.005kg.

U_1 = initial velocity of the bullet =?

M_2 = mass of the block = 1kg.

U_2 = the initial velocity of the block = 0m/s, since it was stationary.

Initial momentum= $(0.005 \times U_1) + (1 \times 0) = 0.005U_1$.

Final momentum after collision:

Since the bullet got embedded or stuck inside the block, then the bullet and the block will move with the same common velocity of $40ms^{-1}$.

The final momentum = $(1 + 0.005)40 = 1.005 \times 40$

But since initial momentum = final momentum, then $0.005U_1 = 1.005 \times 40$,

=> $U_1 = \frac{1.005 \times 40}{0.005} = 0.008m/s$

(b)

Let movement or quantities to the right hand side be positive and let that to the left hand side be negative.

Then in this case,

M_1 = 0.005kg.

U_1 = 0.008m/s.

M_2 = 1kg

U_2 = 5m/s = -5m/s, since movement is to the left.

Initial momentum = $m_1u_1 + m_2u_2$

= $(0.005 \times 0.008) + (1 \times -5) = -5$

Let V_2 = the common velocity in this case, when the two bodies joined together r=> the final momentum = $(1 + 0.005)V_2 = 1.005V_2$.

Since initial momentum = final momentum, then $1.005V_2 = -5$,

=> $V_2 = \frac{-5}{1.005}$ = - 4.9m/s.

<u>Elastic and inelastic collisions:</u>

- A perfectly elastic collision is the type of collision, in which both momentum and kinetic energy are conserved.
- This implies that for an elastic collision,

(a) $m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$

i.e. the total momentum before and after collision are equal.

(b) $\frac{1}{2}m_1u_1{}^2 + \frac{1}{2}m_2u_2{}^2 = \frac{1}{2}m_1v_1{}^2 + \frac{1}{2}m_2v_2{}^2$

i.e. the sum of the kinetic energy before collision = that after collision.

(c) $U_1 - U_2 = -(V_1 - V_2)$, which also implies that the relative velocity of the two bodies remains constant in magnitude but opposite in sign.

- A perfectly inelastic collision is the type of collision in which momentum is conserved but kinetic energy is not.
- This implies that for an inelastic collision,
  $m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$
- In this case, the bodies move with the same velocity after collision => $V_1 = V_2$.

Application of conservation of momentum:

- Such an application can be seen in the recoil of a gun.
- When a gun is fired, there is a backward movement of the gun which is referred to as its recoil.

Let M_g = mass of the gun.

V_g = velocity of the gun.

M_B = Mass of the bullet.

V_B = velocity of the bullet.

- Before the gun is fired, the total momentum of the gun and the bullet is zero, since both are stationary, => the initial momentum = 0.

The momentum of the gun and the bullet after firing occurs = $M_g V_g + M_B V_B$, => the final momentum = $M_g V_g + M_B V_B$.

- Since the final momentum = the initial momentum, then $0 = M_g V_g + M_B V_B$
  => $-M_g V_g = M_B V_B$.

- This implies that the momentum of the gun is equal, but opposite to that of the bullet.

- And for this reason, as the bullet leaves the gun, the gun recoils or gives a kick.

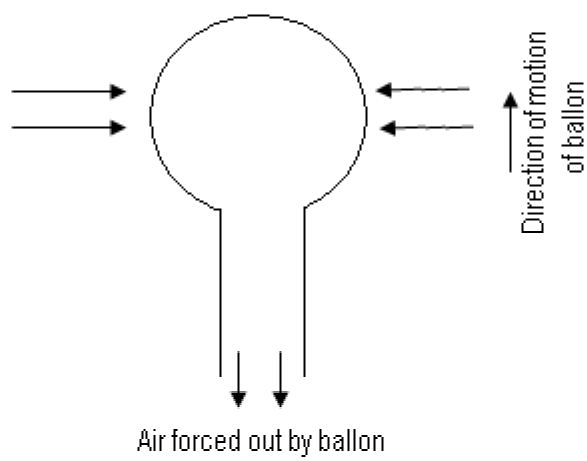
- From $-M_g V_g = M_B V_B$ =>

$-\frac{M_B}{M_g} = \frac{V_g}{V_B}$ or $\frac{M_g}{M_B} = \frac{V_B}{V_g}$.

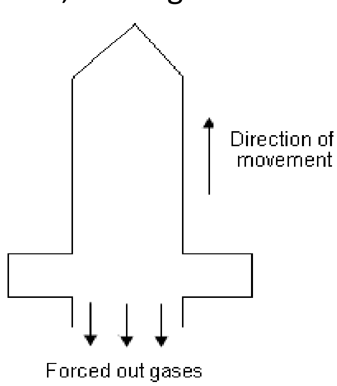
- Therefore if the mass of the bullet is very small compared to that of the guns, then the velocity of the bullet is very high and vice versa.

- If the mass of the bullet is large (say a canon ball), then the gun tends to give a violet kick.

Motion of rocket and jet planes:



- An inflating balloon is observed to move about, when the air it contains is allowed to escape.
- The deflating balloon forces air out through its small opening and as this air comes out, it exerts an equal and opposite force on the balloon causing it to move about.
- The movement of a rocket is based on this principle.
- In the rocket, the fuel is burnt into gases.
- These gases are then forced out at very high speed through a small opening called nozzle of the rocket, by the rocket`s engines.
- This forced out or ejected gases in turn exert an equal but opposite force on the rocket, causing its acceleration.



- The jet plane needs air to burn its fuel and as such, it cannot move higher than a certain level, but this is not so with a rocket, which can move to any height, since it carries its needed oxygen supply along.

(Q1) A bullet leaves a rifle with a velocity V_b = 900ms^{-1}. Find the recoil velocity of the rifle, if the mass of the rifle (M_r) is 500 times as large as the M_b of the bullet.

Soln:

The momentum of the rifle before the bullet was fired is zero.

=> $M_bV_b + M_rV_r = 0$,

=> $V_r = -\frac{(M_b)(V_b)}{M_r}$.

But $M_r = 500M_b$ => $V_r = \frac{(M_b)\,(V_b)}{500M_b}$

=> $V_r = \frac{V_b}{500}$.

=> $V_r = \frac{-(900)}{500} = -1.8m/s.$

The minus sign indicates that the direction of the recoil velocity of the rifle, is opposite to this direction of movement of the bullet from the gun.

In short, the gun recoils as the bullet leaves the gun.

<u>The equations of motion</u>:

With reference to these equations, notice must be taken of the following:

(a) u = the initial velocity.
(b) v = the final velocity.
(c) a = acceleration/ deceleration (retardation).
(d) t = time.
(e) s = distance.

The equations of motion are:

(1) $v = u \pm at$.

- If a = acceleration, then v = u + at, but if a = deceleration or retardation, then

$v = u - at$.

(2) $s = ut \pm \frac{1}{2}at^2$.

- If a = acceleration, then $s = ut + \frac{1}{2}at^2$, but if it is deceleration or retardation, then $s = ut - \frac{1}{2}at^2$

(3) $v^2 = u^2 \pm 2as$.

- If a = acceleration, then $v^2 = u^2 + 2as$, but if it is deceleration or retardation, then $v^2 = u^2 - 2as$.

(4) $s = vt$.

- This equation is only used when the velocity is uniform or constant i.e. when acceleration is zero.

(5) $S = \frac{1}{2}(u + v)\,t$ or $s = (\frac{u+V}{2})t$

- If v is greater than u, then the acceleration a is given by $a = \frac{v-u}{t} => t = \frac{v-u}{a}$.
- But if u is greater than v, then deceleration a is given as $a = \frac{u-v}{t} => t = \frac{u-v}{a}$.

