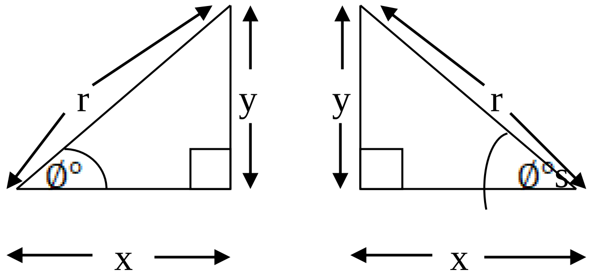
CHAPTER FOUR

TRIGONOMETRY

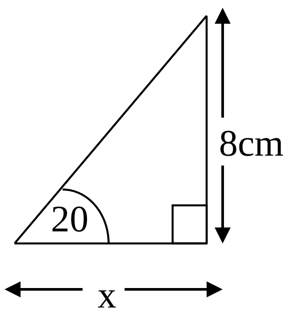


Consider the above two figures. For any one of them, the following facts must be noted:

1. When the length y is divided by the length x, we always get the tangent or the tan of the angle ∅, ie for any of the above figures $\tan \emptyset = \frac{y}{x}$.
2. When the length r is multiplied by the cosine or the cos of the angle ∅, we always get the length $x, ie\ r\ x\ cos\emptyset = x, \Longrightarrow \cos\emptyset = \frac{x}{r}$
3. When the length r is multiplied by the sine or sin of the angle ∅, we always get the length $y\ ie\ r\ x\ sin\emptyset\ =\ y, \Longrightarrow Sin\emptyset = \frac{y}{r}$

The use of tangent:

Q1.



Find the value of x.

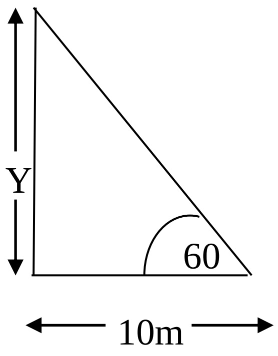
Soln.

Y = 8cm and

$\emptyset = 20°, \Longrightarrow tan\emptyset = \frac{y}{x} \Rightarrow tan20° = \frac{8}{x} \Rightarrow x \times tan20° = 8, \Longrightarrow x\ tan20° = 8.$

Dividing through using tan 20° => $\frac{x \tan 20^0}{\tan 20°} = \frac{8}{\tan 20°} \Longrightarrow x = \frac{8}{\tan 20^0} = \frac{8}{0.364}$, (since tan $20°) = 0.364 \therefore x = 219cm.$

Q2.



Find the length y.

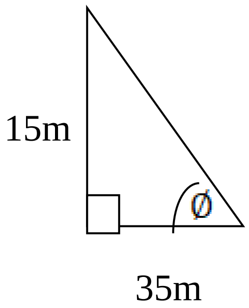
Soln.

$\emptyset = 60°$ *and* $x = 10m$. Since $\tan \emptyset = \frac{y}{x} \Longrightarrow \tan 60° = \frac{y}{10}, \Longrightarrow 10 \times \tan 60° = y, \Longrightarrow$ $y = 10 \times \tan 60°, => y = 10 \times 1.730, (since \tan 60° = 1.730) \Rightarrow y = 10 \times 1.730 = 17.3$

.

$\therefore$Y = 17.30m.

Q3.



Calculate the angle Ø°

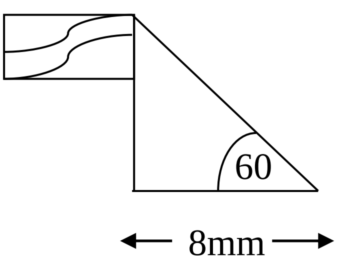
Soln.

$y = 15m$ *and* $x = 35m. From\ tan\emptyset = \frac{y}{x} \Longrightarrow \tan \emptyset = \frac{15}{35},$

$\Rightarrow \tan \emptyset = 0.428, \Rightarrow \emptyset = tan^{-1} 0.428,$

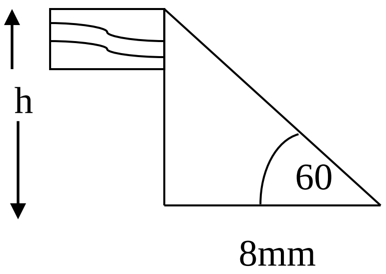
$=> \emptyset = 23°$ approx.

Q4.

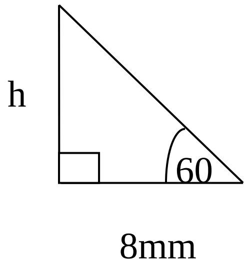


Calculate the height of the flag pole shown above.

Soln.



Let h = height of the flag pole. This can be shown in the form of a diagram as



From the figure tan $\emptyset = \frac{h}{8}$,

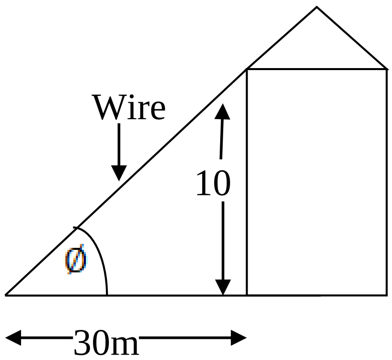
$\therefore \tan 60^\circ = \frac{h}{8} \Longrightarrow 8 \times \tan 60^\circ = h \,, \Rightarrow h = 8 \times \tan 60^\circ,$

$but\ since\ \tan 60^\circ = 1.730 \Longrightarrow h = 8 \times 1.730, => h = 13.84.$

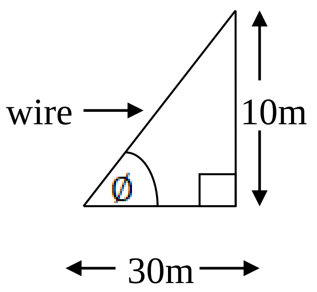
$The\ height\ of\ flag\ pole = 13.84mm$

Q5. One end of a wire is fixed to a point 10m up a building. The other end is fixed to the ground at a point 30m away from the building. Find the angle which the wire makes with the ground.

Soln.



The above diagram can be represented as shown next:.

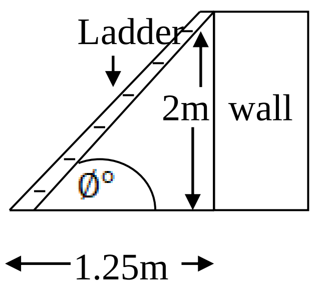


Let Ø = the angle made by the wire with the ground.

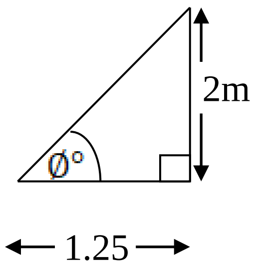
$tan\emptyset° = \frac{10}{30} \Longrightarrow tan\ \emptyset = 0.30 \Longrightarrow \emptyset = tan^{-1}0.30, \Longrightarrow \emptyset = 17° \therefore$
$the\ wire\ makes\ an\ angle\ of\ 17°,$
with the ground.

Q6. A ladder leans against a wall. The foot of the ladder is on the same horizontal level as the foot of the wall and is 1.25m away from it. The top of the ladder just reaches the top of the wall which is 2m high. Calculate the angle between the ladder and the ground.

Soln



The above figure can be simplified as shown next:

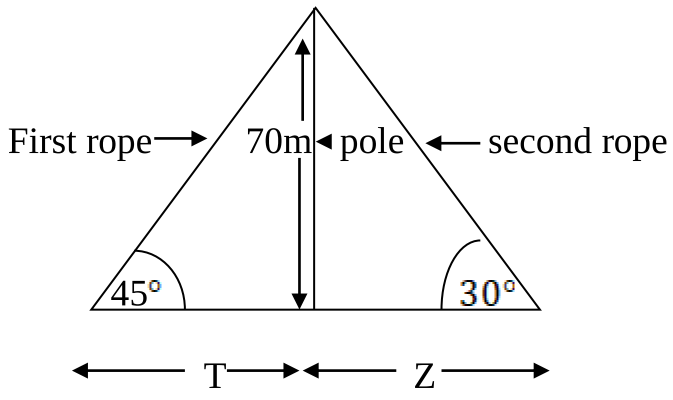


Let Ø = the angle between the ground and the ladder.

Then tan $\emptyset° = \frac{2}{1.25} => tan\ \emptyset° = 1.6, \Longrightarrow \emptyset = tan^{-1}1.6 \Rightarrow \emptyset = 58°$The angle between the ladder and the ground = 58°

Q7. A pole is 70m long and stands on a leveled ground. One end of a first rope is tired to the top of this pole while the other end is fixed to a point on the ground, so that it makes an angle of 45° with the ground. One end of a second rope is fixed to the top of the pole, and its other end is fixed to the ground so that it makes an angle of 30° with the ground. Calculate the distance between the points where the two ropes are fixed to the ground, if they are opposite to each other.

