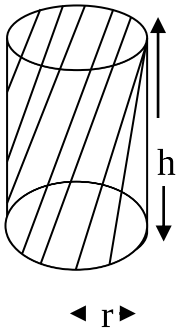
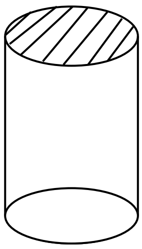
CHAPTER EIGTH

CYLINDERS AND CONES

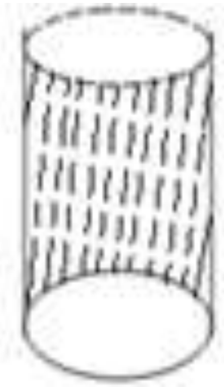
The cylinder:



- The above figure is known as a cylinder.
- The height of this cylinder is h and its radius is r.
- The shaded portion is called the total surface area of the cylinder, also referred to as the area of the cylinder.
- The area of a cylinder is made up of three parts and these are:
  1. The top circular flat surface area, which is also referred to as the top surface area, and which is indicated in the next diagram, by means of shading:

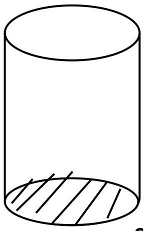


- The flat top circular surface area $= \pi r^2$, since it is circular in shape where r = the radius.
  2. The curved surface area, which is indicated by means of shading, in the next figure:



- The curved surface area = 2 πrh, where h = the height.

The bottom circular surface area, which is indicated in the next diagram by means of shading:

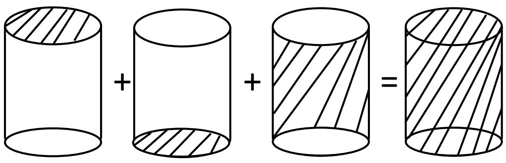


- The bottom surface area = $= \pi r^2$, since it is also circular in shape.

The area of a cylinder:

The total surface area of a cylinder is therefore had by adding together all these three surface areas,

i.e



$\pi r^2 + \pi r^2 + 2\pi rh = 2\pi r^2 + 2\pi rh = 2\pi r(r+h)$.

Q1. The height of a cylinder is 5cm and its radius is 2cm. Calculate

a. its flat top circular area.

b) its flat bottom circular area.

c) its curved surface area.

d)its total surface area. [Take $\pi = 3.14$].

Soln.

h = 5cm, r = 2cm and $\pi = 3.14$.

a. The flat top surface area = $\pi r^2 = 3.14 \times 2^2 = 3.14 \times 4 = 12.56cm^2$

b. The flat bottom surface area = $\pi r^2 = 3.14 \times 2^2 = 3.14 \times 4 = 12.56cm^2$

c. The curved surface area = $2\pi rh = 2 \times 3.14 \times 2 \times 5 = 62.8cm^2$

d. The total surface area = top surface area + bottom surface area + curved surface area = 12.56$cm^2 + 12.56cm^2 + 62.8cm^2 = 87.9cm^2$.

N/B: Also the total surface area = 2 $\pi r(r+h) = 2 \times 3.14 \times 2(2+5) = 12.56(7)$ = 87.9cm^2.

Q2. A cylinder has a height of 40m and a diameter of 12m. Determine

a. its bottom circular area .
b. its curved surface area.
c. Its total surface area.

[Take $\pi = 3.142$]

Soln:

Since d =12m$\Longrightarrow r = \frac{12}{2} = 6m.$

Also $\pi = 3.142$ *and* $h = 40m.$

a. The bottom circular surface area
= $\pi r^2 = 3.142 \times 6^2 = 3.142 \times 36 = 113m^2$

b. The curved surface area = 2$\pi rh = 2 \times 3.142 \times 6 \times 40 = 1508m^2$

c. The total surface area =
2$\pi r(r+h) = 2 \times 3.142 \times 6(6+40) = 1734$m^2.

Q3. A water storage tank is to be constructed using aluminum. If it is to have a diameter of 40m and a height of 120m, determine the amount of aluminum that will be needed to construct

a. its curved surface area .
b. the whole tank. [Take π *or pie* $= 3.14$].

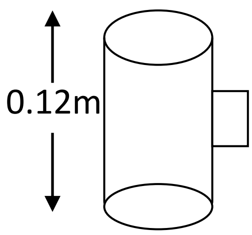
Soln.

Since d = 40m $\Longrightarrow r = \frac{40}{2} = 20m.$ Also pie = 3.14 and h = 120m.

a. The amount of aluminum which is needed to construct the curved surface area = the curved surface area =
2$\pi rh = 2 \times 3.14 \times 20 \times 120 = 15072m^2$

b. The amount of aluminum needed to construct the whole tank = the total surface area = 2$\pi r(r+h) = 2 \times 3.14 \times 20(20+120)$
$= 126(140) = 17640m^2$

Q4.



$\overleftrightarrow{8cm}$

The given figure is that of a drinking cup, which is to be constructed using plastic. If it is to be 0.12m long and have a diameter of 8cm, determine the quantity of plastic needed for its construction. [Take $\pi = 3.142$].

N/B:

- A drinking cup has no top surface area $\Longrightarrow plastic$ will only be needed to construct the curved surface area and the bottom surface area.
- Also since the height is given in metres and the diameter in centimetres, the metres must be converted into centimetres.

Soln.

h = 0.12m = 0.12 x 100 = 12cm.

D = 8cm $\Longrightarrow r = 4cm.$

The amount of plastic needed to construct the curve surface area $= 2\pi rh = 2 \times 3.142 \times 4 \times 12 = 302cm^2$.

The amount of plastic needed to construct the bottom surface area = bottom surface area = $\pi r^2 = 3.142 \times 4^2 = 3.142 \times 16 = 50cm^2$.

The quantity of plastic needed to construct the cup = amount of plastic needed to construct the curved portion + the amount of plastic needed to construct the bottom surface = 302 + 50 = 352cm^2.

Q5. The curved surface area of a cylinder of height 80cm is 2880cm^2. Calculate

i. Its total surface area .
ii. Its circular top surface area. [Take $\pi = 3.14$]

Soln.

The curved surface area = 2πrh, and since the curved surface area of the cylinder is given as 2880$cm^2 \Longrightarrow 2\pi rh = 2880, \Longrightarrow 2 \times 3.14 \times r \times 80 = 2880,$

$$\Longrightarrow 502r = 2880, \Longrightarrow r = \frac{2880}{502} \Longrightarrow r = 5.7cm.$$

i. The total surface area $= 2\pi r(r + h) = 2 \times 3.14 \times 5.7(5.7 + 80) = 36(85.7) = 3085cm^2$
ii. The top circular surface area = $\pi r^2 = 3.14 \times 5.7^2 = 102cm^2$.

N/B: Since in the question the heights as well as the curved surface areas were given, we must first determine the radius.

- In the next question, the curved surface area is given as well as the radius. We must therefore first determine the height.

Q6. The curved surface area of a cylinder whose radius is 5cm is 628cm^2.Determine its total surface area.

Soln.

r = 5cm and h = ?

Since the curved surface area = 628cm^2, then

$2\pi rh = 628 \Longrightarrow 2 \times 3.14 \times 5 \times h = 628, \Longrightarrow 31.4h = 628 \Longrightarrow h = \frac{628}{3.14} = 20.$

Total surface area = $2\pi r(r + h) = 2 \times 3.14 \times 5(5 + 20) = 31.4(25) = 785cm^2$

Q7. A cylinder has a top surface area of 12.56cm^2 and a height of 0.8m. Calculate

a. its curved surface area.
b. its total surface area.

[Take $\pi = 3.142$]

Soln.

Top surface area = 12.56cm^2, h = 0.8m = 0.8m x 100 = 80cm.

$\pi = 3.142 \; and \; r = ?$

The top surface area is given by πr^2, and since this = 12.56cm^2, then

$\pi r^2 = 12.56, \Longrightarrow r^2 = \frac{12.56}{3.142} = 4.$

Since $r^2 = 4 \Rightarrow r = \sqrt{4} = 2.$

a. Curved surface area = $2\pi rh = 2 \times 3.142 \times 2 \times 80 = 1005cm^2$
b. The total surface area = $2\pi r(r + h) = 2 \times 3.142 \times 2(2 + 80) = 12.56(82) = 1030cm^2$.

Q8. A cylinder made of copper of height 80cm and radius 4cm, was melted and converted into a new cylinder whose radius is 6cm. Determine the height of the new cylinder.

Soln.

Amount of copper used in making the first cylinder

= the total surface area

= $2\pi r(r+h) = 2 \times 3.142 \times 4$(4+80) = 25(84) = 2100

For the new cylinder r = 6cm.

Let h = the height of the new cylinder. The amount of copper used in making the new cylinder = its total surface area = $2\pi r(r+h) = 2 \times 3.14 \times 6(6+h)$

$= 38(6+h) = 228 + 38h$. But since the amount of copper used in making the first cylinder = that used in making the second one,
$\Rightarrow 228 + 38h = 2100, \Rightarrow 38h = 2100 - 228 = 1872, \Rightarrow h = \frac{1872}{38} = 49cm.$

The volume of cylinder:

- The volume of a cylinder is the amount of gas, liquid or solid which it can contain or hold.
- The volume of a cylinder is given by $v = \pi r^2 h,$ where r = the radius and h = the height.

Q1. A cylinder has a height of 80cm and a diameter of 20cm. Calculate

a. its volume
b. the volume of air it will contain when it is
   i. full ii. half full.

[Take $\pi = 3.143$]

Soln.

d = 20cm $\Rightarrow r = 10cm.$

a. Volume $= \pi r^2 h = 3.14 \times 10^2 \times 80 = 25120cm^3$.
b. i. The volume of air it will contain when it is full = 25120cm^3.

ii. The volume of air it will contain when it is half full $= \frac{1}{2} \times 25120 = 12560cm^2$.

Q2. A cylinder is to be constructed in order to have a volume of 5540cm^3. If it is to have a radius is 20cm, calculate its height.

Soln.

$v = 5540cm^3, r = 20cm\ and\ h = ?$

Since $v = \pi r^2 h,$ then 5540 = 3.14$\times 20^2 \times h, \Rightarrow 5540 = 1256h \Rightarrow h = \frac{5540}{1256} = 4.4,$

∴ the height = 4.4cm

Q3. A cylindrically shaped water tank, can hold 7000cm^3 of water when it is full. If it has a height of 50cm, determine its radius.

Soln.

$v = 7000cm^3, h = 50cm\ and\ r = ?$

Since $v = \pi r^2 h,$ then 7000 =
$3.14 \times r^2 \times 50, \Rightarrow 7000 = 157r^2, \Rightarrow r^2 = \frac{7000}{157}, \Rightarrow r^2 = 44.5, \Rightarrow r = \sqrt{44.5} =>$
$r = 6.6cm$

Q4. Water for sale is stored in a cylindrically shaped tank, of height 120m and diameter 40m. If the tank is full and a bucket whose volume id 200m^3, is used to sell the water at a price of ¢2 per bucket, calculate the total amount expected if all the water was sold. [Take $\pi = 3.142$]

Soln.

D = 40m $\Rightarrow r = 20m.$

Also h = $120m\ and\ \pi = 3.142$

The amount of water the tank will contain when full = the volume of the tank = $\pi r^2 h = 3.142 \times 20^2 \times 120 = 3.142 \times 400 \times 120 = 150816m^3.$

The volume of the bucket used in selling the water = 200m^3 $\Rightarrow$ the number of buckets of water which can be had from the tank $= \frac{150816}{200} = 754$ buckets.

Since the price of water per bucket = ¢2, then the total amount had = 754 x 2 = ¢1508.

Q5. A petrol tank is cylindrical in shape, has a radius of 80m and is 300m long. It is being filled with petrol and 900m^3 of petrol is pumped into the tank every minute. How long will it take to fill this tank. [Take $\pi = 3.14$]

Soln.

h = 300m and r = 80m

Amount of petrol the tank will contain when full $= \pi r^2 h = 3.14 \times 80^2 \times 300 = 6028800m^3, \Rightarrow$ the volume to be filled with petrol = $6028800m^3$

Since $900m^3$ of petrol is pumped into the tank every minute, then the number of minutes needed to fill the tank = $\frac{6028800}{900} = 6699$ minutes.

Q6. The total surface area of a closed circular cylinder of radius 3.5cm is $1320cm^2$. Calculate the volume of the cylinder.

Soln

Area of the cylinder = $1320cm^2$.

Radius = r = 3.5cm

Height = h = ?

We must first find the height

Area of cylinder = $2\pi r(r + h)$.

Since the area of the given cylinder = $1320cm^2$, then

$2\pi r(r + h) = 1320, \Rightarrow 2 \times 3.14 \times 3.5(3.5 + h) = 1320, \Rightarrow 77 + 22h = 1320,$

$\Rightarrow 22h = 1320 - 77 \Rightarrow 22h = 1243, \Rightarrow h = \frac{1243}{22}$

$\Rightarrow h = 56.5.$

Volume of cylinder = $\pi r^2 h = 3.14 \times 3.5^2 \times 56.5 = 2173cm^3$.

Q7. The volume of a cylinder is $126cm^3$. If its height is 10cm, calculate

a. Its total surface area.
b. its curved surface area.

[Take $\pi = 3.14$]

Soln.

Volume of cylinder = $\pi r^2 h$

Since the volume is given as $126cm^3$, then $\pi r^2 h = 126,$

$\Rightarrow 3.14 \times r^2 \times 10 = 126 \Rightarrow 31.4r^2 = 126, \Rightarrow r^2 = \frac{126}{31.4} = 4$

If $r^2 = 4 \Rightarrow r = \sqrt{4} = 2$cm.

a. Total surface area

$= 2\pi r(r + h) = 2 \times 3.14 \times 2(2 + 10)$

= 12.6(12) = 12.6 x 12 = 151cm^2.

