CHAPTRE TWO

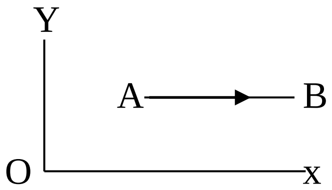
VECTORS

- A vector is a physical quantity which has both magnitude and direction.
- Example are
  a. A force of 20N acting North.
  b. A velocity of 5km/h East.

Types of vectors:

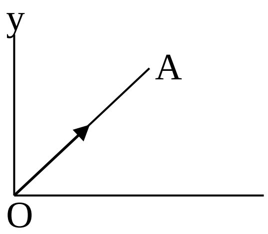
- In general the are two types and these are
  i. Free vector.
  ii. Position vector.

Free vector:



- A free vector is a vector which does not pass through any specific position.
- They are usually represented by small letters e.g $e.g\ \underset{\sim}{a},\underset{\sim}{b}\ and\ \underset{\sim}{c}$

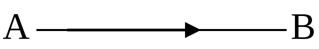
Position vector :



This is a vector which passes through the origin or a specified point.

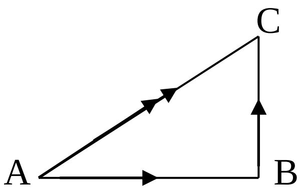
Vector notation:

- A vector may be represented by a line segment as shown next:



- This given vector can be represented by $\overrightarrow{AB}, \overline{AB}, \underline{AB}, \widetilde{AB}, \underset{\sim}{AB}$.

The Triangle law:



According to the triangle law,

$\overline{AC} = \overline{AB} + \overline{BC} \Rightarrow \overline{AB} = \overline{AC} - \overline{BC}\ and\ \overline{BC} = \overline{AC} - \overline{AB}$

The unit vector:

- This is a vector whose magnitude is one in the direction under consideration.
- The unit vector along a vector $\vec{a}$is written as $\hat{a}$
- Also the unit vector along a vector $\overrightarrow{b}$ is written as $\hat{b}$
- The unit vector along the vector $\overline{BC}$ is written as $\widehat{BC}$
- Consider the vector $A \longrightarrow B = 1$
- The vector is written as $\overrightarrow{AB}$ and its unit vector is written as $\widehat{AB}$.

Equal vectors:

- Two vectors are said to be equal if their magnitudes and directions are equal
- Example are $\overline{AB} = 50km/hE\ and\ \overline{CD} = 50km/h\ E.$

The negative vector:

- The negative of the vector $\underset{\sim}{a}$ *is* written as –a
- If $-\underset{\sim}{a}$ *is* the negative vector of the vector $\underset{\sim}{a},\ then\ \underset{\sim}{a}+(-\underset{\sim}{a}) = \underset{\sim}{o}.$
- The vector $-\underset{\sim}{a}$ *is a vector* of the same magnitude as $\underset{\sim}{a}$, but it is opposite in direction.
- It must be noted that $\overline{AB} + \overline{BA} = \underset{\sim}{o}.$
- Also if $\underset{\sim}{b} = \overrightarrow{CD},\ then\ -\underset{\sim}{b} = \overrightarrow{DC},\ and\ \overline{CD} + \overline{DC} = \underset{\sim}{o}.$
- If we consider a vector $\overline{CD}$, then its negative vector is $\overline{DC}$.

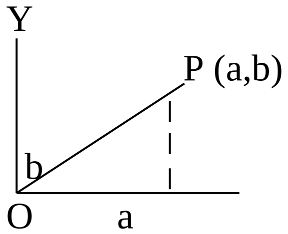
The zero vector (null vector):

- This is a vector where magnitude is zero and its direction is undefined.
- It is represented by $\underset{\to}{0} = \binom{0}{0}$

Notation of the magnitude of a vectors:

- If $\overline{AB}$ is a vector, then its magnitude is written as $|\overrightarrow{AB}|$
- Similarly the magnitude of the vector $\vec{b}$ is written as $|\vec{b}|$

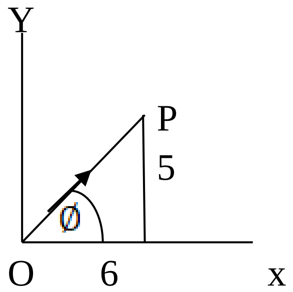
- If $\overline{OP} = \binom{a}{b}, then\ its$ magnitude $= |\overline{OP}| = \sqrt{a^2 + b^2}$



Q1. i. If OP = $\binom{6}{5}, find\ the$ magnitude of $\overline{OP}$.

ii. Find Ø the angle between $\overline{OP}$and the x – axis

Soln.



i. $\overline{OP} = |\sqrt{6^2 + 5^2} = \sqrt{61} = 7.8$

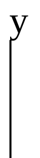
ii. $\tan Ø = {}^{5}/_{6} \Longrightarrow \tan Ø = 0.83 \Longrightarrow Ø = tan^{-1}0.83 \Longrightarrow Ø = 40.$

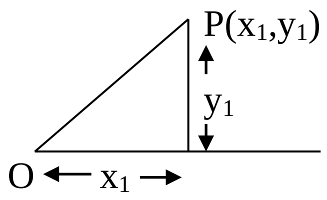
Scalar multiplication of vector:

- If ^ is the scalar and $\overline{a}$ is the vector, then the scalar x the vector $= {}^\wedge\vec{a}$
- When a scalar multiplies a vector, the product is also a vector, and for this reason ${}^\wedge\overline{a}$ is also a vector.
- The vector ${}^\wedge\underset{\sim}{a}$ is parallel to $\underset{\sim}{a}$, and is in the same direction as $\underset{\sim}{a}$,but has ^ times the magnitude of $\underset{\sim}{a}$.
- For example the vectors $\vec{a}\ and\ 2\vec{a}$ have the same direction.

  i.e 

- But the vectors $\vec{a}$ and and $-2\vec{a}$ are opposite in direction.
- ${}^\wedge(\vec{a} + \vec{b}) = {}^\wedge\vec{a} + {}^\wedge\vec{b}$, e.g $6(\underset{\sim}{a}+\underset{\sim}{b}) = 6\underset{\sim}{a}+6\underset{\sim}{b}$
- Also (2 + 4) $\vec{a} = 2\vec{a} + 4\vec{a}$
- Finally ${}^\wedge{}_1({}^\wedge{}_2\vec{a}) = {}^\wedge{}_1{}^\wedge{}_2\vec{a}$, e.g $3(2\vec{a}) = 6\vec{a}$

N/B: 



- If $P(x_1,y_1)$ is a point in the x – y plane, then the position vector of P relative to the origin, O is defined by $\overrightarrow{OP} = \binom{x_1}{y_1}$
- Also if A = (0,6), then $\overrightarrow{OA} = \binom{0}{6}$

Q2. Find the numbers m and n such that

$M\binom{3}{5} + n\binom{2}{1} = \binom{4}{9}$

Soln.

$$M\binom{3}{5} + n\binom{2}{1} = \binom{4}{9} \Rightarrow \binom{3m}{5m} + \binom{2n}{n} = \begin{matrix}4\\9\end{matrix}$$

$\Rightarrow 3m + 2n = 4 \ldots \ldots eqn(1).$

$5m + n = 9 \ldots \ldots . \ldots eqn(2)$

Solve eqns (1) and (2) simultaneously

$\Rightarrow m = 2 \; and \; n = -1$

Q3. If mp + nq = $\binom{4}{3}, find \; m \; and \; n$ where m and n are scalar, given that p = $\binom{2}{3} \; and \; q = \binom{2}{5}$

Soln.

p = $\binom{2}{3} \; and \; q = \binom{2}{5} \; but \; mp + nq = \binom{4}{3}$

$$\Rightarrow m\binom{2}{3} + n\binom{2}{5} = \binom{4}{3} \Rightarrow \binom{2m}{3m} + \binom{2n}{5n} = \binom{4}{3}$$

$\Rightarrow 2m + 2n = 4 - (1)$

$3m + 5n = 3 - (3)$

Solve eqns (1) and (2) simultaneously to get the values of m and n.

Q4. If r = $\binom{3}{1} and \; s = \binom{-2}{1}$, evaluate 6(r + 25)

Soln.

Consider 6(r + 2s), solve what is inside the bracket first

$\Rightarrow r + 2s = \binom{3}{1} + \binom{-2}{1} = \binom{3}{1} + 2\binom{-4}{2} \Rightarrow r + 2s = \binom{3+\bar{4}}{1+2} = \binom{-1}{3} \Rightarrow$
$6(r + 2s) = 6\binom{-1}{3} = \binom{-6}{18}$

Q5. If p = $\binom{1}{2}, q = \binom{-2}{3}$ $and\ r = \binom{1}{1}$, find 2p – q + r

Soln.

2p – q + r =
$2\binom{1}{2} - \binom{-2}{3} + \binom{1}{1} = \binom{2}{4} - \binom{-2}{3} + \binom{1}{1} = \binom{2+2+1}{4-3+1} = \binom{5}{2} \Rightarrow 2p - q + r = \binom{5}{2}$.

Q6. If the vector p = $\binom{2}{3}, q = \binom{2}{5} and\ r = \frac{1}{2}(q - p),$

Find the vector r.

Soln.

r =
$\frac{1}{2}(q - p) \Rightarrow r = \frac{1}{2}\left\{\binom{2}{5} - \binom{2}{3}\right\} \Rightarrow r = \frac{1}{2}\begin{Bmatrix}2-2\\5-3\end{Bmatrix} = \frac{1}{2}\begin{Bmatrix}0\\2\end{Bmatrix} = \begin{Bmatrix}\frac{1}{2}(0)\\\frac{1}{2}(2)\end{Bmatrix} = \begin{Bmatrix}0\\1\end{Bmatrix} \Rightarrow r = \begin{Bmatrix}0\\1\end{Bmatrix}$

N/B: Given the points A and B, then $\overrightarrow{AB} = B - A$.

Examples: If A =
$\binom{5}{2}$ $and\ B = \binom{10}{6}, then\ \overrightarrow{AB} = B - A = \binom{10}{6} - \binom{5}{2} = \binom{10-5}{6-2} = \binom{5}{4}$

Also if C =
$\binom{4}{2}$ $and\ D = \binom{6}{1}, then\ \overrightarrow{CD} = D - C = \binom{6}{1} - \binom{4}{2} = \binom{6-4}{1-2} = \binom{2}{-1} \Rightarrow \overrightarrow{CD} = \binom{2}{-1}$

Q7. If A = (4, 5) and B = (6, 2), find $\overline{AB}$

Soln.

