CHAPTER THREE

INEQUALITIES IN ONE VARIABLE

Symbols used:

-There are four symbols used and these are

1. **<** 2. **≤** 3. **>** 4. **≥**

The meaning of the symbols <

Less than < greater than, greater than > less than.

For these two symbols, the sharp or pointed edge always points to the less or small value, while the other side points towards the great or big value.

Q1. Give the meaning of the following inequalities:

(a) x < 4

Solution

X is less than 4 or 4 is greater than x.

(b) 2 < 4

Solution

2 is less than 4 or 4 is grater than 2.

(c) y > x

Solution

y is greater than x or x is less than y.

(d) 5 > 2

Solution

5 is greater than 2 or 2 is less than 5.

(e) 9x >10

Solution

9x is greater than 10 or 10 is less than 9x.

The meaning of the symbol $\leq and \geq$

1. greater than or equal to ≥ less than or equal to

2) less than or equal to ≤ greater than or equal to

- With respect to these two symbols, the sharp edge points towards the less than or equal to value, while the other side points towards the greater than or equal to value.

Q2. Given the meaning of the following inequalities:

(a) $b \leq 5$

Solution

b is less than or equal to 5, or 5 is greater than or equal to b.

(b) $x \leq 2$

Solution

x is less than or equal to 2, or 2 is greater than or equal to x.

(c) $5 \leq y$

Solution

5 is less than or equal to y or y is greater than or equal to 5.

(d) $4 \geq y$

Solution

Y is less than or equal to 4 or 4 is greater than or equal to y.

(e) $2x \geq 6$

Solution

6 is less than or equal to 2x or 2x is greater than or equal to 6.

Q3. List the members of the following sets:

(a) $Z = \{x: x > 2\}$

Solution

x : x > 2 => x is greater than 2. The members are all the numbers greater than 2
=> Z = {3, 4, 5, 6 ……}

(b) $Y = \{x : x \geq 2 \}$

Solution

$x \geq 2$ => x is greater than or equal to 2,
=> Y = {2, 3, 4, 5 ……..}

(c) $A = \{ n: n > 5\}$

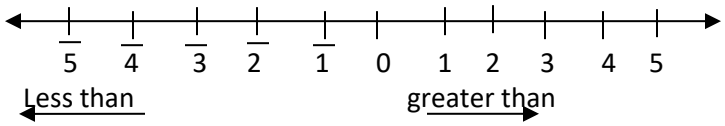
Solution

n > 5 => n is greater than 5
=> A = {6, 7, 8, 9 ……)

(d) N = {n: n ≥ 5}

Solution

N ≥ 5 => n is greater than or equal to 5,
=>N = {5, 6, 7, 8 ……..)
NB:



-The numbers on the left hand side of the number line, are always less than those on the right hand side.
For examples

1. -5 is less than -4
2. -5 is less than -2
3. -4 is less than -2
4. -4 is less than -3
5. - 1 is greater than - 3
6. - 2 is greater than – 5

Q4. List the members of the following sets:

(a) X = { x : x ≤ 2}

Solution

x ≤ 2 => x is less than or equal to 2.
=> X = {2, 1, 0, -1, -2, -3 ……..}.

(b) Y = { x : x < 2}

Solution

x < 2 = > x is less than 2,
=> Y = {1, 0, -1, -2, -3 ……..}.

(c) Y = {n : n ≤ - 2}

Solution

n ≤ - 2 => n is less than or equal to -2.
=> Y = {-2, - 3, - 4, - 5 ……..}.

(d) M = { n : n < - 2}

Solution

n < - 2 => n is less than – 2.
=> M = { - 3, - 4, - 5, - 6 ……..}.

(e) Z = { x : x > - 4}

Solution

x > - 4 => x is greater than – 4,
=> Z = {- 3, - 2, - 1, 0, 1, 2 ……..}

(f) Z = { x: x ≥ - 4}

Solution

$x \geq -4$ => x is greater than or equal to – 4
=> Z = {- 4, - 3, - 2, - 1, 0, 1, 2, 3……}.

(g) $Y = \{x: x \leq -4\}$

Solution

$x \leq -4$ => x is less than or equal to – 4
=> Y = {- 4, - 5, - 6, - 7 ……..}.

(h) $Y = \{x : x < -4\}$

Solution

$x < -4$ => x is less than – 4
=> Y = { -5, -6, - 7 ……..}.

Q5.Determine the members of each of the following given inequalities:
(a) $Y = \{x: 2 < x < 5\}$

Solution

$2 < x < 5$ => x is greater than 2 but less than 5
=> Y = {3, 4}.

(b) $Y = \{x: 2 \leq x \leq 5\}$

Solution

$2 \leq x \leq 5$ => x is greater than or equal to 2, and x is less than or equal to 5.
=> Y = {2, 3, 4, 5}.

(c) $X = \{x: 2 \leq x < 5\}$

Solution

$2 \leq x < 5$ => x is greater than or equal to 2 and x is less than 5.
=> X = {2, 3, 4}.

(d) $M = \{x: 2 < x \leq 5\}$

Solution

$2 < x \leq 5$ => x is greater than 2 and x is less than or equal to 5.
=> M = { 3, 4, 5 }.

(e) $Y = \{x: -2 \leq x \leq 4\}$

Solution

$-2 \leq x \leq 4$ => x is greater than or equal to – 2, and x is less than or equal to 4.
=> Y = {- 2, - 1, 0, 1, 2, 3, 4}.

(f) $Y = \{x: -2 \leq x < 4\}$

Solution

$-2 \leq x < 4$ => x is greater than or equal to -2 and x is less than 4.
=> Y= {-2, -1, 0, 1, 2, 3}.

