CHAPTER THIRTEEN

SURDS

When a square roots sign is squared or raised to the second power, the square root sign disappears.

(1) $(\sqrt{2})^2 = 2$ (2) $(\sqrt{3})^2 = 3$

(3) $(\sqrt{a})^2 = a$ (4) $(\sqrt{b})^2 = b$

(5) $2(\sqrt{3})^2 = 2\ (3) = 6$

(6) $5(\sqrt{2})^2 = 5\ (2) = 10$

(7) $2(\sqrt{a})^2 = 2\ (a) = 2a$

(8) $(2\sqrt{3})^2 = 2^2\ (\sqrt{3})^2 = 4\ (3) = 12$

(9) $(5\sqrt{2})^2 = 5^2\ (\sqrt{2})^2 = 25\ (2) = 50$

(10) $(a\sqrt{b})^2 = a^2\ (\sqrt{b})^2 = a^2\ (b) = a^2b$

The Perfect Squares:

The Perfect squares are:

(1) 2 x 2 = 4 (2)3 x 3 = 9

(3) 4 x 4 = 16 (4) 5 x 5 = 25

(5) 6 x 6 = 36 (6)7 x 7 = 49

(7) 8 x 8 = 64 (8) 9 x 9 = 81

(9) 10 x 10 = 100

-- In surd manipulation, a number which is a multiple of a perfect square must be converted into the multiple of that perfect square

Examples:

1. $\sqrt{8} = \sqrt{4 \times 2} = \sqrt{4} \times \sqrt{2} = 2 \times \sqrt{2} = 2\sqrt{2}$.

2. $\sqrt{12} = \sqrt{4 \times 3} = \sqrt{4} \times \sqrt{3} = 2 \times \sqrt{3} = 2\sqrt{3}$.
3. $\sqrt{32} = \sqrt{16 \times 2} = \sqrt{16} \times \sqrt{2}$
   $= 4 \times \sqrt{2} = 4\sqrt{2}$

4. $\sqrt{27} = \sqrt{9 \times 3} = \sqrt{9} \times \sqrt{3} = 3\sqrt{3}$

5. $\sqrt{69} = \sqrt{9 \times 7} = \sqrt{9} \times \sqrt{7} = 3\sqrt{7}$

6. $\sqrt{80} = \sqrt{16 \times 5} = \sqrt{16} \times \sqrt{5} = 4\sqrt{5}$

(7) $\sqrt{50} = \sqrt{25 \times 2} = \sqrt{25} \times \sqrt{2} = 5\sqrt{2}$

(8) $\sqrt{147} = \sqrt{49 \times 3} = \sqrt{49} \times \sqrt{3} = 7\sqrt{3}$

(9) $\sqrt{125} = \sqrt{25 \times 3} = \sqrt{25} \times \sqrt{3} = 5\sqrt{3}$

How to determine whether a number is a multiple of a perfect square:

-- The perfect squares 4, 9, 16, 25, 36, 49, 64, 81 and 100 are what we shall make use of.

-- When a number is given and we want to know whether it is a multiple of a perfect square, we start dividing the numbers with the perfect squares in turn, starting with the highest which is 100.

-- If we get an answer which is a whole number but not a decimal, then that particular number is a multiple of a perfect square.

-- Far example if we are given $\sqrt{448}$ and we want to know whether or not we can break it down, we first divide 448 by 100 which gives us 4.48.

-- Since this is a decimal, then we try the next perfect square which is 81.

-- Dividing 448 by 81 gives us 5.5 which is also a decimal. We then try the next perfect square which is 64.

-- Dividing 448 by 64 gives us 7, which is a whole numbers.

- This implies that $\sqrt{448}$
  $= \sqrt{64 \times 7} = \sqrt{64} \times \sqrt{7} = 8\sqrt{7}$.

Example (2) Let us now determine whether $\sqrt{294}$ can be simplified or broken down.

- Dividing 294 by 100 gives us 2.94 which is a decimal.

- Dividing 294 by 81 gives us 3.6 which is a decimal.

- Diving 294 by 64 gives us 4.59 which is also a decimal.

- Dividing 294 by 49 gives us 6 which is a whole number

- This implies that √294
  $= \sqrt{49 \times 6}$ = √49 x √6 = 7√6.

Examples (3) Now let us determine whether √150 can be simplified or broken down.

- Dividing 150 by 100 gives us 1.50 which is a decimal

- Dividing 150 by 81 gives us 1.85 which is a decimal.

- Dividing 150 by 49 gives us 3.06 which is a decimal.

- Dividing 150 by 36 gives us 4.2 which is a decimal.

- Dividing 150 by 25 gives us 6 which is a whole number.

- This implies that √150
  $= \sqrt{25 \times 6}$ = √25 x √6 = 5√6.

NB: If we divide a given number by all the perfect Squares we are supposed to use, and in each case get a decimal, then that number cannot be simplified or broken down.

For example, assume that we want to know whether or not √271 can be simplified or broken down.

-- Dividing 271 by 100 gives us 2.71 which is a decimal.

-- Dividing 271 by 81 gives us 3.34 which is a decimal

- Dividing 271 by 64 gives us 4.2 which is decimal.

- Dividing 271 by 49 gives us 5.5 which is decimal.

-- Dividing 271 by 36 gives us 7.5 which is a decimal.

-- Dividing 271 by 25 gives us 10.8 which is decimal.

-- Dividing 271 by 16 gives us 16.9 which is decimal.
-- Dividing 271 by 9 gives us 30.1 which is a decimal.

-- Dividing 271 by 4 gives us 67.7 which is also a decimal.

Since when all the perfect squares concerned, when used to divide the given number gave us decimals as our answer, then $\sqrt{271}$ cannot be simplified and should be left as $\sqrt{271}$.

Addition of surds:

-- In surd addition, we can only add if the numbers under the square root signs are the same.

-- If they are not the same, then nothing can be done.