A = $(4,5) \Rightarrow A = \binom{4}{5}$. Also B = (6, 2)
$\Rightarrow B = \binom{6}{2}. \overline{AB} = B - A = \binom{6}{2} - \binom{4}{5} = \binom{6-4}{2-5} = \binom{2}{-3} \Rightarrow \overline{AB} = \binom{2}{-3}$.

N/B: If $\overline{AB} = \binom{4}{2} \Rightarrow \overline{BA} = -\overline{AB} = -\binom{4}{2} = \binom{-4}{-2}$

Also if $\overrightarrow{CD} = \binom{-2}{5} \Rightarrow \overrightarrow{DC} = -\overrightarrow{CD} = -\binom{-2}{5} = \binom{2}{-5}$

Q8. If A and B are the points (2, 1) and (1, 2) respectively, find $\overline{AB}$ and $\overline{BA}$

Soln.

A = $\binom{2}{1}$ and $B = \binom{1}{2} \Rightarrow \overrightarrow{AB} = B - A = \binom{1}{2} - \binom{2}{1} = \binom{1-2}{2-1} = \binom{-1}{1}$

$\overline{BA} = -\overline{AB} = -\binom{-1}{1} = \binom{1}{-1}$.

Q9. Given A = $\binom{4}{1}$ and $B = \binom{-5}{4}$ $and\ the\ scalar\ as\ 2,$

evaluate i. $\overrightarrow{2A}$ ii. $2\overrightarrow{AB}$ iii. 2(A – B)

Soln.

i. A = $\binom{4}{1} \Rightarrow 2A = 2\binom{4}{1} = \binom{8}{2}$

ii. A = $\binom{4}{1}$ and $B = \binom{-5}{4}$, then

$\overline{AB} = B - A = \binom{-5}{4} - \binom{4}{1} = \binom{-5-4}{4-1} = \binom{-9}{3} \Rightarrow \overline{AB} = \binom{-9}{3}$

Since $\overline{AB} = \binom{-9}{3}$, $then\ 2\overline{AB} = 2\binom{-9}{3} = \binom{-18}{6}$

iii. 2(A – B) = ? but A = $\binom{4}{1}$and B =$\binom{-5}{4}$

A – B = $\binom{4}{1} - \binom{-5}{4} = \binom{4+5}{1-4} = \binom{9}{-3}$

Since A – B =$\binom{9}{-3} \Rightarrow 2(A - B) = 2\binom{9}{-3} = \binom{18}{-6}$

Q10. If A = (2, 4) and B = (4, 9), find$|\overline{AB}|$ie the magnitude of AB.

Soln.

A = $\binom{2}{4}$ and $B = \binom{4}{9} \Rightarrow \overline{AB} = B - A = \binom{4}{9} - \binom{2}{4} = \binom{2}{5} \Rightarrow \overline{AB} = \binom{2}{5}$. $|\overline{AB}| = \sqrt{2^2 + 5^2} = \sqrt{4 + 25} \Rightarrow AB = \sqrt{29} = 5.4$

Q11. If A = (-5, 2) and B(-8 – -9),

i. Find the vector $\overrightarrow{BA}$

ii. Calculate the length of $\overline{BA}$

Soln.

i. A = $\binom{-5}{2}$ *and* $B\binom{-8}{-9} \Longrightarrow AB = B - A = \binom{-8}{-9} - \binom{-5}{2} = \binom{-8+5}{-9-2} = \binom{-3}{-11} \Longrightarrow \overline{AB} = \binom{-3}{-11}, but\ \overrightarrow{BA} = -AB \Longrightarrow \overline{BA} = \binom{3}{11}$

ii. The length of $\overline{BA}$ = the magnitude of $\overline{BA} \Longrightarrow length\ of\ \overline{BA} = \sqrt{3^2 + 11^2} = \sqrt{9 + 121} = \sqrt{130} = 11.4 \Longrightarrow$ $the\ length\ of\ \overline{BA}$ =11.4

Q12. If C = (4, 1) and D = (2, 6),

a. find the vector $\overline{CD}$

b. calculate the length of $\overline{DC}$

Soln.

a. $\overline{CD} = D - C = \binom{2}{6} - \binom{4}{1} = \binom{2-4}{6-1} = \binom{-2}{5}.\overline{DC} = -\binom{-2}{5} = \binom{2}{-5} \Longrightarrow \overline{DC} = \binom{2}{-5}$

.

b. The length of $\overline{DC} = \sqrt{2^2 + (-5)^2} = \sqrt{4 + 25} = \sqrt{29} = 5.4$

Q13. If C = (1, 3) and D = (2, 4) find $|\overline{CD}|$

Soln.

$\vec{C} = \binom{1}{3} and\ \vec{D} = \binom{2}{4} \Longrightarrow \overline{CD} = \vec{D} - \vec{C} = \binom{2}{4} - \binom{1}{3} = \binom{1}{1} \therefore$ $\overline{CD} = \binom{1}{1} \Longrightarrow \overline{CD} = \sqrt{1 + 1} = \sqrt{2} = 1.4$

Q14. If $\vec{p} = \binom{2}{1}$ $and\ \vec{q} = \binom{-3}{2}, evalute$

i. $|\vec{P} + \vec{q}|$ ii. $|\overline{pq}|$

Soln.

i. $\vec{P} + \vec{q} = \binom{2}{1} + \binom{-3}{2} = \binom{-1}{3}, \therefore \vec{p} + \vec{q} = \binom{-1}{3} \Longrightarrow |\vec{p} + \vec{q}| = \sqrt{(-1)^2 + 3^2} = \sqrt{1 + 9} = \sqrt{10} = 3.2$

ii. $\overrightarrow{pq} = q - p = \binom{-3}{2} - \binom{2}{1} = \binom{-5}{1} \Rightarrow \overrightarrow{pq} = \binom{-5}{1}$.

$|pq = \sqrt{(-5)^2 + 1^2} = \sqrt{25 + 1} = \sqrt{26} = 5.1$

Q15. If Q is the point (2,4) and $\overrightarrow{QR} = \binom{1}{3}$, find the coordinates of R.

Soln.

Q = (2, 4) and $\overrightarrow{QR} = \binom{1}{3}$, then the coordinates of R

= (2+1, 4+3) = (3, 7)

The coordinates of R = (3, 7)

Q16. If $\overline{z} = (1,2) and\, \overline{zy} = \binom{-1}{3}$, find the coordinates of y.

Soln.

Since z = (1,2) and $\overline{zy} = \binom{-1}{3}$, then the coordinates of

y = $(1 + \overline{1}, 2 + 3) = (0,5) \Rightarrow the\ coordinates\ of\ y = (0,5)\ or\ \binom{0}{5}$.

Q17. If A = (1, 5) and $\overline{BA} = \binom{-2}{-3}$, find the coordinates of B.

N/B: Since the point given is A and the vector given is $\overline{BA}$, then $\overline{BA}$ must first be changed into $\overline{AB}$

Soln.

Since A is given as (1, 5), we must find$\overrightarrow{AB}$, $but\ since \overline{BA} = \binom{-2}{-3}$, then

$\overrightarrow{AB} = -\overline{BA} \Rightarrow \overline{AB} = -\binom{-2}{-3} \Rightarrow \overline{AB} = \binom{2}{3}$.

Now since A = (1, 5) and$\overline{AB} = \binom{2}{3}$, then

B = (1+2, 5+3)$\therefore B = (3,8)$.

Q18. If Q = (4, 1) and $\overline{RQ} = \binom{2}{-3}$, $find\ the$ coordinates of R.

Soln.

Since Q = (4, 1) and $\overline{RQ} = \binom{2}{-3}$, we must first find $\overline{QR}$.

$\overline{QR} = -\overline{RQ} \Longrightarrow \overline{QR} = -\begin{pmatrix} 2 \\ -3 \end{pmatrix} \Longrightarrow \overline{QR} = \begin{pmatrix} -2 \\ 3 \end{pmatrix}$

Now Q = (4, 1) and $\overline{QR} = \binom{-2}{3}, then\ the$

$coordinates\ of\ R = (-2+4, 1+3) = (2,4)$

Q19. If C = $\binom{1}{3}\ and\ \overline{DC} = \binom{-1}{2}, find\ the$

$coordinates\ of\ D.$

Soln.

$$C \ = \begin{pmatrix} 1 \\ 3 \end{pmatrix} \Longrightarrow C = (1,3). Since\ \overline{DC} = \begin{pmatrix} -1 \\ 2 \end{pmatrix} \Longrightarrow \overline{CD} = -\overline{DC} \Longrightarrow \overline{CD} = -\begin{pmatrix} -1 \\ 2 \end{pmatrix} = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$$

Now C = $\binom{1}{3}\ and \overline{CD} = \ \binom{1}{-2} \Longrightarrow the\ coordinates\ of\ D$

$= \left(1+1, 3+\bar{2}\right) = (2,1)$

Q20. If A = (1,2),$\overline{AB} = \binom{3}{4}\ and\ \overline{AC} = \binom{5}{-3}, find\ the\ coordinates\ of\ B\ and\ C.$

Soln.

Since A = (1, 2) and
$\overline{AB} = \binom{3}{4}, then\ the\ coordinates\ of\ B = (1+3, 2+4) = (4,6)$

Also Since A = (1, 2) and
$\overline{AC} = \binom{5}{-3} then\ the\ coordinates\ of\ C = \left(1+5, 2+\bar{3}\right) = (6,-1)$

Q21. Given B(4,2), $\overline{BC} = \binom{-1}{-5} and\ \overline{BD} = (1,3),$

determine the coordinates of C and D.

Soln.

Since B = (4, 2) and $\overline{BC} = \binom{-1}{-5} \Longrightarrow the\ coordinates$

$$of\ C = \left(4+\bar{1}, 2+\bar{5}\right) = (4-1, 2-5) = (3,-3). B = (4,2) and\ BD = (1,3) \Longrightarrow coordinates of\ D = (4+1, 2+3) = (5,5)$$

Q22. If A is the point (2, 3),
$\overline{BA} = \binom{2}{-3}$ and $\overline{CA} = \binom{-1}{-5}$, $determine\ the\ coordinates\ of\ B\ and\ C$

N/B: Since the point given is point A, then $\overline{BA}$ must be changed into $\overline{AB}$. Also $\overline{CA}$ must be changed into $\overline{AC}$.

