CHAPTER TEN
THE STRAIGHT LINE

The gradient:

The gradient of a line which passes through the points (x_1,y_1) and (x_2, y_2) is given by $\frac{y_2 - y_1}{x_2 - x_1}$. The gradient is also called the slope.

Q1. Find the gradient of the line passing through the points (2, 5) and (7, 4).

Solution

Let (x_1 , y_1) = (2, 5)=>x_1 = 2 and y_1 = 5.
Also let (x_2, y_2) = (7 4) => x_2, = 7 and y_2 = 4.
The gradient = $\frac{y_2 - y_1,}{x_2 - x_1,}$ = $\frac{4 - 5}{7 - 2}$ = $\frac{-1}{5}$ = -0.2.

Q2. Find the slope or the gradient of the line which joins the points (-5, 2) and (8, -4).

Solution

Let (x_1 , y_1) = (-5, 2) =>x_1 = -5 and y_1, = 2.
Also let (x_2, y_2) = (8,- 4)
=> x_2 = 8 and y_2 = - 4.
The slope = $\frac{y_2 - y_1}{x_2 - x_1}$ = $\frac{- 4 - 2}{8 - (-5)}$ = $\frac{- 6}{8+5}$ = $\frac{- 0.46}{13}$.

Q3. Determine the slope of the line which joins the points (- 2, - 4) and (- 6, - 8).

Solution

Let (x_1, y_1) = (-2, - 4) => x_1 = -2 and y_1, = - 4.
Also let (x_2, y_2) = (- 6, - 8) => x_2 = - 6 and y_2 = - 8.
The gradient = $\frac{y_2 - y_1,}{x_2 - x_1,}$ = $\frac{- 8 - (-4)}{- 6 - (-2)}$ = $\frac{- 8 + 4}{- 6 + 2}$ = $\frac{- 4}{- 4}$ = 1.

Q4. The gradient of the line which passes through the points (2,4) and (6,Q) is 2. Find the value of Q.

Solution

Let (x_1, y_1) = (2,4) => x_1 = 2 and y_1= 4.
Also let (x_2, y_2) = (6,Q) => x_2 = 6 and y_2 = Q.
The gradient = $\frac{y_2 - y_1,}{x_2 - x_1,}$ = $\frac{Q - 4}{6 - 2}$
Since the graduate = 2, then 2 = $\frac{Q - 4}{6 - 2}$ => 2 = $\frac{Q - 4}{4}$
=>2 x 4 = Q – 4 => 8 = Q - 4
=> 8 + 4 = Q => Q = 12.

Q5. If the slope of the line which joins the points (1, 6) and (m, - 4) is 5, determine the value of m.

Solution

Let $(x_1, y_1) = (1, 6) \Rightarrow x_1 = 1$ and $y_1 = 6$.

Also let $(x_2, y_2) = (m, -4) \Rightarrow x_2 = m$ and $y_2 = -4$.

Slope $= \frac{y_2 - y_1,}{x_2 - x_1,} = -\frac{4 - 6}{m - 1}$

Since the slope = 5, then $5 = -\frac{4 - 6}{m - 1}$

$\Rightarrow 5(m - 1) = -10$

$\Rightarrow 5m - 5 = -10 \Rightarrow 5m = -10 + 5,$

$\Rightarrow 5m = -5 \Rightarrow m = \frac{-5}{5} = -1.$

Q6.Determine the gradient of the line which joins the points $(^1/_2, ^1/_4)$ and $(^2/_3, ^3/_5)$.

Solution

Since ½ = 0.2, ¼ = 0.25, $^2/_3 = 0.66$ and $^3/_5 = 0.6$, then

the line joins the points (0.2, 0.25) and (0.66, 0.6).

Let $(x_1, y_1) = (0.2, 0.25) \Rightarrow x_1 = 0.2$ and $y_1 = 0.25$.

Also let $(x_2, y_2) = (0.66, 0.6)$,

$\Rightarrow x_2 = 0.66$ and $y_2 = 0.6$.

The gradient $= \frac{y_2 - y_1,}{x_2 - x_1,} = \frac{0.6 - 0.25}{0.66 - 0.2} = \frac{0.35}{0.46} = 0.8.$

Q7. Determine the slope of the line joining the points $(^{-4}/_5, ^{-1}/_3)$ and $(^3/_7, ^{-1}/_6)$.