Examples

(1) $a\sqrt{b} + c\sqrt{b} = (a + c)\sqrt{b}$

(2) $5\sqrt{2} + 4\sqrt{2} = (5 + 4)\sqrt{2} = 9\sqrt{2}$

(3) $6\sqrt{7} + 2\sqrt{7} = (6 + 2)\sqrt{7} = 8\sqrt{7}$

(4) $3\sqrt{5} + 2\sqrt{5} = (3+ 2)\sqrt{5} = 5\sqrt{5}$

(5) $2\sqrt{7} + 3\sqrt{7} = (2 + 3)\sqrt{7} = 5\sqrt{7}$

(6) $2\sqrt{3} + \sqrt{3} = 2\sqrt{3} + 1\sqrt{3} = (2 + 1)\sqrt{3} = 3\sqrt{3}$

(7) $5\sqrt{2} + \sqrt{2} = 5\sqrt{2} + 1\sqrt{2} = (5 + 1)\sqrt{2} = 6\sqrt{2}$

(8) $2\sqrt{5} + 4\sqrt{5} + 3\sqrt{5} = (2 + 4 + 3)\sqrt{5} = 9\sqrt{5}$

(9) $5\sqrt{2} + 2\sqrt{2} + 4\sqrt{2} = (5 + 2 + 4)\sqrt{2} = 11\sqrt{2}$

(10) $\sqrt{3} + 4\sqrt{3} + 6\sqrt{3} = 1\sqrt{3} + 4\sqrt{3} + 6\sqrt{3} = (1 + 4 + 6)\sqrt{3} = 11\sqrt{3}$

(11) $2\sqrt{3} + 4\sqrt{2} = 2\sqrt{3} + 4\sqrt{2}$

(12) $5\sqrt{7} + 2\sqrt{6} = 5\sqrt{7} + 2\sqrt{6}$

(13) $4\sqrt{2} + 3\sqrt{5} = 4\sqrt{2} + 3\sqrt{5}$

(14) $5\sqrt{2} + 3\sqrt{3} + 2\sqrt{7} = 5\sqrt{2} + 3\sqrt{3} + 2\sqrt{7}$

Simplify each of the following:

(Q1) $2 + 3\sqrt{5} + 6 + 4\sqrt{5}$

Solution

$2 + 3\sqrt{5} + 6 + 4\sqrt{5} = 2 + 6 + 3\sqrt{5} + 4\sqrt{5}$

$= 8 + (3 + 4)\sqrt{5} = 8 + 7\sqrt{5}$

(Q2) $6\sqrt{7} + 1 + 4\sqrt{7} + 3 + 2\sqrt{7}$

Solution

$6\sqrt{7} + 1 + 4\sqrt{7} + 3 + 2\sqrt{7} = 6\sqrt{7} + 4\sqrt{7} + 2\sqrt{7} + 1 + 3$

$= (6 + 4 + 2)\sqrt{7} + 4 = 12\sqrt{7} + 4$

(Q3) $2\sqrt{3} + 5\sqrt{7} + 5\sqrt{3} + 2$

Solution

$2\sqrt{3} + 5\sqrt{7} + 5\sqrt{3} + 2 = 2\sqrt{3} + 5\sqrt{3} + 5\sqrt{7} + 2$

$= (2 + 5)\sqrt{3} + 5\sqrt{7} + 2 = 7\sqrt{3} + 5\sqrt{7} + 2$

(Q4) $2 + 5\sqrt{2} + \sqrt{3} + 6\sqrt{2} + 7\sqrt{3} + 6$

Solution

$2 + 5\sqrt{2} + \sqrt{3} + 6\sqrt{2} + 7\sqrt{3} + 6 = 2 + 5\sqrt{2} + 1\sqrt{3} + 6\sqrt{2} + 7\sqrt{3} + 6$