Soln



Let T = the distance between the foot of the pole and the point where the first rope makes an angle of 45° with the ground. Also let Z = the distance from the foot of the pole to the point where the second rope is fixed to the ground and makes an angle of 30° with the ground as shown in the diagram. Then the total distance between the two points = T + Z. We must therefore find T and Z

The original figure can be broken into two parts as shown next:

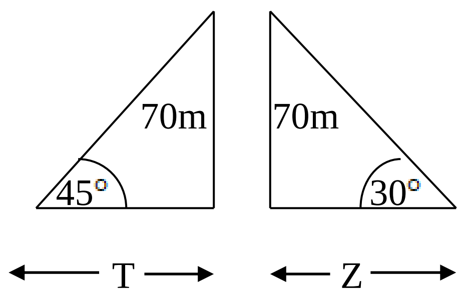


Figure (1) Figure (2)

From fig. (1), tan 45° $= \frac{70}{T} \Longrightarrow T \times \tan 45° = 70°$,

$\therefore T \tan 45° = 70°$

Divide through using tan 45°. i.e

$\frac{T\ \tan 45°}{\tan 45°} = \frac{70}{\tan 45°} \Rightarrow T = \frac{70}{\tan 45°}$, but tan 45° $= 1$

$\Rightarrow T = \frac{70}{1} = 70m.$

From fig. (2), tan 30° $= \frac{70}{Z} \Longrightarrow Z \times \tan 30° = 70,$

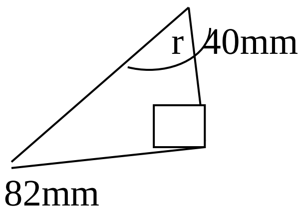
$\Rightarrow Z \tan 30° = 70$'s

Divide through using tan 30° $\Longrightarrow \frac{Z \tan 30°}{\tan 30°} = \frac{70}{\tan 30°}$

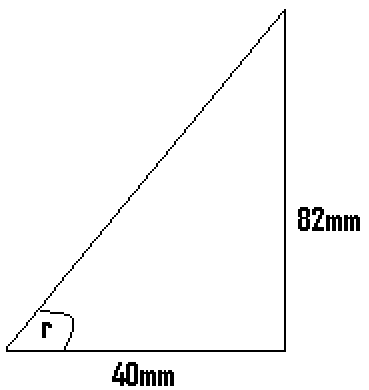
$\Rightarrow Z = \frac{70}{\tan 30°}, but \tan 30° = 0.58, \therefore Z = \frac{70}{0.58} = 121,$

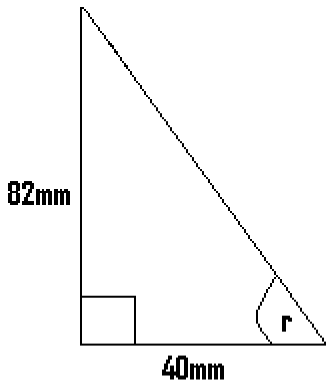
$\Rightarrow Z = 121m.$ Distance between the two points = T + Z = 70 + 121 = 191m.

Q8. Calculate the angle marked r° in the figure below.



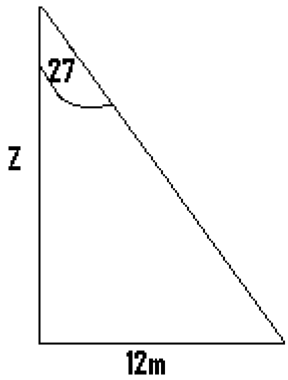
Rotate the figure to get any of the figures below (1.e rotate so that the location of angle rests horizontally).





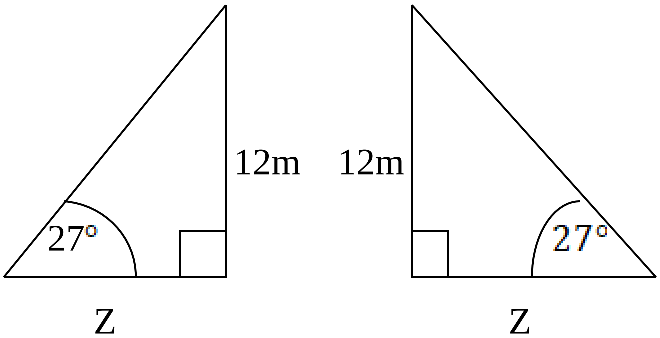
By using any of the above figures, tan
r° $= \frac{82}{40} \Longrightarrow tanr^{\circ} = 2.05, => r^{\circ} = tan^{-1}205 \Rightarrow r = 64.^{\circ}$

Q9. Calculate the length marked Z in the figure given next:



Soln.

Rotate the above to get any of the following:



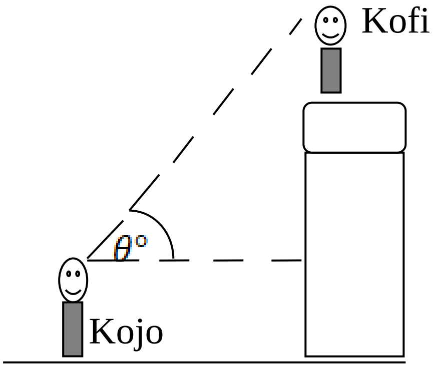
From any of the above figures, tan
$27^{\circ} = \frac{12}{z} \Longrightarrow Z \times \tan 27^{\circ} = 12, \Rightarrow Z \tan 27^{\circ} = 12.$

Divide through using tan 27°

$$\frac{Z \tan 27^{\circ}}{\tan 27^{\circ}} = \frac{12}{\tan 27^{\circ}}$$

$\Rightarrow$Z $= \frac{12}{\tan 27^{\circ}}$, but since tan 27° $= 0.510 \Longrightarrow Z = \frac{12}{0.510} = 23.5m$

Angle of elevation:

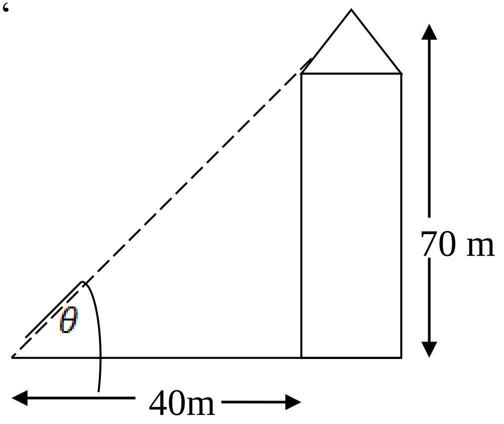


Consider two friends Kojo and Kofi. Kofi is standing on top of a building while Kojo stands on the ground as shown in the diagram. In order for Kojo to look at Kofi, he must turn his eyes through angle θ. This angle θ is referred to as the angle of elevation of Kofi from Kojo.

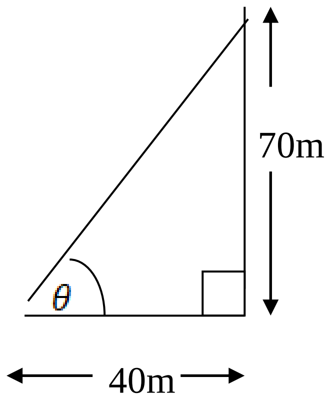
Q1. A boy stood 40m away from the foot of a tower, and found the angle of elevation of the top of the tower. If the tower is 70m high, calculate the angle of elevation of the top of the tower, if the boy's height is to be neglected.

Soln.

‘



N/B: Since the boy's height is negligile or to be neglected, then we can work without taking it into consideration. The above figure can be represented as

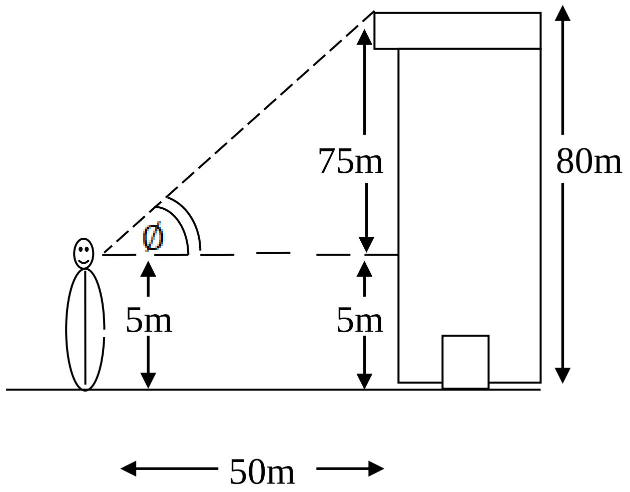


Where θ = the angle of elevation of the top of the tower.

Tan $\theta = \frac{70}{40} = 1.75 \Rightarrow$ tan $\theta = 1.75, \Rightarrow \theta = tan^{-1}1.75 \Rightarrow \theta = 60°$

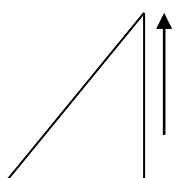
Q2. A boy of height 5m, stood 50m away from the foot of a building, which is 80m of height. What will be the angle of elevation of the top of this building from the boy.

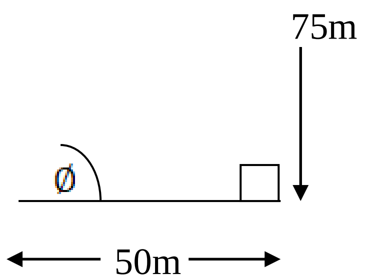
Soln



N/B: In this case the boy`s height is not negligible but 5m. For our calculation, we must not use the 80m but rather

80 – 5 = 75m. Our diagram therefore becomes as shown next:





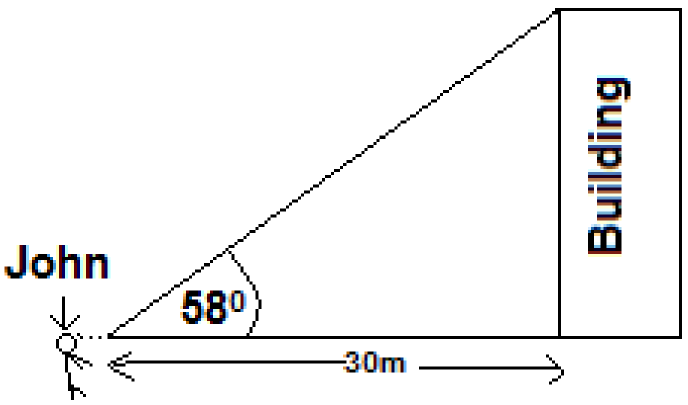
Where Ø = the angle of elevation. Tan

$\emptyset = \frac{75}{50} = \frac{3}{2} = 1.5, \Rightarrow \emptyset = tan^{-1}1.5, \Rightarrow \theta = 57.^{\circ}$

Q3. John stood 30m away from the foot of a building, and observed the angle of elevation of the top of the building to be 58°.

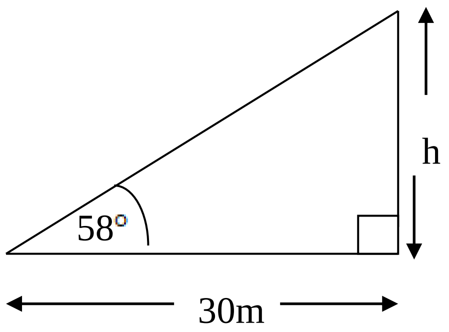
If John's height is negligible, or insignificant, calculate the height of the building.