Solution

$^{-4}/_5 = -0.8$, $^{-1}/_3 = -0.33$, $^3/_7 = 0.4$ and $^{-1}/_6 = -0.17$.

The line therefore joins the points (- 0.8, - 0.33) and

(0.4, - 0.17).

Let $(x_1, y_1) = (-0.8, -0.33) \Rightarrow x_1 = -0.8$ and $y_1 = -0.33$.

Also let $(x_2, y_2) = (0.4, -0.17) \Rightarrow x_2 = 0.4$ and $y_2 = -0.17$.

Slope $= \frac{y_2 - y_1,}{x_2 - x_1,} = \frac{-0.17 - (-0.33)}{0.4 - (-0.8)} = -\frac{0.17 + 0.33}{0.4 + 0.8}$

$= \frac{0.16}{1.2} = 0.13$

Types of equations of straight line:

There are two types of equations of the straight line

and these are (1) y = mx (2) y = mx + c

Lines with equation of the form y = mx:

- In the equation y = mx, m = the slope or the gradient.
- All such lines pass through the origin or the point (0, 0).
- Examples are lines with equations:

(a) y = 2x, in which the slope is 2.

(b) y = 5x in which the slope is 5.

(c) y = -3x in which the gradient = -3.

Lines with equation of the form y = mx + c:

- In y = mx + c, m = the gradient and c = the y intercept i.e. the point where the graph or line cuts the y-axis.
- Examples of such line graphs are

(1) y = 2x +5, in which the slope is 2 and the y intercept is 5.
(2) y = -3x + 4, in which the slope =-3 and the y intercept = 4.

The equation $y - y_1 = m(x - x_1)$:

- The equation of the straight line with slope m and which passes through the point (x_1, y_1), is given by $y - y_1 = m(x - x_1)$.
- This equation is used when the slope of the line as well as one point through which the line passes are given.

Q1. Find the equation of the line which passes through the point (4, 2) and whose slope is 5.

Solution

Since the slope is 5 => m = 5.
Let $(x_1, y_1) = (4, 2) \Rightarrow x_1 = 4$ and $y_1 = 2$.
Using $y - y_1 = m(x - x_1)$
$\Rightarrow y - 2 = 5(x - 4)$,
$\Rightarrow y - 2 = 5x - 20 \Rightarrow y = 5x - 20 + 2$,
$\Rightarrow y = 5x - 18$.

Q2 Determine the equation of the line whose slope is -3, if it passes through the point (- 4, 8).

Solution

The slope = -3 => m = -3.
Let $(x_1, y_1) = (-4, 8) \Rightarrow x_1 = -4$ and $y_1 = 8$.
Since $y - y_1 = m(x - x_1)$, then $y - 8 = -3\{x - (-4)\}$,
$\Rightarrow y - 8 = -3\{x + 4\}$
$\Rightarrow y - 8 = -3x - 12$,
$\Rightarrow y = -3x - 12 + 8$
$\Rightarrow y = -3x - 4$.

Q3. A line which has a gradient of $^{-1}/_{3}$ passes through the point (- 6, -12). Determine its equation.

Solution

The slope of the line = $^{-1}/_{3}$
$\Rightarrow m = {}^{-1}/_{3} = -0.33$.
Let $(x_1, y_1) = (-6, -12)$
$\Rightarrow x_1 = -6$ and $y_1 = -12$.
But $y - y_1 = m(x - x_1)$
$\Rightarrow y - (-12) = -0.33(x - (-6))$,
$\Rightarrow y + 12 = -0.33(x + 6)$
$\Rightarrow y + 12 = -0.33x - 1.98$,
$\Rightarrow y = -0.33x - 1.98 - 12$
$\Rightarrow y = -0.33x - 13.98$.

Q4. Determine the equation of the line whose slope is $^{-1}/_{2}$, if it passes through the point ($^{-3}/_{4}$, $^{-2}/_{5}$).