Soln.

$\overline{BA} = \binom{2}{-3} \Longrightarrow \overline{AB} = -\overline{BA} = -\binom{2}{-3} \Longrightarrow \overline{AB} = \binom{-2}{3}$.

Also $\overline{AC} = -\overline{CA} = -\binom{-1}{-5} = \binom{1}{5}, \therefore \overline{AC} = \binom{1}{5}$

Now since A = (2, 3) and $\overline{AB} = \binom{-2}{3}$, then the coordinates of
$B = (2 + \bar{2}, 3 + 3) = (0,6)$

Also since A = (2,3) and $\overline{AC} = \binom{1}{5}$, $then\ the$ coordinates of C = (2+1, 3+5)= (3, 8)

Q23. The point C is given as (4, 1),
$\overline{CD} = \binom{1}{2}$ and $\overline{DE} = \binom{3}{5}$, $find\ the\ coordinates\ of\ D\ and\ E$.

Soln.

Since C = (4, 1) and $\overline{CD} = \binom{1}{2}$, $the\ the\ coordinates\ of$

$D = (4 + 1, 1 + 2) \Longrightarrow D(5,3)$

Now D = (5, 3) and $\overline{DE} = \binom{3}{5} \Longrightarrow the\ coordinates\ of\ E$

$= (5 + 3, 3 + 5) \Longrightarrow E = (8,8)$

Q24. If the point A is given as
$\binom{3}{4}$ and $\overrightarrow{AB} = \binom{-2}{1}$ and $\overrightarrow{BC} = \binom{5}{1}$, $find\ the\ coordinates\ of\ B$

and C.

Soln.

$A = \binom{3}{4} and\ \overline{AB} = \binom{-2}{1}$ $\Longrightarrow$the coordinates of B = (3+$\bar{2}$, $4 + 1) = (1,5)$.

Now B = (1, 5) and $\overline{BC} = \binom{5}{1} \Longrightarrow the\ coordinates\ of\ C$

= (1+5, 5+1) = (6, 6)

Q25. Given A(4, 1), $\overline{BA} = \binom{1}{6}$ and $\overline{BC} = \binom{2}{2}, find$ the coordinates of B and C.

N/B: The point given is point A and the vector given is $\overline{BA}$.

First find $\overline{AB}$.

Soln.

$\overline{BA} = \begin{pmatrix}1\\6\end{pmatrix}. \overline{AB} = -\overline{BA} = -\begin{pmatrix}1\\6\end{pmatrix} \Rightarrow \overline{AB} = \begin{pmatrix}-1\\-6\end{pmatrix}$

Now A = (4, 1) and $\overline{AB} = \binom{-1}{-6} \Rightarrow the\ coordinates\ of\ B$

$= \left(4 + \overline{1}, 1 + \overline{6}\right) = (3, -5)$

Now B = (3, -5) and $\overline{BC} = \binom{2}{2} \Rightarrow the\ coordinates\ of\ C$

$= (3 + 2, -5 + 2) = (5, -3)$

Q26. B is given as the point (4, 8),
$\overline{CD} = \binom{1}{1}$ and $\overline{BC} = \binom{1}{7}, find\ the\ coordinates\ of\ C\ and\ D.$

Soln.

Since B = (4, 8) and $\overline{BC} = \binom{1}{7} \Rightarrow the\ coordinates\ of\ C$

$= (4 + 1, 8 + 7) = (5, 15) \Rightarrow C(5, 15)$

Now since C = (5, 15) and CD =
$\binom{1}{1}, then\ the\ coordinatesof\ D = (5 + 1, 15 + 1) = (6, 16) \Rightarrow D(6, 16)$

Q27. If A = (1, 3), $\overline{CB} = \binom{5}{5}$ and $\overline{BA} = \binom{1}{2}$, find the coordinates of B and C.

Soln.

$\overline{BA} = \binom{1}{2}\ but\ \overline{AB} = -\overline{BA} = -\binom{1}{2} = \binom{-1}{-2}$s

Now A = (1, 3) and $\overline{AB} = \binom{-1}{-2} \Rightarrow the\ coordinates$

$of\ B = \left(1 + \overline{1}, 3 + \overline{2}\right) = (0, 1) \Rightarrow B = (0, 1)$

Since CB = $\binom{5}{5} \Rightarrow \overline{BC} = -\overline{CB} = -\binom{5}{5} \Rightarrow \overline{BC} = \binom{-5}{-5}$.

Now B(0, 1) and $\overline{BC} = \binom{-5}{-5} \Rightarrow the\ coordinates\ of$

$C = (-5+0, -5+1) = (-5,-4) \Longrightarrow C(-5,-4)$.

Q28. P is the point (4, 1) and Q is (-3, 2). If $\overline{PS} = \binom{-1}{2}$ and $\overrightarrow{QT} = \binom{-2}{-3}$,

i. find the coordinates of S and T.
ii. find also $\overrightarrow{ST}$.

Soln.

i. Since P = (4, 1) and $\overline{PS} = \binom{-1}{2}$, *then the*

$$coordinates\ of\ S = (4+\bar{1}, 1+2) = (3,3) \Longrightarrow S(3,3). Also\ since\ Q = (-3,2) and\ \overrightarrow{QT} = \begin{pmatrix}-2\\-3\end{pmatrix} \Longrightarrow the\ coordinates\ of\ T = (-3+\bar{2}, 2+\bar{3}) = (-5,-1)$$

ii. $\overrightarrow{ST} = T - S = \binom{-5}{-1} - \binom{3}{3} = \binom{-5-3}{-1-3} = \binom{-8}{-4}$

Q29. If A = (4, 3) and B = (1, 1), $\overline{CA} = \binom{2}{1}$ *and* $\overline{DB} = \binom{-3}{-4}$, *find* $\overline{CD}$.

N/B: Before we can find $\overline{CD}$, we must first determine the coordinates of C and D.

Soln.

$$\overline{AC} = -\overline{CA} = -\begin{pmatrix}2\\1\end{pmatrix} = \begin{pmatrix}-2\\-1\end{pmatrix} \therefore \overline{AC} = \begin{pmatrix}-2\\-1\end{pmatrix}. since\ A = (4,3) and\ \overline{AC} = \begin{pmatrix}-2\\-1\end{pmatrix},$$

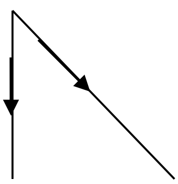
$$then\ the\ coordinates\ of\ C = (-2+4, -1+3) \Longrightarrow C(2,2) = \begin{pmatrix}2\\2\end{pmatrix}$$

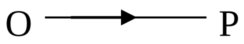
Also

$$\overline{BD} = -\overline{DB} = -\binom{-3}{-4} = \binom{3}{4} \Longrightarrow \overline{BD} = \binom{3}{4}. Since\ B = (1,1) \Longrightarrow$$
$$the\ coordinates\ of\ D = (1+3, 1+4) = (4,5). \overline{CD} = D - C = \binom{4}{5} - \binom{2}{2} = \binom{2}{3}$$

N/B:

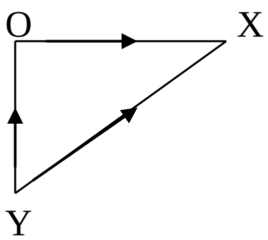
i. M





In the figure drawn, moving from M to O, and then from O to P is the same as moving from M directly to P, since in both cases we end at the same point, which is P. $\Longrightarrow \overline{MO} + \overline{OP} = \overline{MP}$

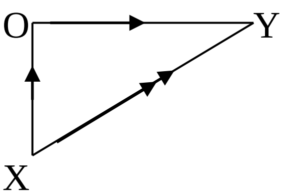
ii.



In the given figure $\overline{YX} = \overline{YO} + OX$

Q1. If $\overline{xo} = \begin{pmatrix} 4 \\ 3 \end{pmatrix}$ $and\ O\overline{y} = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$find|xy|.

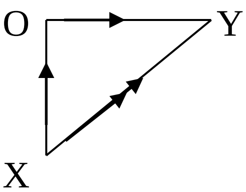
Soln.



$\overline{xy} = \overline{xo} + \overline{oy} \Longrightarrow xy = \begin{pmatrix} 4 \\ 3 \end{pmatrix} + \begin{pmatrix} 2 \\ -1 \end{pmatrix} = \begin{pmatrix} 6 \\ 2 \end{pmatrix} \Longrightarrow |xy| = \sqrt{6^2 + 2^2} \Longrightarrow |xy| = \sqrt{36 + 4} = \sqrt{40} = 6.3$

Q2. If $\overline{ox} = \begin{pmatrix} -2 \\ 3 \end{pmatrix}$ $and\ \overline{oy} = \begin{pmatrix} 4 \\ 1 \end{pmatrix}, find|xy|$

Soln.



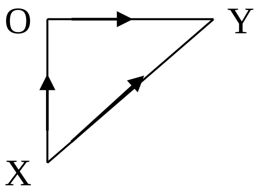
From the diagram, $\overline{xy} = \overline{xo} + \overline{oy}$

Since $\overline{ox} = \begin{pmatrix} -2 \\ 3 \end{pmatrix} \Longrightarrow \overline{xo} = -\overline{ox} = -\begin{pmatrix} -2 \\ 3 \end{pmatrix} = \begin{pmatrix} 2 \\ -3 \end{pmatrix}$.

$Since\ \overline{xy} = \overline{xo} + \overline{oy} \Rightarrow \overline{xy} = \binom{2}{-3} + \binom{4}{1} = \binom{6}{-2} \Rightarrow |xy| = \sqrt{6^2 + (-2)^2} = \sqrt{36+4} = \sqrt{40} = 6.3$

Q3. Given that $\overline{ox} = \binom{-2}{4}$ $and\ \overline{yo} = \binom{-1}{3}. find |xy|$.

Soln.



