CHAPTER ONE

FORCE, WEIGHT AND THE CENTRE OF GRAVITY

Force:

This is the agent which changes a body`s state of rest or uniform motion in a straight line. It generally denotes a push or a pull. It can increase the speed of a moving object in the direction of the force, or change the direction of such an object in the direction of the force. It can also be applied to stop a moving body. Force is measured in Newtons which is written as N, and a Newton is the force which acts on a unit mass of 1kg, causing it to move a unit distance of 1 metre.

The centripetal force:

This is the force which is needed by a body, in order to be able to move in a circular path, and which is directed towards the centre of the circular path. This force can be demonstrated by tying a suitable mass at the end of a string and swinging it round. The pull in the string which is providing the centripetal force can be felt, and it will be noticed that it varies according to the mass, speed and the radius of the circular path.

The total gravitational force gravitational force:

- This is divided into two and these are:
  - (i) The force of gravity.
  - (ii) The centripetal force.

The force of gravity:

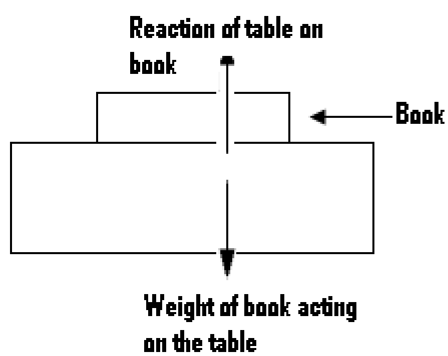
This is the part of the total gravitational force, which attracts an object towards the centre of the earth, and so enables such an object in turn to exert a force on its support. This force exerted on its support, is known as the weight of the object. The weight of a body is therefore the force, which it exerts on anything which freely supports it.

The centripetal force:

In its simplified form, this is the part of the total gravitational force, which is needed to enable a body to move in a circular path. The sum of the force of gravity and the centripetal force is equal to the total gravitational force.

Action and reaction forces:

Sir Isaac Newton pointed out that whenever a force acts on a body, there must be a reaction acting on the same body. In short, action and reaction forces are equal and opposite. For example, consider a book which lies on a table. If it presses on the table with a force which is equal to its weight, then the table will exert an equal upward reaction force on the book.



Our weight and the total gravitational force:

- The total gravitational force acts on everybody or object found on the surface of the earth.
- Since our earth is in a circular motion, then every object or person moves in a circular motion or path, and for this reason, centripetal force is needed.
- Part of the total gravitational force is therefore used to provide this centripetal force.
- The remaining part of the total gravitational force, called the force of gravity is what attracts us towards the centre of the earth, and so enables us to exert a force on our support i.e. enables us to have weight.- It must also be noted that the greater the speed of the circular motion, the greater will be the amount of centripetal force needed, and vice versa.

Application of centripetal force:

- One of the applications of centripetal force is in the separation of suspended particles from a liquid, in which they are suspended by means of a centrifuge.
- The centrifuge normally consists of two tubes which lie horizontal, when they are set in rotation.
- If the suspended particles in the tube are less dense than the surrounding liquid, then the force which acts on them due to the liquid (i.e by the liquid), will be much greater than the centripetal force they require to maintain them in their circular paths.
- They are therefore urged towards the surface of the liquid.
- On the other hand if these particles are denser than the surrounding liquid, then the force which acts on them due to the surrounding liquid will not be enough to maintain them in their respective circular paths, and for this reason, they are urged or moved towards the bottom of the liquid.
- Cream is manufactured in a similar manner, in which the suspended fatty protein particles being less dense, collect together at the centre of a container, if the container is rotated on a turn table.

Weightlessness:

- Assuming we stand on a weighing machine and that by some means the earth`s rotation can be speeded up, then with increasing speed, more and more of the earth`s total gravitational force would be used, to provide the extra centripetal force needed in such a circumstance.
- Our weight (which is equal to the difference between the total gravitational force and the centripetal force), will therefore become less.
- Consequently, a weighing machine will indicate a smaller weight.
- If the earth`s rotational speed continues to increase, then at a certain critical speed, the needed centripetal force would just be equal to the total gravitational force.
- There will be no resultant force left over to provide the force of gravity needed to cause us to have weight.
-The weighing machine will therefore read zero and as such we have become weightless, even though the full gravitational force still continues to act on us.
- By becoming weightless, we say that we are experiencing weightlessness.

- It is a well known fact that astronauts experience weightlessness, when their spacecraft are in orbit above the earth.
- This occurs when all the total gravitational force has been converted into the necessary centripetal force required for their particular mass, speed and orbital radius.
- Weightlessness in space vehicle is highly inconvenient to an astronaut in many ways, for he cannot pour liquid into a cup and neither can he drink from it.
- Also, his movement is controlled and made possible by the use of hand – rails and so on.