Solution

Since the slope = $^{-1}/_{2}$ => m = $^{-1}/_{2}$ = - 0.5.
$^{-3}/_{4}$ = - 0.75 and $^{-2}/_{5}$ = - 0.4,
=> the line passes through the point (- 0.75, -0.4).
Let (x_1, y_1) = (- 0.75, - 0.4)
=> x_1 = - 0.75 and y_1 = -0.4.
From $y - y_1 = m (x - x_1)$
=>$y - (- 0.4) = - 0.5 (x - (-0.75))$,
=> $y + 0.4 = - 0.5(x + 0.75)$
=> $y + 0.4 = - 0.5x - 0.38$,
=> $y = - 0.5x - 0.38 - 0.4$
=> $y = - 0.5x - 0.78$.

Q5. Determine the equation of the line, whose slope is $^{-2}/_{3}$, if it passes through the point ($-1^{1}/_{2}$, $1^{1}/_{3}$).

Solution

Since the slope = $^{-2}/_{3}$ => m = $^{-2}/_{3}$ = - 0.66.
$1^{1}/_{2}$ = $^{-3}/_{2}$ = -1.5 and $1^{1}/_{3}$ = $^{4}/_{3}$ = 1.33,
=> the line passes through the point (-1.5, 1.33).
Let (x_1, y_1) = (-1.5, 1.33) => x_1 = -1.5 and y_1 = 1.33.
From $y - y_1 = m (x - x_1)$
=> $y - 1.33 = - 0.66 \{x - (-1.5)\}$,
=> $y - 1.33 = - 0.66 (x + 1.5)$
=> $y - 1.33 = - 0.66x - 0.99$,
=> $y = - 0.66x - 0.99 + 1.33$
=> $y = - 0.66x + 1.33 - 0.99$,
=> $y = - 0.66x + 0.34$.

The equation $y - y_1 = \frac{y_2 - y_1}{x_2 - x_1} (x - x_1)$

- This formula is used when the gradient is not given, but two points through which the line passes are given.
- Also in this given formula, $\frac{y_2 - y_1}{x_2 - x_1}$ = the slope.

Q1. Find the equation of the line which passes through the points (4, 2) and (5, 3).

Solution

Let (x_1,y_1) = (4,2) => x_1 = 4 and y_1 = 2.
Also let (x_2, y_2) = (5, 3) => x_2 = 5 and y_2 = 3.

But $y - y_1 = \frac{y_2 - y_1}{x_2 - x_1} (x - x_1)$

=> $y - 2 = \frac{3 - 2}{5 - 4} (x - 4)$
=> $y - 2 = {}^{1}/_{1} (x - 4)$
=> $y - 2 = 1 (x - 4)$,

=> $y - 2 = x - 4$ => $y = x - 4 + 2$
=> $y = x - 2$.

Q2. A line joins the points (8, 5) and (- 6, 4). Determine its equation.

Solution

Let $(x_1, y_1) = (8, 5)$ => $x_1 = 8$ and $y_1 = 5$.
Also let $(x_2, y_2) = (- 6, 4)$ => $x_2 = - 6$ and $y_2 = 4$.

Since $y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$

Then $y - 5 = \frac{4 - 5}{- 6 - 8}(x - 8)$
=> $y - 5 = \frac{-1}{-14}(x - 8)$
=> $y - 5 = 0.07 (x - 8)$,
=> $y - 5 = 0.07x - 0.56$
=> $y = 0.07x - 0.56 + 5$,
=> $y = 0.07x + 5 - 0.56$
=> $y = 0.07x + 4.44$.

Q3. Find the equation of the line joining the points (- 6, - 2) and (- 3, 4).

Solution

Let $(x_1, y_1) = (- 6, -2)$ => $x_1 = - 6$ and $y_1 = -2$.
Also let $(x_2, y_2) = (- 3, 4)$
=> $x_2 = -3$ and $y_2 = 4$.
Since $y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$
$=> y - (- 2) = \frac{4 - (-2)}{-3 - (-6)}\{x - (-6)\}$
=> $y + 2 = \frac{4 + 2}{-3 + 6}(x + 6)$,
=> $y + 2 = \frac{6}{3}(x + 6)$,
=> $y + 2 = 2 (x + 6)$
=> $y + 2 = 2x + 12$,
=> $y = 2x + 12 - 2$
=> $y = 2x + 10$.

Q4. A line joins the points (-2, - 6) and (- 4, -1). Determine its equation.

