CHAPTER SIX

LINEAR SEQUENCE

Sequence:

-A sequence is a set of numbers or terms written in a definite order, with a rule or formula for obtaining the terms.

Example(1)
1, 3, 5, 7, 9, 11……………..
- In this given sequence, each term is obtained by adding 2 to the preceding term or the previous term.

Example(2):
1, 3, 9, 27…
In this given sequence, each term is obtained by multiplying the previous term by 3.

The rule of a sequence:

- Each sequence has a rule, and this can be had by making a careful study of the sequence.
- The rule enables us to know or determine any term or other members of the sequence.
- Example(1) 1, 5, 9, 13 …………….. -
For this sequence, the rule is: Any term + 4 = the next term.
- For this reason, the next two terms of this sequence will be 17 and 21.
-Example(2): 3, 9, 27 …….
-The rule is that: Any number multiplied by 3 = the next term.
- For this reason, the next term = $27 \times 3 = 81$.
- After this term, the next two terms are $81 \times 3= 243$, and $243 \times 3 = 729$.
- Example(3): 2, 5, 11, 23 ……………..
- The rule for this sequence is that: (Any term $\times$ 2) + 1 = the next term.
- The next three terms are therefore 47, 95 and 191.

- Example(4) 3, 11, 35 …….. -

The rule is that: (Any number × 3) + 2 = the next term.

-The next three terms are therefore 107, 323 and 971.

- Example(5) $1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8} \ldots \ldots \ldots \ldots \ldots$
- The rule is that: Any number $\times \frac{1}{2}$ = the next term.
- The next two terms are therefore $\frac{1}{8} \times \frac{1}{2} = \frac{1}{16}, and\ \frac{1}{16} \times \frac{1}{2} = \frac{1}{32}\ i.e. \frac{1}{16}$ and $\frac{1}{32}$.
- Example(6): 1, -2, 4, -8, 16, -32 ……………
- The rule is that:(Any term × -2) = the next term.
- The next two terms are (-32 × -2) = 64 and (64 × -2) = -128. i.e. 64 and -128.

  Example(7): 2, 5, 26 …………
- The rule is that:(Any term squared) +1 = the next term.

  Example(8): 25, 20, 15 ……………….
- The rule is that:(Any term – 5) = the next term.

  Example(9): 25, 40, 70 …………
- The rule is that:(any term -5) × 2 = the next term.

  Example(10): {-6, 10, 26 …….}

  The rule is that: any term added to 16 = the next term.

  The next two terms are 42 and 58.

Series:

- When all the terms of a sequence are added together, we get what we call a series.
- For example, consider the sequence 2, 4, 6, 8, 10 ……..
- If the terms of this series are added together, we shall get 2 + 4 + 6 + 8 + 10 ……
- This is what is referred to as a series.
- Since such a series does not contain a fixed number of terms, it is referred to as an infinite series.
- Consider the sequence 2, 4, 8 …….32.
- By adding the terms together, we get the series 2 + 4 + 8 …… + 32.

- Since this series contains a definite number of terms, it is referred to as a finite series.

Types of sequence:

- Two types of sequence shall be considered and these are:

(i) Arithmetic progression (AP), which is also known as linear sequence.

(b) Geometric progression (G.P), which is also known as exponential sequence.

Arithmetic progression:

- In this type, any term differs from the preceding term by a constant known as the common difference.
- This common difference which is represented by d, can either be positive or negative.

  Examples(1) 2, 5, 8, 11 ……………
- In this, the common difference = 5 – 2 = 3 or 8 – 5 = 3.

  Example(2) 3, 13, 23 ……..
- In this, the common difference = 13 – 3 = 10 or 23 – 13 = 10.

  Example (3) -2, - 4, - 6, -8 ……………………
- In this case, d = - 6 – (- 4) = - 6 + 4 = -2.

  - The first term of an A.P is represented by U_1, the second term byU_2 and the third by U_3 and so on.