Differences between weight and mass:

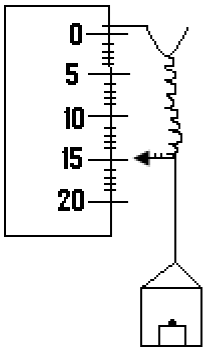
(1) The weight of a body is the force, which it exerts on anything which freely supports it. But the mass of a body is the amount of matter or material within the body.
(2) Weight is measured in Kgf or gf but mass is measured in Kg or g.
(3) Mass is a scalar quantity but weight is a vector quantity.
(4) The mass of a body remains constant throughout the whole universe, but the weight of a body varies from place to place.

Reason why the weight of a body varies from place to place:

- The earth is not a perfect sphere but bulges at the equator, which implies that it is slightly flattened at the equator.
- A body at the poles will therefore be closer to the centre of the earth, where the force of gravity is acting from, than a body placed at the equator.
- The force of gravity which acts on a body placed at the poles will therefore be greater and as such, the body will have more weight.
- If the same body is brought to the equator, the force of gravity which acts on it will be less and as such give it a lesser weight.

The calibration of a spring balance:

- The deformation or the extension of a spring balance is the increase in its length.
-The elasticity of a material (such as a spring), is its ability to regain its shape and size after deformation.
- Provided the elastic limit is not reached, the deformation of the spring can take place.

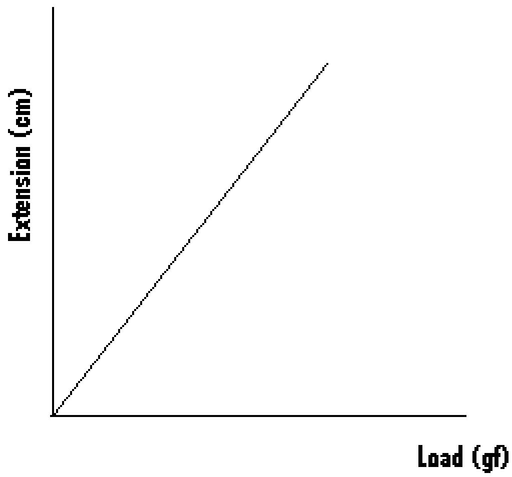


- Forces or weights are often measured by means of a spring balance, and this principle was first investigated by Robert Hooke.
- He showed that when a spring is fixed at one end and a force is applied to the other end, the extension of the spring is proportional to the applied force, provided the elastic limit is not reached (i.e. the force is not large enough to stretch the spring permanently).
- Hookes law states that, provided the elastic limit is not exceeded, the extension of a material is proportional to the applied force.
- To varify Hooke`s law experimentally, a spiral spring with a scale pan and a pointer attached is held vertically by a clamp and a stand.
- Equal weights are then added to the pan, say 10gf at a time, and the corresponding extensions of the spring are calculated from the readings of the pointer on a millimeter scale.

- The results show that for each reading taken, $\frac{\text{extension in cm}}{force\ in\ gf} = constant$,

=> the extension $\propto$ the applied force.

- This relationship may also be verified by plotting a graph of extension against force, which gives rise to a straight line through the origin, which is an example of a calibration graph for a spring.



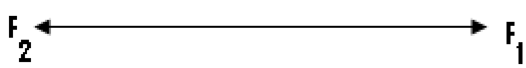
Sensitivity of a spring balance:

If two similar spring balances are connected together, to make a spring twice its length, then the extension for a given load will be twice that provided in a single spring. Therefore the longer the spring, the more sensitive it becomes.

Resultant force:

- When two or more forces are acting together at a point, it is always possible to find a single force which will have the same effect as these forces, called the resultant force.
- Two or more forces acting together at the same point are called cocurrent forces, and since they lie in the same plane, they are said to be coplaner.
- It has been found out that a set of coplaner and concurrent forces, can be replaced by a single force which has the same effect as these forces.
- As already stated, a single force which has the same effect as two forces acting together at a point is called the resultant of the two forces.
- Now, let us consider the resultant of two forces, which have the same line of action and acting at the same point but are parallel:

- For example:



In such a case, the magnitude of the resultant is the difference between the magnitudes of F_1 and F_2 I .e. F_1 - F_2.

(Q1)

In the above diagram, forces 14kgf, 10kgf, 2kgf and 6kgf are acting at a point as indicated. Find the resultant force.

Soln:

Total force acting towards the left = 14 + 10 = 24kgf.