$\overline{xy} = \overline{xo} + \overline{oy}$

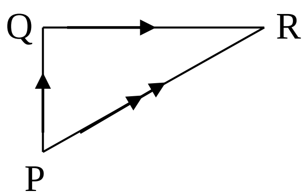
$$Since\ \overline{ox} = \begin{pmatrix}-2\\4\end{pmatrix} \Rightarrow \overline{xo} = -\overline{ox} = -\begin{pmatrix}-2\\4\end{pmatrix} = \begin{pmatrix}2\\-4\end{pmatrix} \Rightarrow \overline{xo} = \begin{pmatrix}2\\-4\end{pmatrix}\ Also\ since\ \overline{yo} = \begin{pmatrix}-1\\3\end{pmatrix} \Rightarrow \overline{oy} = -\overline{yo} = -\begin{pmatrix}-1\\3\end{pmatrix} = \begin{pmatrix}1\\-3\end{pmatrix} \Rightarrow \overline{oy} = \begin{pmatrix}1\\-3\end{pmatrix}$$

$\overline{xy} = \overline{xo} + \overline{oy} \Rightarrow \overline{xy} = \binom{2}{-4} + \binom{1}{-3} = \binom{2+1}{-4+3} = \binom{3}{-7}. |xy| = \sqrt{3^2 + (-7)^2} \Rightarrow |xy| = \sqrt{9+49} = \sqrt{58} = 7.6$

Q4. Given that $\overline{PQ} = \binom{2}{3}$ $and\ \overline{QR} = \binom{4}{1}, find$

a. PR b. |PR|

Soln.



a. $\overline{PR} = \overline{PQ} + \overline{QR} \Rightarrow \overline{PR} = \binom{2}{3} + \binom{4}{1} = \binom{6}{4}$
b. $Since\ \overline{PR} = \binom{6}{4} \Rightarrow |\overline{PR}| = \sqrt{6^2 + 4^2} = \sqrt{36+16} = \sqrt{52} = 7.2$

Q5. If $\overrightarrow{PQ} = \binom{3}{5}$ $and\ \overrightarrow{RQ} = \binom{-1}{-4}, find |PR|$

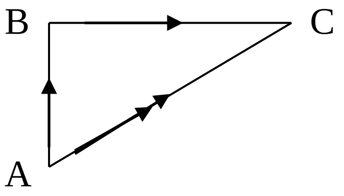
Soln.

$\overline{PR} = \overline{PQ} + \overline{QR}$ Since $\overline{RQ} = \begin{pmatrix}-1\\-4\end{pmatrix} \Rightarrow \overline{QR}, = -\begin{pmatrix}-1\\-4\end{pmatrix} = \begin{pmatrix}1\\4\end{pmatrix}$

$\overline{PR} = \overline{PQ} + \overline{QR} = \binom{3}{5} + \binom{1}{4} = \binom{4}{9} \Rightarrow |\overline{PR}| = \sqrt{4^2 + 9^2} = \sqrt{16 + 81} = \sqrt{97} =$ 9.84

Q6. If $\overline{AB} = \binom{-1}{-3}$ *and* $\overline{BC} = \binom{-2}{4}$, *find* $\overline{AC}$.

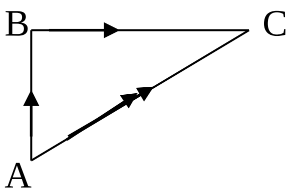
Soln.



$\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC} \Rightarrow \overrightarrow{AC} = \begin{pmatrix}-1\\-3\end{pmatrix} + \begin{pmatrix}-2\\4\end{pmatrix} = \begin{pmatrix}-1+\overline{2}\\-3+4\end{pmatrix} = \begin{pmatrix}-3\\1\end{pmatrix}$

Q7. Given that $\overline{BA} = \binom{6}{3}$ *and* $\overline{BC} = \binom{1}{2}$, *find* $\overline{AC}$.

Soln.



$\overline{AC} = \overline{AB} + \overline{BC}$

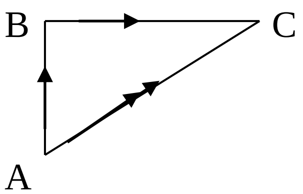
Since AB is not given, we must find it.

Since $\overline{BA} = \binom{6}{3} \Rightarrow \overline{AB} = -\binom{6}{3} = \binom{-6}{-3}$.

From $\overline{AC} = \overline{AB} + \overline{BC} \Rightarrow \overline{AC} = \begin{pmatrix}-6\\-3\end{pmatrix} + \begin{pmatrix}1\\2\end{pmatrix} = \begin{pmatrix}-5\\-1\end{pmatrix}$

Q8. If $\overrightarrow{AB} = \binom{1}{3}$ *and* $\overrightarrow{CB} = \binom{-2}{5}$, *find* $\overrightarrow{AC}$.

Soln.



AC = AB + BC

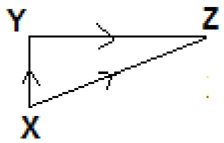
Since $\overline{BC}$ is not given, we have to find it.

From $\overline{CB} = \binom{-2}{5} \Rightarrow \overline{BC} = -\binom{-2}{5} = \binom{2}{-5}$.

Now $\overline{AC} = \overline{AB} + \overline{BC} \Rightarrow \overline{AC} = \binom{1}{3} + \binom{2}{-5} = \binom{3}{-2} \Rightarrow \overline{AC} = \binom{3}{-2}$

Q9. If $\overrightarrow{xy} = \binom{-2}{5}$ *and* $\overrightarrow{zy} = \binom{3}{2}$, *find* $|\overline{xz}|$

Soln.



From the diagram, $\overline{xz} = \overline{xy} + yz$

Since $\overline{zy} = \binom{3}{2} \Rightarrow \overline{yz} = -\overline{zy} = -\binom{3}{2} = \binom{-3}{-2}$

$But\ \overline{xz} = \overline{xy} + \overline{yz} \Rightarrow \overline{xz} = \begin{pmatrix}-2\\5\end{pmatrix} + \begin{pmatrix}-3\\-2\end{pmatrix} = \begin{pmatrix}-5\\3\end{pmatrix}$.

$|xz| = \sqrt{(-5)^2 + 3^2} = \sqrt{25 + 9} = \sqrt{34} = 5.8$

The inverse of a vector or the negative vector.

- If $\overline{AB} = \binom{x}{y}$ *then* $\overline{BA} = -\overline{AB}$ *and* $-\overline{AB} = \binom{-x}{-y}$
- $\overline{AB} = \binom{-x}{-y}$ *is called the inverse or the negative vector of* $\overline{AB}$
- A vector and its inverse have the same magnitude, but have opposite direction
- For example if $\overline{PQ} = \binom{-3}{-4}$, *then its inverse*

  or negative which is $\overline{QP} = -\begin{pmatrix}-3\\-4\end{pmatrix} = \begin{pmatrix}3\\4\end{pmatrix}$
- Also if $\overline{AB} = \binom{1}{-3}$, *then its inverse*

  or negative, which is $\overline{BA} = -\begin{pmatrix}1\\-3\end{pmatrix} = \begin{pmatrix}-1\\3\end{pmatrix}$

The direction of a vector:

- This is the angle φ,which the vector makes with the x-axis
- If P(x, y) and Q(x_2, y_2), then the direction of $\overline{PQ}$ is given by $\tan\varphi = \frac{y_2 - y_1}{x_2 - x_1}$

Q1. Given A(5, 4) and B(3, 1), find the direction of $\overline{AB}$

Soln.

Let (x_1, y_1) = (5, 4)and (x_2, y_2) = (3,1) $\Rightarrow x_1 = 5, y_1 = 4, x_2 = 3$ and $y_2 = 1.$

$\tan\theta = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1-4}{3-5} = \frac{-3}{2} = -1.5.$

$\tan\varphi = -1.5 \Rightarrow \varphi = tan^{-1} - 1.5 \Rightarrow \varphi = -56.$

Q2.

a. Find the magnitude and the direction of the displacement vector $\overline{AB}$, $where$ A and B are the points (2, 1) and (8, 9) respectively.
b. Determine the magnitude of the vector $\overline{BA}$.

Soln.

a. $\overline{A} = \binom{2}{1}$ and $\overline{B} = \binom{8}{9}$. But $\overline{AB} = \overline{B} - \overline{A} = \binom{8}{9} - \binom{2}{1} = \binom{6}{8}$. $Since$ $\overline{AB} = \binom{6}{8} \Rightarrow |\overline{AB}| = \sqrt{6^2 + 8^2} = \sqrt{100} = 10$

Let (2, 1) = (x_1, y_1) and (8, 9) = (x_2, y_2), $then$ $\tan\varphi = \frac{y_2 - y_1}{x_2 - x_1} = \frac{9-1}{8-2} = \frac{8}{6} = 1.33 \Rightarrow \varphi = tan^{-1}1.33 = 53^o$

b. $\overline{A} = \binom{2}{1}$ and $\overline{B} = \binom{8}{9}$ $\overline{BA} = \overline{A} - \overline{B} = \binom{2}{1} - \binom{8}{9} = \binom{-6}{-8}$

$Since$ $\overline{BA} = \binom{-6}{-8} \Rightarrow \overline{BA} = \sqrt{(-6)^2 + (-8)^2} = \sqrt{100} = 10$

Parallel vectors:

- Two vectors are said to be parallel vectors, if one is the scalar multiplication of the other.
- Consider the vectors $\overline{A} = \binom{1}{3}$ and $\overline{B} = \binom{2}{6}$. These are parallel vectors, since one is the scalar multiple of the other, i.e 2 x $\overline{A} = \overline{B}$ or $2\binom{1}{3} = \binom{2}{6}$, $where$ 2 is the $scalar.$
- If the scalar is positive or a positive number, as in the example just given, then the two given vectors are in the same direction.
- But if the scalar is negative, then the two vectors are in the opposite direction
- Also the vectors $\overline{C} = \binom{3}{5}$ and $\overline{D} = \binom{9}{15}$ are parallel vectors, since one is the scalar multiple of the other i.e 3 x C = $\overline{D}$ or $3 \times \binom{3}{5} = \binom{9}{15}$.
- In this case, the scalar is 3 and since it is positive, then the two vectors are in the same direction.

- Now consider $\overline{A} = \binom{4}{5}$ and $\overline{B} = \binom{-16}{-20}$. These are parallel vectors, since one is the scalar multiple of the other i.e $-4 \times \overline{A} = \overline{B}$ $or - 4\binom{4}{5} = \binom{-16}{-20}$.
- In this case, since the scalar is negative i.e – 4, then the two given vectors are in the opposite direction, eventhough they are parallel.

Q1. Determine whether the vector $\overline{B} = \binom{1}{-2}$ and $\overline{C} = \binom{-3}{6}$are parallel to each other, and determine whether they are in the same or opposite in direction.