(Q1) Find the final velocity of a car which moved from rest and accelerated at $6m/s^2$ for 2 seconds.

N/B: If a body starts from rest, then its initial velocity is zero i.e. u = 0.

Soln:

u = 0, t = 2seconds.

$a = 6m/s^2$ and v =?

v = u + at = 0 + (6)(2) = 0 + 12 = 12m/s.

(Q2) A car whose final velocity was 35m/s , had an initial velocity of 20m/s. If it accelerated for 3 seconds, determine its acceleration.

Soln:

v = 35m/s, u = 20m/s,

t = 3 seconds and a =?

From v = u + at => 35 = 20 + a (3),

=> 35 = 20 + 3a => 35 – 20 = 3a,

=> 3a = 15 => a = $^{15}/_{3}$ = 5.

=> The acceleration = 5m/s^2.

(Q3) A vehicle was moving at an initial speed of 50m/s. Because of the application of its brakes, there was a retardation of 3m/s^2 for 5 seconds. Calculate its final velocity.

N/B: Since retardation is negative acceleration, a negative sign must be placed in front of the 3m/s.

Soln:

a = retardation = - 3m/s^2, t = 5 seconds, u = 50, v =?

From v = u + at => v = 50 + (-3) (5), => v = 50 + (-15) => V = 35/s.

(Q4) After a retardation of 4m/s^2 for 5 seconds, the velocity of a particle became 20m/s. At what speed was it moving at first?

Soln:

V = 20m/s, u =?

a = - 4m/s^2, t = 5 seconds. Since v = u = + at,

=> 20 = u + (-4) (5), => 20 = u + (-20),

=> 20 = u – 20 => u = 20 + 20 = 40.

=> The original speed = 40m/s.

N/B: If a moving object comes to rest or it is brought to a stop, then its final velocity, v = 0.

(Q5) After the application of its brakes, a car finally came to rest after being subjected to a retardation of 25m/s^2 for 2 seconds. Calculate its initial velocity.

Soln:

Since the car came to a rest => v = 0.

a = -25m/s^2, t = 2 seconds, u =?

But v = u + at => 0 = u + (-25) (2),

=> 0 = u + (-50) => 0 = u – 50,

=> u = 50.

=> The initial velocity was 50m/s.

(Q6) A bike moving at a velocity of 40m/s, decelerated for 20 seconds before coming to rest. What was its retardation or deceleration?

Soln:

u = 40m/s, v = 0, t = 20 seconds, a =?

From v = u + at => 0 = 40 + a (20),

=> 0 = 40 + 20a => -20a = 40,

=> a = $\frac{40}{-20} = -2ms/s^2$.

The negative sign in front of the acceleration implies that it is negative acceleration or retardation.

N/B: The first equation of motion i.e. v = u$\pm$ at is used in solving problems, which do not involve distance i.e. s.

<u>Application of the second equation of motion i.e. V^2 = U^2$\pm 2as$</u>

(Q1) A bus which is moving at a velocity of 20m/s, travelled a distance of 70m with an acceleration of 5m/s^2. Determine its final velocity.

Soln:

u = 20m/s, a = 5m/s^2, s = 70m, v =?

From v^2 = u^2 + 2as

$\Rightarrow V^2 = (20)^2 + 2(5)(70)$,

$\Rightarrow V^2 = 400 + 700 = 1100$,

$\Rightarrow V = \sqrt{1100} \Rightarrow V = 33.2$m/s.

The final velocity = 33.2m/s.

(Q2) The velocity of a car which was 10m/s became 11.7m/s. If it accelerated at a rate of 6m/s^2, determine the distance travelled.

Soln:

v = 11.7m/s, u = 10m/s and a = 6m/s^2.

But since$v^2 = u^2 + 2as$

$\Rightarrow 11.7^2 = 10^2 + 2(6)(s)$,

$\Rightarrow 137 = 100 + 12s$,

$\Rightarrow 137 - 100 = 12s$, $\Rightarrow 37 = 12s$

$\Rightarrow s = \frac{37}{12} = 3.1$,

=> distance travelled = 3m.

(Q3) A car started from rest and accelerated at 10m/s^{-2} for a distance of 30m. Find its final velocity.

N/B: Since the car started from rest, then its initial velocity = u = 0.

Soln:

u = 0m/s, a = 10m/s^2 and s = 30m. From $v^2 = u^2 + 2as$

$\Rightarrow V^2 = 0^2 + 2(10)(30)$,

$\Rightarrow V^2 = 0 + 600 = 600$,

$\Rightarrow V = \sqrt{600} = 24.5$.

The final velocity = 24.5ms^{-1}.

(Q4) A particle which was initially travelling at $10ms^{-1}$ was subjected to a retardation of $5ms^{-2}$, and finally came to rest after covering a certain distance. Find this distance.

Soln:

Since the body came to rest => the final velocity = v = 0.

Since retardation = $5ms^{-2}$ => a = $-5ms^{-2}$.

u = $10ms^{-1}$, s =?

But from $v^2 = u^2 + 2as$

=> $0^2 = 10^2 + 2(-5)$ s,

=> 0 = 100 – 10S => 10S = 100,

=> S = $\frac{100}{10} = 10.$

=> Distance = 10m.

(Q5) A car is moving along a road at a velocity of $5ms^{-1}$.

(a) What should be the constant deceleration which will be needed, in order to bring it to a halt in a distance of 15m?

(b) How long will it take for it to come to a stop?

Soln:

(a) Since the car came to rest => v = 0.

s = 15m and u = $5ms^{-1}$.

From $v^2 = u^2 + 2as$ => $0^2 = 5^2 + 2(a)(15)$,

=> 0 = 25 + 30a => -30a = 25,

=> a =$\frac{25}{-30} = -0.83,$

=> deceleration = 0..83m/s^2.

(b) Using v = u + at

=> 0 = 5 + (- 0.83) t,

=> 0 = 5 – 0.83t => 0.83t = 5,

=> t = $\frac{5}{0.83} => t = 6.$

The required time = 6 seconds.

(Q6) A car which was moving at 6.33ms^{-1} is to be brought to rest by applying its brakes within a distance of 5m.

(a) Calculate its retardation.
(b) Find the time.

Soln:

(a) u = 6.33ms^{-1}, v = 0, S = 5m.

From $v^2 = u^2 + 2as$

=> $0^2 = 6.33^2 + 2(a)(5)$,

=> o = 40 + 10a => 10a = - 40,

=> a = $\frac{-40}{10} = -4ms^{-2}$.

Since the acceleration is negative => it is retardation.

(b) u = 6.33ms^{-1}, v = 0,

s = 5m, a = - 4ms^{-2}.

From v = u + at => 0 = 6.33 + (- 4)t,

=> 0 = 6.33 – 4t => 4t = 6.33,

=> t = $\frac{6.33}{4} => t = 1.6,$

=> time = 1.6 seconds.

(Q7) A motorist travelling at 144km/h in a car of mass 800kg, reduces his speed uniformly to 36km/h in 10 seconds. Calculate

(a) the deceleration.

(b) the braking force applied to the car.

(c) the distance travelled during the breaking time.

Soln:

m = mass of car = 800kg.

u = 144km/h = 40m/s.

v = 36km/h = 10m/s

a = deceleration.

t = braking time = 10 seconds.

S = distance travelled.

F = braking force.

(a) From V = u + at => a = $\frac{U-V}{t}$ = $\frac{40-10}{10} = 3$ => a = 3m/s^2.

(b) F = ma = 800kg $\times$ $3m/s^2$

= 2400N.

(b) Since the speed of the car was reduced uniformed => it was decelerating

=> a = - 3m/s^2.