(g) $Y = \{x: -2 < x \leq 4\}$

Solution

$2 < z \leq 4$ => x is greater than 2 and less than or equal to 4.
=> Y = { -1, 0, 1, 2, 3, 4}.

(h) $Z = \{n: -6 \leq n \leq -2\}$

Solution

$-6 \leq n < -2$ => n is greater than or equal to - 6 and n is less than or equal to -2.
=>Z = {- 6, -5, - 4, -3, -2}

(i) $Z = \{n: -6 \leq n < -2\}$

Solution

$-6 \leq n < -2$ => n is greater than or equal to - 6 and n is less than or equal to -2.
=>Z = {- 6, -5, - 4, -3}.

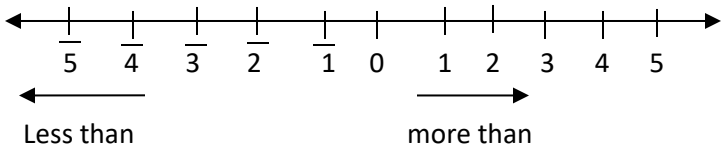
(j) $Z = (n: -6 < n \leq -2\}$

Solution

$-6 < n \leq -2$ => n is greater than -6 and less than or equal to -2.
=> Z = {-5, -4, -3, -2}.

Graphs of inequalities:

NB:



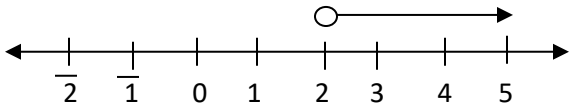
- In the graphical representation of inequalities, if the sign is greater than, then movement is in the right hand side direction.

- If we are dealing with the less than sign, then we move in the left hand side direction as indicated above.

Q1.Represent these inequalities graphically or by means of graphs:
a. $x > 2$

Solution

$x > 2$ => x is greater than 2. i.e {3, 4, 5, 6 ……..}



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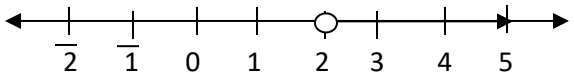


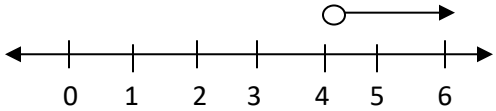
NB: If the circle is not shaded, then number below it or within it is not a member of the set.

b. $x > 4$

Solution

$x > 4$ => x is greater than 4 i.e. {5, 6, 7, 8 ……}

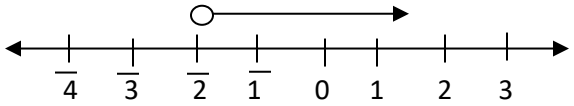
or



c. $Y = \{x: x > -2\}$

Solution

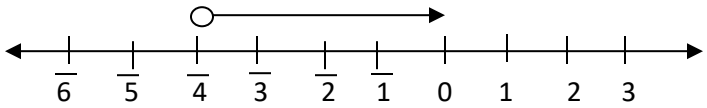
$x > -2$ => x is greater than -2



d. $X = \{n: n > -4\}$

Solution

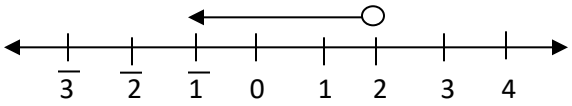
$n > -4$ => n is greater than -4



e. $Y = \{x: x < 2\}$

Solution

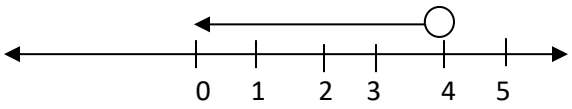
$x < 2$ => x is less than 2



f. x = {n : n < 4}

Solution

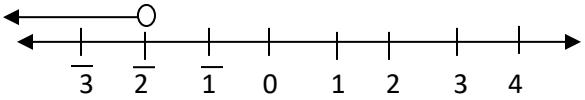
n < 4 => n is less than 4



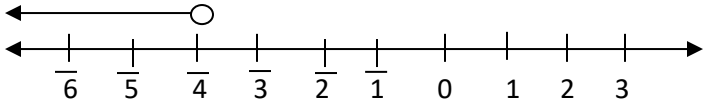
g. x = {n : n < - 2}

Solution

n < - 2 => n is less than -2



h. x = {x : x < - 4}



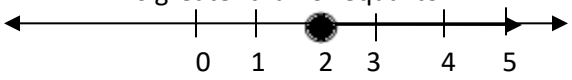
NB: If the circle is shaded, then the number below it is part of the given set.

Q2.Represent each of the following inequalities, by means of a graph

a. x ≥ 2

Solution

X ≥ 2 => x is greater than or equal to 2.