Soln.

$$\overline{B} = \begin{pmatrix} 1 \\ -2 \end{pmatrix} and\, \overline{C} = \begin{pmatrix} -3 \\ 6 \end{pmatrix}\ But - 3 \times \overline{B} = \overline{C}\ i.e - 3\begin{pmatrix} 1 \\ -2 \end{pmatrix} = \begin{pmatrix} -3 \\ 6 \end{pmatrix} i.e$$

One is a scalar multiple of the other $\Longrightarrow$ they are parallel vectors. Since the scalar is negative or a negative number i.e -3, then the two vectors are opposite in direction.

Q2. Determine whether the vectors $\overline{x} = \binom{8}{10}$ and $\overline{y} = \binom{4}{5}$ are parallel to each other, and determine also whether they are in the same direction

Soln.

$$\overline{x} = \begin{pmatrix} 8 \\ 10 \end{pmatrix} and\, \overline{y} = \begin{pmatrix} 4 \\ 5 \end{pmatrix}. 2 \times \overline{y} = \overline{x}\ \ i.e\ 2\begin{pmatrix} 4 \\ 5 \end{pmatrix} = \begin{pmatrix} 8 \\ 10 \end{pmatrix}$$

Since one of the vectors is a scalar multiple of the other, the two given vectors are parallel. Since the scalar = 2 which is positive $\Longrightarrow$ *the* two given vectors are in the same direction.

Determination whether two vectors are parallel – method two:

Let $\vec{A} = \binom{a}{b}$ and $\vec{B} = \binom{c}{d}$ if ad – bc = o, then the two vectors are parallel.

Q1. Show that the vectors$\vec{A} = \binom{4}{2}$ and $\vec{B} = \binom{8}{4}$ are parallel vectors.

Soln.

If $\vec{A}$ is parallel to $\vec{B}$, then (4 x 4) – (2 x 8) = 0 i.e if the left hand side is equal to zero, then they are parallel.

Now L.H.S = (4 x 4) – (2 x 8) = 16 – 16 = 0⟹the two vectors are parallel

Q2. Determine whether or not the vectors $\vec{M} = \binom{-2}{-3}$ *and* $\vec{N}$ *are parallel* vectors. If $\vec{M}$ and $\vec{N}$ are parallel vectors, then

(-2 x 3) – (-3 x 2) = 0

L.H.S = (-6) – (-6) = -6 + 6 = 0

Since L.H.S = 0 ⟹ the two vectors are parallel vectors.

Q3. Determine whether or not $\vec{M} = \binom{4}{1}$ *and* $\vec{N} = \binom{8}{6}$ are parallel vectors.

Soln.

$If\ \vec{M} = \binom{4}{1}\ and\ \vec{N} = \binom{8}{6}$ are parallel vectors, then

(4 x 6) – (1 x 8) = 0

L.H.S = 24 – 8 = 16.

Since the L.H.S $\neq o$ i.e not equal to zero, then the two vectors are not parallel.

Q4. Find the value of x, such that the vector $\overline{A}\binom{4}{9}$will be parallel to the vector $\overline{B}\binom{8}{x}$.

Soln.

$\overline{A} = \binom{4}{9}\ and\ \overline{B} = \binom{8}{x}$. For them to be parallel, then

$(4\ \times\ x) - (9 \times 8) = 0 \Longrightarrow 4x - 72 = 0 \Longrightarrow 4x = 0 + 72 \Longrightarrow 4x = 72 \Longrightarrow x = \frac{72}{4} = 18 \Longrightarrow x = 18$

.

Q5. Given $\vec{C}\binom{8}{x}\ and\ \vec{D}\binom{-4}{-3},\ determine$ the value of x such that the two vectors become parallel.

Soln.

$\vec{C}\binom{8}{x}\ and\ \vec{D}\binom{-4}{-3}$are given. If they are parallel, then

$(8 \times 3) - (-4 \times x) = 0 \Longrightarrow (-24) - (-4x) = 0 \Longrightarrow -24 + 4x = 0 \Longrightarrow 4x = 24 \Longrightarrow x = \frac{24}{4} = 6 \Longrightarrow x = 6$

Q6. Given $\overline{A} = \binom{y}{2}$ and $\overline{B} = \binom{9}{6}$, find the value of y such that the two vectors are parallel.

Soln.

$\overline{A} = \binom{y}{2}$ and $\overline{B} = \binom{9}{6}$. For them to be parallel, then

(y x 6) – (2 x 9) = 0 $\Longrightarrow 6y - 18 = 0 \Longrightarrow 6y = 18 \Longrightarrow y = \frac{18}{6} = 3$

<u>**Perpendicular vectors:**</u>

Consider the vectors $\overline{A} = \binom{a}{b}$ and $\overline{B} = \binom{c}{d}$. if these two vectors are perpendicular, then ac + db = 0

Q1. Show that the vectors $\overline{A}\binom{4}{-2}$ and $\overline{B}\binom{2}{4}$are perpendicular

Soln.

For $\vec{A}\binom{4}{-2}$ and $\vec{B}\binom{2}{4}$ to be perpendicular, then

(4 x 2) + (-2 x 4) = 0 i.e the L.H.S must be equal to zero.

L.H.S = $8 + \overline{8} = 8 - 8 = 0$

Since L.H.S = 0 $\Longrightarrow$ the two vectors are perpendicular.

Q2. Determine whether or not $\overline{B}\binom{8}{2}$ and $\overline{C}\binom{4}{5}$ are perpendicular vectors.

Soln.

Given $\overline{B}\binom{8}{2}$ and $\overline{C}\binom{4}{5}$. If these two vectors are perpendicular, then (8 x 4) + (2 x 5) = 0 i.e

The L.H.S must be equal to zero. L.H.S 32 + 10 = 42.

Since L.H.S $\neq$0, then the two vectors are not perpendicular.

Q3. Given $\overline{A} = \binom{4}{2}$ and $\overline{B} = \binom{-3}{6}$, $determine$ whether these two given vectors are perpendicular vectors.

Soln.

$\overline{A} = \binom{4}{2}$ and $\overline{B} = \binom{-3}{6}$. For these two vectors to be perpendicular, then

$(4 \times \overline{3}) + (2 \times 6) = 0$

L.H.S = (-12) + (12) = 0. Since L.H.S = 0, then the two vectors are perpendicular.

Q4. Are the vectors $\overline{B} = \binom{-5}{-2}$ *and* $\overline{D} = \binom{2}{-3}$ *perpendicualr vectors?*

Soln.

$\overline{B} = \binom{-5}{-2}$ *and* $\overline{D} = \binom{2}{-3}$. If these two vectors are perpendicular, then

$(-5 \times 2) + (-2 \times \overline{3}) = 0$

L.H.S = (-10) + (6) = - 4

Since L.H.S $\neq 0$, then the two given vectors are not perpendicular

Q5. Find the value of x such that the vectors

$\vec{A} = \binom{9}{2}$ *and* $\vec{B} = \binom{x}{-18}$ *will be* perpendicular to each other.

Soln.

$\vec{A} = \binom{9}{2}$ *and* $\vec{B} = \binom{x}{-18}$. If these two vectors are to be perpendicular, then

$(9 \times x) + (2 \times \overline{18}) = 0 \Longrightarrow 9x + (-36) = 0 \Longrightarrow 9x - 36 = 0 \Longrightarrow 9x = 36 \Longrightarrow x = \frac{36}{9}$

$= 4 \Longrightarrow x = 4$

Q6. If $\overline{C} = \binom{-2}{y}$ *and* $\overline{D} = \binom{-8}{4}$ are two vectors, determine the value of y, if these two vectors are perpendicular.

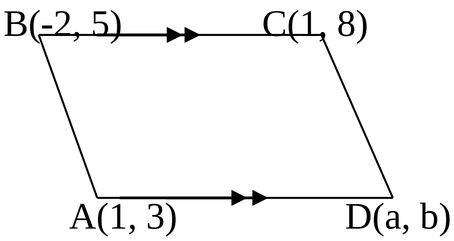
Soln.

If $\overline{C} = \binom{-2}{y}$ *and* $\overline{D} = \binom{-8}{4}$ are perpendicular, then

$(-2 \times \overline{8}) + (y \times 4) = 0 \Longrightarrow 16 + 4y = 0 \Longrightarrow 4y = -16 \Longrightarrow y = \frac{-16}{4} = -4 \Longrightarrow y = -4$

Q7. A parallelogram ABCD has vertices A(1, 3), B(-2, 5) and C(1, 8). Find the coordinates of vertex D.

Soln.



The position vectors of A, B and C are A = $\binom{1}{3}, B = \binom{-2}{5}$ *and* $C = \binom{1}{8}$ *respectively.*

Let the coordinates of D be (a, b).

Since the given figure is a parallelogram, then
$\overline{AB} = \overline{DC}$ *and* $\overline{AB} = B - A = \binom{-2}{5} - \binom{1}{3} = \binom{-3}{2}$

$$\overline{DC} = C - D = \binom{1}{8} - \binom{a}{b} = \binom{1-a}{8-b}$$

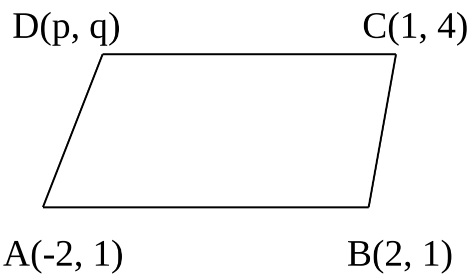
Since $\overline{AB} = \overline{DC} \Rightarrow \binom{-3}{2} = \binom{1-a}{8-b} \Rightarrow -3 = 1 - a \Rightarrow a = 1 + 3 \Rightarrow a = 4$.

Also $2 = 8 - b \Rightarrow b = 8 - 2 = 6$

$\therefore$ *Coordinates of D are* $(4, 6)$.

Q8. The points A (-2, 1), B (2, 1), C (1, 4) and D (p,q) are the vertices of a parallelogram ABCD. Find $\overline{AB}$ *and* $\overline{DC}$ *and* deduce the values of p and q.

. Soln.