Solution

Let $(x_1, y_1) = (-2, - 6)$ => $x_1 = -2$ and $y_1 = - 6$.
Also let $(x_2, y_2) = (- 4, -1)$
=> $x_2 = - 4$ and $y_2 = -1$.
Since $y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$

then $y - (-6) = \frac{-1 - (-6)}{-4 - (-2)}(x - -2)$

$\Rightarrow y + 6 = \frac{-1 + 6}{-4 + 2}(x + 2)$

$\Rightarrow y + 6 = \frac{5}{-2}(x + 2)$

$\Rightarrow y + 6 = -2.5\,(x + 2)$,

$\Rightarrow y + 6 = -2.5x - 5$

$\Rightarrow y = -2.5x - 5 - 6$,

$\Rightarrow y = -2.5x - 11$.

Q5. Find the equation of the line which joins the points $(^1/_2, {}^3/_4)$ and $(^4/_5, {}^3/_{10})$.

Solution

$^1/_2 = 0.5$, $^3/_4 = 0.75$, $^4/_5 = 0.8$ and $^3/_{10} = 0.3$,

$\Rightarrow$ the line joins the points (0.5, 0.75) and (0.8, 0.3).

Let $(x_1, y_1) = (0.5, 0.75)$

$\Rightarrow x_1 = 0.5$ and $y_1 = 0.75$.

Also let $(x_2, y_2) = (0.8, 0.3)$

$\Rightarrow x_2 = 0.8$ and $y_2 = 0.3$.

Using $y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$

$\Rightarrow y - 0.75 = \frac{0.3 - 0.75}{0.8 - 0.5}(x - 0.5)$

$\Rightarrow y - 0.75 = \frac{-0.45}{0.3}(x - 0.5)$

$\Rightarrow y - 0.75 = -1.5\,(x - 0.5)$,

$\Rightarrow y - 0.75 = -1.5x + 0.75$

$\Rightarrow y = -1.5x + 0.75 + 0.75$,

$\Rightarrow y = -1.5x + 1.5$.

Q6) Determine the equation of the line which joins the points $(^{-4}/_5, -1^1/_4)$ and $(^{-3}/_4, 1^1/_3)$.

Solution

$^{-4}/_5 = -0.8$, $-1^1/_4 = {}^{-5}/_4 = -1.25$, $^{-3}/_4 = -0.75$ and

$1^1/_3 = {}^4/_3 = 1.33$

The line therefore joins the points (- 0.8, -1.25) and

(- 0.75, 1.33).

Let $(x_1, y_1) = (-0.8, -1.25)$

$\Rightarrow x_1 = -0.8$ and $y_1 = -1.25$.

Also let $(x_2, y_2) = (-0.75, 1.33)$

$\Rightarrow x_2 = -0.75$ and $y_2 = 1.33$.

From $y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$

$\Rightarrow y - (-1.25) = \frac{1.33 - (-1.25)}{-0.75 - (-0.8)}\{x - (-0.8)\}$

$\Rightarrow y + 1.25 = \frac{1.33 + 1.25}{-0.75 + 0.8}(x + 0.8)$,

$\Rightarrow y + 1.25 = \frac{2.58}{0.05}(x + 0.8)$,

$\Rightarrow y + 1.25 = 52\,(x + 0.8)$,

$\Rightarrow y + 1.25 = 52x + 41.6$

=> y = 52x + 41.6 – 1.25,
=> y = 52x + 40.4.

Method of the determination of the x and the y intercepts of a straight line graph or a line:

If you are given the equation of a line and you are
asked to (1).determine the y intercept, we put x = 0 and solve for y.
2. determine the x intercept, we put y = 0 and solve for x.

Q1. The equation of a line is given as y = 4x + 2. Determine its y and x intercepts.

Solution

y = 4x + 2
For the y intercept, we put x = 0
=> y = 4(0) + 2 = 0 +2 = 2
=> the y intercept = 2.
For the x intercept, we put y = 0
=> 0 = 4x + 2 => 0 – 4x = 2, => - 4x = 2
=> x = $^{-2}/_{4}$ = - 0.5
=> the x intercept = - 0.5

Q2. A line has y = - 6x – 8 as its equation. Find its x and y intercepts.

Solution

y = - 6x – 8
For the y intercept, put x = 0
=> y = - 6 (0) – 8 => y = 0 – 8,
=> y = - 8 => the y intercept = - 8.
For the x intercept, put y = 0
=> 0 = - 6x – 8 => 6x = - 8,
=> x = $^{-8}/_{6}$ => x = -1.33.

