CHAPTER SIX
ELASTICITY

Introduction:

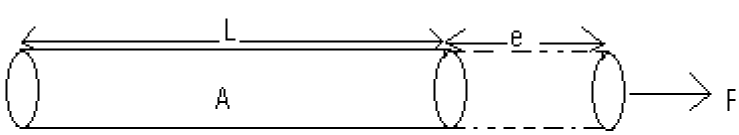
- The elasticity of a material is its tendency to regain its original shape or size, when the external or the deforming forces are removed from it.

Hook`s law:

- This states that provided the elastic limit is not exceeded, the extension of a material is directly proportional to the applied force or load.
- Another way of stating this law is that within the limit of elasticity, the stress is directly proportional to the strain.
- If S_S = Stress and S_n = strain, than $S_S \propto S_n$ => $S_S = E \times S_n$, where E is a constant known as the modulus of elasticity and E = $\frac{S_S}{S_n} = \frac{Stress}{Strain}$.

Young`s modulus of elasticity:

- This is the force which acts per unit area, which is capable of doubling the length of a material within the limit of elasticity.
- Its S.I unit is Nm^{-2} or the pascal (Pa).



- If a force or tension is applied to the end of a wire whose crossectional area is A, then the tensile stress = the force per unit area = $\frac{F}{A}$.
- If the extension of the wire is e and its original length is l, then the tensile strain = the extension per unit length = $\frac{e}{l}$.
- The modulus of elasticity of the material, which is called the Young`s modulus (E) is difined as E = $\frac{tensile\ stress}{tensile\ strain}$
- => E = $\frac{\frac{F}{A}}{\frac{e}{l}}$
- Even though strain has no units since it is a ratio of two lengths, stress has units such as Nm^{-2}.

(Q1) A 2kg mass is attached to the end of a vertical wire of length 2m and diameter 0.64cm, and the extension is 0.60mm. Calculate the Young`s modulus.

Soln:

Mass = m = 2kg.

Length = l = 2m.

Diametre = d = 0.64cm, => r = 0.32cm = $\frac{0.32}{100}$

= 0.0032m.

Extension = e = 0.60mm

= $\frac{0.60}{1000} = 0.0006$m.

Force = m $\times\ g = 2 \times 10 = 20N$.

Area of the wire = πr^2 = 3.14 $\times\ 0.0032^2$ = 3.2 $\times$ $10^{-5}m^2$

Tensile stress = $\frac{F}{A} = \frac{20}{3.2\times10^{-5}}$

= $\frac{20}{0.000032} = 625000Nm^{-2}$

- Tensile strain = $\frac{extension}{length}$

  = $\frac{0.00060}{2} = 0.00030$

  Young`s modulus = $\frac{tensile\ stress}{tensile\ strain}$

  = $\frac{625000}{0.00030} = 19.5 \times 10^8$

  N/B: Young`s modulus E, is calculated from the ratio $\frac{tensile\ stress}{tensile\ strain}$,

  only when the wire is under elastic condition, i.e. when the elastic limit is not exceeded. N/B:

- To convert a length given in millimetres into metres, we either multiply by 10^{-3} or divide by 1000.
- For example, 2mm = 2$\times$ 10^{-3}m or 2mm = $\frac{2}{1000} = 0.002m$.

- To convert a length given in centimetres into metres, we either multiply by 10^{-2} or divide by 100.
- For example, a length of 8cm = 8×10^{-2}m, or 8cm = $\frac{8}{100} = 0.08m$.

Materials:

Strength: This is the resistance of a metal to stress when it is under a steady load.

- Its S.I unit is KN/ m^2.

Tensile strength: This is the maximum load which a metal can sustain before breaking.

Plastic deformation: When an object can no longer regain its original size or shape after the removal of the deforming force, then it is said to have undergone plastic deformation.

Ductile substances: These are substances which lengthen considerably and undergo plastic deformation, until they break.

- Examples of such materials are iron and steel.

Brittle substances: These are substances which break just after the elastic limit is reached, and an example is glass.