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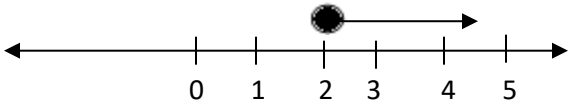
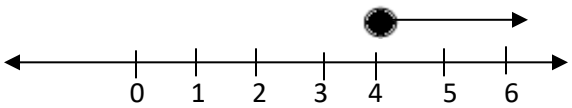


b. x ≥ 4

Solution

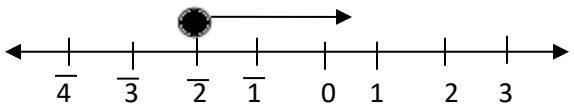
x ≥ 4 => x is greater than or equal to 4



c. Y = {x: x ≥ - 2}

Solution

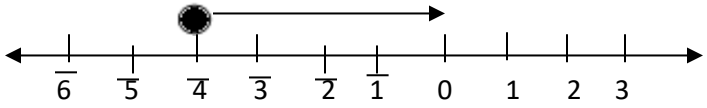
x ≥ -2 => x is greater than or equal to – 2



d. M = {n: n ≥ - 4}

Solution

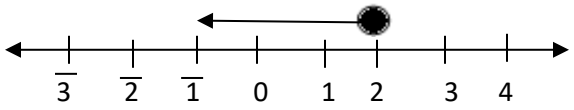
n ≥ - 4 => n is greater than or equal to -4



e. Y = { x: x ≤ 2}

Solution

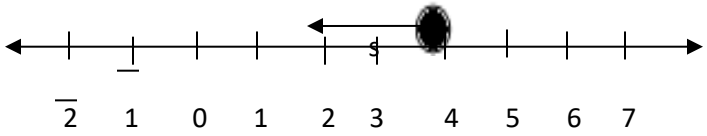
x ≤ 2 => x is less than or equal to 2



f. Y = {x: x ≤ 4 }

Solution

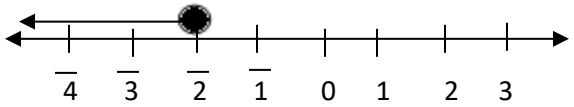
x ≤ 4 => x is less than or equal to 4



g. x ≤ -2

Solution

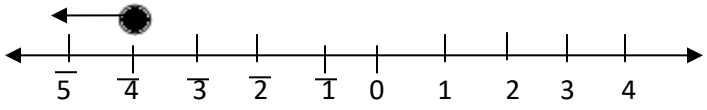
x ≤ -2 => x is less than or equal to - 2



h. x ≤ - 4

Solution

x ≤ - 4 => x is less than or equal to - 4

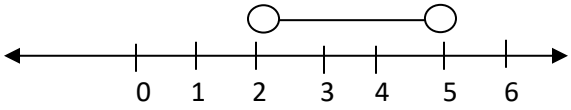


Q3. Represent the following inequalities on graphs and indicate their members:

a. T = {x: 2 < x < 5}

Solution

2 < x < 5 => x is greater than 2 and less than 5.
The members = {3, 4}



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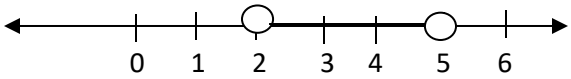
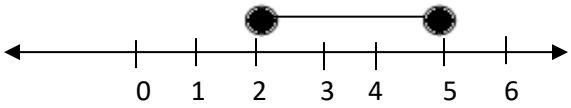


b. W = {x: 2 ≤ x ≤ 5}

Solution

2 ≤ x ≤ 5 => x is greater than or equal to 2 and less than or equal to 5.
The members = {2, 3, 4, 5}



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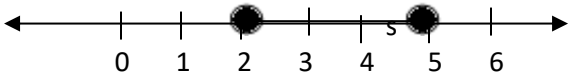
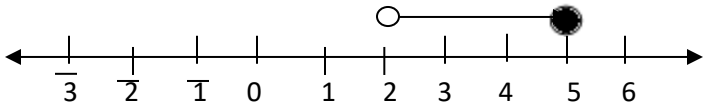


c. W = {x: 2 < x ≤ 5}

Solution

2 < x ≤ 5 => x is greater than 2 and less than or equal to 5.
The members = {3, 4, 5}

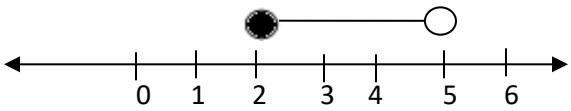


NB: The circle above 2 was not shaded, because 2 is not a member of the set. Since 5 is a member, the circle is shaded.

d. A = {x: 2 ≤ x < 5}

Solution

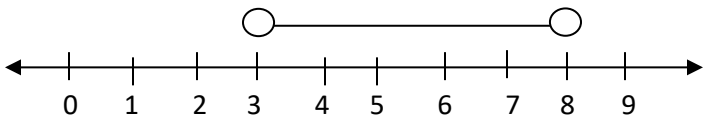
2 ≤ x < 5 => x is greater than or equal to 2 and x is less than 5. The member = {2, 3, 4}



e. W = {x: 3 < x< 8}

Solution

The members = {4, 5, 6, 7}



f. W = {x: 2 ≤ x ≤ 8}

Solution

Members = {2,3, 4, 5, 6, 7, 8}

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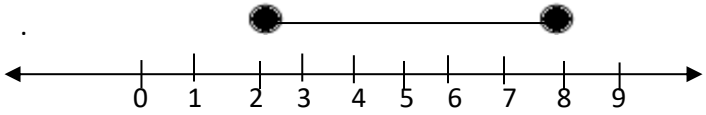
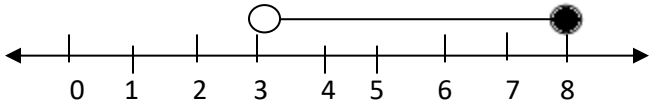


h. W = {n: 3 < n ≤ 8}

Solution

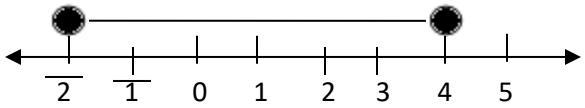
Members = {4, 5, 6, 7, 8}



i. T = {x: -2 ≤ x ≤ 4}

Solution

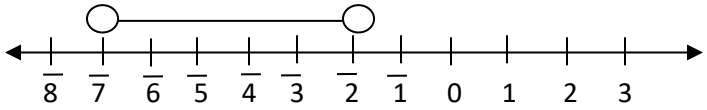
-2 ≤ x ≤ 4 => is greater than or equal to -2 and less than or equal to 4. Members = {-2, -1, 0, 1, 2, 3 4}



