CHAPTER EIGHT

WORK, MECHANICAL ENERGY AND POWER

Work:

- Work is said to be done when the point of application of a force moves.

-Work is given by the product of the force and the distance moved in the direction of the force.

- Work = Force $\times$ distance moved in the direction of the force.

- The S.I unit of work is the Joule (J), and this is the work done when the point of application of a force of 1 Newton (N) moves through 1 metre, in the direction of the force.

- N/B: 1KJ = 1000J = 10^3J and 1MJ = 1000000 J = 10^6J.

(Q1) A man of mass 65kg climbs a ladder 4m high. Calculate the work done.

Soln:

Mass = 65Kg => Force = 65 $\times$ 10 = 650N.

Distance = 4m.

Work done = force $\times$ distance = 650$\times$4 = 2600J.

Energy: Energy is the capacity or the ability to do work, and a body can do work because it possesses energy. Energy which is measured in joules can be obtained in a variety of forms such as mechanical, heat, electric and light energy.

Mechanical Energy:

- There are two kinds of mechanical energy and these are:
  - (I) Kinetic energy.
  - (II) Potential energy.

Kinetic Energy: [K.e].

- This is the energy which a body possesses by virtue of its motion.
- Kinetic energy of a moving body = $\frac{1}{2}mv^2$, where m = mass of the body and v = the velocity of the body.
- If the body is rotating, then it has rotational kinetic energy.
- The kinetic energy of a body depends on its mass, and the square of its speed or velocity.

Potential energy: [P.e]

-This is the energy possessed by a body, by means of its position above the surface of the earth, or when it is raised above the earth`s surface.

- There are two types and these are:
  (I) Gravitational potential energy.
  (II) Elastic potential energy.

Gravitational Potential Energy:

- When a body is lifted up vertically, work is done against its weight.
- The work becomes stored as gravitational potential energy.
- Gravitational potential energy = mgh, where m = the mass of the body or object.

h = its height above the surface of the earth.

g = acceleration due to gravity.

Elastic potential energy:

This is the type of energy possessed by a material which can be stretched, when it is stretched. For example, a stretched spring possesses this type of energy.

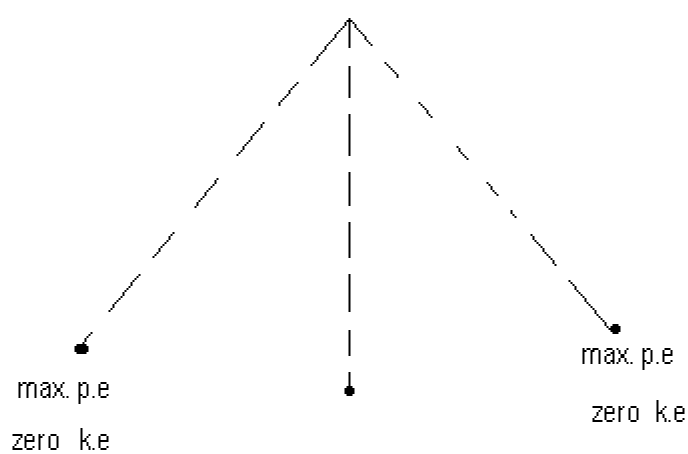
Inter conversion of kinetic and potential energy:

- When a body is released from a height, it falls and decreases in height.

-This decreasing in height causes a corresponding decreasing in its potential energy.

- At the same time the speed of such an object increases, which causes an increase in its kinetic energy.

- The lost or decrease in the potential energy of such an object, is always equal to the gain in its kinetic energy.
- This implies that as the object falls, its potential energy gradually becomes converted into kinetic energy.
- In the beginning when a body is held at a height above the ground, it possesses maximum potential energy and zero kinetic energy.
- When it is released and it is falling, then at any moment or height above the ground it possesses both potential and kinetic energy.
- For as it falls, there is a decrease in height and as such a decrease in its potential energy.
- At the same time its kinetic energy increases, since it gains speed.
- Just before it reaches the ground, the kinetic energy will be at its maximum and the potential energy will be zero.
- This is due to the fact that at this stage, all the potential energy would have been converted into kinetic energy.
- It must also be noted that the bob of swinging pendulum gains kinetic energy, as it loses potential energy, and the lost in potential energy is equal to the gain in kinetic energy.



(Q1) A stone of mass 5kg, is thrown vertically upward to a height of 100m.