Total force acting toward the right = 2 + 6 = 8kgf.

Resultant force = 24 – 8 = 16kgf towards the left hand side direction.

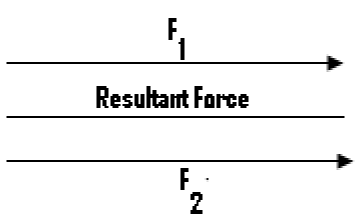
(Q2)

Find the resultant force.

Soln:

Resultant force = 30 – 10 = 20kgf acting right.

- On the other hand if the forces are parallel, the magnitude of the resultant is equal to the sum of the magnitude of these forces.
- Example, consider the diagram next,



The resultant force = $F_1 + F_2$, where F_1 and F_2 are the two parallel forces.

(Q3)

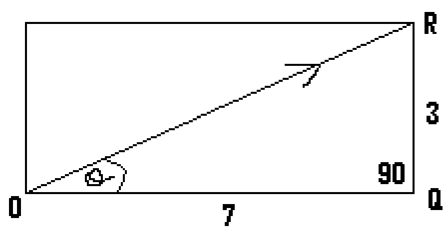
Find the resultant of the two given forces.

Soln:

Resultant force = 40 + 30 = 70kgf acting right.

(Q4) Find the resultant of two forces of 7kgf and 3kgf, acting at a point and at right angle to one another.

Soln:



The resultant force = OR.

From pythagoras theorem,

$OR^2 = OQ^2 + QR^2$

=> $OR^2 = 7^2 + 3^2 = 58$,

=> OR = $\sqrt{58}$ = 7.6kgf.

N/B:

- If : θ = the angle made by the resultant with the 7kgf, then $\tan\theta = \frac{3}{7} => Q = 23^0$.

Graphical representation of forces:

- A scalar quantity is one which has only magnitude but no direction, and an example is a mass of 30kg.
- A vector quantity is one which has magnitude and direction and an example is a force of 15kgf acting at an angle of 30^0 to the surface.

Graphical representation of force:

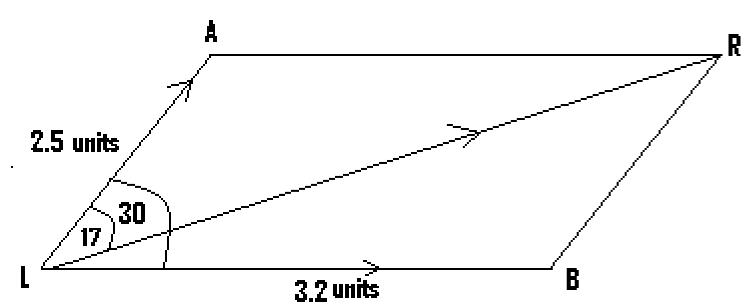
- Vectors can always be represented by straight lines drawn to scale.
- Thus if two forces of 5kgf and 7kgf act at right angle to one another, we can represent them on a diagram using a scale of 1cm to 1kgf by drawing two lines at right angle, which are respectively 5cm and 7cm long.

The parallelogram of forces:

- If two forces acting at a point are represented both in magnitude and direction by the adjacent sides of a parallelogram, their resultant will be represented both in magnitude and direction by the diogonal of the parallelogram drawn from the point.

(Q5) A liner is towed into a habour by two tugs A and B, whose cables make an angle of 30^0. If A exerts a pull of 2.5tf, and B a pull of 3.2tf, find graphically the resultant pull on the liner and the angle it makes with the cable of the weaker tug.

Soln:



- Choose a suitable scale and draw two lines LA = 2.5 units and LB = 3.2units, at an angle of 30^0 to represent the forces acting on the liner.

- On the completion of the parallelogram LARB, the diagonal which represents the resultant will be found to be 5.5units long, making an angle of 17^0 with LA.

=> The resultant pull on the liner = 5.5tf and the angle of weaker tug = 17^0.

The equlibrant:

- When two forces act on a body, they generally produce motion on the body.
- A third force introduced to restore the body in equilibrium, is referred to as the equilibranr of the first two forces.
- The equilibrant will have the same magnitude as well as the same line of action as the resultant of the first two forces.

Three forces acting on a body in equilibrium:

- These forces must all meet at a common point i.e. they must be concurrent.
- They must also lie in the same plane i.e. they must be coplaner.
– In short, three forces acting on a body must be coplaner and cocurrent.

The triangle of forces:

- If three forces acting at a point are in equilibrium, they can be represented in magnitude nad direction, by the three sides of a triangle taken in order.
- The expression "taken in order" means that the arrows following the force direction must follow one another in the same direction round the triangle.

