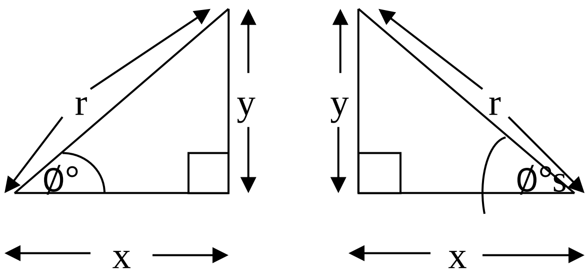
CHAPTER TEN

TRIGONOMETRY

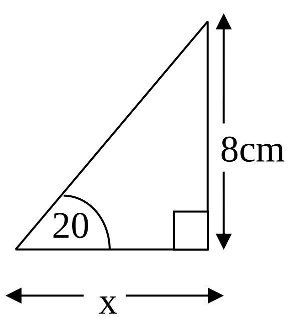


Consider the above two figures. For any one of them, the following facts must be noted:

1. When the length y is divided by the length x, we always get the tangent or the tan of the angle $\emptyset$, ie for any of the above figures $\tan\emptyset = \frac{y}{x}$.
2. When the length r is multiplied by the cosine or the cos of the angle $\emptyset$, we always get the length $x, ie\ r\ x\ cos\emptyset = x, \Longrightarrow \cos\emptyset = \frac{x}{r}$
3. When the length r is multiplied by the sine or sin of the angle $\emptyset$, we always get the length $y\ ie\ r\ x\ sin\emptyset\ =\ y, \Longrightarrow Sin\emptyset = \frac{y}{r}$

<u>The use of tangent:</u>

Q1.



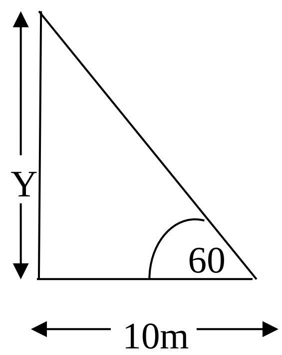
Find the value of x.

Soln.

Y = 8cm and $\emptyset = 20°, \Longrightarrow tan\emptyset = \frac{y}{x} \Rightarrow tan20° = \frac{8}{x} \Rightarrow x \times tan20° = 8, \Longrightarrow$
$x\ tan20° = 8.$

Dividing through using tan 20° => $\frac{x\tan 20^0}{\tan 20°} = \frac{8}{\tan 20°} \Longrightarrow x = \frac{8}{\tan 20^0} = \frac{8}{0.364}$, (since tan $20°) = 0.364 \therefore x = 219cm$.

Q2.



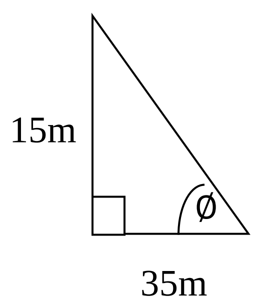
Find the length y.

Soln.

$\emptyset = 60°\ and\ x = 10m.$ Since $\tan\emptyset = \frac{y}{x} \Longrightarrow \tan 60° = \frac{y}{10}, \Longrightarrow 10 \times \tan 60° = y, \Longrightarrow y = 10 \times \tan 60°, => y = 10 \times 1.730, (since \tan 60° = 1.730) \Rightarrow y = 10 \times 1.730 = 17.3.$

∴Y = 17.30m.

Q3.



Calculate the angle Ø°

Soln.

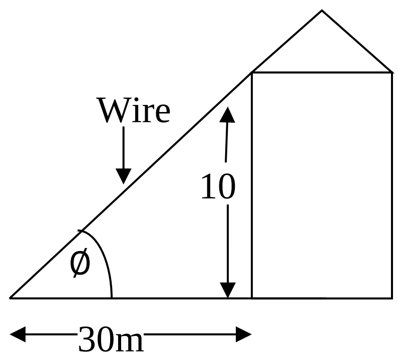
$y = 15m\ and\ x = 35m. From\ tan\emptyset = \frac{y}{x} \Longrightarrow \tan\emptyset = \frac{15}{35},$

$\Rightarrow \tan\emptyset = 0.428, \Rightarrow \emptyset = tan^{-1}\, 0.428,$

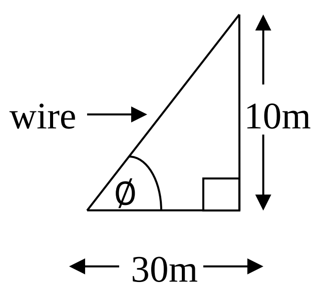
=> $\emptyset = 23°$ approx.

Q4. One end of a wire is fixed to a point 10m up a building. The other end is fixed to the ground at a point 30m away from the building. Find the angle which the wire makes with the ground.

Soln.



The above diagram can be represented as shown next:.

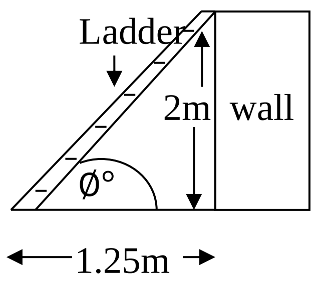


Let Ø = the angle made by the wire with the ground.

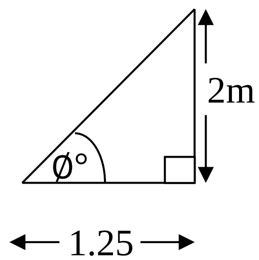
$tan\emptyset° = \frac{10}{30} \Longrightarrow tan\ \emptyset = 0.30 \Longrightarrow \emptyset = tan^{-1}0.30, \Longrightarrow \emptyset = 17° \therefore$
$the\ wire\ makes\ an\ angle\ of\ 17°$, with the ground.

Q5. A ladder leans against a wall. The foot of the ladder is on the same horizontal level as the foot of the wall and is 1.25m away from it. The top of the ladder just reaches the top of the wall which is 2m high. Calculate the angle between the ladder and the ground.

Soln



The above figure can be simplified as shown next:

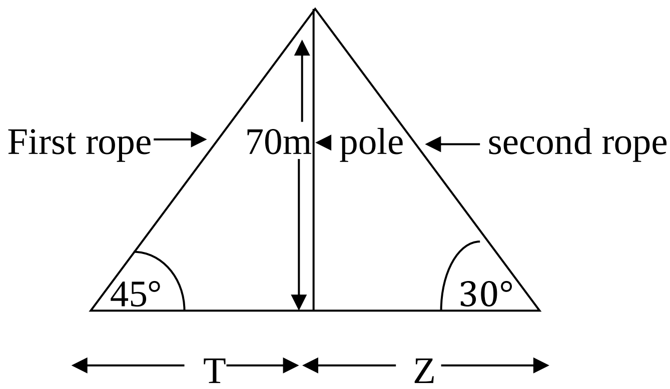


Let Ø = the angle between the ground and the ladder.

Then tan $\emptyset° = \frac{2}{1.25}$ $=> tan\ \emptyset° = 1.6, \Longrightarrow \emptyset = tan^{-1}1.6 \Rightarrow \emptyset = 58°$The angle between the ladder and the ground = 58°

Q6. A pole is 70m long and stands on a leveled ground. One end of a first rope is tired to the top of this pole while the other end is fixed to a point on the ground, so that it makes an angle of 45° with the ground. One end of a second rope is fixed to the top of the pole, and its other end is fixed to the ground so that it makes an angle of 30° with the ground. Calculate the distance between the points where the two ropes are fixed to the ground, if they are opposite to each other.