(b) Curved surface area

= $2\pi r$h = 2 x 3.14 x 2 x 10

= 126cm^2.

Q8. The curved surface area of a cylinder whose diameter is 8cm is 1507cm^2. Determine the volume of water it can contain, when it is filled with water.

N/B: First determine the height.

Soln.

The curved surface area is given by $2\pi rh \text{ and } r = \frac{8}{2} = 4cm$

But since the curved surface area = 1507, then 2πrh = 1507,

$\Rightarrow 2 \times 3.14 \times 4 \times h = 1507, \Rightarrow 25.12h = 1507 \Rightarrow h = \frac{1507}{25.12} \Rightarrow h = 60cm.$

Volume of cylinder = the volume of water the cylinder will contain = $\pi r^2 h = 3.14 \times 4^2 \times 60 = 3.14 \times 16 \times 60 = 3014cm^3$

N/B: The volume of a cylinder is also given by V = either the top or bottom circular surface area x h, where

V = volume and h = height.

Q1. A cylinder has a top circular surface area of 240cm^2. If it has a height of 80cm, calculate its volume.

Soln.

h = 80cm, V =?

V = circular surface area x height

$V = 240 \times 80 = 19200cm^3$

Q2. The amount of water which a cylinder can contain is $5100cm^3$. If it has a circular surface area of $340cm^2$, determine its height. [Take $\pi = 3.142$]

Soln.

$V = 5100cm^3$, circular surface area = $340cm^2$, $\pi = 3.142$ *and* $h = ?$

V = circular surface area x height, $\Longrightarrow$v = 340 x h => 5100 = 340h => h = $\frac{5100}{340} = 15.$

=> The height = 15cm.

Q3. The volume of a cylinder is $126cm^3$. If its height is 10cm, calculate

Soln.

a. its curved surface area
b. its total surface area

Soln.

Volume of cylinder = $\pi r^2 h.$

Since the volume is given as $126cm^3$, then $\pi r^2 h = 126,$

$$\Longrightarrow 3.14 \times r^2 \times 10 = 126, \Longrightarrow 31.4r^2 = 126, \Longrightarrow \frac{r^2 = 126}{31.4} = 4 \Longrightarrow r = \sqrt{4} = 2$$

$\Longrightarrow r = 2$cm.

a. Curved surface area = $2\pi rh = 2 \times 3.14 \times 2 \times 10 = 126cm^2$.
b. Total surface area = $2\pi r(r + h) = 2 \times 3.14 \times 2(2 + 10) = 12.6 \times 12 = 151cm^2$.

Q4. X and Y are two cylindrical tanks with radii 2r and r cm respectively. If the water level in Y is 10cm, what level will the same quantity of water be in X.

(Take $\pi = \frac{22}{7}$).

Considering cylinder Y:

Radius = r cm

Let the level or height of water in this cylinder = h , $\Longrightarrow h = 10cm$

Then the volume of water in cylinder

$Y = \pi r^2 h = \frac{22}{7} \times r^2 \times 10 = \frac{220r^2}{7} = 31.4r^2 cm^3$

Considering cylinder X:

Radius = 2r cm

Let H = the height or the level of water in this cylinder, when the same quantity of water was poured into it (as was done in the first case). Then the volume of water in this cylinder which is given by $\pi r^2 h = \pi(2r)^2 H = \pi \times 2^2 \times r^2 \times H$

$= \pi \times 4 \times r^2 \times H = 4\pi r^2 H = 4 \times \frac{22}{7} \times r^2 \times h$

$= 12.6r^2 H = 12.6r^2 Hcm^3$

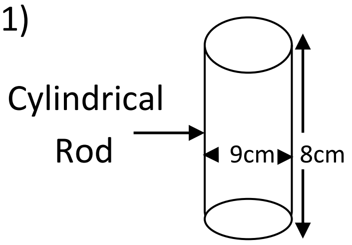
Since the same quantity of water was poured into each of these cylinders, then the volume of water in cylinder Y = the volume of water in cylinder X.

$\Longrightarrow 31.4r^2 = 12.6r^2 H, \Longrightarrow H = \frac{31.4r^2}{12.6r^2} = 4.5,$

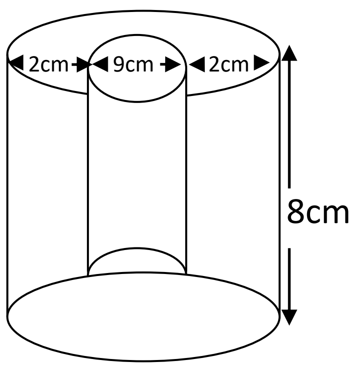
∴ the level or height of water in cylinder X = 4.5cm.

Q5. A strip of metal sheet 2cm thick is wound ten times round a cylindrical rod, of diameter 9cm and height 8cm such that it fits it exactly. Calculate the volume of the resulting object.

N/B: (1)

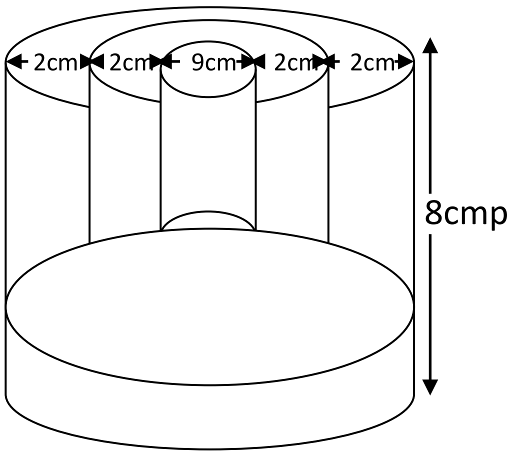


(2) When the first strip of metal is wound round this rod, then the appearance of the resulting object will be as shown next:



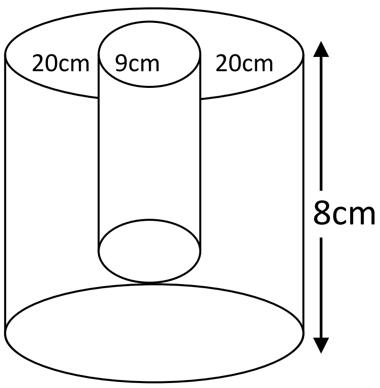
In this case the radius of the strip = 1x2, since the strip of thickness 2cm, was wound once round the cylindrical rod.

1. When the second strip of metal is used to wrap the rod, or when the metal strip is wound two times round the cylindrical rod, then the appearance of the resulting object will be as shown next:



In this case the radius of the metal strip = 2 x 2 = 4cm, since the strip is 2cm thick, and had been wound round the cylindrical rod twice.

Based on these two examples, then the resulting object will look as shown next, with the metal strip. In this case the radius of the metal strip = 2 x 10 = 20cm.



The diameter of the resulting object = 20 + 9 +20 = 49cm,

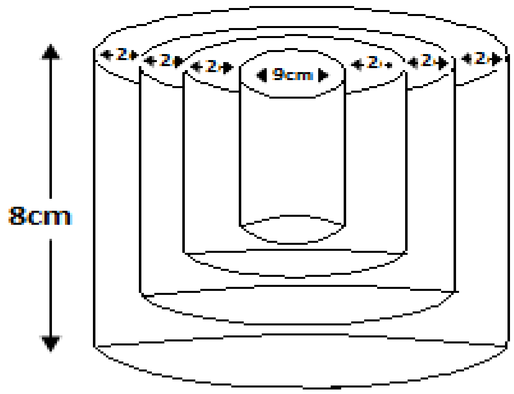
$\Rightarrow$ it's radius $= \frac{49}{2} = 24.5cm$

The height of this resulting object = 8cm

The volume of this object = $\pi r^2 h = \left(\frac{22}{7}\right)(24.5)(24.5)(8) = 15016cm^3$

Method (2)

The figure drawn shows the appearance of the rod, after wrapping it three times with the strip. [Diagram in cm].



Hint:

From the first diagram, when the rod is wrapped once with the strip, the diameter of the resulting structure

= 9 + 2(2) = 9 + 2(1 x 2), where the 1 represents the number of times the strip is wrapped round the rod.

From the second diagram, when the strip is wrapped twice round the rod, the diameter of the resulting structure = 9 + 2(4) = 9 + 2(2 x 2), where the second 2 represents the number of times the strip is wrapped round the rod.

From the third diagram, when the strip is wrapped three times round the rod, then the diameter of the resulting structure = 9+ 2(6) = 9 + 2(3 x 2), where the 3 = the number of times the strip was wrapped round the rod.

From these analysis made, then the diameter of the resulting structure when the rod is wrapped ten times with the rod = 9 + 2(10 x 2) = 9 + 40 = 49cm.

Spheres:

- The volume of a sphere = $\frac{4}{3}\pi r^3$.

- The area of a sphere = $4\pi r^2$.

(Q1) The diameter of a sphere is 12cm. Calculate

(a) its volume
(b) its area

Soln.

(a)
D = 12cm $\Rightarrow$ r = 6cm.
Volume = $\frac{4}{3}\pi r^3 = \frac{4}{3} \times 3.14 \times 6^3$
$= \frac{4}{3} \times 3.14 \times 216 = \frac{2712.96}{3}$
$= 904cm^3$.
(b) The area = $4\pi r^2$
$= 4 \times 3.14 \times 6^2 = 4 \times 3.14 \times 36$
$= 452\ cm^2$.

(Q2) A water tank is spherical in shape and has a radius of 10m. Calculate

(a) the volume of water it will contain when it is full
(b) the amount of zinc needed to construct the tank.

Soln.

(a) Volume of water it will contain when full = the volume of the sphere = $\frac{4}{3}\pi r^3$ =

$\frac{4}{3} \times 3.14 \times 10^3 = \frac{12560}{3} = 4187m^3$.

(b) The amount of zinc needed to construct the sphere,

=> the area of the sphere = $4\pi r^2 = 4 \times 3.14 \times 10^2$

$= 1256m^2$.

N/B: - A hemisphere is a half sphere.

- The volume as well as the area of a hemisphere is half that of a sphere.
- Since the area of a sphere = $4\pi r^2$, then the area of a hemisphere = $\frac{1}{2} \times 4\pi r^2$ $= 2\pi r^2$.
- Also since the volume of a sphere = $\frac{4}{3}\pi r^3$, then the volume of a hemisphere $= \frac{1}{2} \times \frac{4}{3}\pi r^3 = \frac{4}{6}\pi r^3 = \frac{2}{3}\pi r^3$

(Q3) A hemisphere has a radius of 12cm. Calculate

(a) its area.
(b) its volume.

Soln.

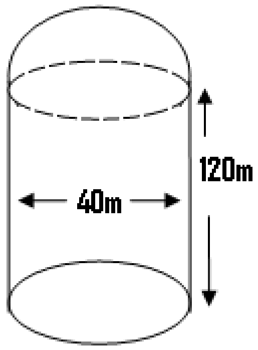
(a) The area of a hemisphere = $2\pi r^2$

= $2 \times 3.14 \times 12^2$ = $2 \times 3.14 \times$ 144 = 904cm^2.

(b)The volume of a hemisphere = $\frac{2}{3}\pi r^3$

= $\frac{2}{3} \times 3.14 \times 12^3$ = 3617cm^3.

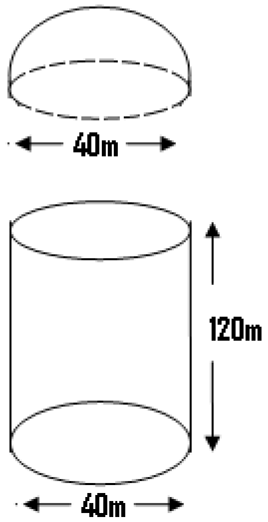
(Q4)



The above shows a petrol tank which is to be constructed. It is to be in the form of a cylinder of height 120m and diameter 40m. The top part of the tank is to be in the form of a hemisphere. Calculate

(a) the volume of air it will contain.
(b) The amount of aluminum which will be needed to construct it.
(c) the cost of building the tank, given that $2cm^2$ of aluminum cost ₵2.

Soln.



The volume of the given figure = the volume of the air it can contain.

Therefore the volume of the figure = volume of hemisphere + the volume of the cylinder.

Soln.

radius of hemisphere = 20m.

volume of hemisphere = $\frac{2}{3}\pi r^3$

= $\frac{2}{3} \times 3.14 \times 20^3$.

= 16747m^3

For the cylinder, r is also = 20m.

The volume of the cylinder = $\pi r^2 h$

= $3.14 \times 20^2 \times 120$

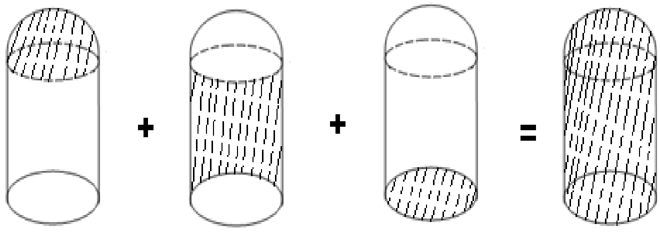
= 150720m^3.