Q3. The equation of a line is given by y – 4x + 2 = 0. Determine its x and y intercepts.
N/B: You must first make y the subject

Solution

y - 4x + 2 = 0 => y = 0 + 4x – 2, => y = 4x – 2.
For the x intercept, we put y = 0
=> 0 = 4x – 2, => 0 - 4x = -2
=> - 4x = -2, => x = $^{-2}/_{-4}$ = 0.5,
=> the x intercept = 0.5.
For the y intercept, put x = 0 => y = 4(0) – 2.
=> y = 0 – 2 => y = -2, => the y intercept = -2.

Method of finding the intecepts from the equation of a line:

If the equation of a line is of the form y = mx + c, then m = the slope and c = the y intercept.

Q1. Determine the gradient and the y intercept of the line y = -3x + 1.

Solution

The slope = -3 and the y intercept = 1.

Q2. The equation of a line is y = -2x – 6. Find its slope and its y intercept.

Solution

The slope = -2 and the y intercept = - 6.

Q3. The equation of a line is given as y -3x = 4. Find its slope as well as the y intercept.
NB: You must first make y the subject of the equation.

Solution

y -3x = 4 => y = 4 + 3x
=> y = 3x + 4.
The slope = 3 and y intercept = 4

Q4.The equation of a line is given by y + 2x – 4 = 0.
Determine its slope and its y intercept.
N/B: Make y the subject first.

Solution

y + 2x – 4 = 0 => y + 2x = 0 + 4,
=> y + 2x = 4 => y = 4 – 2x,
=> y = -2x + 4.
The slope = -2 and the y intercept = 4.

Q5. A straight line has 3x + y – 4 = 0 as its equation. Find its y and x intercepts.

Solution

3x + y – 4 = 0 => 3x + y = 0 + 4,
=>3x + y = 4 => y = 4 – 3x,
=> y = -3x + 4.
For the y intercept, put x = 0 => y = -3(0) + 4 => y = 0 + 4 = 4.
For the x intercept, put y = 0
=> 0 = -3x + 4 => 0 +3x = 4,
=> 3x = 4 => x = $^{4}/_{3}$ = 1.3.

Q6. The equation of a line is given as 2y – 4x + 8 = 0. Find its y and x intercepts.

Solution

2y – 4x + 8 = 0 => 2y – 4x = 0 – 8,
=> 2y – 4x = - 8 => 2y = - 8 + 4x,
=> 2y = 4x – 8. Divide through using 2
$\Rightarrow \frac{2y}{2} = \frac{4x}{2} - \frac{8}{2}$ => y = 2x – 4.
The y intercept = - 4.
For the x intercept, put y = 0.
=> 0 = 2x – 4 => - 2x = - 4,
$\Rightarrow x = \frac{-4}{-2} = 2$ => x intercept = 2.

Parallel lines:

- If two lines are parallel, then they have the same gradient.

Q1. Find the equation of the line which passes through the point (4, 2), and which is parallel to the line y = 3x.

Solution

The equation of the given line is y = 3x, => the gradient = 3.

Since the required line is parallel to this line, then its gradient is also = 3. But this line passes through (4, 2).

Let (x_1, y_1) = (4, 2) => x_1 = 4 and y_1 = 2.

From $y - y_1 = m(x - x_1)$

=> y -2 = 3(x – 4),

=> y – 2 = 3x – 12 => y – 2 = 3x -12,

=> y = 3x – 12 + 2 => y = 3x – 10.

Q2. Find the equation of the line which passes through (- 6, 2) and is parallel to the line y = - 4x.

Solution

From y = - 4x => the gradient of the given line = - 4.

Since our required line is parallel to this line => the gradient of the required line = - 4.

But this line passes through (- 6, 2). Let (x_1, y_1) = (6, 2)

=> x_1= - 6 and y_1 = 2.

From $y - y_1 = m(x - x_1)$

=> y – 2 = - 4 {x – (-6)},

=> y – 2 = - 4 {x + 6}

=> y – 2 = - 4x – 24,

=> y = - 4x – 24 + 2

=> y = - 4x – 22.

Q3. Find the equation of the line passing through (-2, -3) and which is parallel to the line y - 4x -1 = 0.

Solution

The equation of the given line is y – 4x -1 = 0

=> y = 4x + 1,

=> the slope of the given line = 4.