AB = B – 1 = $\binom{2}{1} - \binom{-2}{1} = \binom{4}{0}$

$$\overline{DC} = C - D = \binom{1}{4} - \binom{p}{q} = \binom{1-p}{4-q}$$

Since ABCD is a parallelogram
$\overrightarrow{AB} = \overrightarrow{DC} \Rightarrow 4 = 1 - p \Rightarrow 4 + p = 1 \Rightarrow p = 1 - 4 = -3$

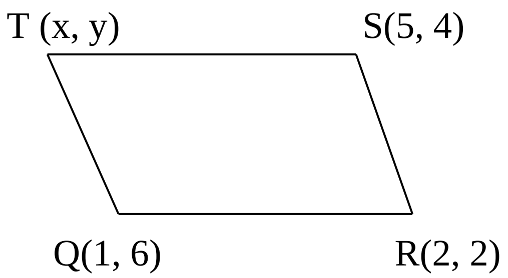
Also $0 = 4 - q \Rightarrow 0 + q = 4 \Rightarrow q = 4$

Q9. The coordinates of the vertexes of a parallelogram QRST are Q (1, 6), R (2, 2), S (5, 4) and T (x, y)

i. Find $\overline{QR}$ *and* $\overline{TS}$ *and* hence find the values of x and y

ii. Calculate the magnitude of $\overline{RS}$

Soln.



i. $\overrightarrow{QR} = R - Q = \binom{2}{2} - \binom{1}{6} = \binom{1}{-4}$ *and* $\overrightarrow{TS} = \binom{5}{4} - \binom{x}{y} = \binom{5-x}{4-y}$

Since QRST is a parallelogram

$\Rightarrow \overrightarrow{QR} = \overrightarrow{TS} => \binom{1}{-4} = \binom{5-x}{4-y} \Rightarrow 1 = 5 - x \Rightarrow 1 - 5 = -x \Rightarrow -4 = -x \Rightarrow x = 4$

.

Also $-4 = 4 - y \Rightarrow -4 - 4 = -y \Rightarrow -8 = -y \Rightarrow 8 = y \Rightarrow y = 8.$

ii. $\overline{RS} = S - R = \binom{5}{4} - \binom{2}{2} = \binom{3}{2}$

$|RS| = \sqrt{3^2 + 2^2} = \sqrt{9 + 4} = \sqrt{13} = 3.6$

Q10. Find the values of x and y such that $\binom{x+3}{2} - \binom{y}{x+y} = \binom{2}{-1}$.

Soln.

$$\binom{x+3}{2} - \binom{y}{x+y} = \binom{2}{-1} \Rightarrow \binom{x+3-y}{2-x-y} = \binom{2}{-1}$$

Equating corresponding component $\Rightarrow x + 3 - y = 2$

$\Rightarrow x - y = 2 - 3 \Rightarrow x - y = -1 \ldots. eqn(1)$

Also $2 - x - y = -1 \Rightarrow -x - y = -1 - 2 \Rightarrow -x - y = -3 \ldots eqn(2)$

Solve eqn (1) and eqn (2) simultaneously $\Rightarrow x = 1\ and\ y = 2$

Q11. If p = $\binom{2}{-2}$ $and\ q = \binom{3}{4}, find\ r\ such\ that \frac{1}{2}p - q - r = \binom{0}{0}$

Soln.

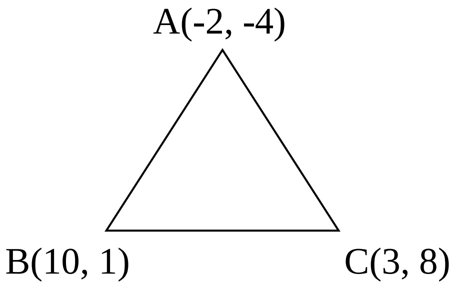
$$\frac{1}{2}p - q + r = \begin{pmatrix}0\\0\end{pmatrix} \Rightarrow \frac{1}{2}\begin{pmatrix}2\\-2\end{pmatrix} - \begin{pmatrix}3\\4\end{pmatrix} + r = \begin{pmatrix}0\\0\end{pmatrix} \Rightarrow \begin{pmatrix}1\\-1\end{pmatrix} - \begin{pmatrix}3\\4\end{pmatrix} + r = \begin{pmatrix}0\\0\end{pmatrix}$$
$$\Rightarrow \begin{pmatrix}1-3\\-1-4\end{pmatrix} + r = \begin{pmatrix}0\\0\end{pmatrix} \Rightarrow \begin{pmatrix}-2\\-5\end{pmatrix} + r = \begin{pmatrix}0\\0\end{pmatrix} \Rightarrow r = \begin{pmatrix}0+2\\0+5\end{pmatrix} \Rightarrow r = \begin{pmatrix}2\\5\end{pmatrix}$$

Q12. Triangle ABC has vertices A(-2, -4), B(10, 1) and C(3, 8).

i. Find the length of the side AB
ii. Show that the triangle is isosceles

Soln.

i.



The length of $\overline{AB}$is the same as the magnitude of $\overline{AB}$.

A = $\begin{pmatrix}-2\\-4\end{pmatrix}$ $and\ B\begin{pmatrix}10\\1\end{pmatrix}$

$$\overline{AB} = B - A = \begin{pmatrix}10\\1\end{pmatrix} - \begin{pmatrix}-2\\-4\end{pmatrix} = \begin{pmatrix}12\\5\end{pmatrix}$$

Since $\overline{AB} = \begin{pmatrix}12\\5\end{pmatrix}, \Rightarrow |\overline{AB}| = \sqrt{12^2 + 5^2} = \sqrt{169} = 13$

Also C = (3, 8) = $\begin{pmatrix}3\\8\end{pmatrix}$ $and\ A = \begin{pmatrix}-2\\-4\end{pmatrix}$

$$\overline{AC} = C - A = \begin{pmatrix}3\\8\end{pmatrix} - \begin{pmatrix}-2\\-4\end{pmatrix} = \begin{pmatrix}5\\12\end{pmatrix} |\overline{AC}| = \sqrt{5^2 + 12^2} = \sqrt{169} = 13 \therefore AC = 13$$

ii. $B = \begin{pmatrix}10\\1\end{pmatrix}$ $and\ C = \begin{pmatrix}3\\8\end{pmatrix}$

$$\overline{BC} = C - B = \begin{pmatrix}3\\8\end{pmatrix} - \begin{pmatrix}10\\1\end{pmatrix} = \begin{pmatrix}-7\\7\end{pmatrix}$$
$$|\overline{BC}| = \sqrt{(-7)^2 + 7^2} = \sqrt{49 + 49} = \sqrt{98} = 9.9$$

Now for the given triangle,$|\overline{BC}| = 9.9, |\overline{AC}| = 13\ and\ \overline{AB} = 13$. Since two lengths of the given triangle are equal, (ie$|\overline{AC}| = 13\ and |\overline{AB}| = 13$), then it is an isosceles triangle

Q13. Given $\overrightarrow{xT} = \binom{-1}{2}$ and $\overrightarrow{YR} = \binom{-2}{-3}$, $x(4,1)$ and $y(-3,2)$

Find $\overline{TR}$.

Soln.

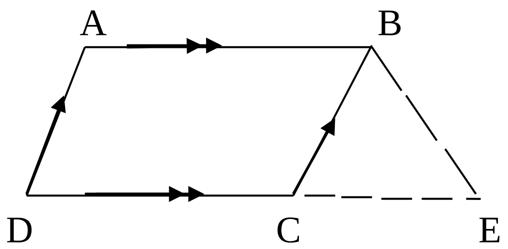
We must first determine the coordinates of T and R

$x = (4,1)$ and $\overline{xT} = \binom{-1}{2} \Rightarrow T(4 + \mathrm{T}, 1 + 2) \Rightarrow T(4 - 1, 1 + 2) \Rightarrow T(3,3)$

Also $Y = (-3,2)$ and $\overline{YR} = \binom{-2}{-3}$

$\Rightarrow R = (-3 + \bar{2}, 2 + \bar{3}) \Rightarrow R(-5,-1) = \binom{-5}{-1}$

Now $T = \binom{3}{3}$ and $R = \binom{-5}{-1} \Rightarrow \overline{TR} = R - T = \binom{-5}{-1} - \binom{3}{3} = \binom{-8}{-4} \therefore TR = \binom{-8}{-4}$

Q14. 

In the given figure, ABCD is a parallelogram. If $\overrightarrow{DA} = \binom{3}{5}$ and $\overrightarrow{CE} = \binom{2}{3}$, find $\overrightarrow{BE}$.

N/B: moving from B to E i.e $\overline{BE}$ is equal to or the same as moving from B to C ie $\overline{BC}$ *plus or in* addition to moving from C to E $\Rightarrow \overrightarrow{BE} = \overrightarrow{BC} + \overline{CE} = \overline{BC} + \binom{2}{3}$.

We must find $\overrightarrow{BC}$. Since ABCD is a parallelogram, then
$\overline{CB} = \overline{DA} \Rightarrow \overline{BC} = -DA \Rightarrow \overline{BC} = -\binom{3}{5}$

$\Rightarrow \overline{BE} = \overline{BC} + \binom{2}{3}, \Rightarrow \overline{BE} = \binom{-3}{-5} + \binom{2}{3} = \binom{-1}{-2} \therefore \overline{BE} = \binom{-1}{-2}$

Q15. If $\overrightarrow{AB} = \binom{4}{-3}$ and $\overline{AC} = \binom{-1}{6}$ are vectors in the same plane, and A is the point (1, 2),

i. Find the coordinates of B and C.
ii. If D is the mid point of BC, show that $\overline{AB} + \overline{AC} = 2\overline{AD}$

Soln.

i. A = (1, 2) and $\overline{AB} = \binom{4}{-3} \Longrightarrow$ the coordinates of

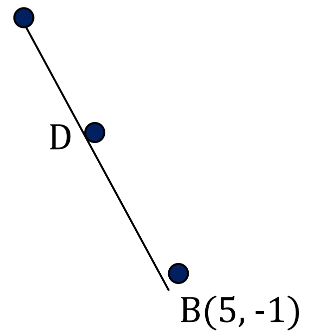
B = $(1+4, 2+\overline{3}) \Longrightarrow B$ has coordinates (5, -1)

Also A = (1, 2) and $\overline{AC} = \binom{-1}{6} \Longrightarrow the$ coordinates of

$C = (1+\overline{1}, 2+6) = (0,8) \Longrightarrow$the coordinates of

C = (0, 8)

ii. C(0, 8)



Since D is the mind point of the line CB.