$= 2 + 6 + 5\sqrt{2} + 6\sqrt{2} + 1\sqrt{3} + 7\sqrt{3}$

$= 8 + (5 + 6)\sqrt{2} + (1 + 7)\sqrt{3}$

$= 8 + 11\sqrt{2} + 8\sqrt{3}$

(Q5) $5^{1}/_{2} + 3\sqrt{2} + 6\sqrt{2} + 7\sqrt{3} + {}^{1}/_{2}$

Solution

$5^{1}/_{2} + 3\sqrt{2} + 6\sqrt{2} + 7\sqrt{3} + {}^{1}/_{2}$

$= 5^{1}/_{2} + {}^{1}/_{2} + 3\sqrt{2} + 6\sqrt{2} + 7\sqrt{3}$

$= 6 + (3 + 6)\sqrt{2} + 7\sqrt{3} = 6 + 9\sqrt{2} + 7\sqrt{3}$

(Q6) $2\sqrt{8} + 3\sqrt{3} + 1$

Solution

$2\sqrt{8} + 3\sqrt{3} + 1 = 2\sqrt{4 \times 2} + 3\sqrt{3} + 1$

$= 2x\sqrt{4}\ x\ \sqrt{2} + 3\sqrt{3} + 1$

$= 2\ x\ 2\ x\ \sqrt{2} + 3\sqrt{3} + 1 = 4\sqrt{2} + 3\sqrt{3} + 1$

(Q7) $3\sqrt{2} + 2 + 2\sqrt{8} + 4\sqrt{2} + 6$

Solution

$3\sqrt{2} + 2 + 2\sqrt{8} + 4\sqrt{2} + 6$

$= 3\sqrt{2} + 2 + 2\sqrt{4 \times 2} + 4\sqrt{2} + 6$

$= 3\sqrt{2} + 2 + 2x\sqrt{4}x\sqrt{2} + 4\sqrt{2} + 6$

$= 3\sqrt{2} + 2 + 2\ x\ 2\ x\sqrt{2} + 4\sqrt{2} + 6$

$= 3\sqrt{2} + 4\sqrt{2} + 4\sqrt{2} + 6 + 2$

$= (3 + 4 + 4)\sqrt{2} + 8$

$= 11\sqrt{2} + 8$

(Q8) $5+3\sqrt{27} + 2 + 6\sqrt{3} + 2\sqrt{2} + \sqrt{12} + 1$

Solution

$5+3\sqrt{27} + 2 + 6\sqrt{3} + 2\sqrt{2} + \sqrt{12} + 1$

$= 5 + 3\sqrt{27} + 2 + 6\sqrt{3} + 2\sqrt{2} + \sqrt{12} + 1$

$= 5 + 3x\sqrt{9}x\sqrt{3} + 2 + 6\sqrt{3} + 2\sqrt{2} + \sqrt{4}x\sqrt{3} + 1$

$= 5 + 3x3x\sqrt{3} + 2 + 6\sqrt{3} + 2\sqrt{2} + 2x\sqrt{3} + 1$

$= 5 + 9\sqrt{3} + 2 + 6\sqrt{3} + 2\sqrt{2} + 2\sqrt{3} + 1$

$= 5 + 2 + 1 + 9\sqrt{3} + 6\sqrt{3} + 2\sqrt{3} + 2\sqrt{2}$
$= 8 + (9 + 6 + 2)\ \sqrt{3} + 2\sqrt{2}$
$= 8 + 17\sqrt{3} + 2\sqrt{2}$

(Q9) $4 + 2\sqrt{32} + 3\sqrt{2} + 1 + 2\sqrt{50}$

Solution

$4 + 2\sqrt{32} + 3\sqrt{2} + 1 + 2\sqrt{50}$
$= 4 + 2\sqrt{16 \times 2} + 3\sqrt{2} + 1 + 2\sqrt{25 \times 2}$
$= 4 + 2x\sqrt{16}\ x\ \sqrt{2} + 3\sqrt{2} + 1 + 2x\sqrt{25}\ x\ \sqrt{2}$
$= 4 + 2\ x\ 4\ x\ \sqrt{2} + 3\sqrt{2} + 1 + 2\ x\ 5\ x\ \sqrt{2}$
$= 4 + 8\sqrt{2} + 3\sqrt{2} + 1 + 10\sqrt{2}$
$= 4 + 1 + 8\sqrt{2} + 3\sqrt{2} + 10\sqrt{2}$
$= 5 + (8 + 3 + 10)\ \sqrt{2}$
$= 5 + 21\sqrt{2}$

(Q10) $3\sqrt{7} + 5 + 2\sqrt{7} + 3\sqrt{16} + 2\sqrt{25} + 4\sqrt{128}$

Solution

$3\sqrt{7} + 5 + 2\sqrt{7} + 3\sqrt{16} + 2\sqrt{25} + 4\sqrt{128}$
$= 3\sqrt{7} + 5 + 2\sqrt{7} + 3\ (4) + 2\ (5) + 4\sqrt{64 \times 2}$
$= 3\sqrt{7} + 5 + 2\sqrt{7} + 12 + 10 + 4x\sqrt{64}\ x\ \sqrt{2}$
$= 3\sqrt{7} + 2\sqrt{7} + 5 + 12 + 10 + 4\ x\ 8\ x\ \sqrt{2}$
$= (3 + 2)\ \sqrt{7} + 27 + 32\sqrt{2}$
$= 5\sqrt{7} + 27 + 32\sqrt{2} = 5\sqrt{7} + 32\sqrt{2} + 27$

NB: You must first check whether 128 is a multiple of a perfect square or not.

(Q11) $5 + \sqrt{243} + 2\sqrt{2} + \sqrt{18} + 4\sqrt{162} + 2\sqrt{4}$

NB: You must first check whether or not 243, 162 and 18 are multiples of perfect squares.

Solution

$5 + \sqrt{243} + 2\sqrt{2} + \sqrt{18} + 4\sqrt{162} + 2\sqrt{4}$
$= 5 + \sqrt{81 \times 3} + 2\sqrt{2} + \sqrt{9 \times 2} + 4\sqrt{81 \times 2} + 2\ (2)$
$= 5 + \sqrt{81}\ x\ \sqrt{3} + 2\sqrt{2} + \sqrt{9 \times 2}\ x\ 4\ \sqrt{81} \times \sqrt{2} + 4$
$= 5 + 9\ x\ \sqrt{3} + 2\sqrt{2} + 3\ x\sqrt{2} + 4\ x\ 9\ x\sqrt{2} + 4$
$= 5 + 9\sqrt{3} + 2\sqrt{2} + 3\sqrt{2} + 36\sqrt{2} + 4$
$= 5 + 4 + 9\sqrt{3} + 2\sqrt{2} + 3\sqrt{2} + 36\sqrt{2}$
$= 9 + 9\sqrt{3} + (2 + 3 + 36)\ \sqrt{2}$
$= 9 + 9\sqrt{3} + 41\sqrt{2}$

NB: $\sqrt{4} = 2$, $\sqrt{25} = 5$, $\sqrt{36} = 6$, $\sqrt{49} = 7$, $\sqrt{64} = 8$,
$\sqrt{81} = 9$ and $\sqrt{100} = 10$

Subtraction of Surds:

In surds subtraction, we can only subtract when the numbers under the square root signs are the same.