From s = ut + $\frac{1}{2}at^2$

=> s = 40 $\times$ 10 + $\frac{1}{2}$ (-3)$(10)^2$

=> S = 400 – 150 = 250m,

=> distance travelled = 250m

<u>Method 2</u>

From $v^2 = u^2 + 2as$ => s = $\frac{V^2-U^2}{2a}$,

=> s = $\frac{10^2 - 40^2}{2(-3)}$ = 250m.

Method 3

S = $\frac{1}{2}(u + v)t = \frac{1}{2}(40 + 10) \times 10$

= $\frac{1}{2} \times 50 \times 10$ =250m.

(Q8) A car of mass 1000kg travelling at 36km/h is brought to rest over a distance of 20m. Find

(a) the average retardation.
(b) the average braking force in Newtons.

Soln:

(a) v = 0m/s, since the car came to rest.

u = 36km/h =10m/s.

s = 20m.

But from V^2= u^2 + 2as => 0^2 = 10^2 + 2(a)(20),

=> 0 = 100 + 40a => - 40a = 100,

=> a = $\frac{100}{-40} => a = -2.5m/s^2$.

The minus sign => it is retardation.

(b) F = ma => F = 1000 $\times$2.5
=> F =2500N.

(Q9) A bullet of mass 20g travelling with a velocity of 16m/s, penetrates a sandbag and is brought to rest in 0.05 seconds. Find

(a) the depth of penetration in metres.

(b) the average retarding force of the sand in Newtons.

Soln:

(a) Since the bullet was brought to rest, then v = 0.

u = 16m/s, t = 0.05.

From v = u + at => 0 = 16 + a (0.05),

=> 0 = 16 + 0.05a => 0.05a = -16,

=> a = $\frac{-16}{0.05} = -320m/s^2$.

For the depth of penetration, we use $v^2 = u^2 + 2as$, where s = the depth of penetration.

=> $0^2 = 16^2 + 2(-320) \times s$,

=> 0 = 256 – 640S, => 640s = 256,

=> s = $\frac{256}{640} = 0.4$.

=> The depth of penetration = 0.4m.

(b) For the average retardation force, we use F = ma, where m = 20g = 0.02kg and a = -320m/s^2.

=> F = 0.02 $\times$ (-320) = - 6.4N => Retarding force = 6.4N

The negative sign implies it is a retarding force.

(Q10) A car of weight 9800N is accelerated from rest to a speed of 10m/s, within a distance of 25m. Determine the constant acceleration force.

N/B: Notice must be taken of the fact that in this case the weight is given in Newtons but not in kg

Soln:

Mass in kg = $\frac{weight\ in\ newtons}{`g`}$

where `g` = acceleration due to gravity = 9.8m/s^2.

=> mass of car = $\frac{9800}{9.8}$ = 1000kg,

=> mass = m = 1000kg .

u = 0m/s, v = 10m and s = 25m.

We must first determine the acceleration, a.

Using $v^2 = u^2 + 2as$,

=> $10^2 = 0^2 + 2(a)(25)$,

=> $100 = 50a$ => $a = 2m/s^2$.

Acceleration force = ma = $1000 \times 2 = 2000N$.

(Q11) A car of weight 9800N starts moving down a hill. Determine its velocity 10m below the starting point.

[Assume that there is no friction, and take $g = 9.8m/s^2$].

N/B: - If the car moved down the hill, then its acceleration = `g`, where $g = 9.8m/s^2$.

But if the car moved up the hill, then its acceleration = -`g` = $-9.8m/s^2$.

Soln:

Since the car starts moving

=> it starts from rest, => u = 0.

Also $a = g = 9.8m/s^2$ and distance = s = 10m.

From $v^2 = u^2 + 2as$

=> $V^2 = 0^2 + 2(9.8)(10)$,

=> $V^2 = 196$ => V = 14m/s.

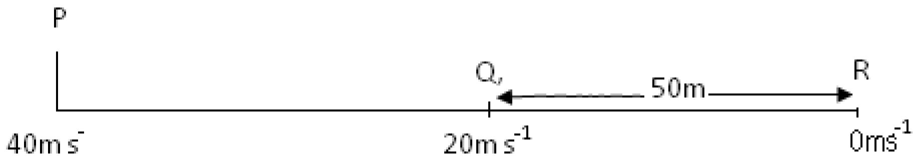
(Q12) A particle moving in a straight line with a uniform deceleration, has a velocity of $40ms^{-1}$ at a point P, $20ms^{-1}$ at a point Q, and comes to rest at R, where QR = 50m.

Calculate,

(I) the distance PQ.

(II) the time taken to cover PR.

Soln:



(1) First consider the distance from Q to R.

Since the velocity at Q is $20ms^{-1}$ => u = initial velocity = $20ms^{-1}$

Also since the particle comes to rest at R => the final velocity = v = 0.

The distance = s = 50m.

To determine the acceleration (retardation) between Q and R, we use the equation

$v^2 = u^2 + 2as$ => $0^2 = 20^2 + 2(a)(50)$,

=> 0 = 400 + 100a => -100a = 400,

=> a = $\frac{400}{-100} = -4$.

=> a = $-4m/s^2$.

The negative sign => the particle is decelerating or retarding.

To find the distance PQ, we use the equation $v^2 = u^2 + 2as$.

Now considering the distance PQ, the velocity at P = $40ms^{-1}$

=> the initial velocity = u = $40ms^{-1}$.

Since the velocity at Q = $20ms^{-1}$.

=> the final velocity = V = $20ms^{-1}$.

The acceleration of the particle between the distance PQ = $- 4m/s^2$,

since the acceleration/ deceleration of the particle is said to be uniform.

Now from $v^2 = u^2 + 2as$ => $20^2 = 40^2 + 2(- 4)s$,

=> 400 = 1600 – 8s => 400 – 1600 = - 8s,

=> -1200 = - 8s, => s = $\frac{-1200}{-8} = 150$.

=> The distance PQ = 150m.

(II) To get the time taken for the particle to cover the distance PR, we consider the distance PR.

The initial velocity in this case occurs at P, and since the velocity at P = 40m/s, then the initial velocity = u = 40m/s.

The final velocity occurs at the point R => v = 0, since the particle comes to rest at the point R. Also a = - 4ms^{-2}.

Now using V = U + at

=> 0 = 40 + (- 4)t => 0 = 40 – 4t,

=> 4t = 40 => t = 10.

The required time = 10 seconds.

<u>The application of the third equation of motion:</u>

i.e. s = ut + $\frac{1}{2}at^2$.

(Q1) A car moved with an initial velocity of 20m/s, and accelerated at a rate of 60m/s^2 for 3 seconds. Calculate the distance travelled.

Soln:

U = 20m/s, t = 3 seconds,

a = 60m/s^2, s =?

From s = ut + $\frac{1}{2}at^2$

=> s = (20) (3) + $\frac{1}{2}$(60)$(3)^2$,

=> s = 60 + $\frac{1}{2}$(60) (9)

=> s = 60 + 270 = 330.

=> Distance travelled = 330m.

(Q2) A particle moved with an initial velocity of $40ms^{-1}$. If it moved for 10 seconds and covered a distance of 650m, calculate

(a) its acceleration.
(b) its final velocity.

Soln:

(a) $u = 40ms^{-1}$, t = 10 seconds,

s = 650m, a =?

From $s = ut + \frac{1}{2}at^2$

=> $650 = 40(10) + \frac{1}{2}(a)(10)^2$,

=> $650 = 400 + \frac{1}{2}(a)(100)$,

=> 650 = 400 + 50a,

=> 50a = 650 – 400 = 250,

=> $a = \frac{250}{50} = 5$.

The acceleration = $5m/s^2$.