j. D = {x: -7 < x < -2}

Solution

-7 < x < - 2 => x is greater than -7 and less than -2 members = {- 6, -5, -4, -3}



k. Y = {x: -7 ≤ x ≤ -2}

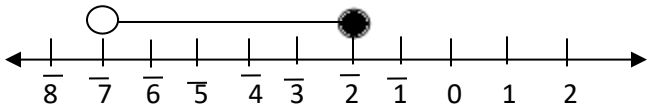
Solution

$-7 \le x \le -2$ => x is greater than or equal to -7 and less than or equal to -2.
Members = {-7, - 6, - 5, - 4, - 3, - 2}

l. $X = \{x: -7 < x \le -2\}$

Solution

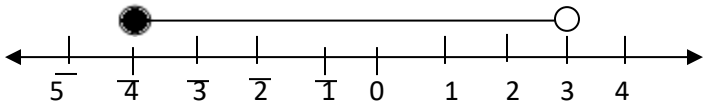
Members = {- 6, - 5, - 4, - 3, - 2}



m. $A = \{x: -4 \le x < 3\}$

Solution

Members = {- 4, - 3, - 2, - 1, 0, 1, 2}

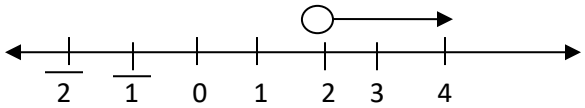


Q4. Solve the following inequalities and represent your answer graphically.

a. $x + 1 > 3$

Solution

$x + 1 > 3 \Rightarrow x > 3 - 1 \Rightarrow x > 2$



b. $x - 2 \ge 4$

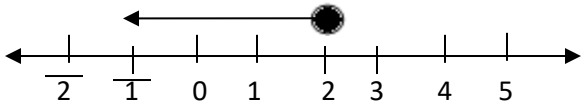
Solution

$x - 2 \ge 4 \Rightarrow x \ge 4 + 2, \Rightarrow x \ge 6$

c. $3x + 2 \le 2x + 4$

Solution

$3x + 2 \le 2x + 4 \Rightarrow 3x \le 2x + 4 - 2,$
$\Rightarrow 3x \le 2x + 2, \Rightarrow 3x - 2x \le 2, \Rightarrow x \le 2$



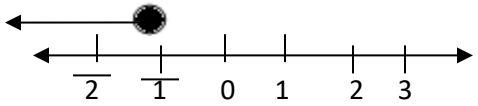
d. $4x - 3 \le 2x - 5$

Solution

$4x - 3 \leq 2x - 5 \Rightarrow 4x \leq 2x - 5 + 3, \Rightarrow 4x \leq 2x - 2,$
$\Rightarrow 4x - 2x \leq -2, \Rightarrow 2x \leq -2.$
Divide through using 2

$\Rightarrow \frac{2x}{2} \leq \frac{-2}{2},$

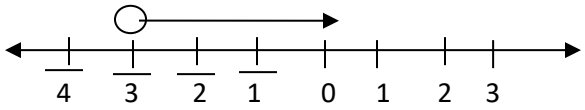
$\Rightarrow x \leq -1$



e. $-2x - 3 > -3x - 6$

Solution

$-2x - 3 > -3x - 6 \Rightarrow -2x > -3x - 6 + 3,$
$\Rightarrow -2x + 3x > -3,$
$\Rightarrow x > -3$

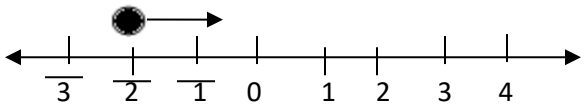


NB: When an inequality is multiplied through by a negative number, or divided through by a negative number, the inequality symbol is reversed.

f. $3x + 4 \leq 4x + 6$

Solution

$3x + 4 \leq 4x + 6 \Rightarrow 3x \leq 4x + 6 - 4,$
$\Rightarrow 3x \leq 4x + 2, \Rightarrow 3x - 4x \leq 2,$
$\Rightarrow -x \leq 2.$
Multiply through using -1
$\Rightarrow x \geq -2$



Q5. Find the truth set of the following inequalities:

a. $\frac{x}{2} + 2 \leq -6$

Solution

Multiply through using 2

$\Rightarrow 2 \times \frac{x}{2} + 2 \times 2 \leq 2x - 6,$

$\Rightarrow x + 4 \leq -12, \Rightarrow x \leq -12 - 4, \Rightarrow x \leq -16.$

Truth set = $\{x: x \leq -16\}$.

b. $\frac{x}{4} - 3 \geq 1$

Solution

Multiply through using 4

$\Rightarrow 4 \times \frac{x}{4} - 4 \times 3 \geq 4 \times 1,$

$\Rightarrow x - 12 \geq 4, \Rightarrow x \geq 4 + 12, \Rightarrow x \geq 16.$

Truth set = $\{x: x \geq 16\}$.

c. $\frac{x}{3} - \frac{1}{2} < x + 5$

Solution

Multiply through using 6, in order to remove the 3 and the 2

$\Rightarrow 6 \times \frac{x}{3} - 6 \times \frac{1}{2} < 6 \times x + 6 \times 5,$

