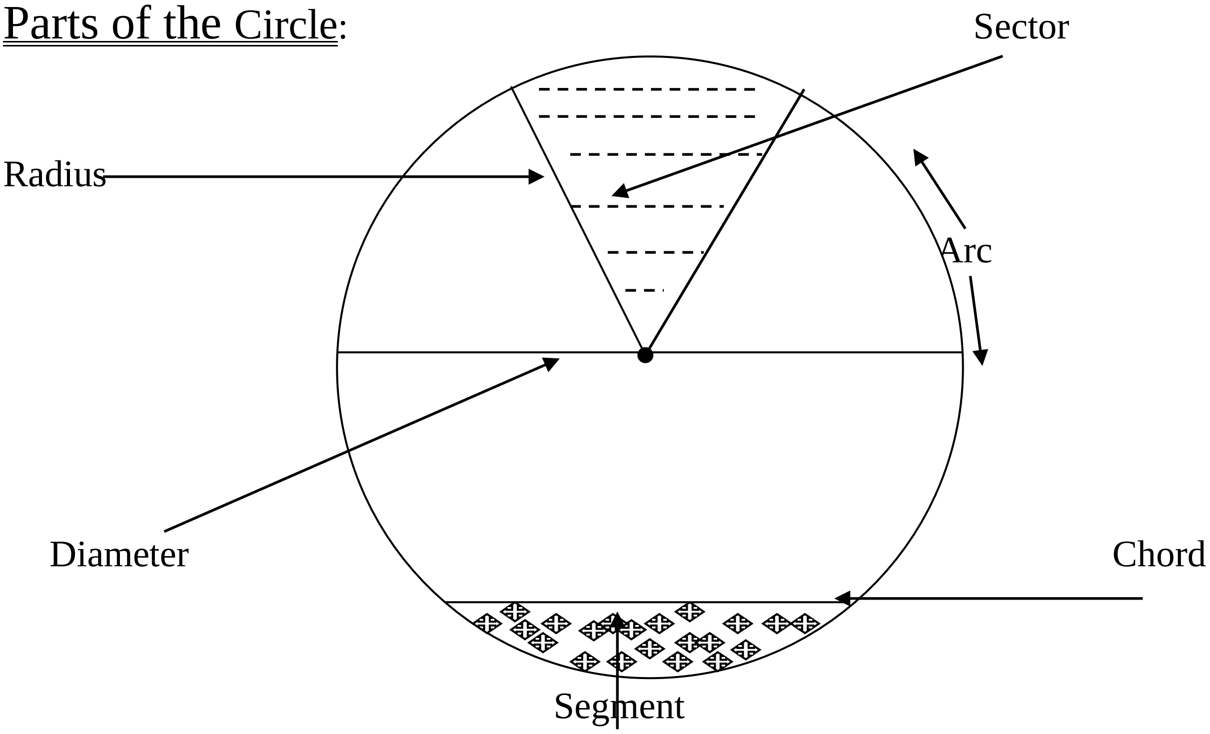
CHAPTER SEVEN

THE CIRCLE

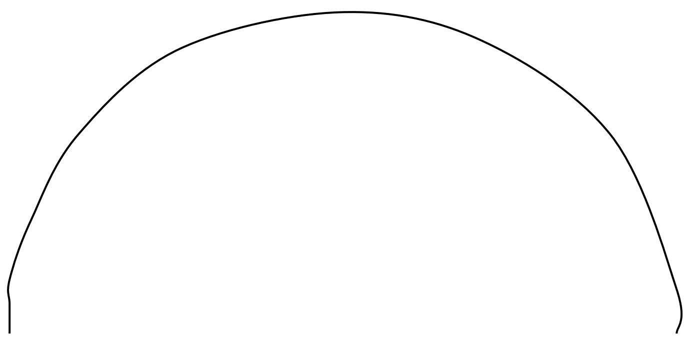
Parts of the Circle:



1) The Circumference: This is the distance around the circle.
2) Chord:This is a straight line which joins two points on the circumference.
3) The diameter: This is a special chord which passes through the centre of the circle.
4) The radius: This is a line drawn from the centre, to a point on the circumference.
5) Arc: This refers to a portion of the circumference.
6) The segment: This is the region between a chord and an arc.
7) The sector: This refers to the region between two radii.

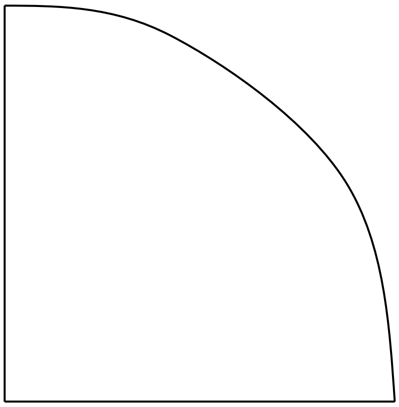
Note:

i. For any circle, the radius × 2 = the diameter i.e. twice the radius gives us the diameter.
ii. Half a circle is referred to as a semi circle.



iii. A quadrant refers to one quarter of the area of a circle.

i.e.



iv. For a circle, C= $2\pi r$, where C= the circumference, r = the radius and $\pi = 3.14 \text{ or } 3.142 \text{ or } \frac{22}{7}$.

Q1) A circle has a radius of 14cm. Determine the distance round it.

Solutions

C = $2\pi r$ => C=2×3.14×14=88cm.

Q2) A city is circular in shape and its diameter is 30km. Determine the distance covered by a man, who walked twice round this city.

Solution

The distance covered by a man who did walk round the city once = the circumference.

D = 30km, =>r =$\frac{30}{2}$ = 15km.

C= $2\pi r$, => C=2×3.14×15= 94km.

Distance covered by walking round the city twice= 2×94=188km.

Q3) A racing bike is travelling round a circular track whose radius is 40km, at a speed of 20km/h. Determine the time it will take to travel

a) once round the track.
b) thrice round the track.

Solution

(a) The distance covered by travelling once round the track = the circumference = $2\pi r$ =2×3.14×40 =251km.

a) the speed of racing bike = 20km/h

∴ If 20km = 1hour

then 251km =$\frac{251}{20}$× 1 = 12.6hrs.

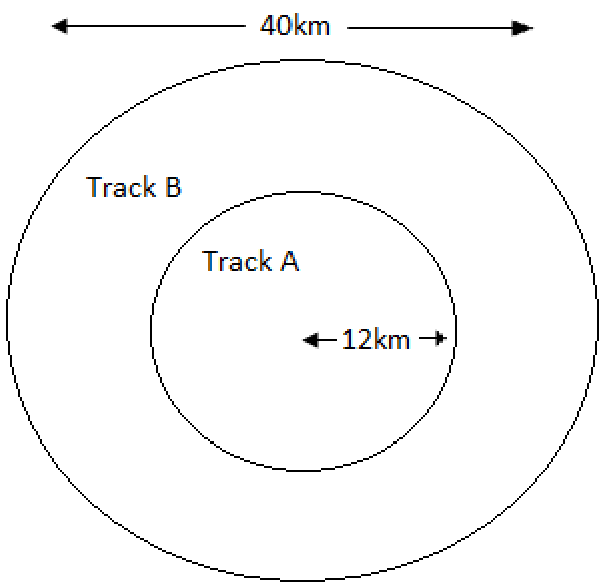
b) Distance travelled by travelling thrice round this track = 3×251= 753km.

Speed of bike = 20km/h

If 20km = 1 hour

Then 753km = $\frac{753}{20}$× 1 = 38hrs.

Q4)



Two cyclists, Addo and John are supposed to travel round two different circular tracks. Addo is to travel in track A at a speed of 40km/h and John is to travel in track B at a speed of 60km/h.

a) Determine which of these men will be the first to complete his journey.
b) Express the distance travelled by Addo as a fraction of the distance travelled by John.

<u>Solution</u>

The distance travelled by Addo = the circumference of track A= $2\pi r$ =2×3.14×12=75km.

Speed of Addo = 40km/h.

If 40km=1 hour

=>75km = $\frac{75}{40}\times 1 = 1.9$.

∴ Time taken by Addo to move round his track = 1.9 hrs.

Distance travelled by John = the circumference of track

B = $2\pi r$ =2×3.14×20 =126km.

Speed of John = 60km/h.

If 60km = 1 hour

=>126km = $\frac{126}{60}\times 1 = 2.1$

John will complete his journey in 2.1 hours..

Addo will finish first

b)Distance travelled by Addo = 75km and distance travelled by John = 126km

Distance travelled by Addo as a fraction of that travelled by John = $\frac{75}{126} = \frac{25}{42}$

Q5) A city which is circular in shape has a length of 420km. Determine the distance walked by Mr. Abu, if he walked from the centre of this city to a point on the city's boundary.

Solution

Length of the city = the circumference = 420km.

Distance travelled by Mr. Abu = the radius =?

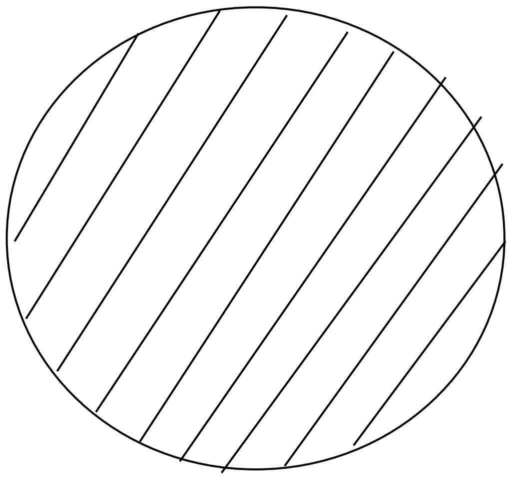
But since C=$2\pi r$ => 420 =2×3.14×r,

=>420 = 6.28r => r = $\frac{420}{6.28}$,

=>r = 67, =>Distance walked by Mr. Abu = 67km.

The area of a circle:

The area of a circle refers to the region within the circle.



For example the shaded portion refers to the area of the given circle.

The area of a circle = πr^2, where r = radius of the circle.

1) A circle has a radius of 7cm. Determine its area.

[Take $\pi = \frac{22}{7}$]

Solution

Area = $\pi r^2 = \frac{22}{7} \times 7^2 = 154\text{cm}^2$

2) A circular plot of land has an area of 255cm^2.

Determine its diameter.

$\left[\text{Take } \pi = 3.142\right]$

Solution

Since $A = \pi r^2 \Rightarrow 255 = 3.14 \times r^2$

$\Rightarrow \frac{255}{3.14} = r^2 \Rightarrow 81 = r^2 \Rightarrow r = \sqrt{81} = 9\text{cm}.$

Diameter = 2r = 2 × 9 = 18cm..

3) A man charges ¢2 for weeding an area of 5m^2.Determine how much he will charge if he weeds a circular field of radius 8m. $\left[\text{Take } \pi = 3.142\right]$.

Solution

Area of circular field to be weeded = $\pi r^2 = 3.142 \times 8^2 = 201\text{m}^2$

If 5m^2 = ¢2

$\Rightarrow 201\text{m}^2 = \frac{201}{5} \times 2 = 80$, $\Rightarrow$ amount charged = ¢80.

1. 4) A man owns a circular plot of land whose diameter is 12m. On a portion of this land which is circular in shape and of diameter 10m, he has planted onion. Determine

i. the fraction of the land on which the onion farm is located.

ii. the percentage of the land on which the onion farm is located

iii. the quantity of land left for future cultivation.

Solution

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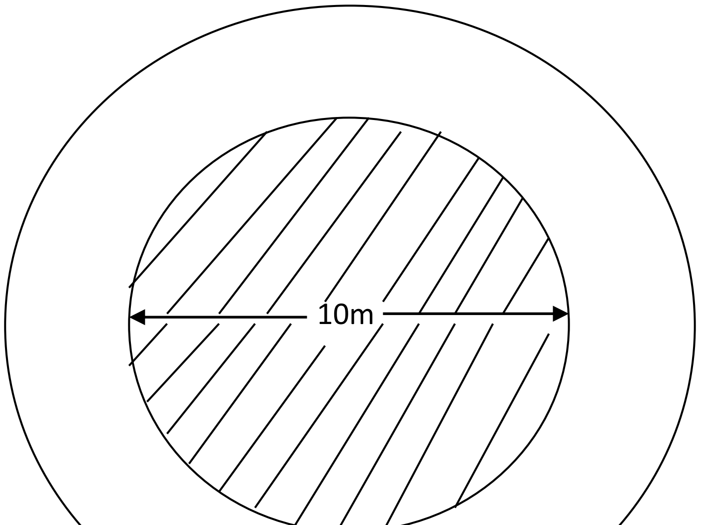


12m

Let the shaded portion represent the onion farm. Since it diameter = 10m => radius = 5m.

The area of the onion farm = $\pi r^2 = 3.14 \times 5^2 = 78.5m^2$

Also since the diameter of the circular field = 12m => its radius = 6m.

The area of this field = $\pi r^2 = 3.14 \times 6^2 = 113m^2$

i) Fraction of the field on which the onion farm is located = $\frac{78.5}{113}$

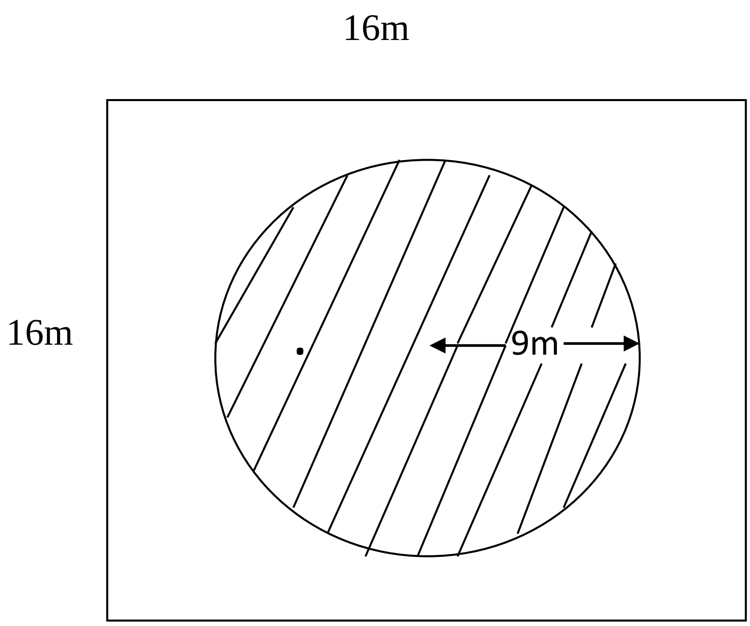
ii) The percentage of the land on which the onion farm is located = $\frac{78.5}{113} \times 100 =$ 69%.

iii) The portion of the land left for future cultivation = 113 - 78.5 = $34.5m^{2.}$

(5) An onion farm which is circular in shape and of radius 9m is situated within a plot of land, which is in the shape of a square of side 16m.

Determine the fraction of the plot on which the onion farm is situated.

<u>Solution</u>

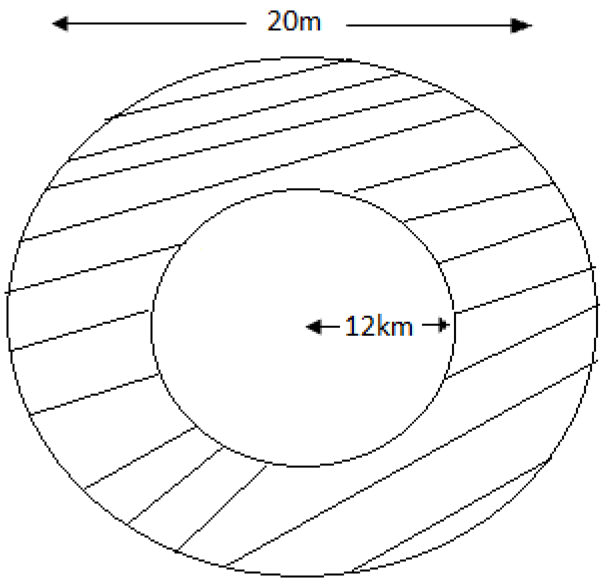


The area of the square plot = $16^2 = 256m^2$

The area of the shaded portion which represents the onion farm = $\pi r^2 = 3.14 \times 9^2 = 254m^2$.

The fraction of the plot on which the onion farm is located = $\frac{254}{256} = \frac{127}{128}$

6)



a) Determine the area of the shaded portion.

b) If 2 gallons of paint are required to paint an area of $15m^2$, how many gallons will be needed to paint the entire shaded portion?

Solution

a) For the small circle, r =12m.

Area of the small circle = $\pi r^2 = 3.14 \times 12^2 = 452m^2$

For the big circle r = $\frac{20}{2}$ =10m.

Area of big circle = $\pi r^2 = 3.14 \times 10^2 = 314m^2$

Area of the shaded portion = Area of big circle – Area of small circle = 452 – 314 = $138m^2$

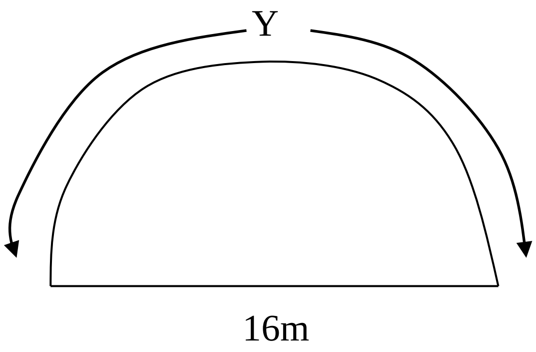
b) $5m^2$ = 2 gallons

$=> 138m^2 = \frac{138}{5} \times 2 = 55$gallons.