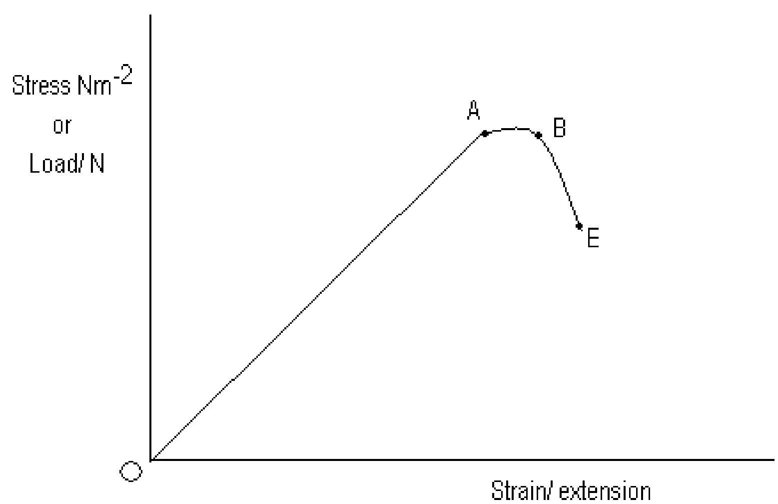
Permissible or safe working stress: This is the stress which a metal can withstand without any fear of it breaking.

Safety factor:

- This is the ratio of the tensile strength to the permissible tensile stress.
- Safety factor = $\frac{tensile\ strength}{permissible\ tensile\ stress}$
- In machines or structures, this permissible tensile stress is always far below the tensile strength and also below the stress which corresponds to the elastic limit.

Load/ extension curves:

(1) For a brittle substance:



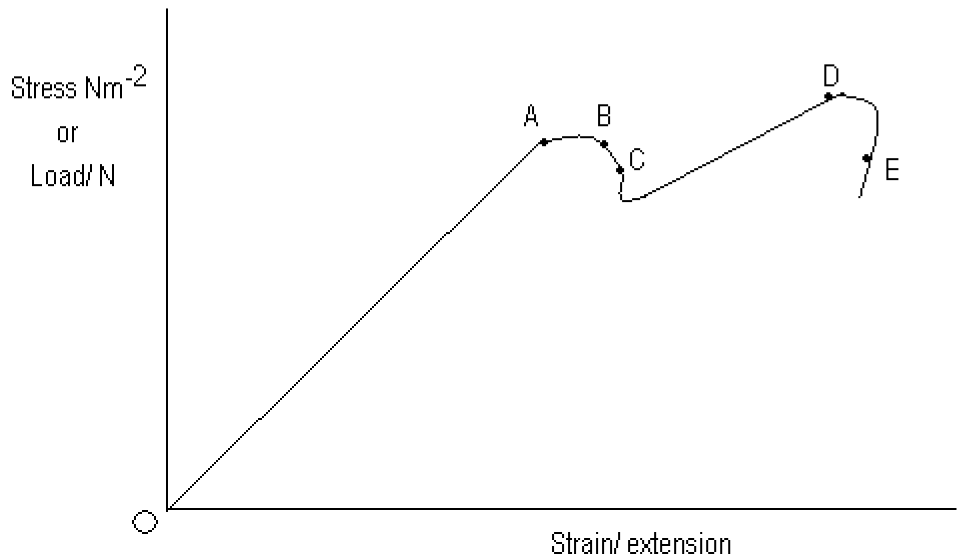
A = the point of proportionality.

B = the elastic limit or the point of elasticity.

E = the point of fracture (the material will break at this point).

OA = Elastic deformation.

(2) For a ductile material:



A = the point of proportionality.

B = the elastic limit.

C = the yield point i.e. the point on the load – extension diagram, at which permanent deformation is first observed.

D = the maximum (breaking) tensile stress point i.e. the point up to which forces can be applied, without unnecessarily extending or breaking the material.

E = The fracture point i.e. the point at which the material breaks.

OB = the elastic deformation stage.

CD = the plastic deformation stage.

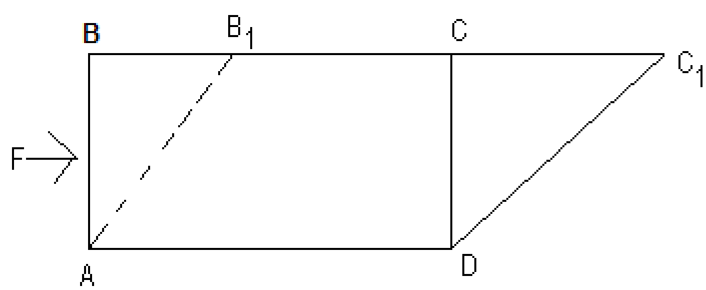
Compressive stress:

- When a material iis subjected to an increased pressure, the material contracts.

- The compressive force on the material is the increased in force per unit area.

Shear stress:

- This is the tangential force which acts per unit area on the free surface of the material.



- In this case, the tangential force tends to make the part on one side of the surface, slide past the part on the other side of the surface.

- In the given figure, AD is fixed and BC has slided to B_1C_1.

(Q2) A railway signal wire 20m long and 4mm in diametre, is stretched 60mm by a force of 5.6KN when the signal is operated. Calculate

(i) the tensile stress.