  - With reference to A.P, the following must be taken notice of:

  U_1= 1st term = a

  U_2 = 2nd term = a + d

  U_3= 3rd term = a + 2d

  U_4= 4th term = a + 3d

  U_n = nth term = a + (n-1)d
- The nth term of an A.P is therefore given by $U_n = a+(n-1)d$.
- This is the formula for finding any term of a linear sequence.

(Q1) Find the 11th term of a linear sequence of the form 4, 9, 14, 19 …………

Soln:

a = 4 and d = 9 – 4 = 5.

From $U_n = a + (n -1)d$

=> $U_{11}= a + (11 – 1)d$

=> $U_{11}= 4 + (11 – 1) \times 5$

=> $U_{11} = 4+5(10) = 4 + 50$

= 54.

(Q2) Find the 20th term of the sequence 4, 6, 8, 10, 12 ……

Soln:

a = 4   and d = 6 - 4 = 2.

Since $U_n = a + (n -1)d$, then

$U_{20} = 4 + (20 – 1) 2$

$U_{20} = 4 + (19) \times 2$

=> $U_{20} = 4 + 2(19)$

=> $U_{20} = 4 + 38$

=> $U_{20} = 42$.

(Q3) Find the 8th term of a linear sequence of the form 47, 42, 37, 32 ………

Soln:

a = 47   and d = 42 – 47 = - 5.

Since $U_n = a + (n -1)d$,

=> $U_8 = 47 + (8 – 1) (-5)$

=> $U_8 = 47 + (-5)(8 - 1)$

=> $U_8 = 47 - 5(7)$

=> $U_8 = 47 - 35 = 12$

(Q4) Show that the 90th term of the sequence 2, 7, 12, 17 ………… is 447.

Soln:

a = 2 and d = 7 – 2 = 5.

From $U_n = a + (n - 1)d$

=> $U_{90} = 2 + (90 - 1)\ 5$

=> $U_{90} = 2 + 5(89)$

=> $U_{90} = 2 + 445 = 447$.

(Q5) Find the 12th term of an A.P of the form 7, $6\frac{1}{4}, 5\frac{1}{2} \ldots \ldots \ldots \ldots \ldots \ldots$

Soln:

a = 7, and d = $6\frac{1}{4} - 7 = 6.25 - 7 = -0.75$.

Since $U_n = a + (n - 1)d$

=> $U_{12} = 7 + (12 - 1)(-0.75)$

=> $U_{12} = 7 + (11)(-0.75)$

=> $U_{12} = 7 + (-8.25)$

=> $U_{12} = 7 - 8.25 = -1.25$.

(Q6) Find the linear sequence whose 8th term is 38 and 22nd term is 108.

Soln:

$U_n = a + (n - 1)d$.

The 8^{th} term =

$U_8 = a + (8 - 1)d$

=> $U_8 = a + 7d$

Since the 8^{th} term = 38

=> $a + 7d = 38$ …………… eqn (1).

Also the 22^{nd} term = 108.

$U_{22} = a + (22-1)d$

=> $U_{22} = a + 21d$.

Since the 22^{nd} term = 108.

=> $a + 21d = 108$ ……………….. eqn (2)

Solving eqn (1) and eqn (2) simultaneously => $a = 3$ and $d = 5$.

The sequence = a, a + d, a + 2d, a + 3d …………………..

= 3, 3+5, 3 + 2(5), + 3 + 3(5) …………….

= 3, 8, 13, 18 ……………………..

(Q7 The fourth term of a linear sequence is 19 and the eleventh term is 54.Find the 8^{th} term.

Soln:

$U_n = a + (n - 1)d$

The fourth term =

$U_4 = a + (4 - 1)d$

=> $U_4 = a + 3d$.

But the fourth term = 19

$\Rightarrow a + 3d = 19$ .... eqn (1)

The 11th term = 54.

$U_{11} = a + (11 – 1)d$

$\Rightarrow U_{11} = a + 10d$.

Since the 11th term = 54

$\Rightarrow a + 10d = 54$ .... eqn (2)

Solving eqn (1) and eqn (2) simultaneously => $d = 5$ and $a = 4$.