(b) To find the final velocity, we use the equation v = u + at,

=> v = 40 + 5(10) = 90.

=> The final velocity = 90m/s.

(Q3)A motor bike which was travelling at a certain speed covered a distance of 21m. Given the time as 3 seconds and its retardation as $2ms^{-2}$, determine the speed at which it was originally moving.

Soln:

s = 21m, u =? , $a = -2m/s^2$, t = 3 seconds.

From $s = ut + \frac{1}{2}at^2$

=> $21 = u(3) + \frac{1}{2}(-2)(3)^2$,

=> 21 = 3u – $\frac{2(9)}{2}$,

=> 21 = 3u – 9, => 21 + 9 = 3u,

=> 3 = 30 => u = $\frac{30}{3} = 10.$

The original velocity = 10m/s.

N/B: - If $ax^2 + bx + c = 0$,

then x = $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

- $ax^2 + bx + c = 0$ is a quadratic in x.

(Q4) A body whose initial velocity was 20m/s moved a distance of 225m. If it was accelerated at a rate of 10m/s^2, find the time.

Soln:

u = 20m/s, s = 225m, a = 10m/s^2, t =?.

From s = ut + $\frac{1}{2}at^2$

=> 225 = 20t + $\frac{1}{2}$ (10)t^2,

=> 225 = 20t + 5t^2

=> 0 = 20t + 5t^2 – 225,

=> 0 = 5t^2 + 20t – 225,

=> 5t^2 + 20t – 225 = 0.

This is a quadratic in t and comparing this with $ax^2 + bx + c = 0$,

- ⇨ a = 5, b = 20, c = -225 and x = t.

t = x = $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

=> t = $\frac{-20 \pm \sqrt{20^2 - 4(5)(-225)}}{2(5)}$

=> t = $\frac{-20 \pm \sqrt{400+4500}}{10}$

=> t = $\frac{-20 \pm \sqrt{4900}}{10}$

=> t = $\frac{-20 \pm 70}{10}$.

Considering the positive sign

=> t = $\frac{-20+70}{10}$ = $\frac{70-20}{10} = \frac{50}{10} = 5$

=> t = 5 seconds.

Considering the negative sign => t = $\frac{-20-70}{10} = \frac{-90}{10} = -9$

But t = -9 seconds is inadmissible.

=> t = 5 seconds.

(Q5) A car decelerated at a rate of 10m/s with an initial velocity of 50m/s. If the distance covered was 120m, find

(a) the time.

(b) the final velocity.

Soln:

(a) Since deceleration is negative acceleration => a = -10m/s^2.

u = 50m/s, s = 120m, t =?.

From s = ut + $\frac{1}{2}at^2$

=> 120 = 50t + $\frac{1}{2}$(-10)t^2

=> 120 = 50t - 5t^2

=> 0 = 50t – 5t^2 – 120,

=> 0 = - 5t^2 + 50t – 120,

=> $-5t^2 + 50t - 120 = 0$, which is a quadratic in t, and by comparing this with $ax^2 + bx + c = 0$, => x = t, a = -5, b = 50 and c = -120.

$t = x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

=> $t = \frac{-50 \pm \sqrt{50^2 - 4(-5)(-120)}}{2(-5)}$

=> $t = \frac{-50 \pm \sqrt{2500 - 2400}}{-10}$

=> $t = \frac{-50 \pm 10}{-10}$.

Considering the positive sign

=> $t = \frac{-50 + 10}{-10} = \frac{-40}{-10} = 4$

Considering the negative sign

=> $t = \frac{-50 - 10}{-10} = \frac{-60}{-10} = 6$.

=> Time = 4 seconds or 6 seconds.

(b) v = u + at

If t = 4seconds => v = 50 + (-10)(4),

=> v = 50 – 40 = 10m/s.

If t = 6seconds => v = 50 + (-10)(6),

=> v = 50 – 60 = -10m/s.

But since we do not have negative velocity, => v = -10m/s is inadmissible => v = 10m/s.

(Q6) A car moving with a velocity of $10ms^{-1}$, accelerated uniformly at $1ms^{-2}$ until it reached a velocity of $15ms^{-1}$. Calculate,

(i) the time taken.

(ii) the distance travelled during the acceleration.

(iii) the velocity reached 100m from the place where the acceleration began.

Soln:

(i) $u = 10ms^{-1}$, $v = 15ms^{-1}$, $a = 1ms^{-2}$.

From v = u + at => t = $\frac{V-U}{a} = \frac{15-10}{1} = 5 => t = 5\ seconnds.$

(ii) For the distance travelled, we use s = ut + $\frac{1}{2}at^2$ =>

$s = 10\times 5 + \frac{1}{2}\times 1\times 5^2 = 50 + 12.5$,

=> s = 62.5m => distance travelled = 62.5.

(iii) For this, we use $v^2 = u^2 + 2as$, where s = 100m, $u = 10ms^{-1}$, $a = 1m/s^2$ and v = the velocity reached.

=> $v^2 = 10^2 + 2(1)(100) = 100 + 200$,=> $v^2 = 300$ => v = $\sqrt{300} = 17.32ms^{-1}$.

(Q7) A car starts from rest and accelerated uniformly at a rate of $2m/s^2$ for 6 seconds. It then maintains a constant speed for half a minute. The brakes are then applied and the vehicle uniformity retarded to rest in 5 seconds. Determine the maximum speed reached in km/h, and the total distance covered in metres.

<u>First stage:</u>

For the first stage of the journey, the car starts from rest and accelerated at a rate of $2m/s^2$ for 6 seconds.

=> u = 0m/s, $a = 2m/s^2$ and t = 6 seconds.

The maximum speed reached is given by v = u + at,

=> v = 0 + (2)(6) = 12.

Maximum speed reached = 12m/s = 43km/h.

For the distance travelled during this first stage, we use s = ut + $\frac{1}{2}at^2$

=> $s = 0 \times 6 + \frac{1}{2} \times 2 \times 6^2$ = 36m.

<u>Second stage:</u>

- For the second stage of the journey, when the speed got to 12m/s, the car travelled at this speed for half a minute (i.e 30 seconds).

- The distance travelled during this stage = Speed $\times$ time

= 12 $\times$ 30 = 360m.

<u>Third stage</u>:

- During the third stage, while the car was still moving at a speed of 12m/s, the brakes were applied and the car uniformly retarded to rest in 5 seconds,

=> u = 12m/s, v = 0m/s and t = 5 seconds.

To find the distance covered, we use $v^2 = u^2 + 2as$,

=> s = $\frac{V^2-U^2}{2a} = \frac{0^2-12^2}{2\times(-2.4)} = 30,$

=> distance travelled during this stage = 30m.

The total distance travelled = 36 + 360 + 30 = 426.

(Q8) (a)(i)What is meant by a displacement – time graph?

(ii)State the quantities which can be obtained from such a graph.

(b) Draw a rough sketch of a displacement – time graph for a body moving with.

(i) Constant velocity.
(ii) Constant acceleration.

Justify your sketch in each case.

(c) After decelerating for 5 seconds at a rate of $4m/s^2$, the velocity of a motor bike became 20m/s.

(i) At what speed was it moving.
(ii) Determine the distance travelled by the bike.
(iii) By means of calculation, show that the time for the deceleration was really 5 seconds.