Examples:
1. $5\sqrt{2} - 3\sqrt{2} = (5 - 3)\sqrt{2} = 2\sqrt{2}$
2. $8\sqrt{3} - 2\sqrt{3} = (8 - 2)\sqrt{3} = 6\sqrt{3}$
3. $5\sqrt{6} - 9\sqrt{6} = (5 - 9)\sqrt{6} = -4\sqrt{6}$
4. $8\sqrt{2} - 2\sqrt{2} - 3\sqrt{2} = (8 - 2 - 3)\sqrt{2} = 3\sqrt{2}$
5. $4\sqrt{3} - 2\sqrt{5} = 4\sqrt{3} - 2\sqrt{5}$
6. $6\sqrt{2} - 7\sqrt{7} = 6\sqrt{2} - 7\sqrt{7}$
7. $4\sqrt{5} - 2\sqrt{3} = 4\sqrt{5} - 2\sqrt{3}$

Simplify each of the following:

(Q1) $2\sqrt{75} - 4\sqrt{3} - 2$
Solution
$2\sqrt{75} - 4\sqrt{3} - 2$
$= 2\sqrt{25 \times 3} - 4\sqrt{3} - 2$
$= 2 \text{ x} \sqrt{25} \text{ x} \sqrt{3} - 4\sqrt{3} - 2$
$= 2 \text{ x } 5 \text{ x } \sqrt{3} - 4\sqrt{3} - 2$
$= 10\sqrt{3} - 4\sqrt{3} - 2$
$= (10 - 4)\sqrt{3} - 2 = 6\sqrt{3} - 2$

(Q2) $5\sqrt{8} - 2 - 2\sqrt{2} - 3\sqrt{3}$
Solution
$5\sqrt{8} - 2 - 2\sqrt{2} - 3\sqrt{3}$
$= 5\sqrt{4 \times 2} - 2 - 2\sqrt{2} - 3\sqrt{3}$
$= 5 \text{ x } \sqrt{4} \text{ x } \sqrt{2} - 2 - 2\sqrt{2} - 3\sqrt{3} = 5 \text{ x } 2 \text{ x } \sqrt{2} - 2 - 2\sqrt{2} - 3\sqrt{3}$
$= 10\sqrt{2} - 2 - 2\sqrt{2} - 3\sqrt{3}$
$= 10\sqrt{2} - 2\sqrt{2} - 3\sqrt{3} - 2$
$= (10 - 2)\sqrt{2} - 3\sqrt{3} - 2$
$= 8\sqrt{2} - 3\sqrt{3} - 2$

(Q3) $-4\sqrt{7} - 3\sqrt{36} - 2 - 2\sqrt{7} - 3\sqrt{72}$
Solution
$-4\sqrt{7} - 3\sqrt{36} - 2 - 2\sqrt{7} - 3\sqrt{72}$
$= -4\sqrt{7} - 3\sqrt{9 \times 4} - 2 - 2\sqrt{7} - 3\sqrt{36 \times 2}$
$= -4\sqrt{7} - 3\text{x } \sqrt{9} \text{ x } \sqrt{4} - 2 - 2\sqrt{7} - 3\text{x}\sqrt{36}\text{x}\sqrt{2}$
$= -4\sqrt{7} - 3\text{x}3\text{x}2 - 2 - 2\sqrt{7} - 3 \text{ x } 6 \text{ x } \sqrt{2}$
$= -4\sqrt{7} - 18 - 2 - 2\sqrt{7} - 18\sqrt{2}$
$= -4\sqrt{7} - 2\sqrt{7} - 18 - 2 - 18\sqrt{2}$
$= (-4 - 2)\sqrt{7} - 20 - 18\sqrt{2}$

= - 6√7 – 18√2 – 20

Combination of addition and subtractions of surds:

Simplify the following:

1). $3\sqrt{2} + 5\sqrt{3} – 2\sqrt{18} + \sqrt{3}$

Solution

$3\sqrt{2} + 5\sqrt{3} – 2\sqrt{18} + \sqrt{3}$
$= 3\sqrt{2} +5\sqrt{3} – 2\sqrt{9 \times 2} + \sqrt{3}$
$= 3\sqrt{2} +5\sqrt{3} – 2x\sqrt{9}x\sqrt{2} + \sqrt{3}$
$= 3\sqrt{2} + 5\sqrt{3} – 2 \times 3 \times \sqrt{2} + \sqrt{3}$
$= 3\sqrt{2} + 5\sqrt{3} – 6\sqrt{2} + \sqrt{3}$
$= 3\sqrt{2} + 5\sqrt{3} – 6\sqrt{2} + 1\sqrt{3} = 3\sqrt{2} - 6\sqrt{2} + 5\sqrt{3} + 1\sqrt{3}$
$= (3 – 6)\sqrt{2} + 5\sqrt{3} + 1\sqrt{3}$
$= - 3\sqrt{2} + (5 +1) \sqrt{3}$
$= - 3\sqrt{2} + 6\sqrt{3}$

(Q2). $3\sqrt{8} – 8 + 2\sqrt{3} – 4\sqrt{2} – 2\sqrt{27} + 5$

Solution

$3\sqrt{8} – 8 + 2\sqrt{3} – 4\sqrt{2} – 2\sqrt{27} + 5$
$= 3\sqrt{4 \times 2} – 8 + 2\sqrt{3} – 4\sqrt{2} – 2\sqrt{9 \times 3} +5$
$= 3x\sqrt{4} \times \sqrt{2} – 8 + 2\sqrt{3} – 4\sqrt{2} – 2x\sqrt{9}x\sqrt{3} + 5$
$=3 \times 2 \times \sqrt{2} – 8 + 2\sqrt{3} – 4\sqrt{2} – 2 \times 3 \times\sqrt{3} + 5$
$= 6\sqrt{2} – 8 + 2\sqrt{3} – 4\sqrt{2} – 6\sqrt{3} + 5$
$= 6\sqrt{2} – 4\sqrt{2} + 2\sqrt{3} – 6\sqrt{3} – 8 +5$
$= (6 – 4) \sqrt{2} + 2\sqrt{3} – 6\sqrt{3} – 3$
$= 2\sqrt{2} + (2 – 6) \sqrt{3} -3$
$= 2\sqrt{2} + (-4)\sqrt{3} – 3$
$= 2\sqrt{2} – 4\sqrt{3} – 3$

