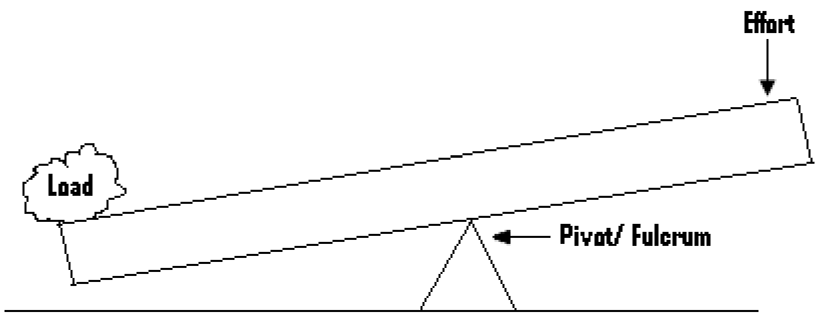
CHAPTER FOUR
MACHINES

Introduction:

A machine is a device by means of which a small force (effort) applied at one end, can be used to overcone a large force (load) at another end.A machine enables work to be done easily, and it does work by taking in energy at one end and feeding it out at another end, possibly in another form.

The Lever:



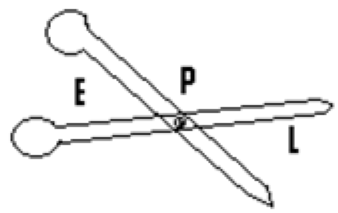
- The lever is a rigid straight bar, which has an immovable point called the pivot or the fulcrum.
- The force which is applied to the lever (machine) is called the effort, and the work it does or the force it overcomes, is referred to as the load.
- In the lever, a small force applied at one point (effort) is used to overcome a greater force or load at another point.

Mechanical advantage (M.A):

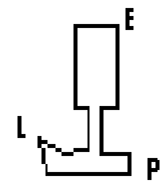
- This is given by the ratio of the load to the effort, i.e. M.A = $\frac{Load}{Effort}$
- When the mechanical advantage is greater than 1, then the implication is that a small effort has been used to overcome a bigger load or resistance.
- Some machines are designed to overcome a load much greater than the effort used, and an example is the car screw jack which is used in the lifting of motor car.
- In such cases, the mechanical advantage of the machine must be greater than 1.
- In certain machines, the mechanical advantage is less than 1 and for this reason, the effort is greater than the load and an example of such a machine is the bicycle.
- Under ordinary conditions, the resistance to the motion of the bicycle along a level road is comparatively small and as such a large mechanical advantage is unecessary.

The mechanical advantage of a lever, and types of levers:

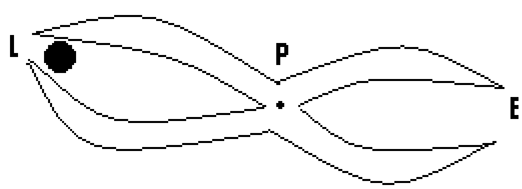
- There are three classes of levers, and these are:
  (1) The first class lever.
  (2) The second class lever.
  (3) The third class lever.
- In the first class lever, the pivot is between the load and the effort, and examples are the pair of scissors, the pincers and the hammer (used to remove nails).



Scissors

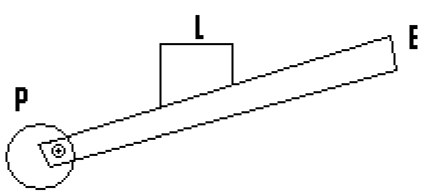


Hammer

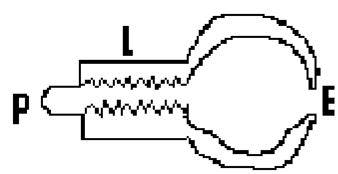


Pincers

- In the second class lever, the load is between the effort and the pivot, and examples are the wheelbarrow and the nut cracker.

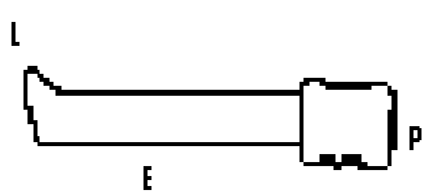


Wheelbarrow

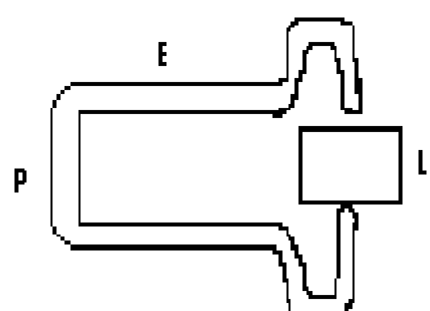


Nut cracker

- In the third class lever, the effort is between the load and the pivot, and examples are the tong and the knife.