Soln



Let T = the distance between the foot of the pole and the point where the first rope makes an angle of 45° with the ground. Also let Z = the distance from the foot of the pole to the point where the second rope is fixed to the ground and makes an angle of 30° with the ground as shown in the diagram. Then the total distance between the two points = T + Z. We must therefore find T and Z

The original figure can be broken into two parts as shown next:

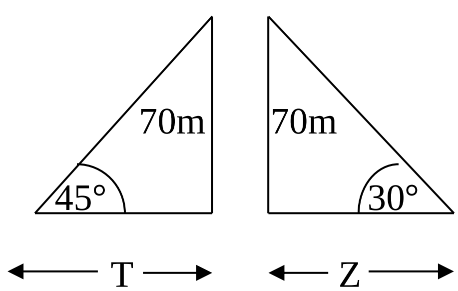


Figure (1) Figure (2)

From fig. (1), $\tan 45° = \frac{70}{T} \Longrightarrow T \times \tan 45° = 70°$,

$\therefore T \tan 45° = 70°$

Divide through using tan 45°. i.e

$\frac{T\ \tan 45°}{\tan 45°} = \frac{70}{\tan 45°} \Rightarrow T = \frac{70}{\tan 45°}$, but $\tan 45° = 1$

$\Rightarrow T = \frac{70}{1} = 70m.$

From fig. (2), $\tan 30° = \frac{70}{Z} \Longrightarrow Z \times \tan 30° = 70,$

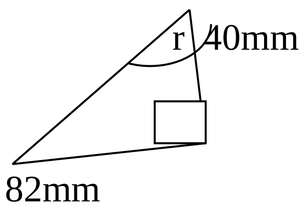
$\Rightarrow Z \tan 30° = 70$’s

Divide through using tan 30° $\Longrightarrow \frac{Z \tan 30°}{\tan 30°} = \frac{70}{\tan 30°}$

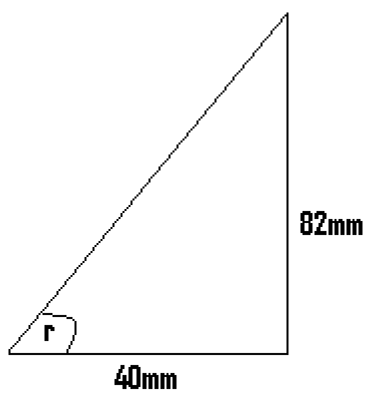
$\Rightarrow Z = \dfrac{70}{\tan 30°}, but \tan 30° = 0.58, \therefore Z = \dfrac{70}{0.58} = 121,$

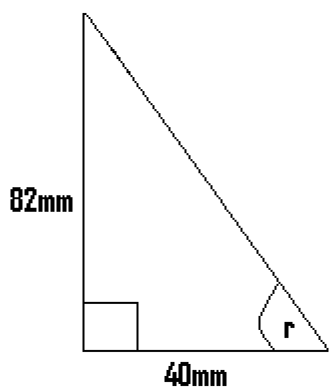
$\Rightarrow Z = 121m.$ Distance between the two points = T + Z = 70 + 121 = 191m.

Q7. Calculate the angle marked r° in the figure below.



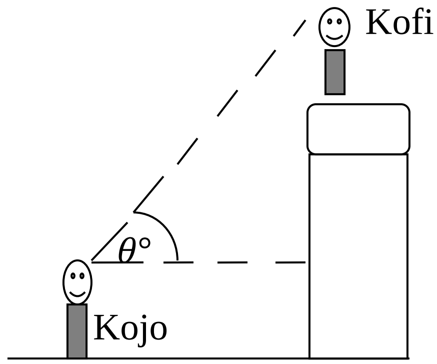
Rotate the figure to get any of the figures below (1.e rotate so that the location of angle rests horizontally).





By using any of the above figures, tan r° $= \frac{82}{40} \Longrightarrow tanr° = 2.05, => r° = tan^{-1}205 \Rightarrow r = 64.°$

Angle of elevation:



Consider two friends Kojo and Kofi. Kofi is standing on top of a building while Kojo stands on the ground as shown in the diagram. In order for Kojo to look at Kofi, he must turn his eyes through angle θ. This angle θ is referred to as the angle of elevation of Kofi from Kojo.

Q1. A boy stood 40m away from the foot of a tower, and found the angle of elevation of the top of the tower. If the tower is 70m high, calculate the angle of elevation of the top of the tower, if the boy's height is to be neglected.

Soln.

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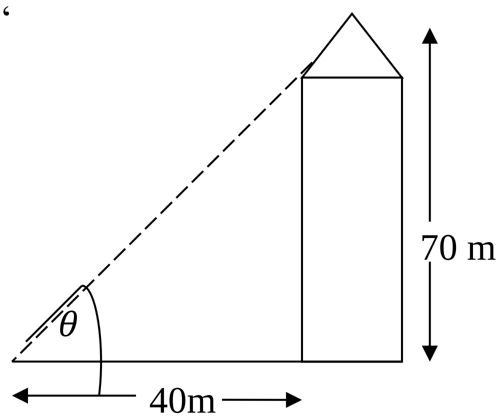
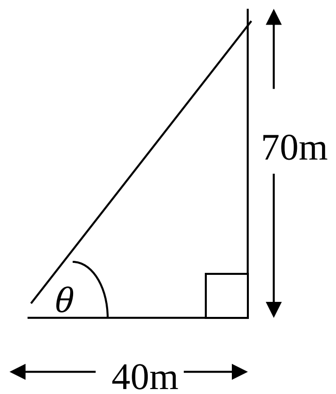


N/B: Since the boy's height is negligile or to be neglected, then we can work without taking it into consideration. The above figure can be represented as

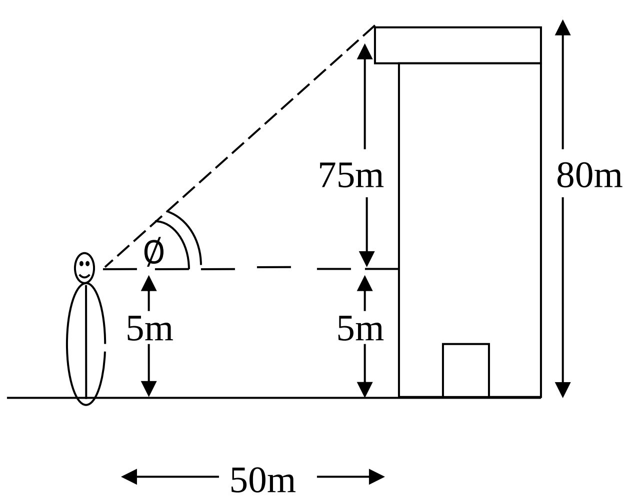


Where $\theta =$ the angle of elevation of the top of the tower.

Tan $\theta = \frac{70}{40} = 1.75 \Longrightarrow \tan\theta = 1.75, \Longrightarrow \theta = tan^{-1}1.75 \Longrightarrow \theta = 60°$

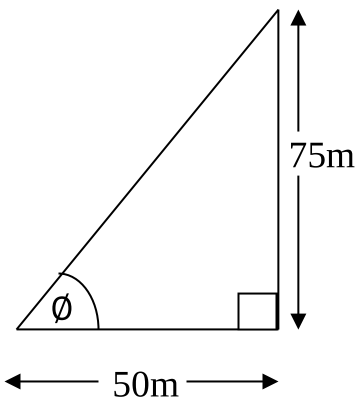
Q2. A boy of height 5m, stood 50m away from the foot of a building, which is 80m of height. What will be the angle of elevation of the top of this building from the boy.

Soln



N/B: In this case the boy`s height is not negligible but 5m. For our calculation, we must not use the 80m but rather

80 – 5 = 75m. Our diagram therefore becomes as shown next:

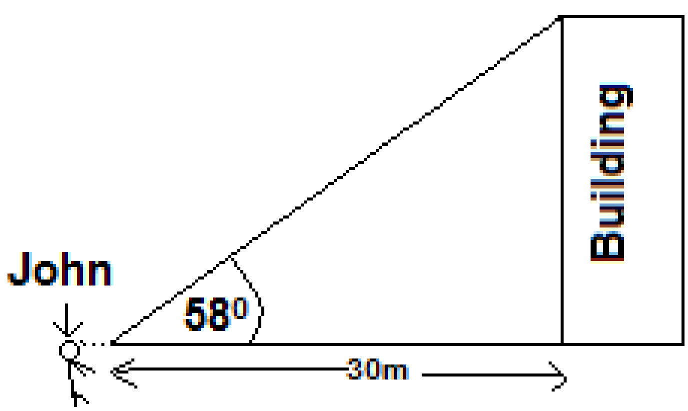


Where Ø = the angle of elevation. Tan $\emptyset = \frac{75}{50} = \frac{3}{2} = 1.5, \Longrightarrow \emptyset = tan^{-1}1.5, \Rightarrow \theta = 57.°$

Q3. John stood 30m away from the foot of a building, and observed the angle of elevation of the top of the building to be 58°.