(Q3). $(2 + 3\sqrt{5}) –(3+ 2\sqrt{5})$

Solution

$(2 + 3\sqrt{5}) – (3+ 2\sqrt{5})$
$= 2 + 3\sqrt{5} – 3 – 2\sqrt{5}$
$= 2 – 3 + 3\sqrt{5} – 2\sqrt{5}$
$= -1 + (3 – 2) \sqrt{5}$
$= -1 + 1\sqrt{5} = -1 + \sqrt{5}$

(Q4) . $1 + 2\sqrt{7} – 4\sqrt{63} + 3\sqrt{12} – 5\sqrt{3} – 4\sqrt{9}$

Solution

$1 + 2\sqrt{7} – 4\sqrt{63} + 3\sqrt{12} – 5\sqrt{3} – 4\sqrt{9}$
$= 1+ 2\sqrt{7} – 4\sqrt{9x7} + 3\sqrt{4x3} – 5\sqrt{3} – 4$ (3)
$= 1 + 2\sqrt{7} – 4x\sqrt{9}x\sqrt{7} + 3x\sqrt{4}x\sqrt{3} – 5\sqrt{3} - 12$
$= 1 + 2\sqrt{7} – 4 \times 3 \times\sqrt{7} + 3 \times 2 \times\sqrt{3} – 5\sqrt{3} – 12$

= 1 + 2√7 – 12√7 + 6√3 – 5√3 – 12
= 1 + (2 – 12) $\sqrt{7}$ + 6√3 -5√3 – 12 = 1 + (-10)$\sqrt{7}$ + (6-5)$\sqrt{3}$ - 12
= 1 - 10$\sqrt{7} + \sqrt{3} - 12$
= 1 – 12 - 10$\sqrt{7} + \sqrt{3}$
= -11 - 10$\sqrt{7} + \sqrt{3}$

(Q5). 6√5 – 2√20 - √45 + 3 + 1 + 5√3 + √125 + 3√5

Solution

6√5 – 2√20 - √45 + 3 + 1 + 5√3 + √125 + 3√5
= 6√5 – 2$\sqrt{4x5}$ - $\sqrt{9 \text{ x } 5}$ + 4+ 5√3 + $\sqrt{25 \times 5}$+ 3√5
= 6√5 – 2x√4 x√5 -√9x√5 + 4 + 5√3 +√25 x √5 + 3√5.
= 6√5 – 2x2x√5 – 3x√5+4 +5√3 +5 x√5 + 3√5
= 6√5 – 4√5 – 3√5 + 4+5√3 + 5√5 + 3√5
= 6√5 – 4√5 – 3√5 + 5√5 + 3√5 + 5√3 + 4
= (6 – 4 – 3 + 5 + 3) √5 + 5√3 + 4
= 7√5 + 5√3 + 4

(Q6). √24 – 3√6 – 216 + √294

Solution

√24 – 3√6 – 216 + √294
=$\sqrt{4 \times 6}$ – 3$\sqrt{6}$ – $\sqrt{36 \times 6}$ + $\sqrt{49 \times 6}$
= √4 x √6 – 3√6 - √36 x √6 + √49 x √6
= 2√6 – 3√6 – 6√6 + 7√6
= (2 – 3 – 6 + 7) √6 = 0√(6) = 0

(Q7). √128 - √50 + $^{2}/_{3}$ √162

Solution

√128 - √50 + $^{2}/_{3}$ √162
= $\sqrt{64 \times 2}$- √50 + $^{2}/_{3}\sqrt{81 \times 2}$
= √64 x√2 - √25 x √2 + $^{2}/_{3}$ x √81 x √2
= 8√2 – 5√2 + $^{2}/_{3}$ x 9 x √2
= (8 -5) √2 + $^{18}/_{3}$ x √2
= 3√2 + 6√2

Multiplication of Surds:

1, a√z x b√y = a x b x √z x √y
= ab x √zy = ab√zy

2. 2√3 x 5√7 = 2 x 5 x √3 x √7
= 10 x $\sqrt{3 \times 7}$= 10√21

3. 5√2 x 6√7 = 5 x 6 x √2 x √ 7
= 30 x $\sqrt{2 \times 7}$ = 30√14

4. $2\sqrt{3} \times 4\sqrt{15} = 2 \times 4 \times \sqrt{3} \times \sqrt{15}$
$= 8 \times \sqrt{3 \times 15} = 8 \times \sqrt{45} = 8 \times \sqrt{9 \times 5}$
$= 8 \times \sqrt{9} \times \sqrt{5} = 8 \times 3 \times \sqrt{5} = 24\sqrt{5}$

5. $2\sqrt{9} \times 4\sqrt{3} = 2 \times 4 \times \sqrt{9} \times \sqrt{3}$
$= 8 \times \sqrt{27} = 8 \times \sqrt{9 \times 3} = 8 \times \sqrt{9} \times \sqrt{3}$
$= 8 \times 3 \times \sqrt{3} = 24\sqrt{3}$

Simplify the following:

(Q1) $\sqrt{10}(2\sqrt{2} + \sqrt{5})$

Solution

$\sqrt{10}(2\sqrt{2} + \sqrt{5}) = \sqrt{10} \times 2\sqrt{2} + \sqrt{10} \times \sqrt{5}$
$= 2 \times \sqrt{2} \times \sqrt{10} + \sqrt{10} \times \sqrt{5}$
$= 2 \times \sqrt{2 \times 10} + \sqrt{10 \times 5}$
$= 2\sqrt{20} + \sqrt{50}$
$= 2\sqrt{4 \times 5} + \sqrt{25 \times 2}$
$= 2 \times \sqrt{4} \times \sqrt{5} + \sqrt{25} \times \sqrt{2}$
$= 2 \times 2 \times \sqrt{5} + 5\sqrt{2} = 4\sqrt{5} + 5\sqrt{2}$