The 8th term of the sequence is given by $U_8 = a + (8-1) d$

$\Rightarrow U_8 = a + 7d \Rightarrow U_8 = 4 + 7(5) = 4 + 35 = 39$.

(Q8) The 10th term of a linear sequence is 16 and the common difference is 2. Determine the first term.

Soln:

$n = 10, d = 2, a = ?$

$U_n = a + (n - 1)d$

$\Rightarrow U_{10} = a + (10 - 1)d$

$\Rightarrow U_{10} = a + 9d = a + 9(2)$

$= a + 18$.

Since the 10th term = 16

$\Rightarrow a + 18 = 16 \Rightarrow a = 18 – 16 = 2 \Rightarrow a = 2$.

The first term = 2.

(Q9) The second and the fourth terms of a linear sequence are 9 and 17 respectively. Find the common difference.

Soln:

The second term = $U_2 = a + (2 - 1)d$

=> $U_2 = a + d$.

Since the second term = 9, => $a + d = 9$ …………………….eqn (1)

The 4th term = $U_4 = a + (4 – 1)d = a + 3d$.

Since the 4th term = 17

=> $a + 3d = 17$ ……………………………eqn (2)

Solving eqn (1) and eqn (2) simultaneously => $d = 4$.

(Q10) Find the nth term of the following sequences:

(a) 2, 4, 6, 8.
(b) -7, - 4, -1 and 2.

N/B: To find the nth term of an A.P, we use the relation $U_n = a + (n - 1)d$.

Soln:

(a) The first term = a = 2.

The common difference = d = 4 – 2 =2.

Since $U_n = a + (n - 1)d$

=> $U_n = 2 + (n - 1)2$,

=> $U_n = 2 + 2(n - 1)$

=> $U_n = 2 + 2n - 2 = 2 – 2 + 2n = 2n$.

=> the nth term = $U_n = 2n$.

(b) $a = -7$ and $d = - 4 – (-7) = - 4 + 7 = 3$.

From $U_n = a + (n - 1)d$

=> $U_n = -7 + (n - 1)3$

=> $U_n = -7 + 3(n - 1)$,

=> $U_n = -7 + 3n - 3$

=> $U_n = -7 - 3 + 3n$,

=> $U_n = -10 + 3n$

=> $U_n = 3n - 10$,

=> the n^{th} term = $3n - 10$.

(Q11) Given the Arithmetic progression 9, 12, 15, 18 ……………

Find (a) the 8^{th} term.

(b) the 9^{th} term.

Soln:

The first term = a = 9.

The common difference = d = 3.

(a) $U_n = a + (n - 1)d$

=> $U_8 = 9 + (8 - 1)3$

=> $U_8 = 9 + (7)3$,

=> $U_8 = 9 + 21 = 30$.

(b) the n^{th} term is given by $U_n = a + (n - 1)d$

=> $U_n = a + (n - 1)d$,

=> $U_n = 9 + (n - 1)\ 3$

=> $U_n = 9 + 3(n - 1)$,

=> $U_n =$ 9 + 3n – 3

=> $U_n =$ 9 – 3 + 3n,

=> $U_n =$ 6 + 3n

=> the n^{th} term = U_n = 6 + 3n.

(Q12) The n^{th} term of a sequence is $5 + \frac{2}{3^{n-2}}$ for n ≥ 1.

What is the sum of the fourth and the fifth terms? Leave your answers in the form $\frac{x}{y}$, where x and y are integers.

Soln:

The n^{th} term = $5 + \frac{2}{3^{n-2}}$.

For the fourth term, n = 4

=> 4^{th} term = $5 + \frac{2}{3^{4-2}}$

$= 5 + \frac{2}{3^2} = 5 + \frac{2}{9}$

$= \frac{5}{1} + \frac{2}{9}$ i.e. find the L.C.M

$\frac{45+2}{9} = \frac{47}{9}$, which is of the form $\frac{x}{y}$ where x and y are integers.

Also x = 47 and y = 9.