KNIFE



TONG

- For a lever M.A = $\frac{L}{E}$
  where M.A = Mechanical advantage, L = Load and E = Effort.

(Q1) By means of a lever, an effort of 10N was applied to raise a stone of mass 20N. Calculate the mechanical advantage.

Soln:

E = Effort = 10N.

L = Load = 20N.

M.A = $\frac{L}{E}$ = $\frac{20}{10} = 2.$

(Q2) A simple machine has a mechanical advantage of 10. Find the effort which will be needed inorder to raise a load of 20N.

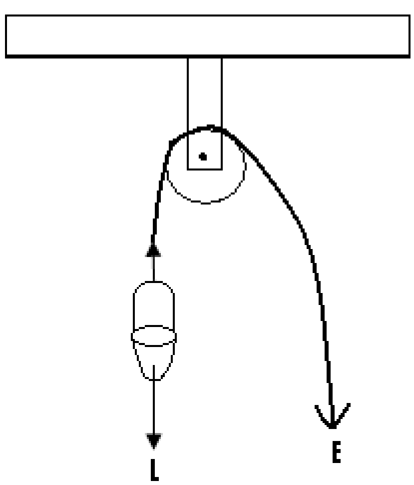
Soln:

From M.A = $\frac{L}{E}$ => E = $\frac{L}{M.A} = \frac{20}{10} = 2.$
=> Effort needed = 2N.

Pulleys:

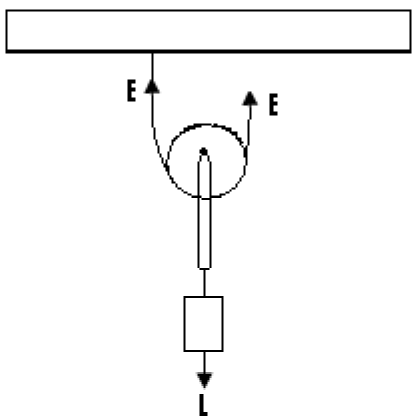
- A pulley is awheel with a grooved rim.
- Several of them may be mounted in a frame work called a block, and the effort is applied to a rope which passes over the pulleys.

The single fixed pulley:



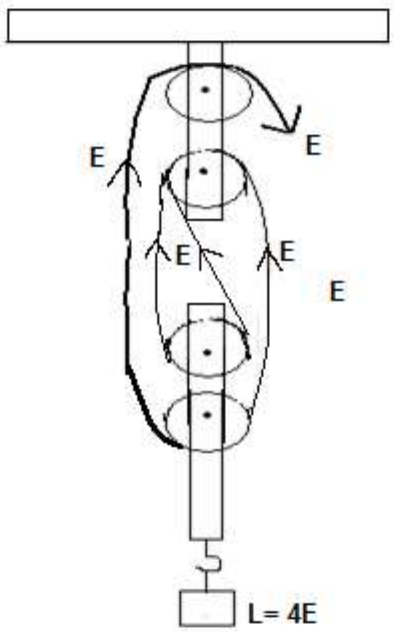
This is often used for the purpose of raising small loads contained in buckets or baskets, to the top of a building during construction or repair work.It can be used to change the direction of application of an effort, so as to make it more convenient to apply the force. For such a pulley, the load = the effort and the mechanical advantage = 1. Even though the effort applied is equal to the load raised, a greater convenience and the ease of being able to stand on the grounds and pull downwards, instead of having to haul the load upward to the top of the building is obtained.

The single moving pulley:



- For this, the tension in the string or rope is equal to the effort applied, so that the total upward pull on the pulley is twice the effort E.
- Suppose a load of 4kgf is supported by the pulley, then since the load is supported by the tension in two sections of the string, the effort needed to be applied must be 2kgf.
  M.A = $\frac{L}{E} = \frac{4}{2} = 2.$

The block and tackle:



- This is by far the most important pulley system of all, being commonly used for lifts and cranes.
- Two blocks are employed containing from two to eight pulleys in each, according to the mechanical advantage required and a single string is used to pass round each pulley.- If an effort E is applied to the free end of the string, then the total upward force on the load will be 4E.