7) A plot of land which is in the form of a semi circle has a diameter of 16m. Determine

a) its perimeter or the distance round it.
b) its area.

Solution



a) Since d = 16m => r = 8m.

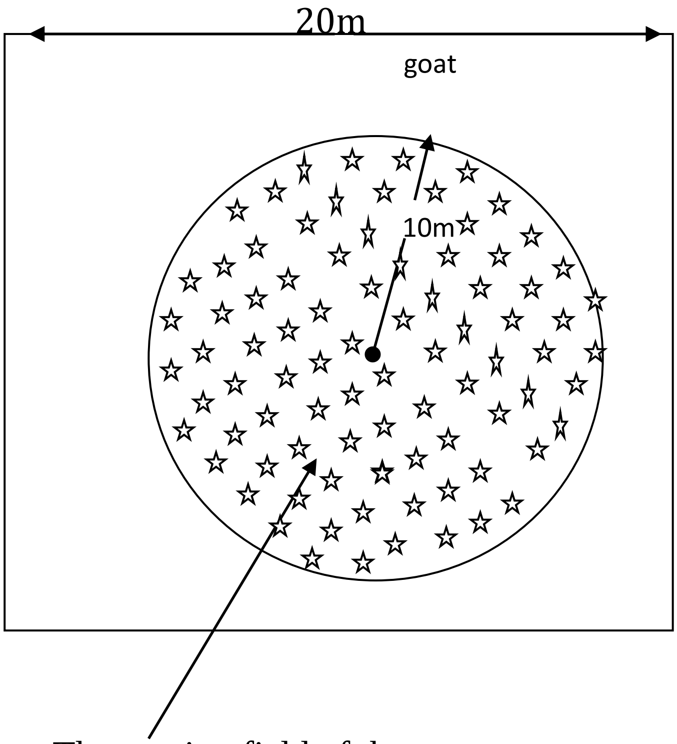
Since Y = half the circumference of the full circle, then Y $= \frac{1}{2} \times 2\pi r = \pi r = 3.14 \times 8 = 25$m.

The perimeter of the given figure = Y + 16 = 25 + 16 = 41m..

b) Since the area of a circle $= \pi r^2$, then the area of the semi circle $= \frac{1}{2} \times \pi r^2 = \frac{1}{2} \times 3.14 \times 8^2 =$ $100m^2$

Q8 A goat is tied at the centre of a square field, of side 20m long by a rope which is 10m long. Find the fraction of the field which the goat is able to graze on. [Take pie =22/7.]

Solution



The grazing field of the goat

Since the field has a square shape, which is of side 20m, then the area of this square field = side squared = $20^2 = 400m^2$.

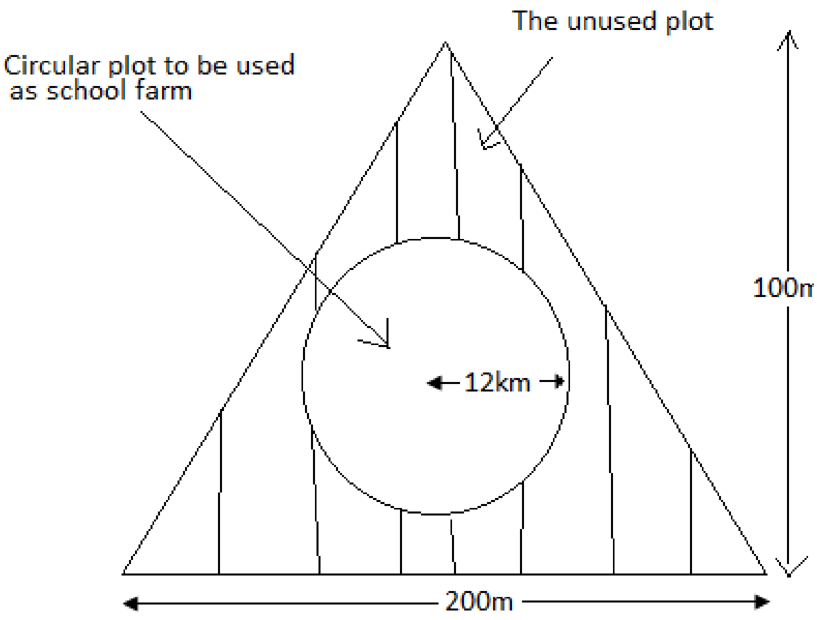
As can be seen from the diagram, the portion of the field which the goat will be capable of grazing upon is given by the area of the circular field of radius 10m, which has been shaded in the diagram.

The area of this grazing field = $\pi r^2 = \frac{22}{7} \times 10^2 = \frac{22}{7} \times 100 = \frac{2200}{7} = 314m^2$.

The fraction of the field which the goat is capable of grazing = $\frac{314}{400} = \frac{157}{200}$

8) The unused plot of a school is triangular in shape, with a base of length 200m and a height which is 100m. A portion of this plot which is circular in shape, and of radius 30m is to be used as the school farm, and the rest sold at a price of ¢5 per $20m^2$ of land. Determine the amount which must be expected from this sale.

Solution



The shaded portion is the plot of land that will be available for sale.

Area of the circular plot to be used as the school farm = $\pi r^2 = 3.14 \times 30^2 = 2826\text{m}^2$

The area of the triangular unused plot = $\frac{1}{2}$ base × height = $\frac{base \times height}{2}$

$= \frac{200 \times 100}{2} = 10000\text{m}^2$.

From the diagram, the area of the shaded portion = the area of the available plot for sale = area of the triangle – area of the circle = 10000 – 2826 = 7174m^2.

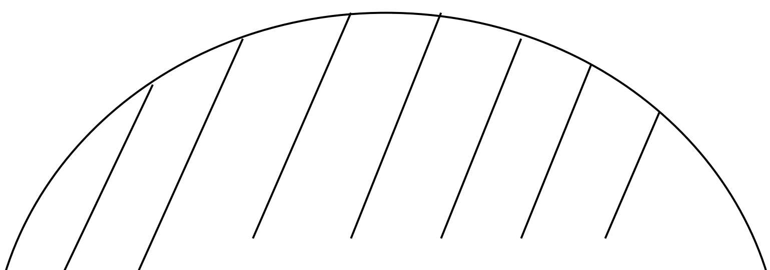
This was sold at ¢5 per 20m^2.

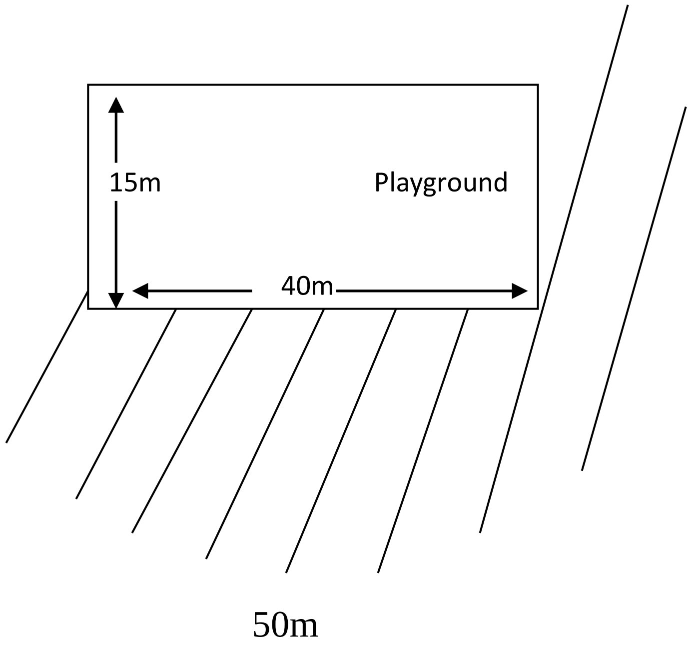
∴ 20m^2 = ¢5

$=> 7174\text{m}^2 = \frac{7174}{20} \times 5 =$ ¢1794.

10) The department of social welfare was given a piece of land, which was in the form of a circle, whose diameter is 50m. Within this plot the department had a rectangular play ground, which is of length 40m and breadth 15m built for children. The rest of the land was used as a cattle grazing ground. Determine the size of land for which the cattle could graze.

<u>Solution</u>





Radius of the circle = 50/2 =25m.

The shaded portion is the part of the plot on which the cattle could graze.

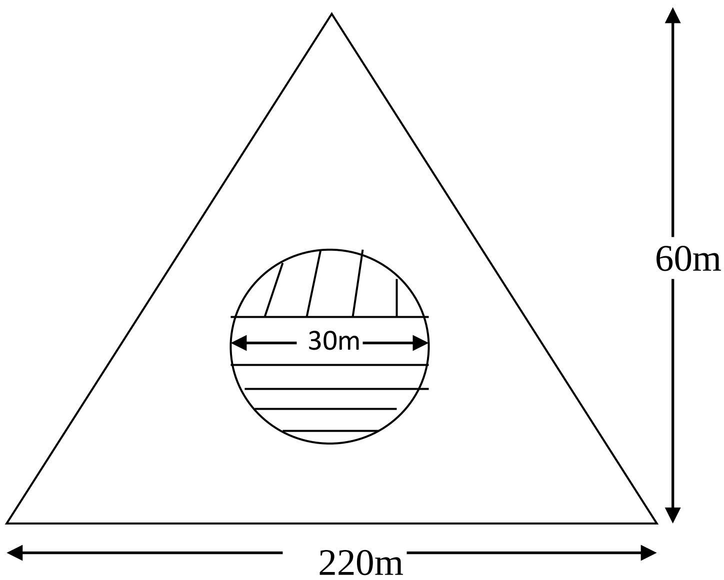
The area of the circle = πr^2 = 3.14 × 25^2 = 1963m^2

The area of the rectangle = length × breadth = 40 × 15 = 600m^2.

The shaded portion = the area on which the cattle could graze = area of the circle – the area of the rectangle = 1963 – 600 = 1363m^2

11) Mr. Abu's farm is in the farm of a triangle whose base is 220m and whose height is 60m. At the centre of the farm is located a circular wooden structure, of diameter 30m, in which the pigs are kept and allowed to roam. Determine the percentage of the land on which the pigs can roam.

<u>Solution</u>



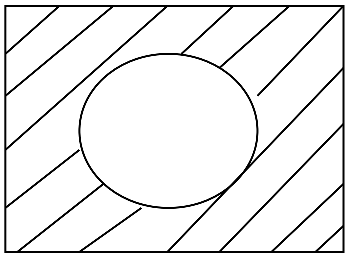
Area of the farm = the area of the triangle = $\frac{base \times height}{2} = \frac{220 \times 60}{2} = 6600m^2$

The area of the portion of the farm on which the pigs can roam = the area of the circle = $\pi r^2 = 3.14 \times 15^2 = 707m^2$

$\therefore$The percentage of the land on which the pigs are allowed to roam = $\frac{7o7}{6600} \times 100 =$ 11%

N/B: Since the diameter of the wooden structure is 30m, then its radius is 15m.

11) 6cm



The above diagram shows a circle which is located within a square whose side is 6cm. If the area of the shaded portion is $16cm^2$, determine the radius of the circle.

<u>Soln</u>

Area of the square = $6 \times 6 = 36cm^2$

Area of the shaded portion = $16cm^2$

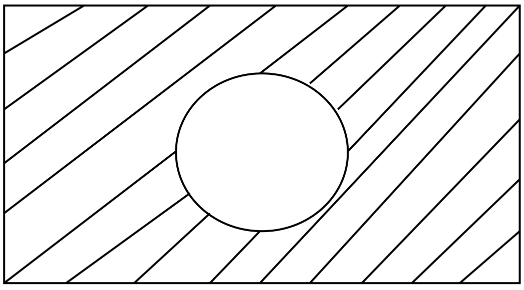
Area of the circle = the area of the square – the area of the shaded portion = $36 - 16 = 20cm^2$

Since the area of a circle = $\pi r^2 \Rightarrow \pi r^2 = 20$, $\Rightarrow 3.14 \times r^2 = 20$, $\Rightarrow r^2 = \frac{20}{3.14}$, $= 6.4$,

$\Rightarrow r^2 = 6.4 \Rightarrow r = \sqrt{6.4} = 2.5$.

$\therefore$ The radius of the circle = 2.5cm.

12)



The figure shows a circle which is located within a rectangle of length 20m and breadth 10cm. Given that the area of the shaded portion is $40m^2$, find the diameter of the given circle.

Soln

Area of rectangle = L×B = 20×10 = $200m^2$

Area of the shaded portion = $40m^2$

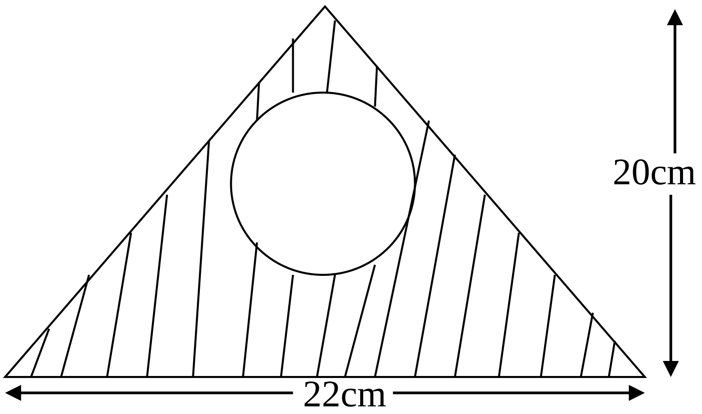
Area of the circle = Area of the rectangle – the area of the shaded portion = 200 – 40 = $160m^2$

Since area of a circle = πr^2, then $\pi r^2 = 160 \Rightarrow 3.14 \times r^2 = 160$, $\Rightarrow r^2 = \frac{160}{3.14}$,

$\Rightarrow r^2 = 51 \Rightarrow r = \sqrt{51} = 7.14$m.

Diameter = 2r = 2(7.14) = 14.3m.

13)



Determine the radius of the given circle, given that the area of the shaded portion is $100cm^2$.

Soln

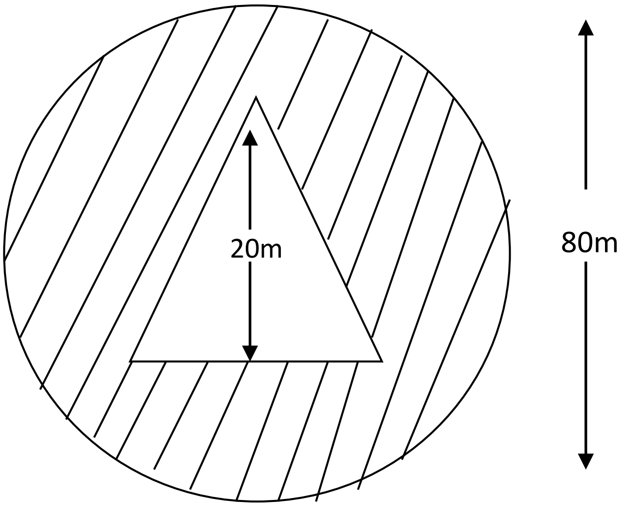
The area of the triangle $= \frac{1}{2} \times$base×height $= \frac{1}{2} \times 22 \times 20 = 220cm^2$.

The area of the circle = Area of the triangle – area of the shaded portion = 220 – 100 = $120cm^2$.

Since area of circle = $\pi r^2 \Rightarrow \pi r^2 = 120$, $\Rightarrow 3.14 \times r^2 = 120$, $\Rightarrow r^2 = \frac{120}{3.14}$,

$\Rightarrow r^2 = 38 \Rightarrow r = \sqrt{38}$, $\Rightarrow r = 6.2$cm.

14)



The given diagram shows a triangular field of height 20m, which is situated within a circular field of diameter 80m. If the area of the shaded portion is $3000m^2$, determine the length of the base of the triangular field.

<u>Soln</u>

Area of the circular field = $\pi r^2 = 3.14 \times 40^2 = 5024m^2$

Let b = the base of the triangular field. Then the area of the triangular field = $\frac{1}{2} \times b \times h = \frac{1}{2} \times b \times 20 = 10b$

Area of the triangular field = area of the circle – the area of the shaded portion = $5024 - 3000 = 2024m^2$.

Since the area of the triangular field = 10b, then 10b = 2024 $\Rightarrow b = \frac{2024}{10} \Rightarrow b = 202.4m$

The base of the triangle = 202.4m.

15)

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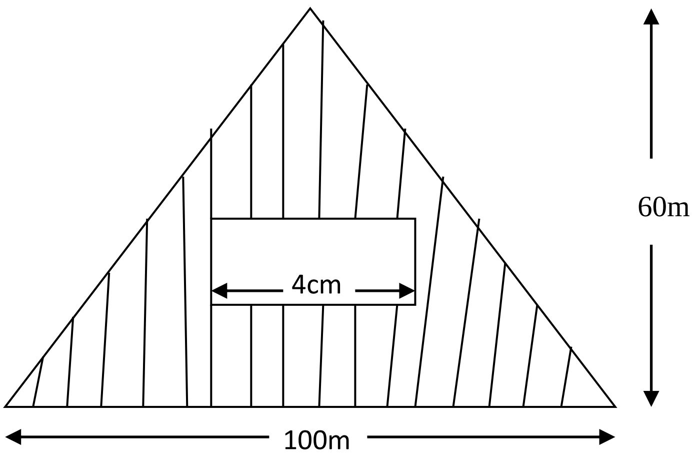


The given diagram shows a rectangular children playing ground of length 40m, which is located within a triangular field of base 100m and height 60m. If the area of the shaded portion is $2000m^2$, determine the breadth of the rectangular field or playing ground.