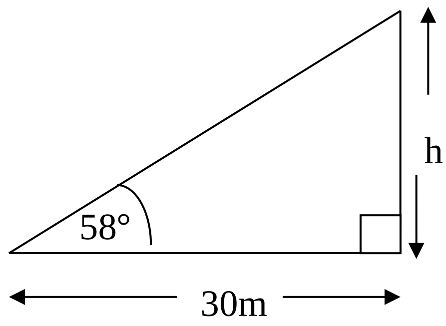
If John's height is negligible, or insignificant, calculate the height of the building.

Soln.



N/B: Since John's height is negligible or to be neglected, we work without using it.

The above diagram can be represented as shown next:



tan 58° $= \frac{h}{30} \Longrightarrow 30 \times tan\ 58° = h$;

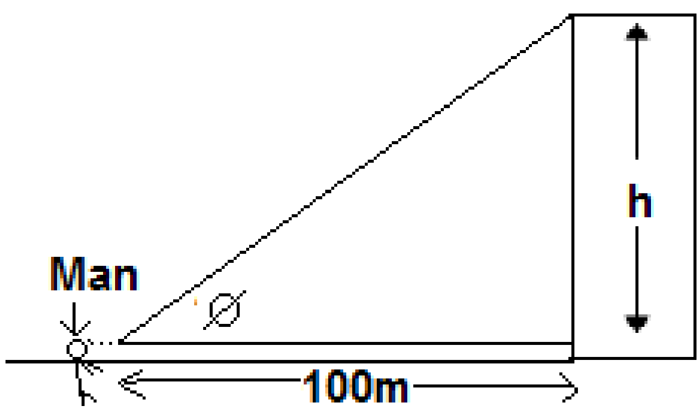
$\Rightarrow$h = 30 x 1.600 = 48m.

$\therefore$ The height of the building = 48m

Q4. A man stood 100m away from a building and measured the angle of elevation of the top of the building. He then moved a distance of 20m towards the building and noted that the angle of elevation of the top of the building has changed to 35°. Calculate the height of the building.

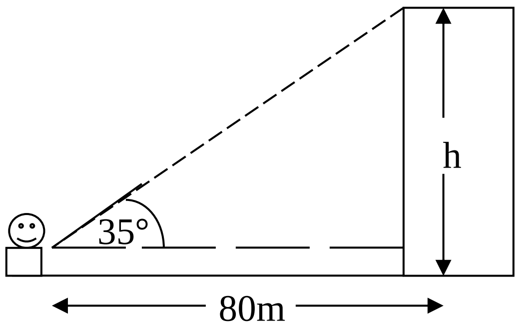
Soln.

First case

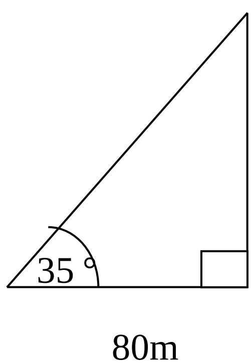


Let Ø = the angle of elevation and h = the height of the building. In the second case, the man moved a distance of 20m towards the building. His distance from the building = 100 – 20 = 80m and the angle of elevation in this case is 35°. Our new diagram

therefore becomes as shown next. Also since the man`s height is not given, then it is negligible.



This diagram can be represented as:

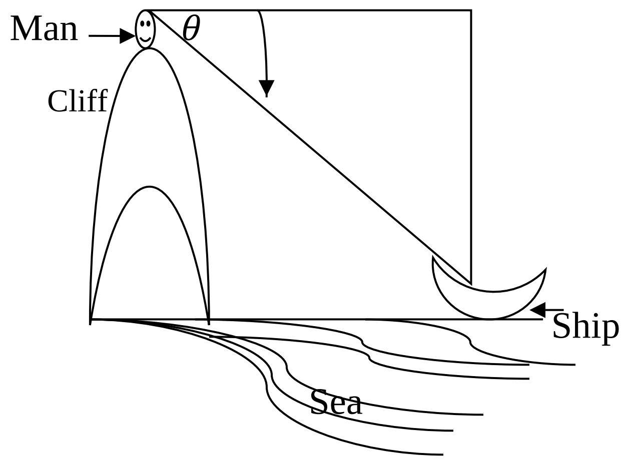


tan 35° $= \frac{h}{80} \Longrightarrow 80 \times tan35° = h,$

$\Rightarrow$ h = 80 x tan 35,

$\Rightarrow$ h = 80 x 0.7 = 56.0,$\Longrightarrow h = 56m$.

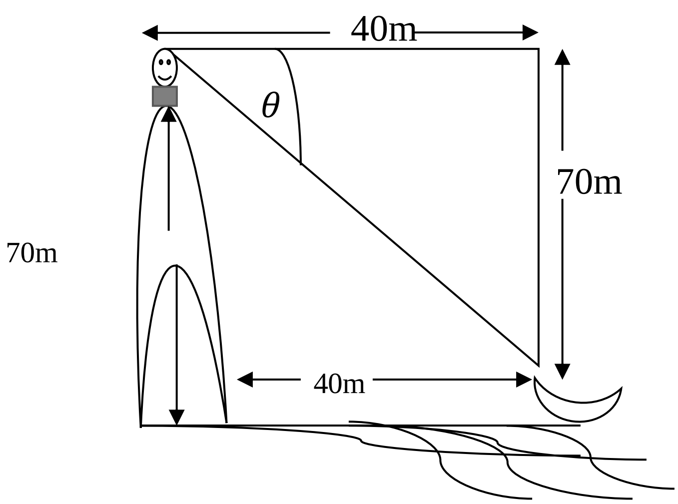
The angle of depression:

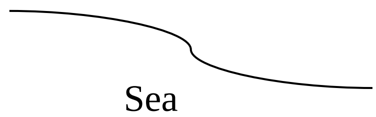


Consider the above figure which shows a man standing on a cliff, and a ship which is on the sea. Before this man can look at the ship from the top of the cliff, he must turn his eyes through an angle θ. This angle, θ is called the angle of depression of the ship from the man.

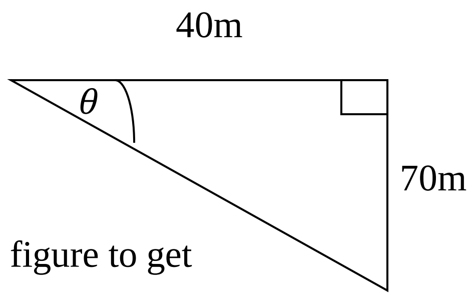
Q1. A man stood on a cliff which is 70m high. The distance between the foot of the cliff, and a canoe which was on the sea was 40m. Find the angle of depression of the canoe fron the man. Neglect the man's height and the canoe's height.

Soln.

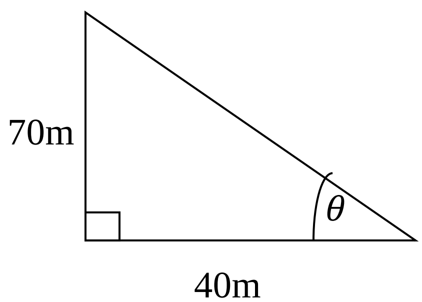




Let $\theta =$ the angle of depression of the canoe from the man. The figure can be represented as shown next:



Rotate this figure to get



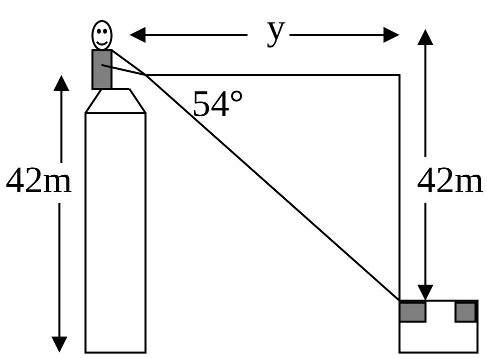
$\tan\theta = \frac{70}{40} = \frac{7}{4} = 1.75.$

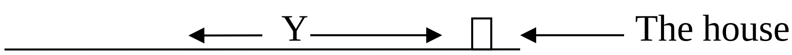
From $\tan\theta = 1.75 \Longrightarrow \theta = tan^{-1}1.75 \Longrightarrow \theta = 60°.$

$\therefore\ Angle\ of\ depression = 60°$

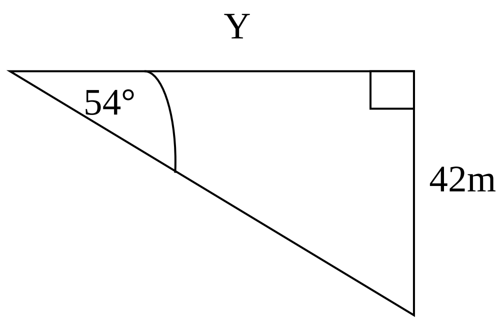
Q2. A man observed from the top of a tower, which is 42m high, that the angle of depression of the top of his house was 54°. Find the distance between his house and the foot of the tower: Assume that the height of the man is negligible.