Volume of air the structure can contain = the volume of the hemisphere + the volume of the cylinder

= 16747 + 150720 = 167467m^3

(b) The amount of aluminum needed to construct it is equal to the area of the hemisphere + the area of the curved surface + the area of the bottom surface.

i.e



(a)(1) The area of the hemisphere = $2\pi r^2$

= $2 \times 3.14 \times 20^2$ = $2 \times 3.14 \times 400$

= $2512 m^2$

(2) The area of the curved surface of the cylinder

= $2\pi rh$ = $2 \times 3.14 \times 20 \times 120$

= $15072 m^2$

(3) The bottom circular surface area = πr^2

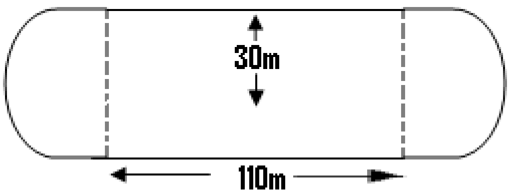
= 3.14×20^2 = 3.14×400 = $1256\ m^2$

$\therefore$ Amount of aluminum needed

= 2512 + 15072 + 1256

= $18840 m^2$.

(Q4)

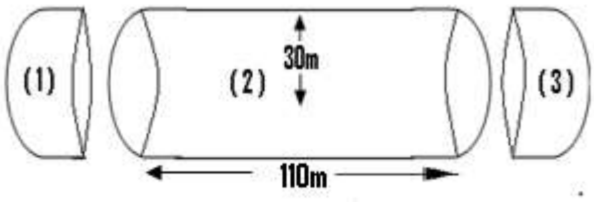


The given figure is that for a storage tank which is to be constructed. It is to have a radius of 30m. The mid portion is cylindrical in shape and of length 110m. Its two ends take the form of hemispheres.

(i) Calculate the volume of the given structure.
(ii) Determine its area.

Soln.

(i)



<u>Volume of figure (1)</u>

r = 30m

Volume of the hemisphere =

$= \frac{2}{3} \times 3.14 \times 30^3$

= $56520m^3$.

<u>Volume of figure (2)</u>

r = 30m and h = 110m

v = $\pi r^2 h$

$= 3.14 \times 30^2 \times 110 = 3.14 \times 900 \times 110$

= $310860m^3$.

<u>Volume of figure (3)</u>

Volume of the hemisphere = $\frac{2}{3}\pi r^3$

$= \frac{2}{3}\pi r^3 = \frac{2}{3} \times 3.14 \times 30^3 = 56520m^3$.

Volume of the given figure = the volume of figure (1) + the volume of figure (2) + the volume of figure (3)

= 56520 + 310860 + 56520 = $423900m^3$.

(b) $\underline{Area\ of\ figure\ (1)}$

The area of the hemisphere

$= 2\pi r^2 = 2 \times 3.14 \times 30^2$

= $5652m^3$.

<u>Area of figure (2)</u>

Area of the curved surface = 2πrh= $2 \times 3.14 \times 30 \times 110$

= $20724m^2$.

<u>Area of figure (3)</u>

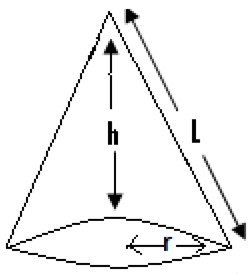
The area of the hemisphere

= $2\pi r^2$

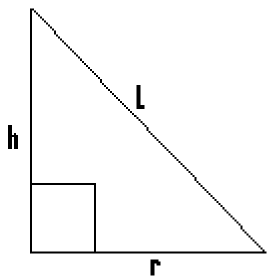
= $2 \times 3.14 \times 30^2 = 5652m^2$.

The area of the given figure = the area of figure (1) + the area of figure (2) + that of figure (3) = 5652 + 20724 + 5652 = $32028m^2$.

Cones



- The given figure is that of a cone, whose height is h.
- The length indicated by l is referred to as the slanted height of the cone, with r being the radius of the cone.
- Half of the given figure drawn can be represented as shown next:



From Pythagoras theorem, $l^2 = h^2 + r^2$

$\Rightarrow l = \sqrt{h^2 + r^2}$.

(Q1) A cone has a radius of 3cm and a height of 4cm. Determine its slanted height.

Soln.

r = 3cm and h = 4cm.

$l = \sqrt{h^2 + r^2}$, where l = slanted height,

$\Rightarrow l = \sqrt{4^2 + 3^2} = \sqrt{16 + 9}$,

$\Rightarrow l = \sqrt{25}$ = 5cm.

(Q2) A cone has a slanted height of 11.2 cm and a radius of 5cm. Find its height.

Soln.

From $l^2 = h^2 + r^2 \Rightarrow l^2 - r^2 = h^2$,

$\Rightarrow h^2 = l^2 - r^2$, $\Rightarrow$ h = $\sqrt{l^2 - r^2}$,

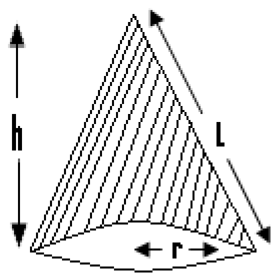
$\Rightarrow$ h = $\sqrt{11.2^2 - 5^2}$, $\Rightarrow$ h = $\sqrt{100}$ = 10cm.

Total surface area of a cone:

* The total surface area of a cone depends on whether it is a solid or a hollow cone.

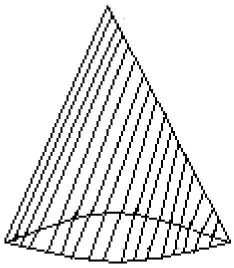
*A hollow cone is a cone without a base and a solid one is one which has a base.

Total surface area of a hollow cone:



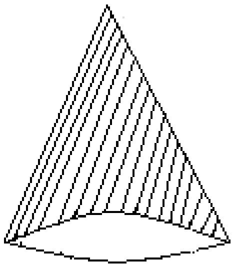
- A hollow cone consists only of the curved surface area, which has been shaded in the given diagram.
- The total surface area of such a cone is given by $\pi r l$, where l = $\sqrt{h^2 + r^2}$.

The total surface area of a solid cone:



- This figure is that of a solid cone, and the shaded portion is what is referred to as its total surface area, which consists of two parts.

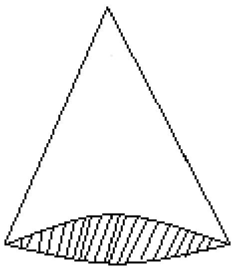
(1) The curved surface area which is shaded in the next figure



- The curved surface area is given by $\pi r l$, where

$l = \sqrt{h^2 + r^2}$.

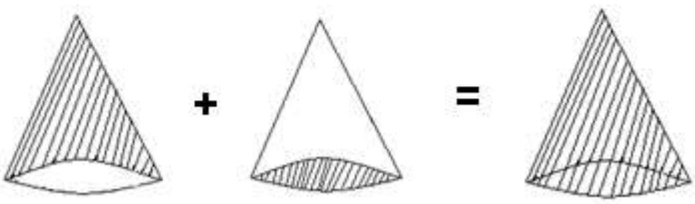
(2) The base circular surface area, which has been shaded in the next figure:



- The base circular surface area is given by πr^2,

where r = the radius of the cone.

- The total surface area of a solid cone is given by



$\Pi r l \quad + \quad \pi r^2 \quad = \quad \Pi r\,(l + r)$.

(Q1) A cone has a slanted height of 12cm and a radius of 6cm. If it is a hollow cone, calculate its total surface area.

[Take π = 3.142]

Soln.

l = 12cm and r = 6cm.

Total surface area = Πrl

= 3.14 $\times 6 \times 12 = 226cm^2$

(Q2) A hollow cone has a height of 4cm and a radius of 3cm. Calculate its total surface area. [Take π = 3.142]

N/B: Since the slanted height is not given, we first have to find it.

Soln.

h = 4cm, r = 3cm and l = ?. Since l = $\sqrt{h^2 + r^2}$

$\Rightarrow l$ = $\sqrt{4^2 + 3^2}$ = $\sqrt{16+9} = \sqrt{25}$,

$\Rightarrow l$ = $\sqrt{25}$$\Rightarrow l$ = 5.

Surface area of a hollow cone = Πrl = 3.14 $\times$ 3 x 5

= 47cm^2

(Q3) A hollow cone is to be constructed using zinc. If it is to have a diameter of 4cm and a height of 6cm, determine the quantity of zinc which will be needed.

Soln.

d = 4cm $\Rightarrow$ r = 2cm, h = 6cm and l = ?. But l = $\sqrt{2^2 + 6^2}$

= $\sqrt{4+36}$, $\Rightarrow l$ = $\sqrt{40}$ = 6.3cm.

The amount of zinc needed = the surface area = πrl

= 3.14 $\times 2 \times 6.3$ = 39.6cm^2.

(Q4) The total surface area of a hollow cone of diameter 6cm, is 47cm^2. Calculate

(a) Its slanted height.
(b) its height.

[Take π = 3.14].

Soln.

(a) d = 6cm $\Rightarrow$ r = 3cm and total surface area = $47cm^2$

Since the total surface area = $\pi r l$, then $\pi r l$ = 47.

$\Rightarrow 3.14 \times 3 \times l = 47, \Rightarrow 47 = 9.42l, \Rightarrow l = \frac{47}{9.42} = 5,$

$\Rightarrow$ slanted length = 5cm.

(b) l = 5cm, r = 3cm and h =?. From $l^2 = h^2 + r^2$

$\Rightarrow h^2 = l^2 - r^2, \Rightarrow h = \sqrt{l^2 - r^2} = \sqrt{5^2 - 3^2},$

$\Rightarrow h = \sqrt{25 - 9} = \sqrt{16}$ = 4cm.

(Q5) A water storage tank is to be built in the form of a hollow cone, using aluminum. It is supposed to have a height of 8m and a slanted height of 10m.If the quantity of aluminum expected to be used is $188.4cm^2$, determine the diameter of this tank.

[Take π = 3.14]

Hint: - First determine the radius and multiply it by 2 to get the diameter.

- Also, the amount of aluminum used = the surface area.

N/B: When the term cone is used without any adjective qualifying it (i.e. solid or hollow), then the assumption is that, it is a solid cone.

(Q6) A solid cone is to be constructed to have a radius of 6cm, and a slanted height of 10cm. Determine

a) its curved surface area.

(b)its base circular surface area.

a) its total surface area.

Soln.

$r = 6\text{cm}$ and $l = 10\text{cm}$.

(a) curved surface area = $\pi r l$

$= 3.14 \times 6 \times 10 = 188.4 cm^2$

(b) The base circular surface area = πr^2

= $3.14 \times 6^2 = 113 cm^2$

(c) The total surface area = curved surface + base circular surface area = 188.4 + 113 = 301.4 cm^2

(Q7) A cone has a radius of 6mm and a height of 8mm. Determine

(a) its curved surface area.
(b) its total surface area.

[Take π = 3.14]

Soln.

r = 6mm and height = 8mm. But $l = \sqrt{r^2 + h^2}$

= $\sqrt{6^2 + 8^2} = \sqrt{36 + 64} = \sqrt{100}$ = 10m.

(a) The curved surface area = $\pi r l$ = $3.14 \times 6 \times 10$

$= 188.4 cm^2$

(b) The total surface area = $\pi r(r + l)$

= $3.14 \times 6(6 + 10)$ = 18.8(16) = $300 cm^2$.

<u>Method 2</u>

The base circular surface area = πr^2

= $3.14 \times 6^2 = 3.14 \times 36 = 113 cm^2$

The total surface area = the curved surface area + the base surface area = 188.4 + 113 = $301 cm^2$

(Q8) A cone whose diameter is 8m, has a curved surface area of $503 m^2$. Determine

(a) its slanted height.
(b) its total surface area.

Soln.

d = 8m $\Rightarrow$ $r = 4m$.

Curved surface area = $\pi r l$. But since curved surface area = 503m^2,

$\Rightarrow \pi r l$ = 503, $\Rightarrow$ $3.14 \times 4 \times l = 503$,

$\Rightarrow 12.56l$ = 503 , $\Rightarrow l = \frac{503}{12.56}$

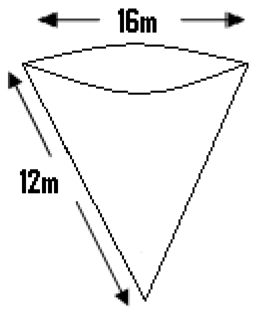
$\Rightarrow l$ = 40m

(b) Total surface area = $\pi r(r + l)$

= $3.14 \times 4(4 + 40) = 12.56(44)$

= $553m^2$.

(Q9)



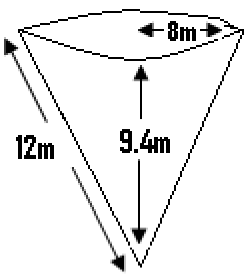
Above shows a structure in the form of a solid cone which is to be painted.

a) Determine its total surface area.

b) If an area of $20m^2$ requires 3 gallons of paint, determine the number of gallons needed to paint the whole structure. [Take π = 3.14

Soln.

(a) d = 16m $\Rightarrow$ r = 8m, l = 12m and h = ?



From $l^2 = h^2 + r^2 \Rightarrow l^2 - -r^2 = h^2$,

$\Rightarrow h^2 = l^2 - r^2 \Rightarrow$ h = $\sqrt{l^2 - r^2}$,

$\Rightarrow$ h = $\sqrt{12^2 - 8^2} = \sqrt{144 - 64} = \sqrt{80}$,

$\Rightarrow$ h = 9.4m

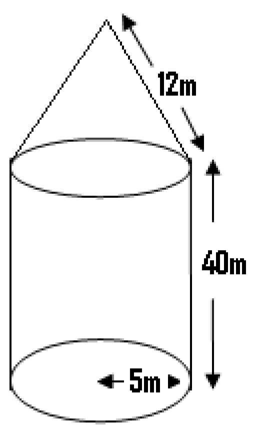
Total surface area = πr(r + l)

= 3.14 $\times$ 8(8 + 12) = 502m^2.