<u>Soln</u>

The area of the triangular field = $\frac{1}{2}\times$ base $\times$ height = $\frac{1}{2}\times100\times60 = 3000m^2$

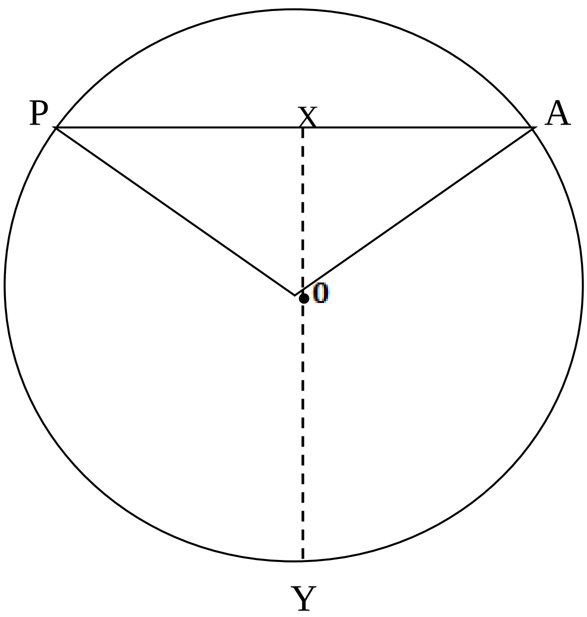
Let B = the breadth of the rectangular field => area of rectangular field= L×B = 40×B = 40B.

Area of the rectangular field = Area of the triangular field – the area of the shaded portion = 3000 – 2000 = $1000m^2$

=> 40B = 1000 => B = $\frac{1000}{40}$ =>B = 25m.

Circle Properties: Chords, Arcs and Sector Areas:

The Chord Properties:



The given figure shows a circle with its centre at a point 0.

The line PA is a chord which is at right angle to the line XY; i.e. line PA is perpendicular to line XY.

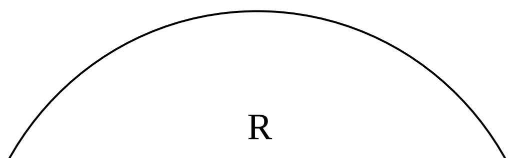
Since XY divide AP into two equal parts, it is referred to as the perpendicular bisector of AP, and for this reason PX=XA.

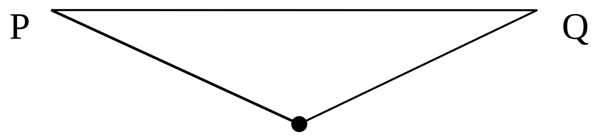
Q1) A chord is 8cm long and is drawn in a circle of radius 6cm. Calculate

a) the distance of the chord from the centre of the circle

b) the angle it subtends at the centre.

N/B

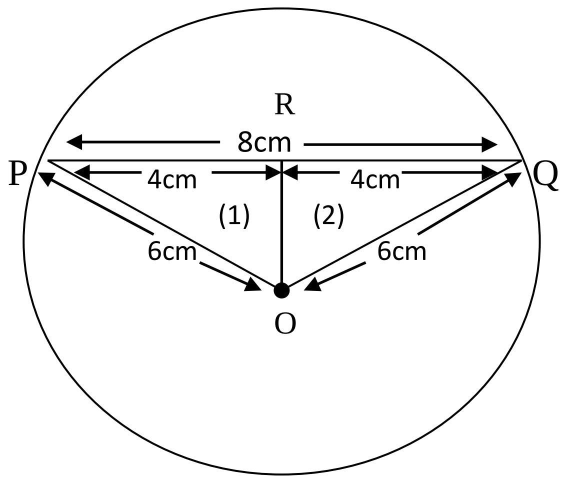




In the above figure, the chord is represented by the line PQ, which implies that PQ = 8cm. Since line RO is the perpendicular bisector of chord PQ, then it will divide it into exactly two equal part, and for this reason RQ = 4cm and PR = 4cm Also with reference to PO and QO, each of them is a radius, and for this reason PO = 6cm and QO = 6cm.

<u>Soln</u>

a)



The distance of the chord from the centre is represented by RO, and for this reason it is the length or the distance that we are required to calculate.
Now consider any of the two triangles for which we shall consider triangle (1) in this case i.e.

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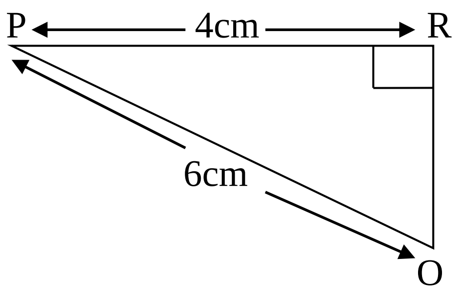
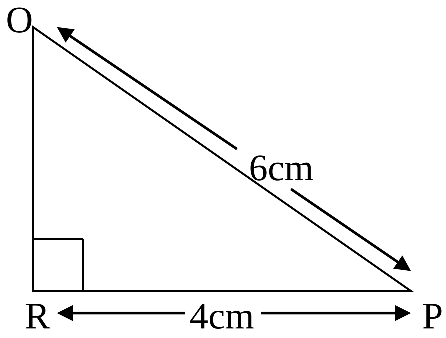


Rotate it to get



From Pythagoras theorem,

$OR^2 + RP^2 = OP^2$

$\Rightarrow OR^2 + 4^2 = 6^2$

$\Rightarrow OR^2 = 6^2 - 4^2$

$\Rightarrow OR^2 = 36 - 16 = 20$

$\Rightarrow OR = \sqrt{20}$, $\Rightarrow OR = 4.47$cm,

$\Rightarrow$ the chord is approximately 4.47cm away from the centre of the circle. To determine the angle subtended at the centre by the chord. We shall draw another diagram for proper analysis

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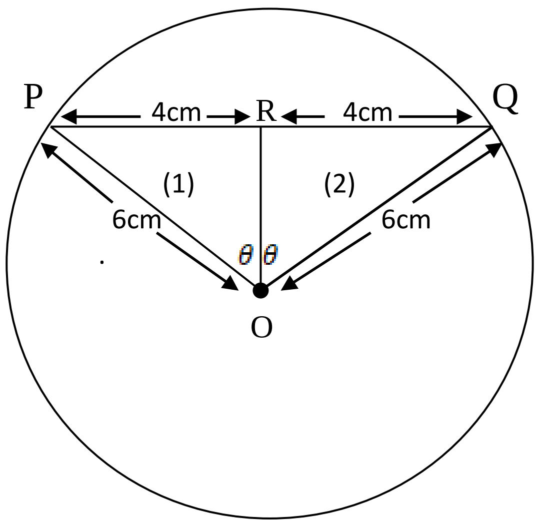
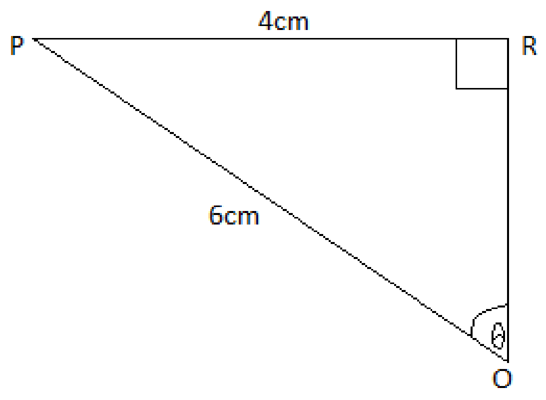
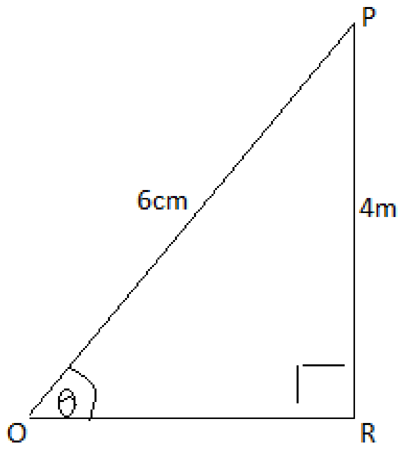


-The angle subtended at the centre of the circle is represented by

$\theta + \theta = 2\,\theta$

-Now consider figure (1) i.e.





Sin $\theta = \frac{4}{6}$ = 0.666 => θ = $\sin^{-1}$ 0.666, => θ = 83^0

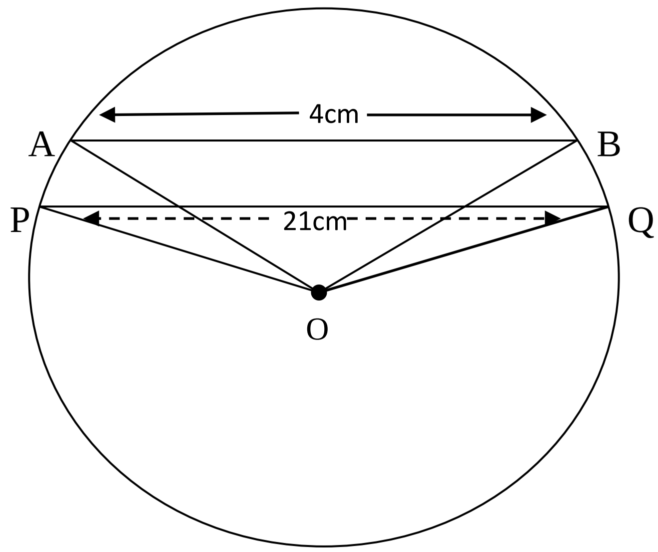
The angle subtended at the centre = 2 θ =2(83) = 2 × 83= 166.

Q2) A circle whose radius is 12cm, has two parallel chords of length 4cm and 21cm.

(a) Calculate the distance of each chord from the centre of the circle.

(b)Determine the distance between these two chords.

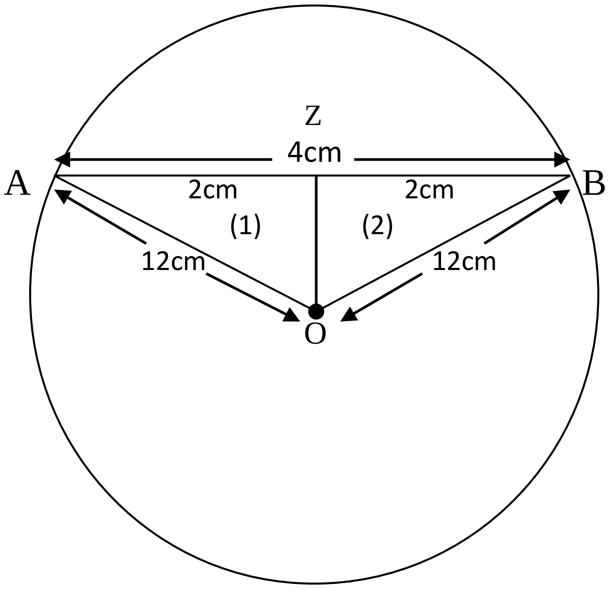
<u>Soln</u>



-Let the two chords be represented by AB and PQ as shown in the diagram.

-We shall consider each one separately.

-Now considering the 4cm length chord i.e

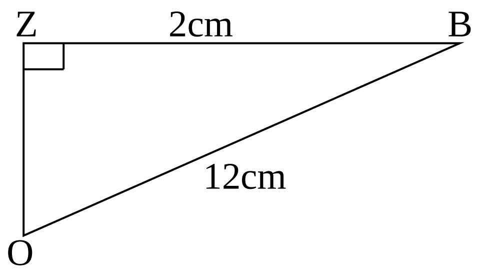


-The distance of the chord AB from the centre is equal to the length ZO

-Since OB and AO are radii, each one of them is equal to 12cm, which is the radius of the given circle.

-Now consider triangle (2)

i.e



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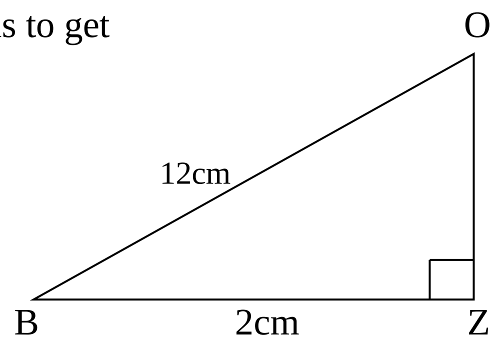
Rotate this to get



From Pythagoras theorem

$OZ^2 + BZ^2 = BO^2$

$\Rightarrow 0Z^2 + 2^2 = 12^2$

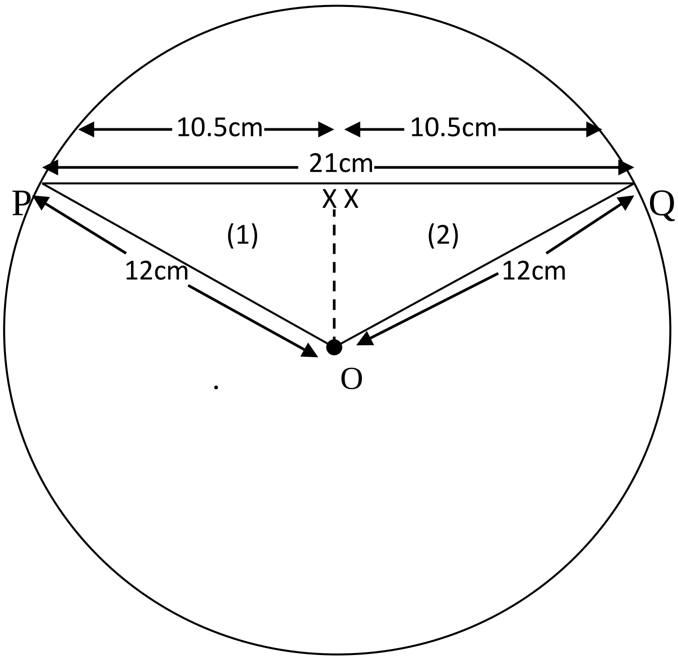
$\Rightarrow OZ^2 = 12^2 - 2^2$

$\Rightarrow OZ^2 = 144 - 4 = 140$

$\Rightarrow 0Z = \sqrt{140} = 11.8$

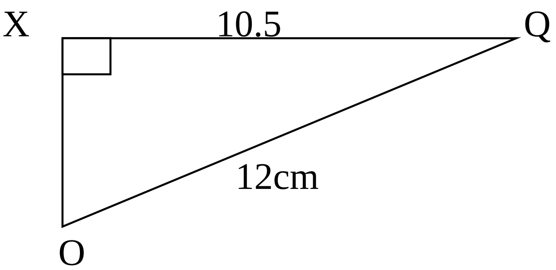
$\Rightarrow$ The distance of the 4cm chord from the centre of the circle = 11.8cm.

We now consider the 21cm long chord i.e.

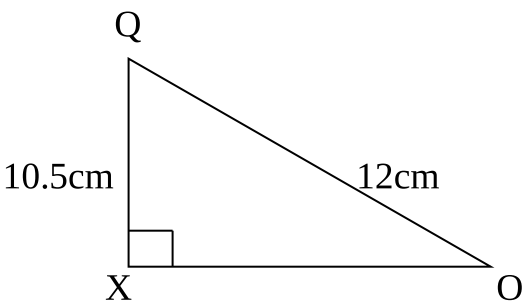


The distance of the 21cm chord from the centre is equal to XO.

Now consider triangle (2) i.e.



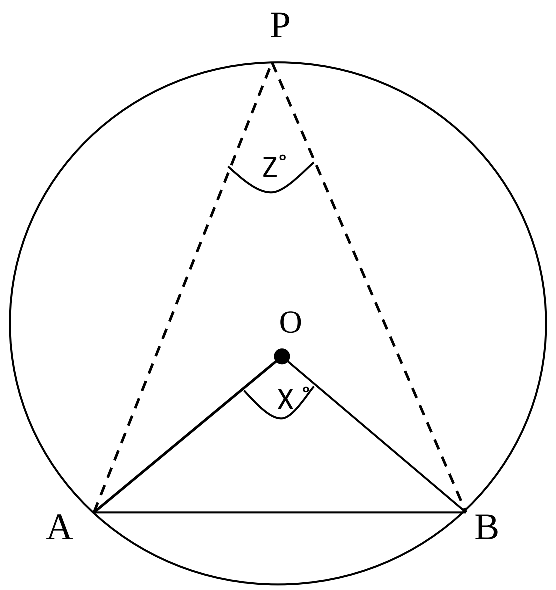
Rotate this to get



From Pythagoras theorem
$OX^2 + XO^2 = OQ^2$

$\Rightarrow 10.5^2 + XO^2 = 12^2$
$\Rightarrow XO^2 = 12^2 - 10.5^2$
$\Rightarrow XO^2 = 144 - 110.25$,
$\Rightarrow XO^2 = 33.75$,
$\Rightarrow XO = \sqrt{33.75}$
$\Rightarrow XO = 5.8$cm.
$\Rightarrow$ The distance of the 21cm chord from the centre is 5.8cm.
The distance between the two chords = 11.8cm – 5.8cm = 6cm.