Soln.



N/B: Since John's height is negligible or to be neglected, we work without using it. O

The above diagram can be represented as shown next:



$\tan 58° = \frac{h}{30} \Rightarrow 30 \times tan\, 58° = h;$

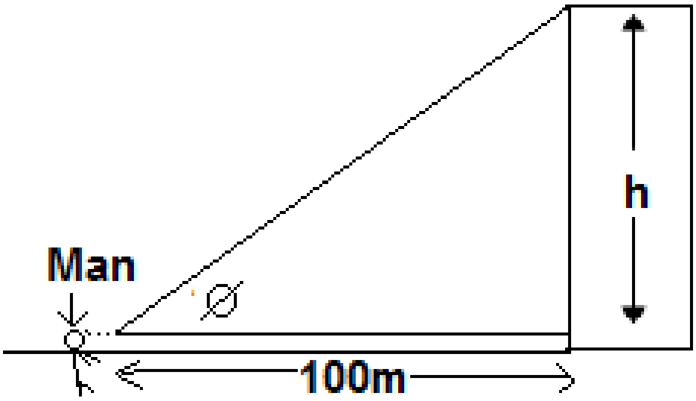
$\Rightarrow$h = 30 x 1.600 = 48m.

∴ The height of the building = 48m

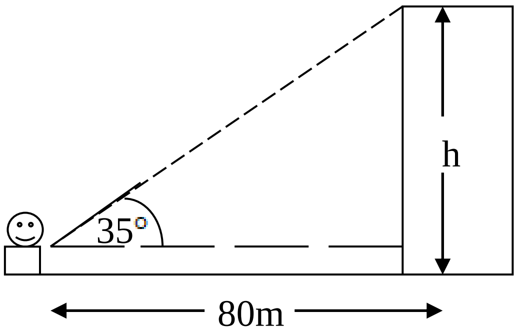
Q4. A man stood 100m away from a building and measured the angle of elevation of the top of the building. He then moved a distance of 20m towards the building and noted that the angle of elevation of the top of the building has changed to 35°. Calculate the height of the building.

Soln.

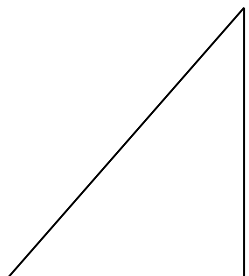
First case

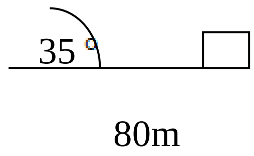


Let Ø = the angle of elevation and h = the height of the building. In the second case, the man moved a distance of 20m towards the building. His distance from the building = 100 – 20 = 80m and the angle of elevation in this case is 35°. Our new diagram therefore becomes as shown next. Also since the man`s height is not given, then it is negligible.



This diagram can be represented as:





tan $35^{\circ} = \frac{h}{80} \Longrightarrow 80 \times tan35^{\circ} = h,$

$\Rightarrow$ h = 80 x tan 35,

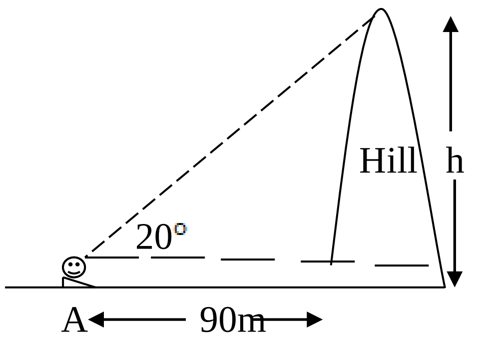
$\Rightarrow$ h = 80 x 0.7 = 56.0,$\Longrightarrow h = 56m.$

Q5. A man stood 90m away from a hill, and observed the angle of elevation of its top part to be 20°. He then moved a certain distance towards the hill, and once again observed the angle of elevation of the top of the hill to be 25°. Calculate

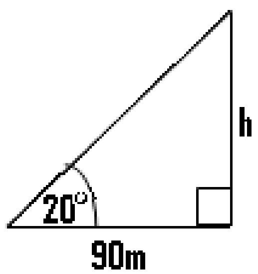
a. the height of the hill.
b. the distance moved by the man towards the hill.

.

Soln.



Let h = the height of the hill, and A = initial position of the man. The above diagram can be represented as shown next:

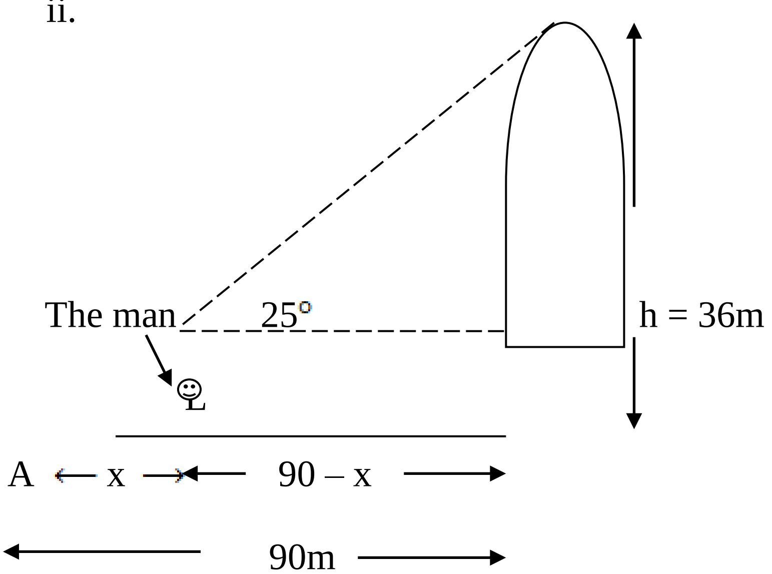


tan $20^{\circ} = \frac{h}{90} \Longrightarrow 90 \times tan20^{\circ} = h$

$\Rightarrow h = 90 \times 0.4 = 36 \Longrightarrow height\ of\ hill = 36m$

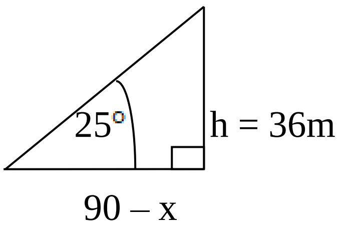
i. In the second case, let x = the distance moved by the man towards the hill. Then his distance from the foot of the hill = 90 – x and the angle of elevation in this case is 25°.

ii.



N/B:A = the man`s original position before moving towrds the hill.

The above diagram can be represted as below



$\tan 25° = \frac{36}{90-x}$, $andsince\ \tan 25 = 0.5 \Longrightarrow 0.5 = \frac{36}{90-x}$,

$\Longrightarrow 0.5(90 - x) = 36, \Rightarrow 0.5 \times 90 - 0.5 \times x = 36,$

$\therefore 45 - 0.5x = 36 \Longrightarrow 45 - 36 = 0.5x, \Longrightarrow 9 = 0.5x$

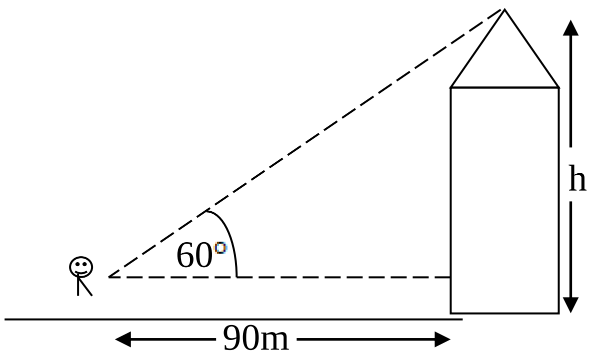
$\Longrightarrow 0.5x = 9, \Longrightarrow x = \frac{9}{0.5} = 18m.$

∴ Distance moved towards the hill = 18m. approx.

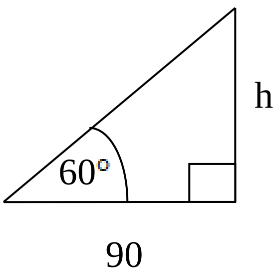
Q6. A man of negligible height stood 90m away from a tower, and observed the angle of elevation of its top part to be 60°. If he moved 20m towards the tower, what will be the new angle of elevation of the top of the tower.

Soln.

First find the height of the tower.



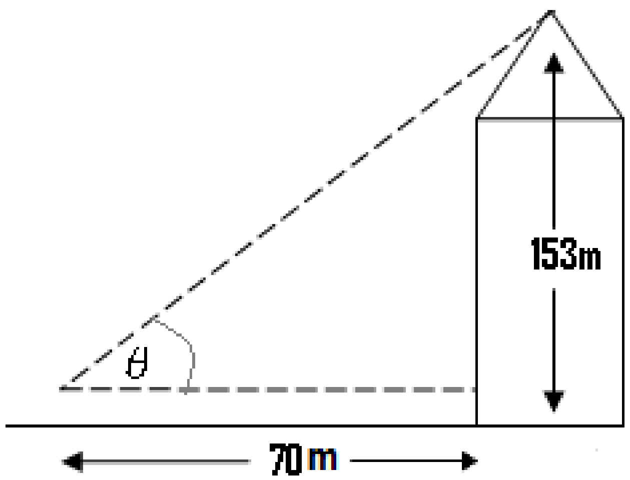
The above diagram can be represented as below:



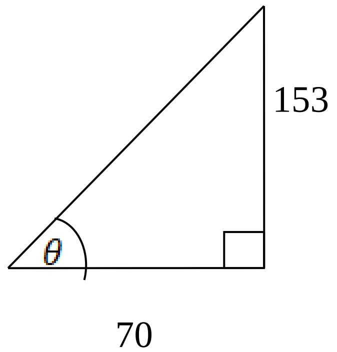
tan 60° $= \frac{h}{90} \Longrightarrow 90 \times tan\ 60° = h, =>$

$h = 90 \times tan\ 60, \Longrightarrow h = 90 \times 1.7 \Longrightarrow h = 153m$

In the second case, the man moved 20m towards the tower. His distance from the foot of the tower now = 90 – 20 = 70m.Let θ be the angle of elevation in this case.



This can be represented as shown next:.



