CHAPTER TWELVE
CHANGE OF SUBJECT

Introduction:

To make a letter the subject of a given equation is to let it stand alone, on one side of the equal to symbol.

Q1. Given that c = 2πr,

i) make r the subject.
ii) calculate r when c = 20 and π = 3.14 .

Soln.

c = 2πr

Divide through using 2π $\Rightarrow \frac{c}{2\pi} = \frac{2\pi r}{2\pi} \Rightarrow \frac{c}{2\pi} = r \Rightarrow$ r = $\frac{c}{2\pi}$

When c = 20 and π = 3.14

$\Rightarrow$ r = $\frac{c}{2\pi} = \frac{20}{2(3.14)} = \frac{20}{6.28}$

= 3.2 $\Rightarrow$ r = 3.2.

Q2. Given that M = RVL,

i) make V the subject.
ii) Calculate V when M = 50, R = 20 and L = 10.

Soln.

i) M = RVL

Divide through using RL $\Rightarrow \frac{M}{RL} = \frac{RVL}{RL}, \Rightarrow \frac{M}{RL} = V \Rightarrow$ V = $\frac{M}{RL}$.

ii) When M = 50, R = 20 and L = 10 $\Rightarrow$ V = $\frac{M}{RL} = \frac{50}{(20)(10)}$

= $\frac{50}{200} = 0.25.$

Q3. You are given the formula 2RV^2 = mg.

i) Make R the subject.
ii) Calculate R when V = 3, M = 5 and g = 2.

Soln.

i) $2\mathrm{R}V^2 = mg.$

Divide through using $2V^2 \Longrightarrow \dfrac{2RV^2}{2V^2} =$ R = $\frac{mg}{2V^2}$

ii) If V = 3, m = 5 and g = 2

$\Rightarrow$ R = $\frac{Mg}{2V^2} = \frac{(5)(2)}{(2)(3)^2} = \frac{10}{(2)(9)} = \frac{10}{18} \Longrightarrow$ R = 0.55 .

Q4. If $5b^2r^3v = 2N, make\ v\ the\ subject.$

Soln.

$5b^2r^3v = 2N. Dividethrough\ using\ 5b^2r^3 \Longrightarrow \frac{5b^2r^3v}{5b^2r^3} = \frac{2N}{5b^2r^3}$

$\Longrightarrow$ v = $\frac{2N}{5b^2r^3}$.

Q5. Given that a + b = 2R,

i) make a the subject.
ii) calculate a, when b = 3 and R = 10.
iii) make R the subject.
iv) calculate R when a = 3 and b = 5.

Soln.

i) a + b = 2R $\Longrightarrow$ a = 2R – b
ii) When b = 3 and R = 10 $\Rightarrow$ a = 2R – b, $\Rightarrow$ a = 2(10) – 3
= 20 – 3 = 17.
iii) To make R the subject, a + b = 2R. Divide through using 2 $\Rightarrow \frac{a+b}{2} = \frac{2R}{2} \Rightarrow \frac{\mathrm{a+b}}{2} = \mathrm{R}$
$\Rightarrow$ R = $\frac{a+b}{2}$.
When a = 3 and b = 5
$\Rightarrow$ R = $\frac{a+b}{2} = \frac{3+5}{2} = \frac{8}{2} = 4$.

Q6. Given that 2V + 3R = 4b,

i) make V the subject.
ii) calculate V when R = 3 and b = 1.

Soln.

i) 2V + 3R = 4b $\Rightarrow$ 2V = 4b – 3R

Divide through using 2 $\Rightarrow \frac{2v}{2} = \frac{4b-3R}{2} \Rightarrow \mathrm{V} = \frac{\mathrm{4b - 3R}}{2}$

ii) When R = 3 and b = 1

$\Rightarrow$ V = $\frac{4(1)-3(3)}{2} = \frac{4-9}{2} = \frac{-5}{2} = -2.5.$

Q7. You are given the formula 2V + 3R = 4b,

(i) make R the subject.

(ii) make b the subject.

Soln.

(i) 2V + 3R = 4b

$\Rightarrow$ 3R = 4b – 2V

Divide through using 3

$\Rightarrow \frac{3R}{3} = \frac{4b-2V}{3} \Rightarrow \mathrm{R} = \frac{\mathrm{4b-2V}}{3}$

(ii) 2V + 3R = 4b .

Divide through using 4

$\Rightarrow \frac{2V+3R}{4} = \frac{4b}{4} \Rightarrow \frac{\mathrm{2V+3R}}{4} = \mathrm{b}$

$\Rightarrow$ b = $\frac{2V+3R}{4}$.

Q8. Given that the three quantities V, u and t are connected by the formula V = 3u + at^2, calculate a when V = 10, t = 1 and u = 3.

N/B: Before you can calculate a, you must first make a the subject.

Soln.

V = 3u + a$t^2 \Rightarrow \mathrm{V - 3u = at^2}$

Divide through using $t^2 \Rightarrow \frac{\mathrm{V-3u}}{\mathrm{t^2}} = \frac{\mathrm{at^2}}{\mathrm{t^2}} \Rightarrow \frac{\mathrm{V-3u}}{\mathrm{t^2}} = \mathrm{a}$

$\Rightarrow$ a = $\frac{V-3u}{t^2}$.

When V = 10, t = 1 and u = 3

$\Rightarrow$ a = $\frac{V-3u}{t^2} = \frac{10-3(3)}{1^2} = \frac{10-9}{1} = \frac{1}{1}$

= 1 .

N/B: If $a^2 = 4 \Rightarrow \mathrm{a} = \sqrt{4} = 2$.

If $x^2 = 25 \Rightarrow \mathrm{x} = \sqrt{25} = 5$.

Q9. If 2V = u – ga^2,

(i) make a the subject.

(ii) calculate a when V = 3, u = 30 and g = 1.

Soln.

(i) 2V = u – $ga^2 \Rightarrow 2\mathrm{V} + \mathrm{ga}^2 = \mathrm{u}$

$\Rightarrow ga^2 = u - 2V$.

Divide through using g

$\Rightarrow \frac{ga^2}{g} = \frac{u - 2v}{g} \Rightarrow \mathrm{a}^2 = \frac{\mathrm{u}-2\mathrm{V}}{\mathrm{g}} \Rightarrow \mathrm{a} = \sqrt{\frac{\mathrm{u} - 2\mathrm{v}}{\mathrm{g}}}$.

(ii) When V = 3, u = 30 and g = 1

$\Rightarrow$ a = $\sqrt{\frac{30 - 2(3)}{1}} \Rightarrow \mathrm{a} = \sqrt{\frac{30 - 6}{1}}$

$\Rightarrow$ a = $\sqrt{\frac{24}{1}} \Rightarrow a = \sqrt{24} = 4.9$.

Q10. If $2\mathrm{RV}^2 = \mathrm{mg}$, calculate V when $\mathrm{m} = 50, \mathrm{g} = 4$ and $\mathrm{R} = 1$.

Soln.

$2\mathrm{R}V^2 = mg$.

Dividing through using 2R $\Rightarrow \frac{2RV^2}{2R} = \frac{mg}{2R} \Rightarrow \mathrm{V}^2 = \frac{\mathrm{mg}}{2\mathrm{R}}$

$\Rightarrow$ V = $\sqrt{{mg}/{2R}}$.

When m = 50, g = 4 and R = 1

$\Rightarrow$ V = $\sqrt{\frac{(50)(4)}{2(1)}} = \sqrt{\frac{200}{2}} = \sqrt{100}$

$\Rightarrow$ V = 10.

Q11. The movement of a particle is such that its final velocity V, its initial velocity u, its acceleration a and its time t, are connected by the formula V = 3u + at^2 . Calculate the time in seconds when V $= 115m/s$, $u = 5m/s$ and a $= 4m/s^2$.

Soln.

V = 3u + a$t^2 \Rightarrow V - 3u = at^2$

Divide through using a $\Rightarrow \frac{v-3u}{a} = \frac{at^2}{a}, \Rightarrow \frac{V-3u}{a} = t^2,$

$\Rightarrow t^2 = \frac{V-3u}{a} \Rightarrow t = \sqrt{\frac{V-3u}{a}}$.

But V = 115, u = 5 and a = 4

$\Rightarrow$ t = $\sqrt{\frac{115-3(5)}{4}} = \sqrt{\frac{100}{4}} = \sqrt{25}$

= 5 $\Rightarrow$ t = 5seconds.