Since the required line is parallel to this line, then the slope of the required line = 4 and it passes through (-2, -3).

Let (x_1, y_1) = (-2, -3) => x_1 = -2 and y_1 = -3.

From $y - y_1 = m(x - x_1)$ => y – (-3) = 4{x – (-2)},

=> y + 3 = 4(x + 2)

=> y + 3 = 4x + 8,

=> y = 4x + 8 – 3

=> y = 4x + 5.

Q4. Determine the equation of the line which passes through (-4, 6) and which is parallel to the line y + 2x = -3.

Solution

y + 2x = -3 => y = -3 – 2x, => y = -2x – 3 => the slope of the given line = -2.

But our required line is parallel to this line => the slope of the required line = -2 and it passes through (-4, 6).

Let (x_1, y_1) = (- 4, 6) => x_1 = - 4 and y_1 = 6.

But $y - y_1 = m(x - x_1)$

=> y – 6 = -2 (x - - 4),

=> y – 6 = - 2 (x + 4)
=> y – 6 = -2x – 8,
=> y = -2x – 8 + 6
=> y = -2x – 2.
N/B: -3 and $\overline{3}$ are the same. Also $\overline{2}$ and -2 are the same.

Q5. Find the equation of the line which passes through the point (4, 5), and is parallel to the line joining the points (- 4, 8) and (6, 2).
NB: You must first find the gradient of the line which joins the points (- 4, 8) and (6, 2).

Solution

Let $(x_1, y_1) = (-4, 8)$ and $(x_2, y_2) =$
=> (6, 2) => $x_1 = -4$, $y_1 = 8$, $x_2 = 6$ and $y_2 = 2$.
The gradient of this line =
$\frac{y_2 - y_1}{x_2 - x_1} = \frac{2 - 8}{6 - (-4)} = \frac{-6}{6 + 4} = \frac{-6}{10} = -0.6.$

Since our required line is parallel to this line, then the gradient of the required line is = - 0.6 and it passes through (4, 5).
Let $(x_1, y_1) = (4, 5)$ => $x_1 = 4$ and $y_1 = 5$. But $y - y_1 = m(x - x_1)$ => $y - 5 = -0.6(x - 4)$,
=>,$y - 5 = -0.6x + 2.4$
=> $y = -,0.6x + 2.4 + 5$,
=> $y = -0.6x + 7.4$.

Q6. Determine the equation of the line which passes through (6, 2), and is parallel to the line which passes through the points (8, 4) and (3, 1).

Solution

Let $(x_1, y_1) = (8, 4)$ and $(x_2, y_2) = (3, 1)$
=> $x_1 = 8$, $y_1 = 4$, $x_2 = 3$ and $y_2 = 1$,
=> the slope of the line which joins the points (x_1,y_1) and (x_2,y_2)
$= \frac{y_2 - y_1}{x_2 - x_1} = \frac{1 - 4}{3 - 8} = \frac{-3}{-5} = \frac{3}{5} = 0.6.$
But our required line is parallel to this line
=> the gradient of the required line = 0.6, and it also passes through (8,4).
Let $(x_1, y_1) = (8, 4)$ => $x_1 = 8$ and $y_1 = 4$.
But $y - y_1 = m(x - x_1)$
=> $y - 4 = 0.6(x - 8)$,
=> $y - 4 = 0.6x - 4.8$
=> $y = 0.6x - 4.8 + 4$,
=> $y = 0.6x - 0.8$

Q7.Determine whether the line passing through (8,4) and (10,2), is parallel to the line which passes through (6,2) and (4,3).
NB: If two lines are parallel, then they must have the same gradient.
- Determine the gradient of the first line, and then that of the second one.
- If they are the same, then the two lines are parallel.

For the first line:
Let $(x_1, y_1) = (8, 4) \Rightarrow x_1 = 8$ and $y_1 = 4$.
Also let $(x_2, y_2) = (10, 2) \Rightarrow x_2 = 10$ and $y_2 = 2$.
Gradient of the first line $= \frac{y_2 - y_1}{x_2 - x_1} = \frac{2 - 4}{10 - 8} = \frac{-2}{2} = -1$.
For the second line
Let $(x_1, y_1) = (6, 2) \Rightarrow x_1 = 6$ and $y_1 = 2$.
Also let $(x_2, y_2) = (4, 3) \Rightarrow x_2 = 4$ and $y_2 = 3$.
Gradient of this line $= \frac{y_2 - y_1}{x_2 - x_1} = \frac{3 - 2}{4 - 6} = \frac{1}{-2} = \frac{-1}{2}$

Since the gradients of the two lines are not equal => they are not parallel.