(ii) the tensile strain.

(iii) Young`s modulus of elasticity of the wire.

Assume that the stretch was within the elastic limit.

Soln:

Applied force = F = 5.6KN.

= $5.6 \times 1000 = 5600N$.

Length of wire = L = 20m.

Extension of the wire = e = 60mm = $\frac{60}{1000} = 0.06m$.

Diameter of the wire = D = 4mm = $\frac{4}{1000} = 0.004m$

=> radius = r = $\frac{0.004}{2} = 0.002m$

Cross – sectional area of the wire = $\pi r^2 = 3.14 \times 0.002^2$

= 1.26×10^{-5}

Let S_s = tensile stress,

S_n = tensile strain and Y = Young`s modulus of elasticity.

$S_S = \frac{F}{A} = \frac{5600}{12.56 \times 10^{-5}}$

= $\frac{5600 \times 10^5}{12.56} = \frac{5600}{12.56} \times 10^5$

= 446×10^5

(I) $S_n = \frac{e}{l}$,

where e = the extension and L = the length.

=> $S_n = \frac{0.06}{20} = 3 \times 10^{-3}$

= 0.003

(III) Y = $\frac{S_S}{S_n} = \frac{446\times10^5}{0.003}$

= $\frac{446\times10^5}{3\times10^{-3}} = \frac{446}{3} \times 10^5 \times 10^3$

= 149×10^8 Nm^{-2}.

N/B: Care must be taken when converting areas given in one unit into another unit.

(1) To convert an area given in millimetres squared(mm^2) into metres squared,we multiply by $10^{-6.}$ ·For this reason, an area of $2mm^2 = 2 \times 10^{-6}m^2 = 0.000002m^2$.

- Also an area of $35mm^2 = 35 \text{ x } 10^{-6}m^2 = 0.000035m^2$

(II) To convert an area given in centimetres squared into metres squared, we multiply by 10^{-4}.

An area of $2cm^2 = 2 \times 10^{-4}m^2 = 0.0002m^2$.

- Also an area of $12cm^2 = 12 \text{ x } 10^{-4}m^2 = 0.0012m^2$

(Q3) A vertical wire of unistretched (unstressed) length 500mm, and a crossectional area of $2.5mm^2$, extends 0.2mm, when a vertical force of 200N is applied. Calculate for the load,

(i) the stress in the wire.

(ii) the strain.

Soln:

F = the vertical force applied = 200N.

L = length of wire = 500mm = 500×10^{-3}m.

E = extension of the wire = 0.2mm = 0.2×10^{-3}m.

A = crossectional area of wire = $2.5mm^2 = 2.5 \times 10^{-6}m^2$.

(I) Stress in wire = $S_n = \frac{F}{Area} = \frac{200}{2.5\times10^{-6}} = \frac{200}{2.5} \times 10^6$

$= 80 \times 10^6$

$= 8 \times 10 \times 10^6 = 8 \times 10^7$ Nm^{-2}.

(II) $S_n = \frac{e}{l} = \frac{0.2\times 10^{-3}}{500 \times 10^{-3}}$

$= \frac{0.2}{500} \times 10^{-3} \times 10^3 = \frac{0.2}{500} = 4 \times 10^{-4}$.

(Q4) A wire 15m long and 3mm in diametre, is stretched 60mm within its elastic limit by force of 5.6KN. Find

(I) the tensile strain.

(II) the tensile stress.

(III) the Young`s modulus of elasticity for the wire.

(IV) Explain why it is necessary to know the elastic limit of a material.

Soln:

$F = 5.6KN = 5.6 \times 10^3 N = 5600N$.

$L = 15m$, $E = 60mm = 60 \times 10^{-3} m$.

$D = 3mm = 3 \times 10^{-3} m$

=> radius = $\frac{3 \times 10^{-3}}{2}$

=> $r = 1.5 \times 10^{-3} m$ (or $r = \frac{1.5}{1000} = 0.0015m$)

Crossectional area of wire = $\pi r^2 = 3.14 \times (1.5 \times 10^{-3})^2$

$= 3.14 \times 1.5^2 \times 10^{-6} = 7.1 \times 10^{-6} m^2$

N/B:{Also the crossectional area of wire = $\pi r^2 = 3.14 \times 0.0015^2$

$= 7.1 \times 10^{-6} M^2$}.