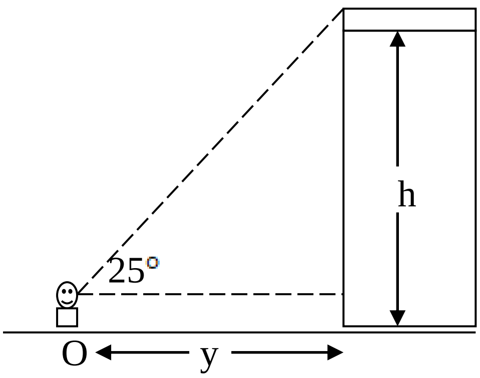
$\tan\theta = \frac{153}{70} = 2.1, \therefore \tan\theta = 2.1 \Rightarrow \theta = \tan^{-1} 2.1,$

$\Rightarrow \theta = 65^{\circ}$ approximately.

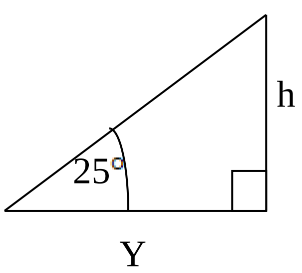
Q7. A man found from a point O that the angle of elevation from his eyes of a corner of a flat roof of a building is 25°. When he walks 10m from O towards the building, the angle of elevation of the same roof corner increased to 35°

i. How far is the point O from the building
ii. Find the height of the building

Soln



In the first case. Let y = the distance from O to the building. Also let h = height of the building.

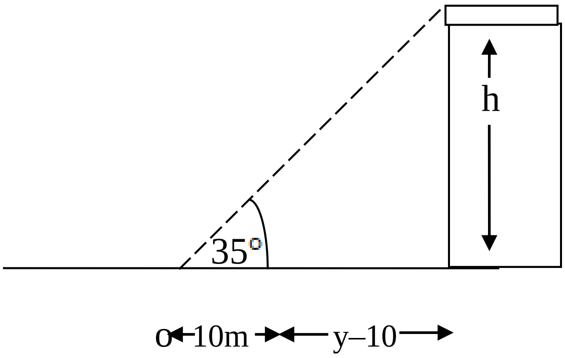


$\tan 25° = \frac{h}{y} \Longrightarrow y \times \tan 25 = h$

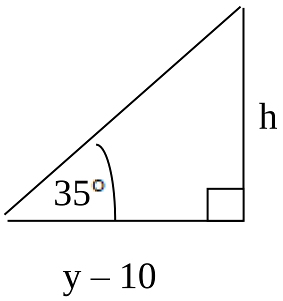
$\therefore h = y \times \tan 25$

$\Rightarrow h = y \times 0.5 \Longrightarrow h = 0.5y \ldots\ldots\ldots\ldots\ldots\ldots\ldots eqn(1)$

In the second case, the man moved 10m towards the building. His distance away from the building now = y – 10 and the angle of elevation in this case is 35°.



This diagram can be shown as:



$\tan 35° = \frac{h}{y-10}, but \tan 35^{0} = 0.7,$

$\therefore 0.7 = \frac{h}{y-10} \Longrightarrow 0.7(y - 10) = h.$

$\therefore 0.7y - 0.7 \times 10 = h \Longrightarrow 0.7y - 7 = h - eqn\ (2).$

From eqn(1) h = 0.5y and from eqn(2) h = 0.7y – 7

Equating eqn(1) and eqn(2) [Since they are equal.]

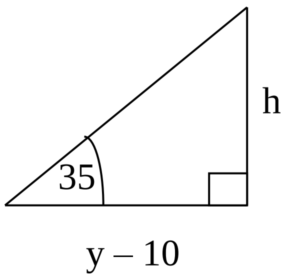
$\Longrightarrow 0.5y = 0.7y - 7, \Longrightarrow 7 = 0.7y - 0.5y,$

$=> 7 = 0.2y \Longrightarrow y = \frac{7}{0.2}, \therefore y = 35.$

b) From eqn(1), h = 0.5y $\Longrightarrow h = 0.5(35)$

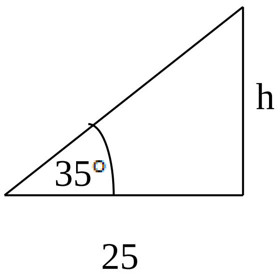
$= 17.5m$,∴ height of building = 17.5m.

(2) consider the last diagram i.e .



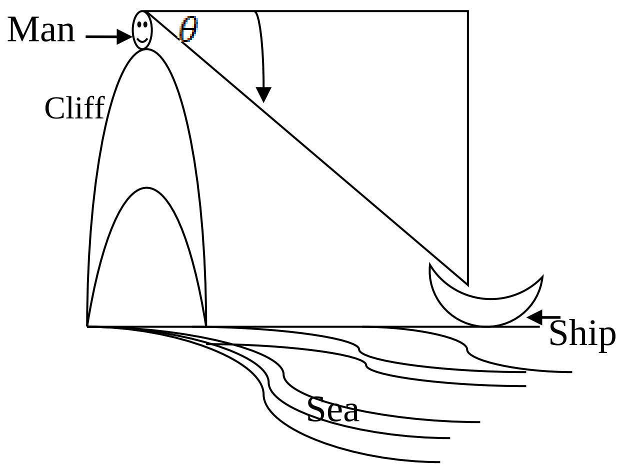
Since y = 35m then y – 10 = 35 – 10 = 25.

The diagram becomes as shown next:



tan 35° $= \frac{h}{25} \Longrightarrow h = 25 \times \tan 35$, => $h = 25 \times 0.7 = 17.5m$

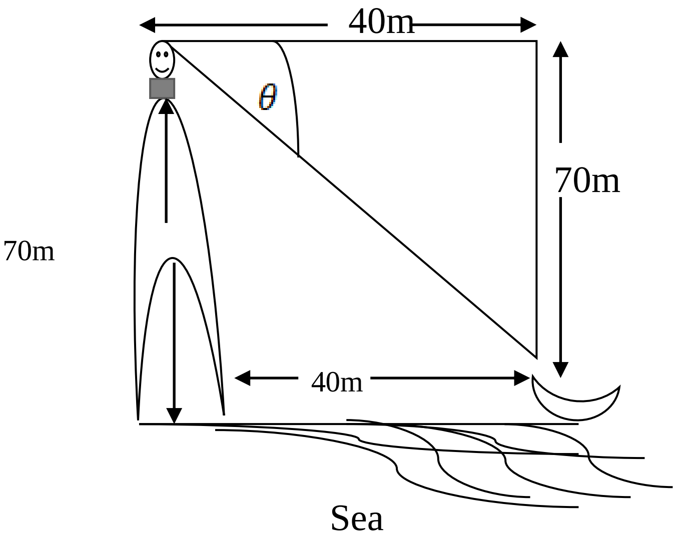
<u>The angle of depression:</u>



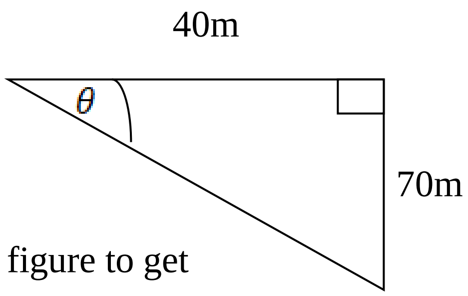
Consider the above figure which shows a man standing on a cliff, and a ship which is on the sea. Before this man can look at the ship from the top of the cliff, he must turn his eyes through an angle θ. This angle, θ is called the angle of depression of the ship from the man.

Q1. A man stood on a cliff which is 70m high. The distance between the foot of the cliff, and a canoe which was on the sea was 40m. Find the angle of depression of the canoe fron the man. Neglect the man's height and the canoe's height.

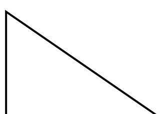
Soln.

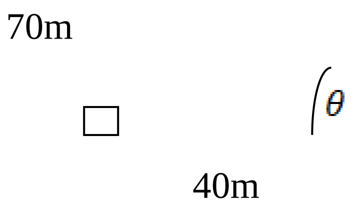


Let $\theta =$ the angle of depression of the canoe from the man. The figure can be represented as shown next:



Rotate this figure to get





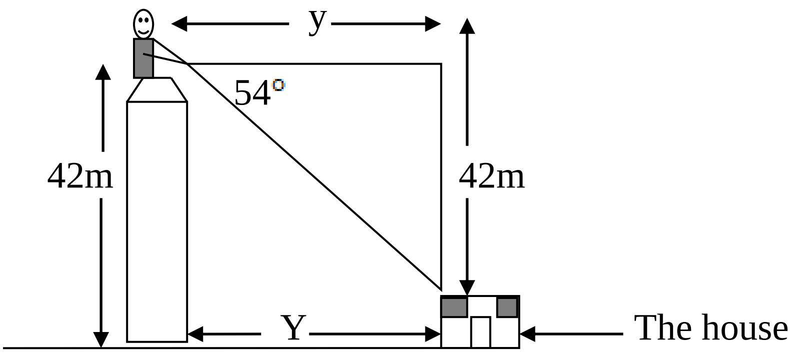
$\tan\theta = \frac{70}{40} = \frac{7}{4} = 1.75.$

From $\tan\theta = 1.75 \Longrightarrow \theta = tan^{-1}1.75 \Longrightarrow \theta = 60°.$

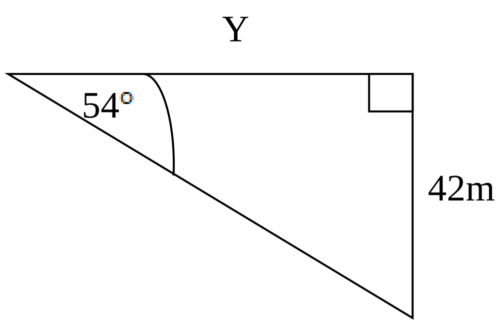
$\therefore Angle\ of\ depression = 60°$

Q2. A man observed from the top of a tower, which is 42m high, that the angle of depression of the top of his house was 54°. Find the distance between his house and the foot of the tower: Assume that the height of the man is negligible.