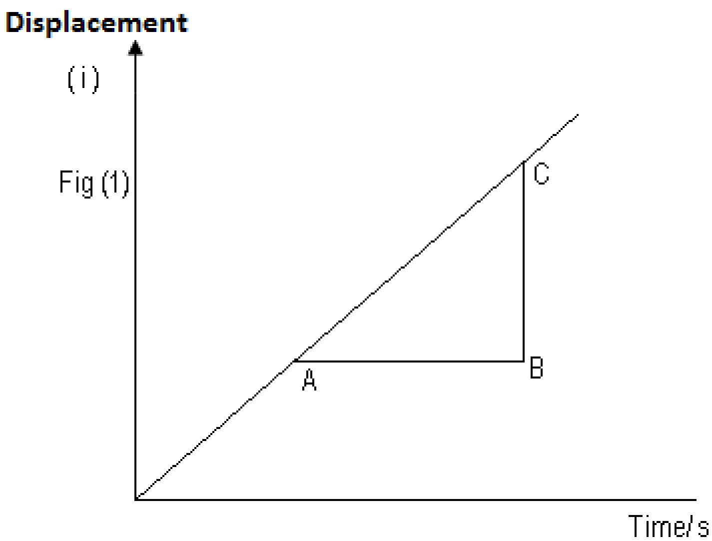
Soln:

(a) (i) A displacement – time graph is a graph of displacement, plotted on the vertical axis against time on the horizontal axis.

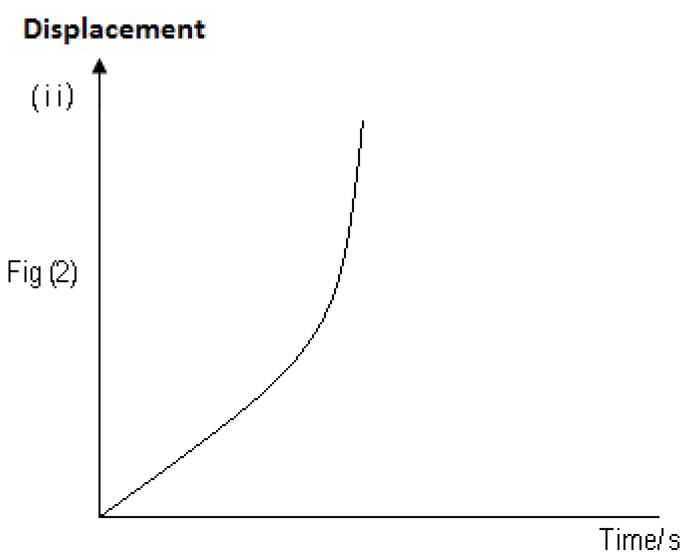
(ii) The following information can be obtained from such a graph.

(1) The displacement for a given time.
(2) The time interval corresponding to a given displacement.
(3) The velocity after any time.

(b)



The displacement – time graph for an object moving with a constant velocity



The displacement – time graph drawn is that for a body, moving with constant acceleration.

- Theoretically, fig. (1) is obtained using the expression s = vt, where v is a constant.

- The expression s = vt is of the form y = mx, and for this reason, a graph of s against t yields a straight line, which passes through the origin, as can be seen in fig. (1).
- The gradient in this case = v.
- Fig. (2) is also obtained from S = Ut + $\frac{1}{2}at^2$
- By plotting s against t, {(i.e. s against (t + t^2)}, we obtain a parabolic curve,.
- The curve passes through the origin because s = 0 when t = 0.

(c) (i) v = 20m/s, u = ?
a = - 4m/s^2, t = 5 seconds.
From v = u + at => 20 = u + (- 4)(5),
=> 20 = u – 20 => u = 20 + 20,
=> u = 40m/s.

(ii) $v^2 = u^2 + 2as$

=> $20^2 = 40^2 + 2(-4)s$,

=> 400 = 1600 – 8s,

=> 400 – 1600 = - 8s,

=> -8s = - 1200,

=> s = 150m.

(iii) From s = ut + $\frac{1}{2}at^2$

=> 150 = 40t + $\frac{1}{2}$(- 4)t^2

=> 150 = 40t – $2t^2$

=> $2t^2$ – 40t + 150 = 0, which is a quadratic in t

Comparing $2t^2$ – 40t + 150 = 0 with $ax^2 + bx + c = 0$,

=> a = 2, b = - 40 and c = 150.

But x = $\frac{-b \pm \sqrt{b^2-4ac}}{2a}$

=> t = $\frac{-b \pm \sqrt{b^2-4ac}}{2a}$

=>t = $\frac{-(-40)\pm\sqrt{(-40)^2-4(2)(150)}}{2(2)}$

=> t = $\frac{40\pm\sqrt{1600-1200}}{4}$

=> t = $\frac{40\pm 20}{4}$.

Considering the positive sign

=> t = $\frac{40+20}{4}$ =15 seconds.

Considering the negative sign

=> t = $\frac{40-20}{4}$ = $\frac{20}{4}$ = 5seconds

=> t = 5 seconds as required

N/B: (1) When a body is thrown vertically upward, then it acceleration = -`g`, where g = the acceleration due to gravity.

(III) The maximum height reached by such a body, occurs when v = 0, where v = the final velocity.

(Q1) A stone is thrown vertically upward with an initial velocity of 20m/s. Neglecting air resistance, determine

(a) the maximum height reached by the stone.
(b) the time taken to reach this height.

[Take g = 10m/s^2].

Soln:

(a) a = - g = -10m/s^2.
v^2= u^2 + 2as, where s = the height above the ground reached by the stone.
Since at the maximum height reached by the stone v = 0,
=> 0^2 = u^2 + 2as,
=> 0 = 20^2 + 2(-10)s,
=> 0 = 400 – 20S => 20S = 400,
=> s = $\frac{400}{20} = 20.$

=> Maximum height attached by the stone = 20m.

(b) For the time taken to reach this height, we use the equation v = u + at,

=> 0 = 20 + (-10)t, => 0 = 20 – 10t,

=> 10t = 20 => t = $\frac{20}{10} = 2$.

=> Time = 2 seconds.

N/B: If a body falls to the ground, then its acceleration = \`g\` = 10m/s^2.

(Q2) A stone falls to the ground with an initial velocity of 20m/s, and took 3 seconds to get to the ground. Determine

(a) its velocity just before it fell to the ground.

(b) the distance it moved before falling to the ground.

Soln:

(a) u = 20m/s, t = 3s, a = 10m/s^2.

Its velocity just before it falls to the ground = the final velocity = v.

From v = u + at, => v = 20 + (10)(3), => v = 50m/s.

(b) From v^2 = u^2 + 2as => 50^2 = 20^2 + 2(10)s,

=>2500 = 400 + 20s => 2100 = 20S,

=> s = $\frac{2100}{20} = 105m$.

(Q3) A stone was released from a height of 80m above the ground. Determine

(a) the velocity with which it fell to the ground.

(b) the time it took to fall to the ground.

N/B: Since the stone was released from a height of 80m, then its initial velocity = 0.

Soln:

(a) u = 0m/s, s = 80m, a = 10m/s^2.

The velocity with which it fell to the ground = v =?.

From v^2 = u^2 + 2as => v^2 = 0^2 + 2(10)(80), => V^2 = 1600,

=> V = 40m/s.

(b) From s = ut + $\frac{1}{2}at^2$

=> 80 = (0)(t) + $\frac{1}{2}$(10)t^2,

80 = 5t^2 => t^2 = $\frac{80}{5} = 16$,

=> t = $\sqrt{16}$ = 4 seconds.

(Q4) An object falling to the ground with a velocity of 25m/s, took 2 seconds to get to the ground. Determine the distance it traveled, before reaching the ground.

[Take g = 10m/s^2].