The Angle Property of Chords:

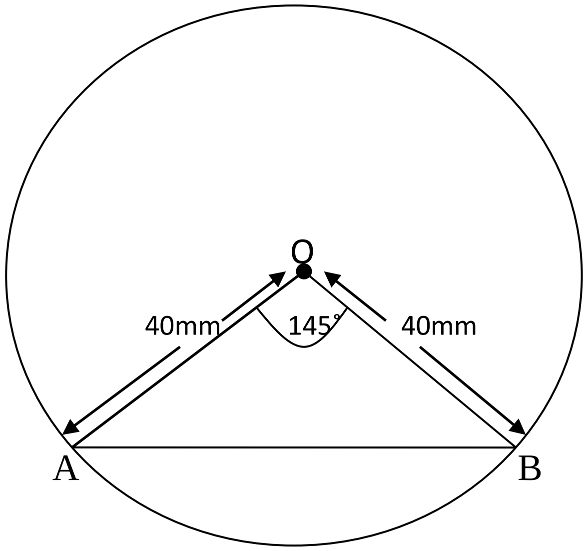


- The given figure shows a circle with its centre at O, with AB being a chord.
- The angle AOB = X°, and it is referred to as the angle subtended at the centre by the chord AB.
- The angle APB = Z°, and it is referred to as the angle subtended by the chord AB at the circumference.
- The angle subtended by a chord at the centre, is always twice that subtended at the circumference.

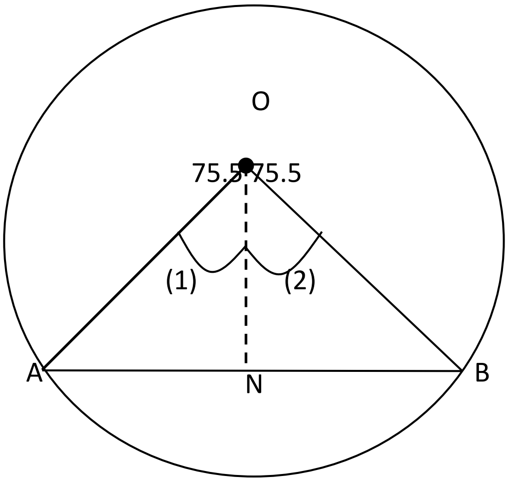
- For example if the angle subtended at the circumference is 50˚, then that subtended at the centre = 100˚.
- Also if the angle subtended at the circumference is 20˚, then that subtended at the centre = 40˚.
- Finally if the angle subtended at the centre is 60˚, then that subtended at the circumference = 30˚

Q1) A chord AB subtends an angle of 145˚, at the centre of a circle where radius is 40mm. Calculate the length of AB.

Soln



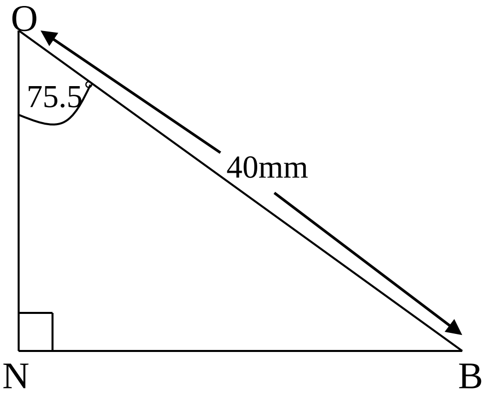
- By drawing the perpendicular bisector of AB, our diagram will look as shown next



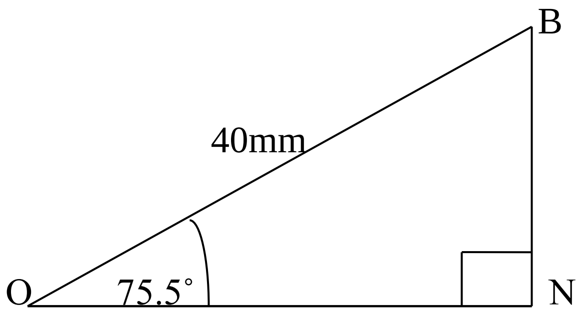
- N/B: Since ON is the perpendicular bisector of AB, then it will divide the angle 145˚ into two equal parts of 75.5˚ each as shown in the diagram.

- The length of the chord we are supposed to determine i.e. chord AB = twice the length of AN or twice the length of NB.
- In other words AB = 2 AN or AB = 2NB, since the perpendicular bisector ON, divides AB into two equal parts, due to which AN = NB.

Now consider triangle (2)

i.e 

Rotate it to get



NB = 40 sin 75.5° = 40 × 0.968,

=>NB = 38.72.

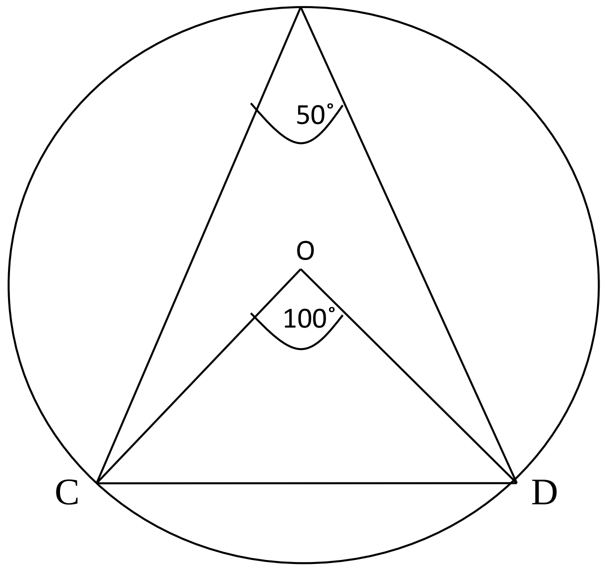
But AB = 2NB = 2(38.72) = 77.44,

=> the length of the chord AB = 77.44mm

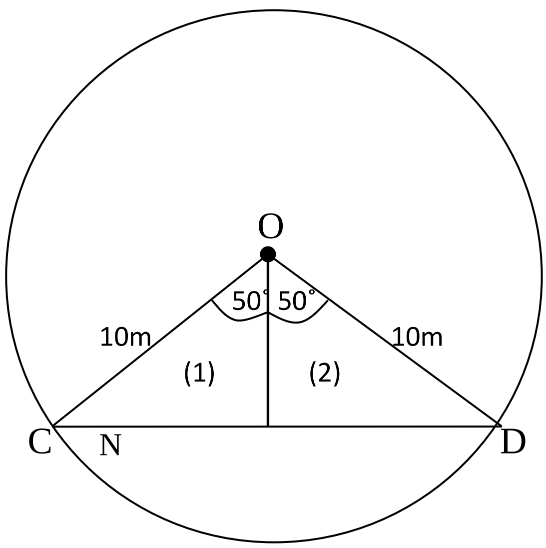
Q2) A chord CD subtends an angle of 50° at the circumference of a circle of radius 10m. Calculate the length of this chord.

NB: Since the angle subtended at the circumference = 50˚, then that subtended at the centre = 2 × 50 = 100˚.

<u>Soln</u>



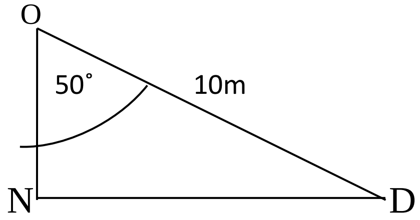
By considering only the angle subtended at the centre, and drawing the perpendicular bisector of CD we shall get the next figure:



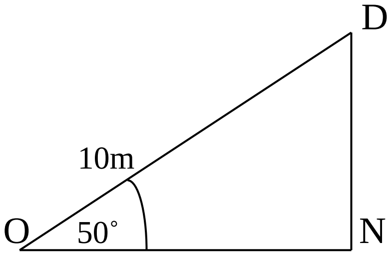
ON = the perpendicular bisector

Now considering triangle (2) or $\triangle$NOD.

i



Rotate to get



ND = 10 sin 50˚ = 10 × 0.76 = 76m.

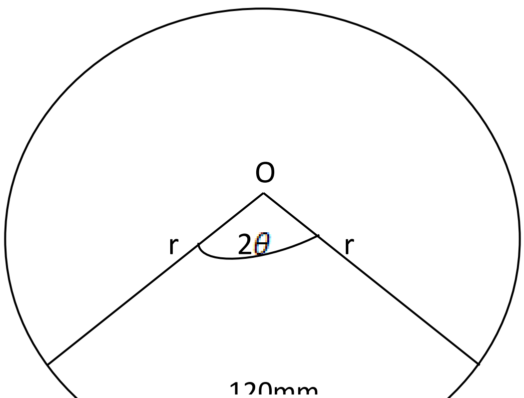
But the length of the chord = CD and CD = 2 × ND = 2 × 76 = 152,

=> the length of the chord = 152m.

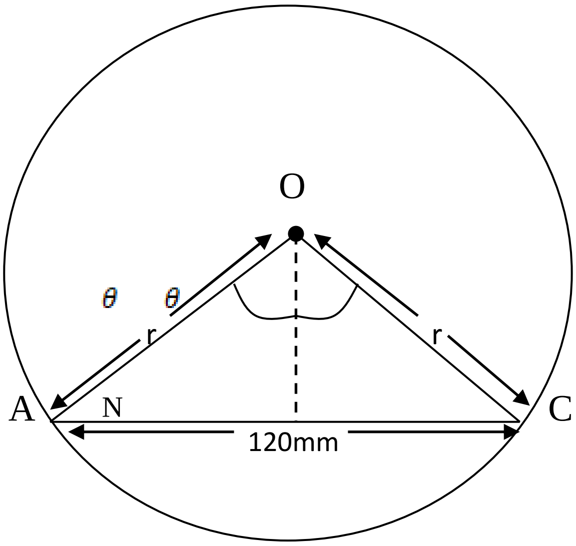
Q3) A chord of a circle is 120mm long. The perpendicular distance of this chord from the centre of the circle is 25mm. Calculate

a) the radius of the circle.
b) the angle subtended by the chord at the centre of the circle.
c) the angle subtended at the circumference.

<u>soln</u>



Let AC be the chord and let 2θ = the amgle subtended at the centre. Also let the radius of the circle = r, as shown in the diagram

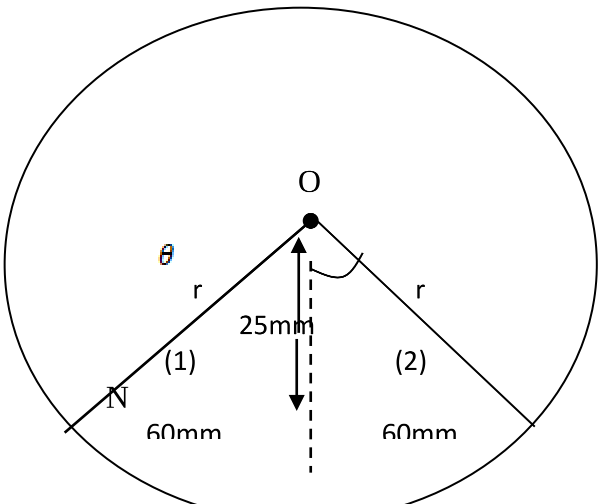


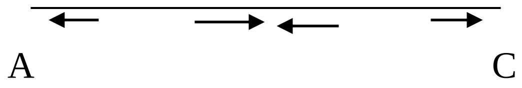
N/B

- In this figure, since ON is the perpendicular bisector of the chord AC, then it will bisect the angle subtended at the centre i.e 2θ into two equal parts of θ.
- Now the perpendicular distance of the chord from the centre of the circle is 25mm.
- This implies that the perpendicular bisector ON is of length 25mm.

- Lastly since the perpendicular bisector will divide the chord AC into two equal parts, then AN = 60mm and NC = 60mm.

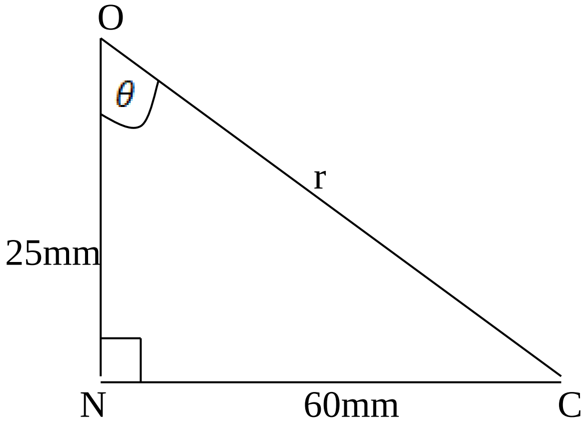
- The facts are indicated in the next figure:



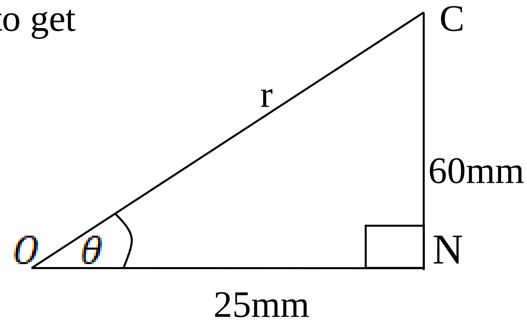


Now considering triangle (2)

i.e



Rotate this to get



$\tan\theta = \frac{60}{25}$ = 2.4, and since

$\tan\theta = 2.4$, => $\theta = \tan^{-1} 2.4$, => $\theta(teta) = 68^\circ$.

a) From the diagram, $\cos\theta = \frac{25}{r}$ => $\cos 68^\circ = \frac{25}{r}$,

=> r cos 68 = 25 => $r = \frac{25}{\cos 68}$,

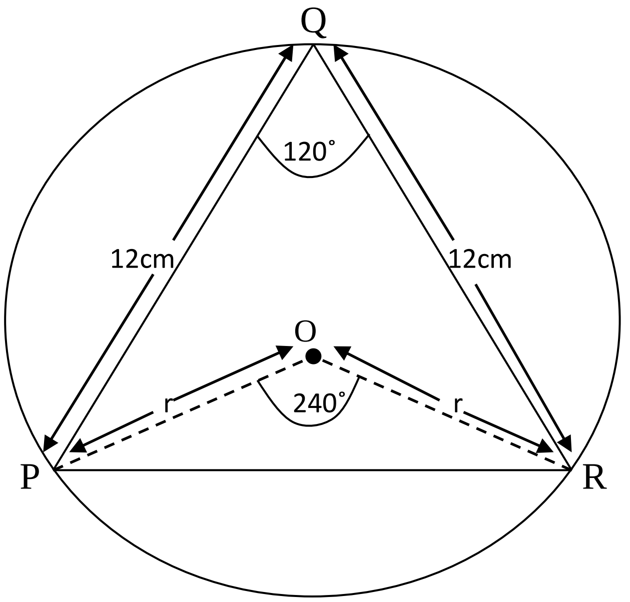
=> $r = \frac{25}{0.36}$ = 66.6mm

a) The angle subtended at the centre =2 (teta) = 2(68) =136.

b) The angle subtended at the circumference = half that subtended at the centre = 136/2 = 68.

Q4) Two equal chords PQ and QR, each 12cm long meet at a point Q on a circle, such that angle PQR = 120˚. Find the radius of the circle.

<u>Soln.</u>

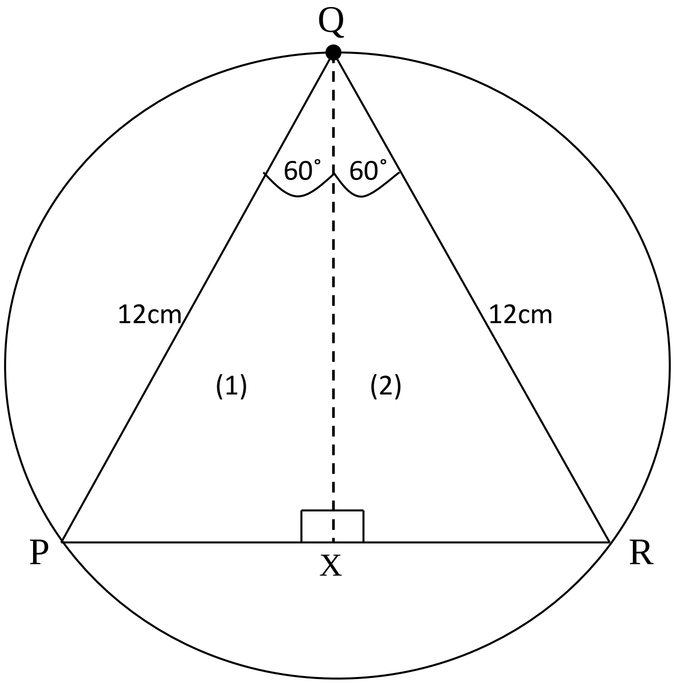


From the figure drawn, the angle subtended by the chord PR at the circumference = 120,˚ => the angle subtended by the same chord at the centre O, is equal 2× 120˚ = 240˚

We must first determine the length of chord PR.