Soln.

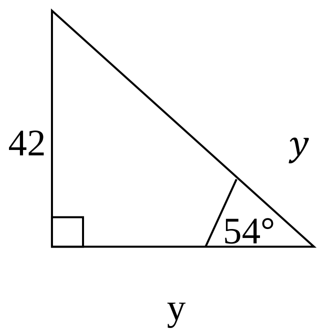




Let y = the distance between the foot of the tower and the man`s house.



Rotate this figure to get

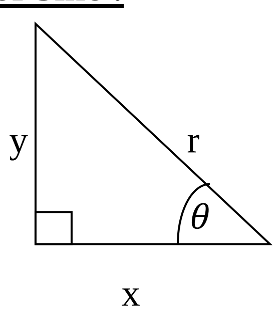


$$\tan 54° = \frac{42}{y} \Rightarrow y\ tan 54° = 42$$

$$y = \frac{42}{\tan 54} \Longrightarrow y = \frac{42}{1.37} \Longrightarrow y = 31m$$

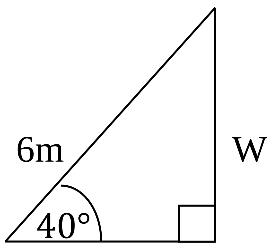
∴ Distance between the boys house and the foot of the hill is 31m.

<u>The use of Sine :</u>



Sin $\theta = \frac{y}{r}$

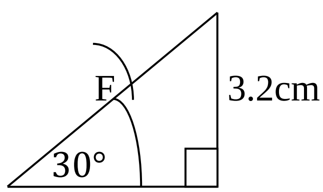
Q1. Calculate the length marked W in the figure given:



Soln.

Sin $40° = \frac{W}{6} \Longrightarrow 6 \times \sin 40 = W, \Rightarrow W = 6 \times \sin 40 \Longrightarrow W = 6 \times 0.642 = 3.84m.$

Q2.



Calculate F.

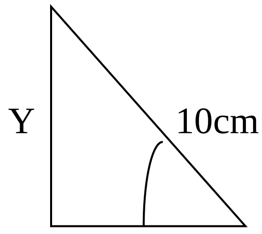
Soln.

Since $\sin 30° = \frac{3.2}{F}, \Rightarrow F \times \sin 30° = 3.2.$

Divide through by $\sin 30° \Longrightarrow \frac{F \sin 30°}{\sin 30°} = \frac{3.2}{\sin 30°}$,

$\Rightarrow F = \frac{3.2}{\sin 30°} = \frac{3.2}{0.5} = 6.4cm.$

Q3. Calculate the value of Y.

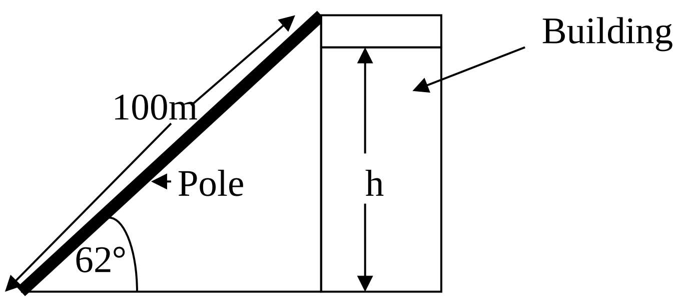


Soln.

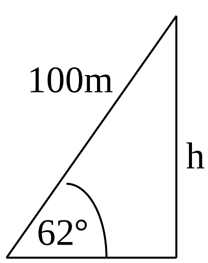
Sin 50° $= \frac{Y}{10} \Longrightarrow y = 10 \times \sin 50°, \Longrightarrow y = 10 \times 0.766$ = 7.7,

$=> y = 7.7cm$

Q4. A pole leans against a building. The pole is 100m long and makes an angle of 62° with the ground. If the top part of the pole reaches the top part of the building, find the height of the building.



Let h = the height of the building. The above diagram can be represented as shown next:



Since sin 62° $= \frac{h}{100} \Longrightarrow 100 \times \sin 62° = h.$

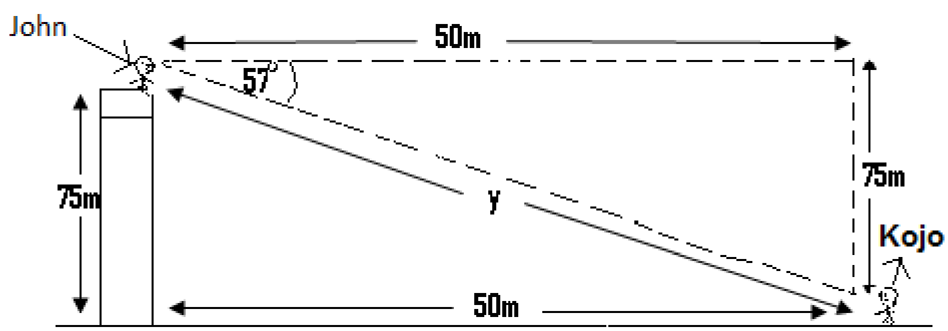
$\therefore h = 100 \times \sin 62° = 100 \times 0.882$,

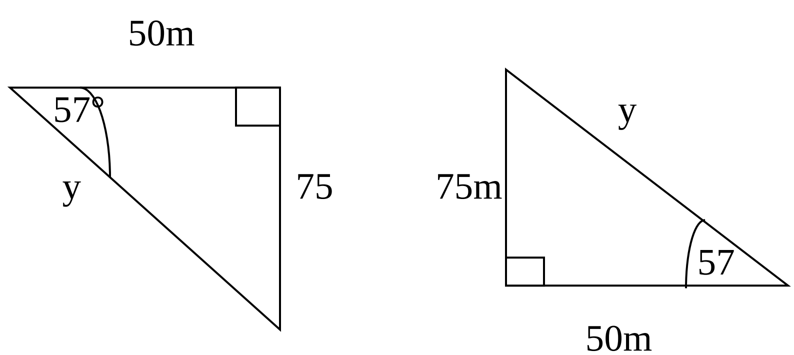
$\Rightarrow h = 88.2m \therefore The\ height\ of\ the\ building\ is\ 88.2m.$

Q5. One day John stood on a storey building which is 75m high. Kojo stood on the ground, at a distance of 50m away from the building. If John observed that the angle of depression of Kojo's head was 57°, find the distance between the two boys.

Soln.

Let y = the distance between the two boys.





Rotate the first figure to get the second one.

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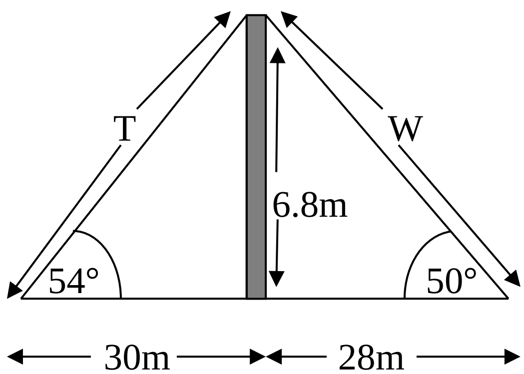
$\text{Sin } 57° = \frac{75}{y} \Longrightarrow y \times \sin 57° = 75 \therefore y = \frac{75}{\sin 57} = \frac{75}{0.84}$

Y = 89 $\therefore$ distance between the two boys = 89m.

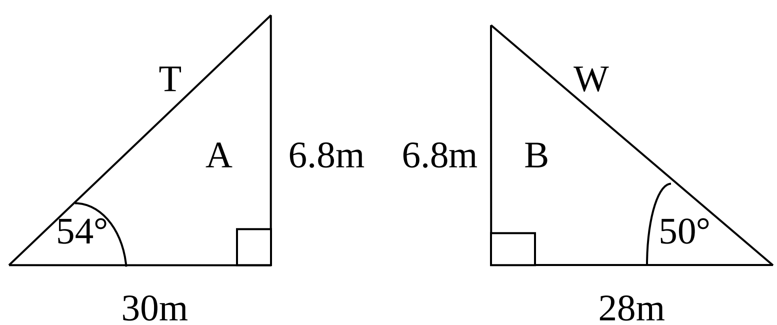
Q6. A pole of length 6.8m long is fixed into the ground.One end of a rope is fixed to the top part of this pole, whilst its other end is fixed to a point on the ground, making an angle of 54° with the ground. One end of a second rope is fixed to the top of the pole, whilst its other end is fixed to a point on the ground, in an opposite direction to the first one. If this second rope makes an angle of 50° with the ground, find the total length of the two ropes. The distance from the foot of the pole to where the first rope is fixed to the ground is 30m, and that between the foot of the pole and where the second rope is fixed to the ground is 28m.