$\Rightarrow 2x - 3 \leq 6x + 30,$

$\Rightarrow 2x \leq 6x + 30 + 3,$

$\Rightarrow 2x \leq 6x + 33,$

$\Rightarrow 2x - 6x \leq 33,$

$\Rightarrow -4x \leq 33.$

Divide through using - 4

$\Rightarrow \frac{-4x}{-4} > \frac{33}{-4},$

$\Rightarrow x > -8.3.$

Truth set = $\{x: x > -8.3\}$.

d. $\frac{2x}{3} + \frac{2}{5} < 4x - 3$

Solution

Multiply through using 15

$\Rightarrow 15 \times \frac{2x}{3} + 15 \times \frac{2}{5} < 15 \times 4x - 15 \times 3,$

$\Rightarrow 10x + 6 < 60x - 45,$

$\Rightarrow 10x < 60x - 45 - 6,$

$\Rightarrow 10x < 60x - 51,$

$\Rightarrow 10x - 60x < -51,$

$\Rightarrow -50x < -51.$

Divide through using -50

$\Rightarrow \frac{-50x}{-50} > \frac{-51}{-50}$

$\Rightarrow x > 1.02.$

Truth set = $\{x: x > 1.02\}$.

e. $\frac{1}{2}x + 1 < \frac{2}{3}x - 6$

Solution

Multiply through using 6

$\Rightarrow 6 \times \frac{1}{2}x + 6 \times 1 \leq 6 \times \frac{2}{3}x - 6 \times 6,$

$\Rightarrow 3x + 6 \leq 4x - 36,$

$\Rightarrow 3x \leq 4x - 36 - 6,$

$\Rightarrow 3x \leq 4x - 42,$

$\Rightarrow 3x - 4x < -,42,$

$\Rightarrow - x \leq - 42.$

Multiply through using -1

$\Rightarrow x \geq 42.$

Truth set = $\{x: x \geq 42\}$.

Q6.Determine the members of the following inequalities and represent your answers graphically:

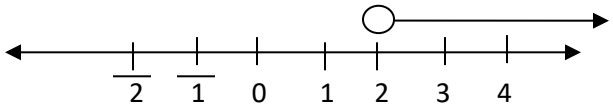
a. $Y = \{x: - 4x < - 8\}$

Solution

$- 4x < - 8 \Rightarrow \frac{- 4x}{- 4} > \frac{- 8}{- 4},$

$\Rightarrow x > 2.$

Members = {3,4,5……..}



b. $A = \{n: 2n - 1 \leq 5\}$

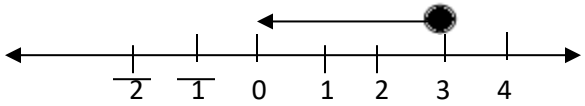
Solution

$2n - 1 \leq 5 \Rightarrow 2n \leq 5 + 1,$

$\Rightarrow 2n \leq 6, \Rightarrow \frac{2n}{2} \leq \frac{6,}{2}$

$\Rightarrow n \leq 3.$

Members = {3, 2, 1, 0, -1, -2, ……}



c. $Y = \{x: 3x > 6 + x\}$

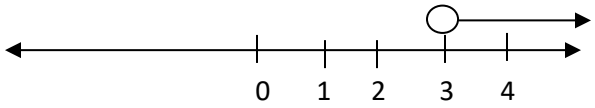
Solution

$3x > 6 + x \Rightarrow 3x - x > 6,$

$\Rightarrow 2x > 6 \Rightarrow \frac{2x}{2} > \frac{6,}{2}$

$\Rightarrow x > 3$

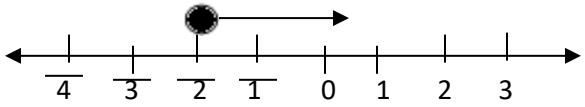
Members = {4, 5, 6, ……}



d. Y = { n : 3n + 4 ≤ 4n + 6}

Solution

3n + 4 ≤ 4n + 6 => 3n ≤ 4n + 6 – 4,
=> 3n ≤ 4n + 2 => 3n – 4n ≤ 2, => - n ≤ 2.
Multiply through using -1
=> n ≥ - 2
Members = {-2, -1, 0, 1, 2, …….}



Q7. Find the solution set of the following inequalities:

a. 2(3 + x) ≤ 3x

Solution

2(3 + x) ≤ 3x
=> 6 + 2x ≤ 3x, => 6 ≤ 3x – 2x,
=> 6 ≤ x => x ≥ 6.
Solution set = {x:x ≥ 6}.

b. 3(x – 1) ≤ 4x + 2

Solution

3(x – 1) ≤ 4x + 2
=> 3x – 3 ≤ 4x + 2,
=>3x ≤ 4x + 2 + 3,
=> 3x ≤ 4x + 5,
=> 3x – 4x ≤ 5,
=> - x ≤ 5, => x ≥ -5.
Solution set = {x: x ≥ - 5}.

c. - (x – 2) + 2x ≤ -2(3+ x)

Solution

- (x – 2) + 2x ≤ -2(3+ x)
=> - x + 2 + 2x ≤ - 6 -2x
=> -x + 2x ≤ - 6 – 2x – 2
=> x ≤ - 6 – 2 – 2x,
=> x ≤ - 8 – 2x,
=> x + 2x ≤ - 8,

=> 3x ≤ - 8 => x ≤ $-\frac{8}{3}$,
=> x ≤ - 2.66.
Solution set = {x: x ≤ - 2.66}.