To find the x component of D, we add the x components of C and B and divide the result by 2 $\Longrightarrow x$ component of D = $\frac{0+5}{2} = 2.5$

Also to find the y components of C and B, add their y components and divide the result by 2 $\Longrightarrow$ the y component of D $\frac{8+T}{2} = \frac{7}{2} = 3.5$

The coordinates of D = $\binom{2.5}{3.5}$

To show that $\overline{AB} + \overline{AC} = 2\overline{AB}$, we evaluate the L.H.S and then the R.H.S, and check if they are equal.

L.H.S = $\overline{AB} + \overline{AC} = \binom{4}{-3} + \binom{-1}{6} = \binom{3}{3}$

But R.H.S = $2\overline{AD} = ?$

$\overline{AD} = D - A = \binom{2.5}{3.5} - \binom{1}{2} = \binom{1.5}{1.5}$

Since $\overline{AD} = \binom{1.5}{1.5} \Longrightarrow 2\overline{AD} = 2\binom{1.5}{1.5} = \binom{3}{3}$

Now since $\overline{AD} + \overline{AC} = \binom{3}{3}$ and $2\overline{AD} = \binom{3}{3}$, then $\overline{AB} + \overline{AC} = 2\overline{AD}$

Q16. A (4, 7) is the vertex of triangle ABC. $\overrightarrow{BA} = \binom{5}{3}$ and $\overrightarrow{AC} = \binom{4}{-3}$.

a. Find the coordinates of B and C.
b. If M is the mid point of the line BC, find $\overrightarrow{AM}$.

Soln.

a. The point A is given as A(4, 7) and
$\overrightarrow{BA} = \binom{5}{3} \Longrightarrow \overrightarrow{AB} = -BA \Longrightarrow \overrightarrow{AB} = -\binom{5}{3} \Longrightarrow \overrightarrow{AB} = \binom{-5}{-3}$

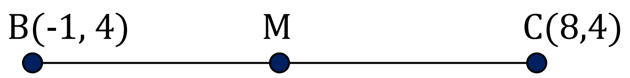
Now A(4, 7) and $\overrightarrow{AB} = \binom{-5}{-3}$ $\Longrightarrow$the coordinates of B is given by B $(4+\overline{5}, 7+\overline{3})$
$\therefore B(-1{,}4) \Longrightarrow B\ has$ coordinates (-1, 4)

Also A(4, 7) and $\overrightarrow{AC} = \binom{4}{-3}$ $\Longrightarrow$the coordinates of

C = $(4+4, 7+\overline{3}) \Longrightarrow C(8{,}4) \Longrightarrow$ coordinates of

C = (8, 4)

b.

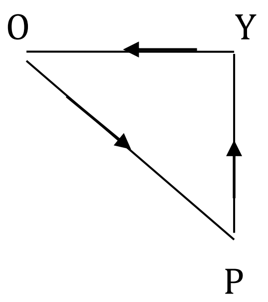


Let M be the mid point of the line BC. Then the coordinates of M =
$\left(\frac{-1+8}{2}, \frac{4+4}{2}\right) = \left(\frac{7}{2}, \frac{8}{2}\right) \Longrightarrow$ M(3.5, 4) $\Longrightarrow$ M has coordinates (3.5, 4), Now A =
$\binom{4}{7}$ and M = $\binom{3.5}{4}$

But $\overrightarrow{AM}$ = M − A = $\binom{3.5}{4} - \binom{4}{7} = \binom{-0.5}{-3}$

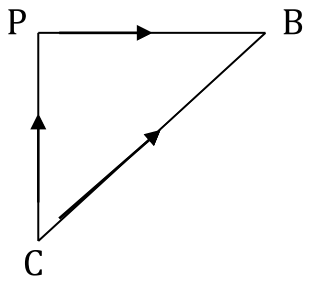
N/B:

i.



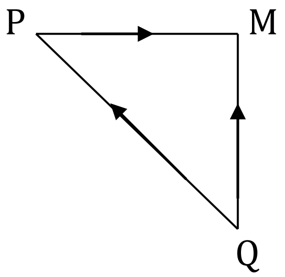
- Considering the given figure under vectors, movement from a particular point through a distance back to the starting point is considered as zero movement.
- Therefore moving from O to P, then from P to Y and finally from Y back to O, is considered as zero movement.
- For this reason $\overline{OP} + \overline{PY} + \overline{YO} = 0$, and this is known as the triangle law

Example (2).



- Consider the given figure
- According to the triangle law $\overrightarrow{BC} + \overline{CP} + \overline{PB} = O$

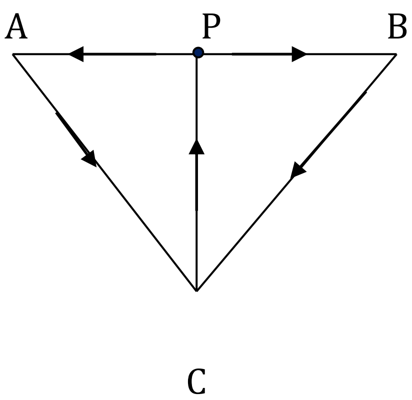
Examples (3)



According to the triangle law $\overline{QM} + \overline{MP} + \overline{PQ}$

Q17. In $\Delta ABC, \overline{AB} = \binom{-2}{6}$ *and* $\overline{CA} = \binom{3}{-4}$. If P is the mid point of $\overline{AB}$, *express* $\overline{CP}$ as a column vector.

Soln.



From ΔABC, since P is the mid point of

$\overline{AB}, and\ \overline{AB} = \binom{-2}{6}, then\ \overline{AP} = \frac{1}{2}(\overline{AB}) = \frac{1}{2}\binom{-2}{6} = \binom{-1}{3} \Longrightarrow \overrightarrow{AP} = \binom{-1}{3}$

From the triangle law,

$\overline{AC} + \overline{CP} + \overline{PA} = 0 \Longrightarrow \overline{CP} + \overline{PA} = -\overline{AC} \Longrightarrow \overline{CP} = -\overline{AC} - \overline{PA}$

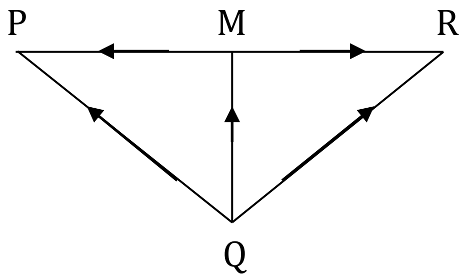
But $\overline{AC} = -(CA) = -\binom{3}{-4} = \binom{-3}{4}\ and\ \overline{PA} = -\overline{AP} = -\binom{-1}{3} = \binom{1}{-3}$. Since $\overline{CP} = -\overline{AC}$ - $\overline{PA}$,

then $\overline{CP} = -\binom{-3}{4} - \binom{1}{-3} = \binom{3}{-4} - \binom{1}{-3} = \binom{3-1}{-4+3} = \binom{2}{-1}$

Q18. ΔPQR i.e triangle PQR is an Isosceles triangle in which $|\overline{PQ}| = |\overline{QR}|\ and\ M$ is the mid point of PR.

a. Show that $\overline{QP} + \overline{QR} = 2QM$
b. If $\overline{PQ} = \binom{-3}{4}\ and\ \overline{QR} = \binom{4}{3}$,
i. express $\overline{QM}\ as\ a\ column\ vector$.
ii. calculate $|\overline{QM}|$

Soln.

a. 

consider the figure above. As already given $|\overline{PQ}|$

$= |\overline{QR}|$.

since M is the mid point of PR $\Longrightarrow |\overline{PM}| = \overline{MR}\ and\ PM = \frac{1}{2}|\overline{PR}|$ From the above figure $\overrightarrow{QR} = \overrightarrow{QM} + \overrightarrow{MR} \ldots .eqn(1)$

Also $\overrightarrow{QP} = \overrightarrow{QM} + \overrightarrow{MP} \ldots \ldots .eqn\ (2)$

$Add\ eqn(1) and\ eqn(2) \Longrightarrow \overrightarrow{QR} + \overrightarrow{QP} = 2\overrightarrow{QM} + \overrightarrow{MR} + \overrightarrow{MP}$

But $\overrightarrow{MR} + \overrightarrow{MP} = 0$, since they are of the same magnitude and one is the inverse or the negative of the other. From

$\overline{QR} + \overline{QP} = 2\overline{QM} + \overline{MR} + \overline{MP} \Longrightarrow \overline{QR} + \overline{QP} = 2\overline{QM} + 0 \Longrightarrow \overline{QR} + \overline{QP} = 2\overline{QM}$

b. Since $\overline{PQ} = \binom{-3}{4}$, *then* $\overline{QP} = -\binom{-3}{4} = \binom{3}{-4}$

But $\overline{QR} + \overline{QP} = 2\overline{QM} \Rightarrow \binom{4}{3} + \binom{3}{-4} = 2QM \Rightarrow \binom{7}{-1} = 2QM$

Multiply through using ½

$\Rightarrow \frac{1}{2}\binom{7}{-1} = \frac{1}{2} \times 2QM = 0.5\binom{7}{-1} => 0.5 \times 2QM \Rightarrow \binom{3.5}{-0.5} = QM$

$|\overline{QM}| = \sqrt{3.5^2 + (-0.5)^2} = \sqrt{12.5} = 3.5$

Questions:

Q1. Find the values of K and M such that

$$K\binom{2}{3} + M\binom{4}{5} = \binom{16}{21}$$

Ans: K = 2 and M = 3

Q2. Determine the values of Q and R such that

$$Q\binom{1}{3} + R\binom{2}{5} = \binom{5}{12}$$

Ans: Q = -1 and R = 3

Q3. Given that $x\binom{2}{3} - Y\binom{3}{1} = \binom{2}{10}$, *find the valuesof x* and y.

Ans: x = 4 and y = 2

Q4. Given A (6, 4) and B(3, 2), evaluate i. $\overline{AB}$ ii. $\overline{BA}$

Ans: i. $\overline{AB} = \binom{-3}{-2}$ ii. $\binom{3}{2} = \overline{BA}$

Q5. If $x = \binom{-3}{4}$ *and* $y = \binom{6}{2}$, *evaluate*

i. $\overline{xy}$ Ans: $\binom{9}{-2}$

ii. *The magnitude of* $\overline{xy}$ Ans: 9.2

iii. $\overline{yx}$ Ans: $\binom{-9}{2}$

Q6. Given that x = (2, 4) and y = (4, 9), determine the length of $\overline{xy}$

Ans: 5.4

Q7. Given that $\overline{x} = \binom{3}{4}$ *and* $\overline{y} = \binom{-2}{1}$, evaluate

a. $3\overline{xy}$ Ans: $\binom{-15}{-9}$

b. $4(\overline{x}-\overline{y})$Ans: $\binom{20}{12}$

c. $|\overline{xy}|$ Ans: 5.8

d. $|\overline{x}+\overline{y}|$Ans: 5.1

e. $|\overline{x}-\overline{y}|$ Ans: 5.8

Q8. Given $P(2{,}4)$ *and* $\overline{PQ} = \binom{3}{6}$, *determine* the coordinates of Q.