Soln.



Let T = the length of the first rope and let W = the length of the second rope. Then the total length of the ropes =

T + W. This diagram may be broken into two parts i.e



From fig. (A), $\sin 54^0 = \frac{6.8}{T}$, $\Rightarrow$ T sin $54^0 = 6.8$.

Devide through using sin 54^0

$\Rightarrow \frac{\boldsymbol{T} \sin 54^0}{\sin 54^0} = \frac{6.8}{\sin 54^0}$ $\Rightarrow$ **T =** $\frac{6.8}{0.8}$ $\Rightarrow$ **T = 8.5.**

From fig. (B), sin $\mathbf{50^0}$ **=** $\frac{\mathbf{6.8}}{\boldsymbol{W}}$ **,**

$\Rightarrow$ **W sin** $\mathbf{50^0 = 6.8.}$

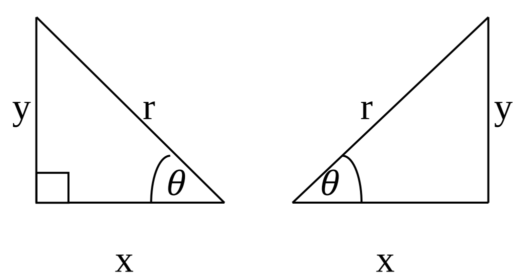
Divide through using sin 50^0 $i.e$ $\frac{\boldsymbol{W} \sin 50^0}{\sin 50^0} = \frac{6.8}{\sin 50^0}$

$\Rightarrow$ **W =** $\frac{6.8}{0.77} = \boldsymbol{8m}.$

The length of the rope **=T + W = 8.5 + 8 = S16.5m.**

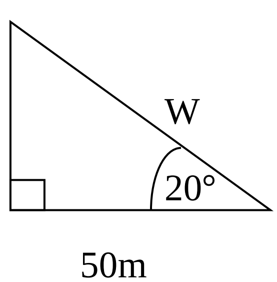
N/B:Sin 50 = 0.77

<u>The use of cosine :</u>



$\cos\theta = \frac{x}{r}$

Q1.



Find the length marked W.

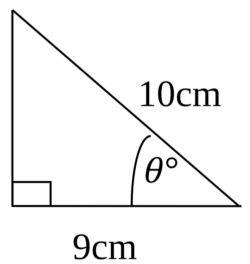
Soln.

Cos 20 ° $= \frac{50}{W} \Longrightarrow W \times cos20° = 50.$

Divide through using cos 20

$\frac{W\ cos\ 20°}{\cos 20°} = \frac{50}{\cos 20°} \Longrightarrow W = \frac{50}{\cos 20} = \frac{50}{0.94} = 53 \Longrightarrow W = 53m.$

Q2.



Calculate the angle marked $\theta°$

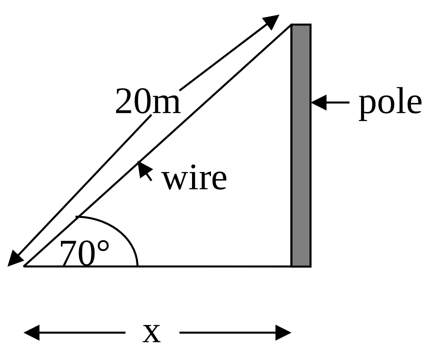
Soln.

Cos $\theta = \frac{9}{10} = 0.9, \Longrightarrow \cos\theta = 0.9 \Longrightarrow \theta = cos^{-1}0.9,$

$\Longrightarrow \theta = 25°.$

Q3. A wire is 20m long and supports a vertical pole. One end of the wire is fixed to the top of the the pole, while the other endis fixed to a point on the ground. If the wire makes an angle of 70 with the ground, calculate the distance between the foot of the pole and the point where it is fixed to the ground.

Soln



let x = the distance between the foot of the pole, and the point where the wire is fixed to the ground.

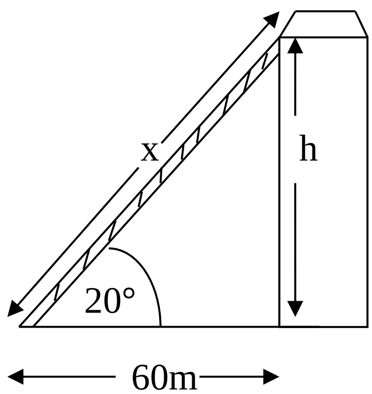
$\cos 70° = \frac{x}{20} \Longrightarrow 20 \cos 70° = x,$

$\Rightarrow x = 20 \cos 70 \Longrightarrow x = 20 \times 0.34 = 6.8m.$

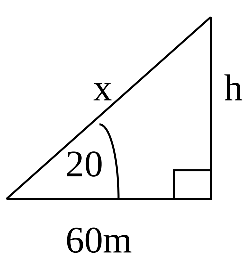
Q4. A ladder leans against a building. The foot of the ladder is 60m away from the bottom of this building. If the angle between the ladder and the ground is 20^0, calculate

i. the length of the ladder.
ii. the height of the building.

Soln.



Let x = the length of the ladder and h = the height of the building.



i. $\text{Cos } 20° = \frac{60}{x} \Longrightarrow x\, cos 20 = 60.$

Divide through using cos 20 $\Longrightarrow \frac{x \cos 20°}{\cos 20°} = \frac{60}{\cos 20°}$

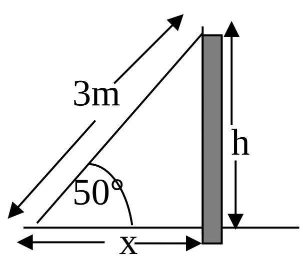
=>$x = \frac{60}{0.9} \Longrightarrow x = 67m.$

ii. $\tan 20° = \frac{h}{60} \Longrightarrow 60 \tan 20 = h, \Longrightarrow$ height of building = 24m.

Q5. A flag pole is supported by means of a rope which is 3m long and makes an angle of 50^0 with the ground. The rope has its one end fixed to the top of the pole and its other end fixed to a point on the ground. Calculate

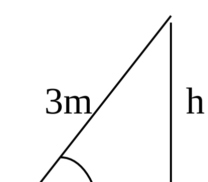
i. the distance between the bottom of the pole and the point at which the rope is fixed to the ground.
ii. the height of the pole.

Soln.



Let h = the height of the pole and x = the distance between the point on the ground, where the rope is fixed and the bottom of the pole.

.



i.Cos $50° = \frac{x}{3} \Longrightarrow 3 \times \cos 50° = x$

$\therefore x = 3 \cos 50 = 3 \times 0.6 \Rightarrow x = 1.8m$

N/B: $\tan \theta = \frac{h}{x}$

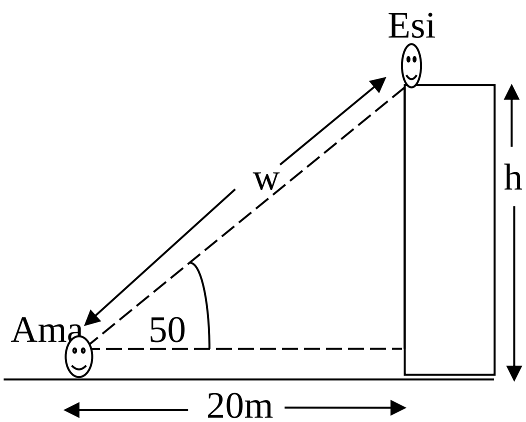
ii. $\text{Tan } \theta = \frac{h}{1.8} \Longrightarrow \tan 50° = \frac{h}{1.8}$

$\Longrightarrow 1.8 \tan 50 = h, \Rightarrow h = 1.8 \times 1.2,$
$\Rightarrow h = 2.16m \Longrightarrow$ height of pole = 2.16m.