(a) Calculate its potential energy and its kinetic energy at the maximum height.
(b) Calculate its kinetic energy and hence its velocity, after a fall of 50m.
[Take $g = 10ms^{-2}$]

Soln:

(a) The kinetic energy at the maximum height or at the top = 0, since the stone will not be in motion at the maximum height.

- At the maximum height, the height of the stone above the ground = h = 100m. Potential energy at the maximum height = $m \times g \times h$
  $= 5 \times 10 \times 100 = 5000J$.

(b) Height of the stone above the ground = 100m. After a fall of 50m, then the height of the stone above the ground = 100 – 50 = 50m.

=> Potential energy possessed by the stone when it is 50m above the ground = $m \times g \times h$

$= 5 \times 10 \times 50 = 2500J$.

The maximum potential energy of the stone (i.e. the p.e. at the maximum height) = 5000J.

But since at any particular height of the stone, k.e + p.e = maximum p.e (or the maximum k.e),

=> k.e at a height of 50m above the ground + p.e at the same height = maximum p.e.

=> K.e + 2500 = 5000,

=> k.e = 5000 – 2500,

=> k.e = 2500J.

Since k.e = $\frac{1}{2}mv^2$

=> $\frac{1}{2}mv^2 = 2500$, => $\frac{1}{2}(5)v^2 = 2500$,

=> $2.5v^2 = 2500$ => $v^2 = \frac{2500}{2.5}$,

=> $v^2 = 1000$ => $v = \sqrt{1000} = 32ms^{-1}$.

(Q2) A stone of weight 3.0kg falls from the top of a building which is 25.0m high. Calculate

(a) the potential energy of the stone at the start of the fall.

(b) the velocity half way through its fall, and the Kinetic energy at that point.

Soln:

(a) At the start of the fall, the height of the stone, h = 25m.
P.e = mgh = $3\times10\times25$ = 750J.
=> Maximum p.e = 750J.

(b) Half way through its fall, the height of the stone above the ground = $\frac{25}{2} = 12.5$m.

p.e of the stone half way through its fall = mgh = $3\times10\times12.5$ = 375J.

But k.e (half way through its fall) + p.e (half way through its fall) = the maximum p.e (or the maximum k.e),

=> p.e + k.e = 750,

=> 375 + k.e = 750,

=> k.e = 750 – 375 => k.e = 375J.

Since k.e = 375J, => $\frac{1}{2}mv^2$ = 375, => $\frac{1}{2}\times 3\times v^2$ = 375,

=> $1.5v^2$ = 375 => $v^2 = \frac{375}{1.5}$,

=> v^2 = 250 => v = $\sqrt{250}$

= $15.8ms^{-1}$.

(Q3) A bird of mass 2.0kg flies at a height of 20.0m above the ground with a speed of $10m^{-1}$. Calculate

(a) its potential energy.

(b) its kinetic energy.

Soln:

(a) P.e = mgh = 2×10×20= 400J.

(b) Kinetic energy = $\frac{1}{2}mv^2$ = $\frac{1}{2}$ ×2×10^2 = 100J.

(Q4) A body of mass 2kg was observed falling to the ground, with a speed of 10ms^{-1}. If its speed just before hitting or touching the ground was 30ms^{-1}, determine

(a)its maximum k.e

(b) its k.e when it is 4m above the earth`s surface.

(c) its velocity when it is 4m above the earth`s surface.

(d) the distance it will travel or cover before it just gets to the ground.

N/B: - The maximum k.e occurs when the speed is greatest.

In this case, the greatest speed occurs just before the body touches the ground.

Soln:

(a) Since mass = m = 2kg and v = 30ms^{-1}

=> Maximum k.e = $\frac{1}{2}mv^2$= $\frac{1}{2}$ ×2×30^2 = 900J.

(b) Potential energy of the body, when it is 4m above the earth`s surface = mgh

= 2×10×4 = 80J.

Since p.e (of the body when it is 4m above the earth`s surface) + k.e (of the body when it is 4m above the surface of the earth) = the maximum k.e,

=> 80 + k.e = 900, => k.e = 900 – 80 = 820,

=> k.e of the body, when it is 4m above the ground = 820J.

(c) Since k.e = 820J,

=> $\frac{1}{2}mv^2$ = 820,

=> $\frac{1}{2} \times 2 \times v^2 = 820$ => $v^2 = 820$,=> $v = \sqrt{820} = 29ms^{-1}$.

(d) $u = 10ms^{-1}$, $v = 30ms^{-1}$

Since the body falls to the ground, then 'g' = a

=> $10ms^{-2} = a$.