Ans: (5, 10)

Q9. Given $P(3{,}6)$ *and* $\overline{QP} = \binom{-1}{-2}$, determine the coordinates of Q.

Ans: (4, 8).

Q10. Given A(3, 2), $\overline{AB} = \binom{1}{5}$ *and* $\overline{AC} = \binom{4}{6}$, determine the coordinates of

a. the point B Ans: (4,7)
b. The point C Ans: (7,8)

Q11. Given A(2, 1), $\overline{BA} = \binom{-2}{3}$ *and* $CA = \binom{4}{1}$, determine the coordinates of

a. the point B Ans: $\binom{4}{-2}$
b. the point C Ans: $\binom{-2}{0}$

Q12. Given x (-2, 1), $\overline{yx} = \binom{-3}{2}$ *and* $\overline{xz} = (-3,-4)$, determine the coordinates of

a. the point Y. Ans: $\binom{1}{-1}$
b. the point Z Ans: $\binom{-5}{-3}$

Q13. Given C(2, 3), $\overrightarrow{CD} = \binom{2}{1}$ *and* $\overrightarrow{DE} = \binom{5}{4}$, *find the* coordinates of

i. point D. Ans: (4, 4)
ii. point E. Ans: (9, 8)

Q14. Given A(3,2), $\overline{BA} = \binom{4}{-5}$ *and* $\overline{BC} = \binom{1}{2}$, determine the coordiantes of

a. point B. Ans: $\binom{-1}{7}$
b. point C Ans:$\binom{0}{9}$

Q15. Given A(4,2), $\overline{CB} = \binom{1}{3}$ *and* $\overrightarrow{BA} = \binom{5}{6}$, determine the coordinates of

a. the point B. Ans: (-1, -4)
b. the point C Ans: (-2, -7)

Q16. Given x(2, 4), y =(3,6), $\overline{xp} = \binom{2}{1}$ and $\overline{yz} = \binom{1}{1}$. Determine the coordinate of

i. Point P Ans: (4,5)
ii. Point Z Ans:(4, 7)

Q17. Given x(2, 3), y(4, 1), $\overrightarrow{Bx} = \binom{-2}{-1}$ and $\overline{y\vec{m}} = \binom{-6}{5}$,

find the coordinates of

a. the point B. Ans: (4, 4)
b. the point M. Ans (-2, 6)

Q18. If $\overline{PQ} = \binom{1}{2}$ and $\overline{QR} = \binom{2}{5}, find$

i. $\overline{PR}$. Ans: $\binom{3}{7}$ ii. $|\overline{PR}|$ Ans: 7.6

Q19. If $\overline{PQ} = \binom{1}{2}$ and $\overline{RQ} = \binom{3}{4}$ find

i. $\overline{PR}$ $Ans: \binom{-2}{-2}$ ii. $|\overline{PR}|$ $Ans: 2.8$

Q20. Given that $\overrightarrow{QO} = \binom{-3}{4}$ and $\overrightarrow{OP} = \binom{-1}{2}, find$

i. $\overrightarrow{QP}$ $Ans: \binom{-4}{6}$
ii. $|\overrightarrow{QP}|$ $Ans: 7.2$

Q21. Given that $\overline{xY} = \binom{2}{1}, \overline{OY} = \binom{4}{3}, find\ \overline{xO}$.

Ans: $\binom{-2}{-2}$

Q22. If M = $\binom{2}{1}$ and $k = \binom{5}{3}$, evaluate

i. 4(3M + 3k) Ans: $\binom{64}{36}$
ii. 2(2M – k) Ans: $\binom{-2}{-2}$

Q23. If P = $\binom{2}{3}, q = \binom{-2}{5}$ and $r = \binom{5}{1}, evaluate\ 3p - 2q + r.$ $Ans: \binom{15}{0}$

Q24. Given $P = \binom{1}{2}, q = \binom{-2}{-3}$ and $r = \frac{1}{3}(2p + q)$,

$evaluate\, r.$ $\qquad Ans: r = \begin{pmatrix} 0 \\ 1/3 \end{pmatrix}$

Q25. If $x = (4{,}10), \overline{xy} = \binom{6}{2}$ $and\ \overline{xO} = \binom{-2}{-8}$,

a. determine the coordinate of Y and O.
   Ans: Y(10, 12) and O(2, 2)
b. If P is the mid point of the line YO, determine the coordinates of P.
   Ans: P (6, 7)

Q26. Find the direction of the displacement vector $\overrightarrow{AB}$, where A and B are the points (8, 4) and (6, 2) respectively. Ans:$45°$

Q27. Find the direction of the displacement vector $\overline{AB}$, where A and B are the points (2, -4) and (-6, -10) respectively. Ans: $37°$

Q28. Determine whether or not these pairs of vectors and parallel.

a. $\overline{A} = \binom{2}{4}$ $and\ \overline{B} = \binom{6}{12}$
   $Ans: They\ are\ parallel$
b. $\overline{x} = \binom{3}{1}$ $and\ \overline{Y} = \binom{9}{3}$
   Ans: They are parallel
c. $\overline{x} = \binom{4}{3}$ $and\ \overline{Y} = \binom{8}{9}$
   Ans: They are not parallel
d. $\overline{A} = \binom{1}{2}$ $and\ \overline{B} = \binom{5}{10}$
   Ans : They are parallel

Q29. Given that the vector $\overline{A} = \binom{x}{2}$ $and\ \overline{B} = \binom{16}{8}$ are parallel vectors, determine the value of x. Ans: 4

Q30. If $\overline{P} = \binom{5}{3}$ $and\ \overline{Q} = \binom{25}{y}$, find the value of y, so that P and Q become parallel vectors. Ans: 15

Q31. Determine whether or not the following pairs of vectors are perpendiculars

a. $\overline{x} = \binom{8}{2}$ $and\ \overline{Y} = \binom{8}{10}$

Ans: They are not perpendicular

b. $\overline{x} = \begin{pmatrix} 12 \\ 6 \end{pmatrix}$ $and\, \overline{Y} = \begin{pmatrix} -3 \\ 6 \end{pmatrix}$

Ans: They are perpendicular

c. $\overline{A} = \begin{pmatrix} 4 \\ -18 \end{pmatrix}$ $and\, \overline{B} = \begin{pmatrix} 36 \\ 8 \end{pmatrix}$

Ans: They are perpendicualr

d. $\overline{A} = \begin{pmatrix} 4 \\ 2 \end{pmatrix}$ $and\, \overline{B} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$

Ans: They are not perpendicular

Q32. Given that $\overline{C}\begin{pmatrix} -8 \\ 4 \end{pmatrix}$ $and\, \overline{D}\begin{pmatrix} x \\ -12 \end{pmatrix}$ are two perpendicular vector, find x.

Ans: -6

Q33. A parallelogram ABCD has vertices

A(2, 6), B(-4, 10) and C(2, 16). Find the coordinates of vertice D. Ans: (8, 12)

Q34. The points A(-4, 2), B(4, 2), C(2, 8) and D(x,y)

are the vertices of a parallelogram ABCD.

i. Find $\overline{AB}$ $and\, \overline{DC}$.

Ans:$\overline{AB} = \begin{pmatrix} 8 \\ 0 \end{pmatrix}$ $and\, \overline{DC} = \begin{pmatrix} 2 - x \\ 8 - y \end{pmatrix}$

ii. Determine the values of x and y

Ans: (-6, 8) ie x = -6 and y =8

Q35. The coordinates of the vertices of a parallelogram QRST are Q(2,12), R(4,4), S(10,8) and T(x, y).

a. Determine the values of x and y

Ans: x = 8 and y = 16 ie (8, 16)

b. Calculate the magnitude of $\overrightarrow{RS}$Ans: 7.2

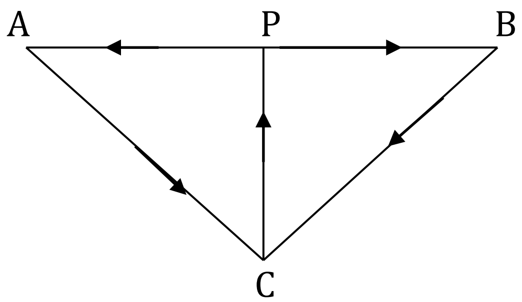
Q36. Find the values of x and y such that $\begin{pmatrix} 3x + 1 \\ 4 \end{pmatrix} + \begin{pmatrix} y \\ x - y \end{pmatrix} = \begin{pmatrix} 6 \\ 7 \end{pmatrix}$

Ans: x = 2 and y = -1

Q37. The triangle ABC has vertices A(-4, -8), B(20, 2) and C(6, 16). Find the length of

a. AB Ans: 26
b. BC Ans: 19.8

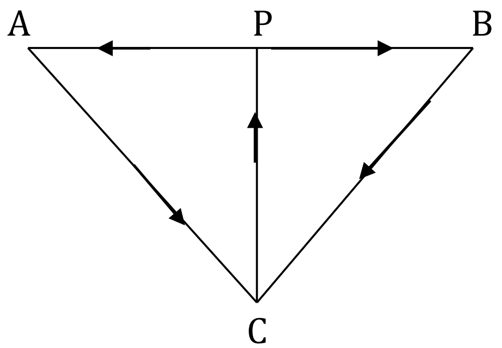
Q38.



In $\Delta ABC, \overline{AB} = \binom{-2}{6}$ $and\ \overline{CA} = \binom{3}{-4}$. If P is the mid point of $\overline{AB}$, $express\ \overline{CP}\ as\ acolumn\ vector$.

$Ans: CP = \binom{2}{-1}$

Q39.



In $\Delta ABC, \overline{AB} = \binom{4}{2}, \overline{AC} = \binom{3}{5}$ $and\ P\ is\ the$ mid point of AB. Express CP as a column vector.

Ans: $CP\ = \binom{-1}{-4}$.