- M.A = $\frac{4E}{E}$ = 4.
  - In practice, however, the practical mechanical advantage is always less than 4, since extra effort must be applied to overcome friction and the weight of the moving pulley block and string.

The velocity ratio (V.R):

- This is the ratio of the distance moved by the effort, to the distance moved by the load at the same time.
- The velocity (the speed) ratio of a machine = $\frac{distance\ moved\ by\ the\ effort}{distance\ moved\ by\ the\ load\ in\ the\ same\ time}$
- Also, V.R.= $\frac{effort\ distance}{load\ distance}$

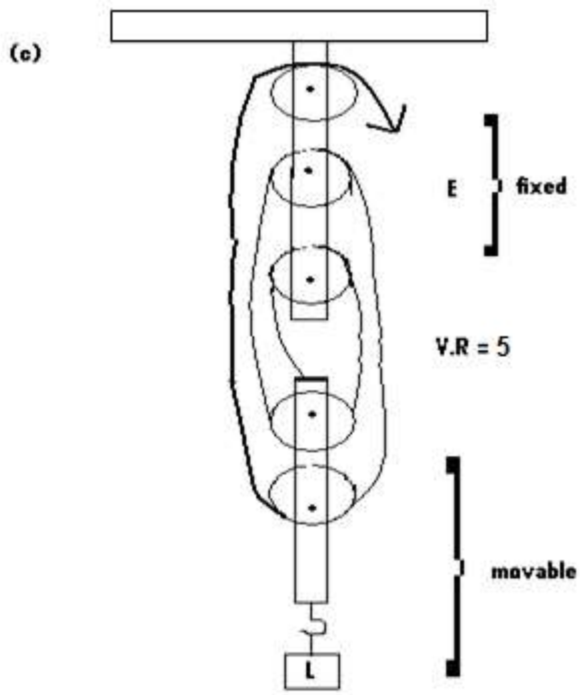
(Q1) During the use of a simple machine, the effort moved through a distance of 150cm, and the load moved through a distance of 30cm. Find the velocity ratio.oSoln:

V.R = $\frac{effort\ distance}{load\ distance} = \frac{150}{30} = 5.$

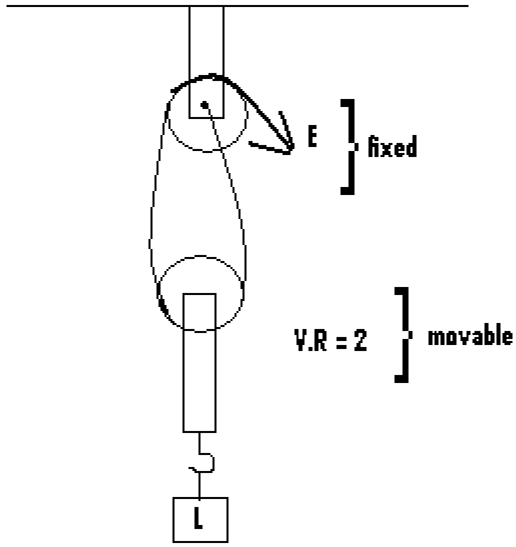
N/B: V.R has no units.

- The velocity ratio of a single pulley is 1, and that of a movable pulley is 2.

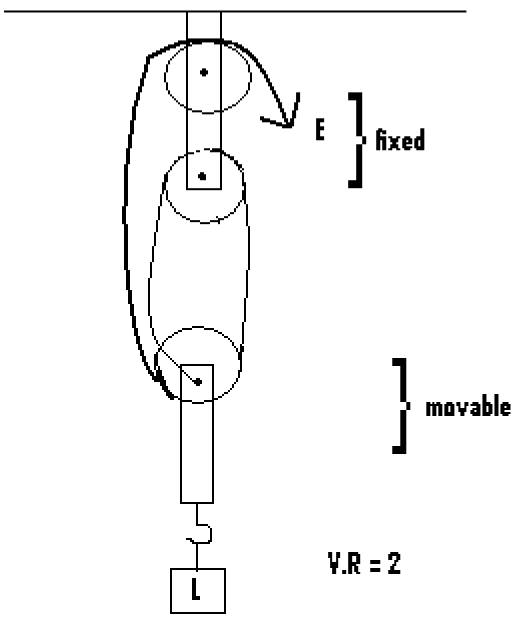
<u>The velocity ratio of the block and tackle:</u>



(a)



(b)



- The velocity ratio for a block and tackle system of pulleys, is obtained by counting the number of strings supporting the movable block of pulleys.
- In the first figure, the movable block is supported by two strings and for this reason, the velocity ratio = 2.
- In the second figure, the movable block is supported by three strings and as such it velocity ratio = 3.