(Q2).2 $(2\sqrt{6} - 3\sqrt{24} + 4\sqrt{10})$

Solution

$2(2\sqrt{6} - 3\sqrt{24} + 4\sqrt{10})$
$= 2 \times 2\sqrt{6} - 2 \times 3\sqrt{24} + 2 \times 4\sqrt{10}$
$= 4\sqrt{6} - 6\sqrt{24} + 8\sqrt{10}$
$= 4\sqrt{6} - 6\sqrt{4 \times 6} + 8\sqrt{10}$
$= 4\sqrt{6} - 6 \times \sqrt{4} \times \sqrt{6} + 8\sqrt{10}$
$= 4\sqrt{6} - 6 \times 2 \times \sqrt{6} + 8\sqrt{10}$
$= 4\sqrt{6} - 12\sqrt{6} + 8\sqrt{10}$
$= (4 - 12)\sqrt{6} + 8\sqrt{10}$
$= -8\sqrt{6} + 8\sqrt{10}$

(Q3). $2\sqrt{2}(2\sqrt{6} - 3\sqrt{25} + 4\sqrt{16})$

Solution

$2\sqrt{2}(2\sqrt{6} - 3\sqrt{25} + 4\sqrt{16})$
$= 2\sqrt{2} \times 2\sqrt{6} - 2\sqrt{2} \times 3\sqrt{25} + 2\sqrt{2} \times 4\sqrt{16}$
$= 2 \times 2 \times \sqrt{2 \times 6} - 2 \times 3 \times \sqrt{2 \times 25} + 2 \times 4 \times \sqrt{2 \times 16}$
$= 4\sqrt{12} - 6\sqrt{50} + 8\sqrt{32}$
$= 4\sqrt{4 \times 3} - 6\sqrt{25 \times 2} + 8\sqrt{16 \times 2}$
$= 4 \times \sqrt{4} \times \sqrt{3} - 6 \times \sqrt{25} \times \sqrt{2} + 8 \times \sqrt{16} \times \sqrt{2}$
$= 4 \times 2 \times \sqrt{3} - 6 \times 5 \times \sqrt{2} + 8 \times 4 \times \sqrt{2}$
$= 8\sqrt{3} - 30\sqrt{2} + 32\sqrt{2}$
$= 8\sqrt{3} + 32\sqrt{2} - 30\sqrt{2}$
$= 8\sqrt{3} + (32 - 30)\sqrt{2}$
$= 8\sqrt{3} + 2\sqrt{2}$

N/B:

1). (a + b) (c + d) = a x c + a x d + b x c + b x d
= ac + ad + bc + bd.

(2) (a – b) (c + d) = a x c + a x d – b x c – b x d
= ac + ad – bc – bd

(3) (a + b) (c – d) = a x c – a x d + b x c – b x d
= ac – ad + bc – bd

(4) (a – b) (c – d) = a x c – a x d – b x c + b × d
= ac – ad – bc +bd

Examples:

1. (2 + 3) (4 + 5) = 2 x 4 + 2 x 5 + 3 x 4 + 3 x 5
= 8 + 10 + 12 + 15 = 45

2. (2 + 3) (4 – 5) = 2 x 4 – 2 x 5 + 3 x 4 – 3 x 5
= 8 – 10 + 12 – 15 = -5

3. (2 – 3) (4 + 5) = 2 x 4 + 2 x 5 – 3 x 4 – 3 x5
= 8 + 10 – 12 – 15 = -9

4. (2 – 3) (4 – 5) = 2 x 4 – 2 x 5 – 3 x 4 + 3 x5
= 8 – 10 – 12 + 15 = 1

Evaluate the following:

(1) (√2 + √3) (√4 + √5)

Solution

(√2 + √3) (√4 + √5)
= √2 x√4 + √2 x √5 + √3 x√4 + √3 x √5
= √8 + √10 + √12 + √15
= $\sqrt{4x2}$ + √10 + $\sqrt{4\ x\ 3}$ + √15
= √4 x √2 + √10 + √4 x √3 + √15
= 2√2 + √10 + 2√3 + √15

(2) (√3 + √6) (√4 - √2)

Solution

(√3 + √6) (√4 - √2)
= √3x√4 - √3x√2 + √6x√4 - √6x√2
= √12 - √6 + √24 - √12
= $\sqrt{4x3}$ - √6 + $\sqrt{4x6}$ - $\sqrt{4x3}$
= √4x√3 - √6 + √4x√6 - √4x√3
= 2√3 - √6 + 2√6 – 2√3
= 2√3 – 2√3 - √6 + 2√6
= (2 – 2) √3 – 1√6 +2√6

= 0√3 – 1√6 + 2√6
= 0 - 1√6 + 2√6 = -1√6 + 2√6
= (-1 + 2) √6 = 1√6 = √6

(3). (√5 - √3) (√6 + √2)

Solution

(√5 - √3) (√6 + √2)
= √5x√6 + √5x√2 - √3x√6 - √3x√2 = $\sqrt{30} + \sqrt{10} - \sqrt{18} - \sqrt{6}$
= √30 + √10 - $\sqrt{9x2}$ - √6
= √30 + √10 - √9 x √2 - √6
= √30 + √10 – 3√2 - √6

(4). (√2 - √6) (√3 - √2)

Solution

(√2 - √6) (√3 - √2)
= √2 x √3 - √2 x√2 - √6 x √3 + √6 x √2
= √6 - √4 - √18 +$\sqrt{12}$
= √6 – 2 - $\sqrt{9\ x\ 2}$ + $\sqrt{4\ x\ 3}$
= √6 – 2 - √ 9 x √2 + √4 x √3
= √6 – 2 – 3√2 + 2√3

Solution

(5). (2√3 + 4√2) (4√4 + √2) .
(2√3 + 4√2) (4√4 + √2)
= 2√3 x 4√4 + 2√3 x √2 + 4√2 x 4√4 + 4√2 x √2
= 8√12 + 2√6 + 16√8 + 4√4
= 8$\sqrt{4\ x\ 3}$ + 2√6 + 16$\sqrt{4x2}$ + 4(2)
= 8 x √4 x √3 + 2√6 + 16 x √4 x √2 + 8
= 8 x 2 x √3 + 2√6 + 16 x 2 x√2 + 8
= 16√3 + 2√6 + 32√2 + 8