For the fifth term, n = 5 => the 5^{th} term = $5 + \frac{2}{3^{5-2}}$

$= 5 + \frac{2}{3^3} = 5 + \frac{2}{27}$,

$= \frac{5}{1} + \frac{2}{27}$ i.e. find the L.C.M

$\frac{135+2}{27} = \frac{137}{27}$

Which is of the form $\frac{x}{y}$ where x and y are integers.

Also x = 137 and y = 27.

The sum of the 4th term and the 5th term = $\frac{47}{9} + \frac{137}{27}$ i.e. find the L.C.M

$\frac{141+137}{27} = \frac{278}{27}$, which is of the form $\frac{x}{y}$, $where\ x = 278\ and\ y = 27.$

Also x and y are integers.

(Q13) The 43rd term of an A.P is 26. Find the first term of the progression, given that the common difference is $\frac{1}{2}$.

Soln:

a = ? and d = $\frac{1}{2}$ = 0.5.

The 43rd term = U_{43}

= a +(43 – 1) 0.5 = a + (42) 0.5,

= a + 21.

But since the 43rd term = 26, => a + 21 = 26, => a = 26 – 21 = 5.

The first term = 5.

(Q14) In an Arithmetic Progression, the thirteenth term is 27 and the seventh term is three times the second term. Find the first term, the common difference and the tenth term of the sequence.

Soln:

Let a = the first term and d = the common difference.

From U_n = a + (n -1)d, => the 13th term = U_{13} = a + (13 -1)d,

=> U_{13} = a + 12d.

Since the 13th term = 27, then a + 12d = 27 …………………. eqn (1)

The 7th term is given by

$U_7 = a + (7 - 1) d = a + 6d$.

The second term is given by $U_2 = a + (2-1)d = a + d$.

Since the 7th term is three times the second term, then $a + 6d = 3(a + d)$

$\Rightarrow a + 6d = 3a + 3d$,

$\Rightarrow 6d - 3d = 3a - a$,

$\Rightarrow 3d = 2a \Rightarrow d = \frac{2a}{3}$............. eqn (2)

Substitute $d = \frac{2a}{3}$ into eqn (1)

i.e. $a + 12d = 27$, $\Rightarrow a + 12\left(\frac{2a}{3}\right) = 27$,

$\Rightarrow a + 12 \times \frac{2a}{3} = 27$

$\Rightarrow a + 8a = 27 \Rightarrow 9a = 27$,

$\Rightarrow a = \frac{27}{9} = 3$.

To find the value of d, substitute a = 3 into eqn (2)

i.e. $d = \frac{2a}{3} \Rightarrow d = \frac{2(3)}{3} \Rightarrow d = 2$.

The 10th term is given by $U_{10} = a + (10 - 1) d$

$= a + 9d = 3 + 9(2), = 3 + 18 = 21$.

(Q15) In a linear sequence, the ratio of the 2nd term to the 3rd term is 4.If the difference between the fourth and the fifth term is 9, determine

(a) the first term
(b) the fifth term

Soln:

The 2^{nd} term = a + (2 – 1)d = a + d.

The 3^{rd} term = a + (3 -1)d = a + 2d.

Since the ratio of the 2^{nd} term to the 3^{rd} term is 4,

=> $\frac{a+2d}{a+d} = \frac{4}{1} = 4, => a + 2d = 4(a + d)$

=> a + 2d = 4a + 4d,

=> a – 4a = 4d – 2d,

=> - 3a = 2d

=> a = $\frac{2d}{-3} => a = \frac{-2d}{3} \dots\dots\dots\dots\dots. eqn\ (1)$

a) Since the difference between the 5^{th} term and the 6^{th} term is 9, then d = 9.

Substitute d = 9 into eqn (1)

i.e. a = $\frac{-2}{3}d => a = \frac{-2}{3} \times 9 = -6$.

(b) The fifth term = a + (5 – 1)d

= a + 4d = - 6 + 4(9) = - 6 + 36 = 30.

N/B: The ratio associated with the 3^{rd} term is 4, and that associated with the 2^{nd} term is 1.