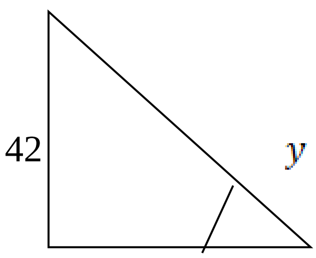
Soln.



Let y = the distance between the foot of the tower and the man`s house.

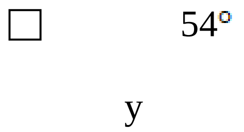


Rotate this figure to get



$\tan 54° = \frac{42}{y} \Rightarrow y\ tan54° = 42$

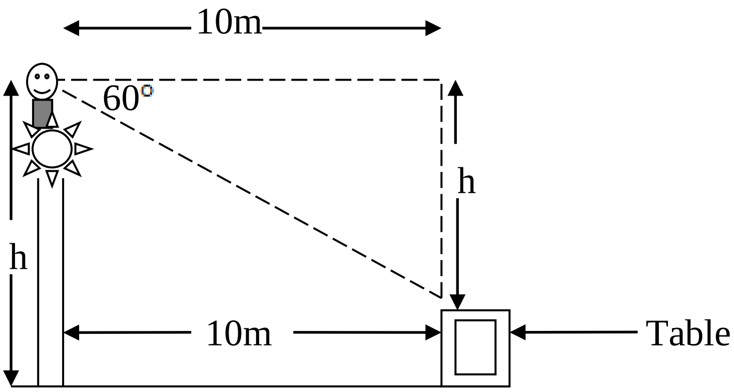
$y = \frac{42}{\tan 54} \Longrightarrow y = \frac{42}{1.37} \Longrightarrow y = 31m$



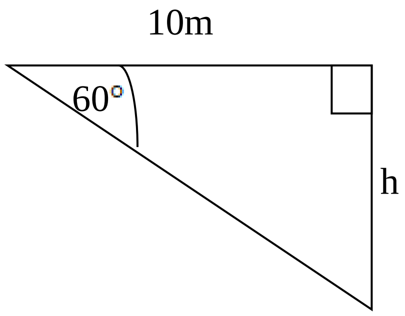
∴ Distance between the boys house and the foot of the hill is 31m.

Q3. A table is placed 10m away from the bottom of a tree. From the top of this tree, Kojo observes the angle of depression of the top of the table as 60°. Calculate the height of the tree, if the heights of Kojo and the table are insignificant.

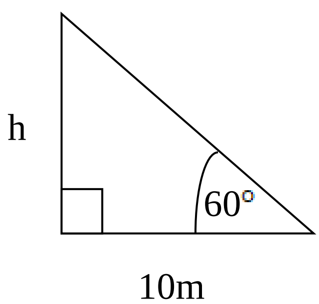
Soln.



Let h = height of the tree.



Rotate this figure to get

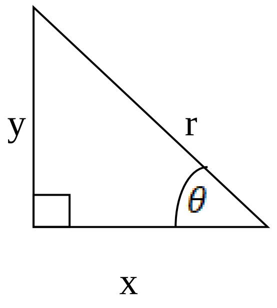


$\tan 60° = \frac{h}{10} \Longrightarrow 10\tan 60 = h\,, \Longrightarrow h = 10\tan 60.$

$\therefore h = 10 \times 1.7 \Longrightarrow h = 17m, \Longrightarrow$the height of the tree

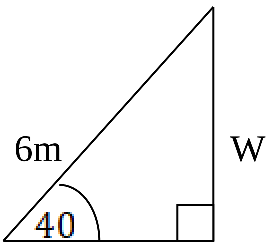
= 17m

The use of Sine :



Sin $\theta = \frac{y}{r}$

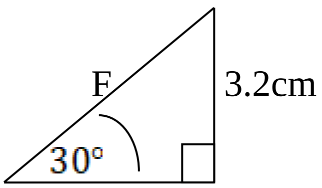
Q1. Calculate the length marked W in the figure given:



Soln.

Sin $40° = \frac{W}{6} \Longrightarrow 6 \times \sin 40 = W, \Rightarrow W = 6 \times \sin 40 \Longrightarrow W = 6 \times 0.642 = 3.84m.$

Q2.



Calculate F.

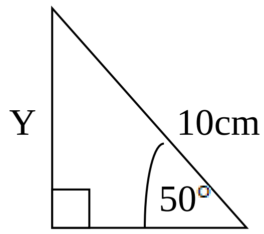
Soln.

Since sin $30° = \frac{3.2}{F}, \Rightarrow F \times \sin 30° = 3.2.$

Divide through by sin $30° \Longrightarrow \frac{F \sin 30°}{\sin 30°} = \frac{3.2}{\sin 30°}$,

$\Rightarrow F = \frac{3.2}{\sin 30^\circ} = \frac{3.2}{0.5} = 6.4cm.$

Q3. Calculate the value of Y.

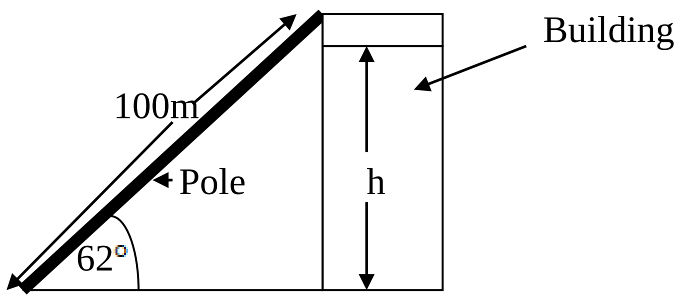


Soln.

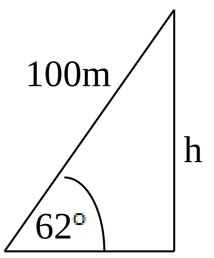
Sin 50° $= \frac{Y}{10} \Rightarrow y = 10 \times \sin 50^\circ, \Rightarrow y = 10 \times 0.766 = 7.7,$

$=> y = 7.7cm$

Q4. A pole leans against a building. The pole is 100m long and makes an angle of 62° with the ground. If the top part of the pole reaches the top part of the building, find the height of the building.



Let h = the height of the building. The above diagram can be represented as shown next:



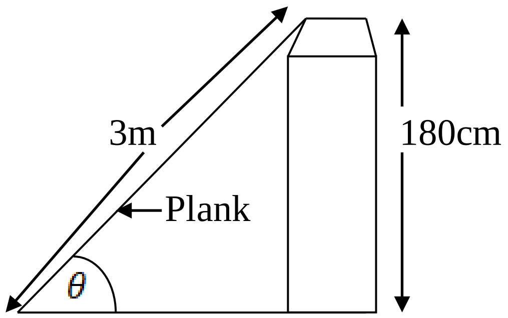
Since sin 62° $= \frac{h}{100} \Rightarrow 100 \times \sin 62^\circ = h.$

$\therefore h = 100 \times \sin 62^\circ = 100 \times 0.882,$

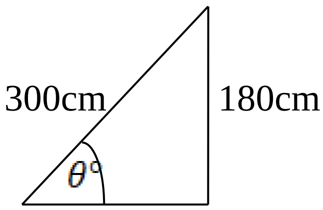
$\Rightarrow h = 88.2m \therefore The\ height\ of\ the\ building\ is\ 88.2m.$

Q5. A plank is 3m long and rests with one of its ends on a horizontal levelled ground.The other end leans against the top of a building which is 180cm high. Calculate the angle between the ground and the plank.

Soln.



Let θ = the angle between the plank and the ground.



Change the 3m into cm $\Longrightarrow 3m = 300cm$. Sin $\theta = \frac{180}{300}$

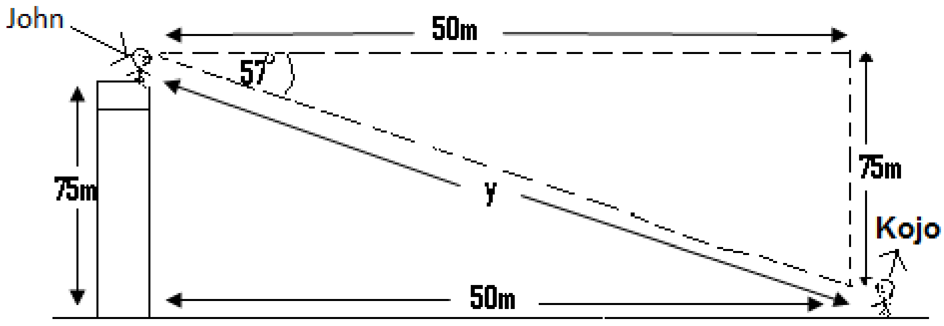
$\Rightarrow$Sin $\theta = 0.6, \Longrightarrow \theta = sin^{-1}0.6 \Longrightarrow \theta = 37$

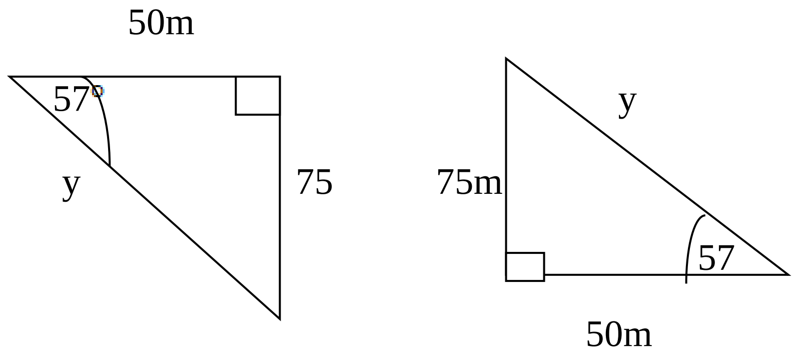
The plank therefore makes an angle of 37° with the ground.

Q6. One day John stood on a storey building which is 75m high. Kojo stood on the ground, at a distance of 50m away from the building. If John observed that the angle of depression of Kojo's head was 57°, find the distance between the two boys.

Soln.

Let y = the distance between the two boys.





Rotate the first figure to get the second one.

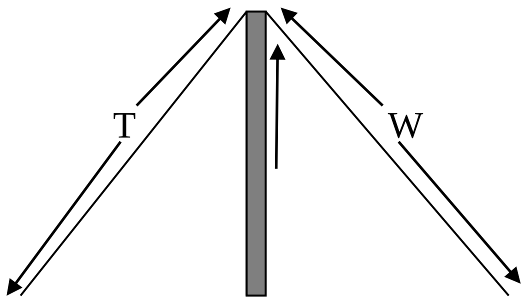
.