(6). (2√5 + 4√3) (3√3 – 2√2)

Solution

(2√5 + 4√3) (3√3 – 2√2)
= 2√5 x 3√3 – 2√5 x 2√2 + 4√3 x 3√3 – 4√3 x 2√2
= 2x3x $\sqrt{5\ x\ 3}$ – 2x2x√5x$\sqrt{2}$ + 4x3x√3x3 – 4x2x√3x$\sqrt{2}$
= 6√15 – 4√10 + 12√9 – 8√6
= 6√15 – 4√10 + 12(3) – 8√6
= 6√15 – 4√10 + 36 – 8√6

(7). (5√3 – 4√2) (√9 + 2√2)

Solution

(5√3 – 4√2) (√9 + 2√2)

= 5√3 x√9 +5√3 x 2√2 – 4√2 x √9 – 4√2 x 2√2
= 5√27 + 10√6 – 4√18 – 8√4
= $5\sqrt{9 \times 3}$+10√6 – 4$\sqrt{9x2}$ – 8(2)
= 5x√9x√3 + 10√6 – 4x√9 x√2 – 16
= 5 x 3 x √3 + 10√6 – 4 x 3 x √2 – 16
= 15√3 + 10√6 – 12√2 – 16

(8). (2√2 – 3√7) (4√6 – 3√3)

Solution

(2√2 – 3√7) (4√6 – 3√3)
= 2√2x4√6 – 2√2x3√3 – 3√7x4√6 + 3√7x3√3
= 8√12 – 6√6 – 12√42 + 9√21
= 8$\sqrt{4x3}$ – 6√6 – 12√42 + 9√21
= 8x√4x√3 – 6√6 – 12√42 + 9√21
= 8x2x√3 – 6√6 – 12√42 + 9√21
= 16√3 – 6√6 – 12√42 + 9√21

(9).i. Simplify 2√3 (2 - √3) + 3√2(√2 – 1).
ii. Hence evaluate your answer to 2.d.p, given that √2 = 1.414 and √3 = 1.732

Solution

2√3 (2 - √3) + 3√2(√2 – 1)
= 2√3 x2 – 2√3x√3 + 3√2x√2 – 3√2 x1
= 4√3 – 2√9 +3√4 – 3√2
= 4√3 – 2(3) + 3(2) – 3√2
= 4√3 – 6 + 6 – 3√2
= 4√3 – 3√2, since – 6 +6 = 0

(ii). If √2 = 1.414 and √3 = 1.732,
then 4√3 – 3√2 = 4(1.732) – 3(1.414)
= 6.93 – 4.242 = 2.69

(10). Simplify (1 - √3) ($^1/_3$ + √3), and leave your answer in the form a + b√3

Solution

(1 - √3) ($^1/_3$ + √3)
= 1 x $^1/_3$ + 1 x √3 - √3 x $^1/_3$ - √3 x√3
= $^1/_3$ + √3 – $^1/_3$ √3 -√9
= 0.33 + 1√3 – 0.33√3 – 3
= 0.33 – 3 + 1√3 – 0.33√3
= - 2.67+(1- 0.33)√3
= - 2.67 + (0.67) √3
Which is of the form a + b√3, where a = - 2.67 and b = 0.67

(11). Simplify (2 + 3√5) (4 – 3√5)

Solution

(2 + 3√5) (4 – 3√5)
= 2 x 4 – 2 x 3√5 + 3√5 x 4 – 3√5 x 3√5

$= 8 – 6\sqrt{5} + 12\sqrt{5} – 9\sqrt{25}$
$= 8 - 6\sqrt{5} + 12\sqrt{5} - 9(5)$
$= 8 + 12\sqrt{5} – 6\sqrt{5} – 9(5)$
$= 8+ (12 – 6)\ \sqrt{5} – 45$
$= 8 + 6\sqrt{5} – 45 = 8 – 45 + 6\sqrt{5}$
$= -37 + 6\sqrt{5}$

(12). Evaluate $(2\sqrt{3} – 4\sqrt{2})^2$

Solution

$(2\sqrt{3} – 4\sqrt{2})^2$
$= (2\sqrt{3} – 4\sqrt{2})\ (2\sqrt{3} – 4\sqrt{2})$
$= 2\sqrt{3}$ x $2\sqrt{3} – 2\sqrt{3}$ x $4\sqrt{2} – 4\sqrt{2}$ x $2\sqrt{3} + 4\sqrt{2}$ x $4\sqrt{2}$
$= 4\sqrt{9} – 8\sqrt{6} – 8\sqrt{6} + 16\sqrt{4}$
$= 4(3) – 8\sqrt{6} – 8\sqrt{6} +16(2)$
$= 12 – 8\sqrt{6} – 8\sqrt{6} + 32$
$= 12 + 32 – 8\sqrt{6} – 8\sqrt{6}$
$= 44 – 8\sqrt{6} – 8\sqrt{6}$
$= 44 – 16\sqrt{6}$

(13) Show that or prove that $2\sqrt{45} + 3\sqrt{8} – 4\sqrt{20} – 5\sqrt{2} = \sqrt{2} – 2\sqrt{5}$
NB: In such a question, either the left hand side of the equation (L.H.S) is simplified to get the right hand side (R.H.S) or vice versa.