(b) $20m^2$ = 3 gallons

$\Rightarrow$ 502m^2= $\frac{502}{20} \times 3$ = 75 gallons.

(Q10)



The given structure is to be constructed using copper. Its lower portion takes the form of a cylinder of height 40m and radius 5m. The upper portion takes the form of a cone (i.e. hollow cone) whose slanted height is 12m.

a) Find the height of the cone.

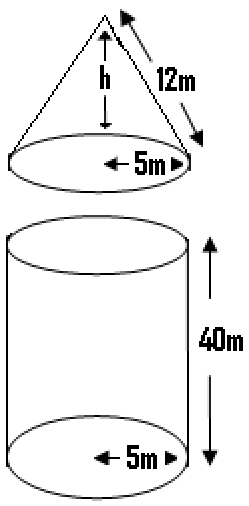
b)Determine the total area of this structure.

c) If 50m^2 of copper cost ¢2, what will be the amount needed to construct the structure.

[Take π = 3.14]

N/B: Separate the structure into two portions i.e. the upper and the lower portion and determine the area of each separately.

Soln.



N/B: The cone and the cylinder will have the same radius or diameter.

a) (i) <u>Considering the cone:</u>

l = 12m, r = 5m and h = ?

From $l^2 = h^2 + r^2 \Rightarrow l^2 - r^2 = h^2$,

$\Rightarrow h^2 = l^2 - r^2 \Rightarrow$ h = $\sqrt{l^2 - r^2}$

= $\sqrt{12^2 - 5^5}$, $\Rightarrow$ h = $\sqrt{144 - 25} = \sqrt{119}$

$\Rightarrow$ h = 11m.

(b) The total area of the cone (which is a hollow one)

= πrl = 3.14$\times 5 \times 12 = 188m^2$

(2) Considering the cylinder:

The total area in this case = the curved surface area + the bottom circular surface area.

The curved surface area of the cylinder = 2πrh

= $2 \times 3.14 \times 5 \times 40 = 1256m^2$

The bottom circular surface area = πr^2

= 3.14 $\times 5^2 = 3.14 \times 25 = 79m^2$

The total area of the given structure = the area of the cone + the curved surface area of the cylinder + the area of the circular base of the cylinder,
$\Rightarrow$ total surface area $= 188 + 1256 + 79 = 2952\text{m}^2$

$Therefore$ the amount of copper needed = 2952m^2.

c) If 50m^2 = ₵2

$\Rightarrow 2952\text{m}^2 = \frac{2952}{50} \times 2$

= ₵188.

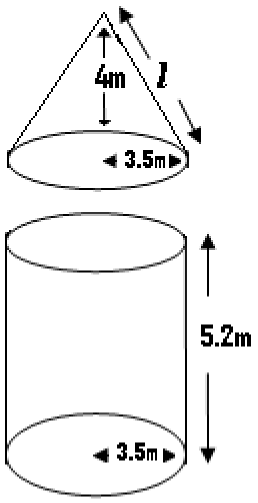
(Q12) The lower portion of a building is cylindrical in shape, and has a height of 5.2m. Its roof takes the form of a cone whose diameter is 7m with a height of 4m. Determine

(a) the slanted height of the roof.
(b) the area of the roof.
(c) the total area of the building.

[Take π = 3.14 and assume that the base of the building forms part of it].

Soln.

Since d = 7m $\Rightarrow$ r = 3.5m.



(a) h = 4m , r = 3.5m and l = ? .

From $l^2 = h^2 + r^2 \Rightarrow l = \sqrt{h^2 + r^2}$,

$\Rightarrow l = \sqrt{4^2 + 3.5^2} = \sqrt{16 + 12.3}$ = 5.3m,

$\Rightarrow$ the slanted height = 5.3m.

(b) The area of the roof = πrl

= 3.14 $\times 3.5 \times 5.3$ = 58m^2.

(c) The curved surface area of the cylinder = 2πrh

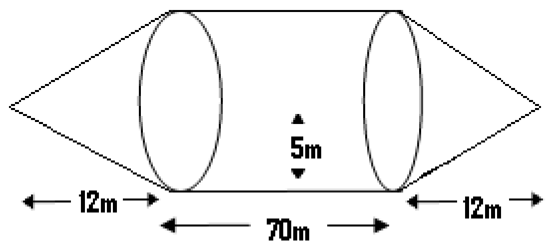
= $2 \times 3.14 \times 3.5 \times 5.2 = 114m$

The bottom circular surface area of the cylinder = πr^2

= $3.14 \times 3.5^2 = 3.14 \times 12.25 = 38.5m^2$.

The total area of the building = area of the roof + area of the curved surface + area of the bottom surface = 58 + 114 + 38.5 = 210m^2.

(Q13)

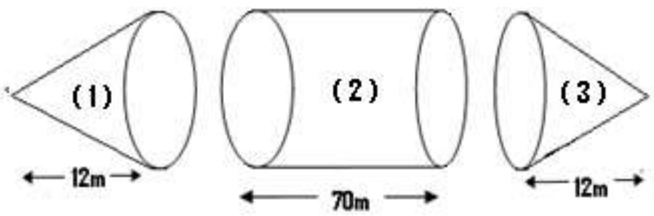


The diagram is that of a water tank which is to be constructed. It is supposed to be cylindrical in shape, of length 70m and radius 5m. Its two ends are modified into cones, each of height 12m as shown in the given diagram. Calculate

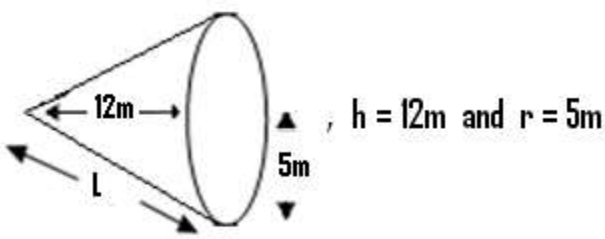
a)the amount of zinc needed to construct it.

b) If one metre squared of zinc cost ₵2, find the cost of constructing the given water tank.

Soln.



The given structure can be divided into three parts as shown in the diagram. Consider figure (1) or cone (1).



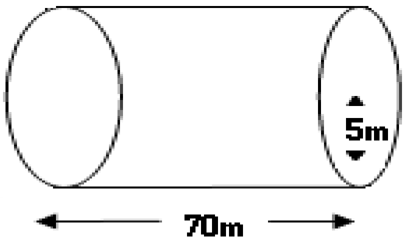
$l = \sqrt{h^2 + r^2} = \sqrt{12^2 + 5^2}$

$l = \sqrt{144 + 25} = \sqrt{169} = 13\text{m}$

The area of figure (1) = the curved surface area = $\pi r l$

= $3.14 \times 5 \times 13 = 204m^2$.

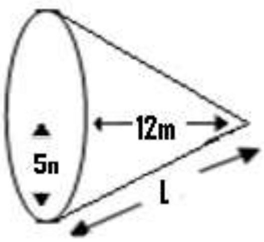
Consider figure (2).



Area of the curved surface = $2\pi rh$

= $2 \times 3.14 \times 5 \times 70 = 2198m^2$.

Consider figure (3) or cone (2)



$l = \sqrt{h^2 + r^2} = \sqrt{12^2 + 5^2}$

$l = \sqrt{144 + 25} = \sqrt{169} = 13\text{m}$

The area of figure (3) = the curved surface area = $\pi r l$

= $3.14 \times 5 \times 13 = 204m^2$.

(a)The total surface area of the given tank or structure

= the curved surface area of cone (1) + curved surface area of the cylinder + curved surface area of cone (2)

= 204 + 2198 + 204 = $2606m^2$.

(Q14) A solid cone of radius 5m and slanted height 20m, made of copper was melted and converted into a cylinder of radius 3m. Determine the height of the cylinder.

N/B: Since the same amount of material used in making the cone is what was converted into the cylinder, then the total area of the cone = the total area of the cylinder.

<u>For the cone:</u>

r = 5m and $l = 20m$.

Total surface area = πr($l + r$)

= 3.14 $\times 5(20 + 5) = 393m^2$.

<u>For the cylinder:</u>

h = ? and r = 3m

Total surface area = 2πr($r +$ h)

= 2 $\times$ $3.14 \times 3(3 + h) = 19(3 + h)$

= 57 + 19h

But since the total surface area of the cone = that of the cylinder, then 57 + 19h = 393, $\Rightarrow$ 19h = 393 – 57 = 336,

$\Rightarrow 19\text{h} = 336$,$\Rightarrow \text{h} = \frac{336}{19} = 17.7.$

$Therefore$ the height of the cylinder = 17.7m

(Q15) A cylinder of diameter 8m and height 70m was converted into a hollow cone of radius 9cm. Determine

(a) the slanted height of the cone.
(b) the height of the cone.

Soln.

<u>For the cylinder:</u>

d = 8m $\Rightarrow \mathrm{r} = 4\mathrm{m}, \text{and } \mathrm{h} = 70\mathrm{m}.$

Total surface area = 2πr(r + h)

= $2 \times 3.14 \times 4(4 + 70) = 1859m^2$

<u>For the hollow cone:</u>

r = 9cm and l = ?

Since the cone is a hollow one, then its total surface area = the curved surface area = π rl = 3.14$\times$ 9 $\times$ l = 28l

Since the area of the hollow cone = that of the cylinder, then 28l = 1859 $\Rightarrow l = \frac{1859}{28}$, $\Rightarrow l$ = 66.

(a) The slanted height of the cone = 66m.

(b) From $l^2 = h^2 + r^2 \Rightarrow l^2 - r^2 = h^2$,

$\Rightarrow h^2 = l^2 - r^2$,$\Rightarrow$ h = $\sqrt{l^2 - r^2}$

$\Rightarrow$ h = $\sqrt{66^2 - 9^2} = \sqrt{4356 - 81}$

= $\sqrt{4275} = 65m.$

<u>The volume of cone:</u>

* The volume of a cone refers to the amount of solid, liquid or gas which it can contain.

*The volume of a cone is given by V = $\frac{1}{3} \times base\ area \times height$,

$\Rightarrow \mathrm{V} = \frac{1}{3} \times \pi r^2 \times h = \frac{1}{3}\pi r^2 h.$

(Q1) A cone has a base radius of 15cm and a height of 25cm. Calculate its volume.

Soln.

r = 15cm and h = 25cm.

V = $\frac{1}{3} \times \pi r^2 h = \frac{1}{3} \times 3.14 \times 15^2 \times 25$,

$\Rightarrow V = \frac{1}{3} \times 3.14 \times 225 \times 25$

$\Rightarrow V = 589\text{cm}^3$.

(Q2) A cone is to be constructed to have a volume of 54000cm^3. If it is to have a height of 0.4m, determine its diameter.

Soln.

V = 54000cm^3

h = 0.4m = 0.4$\times 100 = 40cm$.

Since V = $\frac{1}{3}\pi r^2 h = \frac{1}{3} \times 3.14 \times r^2 \times 40$,

$\Rightarrow 54000 = \frac{1}{3} \times 3.14 \times \text{r}^2 \times 40, \Rightarrow 54000 = \frac{125.5\text{r}^2}{3}$,

$\Rightarrow 54000 = 42\text{r}^2, \Rightarrow r^2 = \frac{54000}{42}, \Rightarrow r^2 = 5400 \Rightarrow \text{r} = \sqrt{5400}$,

$\Rightarrow \text{r} = 77.5\text{cm}, \Rightarrow \text{d} = 2 \times 77.5 \Rightarrow \text{d} = 155\text{cm}$.

N/B: The volume of a cone is also given by

V = $\frac{1}{3} \times base\ area \times height$.

(Q3) A cone has a base area of 240cm^2 and a height of 40cm. Find its volume.

Soln.

V = $\frac{1}{3} \times base\ area \times height$

= $\frac{1}{3} \times 240 \times 40 = 3200cm^3$.

(Q5) A hollow cone has a slanted height of 5cm, a height of 4cm and a surface area of 47cm^2. Determine its volume.

N/B: First determine the radius since V = $\frac{1}{3}\pi r^2 h$, and h is given.

Soln.

Surface area of a hollow cone = πrl.

Since surface area = $47cm^2$,

then $\pi rl = 47, \Rightarrow 3.14 \times r \times 5 = 47$,

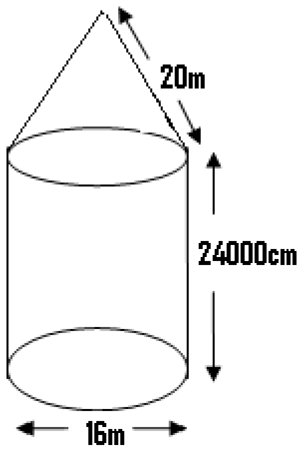
$\Rightarrow 15.7r = 47 \Rightarrow r = \frac{47}{15.7} = 3$,

$\Rightarrow r = 3m$.

Volume = $\frac{1}{3} \times \pi r^2 \times h$

= $\frac{1}{3} \times 3.14 \times 3^2 \times 4 = 38cm^3$.

(Q6)



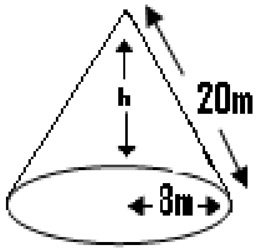
A building is to be constructed in the form of a cylinder which is 24000cm long and whose diameter is 16m. Its roof is to be in the form of a cone of slanted height 20m. Calculate the volume of air it will contain.