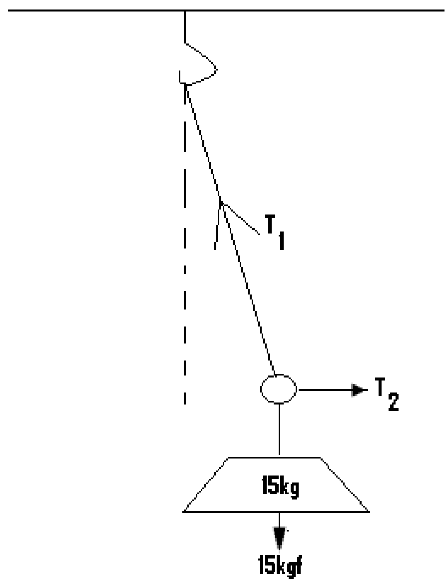
(Q1) A 15kg mass is supported by a thin cord attached to a hook in the ceiling.

Anothercord is attached to the ring of the mass, and pulled horizontally until the

supporting cord makes an angle of 30^0 with the vertical. Determine the tensions in both

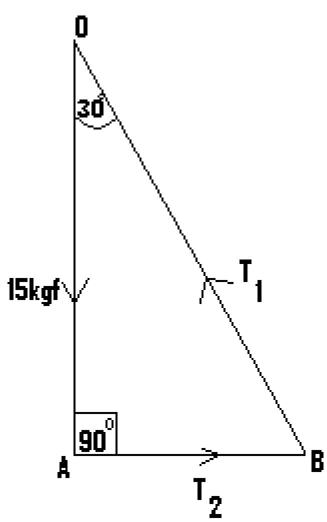
strings.

Soln



Let T_1 and T_2 = the tensions in the cords.

Also let $30^0 = \theta$.



By trignometry, $\cos\theta = \frac{15}{T_1}$

$=> T_1 = \frac{15}{\cos 30^0} = \frac{15}{0.866} = 17.3$kgf.

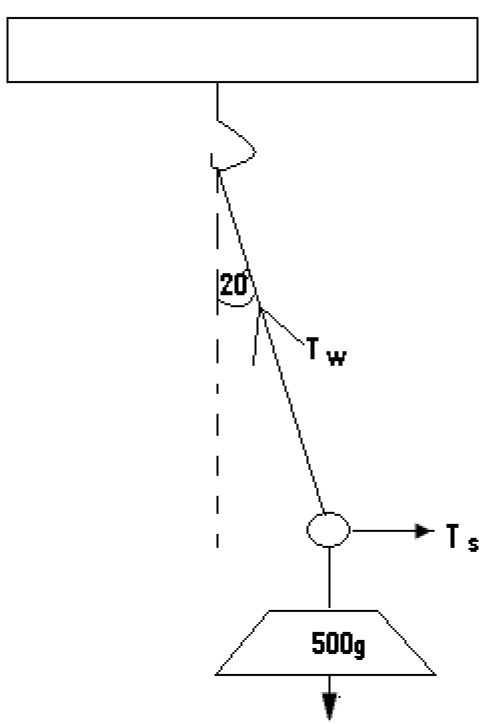
Also $\frac{T_2}{15} = \tan 30^0$

=> $T_2 = 15 \tan 30^0 = 15 \times 0.577$

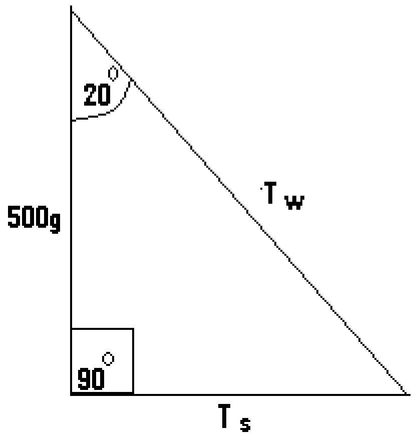
= 8.7kgf.

(Q2) A microphone of mass 500g hangs from the end of a long wire fixedto the ceiling. A horizontal string attached to the microphone exerts a pull which keeps the wire at 20^0 to the vertical. Find the tensions in both the string and the wire.

Soln:



Let T_W = the tensoin in the wire and T_S = The tension in the string.



$T_w = \frac{500}{COs\ 20^0} = 532gf.$

$T_S = T_W \text{ Sin } 20^0 = 500 \times 0.34 = 170gf.$

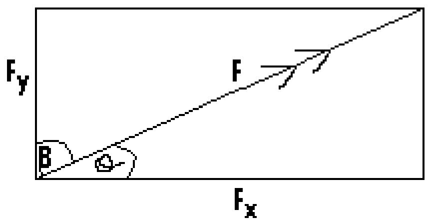
Resolution of forces:-

-As already stated, the resultant of the forces acting at a point at an angle to each other, can be found by applying the principle of parallelogram of forces.