Now considering only an aspect of the diagram

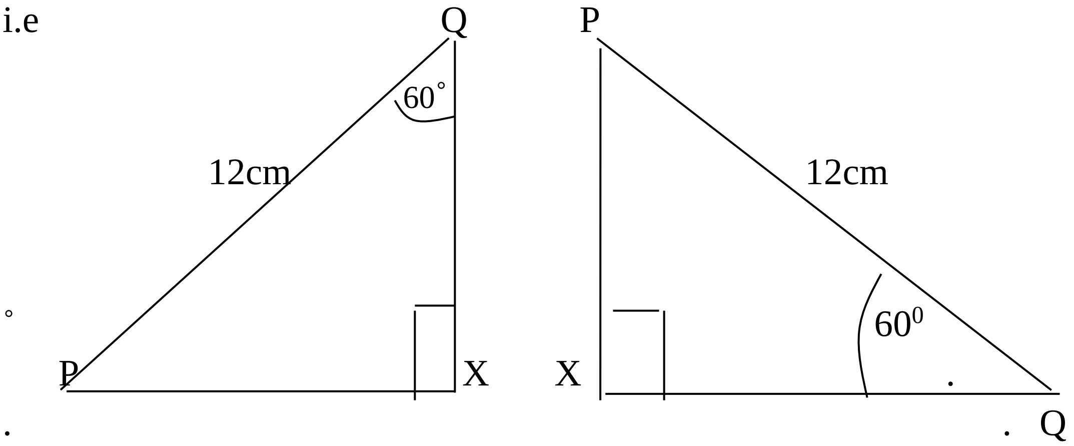
i.e.



Let QX = the perpendicular bisector of PR. Then PR = 2PX.

Now consider triangle (1)

i.e

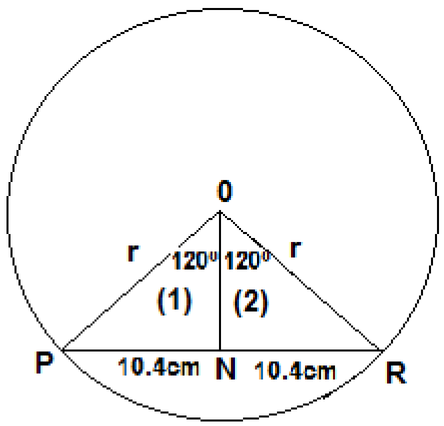


PX = 12 Sin 60˚ = 12 × 0.866 = 10.4,

=> PR = 2 PX = 2× 10.4

=> PR = 20.8cm

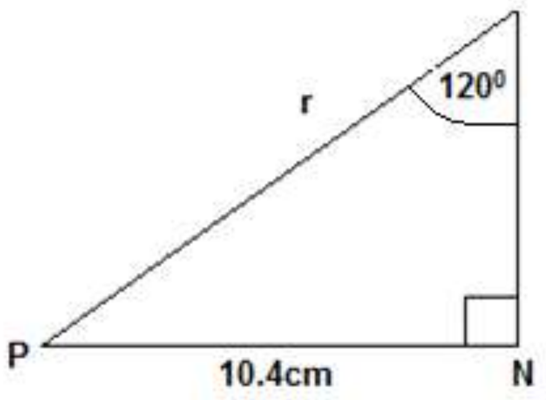
We now consider the second aspect of the diagram, i.e.



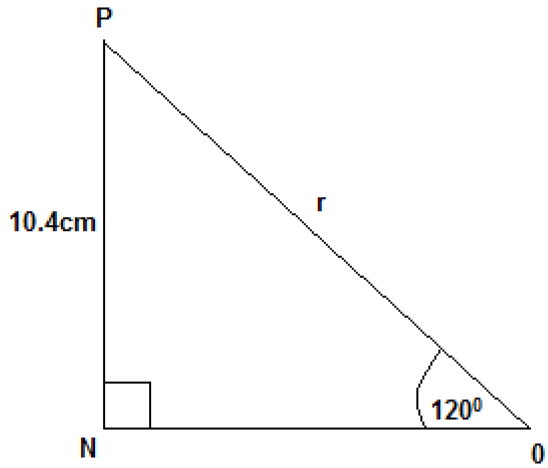
Let ON = the perpendicular bisector of PR

Then since 20.8cm, PN = 10.4cm and NR = 10.4cm

Consider figure (1)



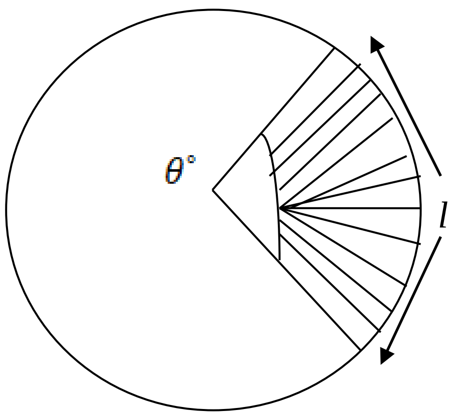
Rotate this to get



Since r sin 120^0 = 10.4

=> r = $\frac{10.4}{\sin 120} = \frac{10.4}{1.73} = 6cm.$

Arcs and Sectors:



- In this figure, the length l is an arc, which is defined as part of the circumference.
- The shaded portion is known as a sector which is the region between two radii, and the angle between these two radii which is θ, is known as the sector angle.
- The following facts must be noted, and these are:
  1. The length of the arc which is equal to l, is proportional to the sector angle i.e $l \alpha \theta$.
  2. The circumference of the circle is proportional to the total angle, within the circle, which is 360˚ i.e C α 360^0, where C = the circumference.
- From these two given facts, then we can conclude that $\frac{L}{C} = \frac{\theta}{360}$.

$=> l = \frac{\theta}{360} \times \text{C}$

$=> l = \frac{\theta}{360} \times$ Circumference.

But since C, or the circumference = 2πr, then $l = \frac{\theta}{360} \times 2\pi r$.

Q1) A circle has a radius of 70mm. If its sector angle = 60˚, find the length of the arc associated with this sector. [Take = 3.14]

<u>Soln.</u>

r = 70mm and θ = 60.˚

From $l = \frac{\theta}{360} \times 2\pi r$

$=> l = \frac{60}{360} \times 2 \times 3.14 \times 70$

= 73.3mm.

Q2) A circle has a sector whose sector angle is 180˚, and whose arc length is 10cm. Determine its circumference.

<u>Soln.</u>

θ = 180˚ and l = 10cm.

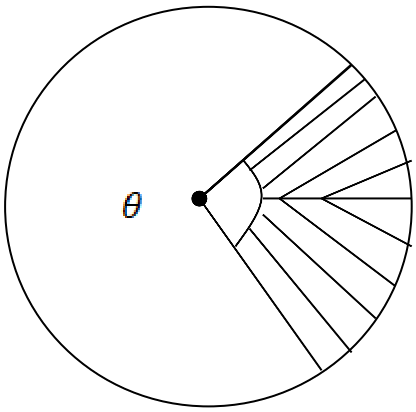
From $l = \frac{\theta}{360} \times C$

$=> 10 = \frac{180}{360} \times C => 10 \times 360^{\circ} = 180 \times C,$

$=> 3600 = 180C, => C = \frac{3600}{180} = 20,$

=> the circumference = 20cm.

The area of a sector and the sector angle:



- Consider the given figure
- The shaded portion is what is referred to as the sector area.
- If the sector angle = θ°, then we can say that

$$\frac{Sector\ area}{Area\ of\ a\ circle} = \frac{\theta}{360^{\circ}}$$

But the area of a circle = πr^2 =>

$$\frac{Sector\ area}{\pi r^2} = \frac{\theta}{360^{\circ}}$$

=> Sector area = $\frac{\theta}{360^{\circ}} \times \pi r^2$ or Sector area = $\frac{\theta}{360^{\circ}} \times$ area of the circle

Q1) Determine the sector area of a circle whose diameter is 16cm, given that the sector angle is 10°. [Take $\pi = 3.14$].

Soln.

D = 16cm => r = 8cm.

Sector area = $\frac{\theta}{360^{\circ}} \times \pi r^{2}$

$= \frac{10}{360^{\circ}} \times 3.14 \times 8^{2} = \frac{10}{360^{\circ}} \times 3.14 \times 64 = 5.58cm^{2}$

Q2) Find the sector angle of a sector whose area is $70cm^2$, if the radius of sector or the circle is 22cm.

<u>Soln.</u>

Sector area = $\frac{\theta}{360^{\circ}} \times \pi r^{2}$

$=> 70 = \frac{\theta}{360^{\circ}} \times 3.14 \times 22^{2}$

$=> 70 \times 360 = 1520\theta, => 25200 = 1520\theta$

$=> \theta = \frac{25200}{1520} = 17^{\circ}$.

Q2) The sector area of a circle is given as $40cm^2$. If the sector angle is 36°, find the radius of the circle.

<u>Soln.</u>

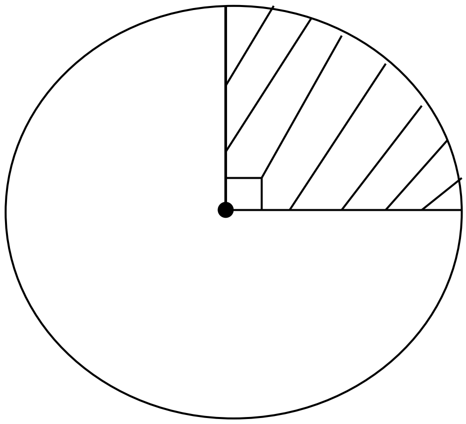
$\frac{Sector\ area}{Area\ of\ a\ circle} = \frac{\theta}{360^{\circ}}$

$=> \frac{40}{\pi r^{2}} = \frac{36^{\circ}}{360}, => \pi r^{2} \times 36^{\circ} = 40 \times 360,$

$=> 3.14 \times r^{2} \times 36 = 14400,$

$=> 113r^{2} = 14400, => r^{2} = \frac{1440}{113} = 127 => r = \sqrt{127} = 11.3cm.$

The area of a quadrant:



- The shaded portion is what is referred to as a quadrant, which may be regarded as a sector, whose sector angle = 90°

The area of a quadrant = $\frac{\theta}{360} \times \pi r^2 = \frac{\theta}{360^\circ} \times$ area of circle

$= \frac{90}{360} \times \pi r^2 = \frac{1}{4}\pi r^2$.

Q1) A circle has a radius of 6cm. Determine the area of its quadrant.

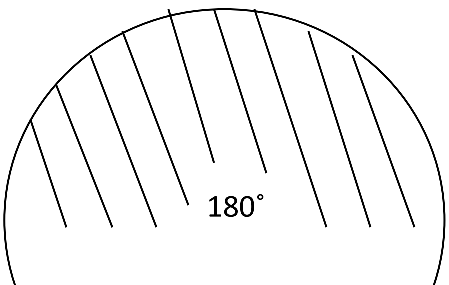
Soln.

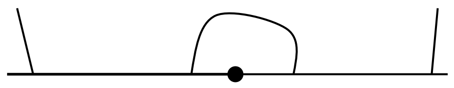
Area of quadrant = $\frac{1}{4}\pi r^2$

$= \frac{1}{4} \times 3.14 \times 6^2$

$= 28.3\text{cm}^2$

The area of a semi circle:





The shaded portion represents the area of a semi circle, which may be regarded as the area of a sector, whose sector angle is 180˚.

The area of a semi circle = $\frac{\theta}{360} \times \pi r^2$

$= \frac{180}{360} \times \pi r^2 = \frac{1}{2} \times \pi r^2$

- The area of a semi circle = half the area of the circle concerned.

Q2) A circle has a diameter of 18cm. Determine the area of its semi circle.

<u>Soln.</u>

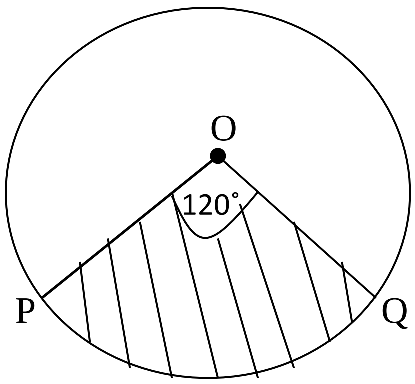
D = 18cm => r = 9cm and θ = 180˚..

Area of semi- circle = $\frac{\theta}{360} \times \pi r^2 = \frac{180}{360} \times \pi r^2$

$= \frac{180}{360} \times 3.14 \times 9^2 = 0.5 \times 3.14 \times 81$

$= 127\text{cm}^2$.

Q3)

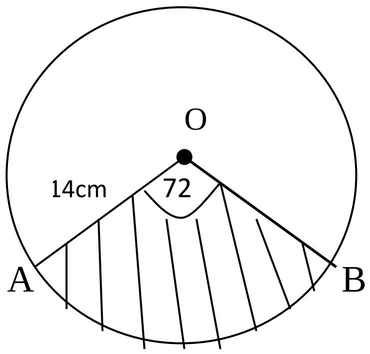


If the area of the given circle is 48cm^2, determine the area of the shaded sector OPQ.

<u>Soln.</u>

Area of sector = $\frac{\theta}{360}$ × area of circle = $\frac{120}{360}$ × 48 = 16cm^2

Q4)



The given diagram shows a circle, with centre O and radius 14cm. The shaded region AOB is a sector with angle AOB = 72˚. Find

i. the length of minor arc AB.

ii. the area of the shaded sector AOB $\left[\text{Take } \pi = \frac{22}{7}\right]$

<u>Soln.</u>

i. The length of the minor arc

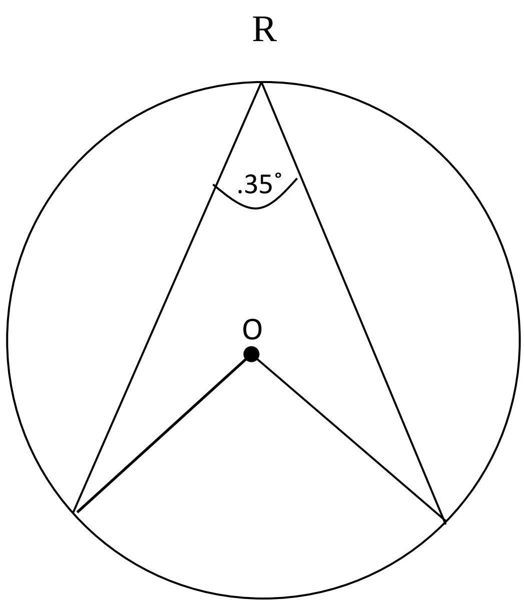
AB = $\frac{\theta}{360^\circ}$ × 2πr = $\frac{72}{360^\circ}$ × 2 × $\frac{22}{7}$ × 14

= 17.6cm

ii The area of the shaded sector AOB = $\frac{\theta}{360^\circ} \times \pi r^2$

= $\frac{72}{360} \times \frac{22}{7} \times 14^2$ = 12

Q5)

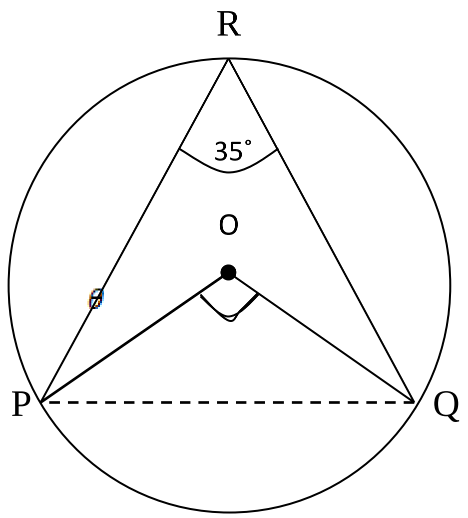


P Q

In the given diagram, P, Q and R are points on the circle with centre O and diameter 14cm. If angle PRQ = 35˚, find the length of

i. the minor arc PQ.
ii. the chord PQ.[Take $\pi = 3.142$]

Soln.



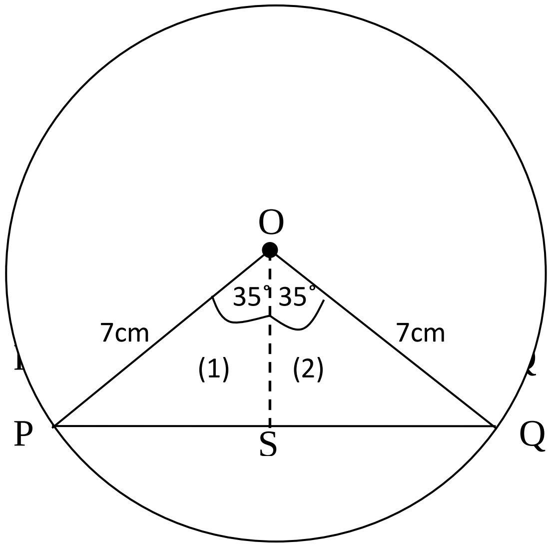
Let θ = the angle subtended at the centre by the chord PQ.

Since the angle subtended at the centre is twice that subtended at the circumference

=> $\theta = 2 \times 35^\circ = 70^\circ$. Since diameter = 14cm => radius, r = 7cm

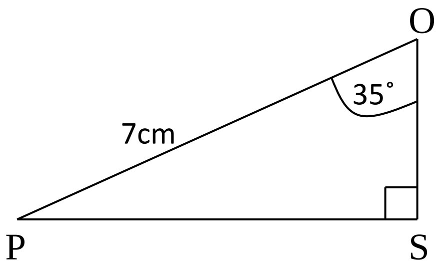
i)The length of the minor arc PQ = $\frac{\theta}{360^\circ} \times 2\pi r$

$= \frac{70}{360} \times 2 \times 3.142 \times 7 = 8.6\text{cm}$



The length of chord PQ = 2 × PS = 2PS

Consider triangle (1) i.e



PS = 7 Sin 35° = 7 × 0.5736 = 4,

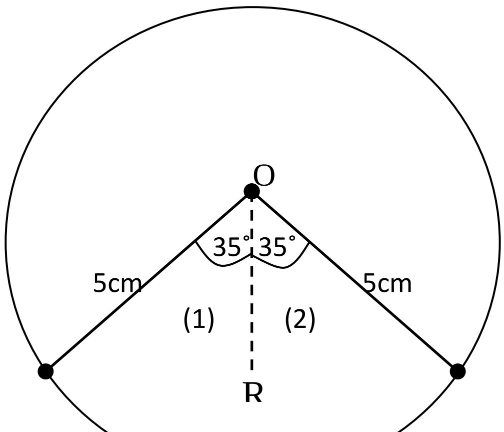
=> length of the chord PQ = 2PS = 2(4) = 8cm.