N/B: If the letter we are required to make the subject appears twice, then we must factorize it by bringing it outside the bracket.

Q12. The variables b and v are connected by the formula 5b - v^2 = vb + 2. Make b the subject.

Soln.

5b – $v^2 = vb + 2 \Rightarrow 5b = vb + 2 + v^2, \Rightarrow 5b - vb = 2 + v^2$. Factorize the b

$\Rightarrow b(5-v) = 2 + v^2$

Divide through using $5 - v \Rightarrow \frac{b(5-v)}{5-v} = \frac{2+v^2}{5-v}$

$\Rightarrow$ b = $\frac{2+v^2}{5-v}$.

Q13. Given that a$v^2 + 2 = 2v^2 - 2c$, *make v the subject*.

Soln.

a$v^2 + 2 = 2v^2 - 2c$

$\Rightarrow$ a$v^2 + 2 - 2v^2 = -2c,$

$\Rightarrow av^2 - 2v^2 = -2c - 2$

$\Rightarrow v^2(a-2) = -2c - 2,$

Divide through using a – 2.

$\Rightarrow \frac{v^2(a-2)}{(a-2)} = \frac{-2c-2}{(a-2)}$

$\Rightarrow v^2 = \frac{-2c-2}{(a-2)} \Rightarrow \text{v} = \sqrt{\frac{-2\text{c}-2}{\text{a}-2}}$.

N/B: (1) If $a^2 = b \Rightarrow \text{a} = \sqrt{\text{b}}$

(2) If $a^3 = b \Rightarrow \text{a} = \sqrt[3]{\text{b}}$

(3) If $a^4 = b \Rightarrow \text{a} = \sqrt[4]{\text{b}}$

(4) If $a^5 = b \Rightarrow \text{a} = \sqrt[5]{\text{b}}$.

Q14. Given that $5b^3 - 1 = v^2$

(i) make b the subject

(ii) calculate b when v = 10.

Soln.

$5b^3 - 1 = v^2 \Rightarrow 5\text{b}^3 = v^2 + 1$,

$\Rightarrow \frac{5b^3}{5} = \frac{v^2+1}{5} \Rightarrow \text{b}^3 = \frac{v^2+1}{5}$,

$\Rightarrow \text{b} = \sqrt[3]{\frac{v^2+1}{5}}$. $\quad If\ v = 10$

$\Rightarrow$ b = $\sqrt[3]{\frac{10^2+1}{5}} = \sqrt[3]{\frac{100+1}{5}} = \sqrt[3]{\frac{101}{5}}$

= $\sqrt[3]{50.5} = \ 3.6$.

Q15. Given that $2b^2 = 10T, calculate\ b\ when\ T = 30.$

Soln.

$2b^5 = 10T \Rightarrow \frac{2\text{b}^5}{2} = \frac{10T}{2}$,

$\Rightarrow \text{b}^5 = \frac{10T}{2} \Rightarrow \text{b} = \sqrt[5]{\frac{10\text{T}}{2}}$

If T = 30 $\Rightarrow$ b = $\sqrt[5]{10(30)/2}$

= $\sqrt[5]{300/2} = \sqrt[5]{150} = 2.6$ approx.

Q16. If $b^3 + 3 = v^4 + 5, make\ b\ the\ subject.$

Soln.

$b^3 + 3 = v^4 + 5 \Rightarrow b^3 = v^4 + 5 - 3, \Rightarrow b^3 = v^4 + 2 \Rightarrow$ b $= \sqrt[3]{v^4 + 2}$.

Change of subject which involves cross multiplication:

There may arise certain questions whose solutions may require cross multiplication.

Q1. Given that $\frac{2V}{T} = b$,

(i) make V the subject.

(ii) calculate v when T = -2 and b = 8.

Soln.

(i) $\frac{2V}{T} = b \Rightarrow \frac{2\mathrm{V}}{\mathrm{T}} = \frac{\mathrm{b}}{1}$.

We now cross multiply $\Rightarrow 2\mathrm{V} \times 1 = \mathrm{T} \times \mathrm{b} \Rightarrow 2\mathrm{V} = \mathrm{Tb}$.

Divide through using 2

$\Rightarrow \frac{2V}{2} = \frac{Tb}{2} \Rightarrow \mathrm{V} = \frac{\mathrm{Tb}}{2}$.

(ii) If T = -2 and b = 8 $\Rightarrow \mathrm{V} = \frac{\mathrm{Tb}}{2} = \frac{(-2)(8)}{2} = \frac{-16}{2} = -8$

$\Rightarrow$ V = - 8 .

Q2. Given that $\frac{T^3}{3} = M$,

(i) make T the subject .

(ii) find T when M = 9.

Soln.

$\frac{T^3}{3} = M \Rightarrow \frac{T^3}{3} = \frac{M}{1}, \Rightarrow$

$T^3 \times 1 = 3 \times M \Rightarrow \mathrm{T}^3 = 3\mathrm{M},$

$\Rightarrow \mathrm{T} = \sqrt[3]{27} \Rightarrow \mathrm{T} = 3.$

Q3. Given that $5\mathrm{x} = \frac{\mathrm{V}^2}{\mathrm{T}^3}$,

(i) make V the subject.

(ii) calculate V when T = 2 and x = 3.

Soln.

(i) 5x = $\frac{v^2}{T^3} \Rightarrow \frac{5\mathrm{X}}{1} = \frac{\mathrm{V}^2}{\mathrm{T}^3}$,

$\Rightarrow 5\mathrm{x} \times \mathrm{T}^3 = 1 \times \mathrm{V}^2 \Rightarrow 5\mathrm{xT}^3 = \mathrm{V}^2$,

$\Rightarrow \mathrm{V}^2 = 5\mathrm{xT}^3 \Rightarrow \mathrm{V} = \sqrt{5\mathrm{xT}^3}$

(ii) If T = 2 and x = 3

$\Rightarrow \mathrm{V} = \sqrt{5\mathrm{xT}^3} = \sqrt{5(3)(2)^3} = \sqrt{5(3)(8)} = \sqrt{120} = 10.9$.

N/B: When cross multiplying, and there is a minus or a plus sign between a letter and a number, or between two letters, they must be placed inside a bracket.

Q4. Given that T – 1 = $V/_{2a}$, $calculate\ T, when\ V = 8\ and\ a = 4$.

Soln.

T – 1 = $\frac{V}{2a} \Rightarrow \frac{T-1}{1} = \frac{V}{2a}$.

Cross multiply $\Rightarrow 2\mathrm{a}(\mathrm{T}-1) = 1 \times \mathrm{V}$,

$\Rightarrow 2\mathrm{aT} - 2\mathrm{a} = \mathrm{V} \Rightarrow 2\mathrm{aT} = \mathrm{V} + 2\mathrm{a}$,

$\Rightarrow \frac{2\mathrm{aT}}{2\mathrm{a}} = \frac{V+2a}{2a} \Rightarrow \mathrm{T} = \frac{\mathrm{V}+2\mathrm{a}}{2\mathrm{a}}$

When V = 8 and a = 4 $\Rightarrow$

T = $\frac{8 + 2(4)}{2(4)} = \frac{8+8}{8} = \frac{16}{8}$

$\Rightarrow \mathrm{T} = 2$.

Q5. Given that $\frac{a + b}{v} = \frac{2b}{T}$,

make a the subject.

Soln.

$\frac{a\ +\ b}{v} = \frac{2b}{T}$.⇒ T(a+b) = 2b × v, ⇒ Ta + Tb = 2bv ⇒ Ta = 2bv – Tb.

Divide through using T ⇒ $\frac{Ta}{T} = \frac{2bv - Tb}{T}$ ⇒ a = $\frac{2bv - Tb}{T}$

Change of subject involving fractions:

N/B: When fractions are available, they must be removed by multiplying through the given formula with a number which can remove them.

Q1. Given that $\frac{1}{2}\ at = 5V, make$

a) s the subject. (b) V the subject

Soln.