- At times, it becomes necessary to convert the resultant into two forces, in directions different from the resultant force.

- These two forces are called the components of the resultant, and when the resultant force has been broken down into two components, it is said to have been resolved into two components.

Determination of the rectangular components of a force:

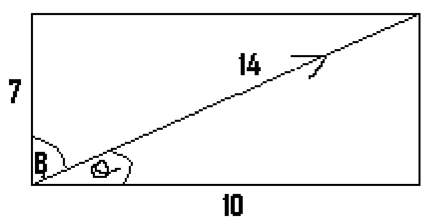


- The resultant force or the inclined force F, can be replaced by its rectangular components, F_X and F_Y.

- $F_x = F\text{Cos}\ \theta$ and $Fy = F\sin\theta$

(Q1) Two froces of 7kgf and 10kgf act at a point so as to produce a resultant of 14kgf. Find the angle between the resultant and the 7kgf component.

Soln:



$\text{Cos}\ \theta = \frac{10}{14} = 0.714$

$\Rightarrow \text{Cos}\ \theta = 0.714 \Rightarrow \theta = \text{Cos}^{-1}\text{o}.714,$

$\Rightarrow \theta = 56^0.$

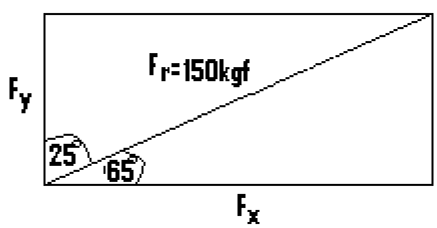
But since $B + \theta = 90^0 \Rightarrow B + 56^0 = 90, \Rightarrow B = 90 - 56 = 34^0$

The angle between the resultant and the 7kgf component = B = 34^0.

(Q2) Two forces acting at a point, make angles of 25^0 and 65^0 respectively with their resultant which is of magnitude 150kgf. Determine the magnitude of the component forces.

Soln:

Ler F_r = Resultant force



Let F_y = the y component and F_x = the x component.

$F_x = 150 \text{ Cos } 65 = 150^0 \times 0.4$,

=> F_x = 60kgf making an angle of $65^0$with F_r.

$F_Y = F_r \text{ Sin } 65^0 = 150 \times 0.9$

= 135kgf making an angle of 25^0 with the resultant.

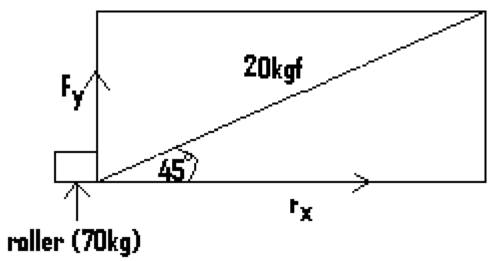
N/B: Also Fy = 150 Cos 25^0 => Fy = 15 x 0.9 = 135.

(Q3) A man using a 70kg garden roller on a level ground, exerts a force of 20kgf at 45^0 to the ground. Determine the vertical force of the roller on the ground.

(a) If he pulls the roller.

(b) If he pushes the roller.

Soln:



Let F_y = the vertical force on the roller.

$F_y = 20\text{Sin } 45^0 = 20 \times 0.7 = 14\text{kgf}.$

(a) If he pulls the roller, then the vertical force on the roller = 70 – 14 = 56kgf.

(b) If he pushes the roller, then the vertical force

The centre of gravity:

- The centre of gravity of a body is the position through which the weight of the body is supposed to act.
- Every body or object has small gravitational forces acting on it.
- All these small gravitational forces together act as a single force pulling at just one point which is called the centre of gravity of the body.

Types of equilibrium:

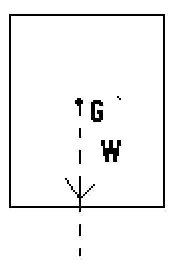
-There are three types and these are:

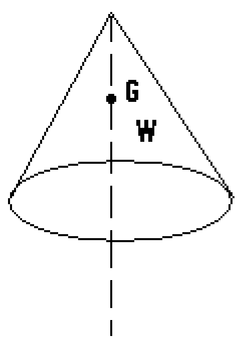
(I) Stable equilibrium.

(II) Unstable equilibrium.

(III) Neutral equilibrium.