Soln.

d = 16m $\Rightarrow r = 8m, l = 20m$.

The volume of the given structure = the volume of the cone + the volume of the cylinder.

<u>Consider the cone:</u>



From $20^2 = h^2 + 8^2$,

$\Rightarrow 400 = h^2 + 64 \Rightarrow 400 - 64 = h^2$,

$\Rightarrow 336 = h^2 \Rightarrow h = \sqrt{336}$,

$\Rightarrow h = 18m.$

The volume of the cone

$= \frac{1}{3}\pi r^2 h = \frac{1}{3} \times 3.14 \times 8^2 \times 18$

$= \frac{1}{3} \times 3.14 \times 64 \times 18$

$= 1206m^3$

<u>Consider the cylinder:</u>

h = 24000cm = $\frac{24000}{100} = 240m$

r = 8m.

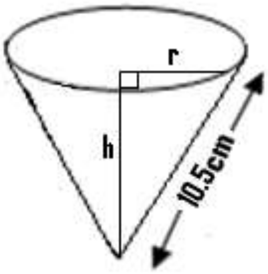
V = πrh = $3.14 \times 64 \times 240 = 48230m^3$

The volume of the given structure = volume of the cone + the volume of the cylinder

= 1206 + 48230 = $49436m^3$

The amount of air the structure will contain = the volume of the given structure = $49436m^3$.

(Q7)



The diagram shows a cone with a slanted height of 10.5cm. If its curved surface area is $115.5cm^2, calculate$

(i) the base radius, r.

(ii) the height h.

(iii) the volume.

Soln.

l = 10.5cm and the curved surface area = $115.5cm^2$.

Since the curved surface area is given by πrl, then

$\pi r l = 115.5 \Rightarrow 3.14 \times r \times 10.5 = 115.5, \Rightarrow 33r = 115.5, \Rightarrow r = \frac{115.5}{33}$,

$\Rightarrow r = 3.5cm.$

Now for a cone, $l^2 - r^2 = h^2$

$\Rightarrow 10.5^2 - 3.5^2 = h^2$,

=> 110.3 – 12.3 = $h^2 \Rightarrow h^2 = 98$,

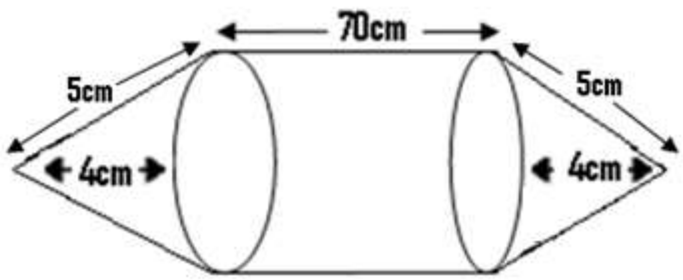
$\Rightarrow h = \sqrt{98} = 9.9$

Volume of cone = $\frac{1}{3}\pi r^2 h = \frac{1}{3} \times 3.14 \times 3.5^2 \times 9.9$

= $\frac{1}{3} \times 3.14 \times 12.25 \times 9.9$

$= 1275m^3$.

(Q8)



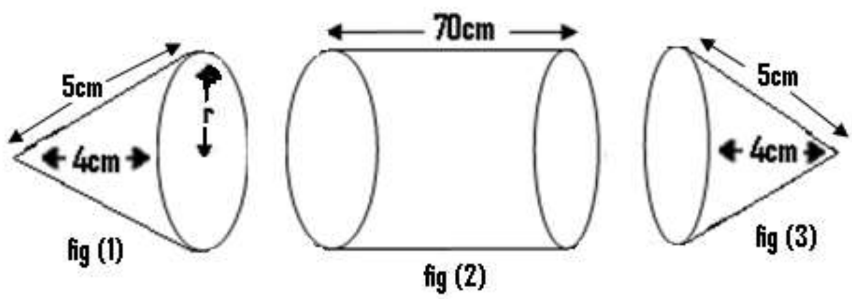
The above structure consists of a cylinder of length 70cm, whose two ends are attached to cones of height 4cm and slanted height 5cm. (a) Determine the volume of mercury it can contain.

(b) If 50cm^3 of mercury cost ₵2, determine the value of the mercury stored in the structure.

Soln.

The given structure can be divided into three parts

i.e.



<u>Consider figure (1) or cone (1)</u>

l = 5cm , h = 4cm and r = ?

From $l^2 = h^2 + r^2 \Rightarrow l^2 - h^2 = r^2$,

$\Rightarrow 5^2 - 4^2 = r^2 \Rightarrow 25 - 16 = r^2$,

$\Rightarrow 9 = r^2 \Rightarrow r = \sqrt{9} = 3$,

$\Rightarrow$radius of the cone and the cylinder $= 3$cm.

The volume of figure (1) = $\frac{1}{3}\pi r^2 h = \frac{1}{3} \times 3.14 \times 3^2 \times 4$

= $\frac{1}{3} \times 3.14 \times 9 \times 4 = 38cm^3$

<u>Consider figure (2)i. e. the cylinder</u>

h = 70cm and r = 3cm.

Volume of the cylinder = $\pi r^2 h = 3.14 \times 3^2 \times 70$

= $3.14 \times 9 \times 70 = 1978cm^3$

<u>Consider figure (3) or cone (2)</u>

l =5cm, h = 4cm and r = ?

From $l^2 = h^2 + r^2 \Rightarrow l^2 - h^2 = r^2$,

$\Rightarrow 5^2 - 4^2 = r^2 \Rightarrow 25 - 16 = r^2$,

$\Rightarrow 9 = r^2 \Rightarrow r = \sqrt{9} = 3$cm.

Volume of cone (2) = $\frac{1}{3} \times \pi r^2 \times h$

= $\frac{1}{3} \times 3.14 \times 3^2 \times 4$ = 38cm^3.

The volume of the given structure = volume of the cone (1) + volume of cylinder + volume of cone (2)

= 38 + 1978 + 38 = 2054cm^3

(b) If 50cm^3 = ¢2

=> 2054cm^3 = $\frac{2054}{50} \times 2$ = ¢82.

(Q9) A cylinder of height 60cm and of radius 4cm, has the same volume as a cone of radius 6cm. Find the height of the cone.

Soln.

<u>For the cylinder:</u>

height = 60cm and radius = 4cm.

Volume = $\pi r^2 h$ =$3.14 \times 4^2 \times 60$

= 3014 cm^3.

<u>For the cone:</u>

height = ? and radius = 6cm.

Volume = $\frac{1}{3}\pi r^2 h = \frac{1}{3} \times 3.14 \times 6^2 \times h$

= 38h cm^3 , where h = height of cone.

Since the volume of the cylinder = the volume of the cone, then 38h = 3014
$\Rightarrow \mathrm{h} = \frac{3014}{38} = 79\mathrm{cm}$.

$\therefore$ The height of the cone is 79cm.

(Q10) A hollow cone whose height is 20cm has a diameter of 8cm. It was filled with water which was later poured into a cylinder whose radius is 5cm. Determine the height of the water within the cylinder. [Take π = 3.14].

Soln.

<u>For the cone:</u>

height = 20cm and diameter = 8cm $\Rightarrow$ radius = 4cm.

The volume of water in the cone

= $\frac{1}{3}\pi r^2 h = \frac{1}{3} \times 3.14 \times 4^2 \times 20$ = 335cm^3.

Therefore volume of water poured from the cone into the cylinder = 335cm^3.

<u>For the cylinder:</u>

N/B: The height of the cylinder = the height of water within the cylinder in this case.

h =height =? and radius = 5cm

Volume = $\pi r^2 h = 3.14 \times 5^2 \times h$

= 78.5h,.

Therefore volume of water poured into the cylinder = 78.5h .

But since the volume of water poured into the cylinder = the volume of water poured from the cone,

$\Rightarrow$ 78.5 h = 335

$\Rightarrow$ h = $\frac{335}{78.5} = 4.3$,

$\Rightarrow$ the height of water within the cylinder = 4.3cm .

(Q11) A hollow cone has a slanted height of 5cm and a radius of 4cm. It was filled with kerosene which was later poured into a cylinder. If the height of kerosene within the cylinder was 10cm, what was the radius of the cylinder?

Soln.

<u>For the cone:</u>

Radius = 4cm, l = 5cm and h = ?

From $l^2 = h^2 + r^2 \Rightarrow l^2 - r^2 = h^2$,

$\Rightarrow h^2 = l^2 - r^2$, $\Rightarrow$ h = $\sqrt{l^2 - r^2} = \sqrt{5^2 - 4^2} = \sqrt{25 - 16}$ = $\sqrt{9}$ = 3 $\Rightarrow$ h = 3cm.

Volume of kerosene within the cone = volume of the cone

= $\frac{1}{3}\pi r^2 h = \frac{1}{3} \times 3.14 \times 4^2 \times 3$ = 50cm^3 $\Rightarrow$ Volume of kerosene poured from the cone = 50cm^3.

<u>For the cylinder:</u>

Height of kerosene = 10cm

$\Rightarrow$ h = 10cm and r =?

Volume of kerosene poured into the cylinder = $\pi r^2 h$

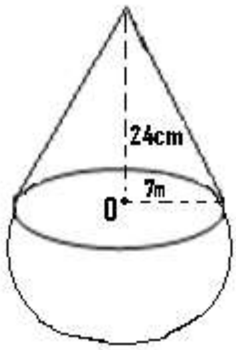
= 3.14 $\times r^2 \times 10 = 31.4r^2$ where r = radius of cylinder.

Since the volume of kerosene poured from the cone = the volume of kerosene poured into the cylinder, then

$31.4r^2 = 50 \Rightarrow r^2 = \frac{50}{31.4}$,

$\Rightarrow r^2 = 1.6 \Rightarrow r = \sqrt{1.6} = 1.26$

(Q12)



The diagram shows a hollow metallic structure which is in the form of a cone, mounted on a hemispherical base. The vertical height of the cone is 24m and the base radius is 7m. Calculate

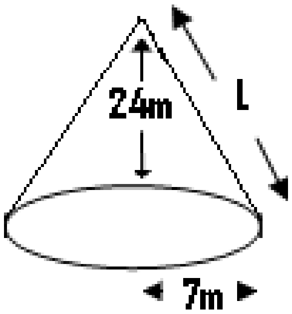
a) the area of the given structure.

b) the volume of air which the given structure contains.

N/B: - The area of the given structure = the area of the curved surface area of the cone + the area of the hemisphere.

- Also the volume of air within the given structure = the volume of the cone + the volume of the hemisphere.
- The radius of the cone and the hemisphere will be the same

(a) Consider the cone:



h = 24m , r = 7m and l = ? .

But l = $\sqrt{h^2 + r^2} \Rightarrow l = \sqrt{24^2 + 7^2}$,

$\Rightarrow l$ = $\sqrt{576 + 49} = \sqrt{625} = 25$.

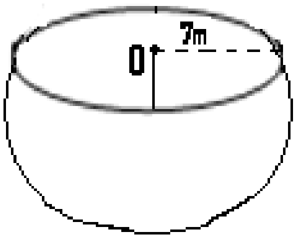
The curved surface area of the cone = $\pi r l$

= $3.14 \times 7 \times 25 = 550 m^2$.

Consider the hemisphere:

r = 7m and area = ? The area of the hemisphere = $2\pi r^2$

= 2 x 3.14 x 7^2 = 2 x 3.14 x 49= 308m^2.



The area of the given structure = curved surface area of the cone + the area of the hemisphere = 550 + 308 = 858m^2.

(b) The volume of the cone

= $\frac{1}{3}\pi r^2 h = \frac{1}{3} \times 3.14 \times 7^2 \times 24$

= $1231 m^3$.

The volume of the hemisphere = $\frac{2}{3}\pi r^3$

= $\frac{2}{3} \times 3.14 \times 7^3$

= $\frac{2}{3} \times 3.14 \times 343 = 718 m^3$.

The volume of the given structure = the volume of the cone + the volume of the hemisphere

= 1231 + 718 = 1949m^3.

N/B: The volume of a cuboid

= L $\times B \times H$,

Where L = the length, B = breadth and H = the height.

* The volume of a cuboid = A$\times H$, $where\ A = the\ area\ of\ one\ face\ of\ the\ cuboid$ and

H = the height.

* The area of one face of a cube = l^2, where l = the length of one of its sides.

*The volume of a cube = l^3, where l = the length of its side.

*The volume of a cube = A $\times$ l, where A = the face area of $one\ of\ its\ sides\ and\ l = the\ length\ of\ its\ side$.

(Q1) A water storage tank which is in the form of a cuboid has a length of 12m, a breadth of 10m and a height of 5m. This tank is filled with water. If the water in this tank is transferred into another tank which is cylindrical in shape and whose radius is 4m, what will be the height of the water within the cylinder?

Soln.

<u>Cuboid:</u>

l = 12m , b = 10m and h = 5m.

Volume = $l \times b \times h = \ 12 \times 10 \times 5$

$= 600m^3$.

There the volume of water within the cuboid = 600m^2.

<u>Cylinder:</u>

N/B: The height of the cylinder in this case = the height of water within the cylinder.

Since volume = $\pi r^2 h$,

$\Rightarrow$Volume = 3.14$\times$ $4^2 \times h$ (=50.2h), where h = the height of the cylinder.

$Since$ volume = 50.2h => the volume of water within the cylinder = 50.2h.

Since the volume of water within the cuboid = the volume of water within the cylinder, then 50.2h = 600

$\Rightarrow \mathrm{h} = \frac{600}{50.2} = 12$,

$\Rightarrow$ the height of water within the cylinder = 12m.