- Because the effort has to overcome the weight of the movable block, the load as well as friction, the mechanical advantage of the block and tackle system is much less than the velocity ratio.
- However, the higher the velocity ratio, the higher the mechanical advantage.

The efficiency of a machine:

- The efficiency of a machine is the ratio of its work output to its work input.

- Efficiency = $\frac{work\ output}{work\ input} \times 100$

- Also the efficiency of a machine = $\frac{useful\ energy\ output}{energy\ input} \times 100$

- The efficiency is usually expressed as a percentage.

- The efficiency of a machine can never be 100%, because part of the work input is used to overcome the friction which operates between the moving parts.

- Apart from that, part of the work input is wasted or used in raising the moving parts of the machine.

- By reducing friction in a machine, its efficiency improves and as such energy is saved.

(Q1) A machine needs 100J of energy to produce an output of 80J. Calculate its efficiency.

Soln:

Work output = 80J.

Work input = 100J.

Efficiency = $\frac{work\ output}{work\ input} \times 100$

= $\frac{80}{100} \times 100$ = 80%.

(Q2) A man operating a machine puts in 60J of energy and gets an output of 50J of energy. Determine the machines efficiency.

Soln:

Efficiency = $\frac{work\ output}{work\ input} \times 100$

= $\frac{50}{60} \times 100$= 83%.

(Q3) The efficiency of a machine is 40%. If the output is 12J, find the work input.

Soln:

Efficiency = 40% = 0.4

Efficiency = $\frac{output}{input}$

=> 0.4 = $\frac{12}{input}$ $=> input = \frac{12}{0.4}$= 30J.

N/B:

- In calculating the efficiency, if the efficiency is not given, we use the formula Efficiency = $\frac{output}{input} \times 100$
- On the other hand if the efficiency is given, then we use efficiency = $\frac{work\ output}{work\ input}$

(Q4) The efficiency of a machine is 60%. Calculate

(a) the work done by the machine, if its input energy is 540J.
(b) the amount of energy needed by the machine to do 90J of work.

Soln:

(a) efficiency = 60% = 0.6

input energy = 540J

output energy = ?

efficiency = $\frac{work\ output}{work\ input}$

=> 0.6 = $\frac{output}{540}$

=> output = 0.6 $\times 540 = 324$J.

(b) The amount of energy needed by the machine = the work input.

Efficiency = $\frac{output}{input}$ => 0.6 = $\frac{90}{input}$

Input = $\frac{90}{0.6}$ = 150J.

(Q5) In a machine, 200J of energy was obtained or wasted as heat. If the input is 600J, calculate its efficiency.

Soln:

Input = 600J.

Output = 600 – 200 = 400J.

Efficiency = $\frac{work\ output}{work\ input} \times 100$ = $\frac{400 \times 100}{600} = 66.7\%$.

N/B: The energy lost or wasted = input – output,

=> energy lost + output = input.

(Q6) In a machine, 20J of energy appears as heat. If it was used to overcome or do a work of 60J, calculate the efficiency.

Soln:

Energy wasted = 20J.

Output energy = 60J.

Input energy = energy lost + output energy = 20 + 60 = 80J

Efficiency = $\frac{output}{input} \times 100$

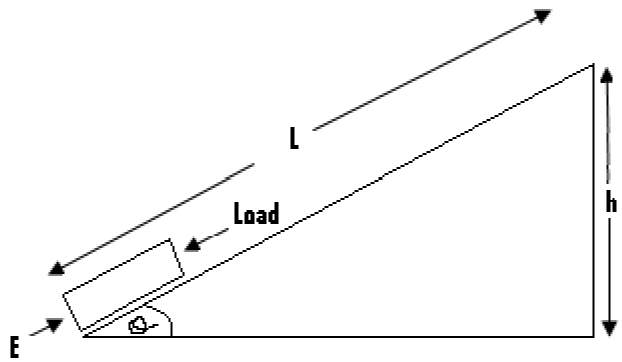
= $\frac{60}{80} \times 100$ = 75%

<u>The relation beween mechanical advantage, velocity ratio and efficiency:</u>

- This relation is that efficiency = $\frac{M.A}{V.A} \times 100\%$
  => M.A = $\frac{V.R \times efficiency}{100\%}$

- M.A is always less than V.R, and M.A becomes equal to the V.R for a machine, only when its efficiency is 100% i.e. a perfect machine.