$(a)\frac{1}{2}\text{at} = 5\text{V}.\text{Multiply through using } 2 \Rightarrow 2 \times \frac{1}{2}at = 2 \times 5V$

$\Rightarrow \text{at} = 10\text{V}.$

Divide through using a

$\Rightarrow \frac{\text{at}}{\text{a}} = \ \Rightarrow \text{t} = \frac{10\text{V}}{\text{a}}.$

(b)From at = 10V

Divide through using 10

$$\Rightarrow \frac{\text{at}}{10} = \frac{10\text{V}}{10} \Rightarrow \frac{\text{at}}{10} = \text{V}$$

$$\Rightarrow \text{V} = \frac{\text{at}}{10}.$$

Q2. Given that $\frac{1}{2}a^2t = \frac{1}{3}b,\ calculate\ a\ when\ b = 45\ and\ t = 3.$

Soln.

$$\frac{1}{2}a^2t = \frac{1}{3}b$$

Multiply through using 6

$$\Rightarrow 6 \times \frac{1}{2}\text{a}^2\text{t} = 6 \times \frac{1}{3}\text{b},$$

$$\Rightarrow 3\text{a}^2\text{t} = 2\text{b}.$$

Divide through using 3t $\Rightarrow \frac{3a^2t}{3t} = \frac{2b}{3t} \Rightarrow a^2 = \frac{2b}{3t}, \Rightarrow a = \sqrt{\frac{2b}{3t}}$.

When b = 45 and t = 3

$\Rightarrow a = \sqrt{\frac{2(45)}{3(3)}} = \sqrt{\frac{90}{9}} = \sqrt{10}$

$\Rightarrow a = 3.1$.

Q3. The base b, volume v and the radius r of a certain structure, are connected by the formula

$b^2 - \frac{v^3}{5} = \frac{3r}{10}$. Calculate the volume when the base is 2cm and the radius is 10cm.

Soln.

$b^2 - \frac{v^3}{5} = \frac{3r}{10}$.

Multiply through using 10

$\Rightarrow 10 \times b^2 - 10 \times \frac{v^3}{5} = 10 \times \frac{3r}{10}$

$\Rightarrow 10b^2 - 2v^3 = 3r,$

$\Rightarrow 10b^2 = 3r + 2v^3$

$\Rightarrow 10b^2 - 3r = 2v^3,$

$\Rightarrow 2v^3 = 10b^2 - 3r.$

$\Rightarrow \frac{2v^3}{2} = \frac{10b^2}{2} - \frac{3r}{2}$

$\Rightarrow V^3 = 5b^2 - 1.5r,$

=> V = $\sqrt[3]{5b^2 - 1.5r}$.

When b = 2 and r = 10

$\Rightarrow V = \sqrt[3]{5(2)^2 - 1.5(10)}$

$\Rightarrow V = \sqrt[3]{5(4) - 15}$

$\Rightarrow V = \sqrt[3]{20 - 15} = \sqrt[3]{5} = 1.7.$

Q4. If $\frac{1}{R} + \frac{V}{R} = 3T, make\ V\ the\ subject.$

Soln.

$\frac{1}{R} + \frac{V}{R} = 3T.$

Multiply through using R

$\Rightarrow \mathrm{R} \times \frac{1}{R} + R \times \frac{V}{R} = R \times 3T$

$\Rightarrow 1 + \mathrm{V} = 3\mathrm{RT} \Rightarrow \mathrm{V} = 3\mathrm{RT} - 1.$

Q5. Given that $\frac{1}{2\mathrm{R}} + \frac{\mathrm{V}^2}{2\mathrm{R}} = \mathrm{N}^2,$

Make V the subject.

Soln.

$\frac{1}{2R} + \frac{V^2}{2R} = N^2.$

Multiply through using 2R

$\Rightarrow 2\mathrm{R} \times \frac{1}{2R} + 2R \times \frac{V^2}{2R} = 2R \times N^2$

$\Rightarrow 1 + \mathrm{V}^2 = 2\mathrm{RN}^2, \Rightarrow \mathrm{V}^2 = 2\mathrm{RN}^2 - 1,$

$\Rightarrow \mathrm{V} = \sqrt{2\mathrm{RN}^2 - 1}\,.$

Q6. Given that $\frac{1}{RV} - \frac{T}{V} = b + 1,$

(a) make b the subject.

(b) make V the subject.

Soln.

a) $\frac{1}{RV} - \frac{T}{V} = b + 1 \Rightarrow \frac{1}{RV} - \frac{T}{V} - 1 = b,$

$\Rightarrow \mathrm{b} = \frac{1}{RV} - \frac{T}{V} - 1\,.$

b) $\frac{1}{RV} - \frac{T}{V} = b + 1$

Multiply through using RV

=>$RV \times \frac{1}{RV} - RV \times \frac{T}{V} = RV \times b + RV \times 1,$

$\Rightarrow 1 - \mathrm{RT} = \mathrm{RVb} + \mathrm{RV}$

$\Rightarrow 1 - \mathrm{RT} = \mathrm{V}(\mathrm{Rb} + \mathrm{R}).$

Divide through using Rb + R $\Rightarrow \frac{1-\ RT}{Rb +\ R} = \ \frac{V(Rb +\ R)}{Rb +\ R}$.

$$\Rightarrow \frac{1 - \mathrm{RT}}{\mathrm{Rb} + \mathrm{R}} = \mathrm{V} \ \Rightarrow \ \mathrm{V} = \ \frac{1 - \mathrm{RT}}{\mathrm{Rb} + \mathrm{R}}$$

Q7. You are given the formula $\frac{2}{v^2} - \frac{2R}{V^2} = 2b + 5$. Calculate b when R = 10 and V = 2.

Soln.

$$\frac{2}{v^2} - \frac{2R}{V^2} \ = 2b + 5\ .$$

Multiply through using V^2

$$\Rightarrow V^2 \times \frac{2}{v^2} - V^2 \times \frac{2R}{V^2} = V^2(2b + 5),$$

$$\Rightarrow 2 - 2\mathrm{R} = 2\mathrm{V}^2\mathrm{b} + 5\mathrm{V}^2$$

$$\Rightarrow 2 - 2\mathrm{R} - 5\mathrm{V}^2 = 2\mathrm{V}^2\mathrm{b},$$

$$\Rightarrow 2\mathrm{V}^2\mathrm{b} = 2 - 2\mathrm{R} - 5\mathrm{V}^2.$$

Divide through using $2\mathrm{V}^2$

$$\Rightarrow \frac{2 - 2\mathrm{R} + 5\mathrm{V}^2}{2\mathrm{V}^2} \ = \frac{2\mathrm{V}^2\mathrm{b}}{2\mathrm{V}^2},$$

$$\Rightarrow \frac{2 - 2\mathrm{R} + 5\mathrm{V}^2}{2\mathrm{V}^2} = \mathrm{b}$$

$$\Rightarrow \mathrm{b} = \frac{2-2\mathrm{R}+5\mathrm{V}^2}{2\mathrm{V}^2}.$$

If R = 10 and V = 2

$$\Rightarrow \mathrm{b} = \ \frac{2 - 2(10) + 5(2)^2}{2(2)^2}$$

= $\frac{2 - 20 + 5(4)}{2(4)} = \frac{2 - 20 + 20}{8}$

= $\frac{2}{8} = 0.25.$

Q8. Given the formula $\frac{2C}{k} + \frac{1}{Q} = 3, make\ C\ the\ subject.$

Soln.

$\frac{2C}{k} + \frac{1}{Q} = 3.$

Multiply through using kQ

$\Rightarrow \mathrm{kQ} \times \frac{2C}{k} + \mathrm{kQ} \times \frac{1}{Q} = \mathrm{kQ} \times 3,$

$\Rightarrow 2\mathrm{CQ} + \mathrm{k} = 3\mathrm{kQ}$

$\Rightarrow 2\mathrm{CQ} = 3\mathrm{kQ} - \mathrm{k}.$

$\Rightarrow \frac{2\mathrm{CQ}}{2\mathrm{Q}} = \frac{3\mathrm{kQ} - \mathrm{k}}{2\mathrm{Q}}$.$\Rightarrow$ C= $\frac{3kQ - k}{2Q}$.

Q9. The variables b, k, w and t are connected by the formula $\frac{b^2}{k} + \frac{1}{w} = 2t. Determine\ the\ value\ of\ k\ when\ w = 10, b = 2\ andt = 3.$

Soln.