Soln:

u = 25m/s, a = 10m/s^2,

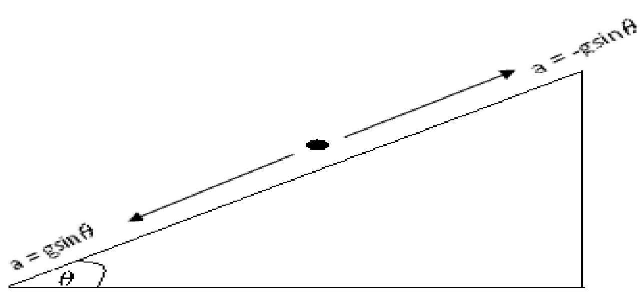
t = 2 seconds and S = ?.

From s = ut + $\frac{1}{2}at^2$

=> s = 25(2) + $\frac{1}{2}$(10)$(2)^2$

=> s = 50 + 20 = 70m.

Motion with reference to the inclined plane:



(I) When a body moves up an inclined plane, its acceleration, a = -gsin θ.

(II) When it moves down the plane, then its acceleration, a = gsinθ

For these reasons, when a body moves up on inclined plane, the following formulae hold.

(a) From v = u + at => v = u + (- g$\sin\theta$)t => v = u – g$\sin\theta$t.

(b) From $v^2 = u^2 + 2as$ => $v^2 = u^2 + 2(-g\sin\theta)s$,

=> $v^2 = u^2 - 2g\sin\theta s$

(c) From s = ut + $\frac{1}{2}at^2$

=> s = ut + $\frac{1}{2}(-g\sin\theta)t^2$ => s = ut - $\frac{1}{2}g\sin\theta t^2$

(c) To find the time taken to reach the maximum height, we consider the equation v = u – g$\sin\theta$t.

At the maximum height, V = 0,

=> 0 = u – g$\sin\theta$t => g$\sin\theta$t = u,

=> t = $\frac{U}{g\sin\theta}$

(e) To determine the maximum height up the plane, we consider the equation

$v^2 = u^2 - 2g\sin\theta s$.

The maximum height occurs when v = 0 => 0 = $u^2 - 2g\sin\theta s$

=> $2g\sin\theta s = u^2$,

=> s = $\frac{U^2}{2g\sin\theta}$

On the other hand, when a body moves down an inclined plane, then a = g$\sin\theta$.

(f) From v = u + at

=> v = u + (g$\sin\theta$)t => v = u+ g$\sin\theta$t.

(g) From $v^2 = u^2 + 2as$

=> $v^2 = u^2 + 2(g\sin\theta)s$

=> $v^2 = u^2 + 2g\sin\theta s$

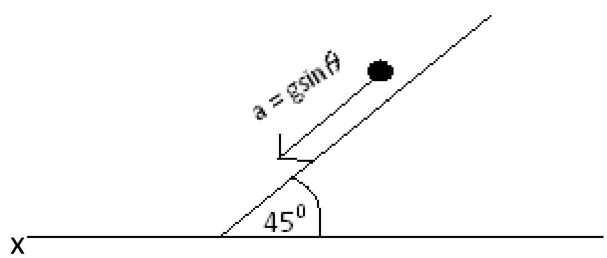
(h) From $s = ut + \frac{1}{2}at^2$

=> $s = ut + \frac{1}{2}(g\sin\theta)t^2$

(Q1) A body which was on an inclined plane, made an angle of 45^0 to the horizontal, and moved along the plane for 10 seconds, in the downwards direction.

(a) Determine its final velocity after 10 seconds.
(b) Find the distance travelled after the 10second period. [Take g = $10ms^{-2}$].

Soln:



(a) Since the body moved from rest => u = 0, t = 10s, v =?,
a = $g\sin\theta$ and θ = 45^0.

From v = u + at => v = u + $g\sin\theta$t,

=> v = 0 + 10sin 45^0(10),

=> v = 0 + 10(0.7)(10) = 0 + 70, => v = $70ms^{-1}$.

(b) From $s = ut + \frac{1}{2}(gsin\theta)t^2$

=> $s = (0)t + \frac{1}{2}(10) \sin 45(10)^2$

=> s = 0 + 5(0.7)(100),

=> s = 0 + 350 = 350m.

(Q2) A ball is thrown up a smooth inclined plane, with an initial velocity of $54kmh^{-1}$.

If the inclination of the plane is 30^0, find

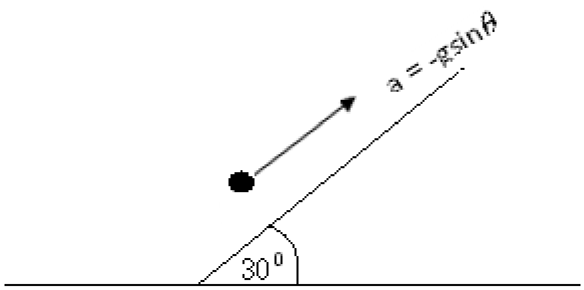
(i) the time taken by the object to reach the maximum height.

(ii) the maximum height reached.

[Take g = 10ms^{-2}].

Soln:

(i)



u = 54km/h, converting this into m/s => u = 15m/s.

At the maximum height, v = 0.

From v = u + at => v = u - gsinθt,

=> 0 = 15 – (10)(sin30^0)t,

=> 0 = 15 – 10(0.5)t => 5t = 15.

=> t = $\frac{15}{5} = 3\ seconds$.

(ii) The maximum height reached is given by h = ut - $\frac{1}{2}g$sinθt^2

=> h = $15 \times 3 - \frac{1}{2} \times 10 \times sin30 \times 3^2$

=> h = 22.5m.

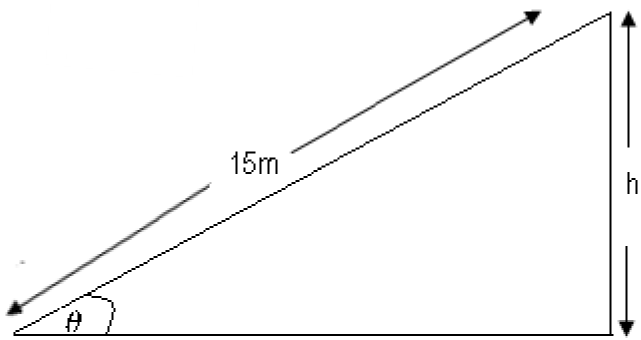
(Q3) An inclined plane of inclination θ has a length of 15m. If a body rolls down the plane freely along the entire length with an acceleration of 5ms^{-2}, calculate

(a) the angle of inclination θ.

(b) the height of the plane.

[Take g = $10ms^{-2}$]

Soln:



Since the ball rolled down the plane with an acceleration of $5ms^{-2}$ => g$\sin\theta$ = 5

(since acceleration down the plane = g$\sin\theta$).

From g$\sin\theta$ = 5 => 10 $\sin\theta$ = 5,

=> $\sin\theta$ $= \frac{5}{10} = 0.5$,

=> $\theta = \sin^{-1} 0.5$ => $\theta = 30^0$

(b) h = 15 $\sin\theta$ = 15 $\sin 30^0$

= $15 \times 0.5 = 7.5$

=> h = 7.5m

N/B: If the height and the length of the inclined plane are given and

(I) the body is said to move or slide down the plane, then a = g.

=> $v^2 = u^2 + 2as$ becomes $v^2 = u^2 + 2gh$, where h = the height.

(II) If the body slides or moves up the plane, then a = - g.

=> $v^2 = u^2 + 2as$ becomes $v^2 = u^2 - 2gh$, where h = the height.

(Q4) A body slides down from rest on a smooth inclined plane, which is 50m long and 5.0m high. Determine the speed with which it reaches the bottom.

Soln:

u = 0, since the body started from rest.

The speed with which it reaches the bottom = its final velocity = v =?.

Also h = 5m.