The inclined plane:



- This figure shows a load being pushed up an inclined plane by an effort, E.
- The distance travelled by the load up the plane is L, and the vertical height moved by the load is h.
- The velocity ratio of inclined plane = $\frac{distance\ moved\ along\ the\ plane}{vertical\ height\ moved}$

  = $\frac{l}{h}$.
- Now, V.R = $\frac{l}{h}$, and from the diagram, $\sin\theta$ = $\frac{h}{l}$ => $\frac{1}{\sin\theta} = \frac{\frac{1}{h}}{l} = \frac{1 \times l}{h} = \frac{l}{h}$.

  => $\frac{l}{h}$ = $\frac{1}{\sin\theta}$.

- Now, from V.R = $\frac{l}{h}$ => V.R = $\frac{l}{h}$ = $\frac{1}{\sin\theta}$

=> V.R = $\frac{1}{\sin\theta}$.

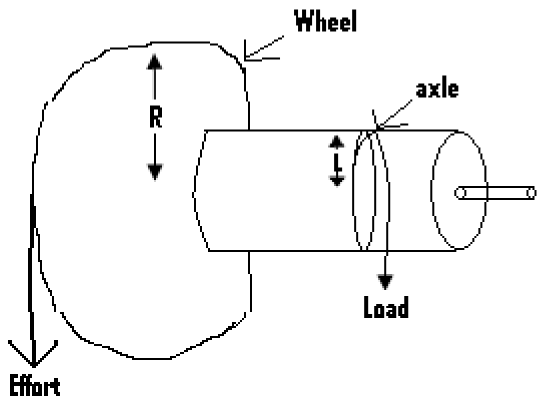
- From this equation, we can see that the smaller the angle of elevation, (or the angle of inclination) which is θ, the greater become the velocity ratio and vice versa. Also, the ratio V.R increases as M.A increases.

- For these reasons, roads up a hill are made meandering so that the angle of elevation can be made as small as possible, in order to increase the velocity ratio.

This will in turn cause an increase in the mechanical advantage, which will enable a relatively smaller force to be applied when climbing the hill.

The wheel and axle:

Figure (1)

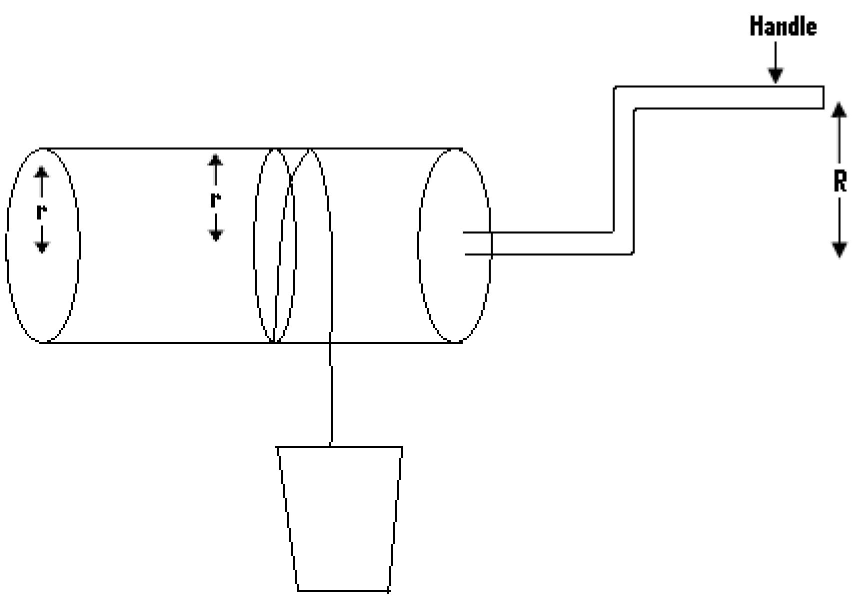


R = the radius of the wheel and r = the radius of the axle.