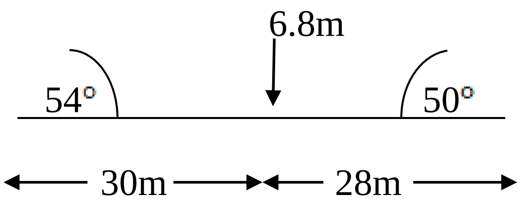
Sin 57° $= \frac{75}{y} \Rightarrow y \times \sin 57° = 75 \therefore y = \frac{75}{\sin 57} = \frac{75}{0.84}$

Y = 89 $\therefore$ distance between the two boys = 89m.

Q7. A pole of length 6.8m long is fixed into the ground.One end of a rope is fixed to the top part of this pole, whilst its other end is fixed to a point on the ground, making an angle of 54° with the ground. One end of a second rope is fixed to the top of the pole, whilst its other end is fixed to a point on the ground, in an opposite direction to the first one. If this second rope makes an angle of 50° with the ground, find the total length of the two ropes. The distance from the foot of the pole to where the first rope is fixed to the ground is 30m, and that between the foot of the pole and where the second rope is fixed to the ground is 28m.

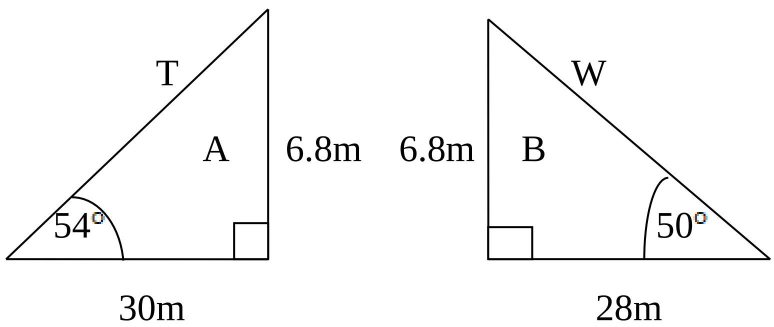
Soln.





Let T = the length of the first rope and let W = the length of the second rope. Then the total length of the ropes =

T + W. This diagram may be broken into two parts i.e



From fig. (A), sin $54^0 = \frac{6.8}{T}$, $\Rightarrow$ T sin 54^0 = 6.8.

Devide through using sin 54^0

$\Rightarrow \frac{T \sin 54^0}{\sin 54^0} = \frac{6.8}{\sin 54^0}$ $\Rightarrow$ **T =** $\frac{6.8}{0.8}$$\Rightarrow$ **T = 8.5.**

From fig. (B), sin 50^0 = $\frac{6.8}{W}$,

$\Rightarrow$ **W sin $50^0 = 6.8$.**

Divide through using sin 50^0 $i.e$ $\frac{W \sin 50^0}{\sin 50^0} = \frac{6.8}{\sin 50^0}$

$\Rightarrow$ **W =** $\frac{6.8}{0.77} = 8m.$

The length of the rope **=T + W = 8.5 + 8 = S16.5m.**

N/B:Sin 50 = 0.77

Q8. A ladder which is 9m long leans against the top part of a wall, of height 7m and makes an angle of 54° with the ground. The distance between the foot of the wall and where the ladder rests on the ground is 5.6m. If the ladder slipped down the wall through a distance of 2m, calculate.

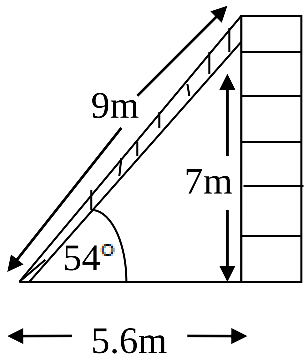
i. the new angle between the ladder and the ground.

ii. the new distance between the foot of the wall and the point where the ladder touches the ground.

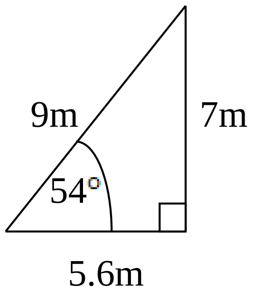
iii. the distance moved on the ground by the ladder.

Soln.

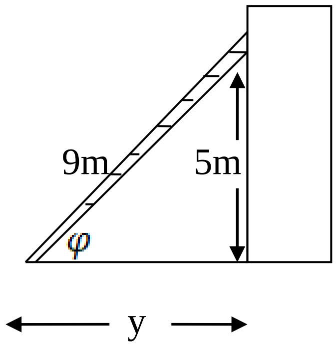
i.

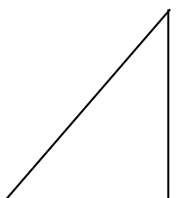


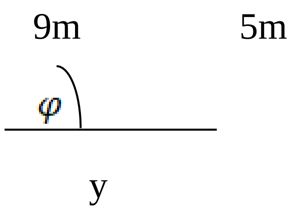
The above can be represented as below.



If the ladder slips 2m down the wall, then it will rest at a distance (7 – 2) = 5m up the wall. Let φ be the new angle made by the ladder with the ground, and also let y = distance between the foot of the wall and where the ladder now rests on the ground. These are shown in the diagram given next:







$\sin\varphi = \frac{5}{9} = 0.5.\therefore \sin\varphi = 0.55, \Longrightarrow \varphi = sin^{-1}0.55,$

$=> \varphi = 34°, \Longrightarrow$ the new angle between ladder and the ground = 34 °.

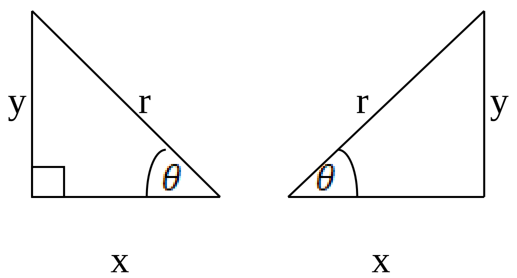
i. From the figure drawn, $\tan\varphi = \frac{5}{y} \Longrightarrow \tan 34° = \frac{5}{y}, \Longrightarrow y \times \tan 34° = 5.$

   Dividing through using tan 34° $\Longrightarrow \frac{y \times \tan 34°}{\tan 34°} = \frac{5}{\tan 34°}, \Longrightarrow y = \frac{5}{\tan 34} = \frac{5}{0.7}, \Longrightarrow y = 7m.$

   The new distance of ladder away from wall = 7m.

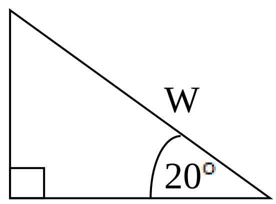
ii. Initial position of ladder away from the wall = 5.6m and final position of ladder away from the wall = 7m..$Therefore\ the$ distance moved by the ladder on the ground = 7 – 5.6m = 1.4m.

The use of cosine :



$\cos\theta = \frac{x}{r}$

Q1.



50m

Find the length marked W.

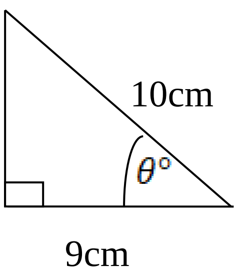
Soln.

Cos 20 ° $= \frac{50}{W} \Longrightarrow W \times cos20° = 50.$

Divide through using cos 20

$\frac{W \cos 20°}{\cos 20°} = \frac{50}{\cos 20°} \Longrightarrow W = \frac{50}{\cos 20} = \frac{50}{0.94} = 53 \Longrightarrow W = 53m.$

Q2.



Calculate the angle marked $\theta°$

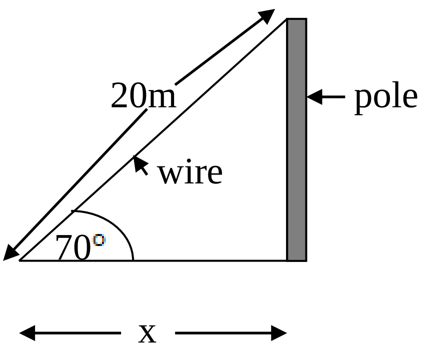
Soln.

Cos $\theta = \frac{9}{10} = 0.9, \Longrightarrow \cos\theta = 0.9 \Longrightarrow \theta = cos^{-1}0.9,$

$\Longrightarrow \theta = 25°.$

Q3. A wire is 20m long and supports a vertical pole. One end of the wire is fixed to the top of the the pole, while the other endis fixed to a point on the ground. If the wire makes an angle of 70 with the ground, calculate the distance between the foot of the pole and the point where it is fixed to the ground.

Soln



let x = the distance between the foot of the pole, and the point where the wire is fixed to the ground.

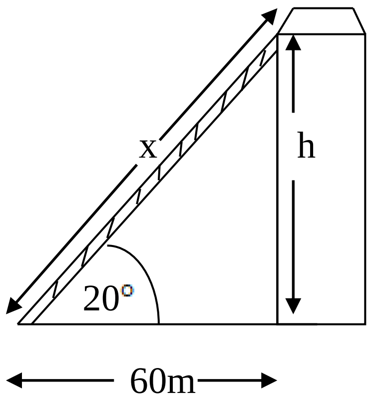
$\cos 70° = \frac{x}{20} \Longrightarrow 20\cos 70° = x,$

$\Rightarrow x = 20\cos 70 \Longrightarrow x = 20 \times 0.34 = 6.8m.$

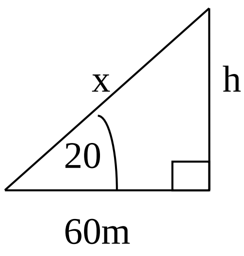
Q4. A ladder leans against a building. The foot of the ladder is 60m away from the bottom of this building. If the angle between the ladder and the ground is 20^0, calculate

i. the length of the ladder.
ii. the height of the building.

Soln.