Q8. Determine whether or not the line passing through the points (2, 2) and (4, 8) is parallel to the line passing through the points (3,6) and (5,12).

Solution

For the first line
Let $(x_1, y_1) = (2, 2)$ and $(x_2, y_2) = (4, 8)$
$\Rightarrow x_1 = 2, y_1 = 2$, and $x_2 = 4, y_2 = 8$.
Slope of this line $= \frac{y_2 - y_1}{x_2 - x_1} = \frac{8 - 2}{4 - 2} = \frac{6}{2} = 3$.

For the second line
Let $(x_1, y_1) = (3, 6)$ and $(x_2, y_2) = (5, 12)$
$\Rightarrow x_1 = 3, y_1 = 6, x_2 = 5$ and $y_2 = 12$.
Slope of this line $= \frac{y_2 - y_1}{x_2 - x_1} = \frac{12 - 6}{5 - 3} = \frac{6}{2} = 3$.
Since the gradient of both lines are the same or equal, then they are parallel.

Q9. Determine whether the line which joins the points $(^1/_2, ^1/_4)$ and $(^2/_3, ^4/_5)$ is parallel to the line which joins the points $(^3/_4, ^2/_5)$ and $(^{-2}/_7, ^{-1}/_3)$

Solution

For the first line
$(^1/_2, ^1/_4) = (0.5, 0.25)$ and $(^2/_3, ^4/_5) = (0.66, 0.8)$.
Let $(x_1, y_1) = (0.5, 0.25)$ and $(x_2, y_2) = (0.66, 0.8)$
$\Rightarrow$ $x_1 = 0.5, y_1 = 0.25$ and $x_2 = 0.66$ and $y_2 = 0.8$.
Slope of this line $= \frac{y_2 - y_1}{x_2 - x_1} = \frac{0.8 - 0.25}{0.66 - 0.5} = \frac{0.5}{0.16} = 3.4$

For the second line
$(^3/_4, ^2/_5) = (0.75, 0.4)$ and $(^{-2}/_7, ^{-1}/_3) = (-0.29, -0.33)$.
Let $(x_1, y_1) = (0.75, 0.4)$ and $(x_2, y_2) = (-0.29, -0.33)$
$\Rightarrow x_1 = 0.75, y_1 = 0.4$ and $x_2 = -0.29$ and $y_2 = -0.33$
Slope of this line $= \frac{y_2 - y_1}{x_2 - x_1} = \frac{-0.33 - 0.4}{-0.29 - 0.75} = \frac{-0.73}{-1.04} = 0.7$.
Since the two lines have different gradients, then they are not parallel.

The distance between two points or the length of a line:

The distance between two points (x_1,y_1) and (x_2,y_2), or the length of the line which joins the points (x_1,y_1) and (x_2,y_2), is given by $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Q1. Find the distance between the points (2, 1) and (8, 7).

Solution

Let $(x_1, y_1) = (2, 1)$ and $(x_2, y_2) = (8, 7) \Rightarrow x_1 = 2, y_1=1,$
$x_2 = 8$ and $y_2 = 7$.
Distance = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(8 - 2)^2 + (7 - 1)^2} = \sqrt{36 + 36}$

$= \sqrt{72} = 8.4$ units.

Q2. Find the length of the line which joins the points (4, 8) and (10, 8).

solution

Let $(x_1, y_1) = (4, 8)$ and $(x_2,y_2) = (10, 8) \Rightarrow x_1 = 4, y_1= 8,$
$x_2 = 10$ and $y_2 = 8$.
Length of line $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(10 - 4)^2 + (8 - 8)^2} = \sqrt{6^2 + 0^2}$

$= \sqrt{36} = 6$units.

Q3. Determine the distance between the points (-3,-2) and (-5,- 8).