$\frac{b^2}{k} + \frac{1}{w} = 2t.$

Multiply through using kw

$\Rightarrow \mathrm{kw} \times \frac{b^2}{k} + \mathrm{kw} \times \frac{1}{w} = \mathrm{kw} \times 2t \Rightarrow \mathrm{W}b^2 + k = 2tkw,$

$\Rightarrow wb^2 = 2tkw - k$

$\Rightarrow wb^2 = k(2tw - 1).$

$\Rightarrow \frac{wb^2}{(2tw-1)} = \frac{k(2tw-1)}{(2tw-1)},$

$\Rightarrow \frac{wb^2}{(2tw-1)} = \mathrm{k} \Rightarrow \mathrm{k} = \frac{wb^2}{(2tw-1)}.$

When $\mathrm{w} = 10, \mathrm{b} = 2$ and $\mathrm{t} = 3$

$\Rightarrow \mathrm{k} = \frac{(10)(2)^2}{(2(3)(10) - 1)} = \frac{(10)(4)}{60-1} = \frac{40}{59}$ = 0.7

Q10. The quantities a, b and c are such that $\frac{2}{a} + \frac{b}{k} = \frac{1}{c}$.

Make k the subject.

Soln.

$\frac{2}{a} + \frac{b}{k} = \frac{1}{c}.$

Multiply through using abc

$$\Rightarrow \mathrm{abc} \times \frac{2}{a} + abc \times \frac{k}{b} = abc \times \frac{1}{c}$$

$$\Rightarrow 2\mathrm{bc} + \mathrm{ack} = \mathrm{ab},$$

$$\Rightarrow \mathrm{ack} = \mathrm{ab} - 2\mathrm{bc}.$$

Divide through using ac $\Rightarrow \frac{\mathrm{ack}}{\mathrm{ac}} = \frac{\mathrm{ab}}{\mathrm{ac}} - \frac{2\mathrm{bc}}{ac}$

$\Rightarrow \mathrm{k} = \frac{\mathrm{b}}{\mathrm{c}} - \frac{2\mathrm{b}}{\mathrm{a}}$.

Q11. Given that $\frac{1}{ab} + \frac{k}{v} = 3$, calculate a, when b = 5, k = 2 and v = 100.

Soln.

$$\frac{1}{ab} + \frac{k}{v} = 3.$$

Multiply through using abv

$$\Rightarrow \mathrm{abv} \times \frac{1}{ab} + \mathrm{abv} \times \frac{k}{v} = \mathrm{abv} \times 3$$

$$\Rightarrow \mathrm{v} + \mathrm{abk} = 3\mathrm{abv},$$

$$\Rightarrow \mathrm{v} = 3\mathrm{abv} - \mathrm{abk}$$

$$\Rightarrow \mathrm{v} = \mathrm{a}(3\mathrm{bv} - \mathrm{bk}),$$

$$\Rightarrow \frac{\mathrm{v}}{3\mathrm{bv} - \mathrm{bk}} = \frac{\mathrm{a}(3\mathrm{bv} - \mathrm{bk})}{3\mathrm{bv} - \mathrm{bk}}$$

$\Rightarrow \frac{\mathrm{v}}{3\mathrm{bv}-\mathrm{bk}} = \mathrm{a}$ $\Rightarrow \frac{\mathrm{v}}{3\mathrm{bv}-\mathrm{bk}}$ = a.

When b = 5, k = 2 and v = 100

$$\Rightarrow \mathrm{a} = \frac{100}{3(5)(100)-(5)(2)} = \frac{100}{1500-10} = \frac{100}{1490} = 0.07$$

$$\Rightarrow \mathrm{a} = 0.07 .$$

N/B:(a) (a + b) (c + d) = $a \times c + a \times d + b \times c + b \times d$

= ac + ad + bc + bd

(a + b) (c - d) = a $\times c - a \times d + b \times c - b \times d$

= ac – ad + bc – bd

(a - b) (c + d) = a$\times c + a \times d - b \times c - b \times d$

= ac + ad – bc – bd

(a - b) (c - d) = a$\times c - a \times d - b \times c + b \times d$ = ac – ad – bc + bd

Negative $\times$ Negative = Positive

i.e $- \times - = +$

(f) Positive $\times$ Negative = Negative

i.e $+ \times - = -$

Q12. Given that u = 1 + $\frac{3v}{ut - w}$,

Make t the subject.

Soln.

u = 1 + $\frac{3v}{ut - w} \Rightarrow \text{u} - 1 = \frac{3v}{ut - w}$,

$\Rightarrow \frac{\text{u} - 1}{1} = \frac{3\text{v}}{\text{ut} - \text{w}}$.

Cross multiply $\Rightarrow (\text{u} - 1)(\text{ut} - \text{w}) = 1 \times 3\text{v}$

$\Rightarrow u \times ut - u \times w - 1 \times ut + 1 \times w = 3v$,

$\Rightarrow \text{u}^2\text{t} - \text{uw} - \text{ut} + \text{w} = 3\text{v}$,

$\Rightarrow \text{u}^2\text{t} - \text{ut} - \text{uw} + w = 3v$,

$\Rightarrow \text{u}^2\text{t} - \text{ut} = 3\text{v} + \text{uw} - \text{w}, \Rightarrow \text{t}(u^2 - u) = 3v + uw - w. \textit{Divide through using } u^2 - u$

$\Rightarrow \frac{\text{t}(\text{u}^2 - \text{u})}{\text{u}^2 - \text{u}} = \frac{3\text{v} + \text{uw} - \text{w}}{\text{u}^2 - \text{u}}$,

$\Rightarrow \text{t} = \frac{3 + \text{uw} - \text{w}}{\text{u}^2 - \text{u}}$

N/B: $a^2 - b^2 = (a + b)(a - b)$.

$n^2 - m^2 = (n + m)(n - m)$.

Q13. Given that $\frac{\text{m}}{\text{n} - \text{y}} = \frac{\text{n}}{\text{m} + \text{y}}$, make y the subject of the relationship.

Soln.

$\frac{m}{n-y} = \frac{n}{m+y}, cross\ multiply$

$\Rightarrow \mathrm{m(m+y) = \ n(n-y)}$

$\Rightarrow \mathrm{m^2 + my = n^2 - ny,}$

$\Rightarrow \mathrm{my = n^2 - ny - m^2}$

$\Rightarrow \mathrm{my + ny = \ n^2 - m^2,}$

$\Rightarrow \mathrm{y(m+n) = \ n^2 - m^2.}$

Divide through using m+n

$\frac{\mathrm{y(m+n)}}{m+n} = \frac{\mathrm{n^2 - m^2}}{m+n}$

$\Rightarrow \mathrm{y} = \frac{\mathrm{n^2-m^2}}{m+n},$

$\Rightarrow$ y = $\frac{(n+m)(n-m)}{m+n}$

$\Rightarrow \mathrm{y = (n-m).}$

Q14. Given that A = P $\left(1+\frac{r}{100}\right), make\ r\ the\ subject.$

Soln.

A = P $\left(1+\frac{r}{100}\right)$

$\Rightarrow \mathrm{A} = \left(\mathrm{P \times 1 + P \times \frac{r}{100}}\right),$

$\Rightarrow \mathrm{A} = \left(\mathrm{P} + \frac{\mathrm{Pr}}{100}\right)$

$\therefore$ A = P + $\frac{Pr}{100}, \Rightarrow \mathrm{A - P} = \frac{Pr}{100}$ which can also be written as $\frac{A-P}{1} = \frac{Pr}{100}$. $By\ cross\ multiplication \Rightarrow 100(\mathrm{A - P}) = 1 \times \mathrm{Pr},$

$\Rightarrow \mathrm{100A - 100P = Pr. Divide\ through\ using\ p} \Rightarrow \frac{\mathrm{100A - 100P}}{\mathrm{p}} = \frac{\mathrm{Pr}}{\mathrm{P}},$

$\Rightarrow \frac{\mathrm{100A-100P}}{\mathrm{p}} = \mathrm{r,}$

$\Rightarrow \mathrm{r} = \frac{\mathrm{100A-100P}}{\mathrm{p}}.$

Change of subject involving the square root sign:

N/B: (i) $(\sqrt{a})^2 = a$, (ii) $(\sqrt{b})^2 = b$, (iii)$(\sqrt{10})^2 = 10$, (4) $(\sqrt{5})^2 = 5$ .This implies that in other to remove the square root sign, the number, (appropriately referred to as the surd) must be raised to the second power.