Let x = the length of the ladder and h = the height of the building.



i. Cos $20^{\circ} = \frac{60}{x} \Longrightarrow x\,cos20 = 60.$

Divide through using cos 20 $\Longrightarrow \frac{x\cos 20^{\circ}}{\cos 20^{\circ}} = \frac{60}{\cos 20^{\circ}}$

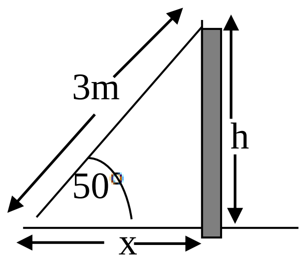
$=> x = \frac{60}{0.9} \Longrightarrow x = 67m.$

ii. tan $20^{\circ} = \frac{h}{60} \Longrightarrow 60\tan 20 = h, \Longrightarrow$ height of building = 24m.

Q5. A flag pole is supported by means of a rope which is 3m long and makes an angle of 50^{0} with the ground. The rope has its one end fixed to the top of the pole and its other end fixed to a point on the ground. Calculate

i. the distance between the bottom of the pole and the point at which the rope is fixed to the ground.
ii. the height of the pole.

Soln.



Let h = the height of the pole and x = the distance between the point on the ground, where the rope is fixed and the bottom of the pole.

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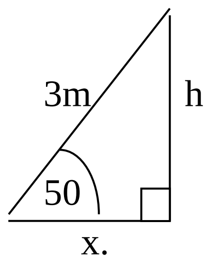


i.Cos $50^{\circ} = \frac{x}{3} \Longrightarrow 3 \times \cos 50^{\circ} = x$

$\therefore x = 3\cos 50 = 3 \times 0.6 \Rightarrow x = 1.8m$

N/B: tan $\theta = \frac{h}{x}$

ii.Tan $\theta = \frac{h}{1.8} \Longrightarrow \tan 50^{\circ} = \frac{h}{1.8}$

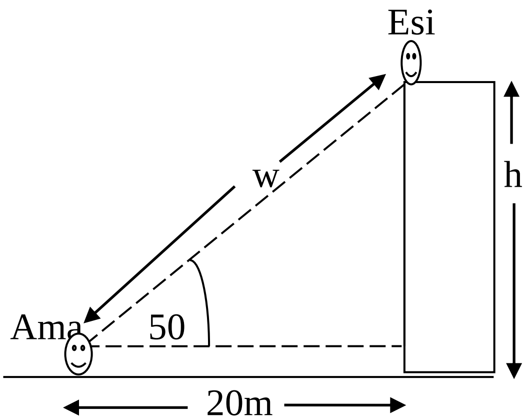
$\Longrightarrow 1.8 \tan 50 = h, \Rightarrow h = 1.8 \times 1.2,$

$\Rightarrow h = 2.16m \Longrightarrow$ height of pole = 2.16m.

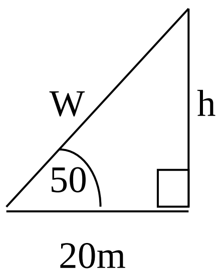
Q6. Ama stood 20m away from a building, as Esi stood on top of the building and the angle of elevation of Esi from Ama was 50^0.

i. Find their distance apart if their heights are negligible
ii. Calculate the height of the building.

Soln.



Let W = their distance apart and h = height of the building.



i. Cos $50° = \frac{20}{W} \Longrightarrow W \cos 50° = 20°.$

$$\frac{W \cos 50}{\cos 50} = \frac{20}{\cos 50} \Longrightarrow W = \frac{20}{0.64} = 31m,$$

$\therefore$ *distance between the two girls* $= 31m.$

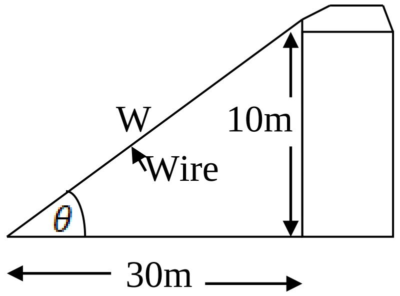
ii. $\tan 50° = \frac{h}{20} \Longrightarrow 20 \tan 50° = h, \therefore h = 20 \tan 50 = 20 \times 1.2 => h = 24.$

The building is 24m high. $\Longrightarrow W = 47m$

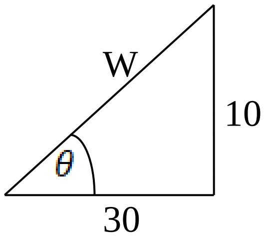
Q7. One end of a wire is fixed to a point 10m up a building. The other end is fixed to the ground at a point 30m away from the building. Calculate

i. the angle the wire makes with the ground
ii. the length of the wire.

Soln.



Let θ = angle the wire makes with the ground and W = length of the wire.



i. $\tan\theta = \frac{10}{30} = 0.3 \Longrightarrow \theta = tan^{-1}0.3,$

⇨ $\theta = 17°.$

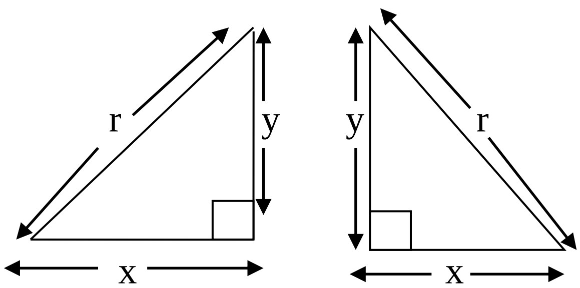
ii. $\cos\theta = \frac{30}{w} \Longrightarrow w\cos\theta = 30.$

But $since\ \theta = 17° \Longrightarrow$W cos 17° $= 30.$

$\frac{W\cos 17}{Cos\ 17} = \frac{30}{\cos 17} \Longrightarrow W = \frac{30}{0.96}$ =>

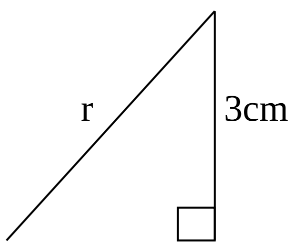
W = 31m.

<u>The Pythagoras theorem:</u>



Pythagoras Theorem holds for all right angle triangles, such as the ones above. From the theorem, $r^2 = x^2 + y^2$.

Q1. Calculate the length marked r in the figure given.



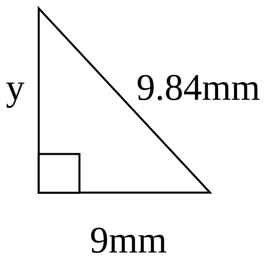
4cm

From Pythagoras theorem ,

$r^2 = 4^2 + 3^2, \Rightarrow r^2 = 16 + 9 \Rightarrow r^2 = 25$

$\Rightarrow r = \sqrt{25} = 5cm.$

Q2. Calculate the length marked y in the diagram given:

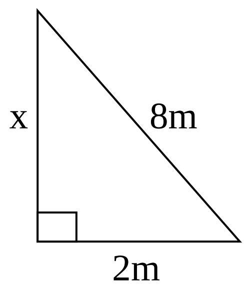


Soln.

$Since\, y^2 + 9^2 = 9.84^2 \Rightarrow y^2 + 81 = 97, \Rightarrow y^2 = 97 - 81$

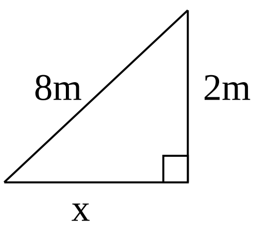
$\Rightarrow y^2 = 16, \Rightarrow y = \sqrt{16} = 4, => y = 4mm.$

Q3. Calculate the length marked x in the given figure:



Soln.

Rotate the right to get the next figure:

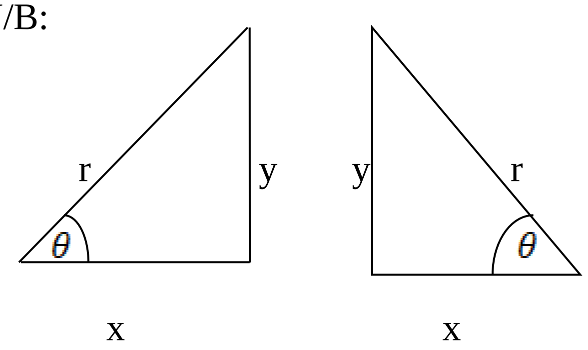


From pythagoras theorem,

$x^2 + 2^2 = 8^2 \Rightarrow x^2 + 4 = 64, \Rightarrow x^2 = 64 - 4 = 60,$

$\Longrightarrow x^2 = 60 \Longrightarrow x = \sqrt{60} = 7.74, \Longrightarrow x = 7.74m$

N/B:



1. $\cos\theta = \frac{x}{r}$ 2. $\sin\theta = \frac{y}{r}$ 3. $\tan\theta = \frac{y}{x}$

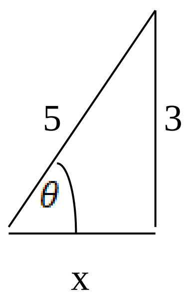
Q1. Given that $\sin\theta = \frac{3}{5}$, find without using tables (or calculator) the values of

i. $\cos\theta$ ii. $\tan\theta$

Soln.

$\sin\theta = \frac{3}{5}$, but

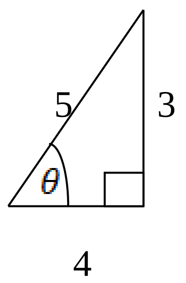
since $\sin\theta = \frac{y}{r} \Longrightarrow y = 3\ and\ r = 5$



From Pythagoras theorem ,

$5^2 = x^2 + 3^2 \Longrightarrow 5^2 - 3^2 = x^2,$

$\therefore 25 - 9 = x^2 \Longrightarrow x^2 = 16, \Longrightarrow x = \sqrt{16} = 4.$

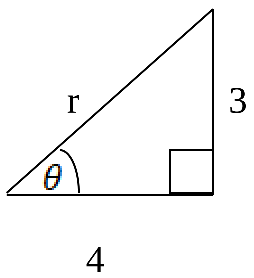


ii. $\cos\theta = \frac{4}{5}$ ii. $\tan\theta = \frac{3}{4}$

Q2. Find without using tables, the values of $\sin\theta$ *and* $\cos\theta$, given that $\tan\theta = \frac{3}{4}$

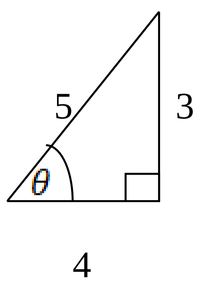
Soln.