Stable equilibrium:

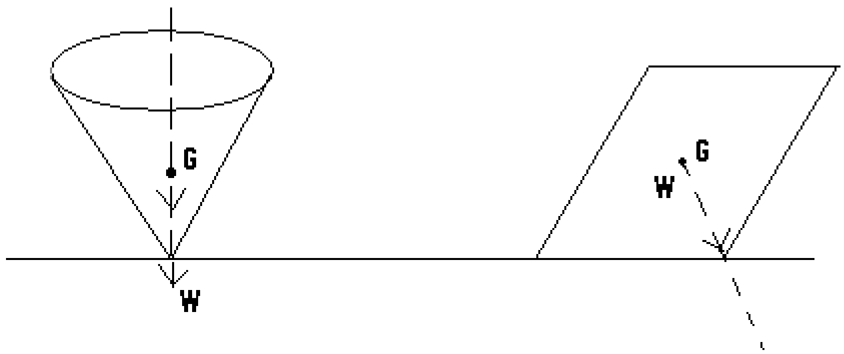




- An object is said to be in stable equilibrium, when the vertical line through which the the centre of gravity acts, passes through the base of the object i.e. the line of action of weight W passes through the base of the object.

- For stability, the vertical line through which the centre of gravity acts must pass through the base of the object.- This is so for the two objects just shown and they are therefore in stable equilibrum state.

Unstable equilibrium:

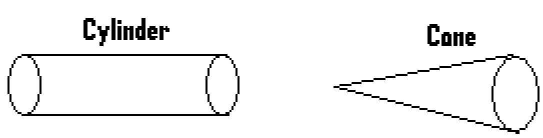


- This occurs when this vertical line passes through the edge of the base.

- In the first figure shown, the cone which is resting on its edge is in a state of unstable equlibrium, since any slight tilt will cause the object to topple over.

- Because such an object can easily topple over if slightly distributed, such a state of equilibrium is said to be unstable.

Neutral equilibrium:



- When a cylinder or a cone lies on its side and it is pushed, it merely rolls along.
- If the ground is a levelled one, the centre of gravity remains at the same horizontal level.
- A body in this state is said to be in a state of neutral equilibrium.

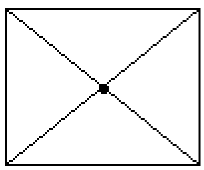
Determination of approximate position of centre of gravity of regularly shaped objects:

- For such objects, the centre of gravity lies at the geometrical centre, which can be determined by mathematical methods.

Uniform metre rule:

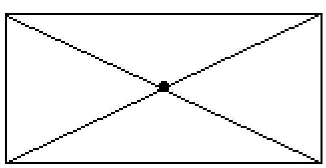
(1) For such an object, the centre of gravity is at the 50cm mark.

(2) Square plate:



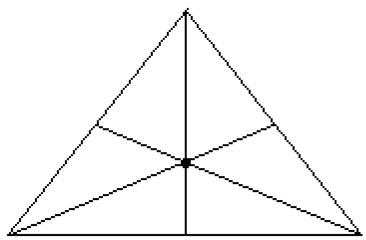
- For a square plate, the centre of gravity is located at the point of intersection of the diagonals.

(3) Rectangle:



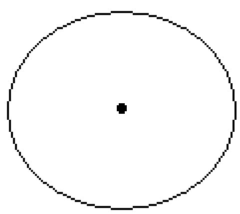
- For a rectangle, the centre of gravity is located at the point of intersection of the diagonals.

(4) Triangular plate:



- The centre of gravity is located at the intersection of the medium.

(5) Circular object:



The centre of gravity is located at the centre of the object.

Experimental determination of the centre of gravity:

- The centre of gravity can be located by:

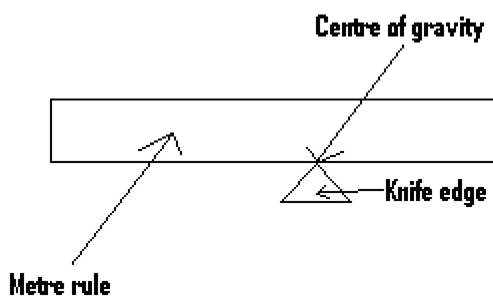
  (a) The balancing method.

  (b) The plumline method.

The balancing method:

- This method is mostly used to locate the centre of gravity of a long thin objects such as a ruler and billard cue.

- It can be used to locate the centre of gravity of a thin sheet of cardboard.
- To locate the centre of gravity of a metre rule, we place the metre rule on a knife (support).
- We then keep on moving the support or knife edge until the ruler is balanced horizontally.
- This point of balance or support where the ruler is balanced is the centre of gravity.