<u>The last term of a linear sequence:</u>

- A linear sequence such as 2, 5, 8, 11 ……., does not have a last term since it is an infinite sequence.
- Another example of a linear sequence which also does not have a last term is 60, 30, 0, -30 ……….
- But a finite sequence such as 2, 5, 11 …….. 95, has a last term which is 95.
- Also the last term of the sequence 3, 6, 9 … 30, is 30.
- The last term of a linear sequence which is represented by l, is given by l = a +(n – 1)d, where n = the number of terms.

(Q1) Find the number of terms of the following linear sequence:

(i) 3, 7, 11 ……. 31.

(ii) 2, -9, -20 ……… -141.

Soln:

(i) a = 3 and d = 4.

But l = a + (n – 1)d, where l = the last term and n = the number of terms.

=> 31 = 3 + (n – 1)4

=> 31 = 3 + 4(n – 1),

=> 31 = 3 + 4n – 4

=> 31 – 3 + 4 = 4n,

=> 4n = 32 => n = $\frac{32}{4}$ = 8

=> there are 8 terms in the sequence.

(ii) a = 2, d = - 9 - 2= - 11 and l = -141.

From l = a + (n – 1)d

=> -141 = 2 + (n – 1)(-11),

=> -141 = 2 + (-11)(n -1),

=> -141 = 2 + (-11n + 11)

=> -141 = 2 + -11n + 11,

=> -141 = 2 – 11n + 11

=> -141 = 13 -11n,

=> 11n = 13 + 141

=> 11n = 154, => n = $\frac{154}{11} = 14.$

=> The sequence has 14 terms.

Application of linear sequence:

Application of linear sequence can be seen in many cases, examples of which have been illustrated in the following few cases.

(Q1) A man started a poultry farm with 250 birds. If this was increased by 60 birds every year, determine the number of birds that he will be having in the eight year.

Soln:

The sequence which can be created from this question is: 250, 250 + 60, 250 + 60 + 60, 250 + 60 + 60 + 60 ……………………..

= 250, 310, 370………………

This is a linear sequence in which a = 250 and d = 60.

The eight year = the eight term.

For the eight term, n = 8.

From $U_n = a + (n – 1)d$

=> $U_8 = 250 + (8 -1)60$,

=> $U_8 = 250 + 60(7)$

=> $U_8 = 250 + 420 = 670$.

=> The number of birds had by the farmer in the 8^{th} year

= 670 birds.

(Q2)Mr. Kwesi Atta deposited ¢500 at a bank. If ¢80 was added to this amount at the end of every month, determine the value of this amount at the end of the twentieth month.

Soln:

The linear sequence which can be had from this question is: 500, 500 + 80, 500 + 80 + 80, + 500 + 80 + 80 + 80…………….

= 500, 580, 660…………………

* This is a linear sequence in which a = 500 and d = 80.

* For the twentieth month, n = 20.

From $U_n = a + (n - 1)d$

=> $U_{20} = 500 + (20 - 1)80$,

=> $U_{20} = 500 + 80(19)$

=> $U_{20} = 500 + 1520 = 2020$.

=> The value of his amount at the end of the twentieth month = ¢2020.

<u>The sum of an A.P. or linear sequence:</u>

* The sum of the first n terms of an A.P. is denoted by $S_n = \frac{n}{2}\{2a + (n-1)d\}$.

* Also $S_n = \frac{n}{2}(a + l)$, where l is the last term.

* The first formula is used when a and d are known, i.e. the first term and the difference.

* The second one is used when used when the difference is not known, but the first term and the last terms are known.

<u>The use of the $S_n = \frac{n}{2}\{2a + (n-1)d\}$:</u>

Q1) Find the sum of the first six terms of the A.P. 3, 5, 7, 9 ……………

Soln:

a = 3 and d = 5 – 3 = 2.

For the first six terms, n = 6.

From $S_n = \frac{n}{2}\{2a + (n-1)d\}$

=> $S_6 = \frac{6}{2}\{2(3) + (6-1)2\}$,

=> $S_6 = 3\{6 + 2(5)\}$

=> $S_6 = 3\{6 + 10\}$,

=> $S_6 = 3\{16\} = 48$.