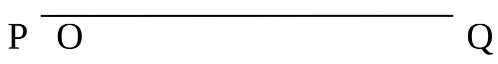
Q6) A chord PQ of a circle of radius 5cm, subtends an angle of 70° at the centre, O. Find

a) the length of the chord PQ.
b) the length of the arc PQ.
c) the area of the sector POQ.
d) the area of the minor segment cut off by PQ.

<u>Soln.</u>

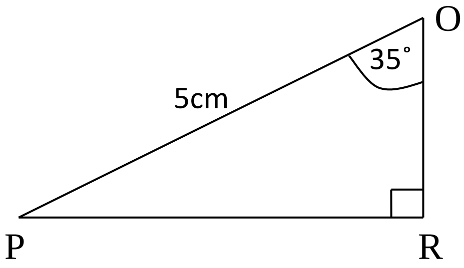
a)





The bisector OR will divide the angle 70˚ into two parts of 35˚ each

Consider triangle (1) i.e the next diagram shown:
PR = 5 sin 35^0
= 5$\times$ 0.05736
= 2.9



The length of chord PQ = 2 × PR = 2 × 2.9 = 5.8cm

b) The length of the arc PQ = $\frac{\theta}{360^{\circ}} \times 2\pi r$, where θ = the angle subtended at the centre = 70˚

=> length of arc PQ = $\frac{70}{360^{\circ}} \times 2 \times 3.14 \times 5$ = 6cm

c) The area of sector POQ = $\frac{\theta}{360^{\circ}} \times \pi r^2 = \frac{70}{360^{\circ}} \times 3.14 \times 5^2$ = 15.3cm^2

d) The area of the minor segment cut off by PQ = area of sector POQ – area of triangle POQ

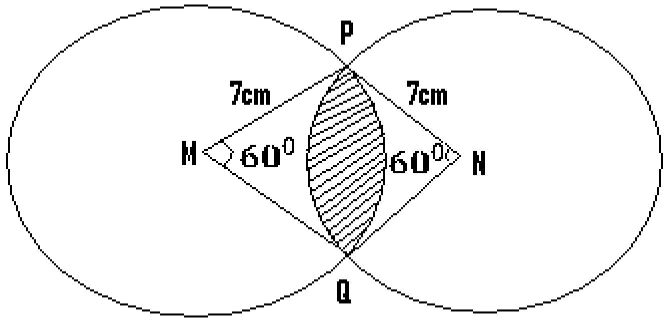
Area of sector POQ = 15.3cm^2

Area of triangle POQ = $\frac{1}{2} r^2 \sin\theta = \frac{1}{2} \times 5^2 \times \sin 70^{\circ}$ = 11.75

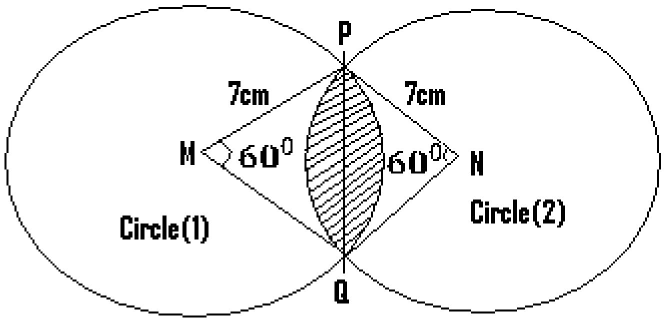
=> Area of minor segment = 15.3 – 11.75 = 3.55cm^2

Q7) In the diagram, M and N are the centres of two circles of equal radii of7cm. These circles intercept at P and Q. If <PMQ = <PNQ = 60^0, *calculate the area of the shadede portion.*

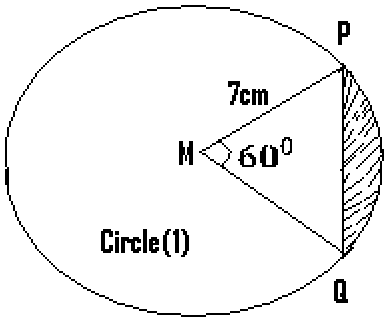
[Take $\pi = \frac{22}{7}$].

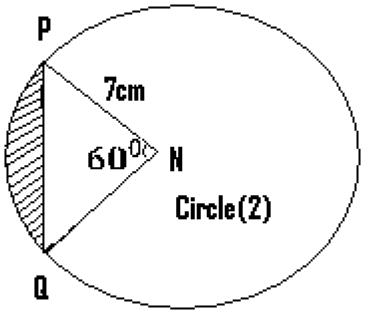


N/B



- Draw a line to join the points P and Q, so as to divide the shaded portion as shown.
- Label the first circle as circle (1) and the second circle as circle (2).
- Finally separate the two circles as shown next





- By so doing the shaded portion whose area we are supposed to find, had been divided into two parts. i.e the one found within circle(1) as well as that found in circle(2).
- Considering circle(1), the area of the shaded portion or segment

  PQ = $\frac{\theta}{360} \times \pi r^2$

$= \frac{60}{360} \times \frac{22}{7} \times 7^2 = 26.7cm^2$

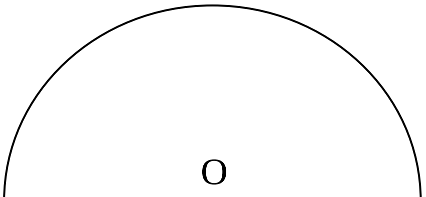
Considering circle(2), the area of the shaded portion or the segment

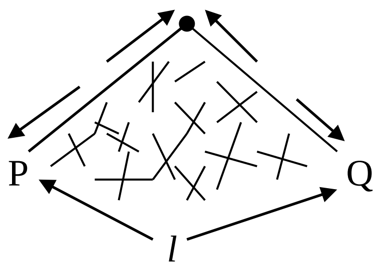
PQ = $\frac{60}{360} \times \pi r^2$

$= \frac{60}{360} \times \frac{22}{7} \times 7^2 = 26.7cm^2$

The required shaded area = the shaded area of circle(1) + the shaded area of circle(2) = 26.7 + 26.7 = 53.4cm

The perimeter of a sector:

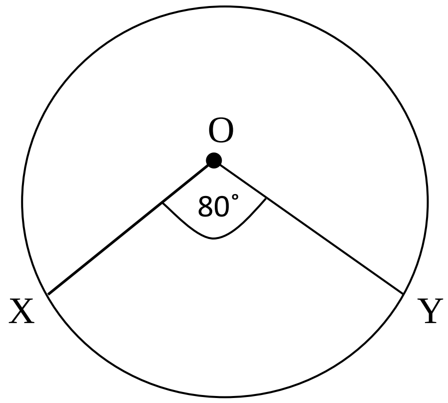




- In the diagram, the shaded portion represents the sector OPQ.
- The perimeter of this sector refers to the distance around it.

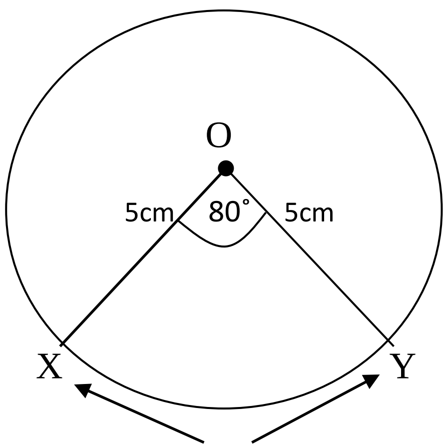
-The perimeter = r + r + *l* = 2r + *l*, where r = the radius of the circle and *l* = the length of the minor arc PQ.

Q1)



The above diagram shows a circle of diameter 10cm. Given that the sector angle of the sector XOY = 80˚, determine the perimeter of this sector.

<u>Soln.</u>



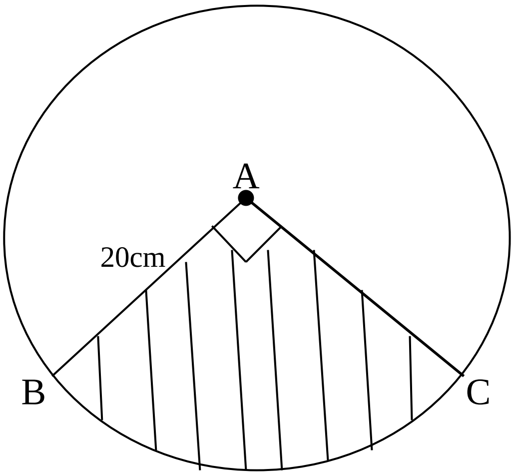
l

The perimeter = 5cm + 5cm + l = 10 + l

But $l = \frac{\theta}{360^\circ} \times 2\pi r = \frac{80}{360^\circ} \times 2 \times 3.14 \times 5 = 7$cm,

=> The perimeter = 10 + l = 10 + 7 = 17cm.

Q2)



In the diagram, A is the centre of the circle with radius 20cm. If the angle BAC = 90˚, find the perimeter of the shaded sector. [Take $\pi = \frac{22}{7}$]

Soln.

The perimeter of the shaded sector = the length of arc BC + 2r, where r = the radius of the circle. Also the sector angle $\theta = 90^\circ$.

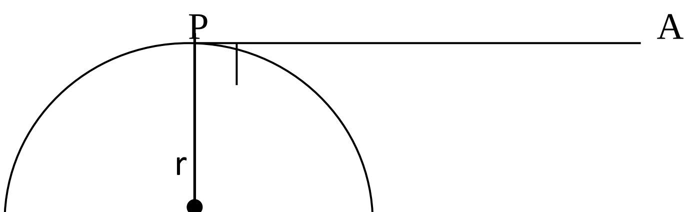
Length of arc BC = $\frac{\theta}{360^\circ} \times 2\pi r = \frac{90}{360^\circ} \times 2 \times 3.14 \times 20 = 31.4$cm.

The perimeter = 31.4 + 2r = 31.4 + 2 (20) = 31.4 + 40 = 71.4cm^2.

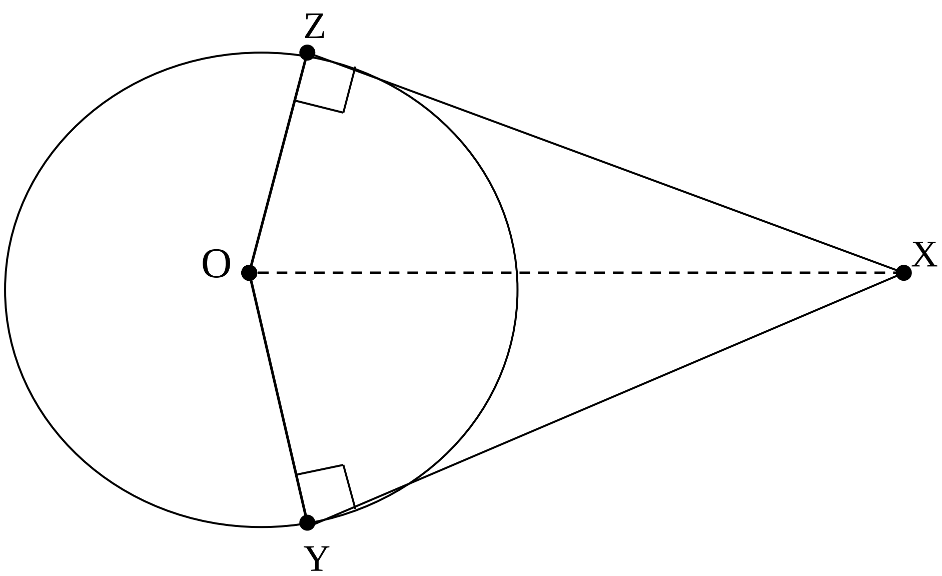
Tangents to a circle:

- A tangent to a circle may be regarded as a straight line, drawn from a point on the circumference.

N/B:



- In the given figure, PA is the tangent and OP is the line drawn from the centre of the circle to the point of contact.
- It will be noted that the angle between these two lines is always 90˚.
- This implies that a tangent to a circle always makes an angle of 90˚ to a line drawn from the centre to the point of contact.

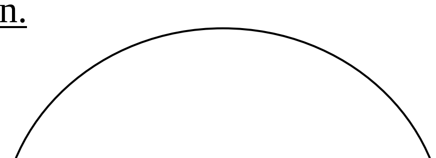


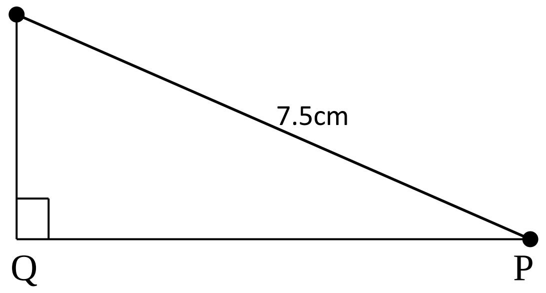
The above diagram shows two tangents ZX and XY, which have been drawn from the same point which is X.

- These two tangents are equal, since the lengths of two tangents from a point outside the circle are equal.

Q1) A point P is 7.5cm away from the centre O, of a circle of radius 4.5cm. Find the length of the tangent to the circle from P.

<u>Soln.</u>





The length of the tangent = QP

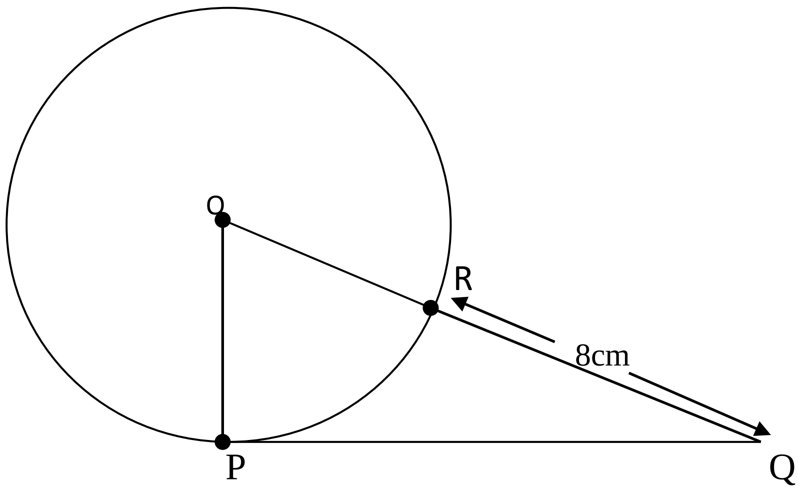
From Pythagoras theorem,

$7.5^2 = 4.5^2 + QP^2$,

$\Rightarrow 7.5^2 - 4.5^2 = QP^2$, $\Rightarrow 56.3 - 20.3 = QP^2$, $\Rightarrow 36 = QP^2$, $\Rightarrow QP = \sqrt{36}$, $\Rightarrow QP = 6cm$.

The length of the tangent to the circle = 6cm

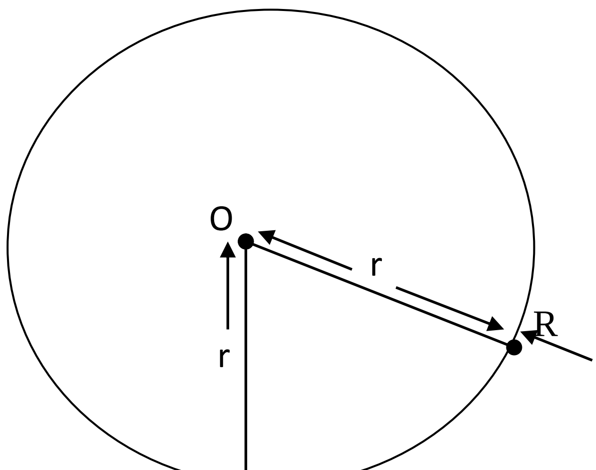
Q2)

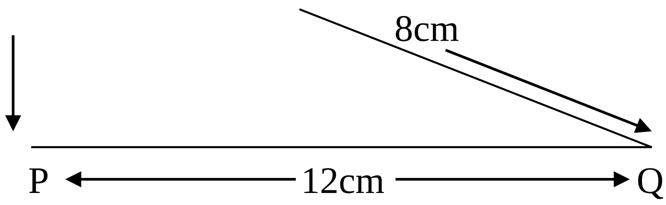


In the diagram, PQ is a tangent to a circle of centre O. Calculate |OR|, if PQ = 12cm.

N/B: QR= r = the radius of the circle.