Ii) If the letter we want to make the subject of a given equation falls under a root sign, the root sign must be removed first.

Q1. If $\sqrt{2b} = w$,

i) make b the subject.
ii) calculate b when w = 6.

N/B: Since the b falls under or is found under the root sign, we must first remove the sign.

Soln.

i) $\sqrt{2b} = w$.

Squaring both sides $\Rightarrow (\sqrt{2\mathrm{b}})^2 = w^2 \Rightarrow 2\mathrm{b} = \mathrm{w}^2$,

$\Rightarrow \mathrm{b} = \frac{w^2}{2}$.

ii)If w = 6 $\Rightarrow \mathrm{b} = \frac{6^2}{2} = \frac{36}{2} = 18$

$\therefore$ b = 18 .

Q2. Given that $\sqrt{b + 5v} = 2Q$,

a) make b the subject.
b) make Q the subject.

Soln.

(a)$\sqrt{b + 5v} = 2Q$

Square both sides $\Rightarrow (\sqrt{b + 5v})^2 = (2Q)^2$

$\Rightarrow \mathrm{b} + 5\mathrm{v} = 2^2\mathrm{Q}^2$,

$\Rightarrow \mathrm{b} + 5\mathrm{v} = 4\mathrm{Q}^2 \Rightarrow \mathrm{b} = 4\mathrm{Q}^2 - 5\mathrm{v}$.

b) $\sqrt{b + 5v} = 2Q$.

Divide through using 2

$$\Rightarrow \frac{\sqrt{b + 5v}}{2} = \frac{2Q}{2}$$

$$\Rightarrow \frac{\sqrt{b+5v}}{2} = \mathrm{Q} \Rightarrow \mathrm{Q} = \frac{\sqrt{b+5v}}{2}.$$

N/B: In the second case, the Q which we want to make the subject does not fall under the root sign. For this reason, the root sign was not removed.

Q3. Given that $\sqrt{T^2-1} = 5b, make$

a) b the subject.
b) T the subject.

Soln.

a)$\sqrt{T^2-1} = 5b$

Divide through using 5
$\Rightarrow \frac{\sqrt{T^2-1}}{5} = \frac{5b}{5} \Rightarrow \frac{\sqrt{T^2-1}}{5} = b$

.b)$\sqrt{T^2-1} = 5b$

Square both sides $\Rightarrow (\sqrt{T^2-1})^2 = (5b)^2 \Rightarrow \mathrm{T}^2 - 1 = 5^2\mathrm{b}^2, \Rightarrow \mathrm{T}^2 - 1 = 25\mathrm{b}^2, \Rightarrow \mathrm{T}^2 = 25\mathrm{b}^2 + 1, \Rightarrow \mathrm{T} = \sqrt{25\mathrm{b}^2+1}$.

Q4. Given that $2\sqrt{b+v^2} = 2T$, calculate b when T = 20 and v = 1.

Soln.

$2\sqrt{b+v^2} = 2T.$

Square both sides

$\Rightarrow (2\sqrt{b+v^2})^2 = (2T)^2$

$\Rightarrow 2^2(\sqrt{b+v^2})^2 = 2^2T^2,$

$\Rightarrow 4(\mathrm{b}+\mathrm{v}^2) = 4\mathrm{T}^2$

$\Rightarrow 4\mathrm{b} + 4\mathrm{v}^2 = 4\mathrm{T}^2,$

$\Rightarrow 4\mathrm{b} = 4\mathrm{T}^2 - 4\mathrm{v}^2$

$$\Rightarrow \frac{4\mathrm{b}}{4} = \frac{4\mathrm{T}^2 - 4\mathrm{v}^2}{4},$$

$$\Rightarrow \mathrm{b} = \frac{4\mathrm{T}^2 - 4\mathrm{v}^2}{4}.$$

When T = 20 and v =1

$\Rightarrow \text{b} = \dfrac{4(20)^2 - 4(1)^2}{4}$

$\Rightarrow \text{b} = \dfrac{4(400) - 4(1)}{4} = \dfrac{1600 - 4}{4}$

$= \dfrac{1596}{4} = 399$.

Q4. If $\pi\sqrt{\frac{l}{g}} = 2T,\ \ calculate\ l\ when\ \pi = 3.14$, g = 6.28 and T = 8

Soln

$\pi\sqrt{\frac{l}{g}} = 2T.$

Square both sides

$\Rightarrow (\pi\sqrt{\frac{l}{g}})^2 = (2T)^2$

$\Rightarrow \pi^2(\sqrt{\frac{l}{g}})^2 = 2^2T^2,$

$\Rightarrow \pi^2\left(\frac{l}{g}\right) = 4T^2$

$\Rightarrow \dfrac{\pi^2 l}{g} = 4T^2,$

$\Rightarrow \dfrac{\pi^2 l}{g} = \dfrac{4T^2}{1}$

$\Rightarrow \pi^2 l\ \times 1 = g \times 4T^2,$

$\Rightarrow \pi^2 l = 4gT^2,$

$\Rightarrow \frac{\pi^2 l}{\pi^2} = \frac{4gT^2}{\pi^2}, \Rightarrow l = \frac{4gT^2}{\pi^2}$

If g = 6.28, T = 8 and π = 3.14

$\Rightarrow l = \dfrac{4(6.28(8)^2}{(3.14)^2} = \dfrac{4(6.28)(64)}{9.9} = 162$.

Q5. You are given the formula $2\pi\sqrt{\frac{2l}{g}} = 3T. Make\ l\ the subject.$

Soln.

$2\pi\sqrt{\frac{2l}{g}} = 3T.$

Square both sides $\Rightarrow$

$(2\pi\sqrt{\frac{2l}{g}})^2 = (3T)^2, \Rightarrow 2^2\pi^2(\sqrt{\frac{2l}{g}})^2 = 3^2T^2, \Rightarrow 4\pi^2(\sqrt{\frac{2l}{g}})^2 = 9T^2, \Rightarrow 4\pi^2 \times \frac{2l}{g} = 9T^2$

$\Rightarrow \frac{8\pi^2 l}{g} = 9T^2, \Rightarrow \frac{8\pi^2 l}{g} = \frac{9T^2}{1}, \Rightarrow 8\pi^2 l \times 1 = 9T^2 \times g, \Rightarrow 8\pi^2 l = 9T^2 g,$

$\Rightarrow \frac{8\pi^2 l}{8\pi^2} = \frac{9T^2 g}{8\pi^2}, \Rightarrow \text{l} = \frac{9T^2 g}{8\pi^2}.$

Q6. If $\sqrt{\frac{2b}{1-TR}} = 3\pi, calculate\ R\ when\ b = 2, T = 5\ and\ \pi = 3.14$.

Soln.

$\sqrt{\frac{2b}{1-TR}} = 3\pi.$

Square both sides $\Rightarrow$

$(\sqrt{\frac{2b}{1-TR}})^2 = (3\pi)^2 \Rightarrow \frac{2\text{b}}{1-\text{TR}} = 3^2\pi^2, \Rightarrow \frac{2\text{b}}{1-\text{TR}} = 9\pi^2 \Rightarrow \frac{2\text{b}}{1-\text{TR}} = \frac{9\pi^2}{1},$

$\Rightarrow 2\text{b} \times 1 = 9\pi^2(1 - TR)$

$\Rightarrow 2\text{b} = 9\pi^2 - 9\pi^2\ TR,$

$\Rightarrow 2\text{b} + 9\pi^2\ TR = 9\pi^2,$

$\Rightarrow 9\pi^2\ TR = 9\pi^2 - 2\text{b},$

$$\Rightarrow \frac{9\pi^2\ TR}{9\pi^2\text{T}} = \frac{9\pi^2 - 2\text{b}}{9\pi^2\text{T}}$$

$\Rightarrow \text{R} = \frac{9\pi^2 - 2\text{b}}{9\pi^2\text{T}}.$

If b = 2, T = 5 and π = 3.14

$\Rightarrow \text{R} = \frac{9(3.14)^2 - 2(2)}{9(3.14)^2(5)} = \frac{9(9.9) - 4}{9(9.9)5} = \frac{89 - 4}{446} = 0.19$

N/B: $a^{1/2} = \sqrt[2]{a}, b^{1/2} = \sqrt[2]{b}, \ 16^{1/2} = \sqrt[2]{16} = 4, \ 25^{1/2} = \sqrt[2]{25} = 5$.