From $v^2 = u^2 + 2as$

=>$30^2 = 10^2 + 2(10)s$,

=> $900 = 100 + 20s$,

=> $900 - 100 = 20s$,

=> $800 = 20s$ => $s = \frac{800}{20}$,

=> s = 40m.

N/B:

- If a body falls to the ground, then its acceleration, a = g, where g = acceleration due to gravity.
- Also if the body is thrown into the air or away from the ground, then its acceleration, a = - g = $-10ms^{-2}$, => a = $-10ms^{-2}$.

(Q5) A boy of mass 70kg climbed a hill 1000m high. Calculate the maximum energy gained by the boy. [Take g = $10ms^{-2}$]

Soln:

Maximum energy gained = the potential energy = mgh = $70 \times 10 \times 1000$ = 700000

= 7×10^5J.

(Q6) If a ball of mass 0.5kg falls through a distance of 20m to the ground, and rebounced to height of 6m, calculate

(a) the velocity of the ball.

(b) the loss of energy due to the collision with the ground. [Take g = $10m/s^2$].

Soln:

(a)

Maximum p.e possessed by the ball before it was allowed to fall = mgh
= $0.5 \times 10 \times 20 = 100J$.
But as the ball falls, this maximum p.e of 100J is converted into k.e and just before the ball drops to the ground, all this p.e of 100J will be changed into k.e.
=> The k.e of the ball, just before it falls to the ground
= 100J => $\frac{1}{2}mv^2$ = 100,
=> $\frac{1}{2} \times 0.5 \times v^2$ = 100,
=> $0.5v^2 = 2 \times 100 = 200$,
=> $v^2 = \frac{200}{0.5} => v^2 = 400$,
=> v = $\sqrt{400} = 20ms^{-1}$
(b) The energy possessed by the ball, when it rebounces to a height of 6m = mgh
= $0.5 \times 10 \times 6 = 30J$.
Since the initial energy possessed by the ball = 100J, => the energy lost due to the collision with the ground = 100 – 30 = 70J.

(Q7) An object of mass 0.5kg is thrown vertically upward, with a velocity of $30.0ms^{-1}$. Calculate its potential energy at its highest height.

N/B:

- In this case, we use $v^2 = u^2 + 2as$.
- S = h i.e. the distance = the height and a = - g.
- The negative sign is brought before the g, since the acceleration due to gravity (i.e. g) acts downwards, and the stone was thrown in the opposite direction.

Soln:

We must first determine the maximum height, h.

From $v^2 = u^2 + 2as$,

=> $v^2 = u^2 + 2(-g)h$,

=> $v^2 = u^2 - 2gh$.

But $u = 30ms^{-1}$ and $g = 10ms^{-2}$ in this case but not $-10ms^{-2}$, (since the negative sign is what has changed the formula $v^2 = u^2 + 2gh$ into $v^2 = u^2 - 2gh$).

Since at the maximum height, v = O => $0 = 30^2 - 2(10)$ h,

=> 0 = 900 – 20h, => 20h = 900,

=> h = $\frac{900}{20} = 45m, \Rightarrow \text{the maximum height} = 45m$.

P.e at the maximum height = $m \times g \times h = 0.5 \times 10 \times 45$= 22.5J.

(Q8) A body of mass 500g is thrown vertically upward from the ground, with an initial velocity of $80ms^{-1}$. Calculate

(i) the potential energy at a point B, which is 80.0cm above the ground.

(ii) the kinetic energy possessed by the body when it is 80m above the ground.

(iii) the total time of the flight. [Take $g = 10m/s^2$].

Soln:

(I) Mass of the body = m = 500g = $\frac{500}{1000} = 0.5kg$.
Initial velocity u = 80m/s.
Height = h = 80cm = $\frac{80}{100} = 0.8, g = 10m/s^2$.
P.e at point B = mgh = $0.5 \times 10 \times 0.8$ = 4J.

Since the point B is 0.8m above the ground, => the p.e at the point which is 0.8m above the ground = 4J.

(II) First determine the maximum height reached by the body.
From $v^2 = u^2 + 2as$
=> $v^2 = u^2 - 2gh$.

At the maximum height, v = 0, $u = 80ms^{-1}$ and $g = 10m/s^2$

=> $0^2 = 80^2 - 2(10)(h)$,

=> 0 = 6400 – 20h, => h = 320,

=> the maximum height reached by the stone = 320m.

Maximum p.e of the stone = mgh = 0.5 $\times$ 10 $\times$ 320 = 1600J.