From $v^2 = u^2 + 2gh$

=> $v^2 = 0^2 + 2(10)(5)$,

=> $v^2 = 100$ => $V = \sqrt{100} = 10$.

The required velocity = $10ms^{-1}$.

(Q5) A motor bike moved up on an inclined plane, which is inclined at an angle of 10^0 with the horizontal. If its initial velocity was 15m/s and its final velocity was 12m/s, find the distance it travelled.

Soln:

Using $v^2 = u^2 - 2g\sin\theta s$

=> $s = \frac{v^2 - U^2}{-2g\sin\theta}$

=> $s = \frac{12^2 - 15^2}{-2\,(10)\sin 10^0} = \frac{144-225}{-3.5}$

=> S = 23m.

<u>Oscillatory Motion:</u>

<u>The simple pendulum:</u>

- This is a heavy body which is suspended by a light inextensible string.
- One complete to and fro movement of the pendulum is called an oscillation or vibration.
- The maximum displacement of the bob from the equilibrium or the rest position is called the amplitude.
- Provided the amplitude is small. i.e. a few degrees, the period or the periodic time, (i.e. the time taken for one complete oscillation), depends only on the length of the pendulum and the acceleration due to gravity.
- The length of the pendulum is the distance from the point of suspension to the centre of the bob, and for a pendulum T = $2\pi\sqrt{\frac{l}{g}}$ where l = the length in metres, g = acceleration due to gravity and T = the period of the pendulum.

- For a pendulum $\frac{T^2}{l}$ = constant or $\frac{T^2}{l} = \frac{4\pi^2}{g}$.
- To verify this experimentally, the time for lets say50 oscillations is found for at least 6 different lengths.
- The result is tabulated as follows:
-

Length l/m	Time for 50 oscillations	Periodic Time,T/S	T^2	$\frac{T^2}{l}$

- The constancy of the values of $\frac{T^2}{l}$ entered in the last column of the table, shows that the square of the period or the periodic time, is proportional to the length of the pendulum.
- This may also be demonstrated by plotting a graph of T^2 against l, for which a straight line through the origin is obtained.
- The value of `g` or the acceleration due to gravity, can be calculated by dividing $4\pi^2$ by the mean value of $\frac{T^2}{l}$ obtained from the last column of the table.

(Q1) Explain how you will show that for a pendulum, $\frac{T^2}{l}$ is always a constant, where T = the period of the pendulum and l = the length of the pendulum.

Soln:

(1) Pendulums of different lengths are taken.
(2) Determine the time taken for each of them to go through lets say 50 oscillations.
(3) Find the period 'T' for each of the pendulums, by dividing the time for the 50 oscillations by 50.
(4) For each pendulum, determine the value of T^2.
(5) Also for each pendulum, determine the value of $\frac{T^2}{l}$, by dividing the period squared by the length.
(6) It will be noted that the value of $\frac{T^2}{l}$ will be the same for each (i.e. constant) for each of the pendulums.
(7) This shows that for any pendulum $\frac{T^2}{l}$ is always a constant.

Questions:

(1) What do you understand by rectilinear motion?

(2) Differentiate between displacement and velocity.

(3) Explain when the velocity of a body is said to be uniform.

(4) Differentiate between acceleration and retardation.

(5) What is a velocity – time graph, and draw the velocity – time graph for an object which is moving with

(i) a constant velocity.

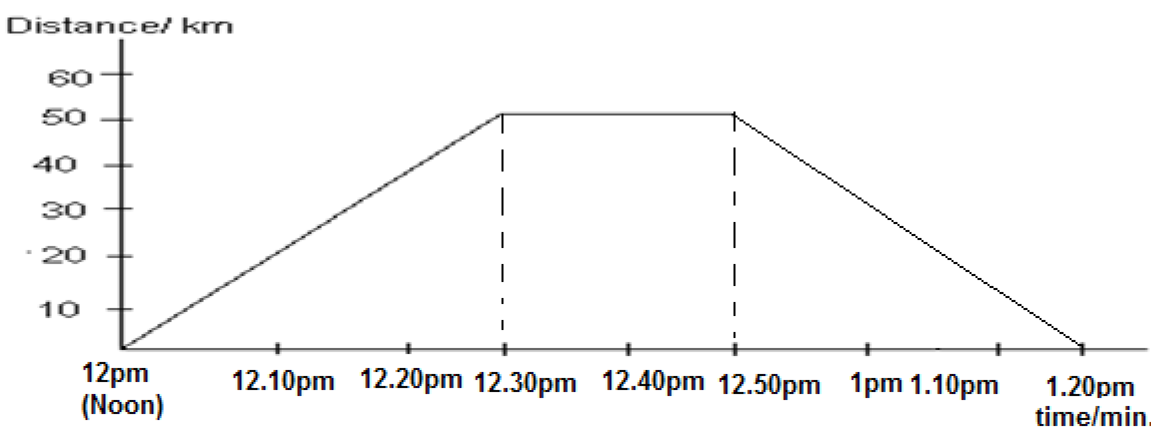
(ii) a uniform acceleration.

(iii) a non uniform acceleration.

(6) Kofi started a journey at exactly 12 noon and travelled uniformly. After 30 minutes, he had covered a distance of 50km. He then rested for 20 minutes and then moved straight towards his destination. If he arrived at his destination at 1.20pm,

(a) represent this on a distance - time graph.

Ans:



(b) Determine the velocities for the outward and the return journeys.

Ans:

The velocity for the outward journey = 100km/h.

The velocity for the return journey = 100km/h.

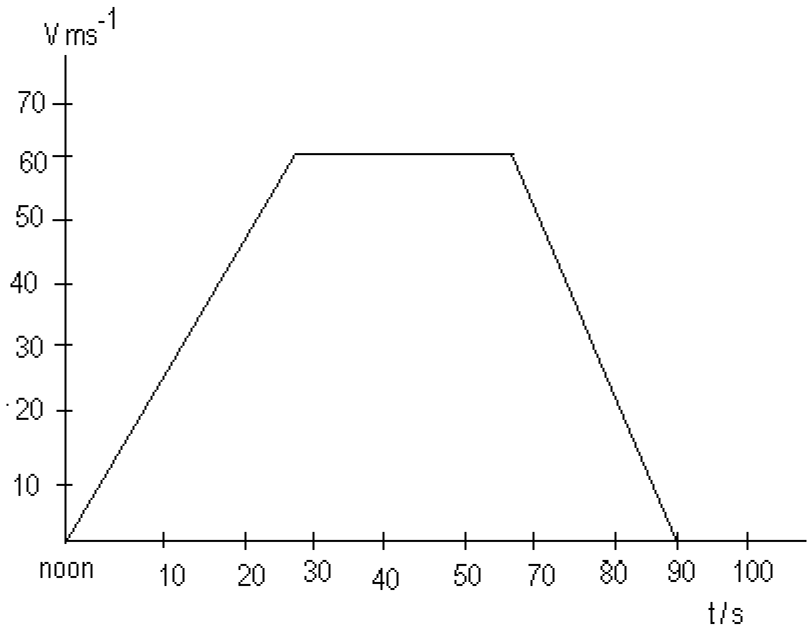
(Q7) A motorist started from rest and attained a velocity of $60ms^{-1}$ in 30 seconds. He kept this velocity for the next 40 seconds and finally came to rest in 20 seconds.

(i) Represent this on a velocity – time graph.

(ii) Determine his acceleration.

(iii) Find the total distance he travelled.