Solution

Let $(x_1, y_1) = (-3,-2)$ and $(x_2, y_2) = (-5,- 8,) \Rightarrow x_1 = - 3,$
$y_1= -2, x_2 = - 5$ and $y_2 = - 8$.
Distance = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{-5 - -3)^2 + (- 8 - -2)^2}$

$= \sqrt{(-5 + 3)^2 + (-8 +2)^2}$

$= \sqrt{(-2)^2 + (-6)^2} = \sqrt{4 + 36}$

$= \sqrt{40} = 6.3$units.

Q4. Find the length of the line, which joins the points $(^1/_2, -^1/_4)$ and $(^2/_5, ^1/_3)$.

Solution

$(^1/_2, -^1/_4) = (0.5, - 0.25)$ and $(^2/_5, ^1/_3) = (0.4, 0.33)$
Let $(x_1, y_1) = (0.5, - 0.25)$ and $(x_2, y_2) = (0.4, 0.33)$
$\Rightarrow x_1 = 0.5, y_1= - 0.25, x_2 = 0.4$ and $y_2 = 0.33$.

The required length

$= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(0.4 - 0.5)^2 + (0.33 - -0.25)^2}$

$= \sqrt{(0.4 - 0.5)^2 + (0.33 + 0.25)^2}$

$= \sqrt{(-0.1)^2 + (0.58)^2}$

$= \sqrt{0.01 + 0.34}$

$= \sqrt{0.35}$ = 0.6 units.

Questions

Q1. Find the gradients of the line, which joins the following pairs of points

a. (8, 6) and (4, 2) . Ans: 1

b. (- 6,-2) and (- 4, 4). Ans: 3

c. (-3,- 6) and (-12, - 4). Ans: - 0.22

Q2. Find the equation of the line which passes through the point (2, 3), and whose slope is 4.
Ans: y = 4x – 5.

Q3. Determine the equation of the line whose slope is $^1/_4$., if it passes through the point (-2, - 4).
Ans: y = 0.25x -3.5.

Q4. Find the equation of the line whose slope is -3, if it passes through the point (- 4, 2).
Ans: y = -3x – 10.

Q5. A line has a slope of $-^1/_2$ and passes through the point ($^1/_4$, $-^3/_4$). Determine its equation.
Ans: y = - 0.5x – 0.63.

Q6. Determine the equation of the line, which passes through the points (8, 4) and (6, 2).
Ans: y = x - 4

Q7. A line joins the points (- 6, 4) and (-2, 12). Find its equation.
Ans: y = 2x + 16.

Q8. Write down the equation of the line which passes through the point (4, 2) and (6, 8).
Ans: y = 3x – 10

Q9. Find the equation of the line which joins the points ($^1/_4$, $^{-1}/_2$) and ($^2/_3$, $^1/_3$).
Ans: y = 2x -1.

Q10. For each of these line graphs, determine the x intercept, the y intercept as well as the slope.
(a) Y = 3x + 4.

Ans:slope = 3, y intercept = 4 and x intercept = -1.33.

(b) y = - 4x + 6.

Ans : slope = - 4, y intercept = 6 and
x intercept = 1.5.

y + 6x – 5 = 0

Ans : slope = - 6, y intercept = 5 and
x intercept = 0.8.

Q11. Determine the equation of the line which passes through the point (4, 3), and which is parallel to the line $y = 2x – 6$.

Ans: $y = 2x - 5$

Q12. Write down the equation of the line which passes through (- 4, 6), and which is parallel to the line $y + 4x – 1 = 0$.

Ans: $y = - 4x - 10$

Q13. Determine the equation of the line which passes through (-3, 2) and which is parallel to the line passing through (8, 4) and (10, 2).

Ans: $y = -x – 1$

Q14. A line passes through (4, 5) and is parallel to the line joining the points (3, 6) and (5, 12). Determine its equation.

Ans: $y = 3x – 7$.

Q15. Write down the equation of the line which passes through the point (- 4, 6), and which is parallel to the line joining the points ($^1/_2$, $^1/_4$) and ($^2/_3$, $^4/_5$).

Ans: $y = 3.4x + 1.95$

Q16. Find the distance between the point (8, 6) and (4, 2).

Ans: 5.7 units

Q17. Find the length of the line which joins the points (- 8, - 6) and (-3, 2)

Ans: 9.4 units

Q18. Determine the length of the line which joins the points ($^1/_2$, $^{-1}/_3$) and ($^2/_5$, $^{-2}/_3$).

Ans: 0.34 units