Sine p.e (at a height of 0.8m above the ground) + K.e (at a height of 0.8m above the ground) = the maximum P.e,

=> 4 + K.e (at the point B, 0.8m above the ground) = 1600J, => K.e = 1600 – 4 = 1596J.

Since K.e = $\frac{1}{2}mv^2$ = 1596, => $\frac{1}{2}mv^2$ = 1596, => $\frac{1}{2} \times 0.5 \ \times v^2$ = 1596,

=> $v^2 = \frac{2\times1596}{0.5} = 6384,$

=> v = $\sqrt{6384} = 80.$

K.e = $\frac{1}{2} \times 5 \times 80^2$

= $\frac{1}{2} \times 5 \times 6400$ = 16000J.

(iii) From v = u + at => 0 = 80 + (-g)t => 0 = 80 – 10t, => 10t = 80 => t = 8 sec.

(Q9) Calculate the power transferred when a pump moves 50kg of water, through a vertical height of 8m in 5 seconds.

Soln:

Mass of water = 50kg.

Force = Mass $\times$ acceleration.

In this particular case, force = mass $\times$ acceleration due to gravity.

=> Force = m $\times$ g = 50 $\times$ 10 = 500N.

Time taken = 5 seconds.

Power = $\frac{work\ done}{time\ taken\ in\ seconds}$

Power = $\frac{Force \times distance}{time}$

Power = $\frac{500 \times 8}{5} = 800$W.

N/B: The mass must be in kilograms, the distance in metres and the time in seconds.

(Q10) An engine raised 200kg of water through a height of 30m in 20 seconds. Calculate

(a) the p.e obtained by the water.

(b)the power at which the engine works.

Soln:

(a) Potential energy = mgh = 200 $\times$ 10 $\times$ 30 = 60,000J.

(b) Force = ma = mg = 200 $\times$ 10 = 2000N.

Distance = 30m, time = 20 seconds

Power = $\frac{Force \times distance}{time\ taken}$

Power = $\frac{200 \times 30}{20} = 300W$.

(Q11) A pump is used to spray water from a storage tank. If 40 litres of water is pumped per minute, and the spray reaches a maximum height of 60m, determine the power of the pump. [Assume that 1 litre of water has a mass of 1kg and g = 10ms^{-1}].

Soln:

The height of water = 60m.

Time = 1 minute = 1 $\times$ 60 = 60 seconds.

Volume of water = 40 litres.

Since the mass of 1 litre of water = 1kg => the mass of 40 litres of water = m = 40kg.

But force = ma = mg = 40 $\times$ 10 = 400N.

Distance = the maximum height reached by the water = the distance = 60m.

Work done = Force $\times$ distance = 400 $\times$ 60 = 24000J.

Power = $\frac{Work\ done\ (J)}{time\ taken}$

= $\frac{24000}{60} = 400W.$

The maximum power therefore = 400W.

(Q12) A 2000 kg car starts from rest and accelerated to a final velocity of $20ms^{-1}$ in 16s. Assuming a constant air resistance of 500N, find the average power developed by the car.

Soln:

Since the car starts from rest => u = 0.

Also v = $20ms^{-1}$, and t = 16s.

Since v = u + at

=> 20 = 0 + 16a => a = $\frac{20}{16}$,

=> a = $1.25ms^{-2}$.

Let F = the force.

Since air resistance or frictional force = 500N,

=> the net force = F – 500.

But since net force = ma,

=> F – 500 = ma,

=> F - 500 = 2000 x 1.25, => F – 500 = 2500,

=> F = 2500 + 500 = 3000N.

The average velocity of the car during this period

= V = $\frac{V+U}{2} = \frac{20+0}{2} = 10ms^{-1}$

Power = F $\times$ V = 3000 $\times$ 10

= 30,000W = 30KW.

(Q13) A body of mass 10kg moves between two points on a horizontal road, and does 500J of work against friction. If the initial speed of the body is 20ms^{-1}, determine the final speed.

Soln:

Work done (against friction)

= force $\times$ distance = ma $\times$ distance= 10 $\times$ a $\times$ distance.

Let distance = s => work done (against friction) = 10 $\times$a$\times$s = 10as.

Since the work done against friction = 500J, => 10as = 500 => as = $\frac{500}{10}$

=> as = 50.

Substitute as = 50 in

$v^2 = u^2 + 2as$

=> $v^2 = 20^2 + 2(50)$,

=> $v^2 = 400 + 100 = 500$,

=> V = $\sqrt{500} = 22.4ms^{-1}$