(Q2) Find the sum of the first twenty terms of the sequence 1, 3, 5, 7 ………….

Soln:

n = 20, d = 2 and a = 1.

$S_n = \frac{n}{2}\{2a + (n-1)d\}$

=> $S_{20} = \frac{20}{2}\{2(1) + (20-1)2\}$,

=> $S_{20} = 10\{2 + 2(19)\}$

=> $S_{20} = 10\{40\}$,

=> $S_{20} = 400$.

(Q3) In an A.P., there are six terms and the first term is 4500. If the common difference is 250, find the sum of this sequence.

Soln:

a = 4500, d = 250 and n = 6.

Since $S_n = \frac{n}{2}\{2a + (n-1)d\}$

=> $S_6 = \frac{6}{2}\{2(4500) + (6-1)250\}$,

=> $S_6 = 3\{9000 + (5)250\}$

=> $S_6 = 3\{9000 + 1250\}$,

=> $S_6 = 3\{10250\}$

=> $S_6 = 30750$.

(Q4) Find the sum of the first 20 terms of the arithmetic progression 16 + 9 + 2 + (-5)+ ……………………

Soln:

a = 16, d = 9 - 16 = -7 and n = 20.

Since $S_n = \frac{n}{2}\{2a + (n-1)d\}$

=> $S_{20} = \frac{20}{2}\{2(16) + (20-1)(-7)\}$,

=> $S_{20} = 10\{32 + (19)(-7)\}$

=> $S_{20} = 10\{32 + (-133)\}$, => $S_{20} = 10\{32 - 133\}$

=> $S_{20} = 10\{-101\} = -1010$.

(Q5) Find the sum of the first five terms of an A.P. of the form 0.03, 0.06, 0.09………………

Soln:

a = 0.03, d = 0.06 – 0.03 = 0.03 and n = 5.

From $S_n = \frac{n}{2}\{2a + (n-1)d\}$

=> $S_5 = \frac{5}{2}\{2(0.03) + (5-1)0.03\}$,

=> $S_5 = 2.5\{0.06 + (4)(0.03)\}$

=> $S_5 = 2.5\{0.06 + 0.12\}$,

=> $S_5 = 2.5\{0.18\} = 0.45$.

=> The sum of the first five terms of the sequence is 0.45.

(Q6) The sixth and the eleventh terms of a linear sequence are respectively 23 and 48. Calculate the sum of the first twenty terms of the sequence.

Soln:

The sixth term is given by $U_6 = a + (6-1)d = a + 5d$.

But since the sixth term = 23, then a + 5d = 23 ……..eqn (1)

The eleventh term is given by $U_{11} = a + (11-1)d = a + 10d$.

But since the eleventh term = 48, then a + 10d = 48 …………eqn (2).

Solving eqn (1) and eqn (2) simultaneously, => a = -2 and d = 5.

Sum of A.P. = $S_n = \frac{n}{2}\{2a + (n-1)d\}$

=> $S_{20} = \frac{20}{2}\{2(-2) + (20-1)5\}$

=> $S_{20} = 10(-4 + 95) = 910$.

(Q7) The 21^{st} term of a linear sequence is $5\frac{1}{2}$, and the sum of the first 21 terms of the same sequence is $94\frac{1}{2}$. Determine

(a) the first term and the common difference.
(b) the sum of the first 30 terms.

Soln:

a) The 21^{st} term of the linear sequence is given by a + (21 – 1)d = a + 20d.

Since the 21^{st} term = $5\frac{1}{2}$

=> a + 20d = $5\frac{1}{2}$

=> a + 20d = 5.5 ……….. eqn (1)

The sum of the first 21 terms of the sequence is given by

$S_{21} = \frac{21}{2}(2a + (21-1)d\}$ = 10.5(2a + 20d) = 21a + 210d.