(Q2) A cuboid whose face surface area is 60cm^2, has a height of 15cm. If it was filled with oil, which was later transferred into a cone of radius 9cm, determine the height of oil within the cone.

Soln.

<u>Cuboid:</u>

Face surface area = A = 60cm^2.

Height = h = 15cm.

Volume = face surface area $\times$ $height$

= 60 $\times$ 15 = 900cm^3,

$\Rightarrow$the volume of oil within or transferred from the cuboid = 900cm^3.

<u>Cone:</u>

Volume = $\frac{1}{3}\pi r^2 h$, where in this case, h = the height of the cone or the height of the oil within the cone.

Volume = $\frac{1}{3} \times 3.14 \times 9^2 \times h$ = 85h.

Therefore the volume of oil within or transferred into the cone = 85hcm^3

Since the volume of oil within the cuboid = the volume of oil within the cone, then 85h = 900 $\Rightarrow$ h = $\frac{900}{85}$ $\Rightarrow$ h = 10.6.

$\therefore$ The height of the oil within the cone = 10.6m .

(Q3) A water tank which is in the shape of a cube of length 5m is filled with petrol, which was later pumped into a cylindrically shaped storage tank. If the height of petrol within this tank is 8m, determine the diameter of this tank.

Soln.

<u>Cube:</u>

Volume = $l^3 = 5^3 = 125m^3 \Rightarrow$ the volume of petrol within the tank whose shape is in the form of a cube $= 125\text{m}^3$.

<u>Cylinder:</u>

Volume = $\pi r^2 h = 3.14 \times r^2 \times 8$

$\Rightarrow$Volume = $25r^2$,

$\Rightarrow$ the volume of petrol within or pumped into the cylinder = $25r^2m^3$

But since the volume of petrol within the tank = the volume of petrol within the cylinder,. then $25r^2 = 125$,$\Rightarrow \text{r}^2 = \frac{125}{25}$,

$\Rightarrow \text{r}^2 = 5$,$\Rightarrow$ r = $\sqrt{5}$ = 2.2,$\Rightarrow$ the radius of the cylinder $= 2.2\text{m} \Rightarrow$ diameter $= 2 \times 2.2 = 4.4\text{m}$.

(Q4) The surface area of the face of a cuboid, whose height is 9cm is $240cm^2$. This cuboid was completely filled with water, which was later pumped into a storage tank, which is in the form of a cube. If the water tank was completely filled with the water, determine the length of the tank.

Soln.

<u>Cuboid:</u>

Face surface area = A =$240cm^2$,

Height = h = 9cm.

Volume = A$\times h = 240 \times 9$

= $2160cm^3$

$\Rightarrow$Volume of water within the cuboid $= 2160cm^3$.

Cube:

Let l = the length of the cube.

$\Rightarrow$ the volume of the cube = l^3 ,

$\Rightarrow$ the volume of water within or pumped into the cube $= l^3$.

Since the volume of water within the cube = the volume of water within the cuboid, then l^3= 2160

$\Rightarrow l = \sqrt[3]{2160}$ = 13

Therefore the length of the cube = 13cm.

(Q5) Water is stored in a tank which is in the form of a cube. The surface area of the cube is $25m^2$. If the water from this full tank is transferred into another storage tank, which is in the form of a cone and the height of the water within the cone is 20m, determine the radius of the cone.

Soln.

Cube:

The surface area of one face = $25m^2 \Rightarrow l^2 = 25m$,

$\Rightarrow l = \sqrt{25}$ = 5.

$\therefore$ The length of the cube = 5m.

The volume of the cube = $l^3 = 5^3 = 125m^3$.

Therefore the volume of water within or transferred from the tank, whose shape is in the form of a cube = $125m^3$.

Cone:

Volume = $\frac{1}{3}\pi r^2 h = \frac{1}{3} \times 3.14 \times r^2 \times 20$,

$\Rightarrow$ Volume = $2Ir^2$, where r = the radius of the cone.

$\therefore$Volume of water within or transferred into the cone = $21r^2$.

But since the volume of water within the cube = volume of water within the cone, then $21r^2 = 125 \Rightarrow r^2 = \frac{125}{21}$,

$\Rightarrow r^2 = 6 \Rightarrow \mathrm{r} = 3.45\mathrm{m}$.

(Q6) An oil tank takes the form of a cube of side 12m. It is to be filled with oil, which is to be transferred into another storage tank, which takes the form of a cylinder of radius 6m. Determine the expected level of oil within this tank.

Soln.

<u>Cube:</u>

Since volume = $l^3 = 12^3 = 1728m^3$,

then volume of oil within or transferred from the cube like tank $= 1728\mathrm{m}^2$.

<u>Cylinder:</u>

N/B: The height of the cylinder = the height or level of oil within the cylinder.

Volume = $\pi r^2 h = 3.14 \times 6^2 \times h$

= 113h, where h = the level or height of oil within the cylinder.

Thereforevolume of oil within or transferred into the cylinder $= 113\mathrm{h}$.

But volume of oil within the cylinder = volume of oil within the cube,

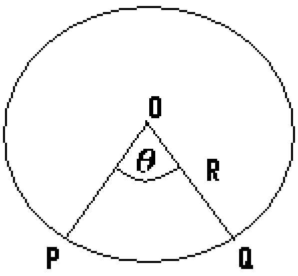
$\Rightarrow$ 113h = 1728,

$\Rightarrow$ h = $\frac{1728}{113} = 15$,

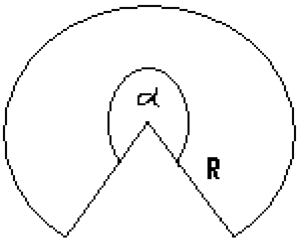
$\Rightarrow$the level of water within the cylinder $= 15\mathrm{m}$.

<u>The formation of cones from the sectors of circles:</u>

- Cones can be formed from a circular sheet of an appropriate material, after the removal of a sector from it.

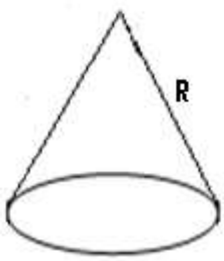


- Above shows a circular material whose center is 0, from which a sector of angle θ is about to be removed.
- After the removal of this sector, the net formed which can be folded to form the cone is shown next:



- If the above is folded to form a cone then the following facts must be noted:

(a) The radius R of the net will become the slanted height of the cone.



This implies that R = l, where l = the slanted height.

(b) The angle represented by ∝ = 360 – θ, where

θ is the angle of the removed sector.

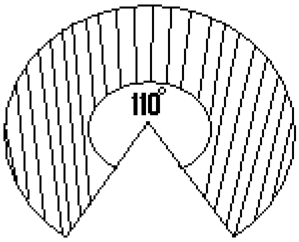
(c) The cone formed is the hollow type and for this reason, its total surface area is given by $\pi r l$.

(d) $\frac{r}{l} = \frac{\theta}{360}$.

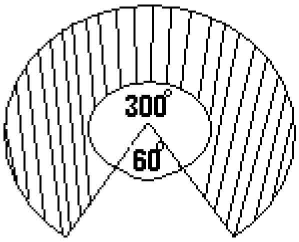
where r = the radius of the cone formed and θ is the angle of the sector used in forming the cone, i.e. the sector angle of the net used to form the cone.

- This fact must be well understood to avoid confusion.

- In the first case for example, if we are told that a sector of angle 110^0 is removed from a circular paper and used in forming the cone, then in this case θ = 110^0, since that is the sector angle of the net used in forming the cone, as shown in the next diagram:

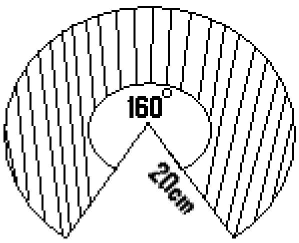


- In the second case, a sector of a certain angle, let`s say 60^0 is removed from the circular sheet and the resultant net folded into the cone.
- In this case, the sector angle of the net = 360 – 60 = 300^0, as shown next in a diagram

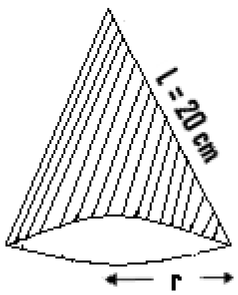


(Q1) A sector of angle 160^0 was removed form a circular paper of radius 20cm, and then folded into a cone. Determine the radius of the cone formed.

Soln.



$\theta = 160^0$, R = l = 20cm



If r = the radius of the cone formed, then

$\frac{r}{R} = \frac{\theta}{360} \Rightarrow \frac{r}{20} = \frac{160^0}{360^0} \Rightarrow 360^0 \times r = 20 \times 160^0,$

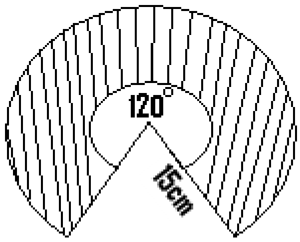
$\Rightarrow 360\,r = 3200 \Rightarrow r = \frac{3200}{360} \Rightarrow r = 8.9\text{cm}$

(Q2) A sector of angle 120^0 was removed from a circular paper of radius 15 cm, and then folded into a cone.

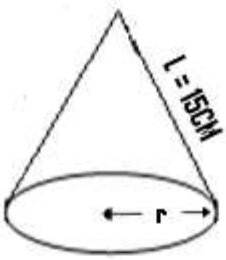
(a) Determine the radius of the cone formed.

(b) What is the surface area of this cone.

Soln.



$\theta = 120^0$ and $R = l = 15cm.$



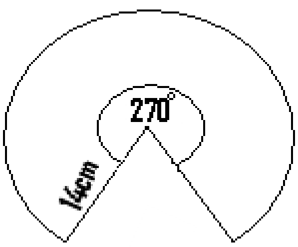
(a) Let r = the radius of the cone formed. From

$\frac{r}{R} = \frac{\theta}{360} \Rightarrow \frac{\mathrm{r}}{15} = \frac{120}{360}$ $, \Rightarrow 360\mathrm{r} = 15 \times 120, \Rightarrow 360\mathrm{r} = 1800, \Rightarrow \mathrm{r} = \frac{1800}{360} = 5$ $, \Rightarrow \text{radius} = 5\text{cm}.$

(b) The surface area = πrl

= $3.14 \times 5 \times 15 = 1236cm^2$.

(Q3)



The diagram shows a sector of a circle whose radius is 14cm. The angle at the centre is 270^0. If this sector is folded to form a cone, find

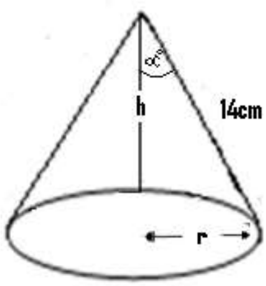
I) the radius of the cone.

Ii) the surface area of the cone.

Iii) the semi vertical angle of this cone.

Soln.

I) $\theta = 270^0$ *and since* $R = 14cm \Rightarrow l = 14cm$.



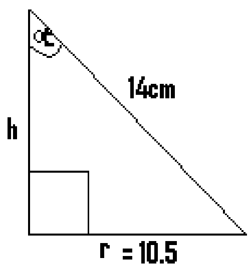
From
$\frac{\theta}{360} = \frac{r}{l} \Rightarrow \frac{270}{360} = \frac{\mathrm{r}}{14}, \Rightarrow 360 \times r = 14 \times 270, \Rightarrow 360\mathrm{r} = 3780 \Rightarrow \mathrm{r} = \frac{3780}{360} \Rightarrow \mathrm{r} = 10.5$

iI) The surface area of the cone = $\pi \mathrm{r} l$.

= $3.14 \times 10.5 \times 14 = 462cm^2$.

iii) Consider half of the cone drawn

i.e.

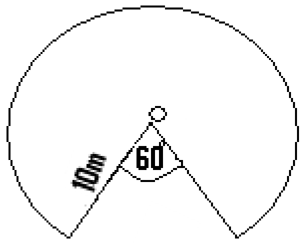


If ∝ = the semi vertical angle, then sin $\propto = \frac{10.5}{14} = 0.75$.

Since $\sin \propto = 0.75, then \ \propto = sin^{-1}0.75$

$\Rightarrow \alpha = 49^0$.

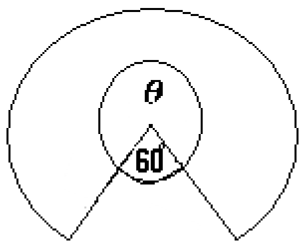
(Q4)



The diagram shows a major sector of a circle whose centre is 0 and whose radius is 10m. If this sector is folded to form a cone, find

(i) the radius of the cone.
(ii) the area of the cone.
(iii) the semi vertical angle of the cone formed.
(iv) the greatest amount of water which the cone can contain.

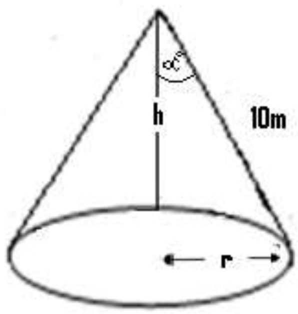
N/B:



If θ = the sector angle of the net folded into the cone, then θ = 360 – 60 = 300^0.

Soln.