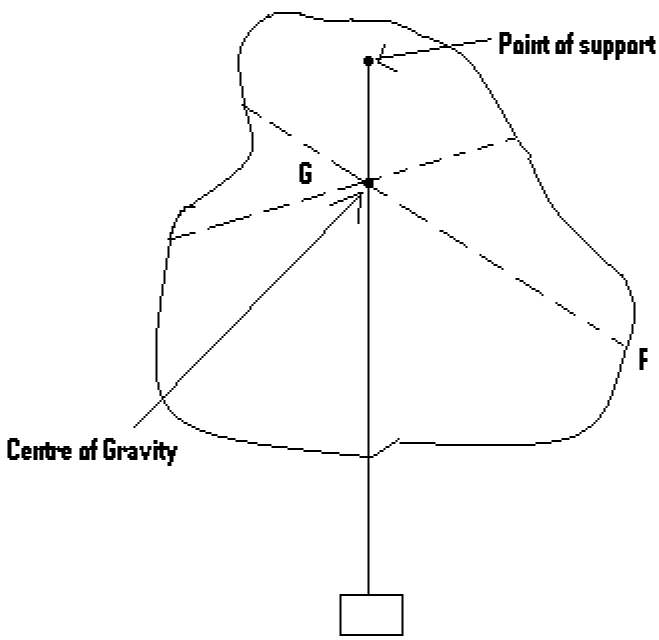


- The body is balanced when the centre of gravity comes to lie immediately below the knife edge.
- To make a check, we turn on the other side and then repeat the experiment.

<u>The plumbline method:</u>

- Is most often the best method used to locate the centre of gravity of objects.
- To locate the centre of gravity of a sheet of cardboard by the plumbline method, we make three small holes at well spaced intervals round the edge of the card.
- A nail or a stout pin is then put through one of the holes and held firmly by a clamp and stand so that the card hangs freely.

- A suitable plumbline made from a length of cotton thread with a loop at one end and a weight tied at the other endis then hung from the pin or the nail.



- We mark the position of the plumline on the cardboard as shown above.
- We then hang the cardboard from another hole and mark the position of the plumbline on the cardboard.
- The centre of gravity is taken as the point of intersection of the marks made on the cardboard i.e G.
- As a check, the procedure is repeated with the third hole.

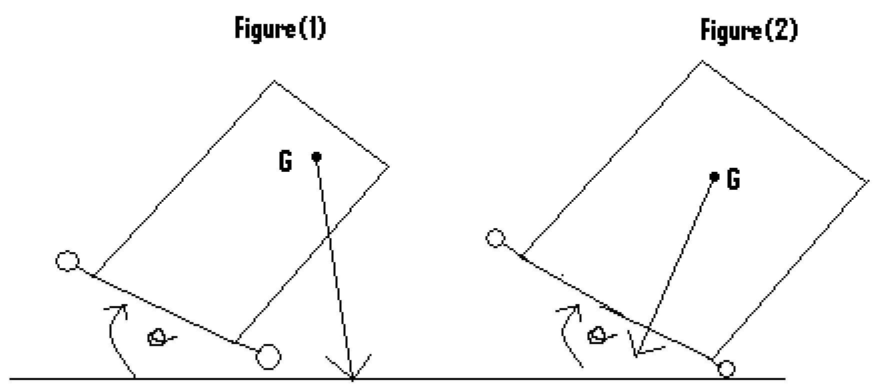
Application or factors which determine the stability of objects:

- The stability of an object such as a box or a car (bus), is determined by two factors which are:

(I) The width of the base.

(II) The location of the position of its centre of gravity.

The effect of the width of the base:



- The wider the base of an object, the greater its stability and vice versa.
- For example, a lorry with a broad base is more stable than one with a narrow base.
- An object becomes less stable as it becomes easier for its line of action ofcentre of gravity to pass outside of its base.
- For example consider the two figures drawn.
- While the first one represents a bus with a narrow base, the second represents one with a broad base.
- If these two buses are tilted through the same angle, i.e. θ, it is possible for the line of action of the centre of gravity to pass outside the base of the first one, causing it to topple.

- But eventhough the second bus is tilted through the same angle i.e θ., this line of action of the centre of gravity still passes through its base since it has a broad base.
- For this reasoit will not topple, and as such it is in a more stable state than the first one.

The effect of the location of the position of the centre of gravity:

- It is also a well known fact that raising the position of the centre of gravity of an object, makes it to become less stable.
- For the lower the location of the position of the centre of gravity of an object, the more stable it becomes.
- Buses or lorries which carry goods or loads on their top are less or not too stable, since they can easily topple over.
- This is due to the fact that putting loads on top of a bus causes arise in the level or the position, at which its centre of gravity is located.
- Cars or buses carrying goods or loads on their top are therefore not quite stable for this reason.
- The carrying of goods or load at the lower portion of a bus, lowers the position or the level at which its centre of gravity is located and makes it very stable.
- For this reason, the state transport corporation buses are loaded at the chassis, in order to bring their centres of gravity as low as possible, so as to enhance their stability.