Since the sum of the first 21 term = $94\frac{1}{2}$ = > 21a + 210d = $94\frac{1}{2}$

=> 21a + 210d = 94.5 ……….. eqn (2)

Solving eqn (1) and eqn (2) simultaneously,=> a = 3.5 and d = 0.1

a) From $S_n = \frac{n}{2}\{2a + (n-1)d\}$

=> the sum of the first 30 terms is given by

$S_{30} = \frac{30}{2}\{2(3.5) + (30-1)0.1\}$

=> $S_{30} = 15(7 + (29)0.1)$

=> $S_{30} = 15(7 + 2.9)$ = 15(9.9)

= 149.

The application of the formula $S_n = \frac{n}{2}(2a + (n-1)d)$:

(Q1) Madam Adjoa Essoun started working on a starting salary of $500. At the end of each month, her salary was increased by $20. Determine the total amount she earned as salary at the end of the eleventh month.

N/B: The linear sequence which can be formed is as follows:

500, 500 + 20, 500 + 20 + 20, 500 + 20+ 20 + 20 ……….. = 500, 520,540,560 ………..

Soln:

a = 500 and d = 20.

At the end of the eleventh month => n = 11.

$S_n = \frac{n}{2}\{2a + (n-1)d\}$.

The total amount earned as salary is given by

$S_{11} = \frac{11}{2}\{2(500) + (11-1)20\}$

=> $S_{11} = 5.5\{1000 + 20(10)\}$,

=> $S_{11} = 5.5(1200) = 6600$,

=> the total amount earned as salary = $6600.

(Q2) Amadu Musah has begun the exportation of goats, and in his first exportation, he exported 200 goats. He expects to increase the number of goats exported by 40 goats every year. Determine the total number of goats he is expected to export in the first twenty years.

N/B: The linear sequence which can be formed or had is as follows: 200, 200 + 40, 200 + 40 + 40, 200 + 40 + 40 + 40 …….. = 200, 240, 280,320…….

Soln:

a = 200 and d = 40.

The twentieth year => $n = 20$.

The total number of goats exported in the twenty years is given by

$S_{20} = \frac{20}{2}\{2(200) + (20 - 1)40\}$

=> $S_{20} = 10(400 + 760)$,

=> $S_{20} = 10(1160) = 11600$.

=> Total number of goats expected to be exported = 11600 goats.

<u>The use of the formula l = a + (n – 1)d:</u>

(Q1) Find the number of terms in the linear sequence 3, 7, 11, 15 ……… 31.

Soln:

$a = 3$, $d = 4$ and the last term, $l = 31$.

From $l = a + (n – 1)d$

=> $31 = a + (n – 1) 4$,

=> $31 = 3 + 4(n – 1)$

=> $31 = 3 + 4n – 4$,

=> $31 – 3 + 4 = 4n$

=> $32 = 4n$,

=> $n = \frac{32}{4} = 8$.

=> There are 8 terms.

(Q2) An arithmetic progression is of the form 2, -9, -20 ……… -141. Determine the number of terms that it contains.

Soln:

$a = 2$, $d = - 9 – 2 = -11$ and $l = -141$.

Since l = a + (n – 1)d

=> -141 = 2 + (n – 1)(-11),

=> -141 = 2 + (-11)(n – 1)

=> -141 = 2 – (11)(n – 1),

=> -141 = 2 – (11n – 11)

=> -141 = 2 – 11n + 11,

=> - 141 = 13 – 11n

=> 11n = 13 + 141,

=> 11n = 154

=> n = $\frac{154}{11} = 14.$

=> There are 14 terms.

(Q3) Find the sum of the linear sequence 50, 47, 44 ……, 14

N/B: We must first determine the number of terms within the sequence.

Soln:

a = 50, d = 47 – 50 = -3 and l = 14(i.e. the last term).

From l = a + (n -1)d

=>14 = 50 + (n – 1)(-3),

=> 14 = 50 +(-3)(n – 1)

=> 14 = 50 – 3(n – 1),

=> 14 = 50 – 3n + 3

=> 14 = 53 – 3n,

=> 3n = 53 - 14

=> 3n = 39, => n = $\frac{39}{3} = 13.$

=>There are 13 terms in the sequence.