$\theta = 300^0$ and R = 10m $\Rightarrow l$ =10m



(i) From

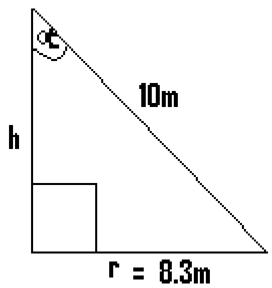
$\frac{r}{R} = \frac{\theta}{360} \Rightarrow \frac{\mathrm{r}}{10} = \frac{300}{360} \Rightarrow 360 \times \mathrm{r} = 10 \times 300, \Rightarrow 360\mathrm{r} = 3000, \Rightarrow \mathrm{r} = \frac{3000}{360} \Rightarrow \mathrm{r} = 8.3\mathrm{m}$

(ii) The area of the cone = $\pi r l$

= $3.14 \times 8.3 \times 10 = 261 m^2$

(iii) Considering half of the cone

i.e



Let

$\propto^0 = \textit{the semi vertical angle}. \sin \propto = \frac{8.3}{10} \Rightarrow \sin \propto = 0.83,$

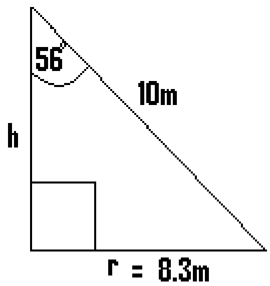
$\Rightarrow \propto = \sin^{-1} 0.83 \Rightarrow \propto = 56^0.$

(iv) The greatest amount of water which the cone can hold = its volume

=

$\frac{1}{3}\pi r^2 h, where\ h =$
$the\ hight\ of\ the\ cone. We\ must\ therefore\ find\ h.$

Since $\propto = 56 \Rightarrow$ the last diagram becomes as shown next:



$\tan 56^0 = \frac{8.3}{h}$

$$\Rightarrow 1.5 = \frac{8.3}{h} \Rightarrow 1.5h = 8.3$$

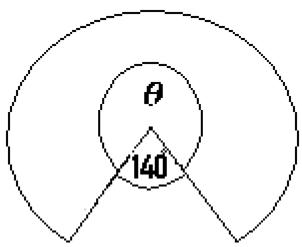
$$\Rightarrow h = \frac{8.3}{1.5} = 5.5m$$

The maximum volume of water the cone can hold =
$\frac{1}{3}\pi r^2 h = \frac{1}{3} \times 3.14 \times 8.3^2 \times 5.5 = 937m^3$.

(Q5) A circular sheet of aluminum plate was taken, and a sector of angle 140^0 and radius of 12cm was removed from it. The remaining portion was then folded with the straight edges coinciding to form a cone. Determine

(i) the quantity of aluminum used in making the cone.
(ii) the vertical angle formed by the cone.
(iii) the perpendicular height of the cone.
(iv) the volume of air within the cone.
[Take π =3.14]

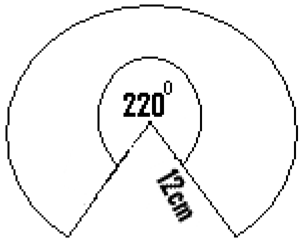
N/B:



If $\theta = the\ sector\ angle\ of\ the\ sector\ used\ in$ forming the cone (i.e. the net), then θ= 360 -140 = 220^0

Soln.

(i) The quantity of aluminum used = the total surface area of the cone = $\pi r l$. We must therefore find r.



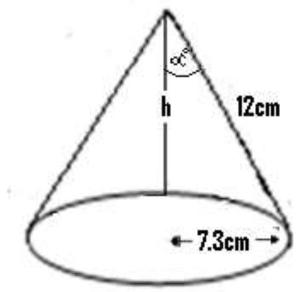
$\theta = 220^0\ and\ R = l = 12cm. From\ \frac{r}{l} = \frac{\theta}{360} \Rightarrow \frac{r}{12} = \frac{220}{360}$

$\Rightarrow 360 \times r = 12 \times 220, \Rightarrow 360r = 2640, \Rightarrow r = \frac{2640}{360}, \Rightarrow r = 7.3cm$.

The quantity of aluminum used = $\pi r l$

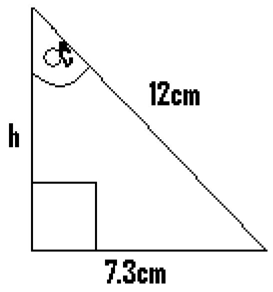
= $3.14 \times 7.3 \times 12 = 275 cm^2$.

(ii)



Consider the half portion of the cone.

i.e.



First determine the value of the semi vertical angle formed

i.e $\propto$:

$\sin \propto = \frac{7.3}{12} = 0.6, \Rightarrow \propto = sin^{-1}0.6,$

$\Rightarrow \propto = 37^0, \Rightarrow$ the vertical angle formed $= 2 \times 37 = 74^0$, since the vertical angle is twice the semi vertical
angle.

$\tan \propto = \frac{7.3}{h} \Rightarrow \tan 37^0 = \frac{7.3}{h} \Rightarrow 0.75 = \frac{7.3}{h} \Rightarrow 0.75\text{h} = 7.3, \Rightarrow \text{h} = \frac{7.3}{0.75} = 9.7\text{cm}$

(iii) The volume of air within the cone = the volume of the cone = $\frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times 3.14 \times 7.3^2 \times 9.7$

= 541cm^3.

N/B: - The vertical angle = twice the semi vertical angle.

- If $\theta =$ the angle of th sector which was folded into the cone, and r is the radius of this sector, i.e. the net, then the area of the sector = $\frac{\theta}{360} \times \pi r^2$.

N/B: r can also be referred to as the radius of the circle plate or board and so on.

(Q1) A sector of angle 150^0, was removed from a thin circular cardboard sheet of radius 20cm. Determine the area of this sector.

Soln

$\theta = 150^0 \; and \; r = 20cm.$

The area of the sector = $\frac{\theta}{360} \times \pi r^2$

$= \frac{150}{360} \times 3.14 \times 20^2$

= 523cm^2

N/B: The formula $\frac{\theta}{360} \times \pi r^2$ can also be written as $\frac{\theta}{360} \times \pi R^2$, where R = the radius.

(Q2) A sector of angle 120^0 was removed from a circular paper, whose diameter was 14cm. Find the area of this sector.

Soln.

d = 14cm $\Rightarrow$ $\mathrm{r} = 7\mathrm{cm}$, and $\theta = 120^0$.

Sector area = $\frac{\theta}{360} \times \pi r^2$

= $\frac{120}{360} \times 3.14 \times 7^2 = 51cm^2$.

(Q3) A sector of area 90cm^2 was removed from a circular plate of radius 8cm. Determine the sector angle.

Soln.

Let θ = the sector angle and r = the radius of the plate.

Since sector area =
$\frac{\theta}{360} \times \pi r^2 \Rightarrow 90 = \frac{\theta}{360} \times 3.14 \times 8^2, \Rightarrow 90 = \frac{\theta}{360} \times 3.14 \times 64, \Rightarrow 90 \times 360 = \theta \times 3.14 \times 64,$

=> 32400 = 200θ $\Rightarrow \theta = \frac{32400}{200} = 162,$

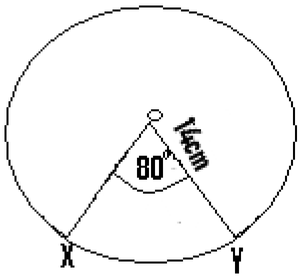
$\Rightarrow \text{the sector angle} = 162^0$.

N/B: - (i) The length of an arc of a sector = $\frac{\theta}{360} \times 2\pi$R ,

Where $\theta = the\ sector\ angle, and\ R = the\ radius\ of\ the$ circle or the circular sheet. (ii) The length of the arc = the circumference of the base of the cone = 2πr.

(iii)The formula $\frac{\theta}{360} \times 2\pi$R can also be written as $\frac{\theta}{360} \times 2\pi$r, where r = the radius of the circle or the circular (sheet, plate or board and so on).

(Q4)



Find the length of arc xy

Soln.

R = 14cm and $\theta = 80^0$.

Length of arc xy = $\frac{\theta}{360} \times 2\pi R$

= $\frac{80}{360} \times 2 \times 3.14 \times 14$

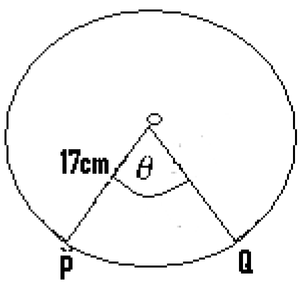
= 19. 5cm

(Q5) A sector of area 427cm^2, is cut out from a thin circular metal sheet of radius 17cm. It is then folded with the straight edges coinciding to form a cone. Calculate

(a) the angle of the sector.
(b) the length of the arc of the sector.
(c) the height of the right circular cone.
(d) the volume of the cone.

Soln.

(a)



R = 17cm and sector area = 427cm^2

Let θ = the sector angle and P Q = the length of the arc.

The area of the sector OPQ = $\frac{\theta}{360} \times \pi R^2 \Rightarrow 427 = \frac{\theta}{360} \times 3.14 \times 17^2, \Rightarrow 427 = \frac{907\theta}{360}$

$\Rightarrow 427 \times 360 = 907\theta$,

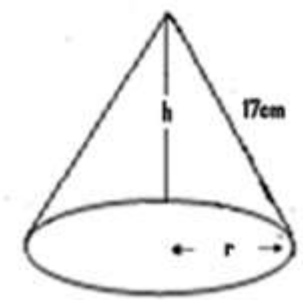
$\Rightarrow 907\theta = 427 \times 360$,

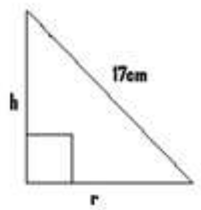
$\Rightarrow \theta = \frac{427 \times 360}{907} = 169^0$.

b)The length of arc PQ = $\frac{\theta}{360} \times 2\pi R$

= $\frac{169}{360} \times 2 \times 3.14 \times 17$

= 50cm.





Since the length of the arc PQ = 2πr, then 2πr = 50,

$\Rightarrow 2 \times 3.14 \times r = 50 \Rightarrow 6.28r = 50, \Rightarrow r = \frac{50}{6.28} = 9.$

Now from Pythagoras theorem,

$17^2 = r^2 + h^2 \Rightarrow 17^2 = 9^2 + h^2, \Rightarrow 17^2 - 9^2 = h^2, \Rightarrow h = \sqrt{17^2 - 9^2}$

=15cm.

(d) The volume = $\frac{1}{3}\pi r^2 h = \frac{1}{3} \times 3.14 \times 9^2 \times 15 = 1003cm^3$

(Q7) Mr. Hansen took a circular copper plate of diameter 12m, and cut out from it a sector whose angle was 240^0. He then had this bent in order to form a cone. Determine

I) the radius of the circular base of the cone.

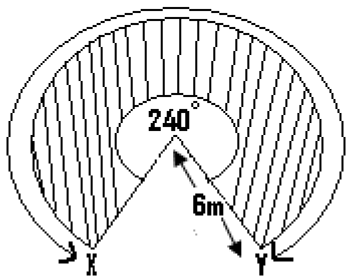
Ii) the height of the cone.

Iii) the vertical angle formed by the cone.

Iv) the amount of kerosene which the cone can contain.

I) the quantity of copper which was used to form this cone.[Take π = 3.14]

Soln.



Since the diameter = 12m $\Rightarrow$ the radius = 6m .

R = l= 6m.

Also θ= 240^0.

When the net of the cone shown is bent into a cone, the length of arc X Y will become the circumference of the cone formed.

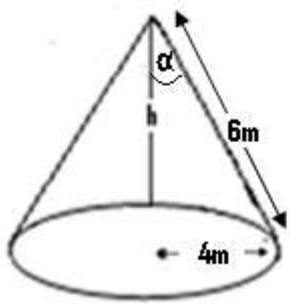
$\Rightarrow \frac{\theta}{360} \times 2\pi R = 2\pi r$

i.e. length of arc = the circumference where r = the base radius of the cone.

Therefore $\frac{240^0}{360} \times 2 \times 3.14 \times 6$

=2 $\times 3.14 \times r, \Rightarrow 25 = 6.28r \Rightarrow r = \frac{25}{6.28} \Rightarrow$ $r = 4, \Rightarrow$ the radius of the cone $= 4$m.

(ii)

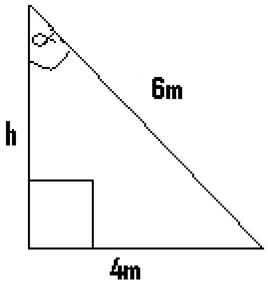


From this figure, r = 4m, l = 6m and h = ?, But since

$l^2 = h^2 + r^2 \Rightarrow l^2 - r^2 = h^2, \Rightarrow \mathrm{h} = \sqrt{l^2 - r^2} \Rightarrow \mathrm{h} = \sqrt{6^2 - 4^2} = \sqrt{36 - 16} = \sqrt{20},$
$\Rightarrow \mathrm{h} = 4.5\mathrm{m}$

(iii) Consider one half of the cone

i.e.



If ∝= the semi vertical angle, then sin ∝ = $\frac{4}{6}$,

$\Rightarrow \sin \propto = 0.66,$

$\Rightarrow \propto = sin^{-1}0.66 \Rightarrow \propto = 42^0.$

The vertical angle $= 2 \times 42 = 84^0$.

(iv) The quantity of kerosene which the cone can hold = the volume = $\frac{1}{3}\pi r^2 h$ = $\frac{1}{3} \times 3.14 \times 4^2 \times 4.5 = 75m^3$.