Q1. Given that $(a + b)^{1/2} = 2v$, make b the subject.

. Soln.

$(a + b)^{1/2} = 2v \Rightarrow \sqrt{a+b} = 2v.$

Square both sides $\Rightarrow$

$(\sqrt{a+b})^2 = (2v)^2$

$\Rightarrow a + b = 2^2v^2, \Rightarrow a + b = 4v^2,$

$\Rightarrow b = 4v^2 - a.$

Q2. If $\left(\frac{T-2v}{3}\right)^{1/2} = \frac{b}{2}$, calculate T when $b = 2$ and $v = 1$.

Soln.

$\left(\frac{T-2v}{3}\right)^{1/2} = \frac{b}{2}.$

$$\Rightarrow \sqrt{\frac{T-2v}{3}} = \frac{b}{2}$$

$\Rightarrow$ Square both sides $\Rightarrow$

$\left(\sqrt{\frac{T-2v}{3}}\right)^2 = \left(\frac{b}{2}\right)^2$

$$\Rightarrow \frac{T-2v}{3} = \frac{b^2}{2^2}, \Rightarrow \frac{T-2v}{3} = \frac{b^2}{4}$$

$\Rightarrow 4(T - 2v) = 3 \times b^2,$

$\Rightarrow 4T - 8v = 3b^2$

$\Rightarrow 4T - 8v = 3b^2, \Rightarrow 4T = 3b^2 + 8v.$

$\therefore \frac{4T}{4} = \frac{3b^2 + 8v}{4} \Rightarrow T = \frac{3b^2 + 8v}{4}.$

= If b = 2 and v = 1

$\Rightarrow T = \frac{3(2)^2 + 8(1)}{4} = \frac{3(4)+8}{4},$

$\Rightarrow T = \frac{12 + 8}{4} = \frac{20}{4} = 5$

$\Rightarrow T = 5.$

N/B: $(\sqrt[3]{a})^3 = a\,,(\sqrt[3]{b})^3 = b$

$(\sqrt[3]{27})^3 = 27, (\sqrt[3]{10})^3 = 10$

These given examples indicate that in order to get rid or remove the cube root sign, we only raise the number (the surd) to the third power.

Q3. Given that $\sqrt[3]{2a+b} = 2V, calculate\ a\ when\ b = 2\ and\ V = 3.$

Soln.

$\sqrt[3]{2a+b}$ = 2V.

Raise both sides to the third power .

$\Rightarrow (\sqrt[3]{2\mathrm{a}+\mathrm{b}})^3 = (2\mathrm{v})^3$

$\Rightarrow 2\mathrm{a}+\mathrm{b} = 2^3\mathrm{V}^3, \Rightarrow 2\mathrm{a}+\mathrm{b} = 8\mathrm{V}^3,$

$\Rightarrow 2\mathrm{a} = 8\mathrm{V}^3 - \mathrm{b}\,, \Rightarrow \frac{2\mathrm{a}}{2} = \frac{8\mathrm{V}^3-\mathrm{b}}{2},$

$\Rightarrow \mathrm{a} = \frac{8\mathrm{V}^3 - \mathrm{b}}{2}$

When b =2 and V = 3

$\Rightarrow \mathrm{a} = \frac{8(3)^3-(2)}{2} = \frac{8(27)-2}{2} = \frac{216-2}{2}$ = 107 .

Q4. If $\sqrt[3]{\frac{l}{2g}} = 3\pi$

a) make l the subject.
b) make π the subject.

N/B: The pie (π) is not found under the cube root sign.

Soln.

a) $\sqrt[3]{\frac{l}{2g}} = 3\pi.$

$Raise\ both\ sides\ to\ the\ third\ power \Rightarrow$

$(\sqrt[3]{\frac{l}{2g}}\,)^3 = (3\pi)^3$

$\Rightarrow \frac{\mathrm{l}}{2\mathrm{g}} = 3^3\pi^3, \Rightarrow \frac{\mathrm{l}}{2\mathrm{g}} = 27\pi^3$

$$\Rightarrow \frac{\mathrm{l}}{2\mathrm{g}} = \frac{27\pi^3}{1}, \Rightarrow \mathrm{l} \times 1 = 2\mathrm{g} \times 27\pi^3 \Rightarrow \mathrm{l} = 54\mathrm{g}\pi^3.$$

b) $\sqrt[3]{\frac{1}{2g}} = 3\pi$

Divide both sides by 3

$$\Rightarrow \frac{\sqrt[3]{\frac{1}{2g}}}{3} = \frac{3\pi}{3}$$

$$\Rightarrow \frac{\sqrt[3]{\frac{1}{2g}}}{3} = \pi \Rightarrow \pi = \frac{\sqrt[3]{\frac{1}{2g}}}{3}$$

Q5. Given that $2\pi\sqrt[3]{\frac{\mathrm{l}}{1-\mathrm{g}}} = 3$, calculate *l* when g = 1 and π = 3.14.

Soln.

$$2\pi\sqrt[3]{\frac{l}{1-g}} = 3.$$

Raise both sides to the third power $\Rightarrow (2\pi\sqrt[3]{\frac{l}{1-g}}\ )^3 = 3^3$

$$\Rightarrow (2\pi)^3 (\sqrt[3]{\frac{l}{1-g}})^3 = 27,$$

$$\Rightarrow 2^3\pi^3\left(\frac{\mathrm{l}}{1-\mathrm{g}}\right) = 27$$

$$\Rightarrow 8\pi^3 \times \frac{\mathrm{l}}{1-\mathrm{g}} = 27,$$

$$\Rightarrow \frac{8\pi^3\mathrm{l}}{1-\mathrm{g}} = 27 \Rightarrow 27(1-\mathrm{g})$$

$$= 8\pi^3\mathrm{l}, \Rightarrow 27 - 27\mathrm{g} = 8\pi^3\mathrm{l}\ ,$$

$$\therefore \frac{27-27\mathrm{g}}{8\pi^3} = \frac{8\pi^3\mathrm{l}}{8\pi^3} \Rightarrow l = \frac{27-27\mathrm{g}}{8\pi^3}$$

When π = 3.14 and g = 1,

$$\Rightarrow \mathrm{l} = \frac{27-27(1)}{8(3.14)^3} = \frac{27-27}{8(31)} = \frac{0}{248}$$

$$\Rightarrow \mathrm{l} = 0\ .$$

N/B: $a^{1/3} = \sqrt[3]{a}$, $10^{1/3} = \sqrt[3]{10}$, $20^{1/3} = \sqrt[3]{20}$, $9^{1/3} = \sqrt[3]{9}$.

Q6. Given that $(\frac{T}{v^2-3})^{1/3}$ = ab², make T the subject.

Soln.

$(\frac{T}{v^2-3})^{1/3} = ab^2$

$\Rightarrow \sqrt[3]{\frac{T}{v^2-3}} = ab^2$

Raise both sides to the third power $\Rightarrow (\sqrt[3]{\frac{T}{v^2-3}})^3 = (ab^2)^3$

$\Rightarrow \frac{T}{v^2-3} = a^3b^6, \Rightarrow \frac{T}{v^2-3} = \frac{a^3b^6}{1},$

$\Rightarrow T \times 1 = a^3b^6(v^2-3),$

$\Rightarrow T = a^3b^6v^2 - 3a^3b^6.$

N/B: $(a^2b^3)^4 = (a^8b^{12})$

$(xy^5)^2 = (x^1y^5)^2 = x^2y^{10}$

Q7. If $(2a-b)^{1/3} = 2v^3$, *make b the subject*.

Soln.

$(2a-b)^{1/3} = 2v^3$

$\Rightarrow \sqrt[3]{2a-b} = 2v^3$

Raise both sides to the third power $\Rightarrow (\sqrt[3]{2a-b})^3 = (2v^3)^3$

$\Rightarrow 2a - b = 2^3v^9, \Rightarrow 2a - b = 8v^9, \Rightarrow 2a = 8v^9 + b \Rightarrow 2a - 8v^9 = b,$

$\Rightarrow b = 2a - 8v^9$

N/B: - In order to remove the fourth root sign, raise the number (surd) to the fourth power. $(\sqrt[4]{a})^4 = a$, $(\sqrt[4]{10})^4 = 10.$

Q8. *Given that* $\sqrt[4]{\frac{N}{2-b}} = 2T$, calculate N when $T = 2$ and $b = 1$.