The V.R = $\frac{\text{the distance travelled by the effort}}{distance\ travelled\ by\ the\ load}$

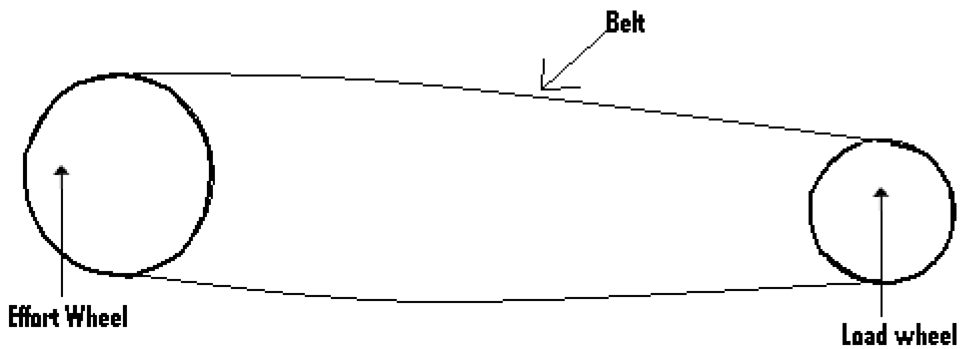
= $\frac{2\pi \times radius\ of\ wheel}{2\pi\ \times radius\ of\ axle} = \frac{R}{r}$.

- The mechanical advantage of a perfect wheel and axle is also = $\frac{R}{r}$.



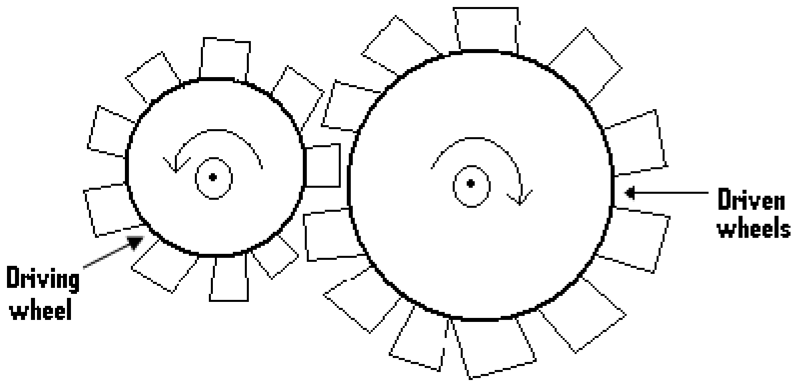
- For this, the V.R = $\frac{2\pi R}{2\pi r} = \frac{R}{r}$.

Pulley belt (Fan belt):



- For this system, the velocity ratio, V.R = $\frac{radius\ of\ the\ load\ wheel}{radius\ of\ the\ effort\ wheel}$
- For this reason if the effort wheel is larger then the load wheel, then the velocity ratio will be less than 1 and as such, the mechanical advantage will also be less than 1.
- For the bicycle, both the velocity ratio and the mechanical advantage are less than 1.
- Since the mechanical advantage is less than 1, the implication is that a much bigger force will be applied to the pedals, in order to move the bicycle along.
- And since the velocity ratio is also less than 1, the implication is that the speed of the load which is the bicycle itself, will be much greater than the speed of rotation of the pedals.
- The mechanical advantage for a pulley system, tends to increase as the load increases.
- As the load increases, the frictional forces becomes less significant, since their magnitude compared to that of the load becomes more and more negligible.
- The mechanical advantage therefore increases with increases in load and its value approaches that of the velocity ratio.
- This implies that the efficiency increase with increase in load.

Gears:



- The principle of the wheel and axle is applied in the gear box, where toothed wheels of different diameters engage to give turning force at low speed (large mechanical advantage) or at high speed (small mechanical advantage), according to which gear is the “driver” and which one is the “driven”.
- To start a car, a much bigger force is required and for this reason, the gear wheel whose velocity ratio is less than 1 is not desirable.
- This is due to the fact that enough force may not be generated to enable the car to move.
- For this reason, the velocity ratio must be greater than 1.
- However, if the velocity ratio is less than 1, and the car is already in motion, the car can move much faster, since the force required to maintain motion is smaller.
- To climb up a hill, the velocity ratio must be grater than 1 since the load to be overcome is larger.
- The speed therefore becomes small.