(v) The quantity of copper needed to form this cone = the total surface area = πrl = 3.14$\times 4 \times 6 = 75m^2$

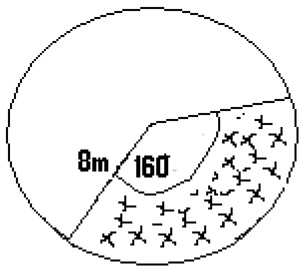
(Q8) A piece of paper is in the form of a circle of radius 8m. If a section of this circular piece of paper, which subtends an angle of 160^0 at the centre is cut and folded to form a cone, determine

(a) the base radius of the cone.
(b) its perpendicular height.
(c) the cost of painting the entire outer surface area of the cone, If it cost ₵3 to paint an area of $2m^2$.

[Take π = $\frac{22}{7}$].

Soln.

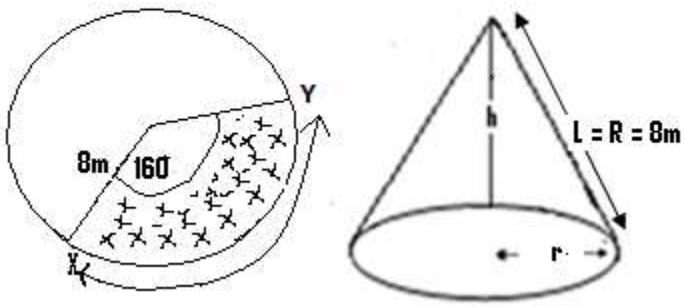
(a)



The shaded part forms the net which was folded to form the cone, ⇒R= 8m and $\theta = 160^0$, where R= the radius of the circular piece of paper and $\theta = the\ sector$ angle.

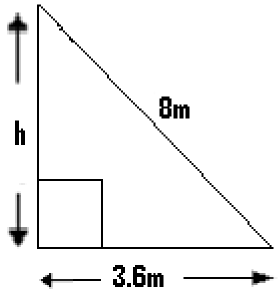
Now let XY = the length of the sector arc, r = the radius of the cone and h = the height of the cone.

i.e.



Since the length of the sector arc i.e. arc XY = the circumference of the circular base of the cone, $\Rightarrow \frac{\theta}{360} \times 2\pi R = 2\pi r$, $\Rightarrow \frac{160}{360} \times 2 \times \frac{22}{7} \times 8 = 2 \times 3.14 \times r$, $\Rightarrow 22 = 6.28r \Rightarrow r = \frac{22}{6.28} \Rightarrow r = 3.6m.$

(b) Consider one half of the cone i.e.



From this diagram $l = 8$m, r = 3.6 and h = ?

But since $l^2 = h^2 + r^2, \Rightarrow l^2 - r^2 = h^2, \Rightarrow \mathrm{h} = \sqrt{l^2 - r^2}$,

$\Rightarrow \mathrm{h} = \sqrt{8^2 - 3.6^2} \Rightarrow \mathrm{h} = \sqrt{64 - 13}$ = 7.

(c) The total surface area of the cone which must be painted = $\pi r l = \frac{22}{7} \times 3.6 \times 8$ = $91m^2$

Since $2m^2 = ¢3$

$\Rightarrow \ 91m^2 = \frac{91}{2} \times 3 = \frac{273}{2} = ¢137$.

Summary of formulae:

(a) **Cylinder:**

(1) Curved surface area = 2πrh.

(2) Circular surface area = πr^2.

(3) Total surface area = 2πr(r + h).

(4) Volume = πr^{2}h.

(5) Volume = circular surface area x height.

N/B:[r = radius and h = height].

(b) **Sphere**:

(1) Surface area = 4πr^2.

(2) Volume = $\frac{4}{3}\pi$r^3.

(c) **Hemisphere**:

(1) Area = 2πr^2

(2) Volume = $\frac{2}{3}\pi$r^3

(d) **Cone**:

(1) Base circular surface area = πr^2.

(2) Curved surface area = πrl.

(3) Surface area of a hollow cone = πrl

(4) Surface area of a solid cone (cone) = πr(r + l).

(5) Volume = $\frac{1}{3}\pi r^2h$

(6) Also volume = $\frac{1}{3}$ x base area x height.

N/B:[r = radius, h = height and l = slanted height].

e) Cuboid:

(1) Volume = surface area x height

(2) Volume == L x B x H where L = length, B = breadth and H = height.

(f)Cube:

Area l^2 and volume = l^3, where l = length or breadth or side.

N/B: (1) $\frac{r}{l} = \frac{\theta}{360}$, where r = the radius of the cone formed, $and\ \theta$ = the sector angle of the net used to form the cone.

(2)The sector area = $\frac{\theta}{360}$ x πr^2, where θ = sector angle and r = the radius of the circular plate or cardboard e.t.c.

(3)The length of a sector arc = $\frac{\theta}{360}$ x $2\pi R$, where θ = the sector angle angle, and R = the radius of the circle or the circular sheet.

(4)The circumference of a cone = $2\pi r$, where r = the base radius of the cone.

Questions:

(Q1) A cylinder has a height of 8cm and a radius of 3cm. Find its

(a) bottom circular surface area. Ans: $28.3cm^2$

(b) curved surface area. Ans: $151cm^2$

(c) total surface area. Ans: $207cm^2$

(d) volume. Ans: $226cm^3$

[Take π =3.142].

(Q2) A cylindrically shaped water tank is to be constructed using zinc. It is to be of height 40m, and and of diameter 16m. Determine the amount of zinc that will be needed to construct

(a) the curved surface portion. Ans: 2010m^2
(b) the whole tank. Ans: 2412m^2
(c) Given that 5m^2 of zinc cost ₵3, determine the total amount expected to be used to construct the whole tank. Ans: ₵1447.
(d) What will be the amount of water the tank will contain when it is half full? Ans: 4019m^3
[Take π=3.14]

(Q3) Given that the curved surface area of a cylinder whose radius is 6cm is 720cm^2, find its

(a) total surface area. Ans: 943cm^2
(b) volume. Ans: 2149cm^3
[Take $\pi = 3.14$].

(Q4) A cylinder is 60m long and contains 8400m^3 of water when it is full. Determine

(a) its radius. Ans: 6.7m
(b) its diameter. Ans 13.4m

[Take π=3.142]

(Q5) A petrol storage tank, built in the form of a cylinder has a height of 0.91m and a top surface area of 16cm^2. Calculate

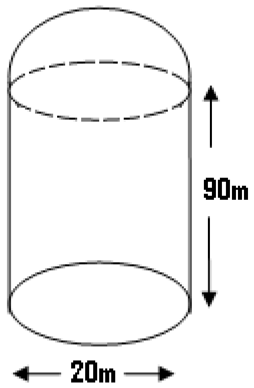
(a) its curved surface area. Ans: 1274cm^2
(b) its total surface area. Ans: 1306cm^2
(c) its volume. Ans: 1420cm^3
(d) If this tank is to be filled with petrol, which is pumped at a rate of 4cm^3 per second, how long will it take to fill the tank. .[Take π=3.14]. Ans 355 seconds.

(Q6) A gas cylinder of radius 18cm has a total surface area of 11480cm^2. Determine the quantity of gas it will contain when it is fully filled.[Take π =3.14]. Ans: 85458cm^3.

(Q7) The area of one of the circular faces of a cylinder is 178cm^2. If it is of height 120cm, determine its volume.

Ans: 21360cm^3.

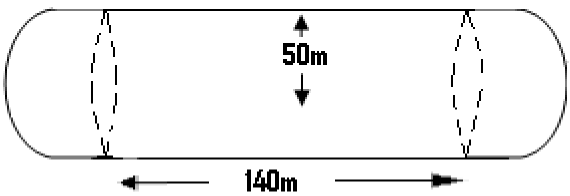
(Q8)



The above structure is to be constructed using aluminum. It consists of a cylinder of height 90m and diameter 20m, joined to a hemisphere as shown in the given diagram.

(a) What will be the total surface area of the given structure? Ans: 6594m^2
(b) Determine the total volume of air which the structure will contain. Ans: 30353m^3
[Take π = 3.14].

(Q9)



The above structure shows a cylinder of radius 50m and height 140m, whose both ends are joined to hemispheres. Determine

(a) the total surface area of the given structure
Ans: 75360m^2
(b) the maximum amount of water which can be stored in the tank.
. Ans: 1622333m^3

(Q10) A cone has a diameter of 8cm and a height of 12cm. Determine its slanted height. Ans: 12.6cm

(Q11) Determine the radius of a cone whose slanted height is 9cm, and whose height is 6cm. [Take π 3.14]. Ans: 6.7cm

(Q12) Find the total surface area of a hollow cone, whose height is 8cm, if it has a radius of 5cm. Ans: 148cm^2

(Q13) The total surface area of a hollow cone of radius 6cm is 264cm^2. Determine

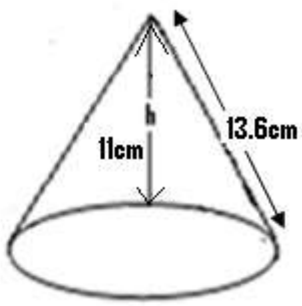
(a) its slanted height. Ans : 14cm
(b) its height. Ans: 12.6cm

(Q14) Determine the total surface area of a solid cone whose slanted height is 12m, if it has a diameter of 16m. Ans: 502m^2

(Q15) The curved surface area of a cone whose radius is 5cm is 620cm^2. Find

(a) its slanted height. Ans: 39cm
(b) its total surface area. Ans: 699cm^2

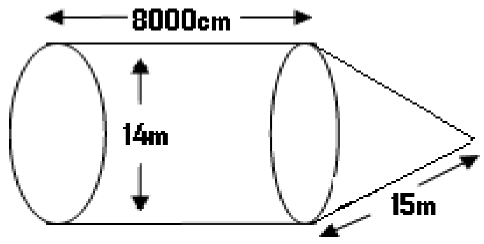
(Q16)



The given figure is that of a cone to be constructed. It is supposed to have a height of 11cm and a slanted height of 13.6cm. Determine

(a) its curved surface area. Ans: 342cm^2.
(b) its total surface area. Ans: 543cm^2.
(c) its volume. Ans: 737cm^3.

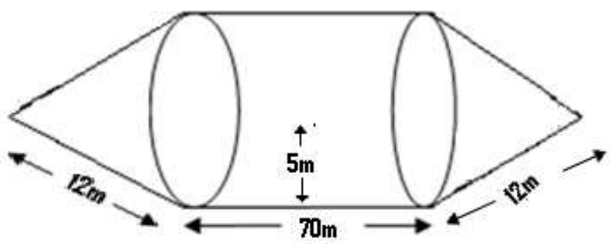
(Q17)



The structure shown is that of a fuel storage tank. It consists of a cylinder, which is attached to a hollow cone. The diameter of the cylinder is 14m and its length is 8000cm. If the cone has a slanted height of 15m, determine

(a) the amount of aluminum which will be needed to construct this structure.
    Ans: $4000m^2$
(b) the maximum volume of water the structure can hold. .
    Ans: $12991m^3$.

(Q18)



The given structure is to be painted and then filled with kerosene.

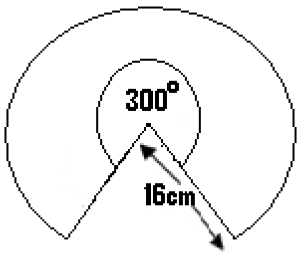
(a) If an area of $6m^2$ requires a gallon of paint, determine the quantity of paint which will be needed to paint the outer surface of the whole structure.
    Ans:434 gallons.
(b) If kerosene is pumped into the structure at a rate of $10m^3$ per second, how long will it take to fill the given structure, given that its mid portion has a cylindrical shape, whilst its two ends are in the form of cones. [Take π=3.142].
    Ans: 612 seconds.

(Q19) A sector of angle 145^0 is to be removed from a circular copper plate of diameter 26cm, and then bent into a cone.

(a) What will be the radius of this cone?
    Ans: 5.2cm

(b) Find its surface area. / . .
Ans: 212.4cm^2.

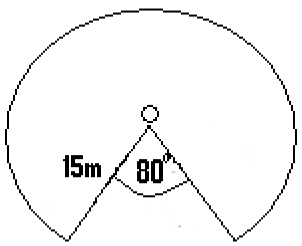
(Q20)



The diagram shows a sector whose radius is16cm, and whose sector angle is 300^0. If this is folded to form a cone, determine

(a) the radius of the cone formed. Ans: 13cm.
(b) What is the surface area of the cone. Ans: 653cm^2
(c) Determine the sami vertical angle. Ans: 54^0.

(Q21)



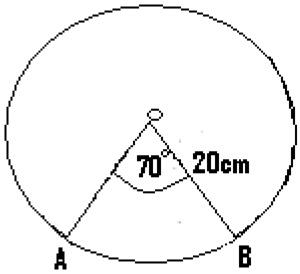
The diagram shows a major sector of a circle whose centre is O and whose radius is 15m. If this sector is folded to form a cone, find

(i) the radius of the cone. Ans: 11.7m
(ii) the area of the cone. Ans: 551m^2
(iii) the vertical angle of the cone formed. Ans: 102^0
(iv) the maximum quantity of water the cone can hold. . .
. Ans: 1333m^3

(Q22) A sector whose area is $120cm^2$ was removed from a circular plate of radius 9cm. What is the sector angle? Ans: 170^0

.

(Q23)



Determine the length of arc AB. Ans: 24cm

(Q24) A sector of area $520cm^2$ is cut out from a thin circular cardboard sheet of radius 19cm. It is then folded to form a cone. Calculate

(a) the sector angle. Ans: $165.^0$
(b) the length of the sector arc. Ans: 55cm
(c) the height of the cone. Ans: 16.8cm
(d) the volume of the cone. Ans: $1331cm^3$.