Questions:

(1) Explain what force is and list three things that force can do.

(2) Differentiate between the force of gravity and the centripetal force.

Ans:

- Force of gravity pulls us towards the centre of the earth, but centripetal force enables us to move in a circular path.

(3) Explain why objects have weights.

Ans:

- Because of the attraction on them by the force of gravity which enables eachin turnto exert a force on its support.

(4) Expplain why it is that during weightlessness, the weight of a body becomes zero.

Ans:

- Because all the force of gravity acting on it has been converted into centripetal force, due to an increase in the velocity of the circular motion.
- None will therefore be left to attract the body towards the earth`s surface so as to give it a weight.

(5) Why is it that the weight of an object or a person, keeps on changing from place to place?

Ans:

- Because the earth is not a perfect sphere but bulges at the equation. A person at the pole will therefore be closer to the centre of the earth, where the force of gravity operates from, than when he is at the equator.
- Therefore the force of gravity acting on him will be greater at the pole than at the equation, which enables him to have a greater weight at the poles than at the equator.

(6) List three differences between weight and mass.

Ans:

(i) Mass is a scalar quantity but weight is a vector quantity.

(ii) Mass is measured in kg or g but weight is measured in kgf or gf.

(iii) Mass of an objetc is constant but its weight is not constant.

(7) Briefly explain how you will verify Hooke`s law by means of an experiment.

Ans:

- A spiral spring with a scale pan and a pointer attached, is held vertically by a clamp and a stand.
- Equal weights are then added to the pan, and the extensions produced noted.
- By plotting a graph of extension against force, we get a straight line through the origin which implies that the extension produced is proportional to the applied force, as stated in Hooke`s law.

(8) Why is it that objects can be balanced at certain points?

Ans:

- Because these points are the centre of gravity of these objects, through which the weights of these bodies act.

(9) Describe the way and manner in which the balancing method is used to determine the centre of gravity of an object.

Ans:

- Attempts are made to balance the object on a knife edge or support at various points.
- The point where the object gets balanced on the knife edge, is its center of gravity.

(10) When is an item said to be in a state of stable equilibrium?

Ans:

- When the vertical line through which the centre of the gravity acts, passes through its base.

(11) Explain why it is important to ensure that, the centre of gravity of a car is made as low as possible.

Ans: The car becomes more stable, by so doing.

(12) What do we mean when we say that an object is in a state of unstable equilibrium?

Ans: When the vertical line through which the centre of gravity acts, passes through a point which is very close to its edge.

(13) List two conditions needed to enhance the stability of let say a lorry.

Ans:

(a) The base must be made wide or broad.

(b) The location of its centre of gravity must be made as low as possible.

(14) Why are buses with narrow bodies or bases less stable?

Ans: - Because an appreciable level of its tilting, will cause the line of action of the centre of gravity to act outside its base, causing it to topple over.

(15) Explain what the following mean:

(a) Concurrent forces.

(b) Coplaner forces.

(c) Resultant force.

Ans:

(a) Concurrent forces are two or more forces which act together at the same point.

(b) Coplaner forces are two or more forces which lie in the same plane, and act together at the same point.

(c) Resultant is a single force, which has the same effect as two or more forces acting together at a point.

(16)

In the above diagram, forces 10kgf, 7kgf, 2kgf, 8kgf and 3kgf are acting at a point as indicated. Determine the resultant force.

Ans: 8kgf acting left.

(17) (a) Determine the resultant of two forces 10kgf and 4kgf, acting at a point and at right angle to one another.

Ans: 10.8kgf.

(b) Find the angle between the resultant and the 10kgf.

Ans: 23^0.

(18) The resultant of two forces acting together at a points are separated by an angle of 90^0 is 20kgf. If the x component of this resultant is 14kgf,

(a) determine its y component.

Ans: 14.3kgf.

(b) find the angle between the resultant and the 14kgf (the x component).

Ans: 45.6^0.

(c) find also the angle between the resultant and the 14.3kgf (the y component).

Ans: 44.4^0.

(19) A bag of cement of mass 50kg is suspended from the top of a mango tree, by means of a cable. A rope was then attached to this bag of cement and pulled horizontally till the cable made an angle of 42^0 with the vertical. Determine the tension in

(a) the cord. Ans: 67kgf

(b) the rope. Ans : 45kgf.