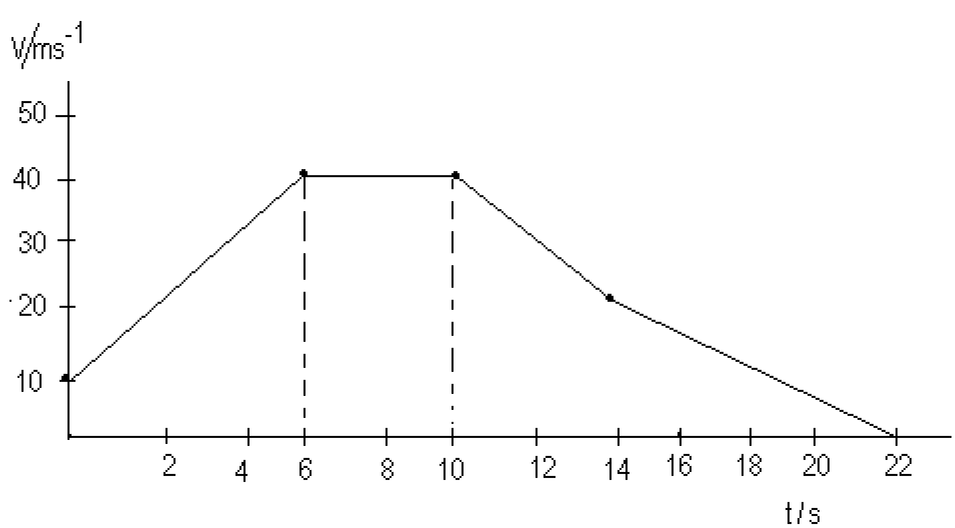
Ans:

(I) 

(II) The acceleration = $2ms^{-2}$.

(III) The total distance travelled = 390m.

(Q8) A cyclist whose initial velocity was 10m/s, accelerated at $5m/s^2$ for 6 seconds. He rode at this speed for 4 seconds. His speed then dropped to 20m/s within the next 4 seconds before finally coming to rest in 8 seconds time. Represent this on a velocity – time graph. Ans:



(Q9) Explain what is meant by inertia and explain also what the law of conservation of momentum means.

(Q10) A body of mass 6g moving with a velocity of $5ms^{-1}$, collided with another body of mass 2g moving with a velocity of $8ms^{-1}$ in the same direction. After the collision, both bodies continued moving in their previous direction of motion. Given that the velocity of the 6g body became $10ms^{-1}$, determine the velocity of the other one.

Ans: -7m/s

(Q11) Two metallic balls, labeled X and Y of masses 4g and 3g respectively, move with velocities of $10ms^{-1}$ and $12ms^{-1}$respectively in opposite direction. After collision, the first ball maintained its original movement direction but moved with a velocity of 2 ms^{-1}. Determine the final velocity of the second ball, and its direction of motion after the collision.

Ans: Velocity = $1.33ms^{-1}$ in the original direction of movement of the second ball.

(Q12) A truck of mass 30kg moving with a velocity of $20ms^{-1}$, collided with a stationary car of mass 12kg. After the collision, both the truck and the car moved in the same direction as that in which the truck was originally moving. If the velocity of the truck became $8ms^{-1}$, determine that of the car.

Ans: $30ms^{-1}$.

(Q13) A bus of mass 30kg, moving at $4ms^{-1}$ collided with a mini bus of mass 10kg, which was moving at $2ms^{-1}$, in the opposite direction. After the collision, the two buses coalased and moved with a common velocity. Determine the common velocity and its direction.

Ans: 2.5m/s in the original direction of motion of the 30kg bus.

(Q14) What is the main difference between perfectly elastic collision and a perfectly inelastic collision?

Ans:

- In elastic collision, both momentum and kinetic energy are conserved.
- But in the inelastic collision, it is only momentum which is conserved.

(Q15) Briefly explain the principle under which the space rocket operates.

(Q16) A particle started from rest and accelerated at a rate of $4m/s^2$ for 5 seconds. Determine its final velocity.

Ans: 20m/s.

(Q17) A car which was moving at a speed of 6m/s, accelerated at $3m/s^2$ for 4 seconds. Determine its speed at the end of this time period.

Ans: 18m/s.

(Q18) After undergoing acceleration for 4 seconds, the velocity of a vehicle changed from 20m/s to 60m/s. Determine the vehicle's acceleration.

Ans: $10ms^{-2}$

(Q19) After undergoing a retardation of $5m/s^2$ for 4 seconds, a vehicle finally came to rest. What was its original velocity?

Ans: 20m/s.

(Q20) An express train which started from rest achieved a velocity of 80m/s, after travelling a distance of 200m. At what rate was it accelerating?

Ans: $16m/s^2$.

(Q21) A motor which was travelling at a velocity of 60m/s, accelerated for 10 seconds at a rate of $3ms^{-2}$. Calculate

(a) its final speed.
Ans: 90m/s.
(b) the distance travelled.
Ans: 750m.

(Q22) A truck was moving at a speed of 40km/h, and has a mass of 2500kg. If the brakes were applied which forced it to come to a halt in a distance of 80m, determine the braking force.

Ans: 25000N or 25kN

(Q23) A jet plane which has a mass of 8×10^2kg, and was moving at an airport at a speed of 144km/h. If the pilot was able to reduce this speed to 36km/h in 10 seconds, determine (a) the retardation.

Ans: 30m/s^2

(b) the breaking force the plane was subjected to.

Ans: 24KN or 24000N.(c) the distance travelled by the plane.

Ans: 250m.

(Q24) A car started from rest and travelled between two towns, which are 50m apart within a time interval of 8 seconds. What was its acceleration?

Ans: 1.6m/s^2.

(Q25) The initial velocity of a car was 5m/s. If it accelerated at a rate of 4m/s^2, and covered a distance of 18m, determine the time.

Ans: 2 seconds.

(Q26) A stone is vertically thrown into the sky with an initial velocity of 25m/s. Determine

(a) the maximum height it will reach.
Ans: 31.3m.

(b) the time taken to reach this height.

Ans: 2.5 seconds.

[Take g = 10m/s^2]

(Q27) An inclined plane makes an angle of 60^0 with the ground. A bag of cement at rest was moved down along the plane for 20seconds. Find

(a) the velocity of the bag of cement, just after the 20 seconds.

Ans: 174m/s.

(b) the distance travelled by the cement bag after the 20 seconds.

Ans: 1732M

(Q28) A stone of mass 30g was pushed from rest up an inclined plane for half a minute. Find

(a) its velocity after the given time interval.

Ans: -243m/s i.e 243m/s up the plane.

(b) the distance travelled by the stone after this given time.
Ans: -3641m i.e. 3641m up the plane.

[Take g = $10m/s^2$ and the angle between the plane and ground = 54^0]

(Q29) A vehicle climbs an inclined plane whose inclination is 20^0 with reference to the horizontal ground. Given the time to be 2 seconds and the initial velocity of the vehicle as 30m/s, determine.

(a) the velocity of the vehicle after the 2 seconds.

Ans: 1.5m/s

(b) the distance travelled by the vehicle at the end of the 2 seconds.

Ans: 9.8m.

(Q30) A block slides down a smooth inclined plane with an initial velocity of 2m/s. The plane is 60m long and 8m high. Find the speed with which it will reach the ground.

Ans: 12.8m/s.

(Q31) An object moved up an inclined plane, whose inclination is 40^0 with an initial velocity of 20m/s. If its final velocity was 17.2m/s, determine the distance travelled.

Ans: 8m.

(Q32) A boy runs up an inclined plane whose angle of inclination is 30^0, with a velocity of $54kmh^{-1}$. If the maximum height of the plane is 150m, determine

(a) the length of the plane.

Ans: 300m.

(b) the time in seconds he takes to ran up the plane.

Ans: 2 seconds.

(c) his final velocity just before he got to the top or the maximum height of the plane, before he stopped running in metres per seconds.

Ans: 139m/s.