(Q1) A pulley system has a velocity ratio of 6, and an efficiency of 60%. Calculate the load this system can overcome with an effort of 1000N.

Soln:

M.A = $\frac{V.R \times Efficiency}{100\%}$

= $\frac{6 \times 60\%}{100\%} = \frac{6 \times 60}{100} = 3.6$

But M.A $= \frac{L}{E} => 3.6 = \frac{L}{1000}$

=> L = 3.6 × 1000 = 3600N.

(Q2) A machine whose velocity ratio is 4, has an efficiency of 80%. The application of an effort of 100N, caussed the load to move through a distance of 5m. Find the work done on the load.

Soln:

Efficiency = $\frac{M.A}{V.R} \times 100\%$

=> 80% = $\frac{M.A}{4} \times 100\%$,

=> 80 = $\frac{M.A}{4} \times 100$

=> 80 = 25 M.A, => M.A = $\frac{80}{25}$

=> M.A = 3.2.

Since M.A = $\frac{L}{E}$ $=>$3.2 = $\frac{L}{100}$,

=> L = 3.2 $\times$ 100 = 320N.

=> Load = 320N.

Since this load moved through a distance of 5m, => work done on the load

= forces (Load) $\times$ distance moved = 320 $\times$ 5 = 1600J.

Questions:

(1) Differentiate between mechanical advantage and velocity ratio.

(2) (a) Draw a diagram of the block and tackle system of pulleys, whose velocity ratio is 3.

(b) Explain how you will determine the velocity ratio of such a pulley.

Ans:

(b) By counting the number of strings which supports the movable block of such a system.

(3) As a result of its overuse, the efficiency of a machine which was 80% had fallen by 30%. Determine (a) the work done by this machine, if its input energy is 420J.

Ans: 210J.

(b) the amount of energy needed to do 70J of work.

Ans: 140J.

(4) In using a machine 120J was the amount of heat produced. Due to heavy noise produced by the machine, 10J was wasted in the form of noise. If the amount of work done by the machine was 700J, (a) determine the amount of energy which was put into the machine. Ans: 570J.

(b) Assuming that no heat and sound was produced by the machine, determine its efficiency. Ans: 100%.

(5) (a) The efficiency of a machine is 70% and its mecahanical advantage is 20. Find its velocity ratio. Ans: 29.

(b) If this machine was used to raise a 50kg bag of cement 80m high, within a time period of one quarter of a minute, determine.

(i) the work done by the machine. Ans: 40,000J.

(ii) the power concerned in this case. Ans: 2667W or 2667J/s.

(6) The efficiency of a machine is 80% and its velocity ratio is 10. In using this machine, the effort appliedwas 2N. Determine the load.

Ans: 16N.

(7) An inclined plane of length 20m, has an angle of inclination of 30^0.

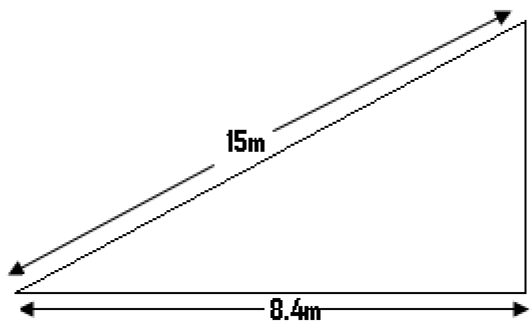
(a) If a 5kg bag of cement is moved from the ground up the plane, determine the work done. Ans: 1000J.

(b) Given that the time taken to push the bag of cement up the plane was 1 minute 20 seconds, determine the power or the rate of doing work.

Ans: 12.5W.

(c) Determine the velocity ratio. Ans: 2.

(8)



The figure drawn shows an inclinedplane. An object of mass 0.3kg was moved up this plane. Determine

(a) the velocity ratio.   Ans: 1.2

(b) the potential energy possed by the object, when it is up the plane. Ans: 3.7J.

(9) explain why roads up a mountain have a meandering nature.

Ans: To make the angle of elevation as small as possible, in order to increase the velocity and the mechanical advantage so that when climbing a hill, a relatively small force can be applied.

(10) Explain why the mechanical advantage and the velocity ratio of a bicycle are both made to be less than 1.

Ans:

(i) So that a much bigger force will be applied on the pedals, in order to move the bicycle along.

(ii) So that the speed of the bicycle becomes greater then that of the pedals.