.Soln





From Pythagoras theorem,

$(r + 8)^2 = r^2 + 12^2$

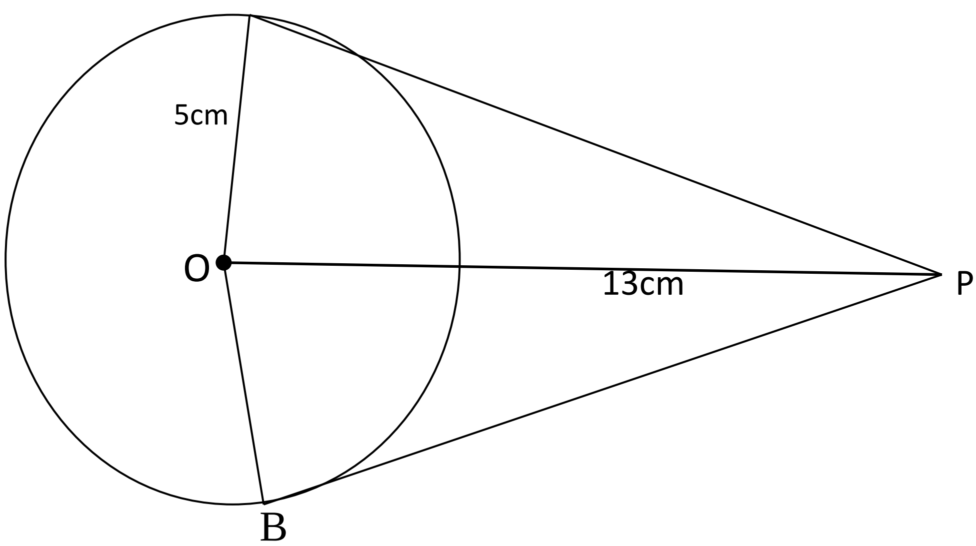
$\Rightarrow r^2 + 16r + 64 = r^2 + 144$

$\Rightarrow r^2 + 16r + 64 - r^2 = 144$

$\Rightarrow r^2 + 16r - r^2 = 144 - 64$

$\Rightarrow r^2 - r^2 + 16r = 80 \Rightarrow 16r = 80 \Rightarrow r = \frac{80}{16} = 5 \Rightarrow r = 5cm.$

Q3)

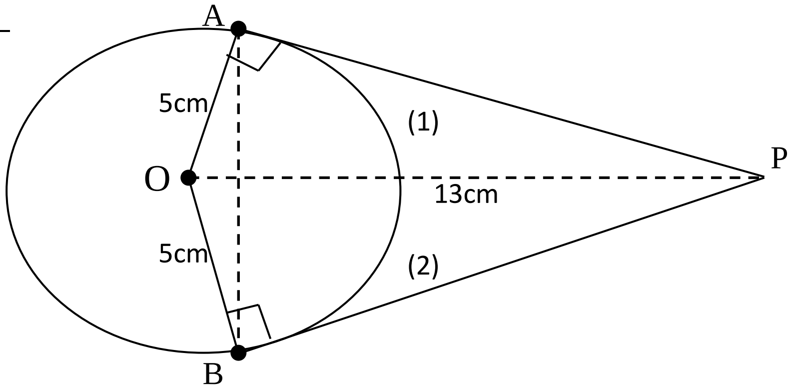


In the diagram given, tangents PA and PB to a circle with centre O are drawn from a point P outside the circle.

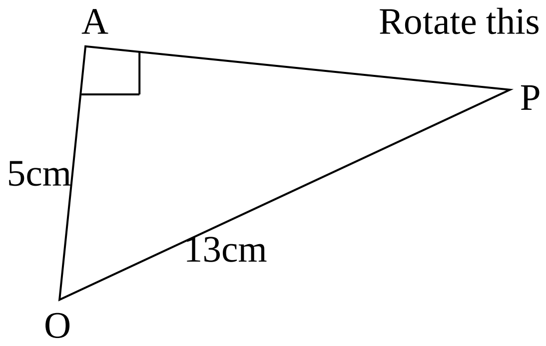
i. If the radius of the circle is 5cm and OP = 13cm, find PA and PB.

ii. If $\angle AOB = 140^\circ$, and AB is joined, determine $\angle APB$ and $\angle ABP$.

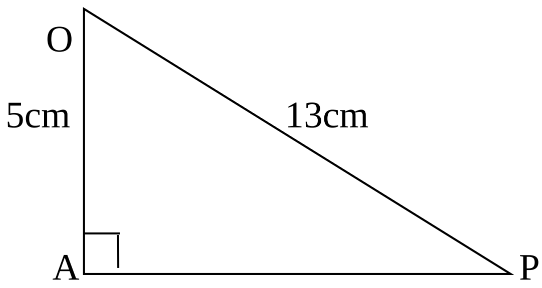
<u>Soln.</u>



Consider triangle (1) i.e



Rotate this to get



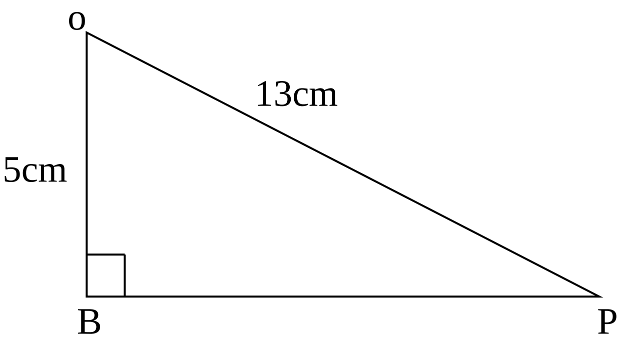
From Pythagoras theorem,

$13^2 = 5^2 + AP^2 \Rightarrow 169 = 25 + AP^2$,

$\Rightarrow 169 - 25 = AP^2$, $\Rightarrow 144 = AP^2$,

$\Rightarrow AP = \sqrt{144} = 12 \quad \Rightarrow AP = 12cm$

Consider triangle (2) i.e

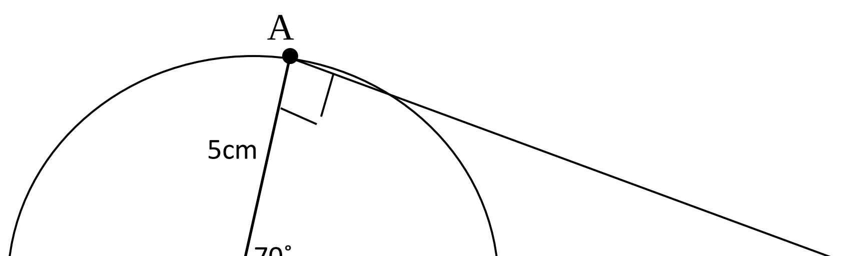


From Pythagoras theorem,

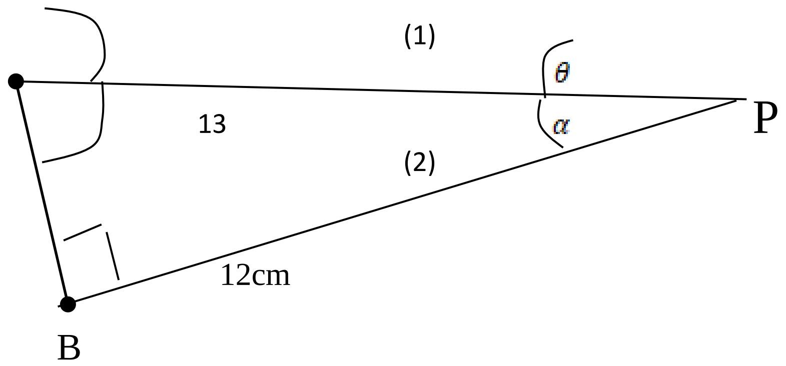
$13^2 = 5^2 + PB^2 \Rightarrow 13^2 - 5^2 = PB^2$

$\Rightarrow 169 - 25 = PB^2 \Rightarrow PB^2 = 144$, $\Rightarrow PB = \sqrt{144} = 12cm$

i.



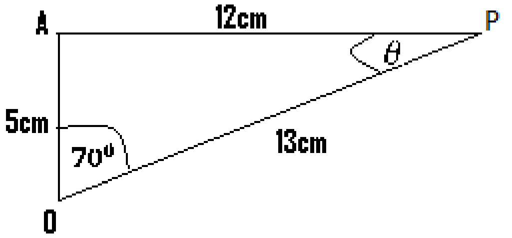
12cm



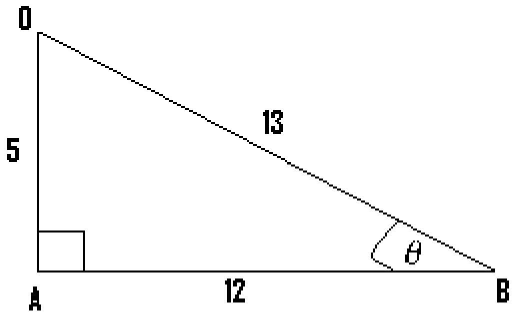
Since <AOB = 140° =><AOP = $\frac{140}{2}$ = 70° and < POB = $\frac{140}{2}$ = 70°

Also <APB = $\theta^\circ + \alpha^\circ$

Consider figure (1)

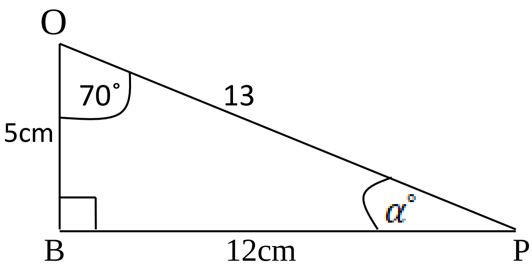


Rotate this to get



tan θ = $\frac{5}{12}$ = 0.42, => θ = tan^{-1} 0.42 = 22.6°

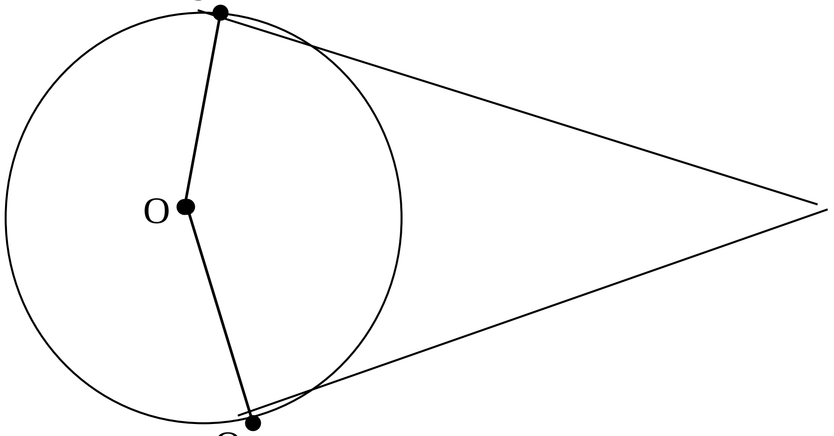
Consider figure (2)



$\tan \alpha = \frac{5}{12} = 0.42 \Rightarrow \alpha = \tan^{-1} 0.42 = 22.6^{\circ}$

$<APB = \theta + \alpha = 22.6 + 22.6 = 45.2^{\circ}$

Q4)



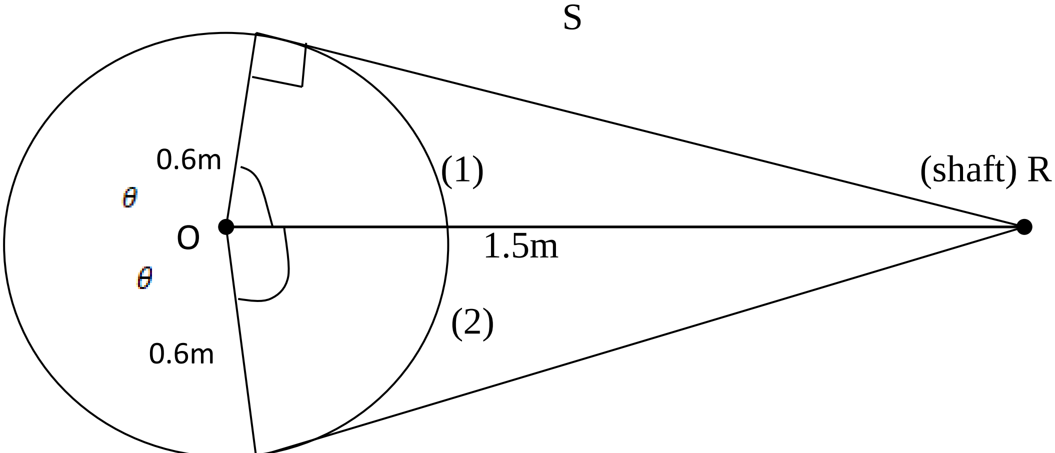
The given diagram shows a belt QRST round a shaft R (of negligible radius), and a pulley of radius 0.6m. The centre of the pulley is O and |OR| = 1.5cm and the straight portions QR and RS of the belt are tangent at Q and S to the pulley. Calculate

i) angle QOS, correct to the nearest degree.

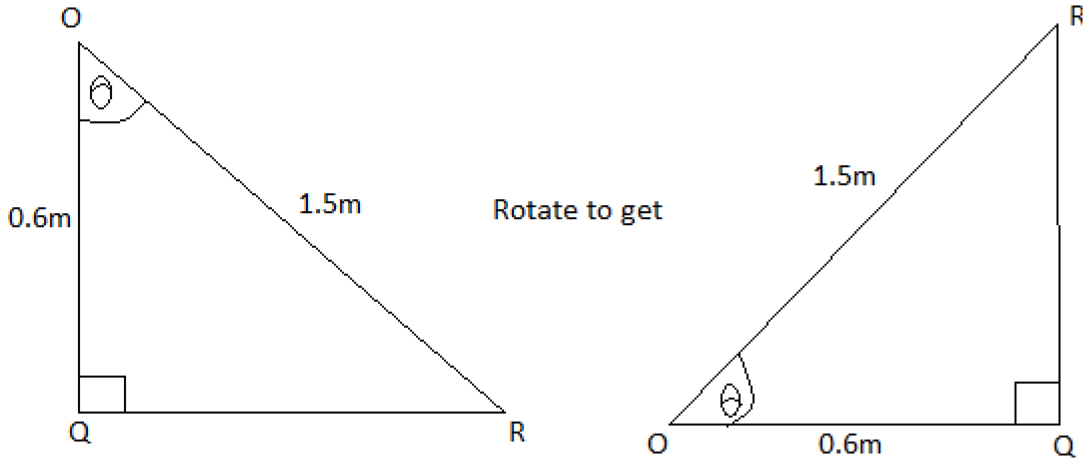
Ii) the length of the belt (QRST) to the nearest metre. [Take π = 3.142]

<u>Soln</u>

i.



Consider triangle (2) i.e

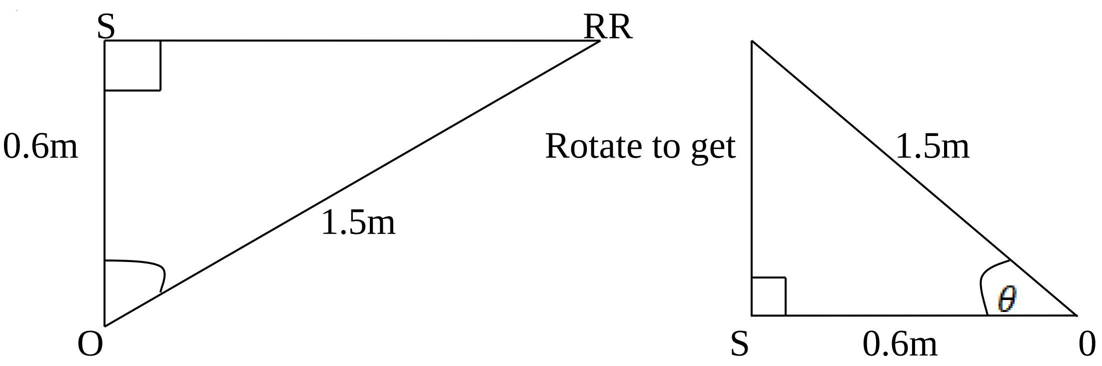


Cos $\theta = \frac{0.5}{1.5} = 0.4$, => $\theta = \cos^{-1} 0.4$ => $\theta = 66.4^{\circ}$

From the original given diagram,< QOS = $\theta + \theta = 2\theta = 2 \times 66.4 = 133^{\circ}$

ii. The length of the belt = SR + RQ + length of arc QTS

Consider triangle (1) i.e



From Pythagoras theorem,

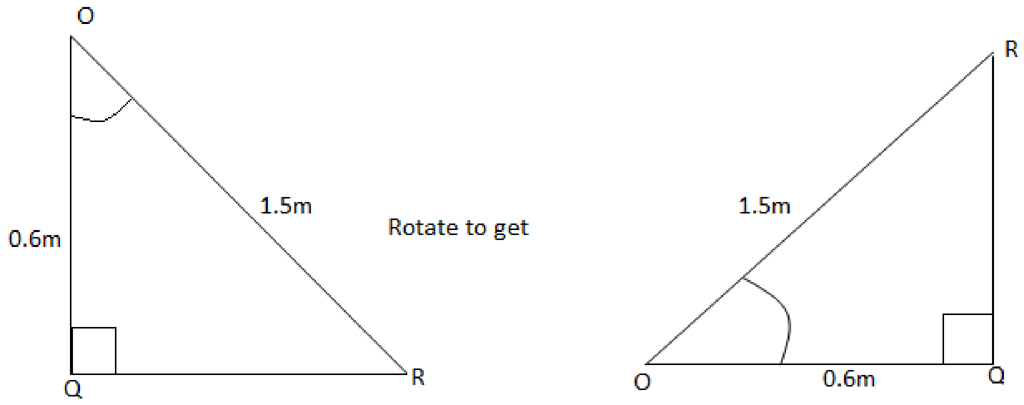
$1.5^2 = 0.6^2 + SR^2$

=> $2.25 = 0.36 + SR^2$

=> $2.25 - 0.36 = SR^2$

=> 1.89 = $SR^{2,}$ => $SR^2 = \sqrt{1.89}$ => SR = 1.37m.