Soln.

$\sqrt[4]{\frac{N}{2-b}} = 2T.$

Raise both sides to the power four.

$\Rightarrow (\sqrt[4]{\frac{N}{2-b}})^4 = (2T)^4$

$\Rightarrow \frac{\mathrm{N}}{2-\mathrm{b}} = 2^4\mathrm{T}^4 \Rightarrow \frac{\mathrm{N}}{2-\mathrm{b}} = 16\mathrm{T}^4,$

$\Rightarrow \frac{\mathrm{N}}{2-\mathrm{b}} = \frac{16\mathrm{T}^4}{1} \Rightarrow 1 \times \mathrm{N} = 16\mathrm{T}^4(2-\mathrm{b}),$

$\Rightarrow \mathrm{N} = 32\mathrm{T}^4 - 16\mathrm{T}^4\mathrm{b}.$

When T = 2 and b = 1

$\Rightarrow \mathrm{N} = 32(2)^4 - 16(2)^4(1)$

$\Rightarrow \mathrm{N} = 512 - 256 = 256\,.$

Q9. The relationship between three variables v, r and l is shown in the formula $(\frac{v}{1+r^2})^{1/4} = 3l.$

Determine the value of v when l = 2 and r = 1.

Soln.

$(\frac{v}{1+r^2})^{1/4} = 3l$

$\Rightarrow \sqrt[4]{\frac{\mathrm{v}}{1+\mathrm{r}^2}} = 3\mathrm{l}.$

Raise both sides to the power four $\Rightarrow (\sqrt[4]{\frac{\mathrm{v}}{1+\mathrm{r}^2}})^4 = (3\mathrm{l})^4$

$\Rightarrow \frac{\mathrm{v}}{1+\mathrm{r}^2} = 3^4\mathrm{l}^4, \Rightarrow \frac{\mathrm{v}}{1+\mathrm{r}^2} = 81\mathrm{l}^4$

$\Rightarrow \frac{\mathrm{v}}{1+\mathrm{r}^2} = \frac{81\mathrm{l}^4}{1}, \Rightarrow \mathrm{v} \times 1 = 81\mathrm{l}^4(1+\mathrm{r}^2)$

$\Rightarrow \mathrm{v} = 81\mathrm{l}^4(1+\mathrm{r}^2).$

When l = 2 and r = 1

$\Rightarrow \mathrm{v} = \; 81(2)^4(1+(1)^2),$

$\Rightarrow \mathrm{v} = 81(16)(1+1) = 81 \times 16 \times 2,$

$\Rightarrow \mathrm{v} = 2592$.

N/B: $(\sqrt[5]{a})^5 = a,\ (\sqrt[5]{20})^5 = 20$

This implies that in order to remove the fifth root sign, the number (surd) must be raised to the fifth power.

Q10. Given that $\sqrt[5]{\frac{a-1}{b}} = 2v, make a the subject.$

Soln.

$\sqrt[5]{\frac{a-1}{b}} = 2v.$

Raise both sides to the fifth power $\Rightarrow (\sqrt[5]{\frac{a-1}{b}}\ )^5 = (2v)^5$

$\Rightarrow \dfrac{\mathrm{a}-1}{\mathrm{b}} = 2^5\mathrm{v}^5, \Rightarrow \dfrac{\mathrm{a}-1}{\mathrm{b}} = 32\mathrm{v}^5$

$\Rightarrow \frac{\mathrm{a}-1}{\mathrm{b}} = \frac{32\mathrm{v}^5}{1}, \Rightarrow 1(\mathrm{a}-1) = \mathrm{b} \times 32\mathrm{v}^5,$

$\Rightarrow \mathrm{a} - 1 = 32\mathrm{v}^5\mathrm{b} \Rightarrow \mathrm{a} = 32\mathrm{v}^5\mathrm{b} + 1$

Q11. If $(\frac{3\pi}{g}\ )^{1/5} = v$

Make π the subject.

Soln.

$(\frac{3\pi}{g}\ )^{1/5} = v$

$\Rightarrow \sqrt[5]{\dfrac{3\pi}{\mathrm{g}}} = \mathrm{v}.$

Raise both sides to the fifth power $\Rightarrow (\sqrt[5]{\frac{3\pi}{\mathrm{g}}}\ )^5 = \mathrm{v}^5$

$\Rightarrow \dfrac{3\pi}{\mathrm{g}} = \mathrm{v}^5, \Rightarrow \dfrac{3\pi}{\mathrm{g}} = \dfrac{\mathrm{v}^5}{1},$

=> $3\pi \times 1 = \mathrm{v}^5 \times\ \mathrm{g}\ \Rightarrow 3\pi = \mathrm{g}\mathrm{v}^5,$

$\Rightarrow \frac{3\pi}{3} = \frac{gv^5}{3} \Rightarrow \pi = \frac{gv^5}{3}$.

Q12. If $2\pi\,(\frac{l}{g})^{1/2} = T$, make l the subject.

Soln.

$2\pi\,(\frac{l}{g})^{1/2} = T$

$\Rightarrow {}^{2\pi}\sqrt{\frac{l}{g}} = T$

Square both sides

$\Rightarrow ({}^{2\pi}\sqrt{\frac{l}{g}})^2 = T^2$

$\Rightarrow (2\pi)^2(\sqrt{\frac{l}{g}})^2 = T^2,$

$\Rightarrow 4\pi^2\left(\frac{l}{g}\right) = T^2$

$\Rightarrow \frac{4\pi^2 l}{g} = T^2, \Rightarrow \frac{4\pi^2 l}{g} = \frac{T^2}{1}$

$\Rightarrow 4\pi^2 l \times l = g \times T^2 \Rightarrow 4\pi^2 l = gT^2,$

$\Rightarrow \frac{4\pi^2 l}{4\pi^2} = \frac{gT^2}{4\pi^2}$

$\Rightarrow l = \frac{gT^2}{4\pi^2}.$

Q13. Given that $\pi(\frac{1}{2b})^{1/3} = a, calculate\ b\ when\ \pi = 3.14\ and\ a = 2$.

Soln.

$\pi(\frac{1}{2b})^{1/3} = a$

$\Rightarrow \pi\sqrt[3]{\frac{1}{2b}} = a.$

Raise both sides to the third power $\Rightarrow (\pi\sqrt[3]{\frac{1}{2b}})^3 = a^3$

$\Rightarrow \pi^3(\sqrt[3]{\frac{1}{2b}})^3 = a^3,$

$\Rightarrow \pi^3\left(\frac{1}{2b}\right) = a^3$

$\Rightarrow \pi^3 \times \frac{1}{2b} = a^3,$

$\Rightarrow \frac{\pi^3}{2b} = a^3 \Rightarrow \frac{\pi^3}{2b} = \frac{a^3}{1},$

$\Rightarrow \pi^3 \times 1 = 2b \times a^3$

$\Rightarrow \pi^3 = 2ba^3,$

$\Rightarrow \frac{\pi^3}{2a^3} = \frac{2ba^3}{2a^3} \Rightarrow \frac{\pi^3}{2a^3} = b,$

$\Rightarrow b = \frac{\pi^3}{2a^3}.$

If π = 3.14 and a = 2

$\Rightarrow b = \frac{(3.14)^3}{2(2)^3} = \frac{31}{2(8)} = \frac{31}{16} = 1.9.$

Q14. If $2(\frac{b+1}{2})^{1/4} = V$, make b the subject.

Soln.

$2(\frac{b+1}{2})^{1/4} = V$

$\Rightarrow 2\sqrt[4]{\frac{b+1}{2}} = V.$

Raise both sides to the fourth power

$\Rightarrow (2\sqrt[4]{\frac{b+1}{2}})^4 = V^4$

$\Rightarrow 2^4(\sqrt[4]{\frac{b+1}{2}})^4 = V^4,$

$\Rightarrow 16\left(\frac{b+1}{2}\right) = V^4$

$\Rightarrow \frac{16}{2}(\mathrm{b}+1) = \mathrm{V}^4,$

$\Rightarrow 8(\mathrm{b}+1) = \mathrm{V}^4$

$\therefore$ 8b + 8 = V^4,

$\Rightarrow 8\mathrm{b} = \mathrm{V}^4 - 8$

$\Rightarrow \frac{8\mathrm{b}}{8} = \frac{\mathrm{V}^4 - 8}{8}$

$\Rightarrow \mathrm{b} = \frac{\mathrm{V}^4 - 8}{8}.$

N/B: If a number is squared or raised the power 2 and we want to remove the power or the exponent, we just find square root of the given number.