(I) $S_n = \frac{e}{l} = \frac{60 \times 10^{-3}}{15} = \frac{60}{15} \times 10^{-3}$

$= 4 \times 10^{-3}$

(II) $S_s = \frac{F}{A} = \frac{5.6\times10^3}{7.1\times10^{-6}} = \frac{5.6}{7.1} \times 10^3 \times 10^6$

$= 0.78 \times 10^9 = 0.78 \times 10 \times 10^8$

$= 7.8 \times 10^8 NM^{-2}$

(III) $Y = \frac{S_S}{S_n} = \frac{7.8\times10^8}{4\times10^{-3}} = \frac{7.8}{4} \times 10^8 \times 10^3$

$= 1.95 \times 10^{11} NM^{-2}$.

(IV) A knowledge of the elastic limit of a material enables us to apply the appropriate amount of load to the material, without any fear of damaging it.

(V) If the amount of load applied is more than necessary, then the material will be deformed permanently or it will break.

(Q5) A tie rod has an ultimate stress of $532MN/M^2$ and a safetyfactor of 4. Determine the cross – sectional area of the rod, if the maximum load is to be 50KN.

Soln:

S.F = safety factor = 4.

F = Maximum force = maximum load (force) applied = 50KN

Tensile strength = ?

PTS = permissible tensile strength or ultimate stress = $532MN/M^2$

$= 532 \times 1000 = 532000KN/M^2$.

Let A = cross – sectional area of tie rod.

S.F = $\frac{Ts}{PTs}$ $=> T_S = sF \times$ PTs

= 4 × 532000KN/M^2 = 2128000 KNM^{-2}

Ts = $\frac{Fmax}{A} => A = \frac{Fmax}{Ts}$

= $\frac{50KN}{2128000} = 23.5 \times 10^{-5}$m^2

N/B: 1MN = 1 mega Newtons

= 1 × 1000 = 1000KN => to convert from mega into kilo, we multiply by 1000.

Modulus of rigidity or shear modulus:

- The shear modulus, G is defined by G = $\frac{\text{shear stress (force per unit area)}}{shear\ strain\ (angular\ displacement)}$

Bulk modulus:

- When a gas or a liquid is subjected to an increased pressure, the substance contracts.
- A change in bulk has occurred and the bulk strain is defined by

  Bulk strain = $\frac{change\ in\ volume}{original\ volume}$
- The bulk stress on the substances is the increased force per unit area.
- The bulk modulus K is defined as K = $\frac{bulk\ stress}{bulk\ strain}$

Some prefixes for multiples and sub multiple units:

Prefix	T tera	G giga	M MEGA	K Kilo	m millo	U micro	n nano	p pico	f femto
Example	TJ 10^{12}J	GJ 10^{9}J	MJ 10^{6}J	KJ 10^{3}J	mm 10^{-3}m	Um 10^{-6}m	nm 10^{-9}m	pm 10^{-12}m	fm 10^{-15}m.

Questions:

(1) State Hooke`s law and explain what you understand by the elasticity of material.

(2) Explain what Young`s modulus of elasticity is.

(3) A vertical cord is used to support a 5kg mass in the air from the ceiling of a building. If the cord has a length of 2m and a radius of 0.3cm, determine the Young`s modulus if 0.42mm is the extension produced.

Ans: 8.4×10^{9}.

(4) Differentiate between strength and tensile strength.

(5) Explain what the following refer to:

(a) Ductile substances.

(b) Brittle substances.

(c) The safe working stress.

(d) The safety factor.

(6) Draw the load/ extension curve for these materials.

(a) Iron

(b) Glass.

N/B:

- Since iron is a ductile substance or material, its load/ extension curve will be that for a ductile substanc or material.
- Also because glass is a brittle substance or material, it load/ extension curve will be that for a brittle substance.

(7) What do you ubderstand by the elastic limit of a material, and explain its importance.

Ans:

- The elastic limit of a material is the limit beyond which when the material is stretched, its extension will no longer be proportional to the applied force.
- Its importance is that, it enables us to apply the appropriate amount of load to the material, without any fear of damaging it.

(8) An electric wire of length 3000cm and diameter 6mmis used in the construction of a device. If it is stretched 80mm by a force of 7KN when the device is in operation, determine

(a) the tensile stress.

Ans: $24.8 \times 10^{7}NM^{-2}$.

(b) the tensile strain.

Ans: 2.7×10^{-3}

(c) the Young`s modulus of elasticity. Ans: $1.0 \times 10^{9} NM^{-2}$.

(9) A wire, 20m long and of diametr 4.0mm is stretched 80mm by a force of 6KN.

Calculate

(I) the tensile strain.

Ans: 4×10^{-3}

(II) the tensile stress.

Ans: $4.8 \times 10^{8} NM^{-3}$.

(III) Young`s modulus of elasticity.

Ans: 1.2×10^{11} NM^{-2}.

(10) Briefly explain what the following mean:

(a) Bulk modulus.

(b) Shear modulus.