We then determine the sum of these 13 terms. The sum of these 13 terms or the first 13 terms of this sequence is given by $S_{13} = \frac{13}{2}\{2(50) + (13-1)(-3)\}$

=> $S_{13} = 6.5\{100 + 12(-3)\}$,

=> $S_{13} = 6.5\{100 + (-36)\}$

=> $S_{13} = 6.5\{100 - 36\}$,

=> $S_{13} = 6.5(64)$ = 416.

Questions:

(1) Find the 7^{th} and the 8^{th} term of the linear sequence 6,12,18……………….

Ans:

7^{th} term = 42 and 8th term = 48.

(2) Determine the 21^{st} term and the 50^{th} term of the sequence of the form 34,30,26,22……………………

Ans:

21^{st} term = -46 and the 50^{th} term = -162.

(3) Find the 10^{th} term of the linear sequence 2, $2\frac{1}{2}, 3, 3\frac{1}{2} \ldots \ldots \ldots \ldots \ldots \ldots$

Ans:

10^{th} term = 6.5 or $6\frac{1}{2}$.

(4) A linear sequence is given as 4, 3.8, 3.6, 3.4 ………. Find

(a) its 31^{st} term.

(b) the difference between the 31^{st} term and the 60^{th} term.

Ans:

(a) 31^{st} term = -2.

(b) The difference = 5.8.

(5) The second term of a linear sequence is 8, and its fifth term is 20. Determine the first term and the common difference.

Ans:

The first term = 4 and the common difference = 4.

(6) For a linear sequence, the third term is 9 and the sixth term is 18.Determine the first term and the common difference.

Ans:

The first term = 3 and the common difference = 3.

(7) Determine the n^{th}term of the following sequences:

(a) 6, 12, 18……………….

(b) -10, -6, -2……………..

Ans:

(a) The n^{th} term is given by $U_n = 6n$.

(b) $U_n=4n - 14$.

(8) The first term of an arithmetic progression is 5, and its common difference is -2. Determine the n^{th} term.

Ans:

$U_n = 7 - 2n$ or $U_n = -2n + 7$

(9) The n^{th} term of a sequence is $6 + \frac{1}{2}n$.Find

(a) the sum of the 4^{th} term and the sixth term.

(b) the product of the 8^{th} term and the 10^{th} term.

Ans:

(a) 17 (b) 110.

(10) In an A.P, the third term is 6 and the fifth term is five times the first term. Find the first term and the common difference.

Ans:

The first term = 2 and the common difference = 2.

(11) Find the number of terms of the sequence 5, 10, 15 ………………. 60.

Ans:12 terms.

(12) An A.P has a last term of 25 and a first term of 15. How many terms does it contain if the common difference is 2?

Ans: 6 terms.

(13) A farmer had 400 goats and added 20 goats each year. How many goats did he have in the 9^{th} year.

Ans:580 goats.

(8) Find the sum of the first 60 terms of the sequence 1, 3, 5, 7 …………

Ans:3600.

(14) There are twenty terms in an A.P and the first term is 520. If the common difference is 40, find the sum of this sequence. Ans: 18000.

(15) Determine the sum of the first thirty one terms of the arithmetic progression 20 + 16 + 12 + 8 ……………

Ans: -1240.

(16) Find the sum of the first 4 terms of a linear sequence, whose fifth term is 10 and eight is 16.

Ans20.

(17) The fifth term of a linear sequence is 10, and the sum of the first three terms is 12. Determine

(a) the first term and the common difference.

(b) the sum of the first seven terms.

Ans:

(a) The first term is 2 and the common difference is 2.
(b) 56.

(18) Mr Kumi opened a fruit shop and sold 50 oranges on the first day. If the number of oranges sold increased by 10 everyday, find the total number of oranges he had sold altogether by the end of the tenth day.

Ans: 950 oranges.

(19) Find the number of terms in the linear sequence 5, 10, 15…………………..250.

Ans:50 terms

(20) An arithmetic progression is of the form 3, -1, -5, -9, -13 ……………. -29. How many terms does it contain?

Ans 9 terms.

(21) Determine the sum of the sequence 14, 10, 6, 2 ……………. -22.

Ans: -40.