d. $3(x - 2) + 2(x + 3) \geq 2(x - 6)$

Solution

$3(x - 2) + 2(x + 3) \geq 2(x - 6)$
$\Rightarrow 3x - 6 + 2x + 6 \geq 2x - 12,$
$\Rightarrow 3x + 2x - 6 + 6 \geq 2x - 12,$
$\Rightarrow 5x \geq 2x - 12$ (since $-6 + 6 = 0$),
$\Rightarrow 5x - 2x \geq -12,$
$\Rightarrow 3x \geq -12 \Rightarrow x > -\frac{12}{3}, \Rightarrow x \geq -4.$

e. $-4(x + 2) + (x - 1) < -2(x - 1)$

Solution

$-4(x + 2) + (x - 1) < -2(x - 1)$
$\Rightarrow -4x - 8 + x - 1 < -2x + 2,$
$\Rightarrow -4x + x - 8 - 1 < -2x + 2,$
$\Rightarrow -3x - 9 < -2x + 2,$
$\Rightarrow -3x < -2x + 2 + 9,$
$\Rightarrow -3x < -2x + 11,$
$\Rightarrow -3x + 2x < 11,$
$\Rightarrow -x < 11.$
Multiply through using $-1 \Rightarrow x > -11$
Solution set = $\{x: x > -11\}$

f. $x + 2 - 2(x - 1) \leq 9 - 2x$

Solution

$x + 2 - 2(x - 1) \leq 9 - 2x$
$\Rightarrow x + 2 - 2x + 2 \leq 9 - 2x,$
$\Rightarrow x - 2x + 2 + 2 \leq 9 - 2x,$
$\Rightarrow -x + 4 \leq 9 - 2x,$
$\Rightarrow -x + 4 + 2x \leq 9,$
$\Rightarrow -x + 2x \leq 9 - 4 \Rightarrow x \leq 5.$
Solution set = $\{x : x \leq 5\}$.

g. $\frac{-3x}{4} - 3 \leq x + \frac{1}{2}$. N/B: First multiply through using 4.

Solution

$\Rightarrow \frac{-3x}{4} \times 4 - 3 \times 4 \leq x \times 4 + \frac{1}{2} \times 4,$
$\Rightarrow -3x - 12 \leq 4x + 2,$
$\Rightarrow -3x \leq 4x + 2 + 12$
$\Rightarrow -3x \leq 4x + 14,$
$\Rightarrow -3x - 4x \leq 14$
$\Rightarrow -7x \leq 14.$
Divide through using -7
$\Rightarrow \frac{-7x}{-7} \geq \frac{14}{-7} \Rightarrow x \geq -2.$
Solution set = $\{x: x \geq -2\}$.

Q8.Determine the truth sets of the following inequalities and list their members:

a. $\frac{1}{3}(2x + 1) < x + 1$

N/B: Multiply through using 3.

Solution

=> $3 \times \frac{1}{3}(2x + 1) < 3 \times x + 3 \times 1$

=> 1 (2x + 1) < 3x + 3,

=> 2x + 1 < 3x + 3,

=> 2x – 3x < 3 -1,

=> -x < 2 and multiply through using -1 => x > -2.

Truth set = {x: x > -2}.

Members = {-1, 0, 1, 2, 3 ……..}.

b. $\frac{2}{3}(2x + 2) \leq \frac{1}{4}(x + 1)$

Solution

Multiply through using 12

=> $12 \times \frac{2}{3}(2x + 2) \leq 12 \times \frac{1}{4}(x + 1)$

=> 8 (2x + 2) ≤ 3 (x + 1),

=> 16x + 16 ≤ 3x + 3,

=> 16x – 3x ≤ 3 – 16,

=> $13x \leq -13 \Rightarrow \frac{13}{13}x \leq -\frac{13}{13}$,

=> x ≤ - 1. Truth set = {x : x ≤ - 1}.

Members = { -1, -2, -3……}.

c. $\frac{2x - 2}{4} \leq x + 1$

N/B: $\frac{2x - 2}{4} = \frac{1}{4}(2x - 2)$

Solution

From $\frac{2x - 2}{4} \leq x + 1$

$\frac{1}{4}(2x - 2) \leq x + 1$

Multiply through using 4

=> $4 \times \frac{1}{4}(2x - 2) \leq 4 \times x + 4 \times 1$

=> 1 (2x – 2) ≤ 4x + 4,

=> 2x – 2 ≤ 4x + 4

=>2x ≤ 4x + 4 + 2,

=> 2x – 4x ≤ 6

=> -2x ≤ 6.
Divide through using -2 => $\frac{-2x}{-2} \geq \frac{6}{-2}$ => x ≥ -3.
Truth set {x: x ≥ -3}.
Members = {-3, -2, -1, 0, …….).

d. $\frac{2x-1}{3} > \frac{x+1}{2}$

Solution

$\frac{2x-1}{3} > \frac{x+1}{2}$ can also be written as
$\frac{1}{3}(2x-1) > \frac{1}{2}(x+1)$.
Multiply through using 6
=> $6 \times \frac{1}{3}(2x-1) > 6 \times \frac{1}{2}(x+1)$
=>2(2x – 1) > 3(x + 1),
=> 4x – 2 > 3x + 3 => 4x > 3x + 3 + 2,
=> 4x > 3x + 5 => 4x – 3x > 5,
=> x > 5. Truth set = (x: x > 5).
Members = {6, 7, 8, 9 ……)

e.- $4(x-2) > \frac{1}{6}(x+19) + \frac{2}{3}(2x-1)$.