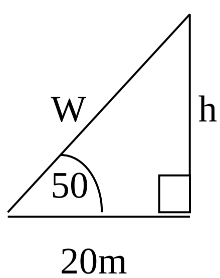
Q6. Ama stood 20m away from a building, as Esi stood on top of the building and the angle of elevation of Esi from Ama was 50^0.

i. Find their distance apart if their heights are negligible
ii. Calculate the height of the building.

Soln.



Let W = their distance apart and h = height of the building.



i. $\text{Cos } 50° = \frac{20}{W} \Longrightarrow W \cos 50° = 20°.$

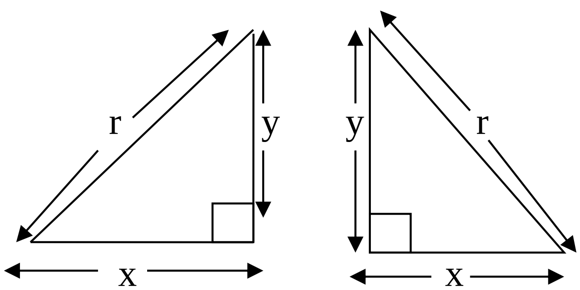
$\frac{W \cos 50}{\cos 50} = \frac{20}{\cos 50} \Longrightarrow W = \frac{20}{0.64} = 31m,$

$\therefore \textit{distance between the two girls} = 31m.$

ii. $\tan 50° = \frac{h}{20} \Longrightarrow 20 \tan 50° = h, \therefore h = 20 \tan 50 = 20 \times 1.2 => h = 24.$

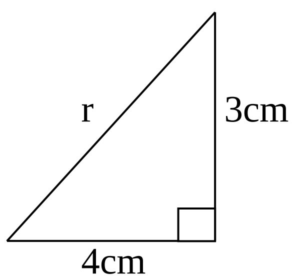
The building is 24m high. $\Longrightarrow W = 47m$

The Pythagoras theorem:



Pythagoras Theorem holds for all right angle triangles, such as the ones above. From the theorem, $r^2 = x^2 + y^2$.

Q1. Calculate the length marked r in the figure given.

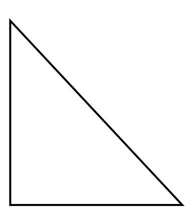


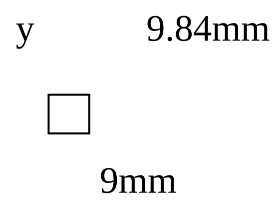
From Pythagoras theorem ,

$r^2 = 4^2 + 3^2, \Longrightarrow r^2 = 16 + 9 \Longrightarrow r^2 = 25$

$\Longrightarrow r = \sqrt{25} = 5cm.$

Q2. Calculate the length marked y in the diagram given:



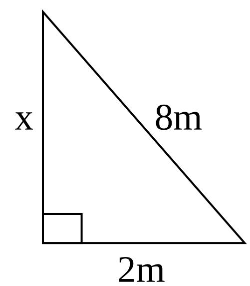


Soln.

$Since\ y^2 + 9^2 = 9.84^2 \Longrightarrow y^2 + 81 = 97, \Longrightarrow y^2 = 97 - 81$

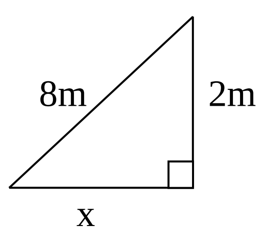
$\Longrightarrow y^2 = 16, \Longrightarrow y = \sqrt{16} = 4, => y = 4mm.$

Q3. Calculate the length marked x in the given figure:



Soln.

Rotate the right to get the next figure:

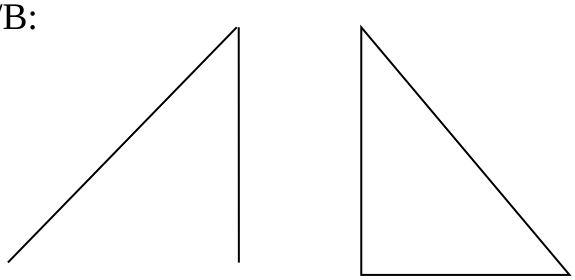


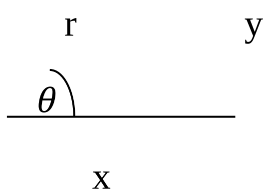
From pythagoras theorem,

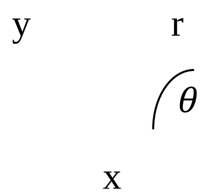
$x^2 + 2^2 = 8^2 \Longrightarrow x^2 + 4 = 64, \Longrightarrow x^2 = 64 - 4 = 60,$

$\Longrightarrow x^2 = 60 \Longrightarrow x = \sqrt{60} = 7.74, \Longrightarrow x = 7.74m$

N/B:







1. $\cos\theta = \frac{x}{r}$ 2. $\sin\theta = \frac{y}{r}$ 3. $\tan\theta = \frac{y}{x}$

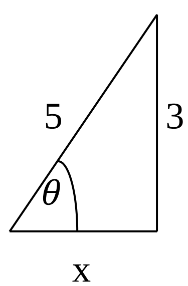
Q1. Given that $\sin\theta = \frac{3}{5}$, find without using tables (or calculator) the values of

i. $\cos\theta$ ii. $\tan\theta$

Soln.

$\sin\theta = \frac{3}{5}$, but

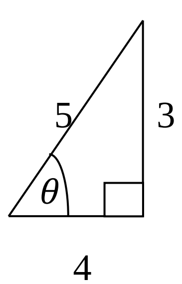
since $\sin\theta = \frac{y}{r} \Longrightarrow y = 3\ and\ r = 5$



From Pythagoras theorem ,

$5^2 = x^2 + 3^2 \Longrightarrow 5^2 - 3^2 = x^2,$

$\therefore 25 - 9 = x^2 \Longrightarrow x^2 = 16, \Longrightarrow x = \sqrt{16} = 4.$

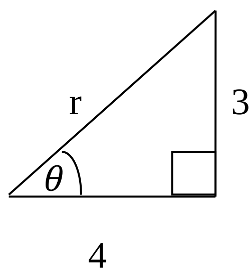


ii. $\cos\theta = \frac{4}{5}$ ii. $\tan\theta = \frac{3}{4}$

Q2. Find without using tables, the values of $\sin\theta$ *and* $\cos\theta$, given that $\tan\theta = \frac{3}{4}$

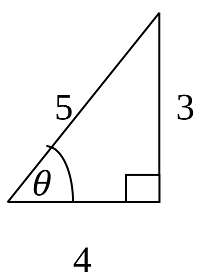
Soln.

Since $\tan\theta = \frac{3}{4}$ *and since* $\tan\theta = \frac{y}{x}, \Longrightarrow y = 3$ *and* $x = 4$.



$From\; r^2 = 4^2 + 3^2 \Longrightarrow r^2 = 16 + 9,$

$=> r^2 = 25 \Longrightarrow r = \sqrt{25} = 5.$

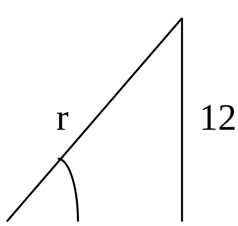


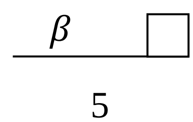
$\cos\theta = \frac{4}{5}$. ii. $\sin\theta = \frac{3}{5}$

Q3. If tan β = $\frac{12}{5}$, find without tables the values of (i) Cos β (ii)Sin β.

<u>Soln:</u>

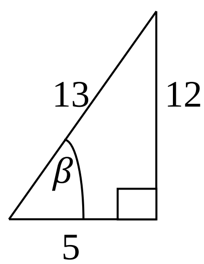
If $\tan\beta = \frac{12}{5}$ *and* $\tan\beta = \frac{y}{x}$, then y = 12 and x = 5.