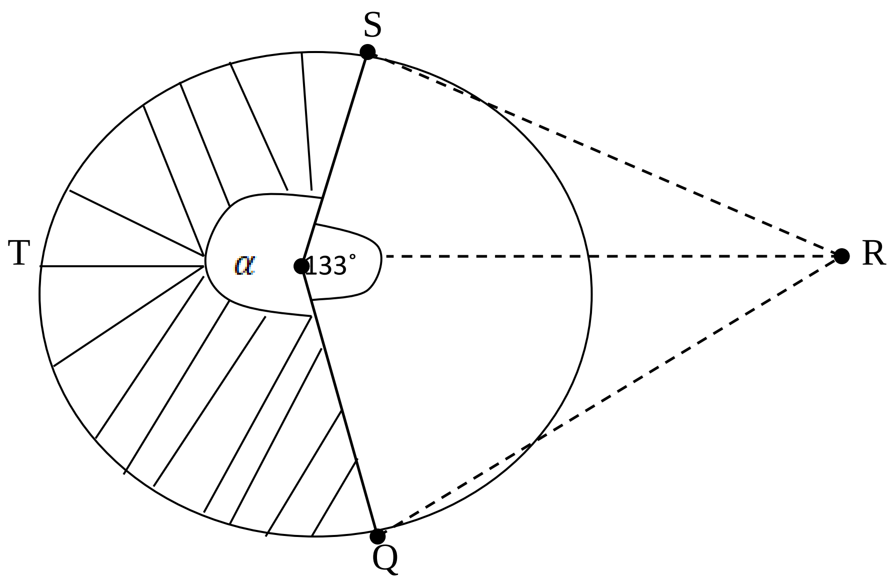
Consider triangle (2) i.e



From Pythagoras theorem,

$1.5^2 = 0.6^2 + RQ^2$

=> $2.25 - 0.36 = RQ^2$, => $1.89 = RQ^2$, => $RQ = \sqrt{1.89} = 1.3$



Let α = the sector angle of shaded sector.

Then the length of arc

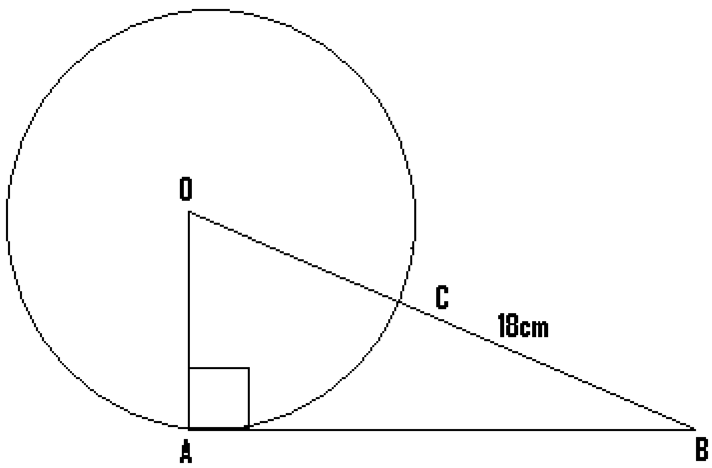
QTS = $\frac{\alpha}{360} \times 2\pi r$

But α = 360˚ - 133˚ = 227˚

=> length of arc QTS = $\frac{227}{360} \times 2 \times 3.142 \times 0.6 = 2.4$m

But length of the belt
= SR + RQ + length of arc QTS
= 1.37 + 1.37 + 2.4 = 5.14m

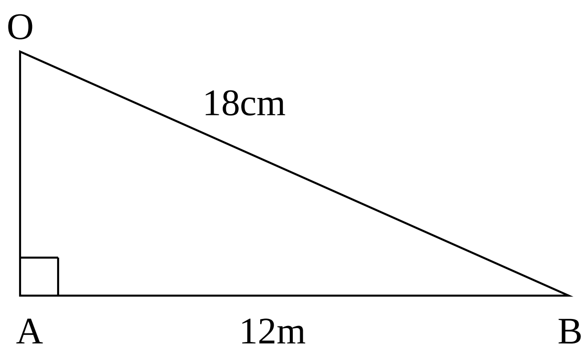
Q5)



Not drawn to scale.
In the diagram $\overline{AB}$ is a straight line touching the centre O at A. If $|\overline{AB}|$ = 12cm, |BC| = 18cm and angle OAB = 90˚, calculate the radius of the circle.
N/B: The radius of the circle = the length of line OA.

<u>Soln.</u>

Consider the triangle, i.e



From Pythagoras theorem,

$18^2 = OA^2 + 12^2$

$\Rightarrow 324 = OA^2 + 144$

$\Rightarrow 324 - 144 = OA^2$

$\Rightarrow OA^2 = 180 \Rightarrow OA = \sqrt{180} = 13.4$cm.

Questions

Q1) A circle has a diameter of 12cm. Determine

a) its circumference. Ans: 38cm
b) its area.Ans: 113cm^2
c) the area of its semi circle. Ans: 56.5cm^2

[Take π= 3.142]

Q2) A driver is driving round a circular road whose radius is 20km, at an average speed of 40km/h.

a) How long will he take to drive round this road. Ans: 3.14hrs.
b) Determine the time he will take to drive three times round this road. . Ans: 9.42hrs
c) If he now drives at a speed of 5km/h, how long will he take to drive round this road? Ans: 25hrs

[Take π 3.142]

Q3) A circular board of radius 4m is to be painted. If it cost ¢5 to paint an area of 6m^2, how much will it cost to paint the whole board? Ans: ¢42

Q4) Mr. Amoo's farm is rectangular in shape. It has a length of 40m and a breadth of 20m. He has now decided to use a portion of this farm which is circular in shape, and of a radius of 8m for the construction of an office building. Determine

a) the fraction of the farm land, on which this office building will be situated. .
Ans:$\frac{201}{800}$ or $\frac{1}{4}$ approx.
b) the percentage of the farm land which is to be used for the construction of this office building. Ans: 25%
c) the percentage of the farm land which will be available for farming, after the construction of this office building. . . Ans: 75%

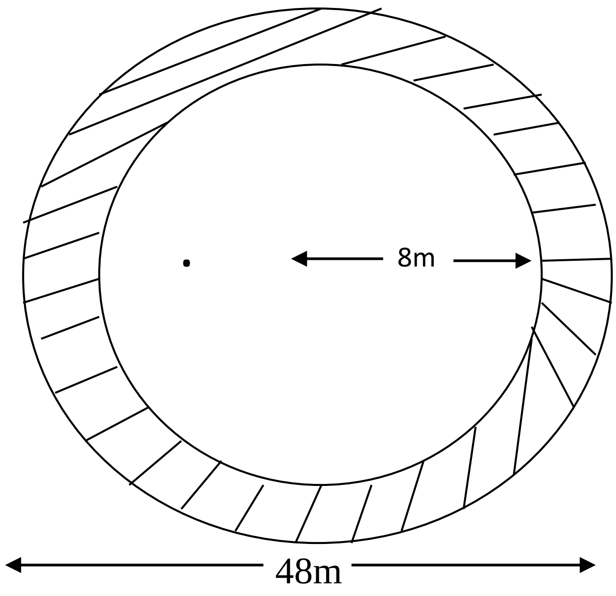
Q5) A town which is shaped in the form of a square has a length or a side of 30km. Its senior high school which is circular in shape and of diameter 8km is located at the centre of the town. Determine

a) the fraction of the town's land on which the school is located.
. Ans: $\frac{1}{8}$ approx

b) the percentage of the town's land on which the school is located. .
Ans: 5.6%

c) the size of the land left for use by the people of the town, due to the presence of the school. Ans: 850km^2 $approx$.

Q6)



Determine the area of the shaded portion. Ans: 1609m^2

Q7) A chord is 4cm long and is drawn in a circle of radius 3cm. Determine

a) the distance of this chord from the centre of the circle.
Ans:2.24cm

b) the angle it subtends at the centre. Ans: 83.6°

[Take π = 3.142]

Q8) A chord XY subtends an angle of 160° at the centre of a circle, whose radius is 50cm. Find the length of XY. Ans:98.5cm.

Q9) A chord QR subtends an angle of 40°, at the circumference of a circle whose radius is 12m. Find the length of this chord. Ans: 15.4m

Q10) A chord of a circle is 140cm long. The perpendicular distance of this chord from the centre of the circle is 20cm. Determine

a) the diameter of the circle. Ans: 145.6cm

b) the angle subtended by the chord at the centre of the circle.
Ans: 148°

c) the angle subtended at the circumference. Ans: 74°. [Take π = 3.142]

Q11) A circle whose diameter is 180cm has a sector angle of 75˚. Determine the length of the sector arc. Ans: 117.8cm

Q12) A circle has a circumference of 160cm. If a sector whose sector angle is 40˚ is removed out of this circle, find the length of the sector arc. [Take π= 3.142]
. . Ans: 17.8cm

Q13) A sector forming part of a circle has a sector angle of 120˚. If the length of the arc associated with the sector is 16m, determine the circumference of this circle.
Ans: 48m

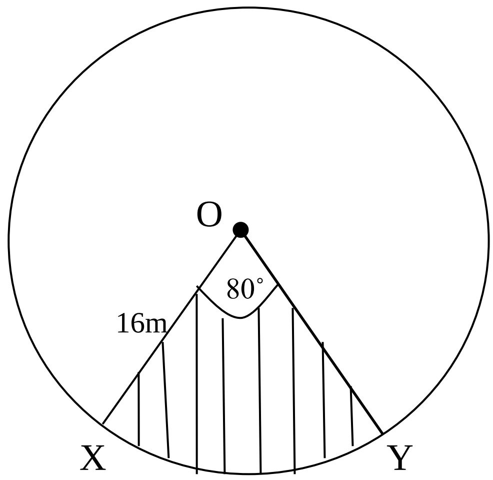
Q14) A circle has a radius of 8cm and a sector angle of 20˚. Determine the area of this sector. Ans: 11cm^2

Q15) Determine the sector angle of a sector whose area is 80cm^2, given that the diameter of the circle of which the sector forms part is 26cm. .
. Ans: 54˚

Q16) The sector area of a circle is given as 60cm^2. If the sector angle is 40˚, find the diameter of this circle. Ans: 26cm

Q17) The diameter of a circle is given as 18cm. Find the area of its quadrant.
. . Ans: 63.6cm

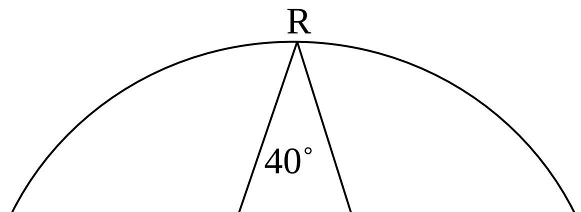
Q18)

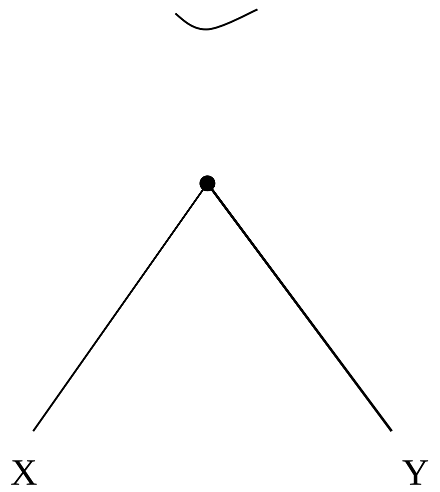


The given diagram shows a circle, whose centre is O and radius is 16m. The shaded portion forms a sector, whose sector angle is 80˚. Determine

a) the length of the minor arc XY. Ans: 22m
b) the area of the shaded portion. Ans: 179cm^2

Q19)

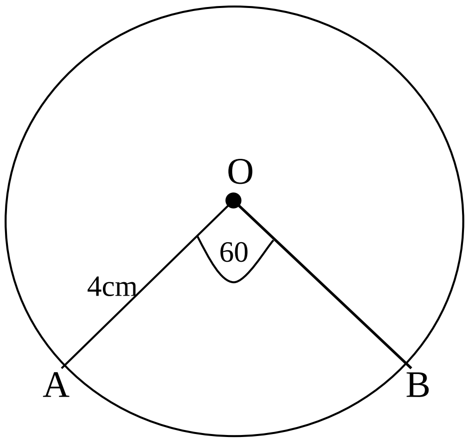




In the given figure, the points X, Y and R lie on the given circle, whose centre is O and diameter is 12mm. If angle XRY = 40˚, find the length of

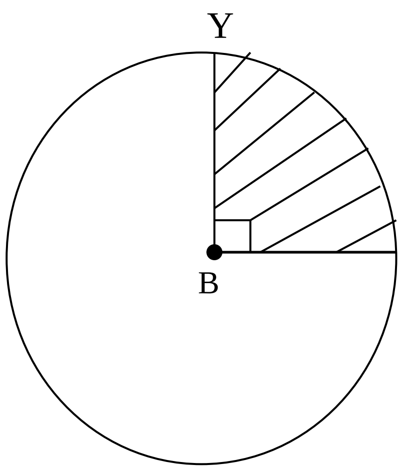
a) the minor arc XY . Ans: 8.4mm
b) the chord XY. Ans: 20.6mm

Q20)



The above shows a circle of radius 4cm. Given that the sector angle of the sector AOB = 60˚, find the perimeter of the given sectors. Ans: 12.2cm

Q21)

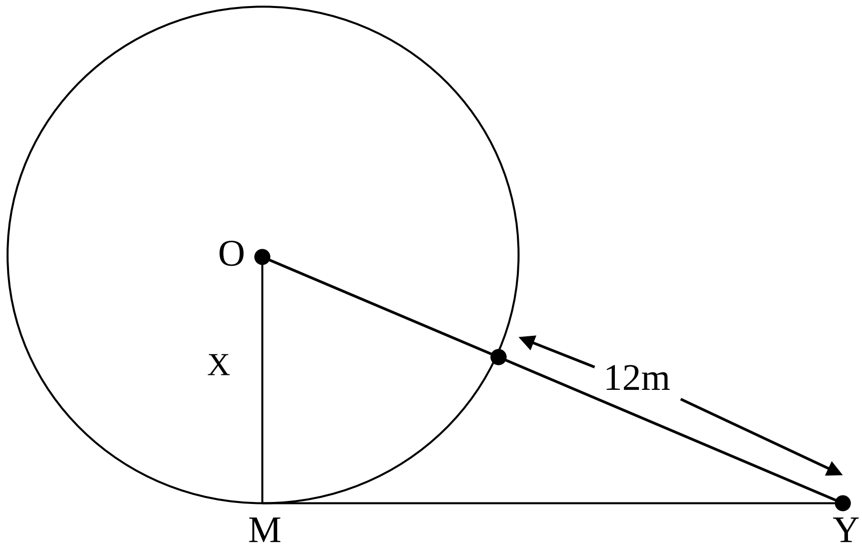


In the given figure, B is the centre of the circle whose diameter is 22cm. Find the perimeter of the shaded sector. Ans: 39cm.

Q22) A point Q is 80mm away, from the centre of a circle of diameter 20mm. What is the length of the tangent to the circle from the point Q. .
Ans: 78.6mm

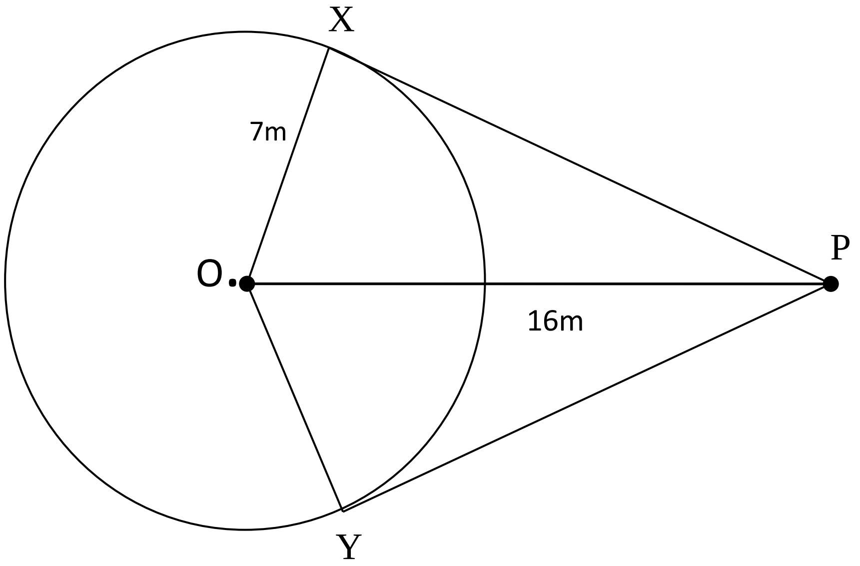
Q23) A point X is positioned at a certain distance away from a circle, whose centre is O and of diameter 22m. If the tangent drawn from this point to the circle has a length of 13m, determine the distance from the point X to the centre of the circle.
Ans: 17m

Q24)



The given figure shows a circle whose diameter is 16cm. If XY = 12m, determine the length of the tangent MY. Ans: 18.3m

Q25)



In the given figure, XP and YP are two tangents drawn to a circular path, whose radius is 7m. The distance between the centre, O of the circle and a point P, which is outside the circular path is 16cm, as indicated in the diagram. If Amu walked from the point X to P, and then moved from P to Y, determine the total distance he walked. Ans: 28.8m.