Examples:

$(\sqrt{a^2})^2 = a$ $\quad$ (2)$(\sqrt{8^2})^2 = 8$

(3) $(\sqrt{3^2})^2 = 3$ $\quad$ (4)$(\sqrt{b^2})^2 = b$

Q1. If (2b + v)$^2 = T, find\ v\ when\ T = 25\ and\ b = 2$.

Soln.

(2b + v)$^2 = T$.

Find the square root of both sides $\Rightarrow \sqrt{(2\mathrm{b}+\mathrm{v})^2} = \sqrt{\mathrm{T}}$

$\Rightarrow 2\mathrm{b} + \mathrm{v} = \sqrt{\mathrm{T}}, \Rightarrow \mathrm{v} = \sqrt{\mathrm{T}} - 2b.$

If T = 25 and b = 2

$\Rightarrow \mathrm{v} = \sqrt{25} - 2(2)$

$\Rightarrow \mathrm{v} = 5 - 4 \Rightarrow \mathrm{v} = 1$.

Q2. Given that $(\frac{\mathrm{l}}{2\mathrm{g}})^2 = 5\mathrm{T}$, make l the subject.

Soln.

$(\frac{l}{2g})^2 = 5T.$

Find the square root of both sides $\Rightarrow$

$$\sqrt{(\frac{l}{2g})^2} = \sqrt{5T}$$

$$\Rightarrow \frac{\mathrm{l}}{2\mathrm{g}} = \sqrt{5\mathrm{T}}, \Rightarrow \frac{\mathrm{l}}{2\mathrm{g}} = \frac{\sqrt{5\mathrm{T}}}{1}$$

$$\Rightarrow \mathrm{l} \times 1 = 2\mathrm{g} \times \sqrt{5\mathrm{T}}, \Rightarrow \mathrm{l} = 2\mathrm{g}\sqrt{5\mathrm{T}}$$

N/B: If a number is raised to the third power and we want to remove the power, we must find the cube root of the number.

Examples: $\sqrt[3]{a^3} = a$ (2) $\sqrt[3]{10^3} = 10$

(3) $\sqrt[3]{b^3} = b$ (4) $\sqrt[3]{20^3} = 20$

Q3. If (2b + 1 $)^3 = 3v$, *make*

1) b the subject.
11) v the subject.

Soln.

(2b + 1 $)^3 = 3v$

Find the cube root of both sides $\Rightarrow \sqrt[3]{(2\mathrm{b}+1)^3} = \sqrt[3]{3\mathrm{v}}$

$$\Rightarrow 2\mathrm{b} + 1 = \sqrt[3]{3\mathrm{v}}, \Rightarrow 2\mathrm{b} = \sqrt[3]{3\mathrm{v}} - 1,$$

$$\Rightarrow \frac{2\mathrm{b}}{2} = \frac{\sqrt[3]{3\mathrm{v}}-1}{2} \Rightarrow \mathrm{b} = \frac{\sqrt[3]{3\mathrm{v}}-1}{2}.$$

(11) (2b + 1 $)^3 = 3v$

Divide through using 3

$$\Rightarrow \frac{(2\mathrm{b} + 1)^3}{3} = \frac{3\mathrm{v}}{3}$$

$$\Rightarrow \frac{(2\mathrm{b} + 1)^3}{3} = \mathrm{v}, \Rightarrow \mathrm{v} = \frac{(2\mathrm{b} + 1)^3}{3}$$

Q4. Given the formula $(\frac{b-1}{v})^3 = T$, *calculate b when* $v = 1$ *and* $T = 8$.

Soln.

$(\frac{b-1}{v})^3 = T$.

Find the cube root of both sides

$$\Rightarrow \sqrt[3]{(\frac{b-1}{v})^3} = \sqrt[3]{T}$$

$$\Rightarrow \frac{b-1}{v} = \sqrt[3]{T}, \Rightarrow \frac{b-1}{v} = \frac{\sqrt[3]{T}}{1} \Rightarrow 1(b-1) = v \times \sqrt[3]{T}, \Rightarrow b-1 = \sqrt[3]{T}V \Rightarrow b = \sqrt[3]{T}V + 1.$$

If v = 1 and T = 8

$$\Rightarrow b = \sqrt[3]{8}(1) + 1 = 2$$

Questions:

Q1. Given that $3m^2v = 15, calculate\ v\ when\ m = 5.$

Ans: 0.2 .

Q2. If 3ab + v = 3w, a) make a the subject.

Ans: a = $\frac{3w - v}{3b}$.

b) find a when w = 10, v = 15 and b = 1.

Ans: a = 5 .

Q3. The variables v, u, s and b are connected by the formula $v^2 = 2us - b$.Make u the subject..

Ans: u = $\frac{v^2 + b}{2s}$

Calculate u when v = 10, b = 20 and s = 15.

Ans: 4

Q4. Given the formula 2a$t^2 - uv = 3$,

a) make t the subject.

Ans: t = $\sqrt{\frac{3 + uv}{2a}}$

b) calculate t when u = 10, v = 5 and a = 1.

Ans: 26.5 .

Q5. Given the formula $\frac{8v}{T} = 3k, determine\ the\ value\ of\ v\ when\ k = 1\ and\ T = 16.$

Ans: 6

Q6. A certain equation is such that $\frac{1 + 2v}{3} = \frac{T}{2}$.Make v the subject.
Ans: v = $\frac{3T-2}{4}\ \ or\ v = \frac{3T}{4} - \frac{1}{2}$.

Q7. If $\frac{1}{3}\ at^2 = v, (i)\ make\ a\ the\ subject.$

Ans: a = $\frac{3v}{t^2}$.

(ii) calculate a when v = 40 and t = 2.

Ans: a = 30 .

Q8. Given the formula$\frac{1}{2}at - \frac{1}{3}v = 2b$,

make a the subject.

Ans: a = $\frac{12b + 2v}{3t},\ or\ a = \frac{12b}{3t} + \frac{2v}{3t}$.

Q9. In a certain formula, $\frac{2}{kQ} + \frac{1}{Q} = v$.

a) Make Q the subject.

Ans: Q = $\frac{2+k}{kv}$.

b) Find Q when k = 2 and v = 1

Ans: Q = 2 . $Q10. If\ \frac{1}{ab} + \frac{v}{c} =$
$2c, make\ v\ the\ subject\ of\ the\ given\ formula.$

Ans: v =$\frac{2abc^2 - c}{ab}$.

Q11. Given that $\sqrt{b+3v} = Q, calculate\ b\ when\ v = 1\ and\ Q = 6$.

Ans: Q = 33

Q12. If $\sqrt{\frac{1}{2\,b - c}} = 2v, make\ v\ the\ subject.$

Ans: v = $\frac{\sqrt{\frac{1}{2b-c}}}{2}$

Q13. Given that 2π$\sqrt{l/g} = T,$

a) make g the subject.

Ans: g = $\frac{4\pi^2 l}{T}$

b) calculate g when π= 3.14. l = 10 and T = 5 .

Ans: g = 79 .

Q14. Given that 2$\sqrt{\frac{1-T}{2}} = v, make\ T\ the\ subject.$

Ans: T = $\frac{4 - 2v^2}{4}$.

Q15. If $(\frac{2b-1}{a})^{1/2} = T/_2, make\ T\ the\ subject.$

Ans: T = $\sqrt{\frac{8b-4}{a}}$.

Q16. If T $(\frac{\pi}{g})^{1/2} = 2v, make\ g\ the\ subject.$

Ans: g = $\frac{T^2\pi}{4v^2}$.

Q17. Given the formula $\sqrt[3]{l/g} = T, calculate\ l\ when\ g = 10\ and\ T = 2$.

Ans: 80 .

Q18. Given the formula $2\pi(\frac{2l}{g})^{1/3} = 2T,\ make\ g\ the\ subject.$

Ans: g = $\frac{2\pi^3 l}{T^3}$.