From Pythagoras theorem ,

$$r^2 = 5^2 + 12^2 \Longrightarrow r^2 = 25 + 144 = 169, \Longrightarrow r^2 = 169 \Longrightarrow r = \sqrt{169} = 13.$$



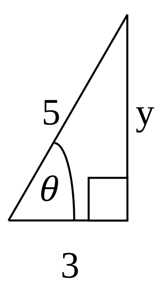
i. $cos\ \beta = \frac{5}{13}$ ii. $sin\ \beta = \frac{12}{13}$

Q4. Given that $\cos\theta = 0.6$, evaluate without tables the values of (i). $\sin\theta$ (ii). $\tan\theta$

$$soln.$$

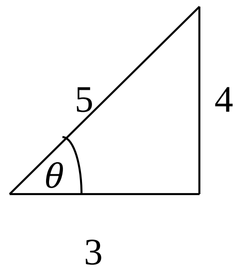
$\cos\theta = 0.6 \Longrightarrow \cos\theta = \frac{6}{10} \Longrightarrow \cos\theta = \frac{3}{5}$.

Since $\cos\theta = \frac{3}{5}\ and \cos\theta = \frac{x}{r} \Longrightarrow x = 3\ and\ r = 5$.



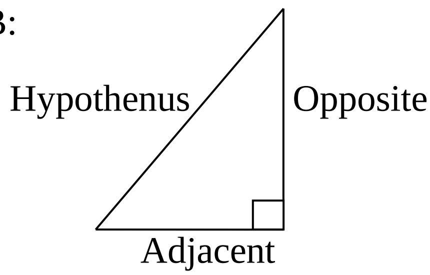
$From\ 5^2 = 3^2 + y^2 \Longrightarrow 5^2 - 3^2 = y^2, \therefore 25 - 9 = y^2$

$\Longrightarrow 16 = y^2, \Longrightarrow y^2 = 16 \Longrightarrow y = \sqrt{16} = 4, \Longrightarrow y = 4$



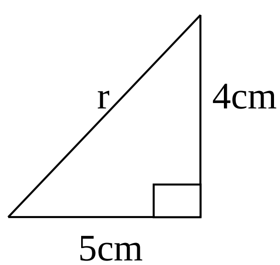
i. $sin\ \theta = \frac{4}{5}$ ii. $tan\ \theta = \frac{4}{3}$

N/B:



Q5. In a right angle triangle, the length of the opposite is 4cm and that of the adjacent is 5cm. Find the length of the hypothenus

Soln.



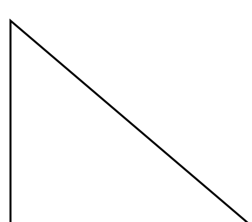
If r = length of the hypothenus, then $r^2 = 5^2 + 4^2, \Rightarrow r^2 = 25 + 16 = 41, \Rightarrow r^2 = 41 \Longrightarrow r = \sqrt{41} = 6.4.$

∴ length of the hypothenus = 6.4cm approx.

Q6. In a right angle triangle, the length of the hypothenus is 5cm and that of the adjacent is 3cm. Calculate.

i. the angle between the hypothenus and the adjacent.
ii. the length of the opposite side.

Soln.



y 5cm

θ

3cm

Let θ = the angle between the hypothenus and the adjacent. Let y = the length of the opposite.

i. $cos\ \theta = \frac{3}{5}$, =>$cos\ \theta = 0.6, \Longrightarrow \theta = cos^{-1}0.6 \Longrightarrow \theta = 53°$.

ii. From pythagoras theorem, $5^2 = y^2 + 3^2$,
$\Longrightarrow 25 = y^2 + 9 \Longrightarrow 25 - 9 = y^2, \therefore 16 = y^2$
$\Longrightarrow y^2 = 16, \Rightarrow y = \sqrt{16} = 4cm$,
$\Longrightarrow$ length of the opposite = 4cm

Q7. Given that $sin\ \theta = \frac{3}{5}$, find the values of

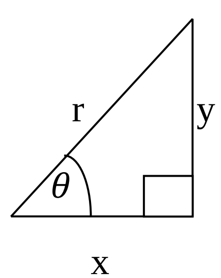
a. $cos\ \theta$ b. $2\ cos\ \theta + 1$

c. $tan\ \theta$ d. $3\ tan\ \theta - \cos\theta$

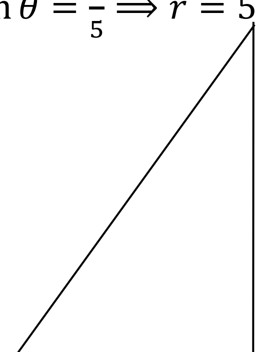
e. $\frac{1+\tan\theta}{2\sin\theta}$

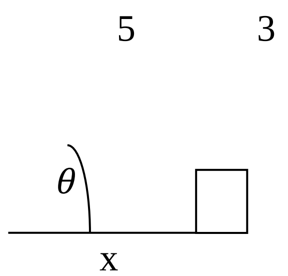
NB: You are not supposed to use tables or calculator.

Soln.



Since $\sin\theta = \frac{3}{5} \Longrightarrow r = 5\ and\ y = 3$.



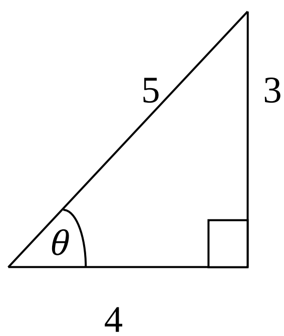


From Pythagoras theorem, $5^2 = x^2 + 3^2$,

$\Longrightarrow 25 = x^2 + 9 \Longrightarrow 25 - 9 = x^2, \Longrightarrow 16 = x^2$

$\Longrightarrow x = \sqrt{16} = 4.$

The diagram just drawn then becomes as shown next:



a. $cos\ \theta = \frac{4}{5} = 0.8$

b. $2\cos\theta + 1 = 2\left(\frac{4}{5}\right) + 1 = 2 \times \frac{4}{5} + 1 = \frac{8}{5} + 1 = 1.6 + 1 = 2.6$

c. $tan\ \theta = \frac{3}{4} = 0.75$

d. $3\tan\theta - \cos\theta = 3(0.75) - \frac{4}{5} = 2.25 - 0.8 = 1.45$

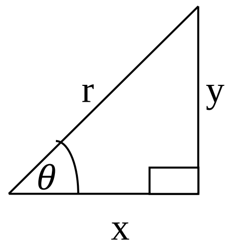
e. $\frac{1+\tan\theta}{2\sin\theta} = \frac{1+\frac{3}{4}}{2\left(\frac{3}{5}\right)} = \frac{1+0.75}{\frac{6}{5}} = \frac{1.75}{1.2} = 1.46$

Q8. Given that $tan\ \theta = \frac{3}{4}$, determine without using tables the values of

a. $sin\ \theta$ b. $cos\ \theta$

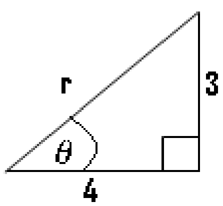
c. $\frac{1-\sin\theta}{\cos\theta}$ d. $\frac{2\cos\theta + 3\sin\theta}{1 + \cos\theta}$

Soln.

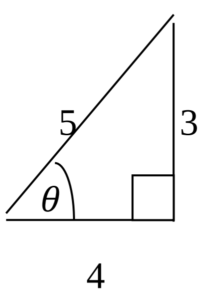


$\tan\theta = \frac{y}{x}$

Since $tan\ \theta = \frac{3}{4}, then$ y = 3 and x = 4.



$From\ r^2 = 4^2 + 3^2 \Longrightarrow r^2 = 16 + 9, \Longrightarrow r^2 = 25 \Longrightarrow r = \sqrt{25} = 5.$



a. $sin\ \theta = \frac{3}{5} = 0.6.$
b. Cos $\theta = \frac{4}{5} = 0.8.$
c. $\frac{1 - \sin\theta}{\cos\theta} = \frac{1 - 0.6}{0.8} = \frac{0.4}{0.8} = 0.5.$
d. $\frac{2\cos\theta + 3\sin\theta}{1 + \cos\theta} = \frac{2(0.8) + 3(0.6)}{1 + 0.8} = \frac{1.6 + 1.8}{1.8} = \frac{3.4}{1.8} = 1.9.$

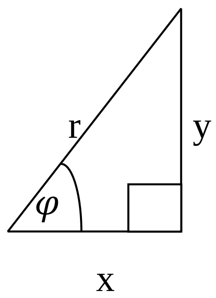
N/B: When asked not to use tables, calculators cannot also be used. Also when asked not to use calculator, then tables i.e the four figure table can also not be used.