Solution

Consider the L. H. S
$2\sqrt{45} + 3\sqrt{8} – 4\sqrt{20} – 5\sqrt{2}$
$= 2\sqrt{9 \times 5} + 3\sqrt{4 \times 2} – 4\sqrt{4 \text{ x } 5} – 5\sqrt{2}$
$= 2\text{x}\sqrt{9}\text{x}\sqrt{5} + 3\text{x}\sqrt{4}\text{x}\sqrt{2} – 4\text{x}\sqrt{4}\text{x}\sqrt{5} – 5\sqrt{2}$
$= 2\text{x}3\text{x}\sqrt{5} + 3\text{x}2\text{x}\sqrt{2} – 4\text{x}2\text{x}\sqrt{5} – 5\sqrt{2}$
$= 6\sqrt{5} + 6\sqrt{2} – 8\sqrt{5} – 5\sqrt{2}$
$= 6\sqrt{5} – 8\sqrt{5} + 6\sqrt{2} – 5\sqrt{2}$
$= (6 – 8)\ \sqrt{5} + 6\sqrt{2} – 5\sqrt{2}$
$= - 2\sqrt{5} + (6 – 5)\sqrt{2}$
$= -2\sqrt{5} + 1\sqrt{2} = -2\sqrt{5} + \sqrt{2}$
$= \sqrt{2} - 2\sqrt{5}$, which is equal to the R.H.S

Questions
(Q1) Evaluate the following:

(a) $(\sqrt{9})^2$ Ans: 9

(b) $(3\sqrt{2})^2$ Ans: 18

(c) $3(\sqrt{2})^2$ Ans: 6

(d) $(3\sqrt{2})^2 + (4\sqrt{2})^2$ Ans: 50

(e) $4(\sqrt{3})^2 – (2\sqrt{2})^2$ Ans: 4

(Q2) Simplify the following:

(a) $\sqrt{16} + \sqrt{320} + \sqrt{75}$
Ans: $4 + 8\sqrt{5} + 5\sqrt{3}$

(b) $2\sqrt{2} + 3\sqrt{3} + \sqrt{27} + \sqrt{8}$
Ans: $4\sqrt{2} + 6\sqrt{3}$

(c) $3 + 2\sqrt{8} + 5\sqrt{72} + \sqrt{27}$
Ans: $3 + 34\sqrt{2} + 3\sqrt{3}$

(d) $\sqrt{147} + 3\sqrt{3} - \sqrt{243} – 2\sqrt{2}$
Ans: $\sqrt{3} – 2\sqrt{2}$

(e) $6 + 4\sqrt{2} + 2\sqrt{64} – 3\sqrt{45} - \sqrt{2}$
Ans: $22 + 3\sqrt{2} – 9\sqrt{5}$

(f) $\sqrt{200} + 20 – 2\sqrt{20} + \sqrt{175} + 3 – 2\sqrt{7}$
Ans: $10\sqrt{2} + 23 – 4\sqrt{5} + 3\sqrt{7}$

(g) $1 – 2\sqrt{5} + \sqrt{125} - \sqrt{567} + 2\sqrt{7} + 2$
Ans: $3 + 3\sqrt{5} – 7\sqrt{7}$

(h) $(2 + 4\sqrt{2}) – (1 – 3\sqrt{2})$
Ans: $1 + 7\sqrt{2}$

(i) $5 + 3\sqrt{3} + 4\sqrt{2} - \sqrt{3} + 4\sqrt{3} – 2\sqrt{2} – 1$
Ans: $4 + 6\sqrt{3} + 2\sqrt{2}$

(j) $3\sqrt{180} + 2 – 2\sqrt{5} – 5 + 2\sqrt{98} – 6\sqrt{2}$
Ans: $16\sqrt{5} + 8\sqrt{2} - 3$

(Q3) Simplify each of the following:

(a) $\sqrt{4}\,(\sqrt{3} + \sqrt{10})$
Ans: $2\sqrt{3} + 2\sqrt{10}$

(b) $2\sqrt{3}\ (2\sqrt{5} + 4\sqrt{12})$
Ans: $4\sqrt{15} + 48$

(c) $3\sqrt{4}\ (2\sqrt{10} - 2\sqrt{5})$
Ans: $12\sqrt{10} - 12\sqrt{5}$

(d) $(2 + \sqrt{5})\ (\sqrt{3} + \sqrt{4})$
Ans: $\sqrt{6} + 2\sqrt{2} + \sqrt{15} + 2\sqrt{5}$

(e) $(2\sqrt{3} + \sqrt{4})\ (\sqrt{2} - 3\sqrt{2})$
Ans: $- 4\sqrt{6} - 4\sqrt{2}$

(f) $(2\sqrt{3} - 4\sqrt{2})\ (3\sqrt{5} + 2\sqrt{6})$
Ans: $6\sqrt{15} + 12\sqrt{2} - 12\sqrt{10} - 16\sqrt{3}$

(g) $(2\sqrt{5} - 3\sqrt{2})\ (4\sqrt{5} - 3\sqrt{6})$
Ans: $40 - 6\sqrt{30} - 12\sqrt{10} + 18\sqrt{3}$

(h) $2\sqrt{2}\ (3\sqrt{2} - 4\sqrt{3}) + 3\sqrt{3}(\sqrt{4} - 2\sqrt{4})$
Ans: $12 - 8\sqrt{6} - 6\sqrt{3}$

(i) $2\sqrt{5}(2\sqrt{2} + 4\sqrt{3}) - 3\sqrt{4}(2\sqrt{2} - 2\sqrt{5})$
Ans: $4\sqrt{10} + 8\sqrt{15} - 12\sqrt{2} + 12\sqrt{5}$

(Q4)a. Simplify $2\sqrt{5}\ (4 - \sqrt{2}) + 3\sqrt{5}(\sqrt{2} + 1)$.
b. Hence evaluate your answer to 2 d.p, given that $\sqrt{5} = 2.34$ and $\sqrt{10} = 3.16$
Ans: a. $11\sqrt{5} + \sqrt{10}$
b. 28.9

(Q5) Evaluate $(1 - 2\sqrt{2})\ (^1/_2 + \sqrt{2})$
Ans: -3.5

(Q6) Evaluate the following:
(a) $(2\sqrt{2} + 3\sqrt{5})^2$
Ans: $12\sqrt{10} + 53$

(b) $(3\sqrt{4} - 2\sqrt{2})^2$
Ans: $44 - 24\sqrt{2}$