Solution

Multiply through using 6
=> $6 \times 4(x-2) > 6 \times \frac{1}{6}(x+19) + 6 \times \frac{2}{3}(2x-1)$
=> -24(x – 2) > 1 (x + 19) + 4 (2x – 1),
=> -24x + 48 > x + 19 + 8x – 4,
=> -24x + 48 – x – 8x > 19 – 4,
=> -24x – x – 8x > 19 – 4 – 48,
=> -33x > -33.
Divide through using -33, => x < 1
Truth set = {x: x < 1}
Members (- 0, -1, -2 ……)

Q9.Simplify the following inequality

$\frac{3x+1}{2} - 1 \geq \frac{2x+3}{4} - 2$

Solution

The given inequality can be written as
$\frac{1}{2}(3x+1) - 1 \geq \frac{1}{4}(2x+3) - 2$
Multiply through using 4
=> $4 \times \frac{1}{2}(3x+1) - 4 \times 1 \geq 4 \times \frac{1}{4}(2x+3) - 4 \times 2$

$\frac{2}{}$ $\frac{4}{}$

$\Rightarrow 2(3x + 1) - 4 \geq 1(2x + 3) - 8,$

$\Rightarrow 6x + 2 - 4 \geq 2x + 3 - 8,$

$\Rightarrow 6x - 2x \geq 3 - 8 - 2 + 4,$

$\Rightarrow 4x \geq -3 \Rightarrow x \geq -\frac{3}{4}$

$\Rightarrow x \geq -0.75.$

Q10. Given that $\frac{-2x + 4}{3} - \frac{5(x + 1)}{2} \leq 2,$

find the value of x.

Solution

$\frac{-2x + 4}{3} - \frac{5(x + 1)}{2} \leq 2,$

$\Rightarrow \frac{1}{3}(-2x + 4) - \frac{5}{2}(x + 1) \leq 2.$

Multiply through using 6

$\Rightarrow 6 \times \frac{1}{3}(-2x + 4) - 6 \times \frac{5}{2}(x + 1) \leq 6 \times 2,$

$\Rightarrow 2(-2x + 4) - 15(x + 1) \leq 12,$

$\Rightarrow -4x + 8 - 15x - 15 \leq 12,$

$\Rightarrow -4x - 15x \leq 12 - 8 + 15,$

$\Rightarrow -19x \leq 4 + 15,$

$\Rightarrow -19x \leq 19.$

Divide through using -19

$\Rightarrow \frac{-19x}{-19} \geq \frac{19}{-19} \Rightarrow x \geq -1.$

Q11. If $\frac{2(x - 1)}{3} - \frac{4(2x + 12)}{5} \leq -14,$

determine the value of x which satisfies the given equation.

Solution

$\frac{2(x - 1)}{3} - \frac{4(2x + 12)}{5} \leq -14$

Can also be written as

$\frac{2}{3}(x - 1) - \frac{4}{5}(2x + 12) \leq -14$

Multiply through using 15

$15 \times \frac{2}{3}(x - 1) - 15 \times \frac{4}{5}(2x + 12) \leq 15 \times 14$

$\Rightarrow 10(x - 1) - 12(2x + 12) \leq -210,$

$\Rightarrow 10x - 10 - 24x - 144 \leq -210,$

$\Rightarrow 10x - 24x - 10 - 144 \leq -210,$

$\Rightarrow -14x \leq -210 + 154,$

$\Rightarrow -14x - 154 \leq -210,$

$\Rightarrow -14x \leq -210 + 154,$

$\Rightarrow -14x \leq -56,$

$\Rightarrow -14x \geq -56$

$\frac{}{-14}$ $\frac{}{-14}$

=> x ≥ 4.

Q12. Solve the inequality

$\frac{4x + 1}{2x - 1} < \frac{5}{2}$, where $x \neq \frac{1}{2}$

and illustrate your answer on a number line

Solution

$\frac{4x + 1}{2x - 1} < \frac{5}{2}$

Cross multiply => 2(4x + 1) < 5(2x – 1),

=> 8x + 2 < 10x – 5

=> 8x – 10x < -5 -2,

=> -2x < -7

=> $\frac{-2x}{-2} > \frac{-7}{-2}$

=> x > 3.5

Q13. Find the truth set of the inequality $\frac{x - 1}{3(x + 1)} \geq \frac{1}{-4}$

Solution

$\frac{x - 1}{3(x + 1)} \geq \frac{1}{-4}$ Cross multiply

=> - 4 (x – 1) ≥ 1 x 3(x + 1),

=> - 4(x – 1) ≥ 3(x +1),

=> - 4x + 4 ≥ 3x + 3,

=> - 4x -3x ≥ 3 – 4,

=> -7x ≥ -1

=> $\frac{-7x}{-7} \leq \frac{-1}{-7}$

=> $x \leq \frac{1}{7}$ => x ≤ 0.14.

Q14. Given that $\frac{2}{3}(2x + 5) \leq 8^2/_3$,

find the value of x and represent your answer graphically.

Solution

$\frac{2}{3}(2x + 5) \leq 8^2/_3$,

=> $\frac{2(2x + 5)}{3} \leq \frac{26}{3}$

Multiply through using 3

$\Rightarrow 3 \times \frac{2}{3}(2x + 5) \leq 3 \times \frac{26}{3}$

$\Rightarrow 2(2x + 5) \leq 26,$

$\Rightarrow 4x + 10 \leq 26,$

$\Rightarrow 4x \leq 26 - 10$

$\Rightarrow 4x \leq 16,$

$\Rightarrow \frac{4x}{4} \leq \frac{16}{4} \Rightarrow x \leq 4$

QUESTIONS

Q1. List the members of the following inequalities

a. $x > 5$ Ans. {6, 7, 8……..}

b. $x \geq 8$ Ans. {8, 9, 10 ……}

c. $x \geq -5$ Ans. {-5, -4, -3, -2, -1, 0, 1, 2, 3…}

d. $x < 3$ Ans. {2, 1, 0, -1, -2 ……}

e. $x < -2$ Ans. {-3, -4, -5, ……..}

f. $x \leq -5$ Ans. {-5, -6, -7 ……..}

Q2. Write down the members of each of these given sets:

a. $X = \{x: 2 \leq x \leq 7\}$
Ans. {2, 3, 4, 5, 6, 7}

b. $Y = \{x: 3 \leq x < 9\}$
Ans. {3, 4, 5, 6, 7, 8}

c. $M = \{x: -3 \leq x < 2\}$
Ans. {-3, -2, -1, 0, 1}

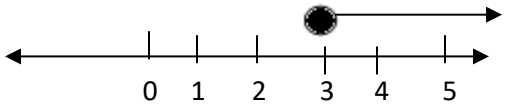
d. $N = \{Y: -7 < Y \leq -1\}$
Ans. {- 6, -5, - 4, -3, -2, -1}

e. $M = \{N: -8 \leq N < -2\}$
Ans. {- 8, -7, -6, -5, - 4, -3}

Q3. Represent the following inequalities graphically:

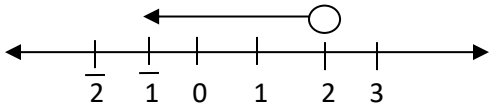
a. $Y = \{x: x > 5\}$
Ans.

b. $Y = \{x: x \geq 3\}$
Ans.



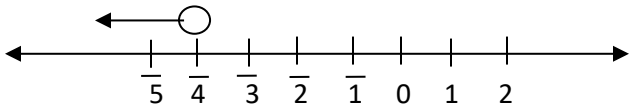
c. $M = \{N: N < 2\}$

Ans.



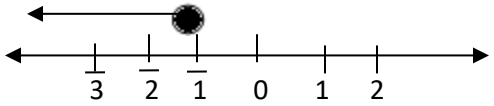
d. M = {Y: Y < - 4}

Ans.



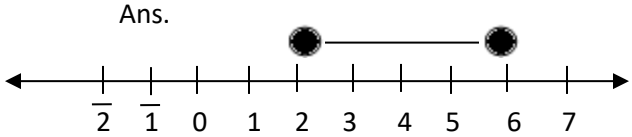
e. M = {Y: Y ≤ - 1}

Ans.



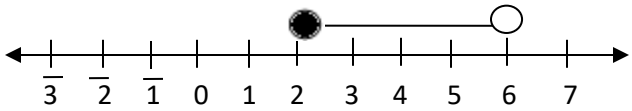
f. X = {Y: 2 ≤ Y ≤ 6}

Ans.



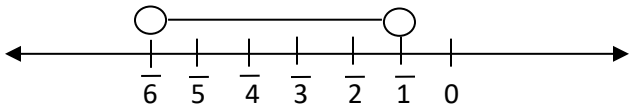
g. X = {Y:2 ≤ Y < 6}

Ans.



h. X {Y: - 6 < Y < -1}

Ans.



Q4.Determine the truth set of each of the following inequalities and list its members:

a.$3x - 4 \geq x + 2$

Ans. $x \geq 3$. Members = {3, 4, 5 …….}

b. $5x + 2 < 8x - 10$

Ans. Truth set = { x: x > 4}

Members = {5, 6, 7……)

c. $2x + 2 + 4x \leq x - 8 \Rightarrow x \leq -2$

Ans. Truth set = (x: x ≤ -2}

Members = {-2, -3, - 4 ……..}

d. $\frac{1}{3}x - 1 \leq 2$

Ans. Truth set{x: x ≤ 9}

Members = {9, 8, 7, 6 ……..}

e. $\frac{x}{2} - 2 \geq 4$

Ans. Truth set = {x : x ≥ 12}

Members = {12, 13, 14 ……..}

f. $\frac{1}{2}x + 1 \geq \frac{4x}{6} - 6$

Ans. Truth set = {x: x ≤ 42}

Members = {42, 41, 40 ……)

g. $4(3 + x) \leq 6x$

Ans. Truth set = {x: x ≥ 6)

Members = {6, 7, 8 ……}

Q5. Find the truth set of each of the following inequalities:

a. $-2(x - 2) + 4x \leq -4(3 + x)$

Ans. {x; x ≤ -2.66}

b. $6(x - 2) + 4(x + 3) \geq 4(x - 6)$

Ans. {x: x ≥ - 4}

c. $-8(x + 2) + 2(x - 1) < -4(x - 1)$

Ans. {x: x > -11}

d. $2x + 4 - 4(x - 1) \leq 18 - 4x$

Ans. {x: x ≤ 5}

e. $\frac{2}{3}(2x + 1) < 2x + 2$

Ans. {x: x > -2}

f. $\frac{4x - 4}{4} \leq 2x + 2$

Ans. {x: x ≥ -3}

g. $\frac{2x - 14}{6} < x + 1$

Ans. {x: x > 5}

.

Q6. Given that $-\frac{4x + 8}{3} - \frac{10(x + 1)}{2} \leq 2$

Find the value of x

Ans. $x \geq -\frac{13}{19}$

Q7. If $\frac{2(x - 1)}{6} - \frac{4(2x + 12)}{10} \leq -14$,

Find the value of x which satisfies the given equation.

. Ans. $x \geq 19$.

Q8. Solve the inequality $\frac{4x + 1}{4x - 2} < \frac{5}{4}$

Ans. $x > 3.5$ or $x > 3^1/_2$

Q9. Given that $2 - \frac{1}{3}(2x - 2) \geq \frac{2x}{3}$

Find the value of x. Ans. $x \leq 2$