Q8. Given that $\cos\varphi = {}^{4}/_{5}$, determine without the use of tables or calculator, the values of the following:

a. $\tan\varphi$ b. $\sin\varphi$

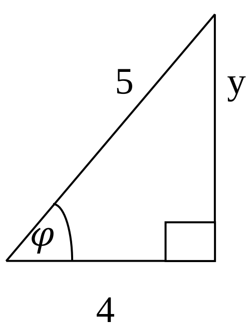
c. $12\tan\varphi$ d. $1+2\tan\varphi$ e. $\frac{16\,tan\varphi}{2-15\sin\varphi}$

soln.



$\cos\varphi = \frac{x}{r}$

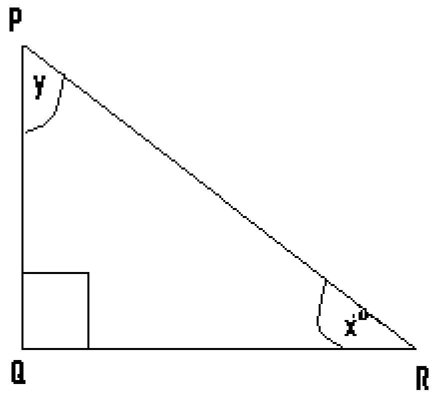
Since $\cos\varphi = \frac{4}{5} \Longrightarrow x = 4\ and\ r = 5.$



From Pythagoras theorem, $5^2 = y^2 + 4^2$,

$\Longrightarrow 25 = y^2 + 16 \Longrightarrow 25 - 16 = y^2, \Longrightarrow 9 = y^2 \Longrightarrow y = \sqrt{9} = 3.$

(Q8)



In the triangle PQR, cos $x^0 = \frac{15}{17}$. Find tan y..

Soln.

Since cosx = $\frac{15}{17}$ ⇒ cos x = 0.88,

⇒ x = cos^{-1} 0.88 ⇒ $x = 42^0$.

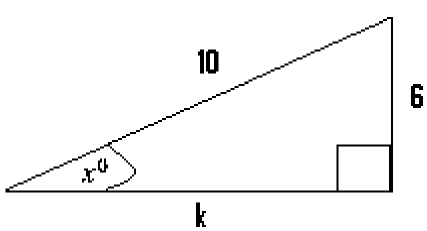
Since the sum of angles within a triangle = 180^0, then x + y + 90 = 180^0, ⇒ $42^0 + y + 90 = 180^0$, ⇒ y = 180 – 90 – 42, ⇒ y = 48^0.

Tan y^0 = tan 48^0 = 1.1.

(Q9) Given that sin x = 0.6 and $0^0 \leq x \leq 90^0$, find 1-tanx and leave your answer in the form $\frac{a}{b}$, where a,b ∈ integers.

Soln.

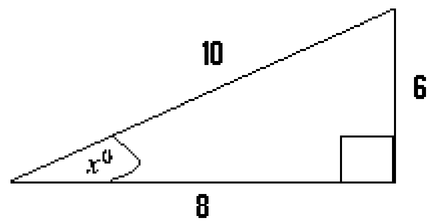
From sin x^0 = 0.6 ⇒ sin x^0 = $\frac{6}{10}$.



From Pythagoras theorem, $10^2 = k^2 + 6^2$ ⇒ $100 = k^2 + 36$,

⇒ $100 - 36 = k^2$, ⇒ $64 = k^2$

$\Rightarrow$ k = $\sqrt{64}$ = 8.



From this second figure, tan x = $\frac{6}{8} = 0.75$,

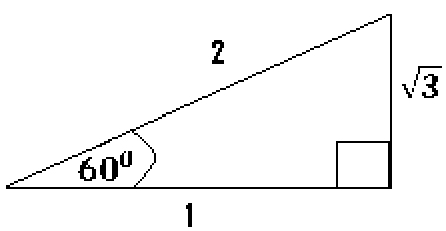
=>1 – tan x = 1 – 0.75,

= 0.25 = $\frac{25}{100} = \frac{1}{4}$ $, which\ is\ of\ the\ form\ \frac{a}{b}\ where\ a = 1\ and\ b = 4.$

The special angles:

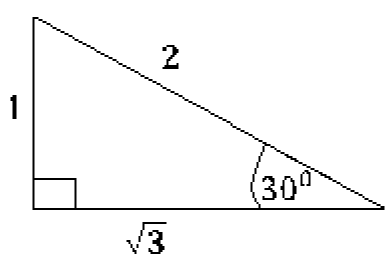
These angles are $30^0, 45^0$ and 60^0. Their values can be had by using three different types of triangles.

(a)



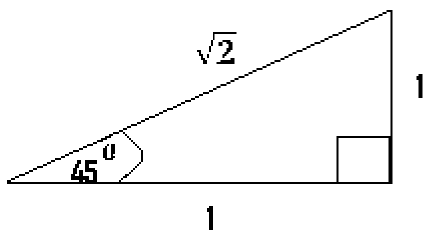
i) cos $60^0 = \frac{1}{2}$ii) sin 60^0 =$\frac{\sqrt{3}}{2}$

(b)



i) Cos $30^0 = \frac{\sqrt{3}}{2}$ ii) sin $30^0 = \frac{1}{2}$ $iii)$ tan $30^0 = \frac{1}{\sqrt{3}}$

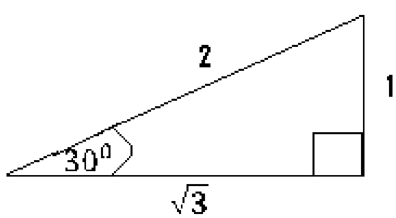
(c)

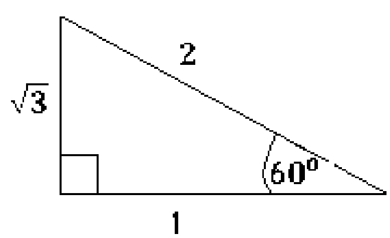


i) Cos $45^0 = \frac{1}{\sqrt{2}}$ ii) $\sin 45^0 = \frac{1}{\sqrt{2}}$ iii) $\tan 45^0 = \frac{1}{1} = 1$

(11) Without using tables or calculator, simplify $\frac{\sin 45^0 + \tan 30^0}{\tan 45 - \cos 60^0}$

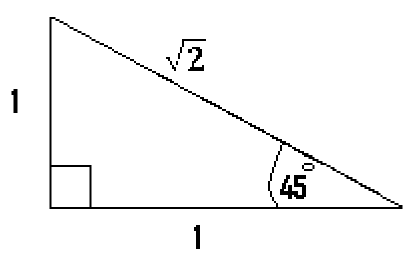
Soln.





$\tan 30^0 = \frac{1}{\sqrt{3}}$

$\cos 60^0 = \frac{1}{2}$



$\sin 45^0 = \frac{1}{\sqrt{2}}$ and $\cos 45^0 = \frac{1}{\sqrt{2}}$.

(i) $2 \tan 30^0 - 4\, cos\ 60^0$

$= 2\left(\frac{1}{\sqrt{3}}\right) - 4\left(\frac{1}{2}\right)$

$= \frac{2}{\sqrt{3}} - \frac{4}{2} = \frac{2}{\sqrt{3}} - 2$

$= \frac{2}{\sqrt{3}} - \frac{2}{1} \quad = \quad \frac{2-2\sqrt{3}}{\sqrt{3}}$

N/B: The L.C.M of $\sqrt{3}$ and 1 = $\sqrt{3} \times 1 = \sqrt{3}$.

(ii) $\frac{3sin45^0 + cos45^0}{2\tan 30^0}$

$= \frac{3\left(\frac{1}{\sqrt{2}}\right) + \frac{1}{\sqrt{2}}}{2\left(\frac{1}{\sqrt{3}}\right)}$

$= \frac{\frac{3}{\sqrt{2}} + \frac{1}{\sqrt{2}}}{\frac{2}{\sqrt{3}}}$, simplify the numerator

$\Rightarrow \frac{3}{\sqrt{2}} + \frac{1}{\sqrt{2}} \frac{3 + 1}{\sqrt{2}} = \frac{4}{\sqrt{2}} => \frac{\frac{3}{\sqrt{2}} + \frac{1}{\sqrt{2}}}{\frac{2}{\sqrt{3}}} = \frac{\frac{4}{\sqrt{2}}}{\frac{2}{\sqrt{3}}}$

$= \frac{4 \times \sqrt{3}}{2 \times \sqrt{2}} = \frac{4\sqrt{3}}{2\sqrt{2}}$.