Since $\tan\theta = \frac{3}{4}$ *and since* $\tan\theta = \frac{y}{x}$, $\Rightarrow y = 3$ *and* $x = 4$.



$From\ r^2 = 4^2 + 3^2 \Rightarrow r^2 = 16 + 9,$

$=> r^2 = 25 \Rightarrow r = \sqrt{25} = 5.$

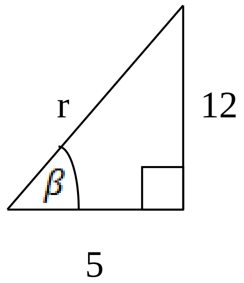


$\cos\theta = \frac{4}{5}$. ii. $\sin\theta = \frac{3}{5}$

Q3. If $\tan\beta = \frac{12}{5}$, find without tables the values of (i) Cos β (ii)Sin β.

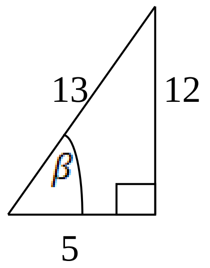
Soln:

If $\tan\beta = \frac{12}{5}$ and $\tan\beta = \frac{y}{x}$, then y = 12 and x = 5.



From Pythagoras theorem ,

$r^2 = 5^2 + 12^2 \Rightarrow r^2 = 25 + 144 = 169, \Rightarrow r^2 = 169 \Rightarrow r = \sqrt{169} = 13.$



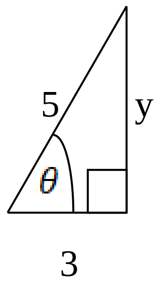
i. $cos\,\beta = \frac{5}{13}$ ii. $sin\,\beta = \frac{12}{13}$

Q4. Given that $\cos\theta = 0.6$, evaluate without tables the values of i. $\sin\theta$ ii. $\tan\theta$

$soln.$

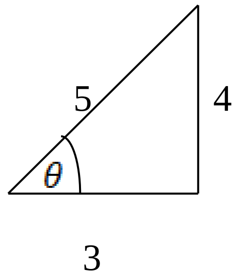
$\cos\theta = 0.6 \Rightarrow \cos\theta = \frac{6}{10} \Rightarrow \cos\theta = \frac{3}{5}.$

Since $\cos\theta = \frac{3}{5}$ $and \cos\theta = \frac{x}{r} \Rightarrow x = 3$ and $r = 5.$



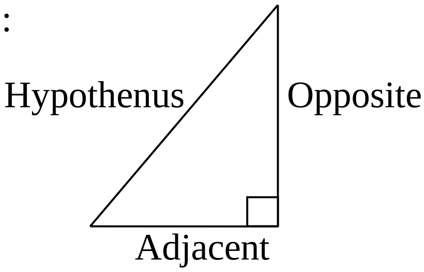
$From\ 5^2 = 3^2 + y^2 \Rightarrow 5^2 - 3^2 = y^2, \therefore 25 - 9 = y^2$

$\Rightarrow 16 = y^2, \Rightarrow y^2 = 16 \Rightarrow y = \sqrt{16} = 4, \Rightarrow y = 4$



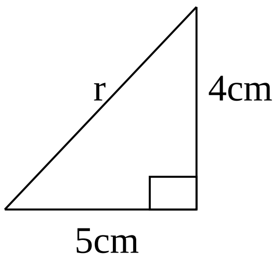
i. $\sin\theta = \frac{4}{5}$ ii. $\tan\theta = \frac{4}{3}$

N/B:



Q5. In a right angle triangle, the length of the opposite is 4cm and that of the adjacent is 5cm. Find the length of the hypothenus

Soln.



If r = length of the hypothenus, then

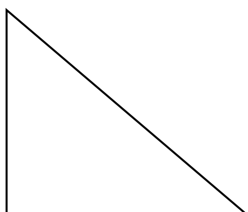
$r^2 = 5^2 + 4^2, \Rightarrow r^2 = 25 + 16 = 41, \Rightarrow r^2 = 41 \Rightarrow r = \sqrt{41} = 6.4.$

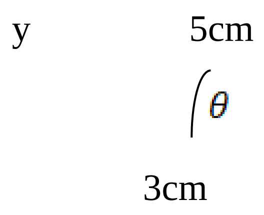
∴ length of the hypothenus = 6.4cm approx.

Q6. In a right angle triangle, the length of the hypothenus is 5cm and that of the adjacent is 3cm. Calculate.

i. the angle between the hypothenus and the adjacent.
ii. the length of the opposite side.

Soln.





Let θ = the angle between the hypothenus and the adjacent. Let y = the length of the opposite.

i. $cos\ \theta = \frac{3}{5}$, =>$cos\ \theta = 0.6, \Rightarrow \theta = cos^{-1}0.6 \Rightarrow \theta = 53°$.

ii. From pythagoras theorem, $5^2 = y^2 + 3^2$,

$\Rightarrow 25 = y^2 + 9 \Rightarrow 25 - 9 = y^2, \therefore 16 = y^2$

$\Rightarrow y^2 = 16, \Rightarrow y = \sqrt{16} = 4cm,$

$\Rightarrow$ length of the opposite = 4cm

Q7. Given that $sin\ \theta = \frac{3}{5}$, find the values of

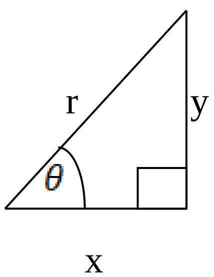
a. $cos\ \theta$ b. $2\ cos\ \theta + 1$

c. $tan\ \theta$ d. $3\ tan\ \theta - \cos\theta$

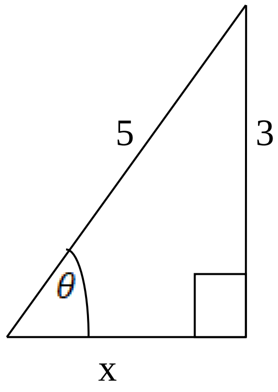
e. $\frac{1+\tan\theta}{2\sin\theta}$

NB: You are not supposed to use tables or calculator.

Soln.



Since $\sin\theta = \frac{3}{5} \Rightarrow r = 5\ and\ y = 3.$

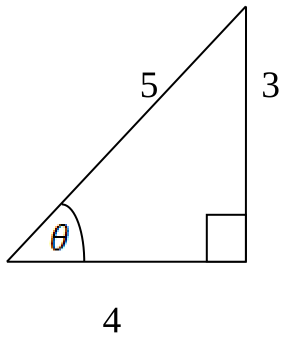


From Pythagoras theorem, $5^2 = x^2 + 3^2$,

$\Longrightarrow 25 = x^2 + 9 \Longrightarrow 25 - 9 = x^2, \Longrightarrow 16 = x^2$

$\Longrightarrow x = \sqrt{16} = 4.$

The diagram just drawn then becomes as shown next:



a. $\cos\theta = \frac{4}{5} = 0.8$

b. $2\cos\theta + 1 = 2\left(\frac{4}{5}\right) + 1 = 2 \times \frac{4}{5} + 1 = \frac{8}{5} + 1 = 1.6 + 1 = 2.6$

c. $\tan\theta = \frac{3}{4} = 0.75$

d. $3\tan\theta - \cos\theta = 3(0.75) - \frac{4}{5} = 2.25 - 0.8 = 1.45$

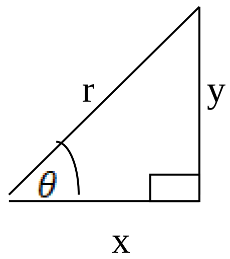
e. $\frac{1+\tan\theta}{2\sin\theta} = \frac{1+\frac{3}{4}}{2\left(\frac{3}{5}\right)} = \frac{1+0.75}{\frac{6}{5}} = \frac{1.75}{1.2} = 1.46$

Q7. Given that $\tan\theta = \frac{3}{4}$, determine without using tables the values of

a. $\sin\theta$ b. $\cos\theta$

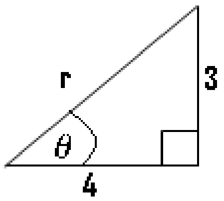
c. $\frac{1-\sin\theta}{\cos\theta}$ d. $\frac{2\cos\theta + 3\sin\theta}{1+\cos\theta}$

Soln.

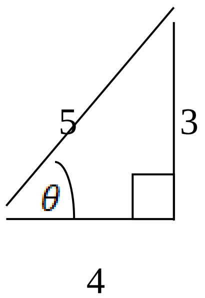


$\tan\theta = \frac{y}{x}$

Since $tan\,\theta = \frac{3}{4}, then$ y = 3 and x = 4.



$From\, r^2 = 4^2 + 3^2 \Rightarrow r^2 = 16 + 9, \Rightarrow r^2 = 25 \Rightarrow r = \sqrt{25} = 5.$



a. $sin\,\theta = \frac{3}{5} = 0.6.$

b. Cos $\theta = \frac{4}{5} = 0.8.$

c. $\frac{1-\sin\theta}{\cos\theta} = \frac{1-0.6}{0.8} = \frac{0.4}{0.8} = 0.5.$

d. $\frac{2\cos\theta+3\sin\theta}{1+\cos\theta} = \frac{2(0.8)+3(0.6)}{1+0.8} = \frac{1.6+1.8}{1.8} = \frac{3.4}{1.8} = 1.9.$

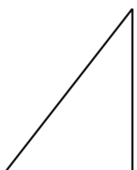
N/B: When asked not to use tables, calculators cannot also be used. Also when asked not to use calculator, then tables i.e the four figure table can also not be used.

Q8. Given that cos $\varphi = {}^{4}/_{5}$, determine without the use of tables or calculator, the values of the following:

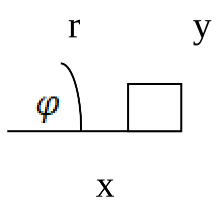
a. tan φ b. sin φ

c. 12 tan φ d. 1+2 tan φ e. $\frac{16\,tan\varphi}{2-15\sin\varphi}$

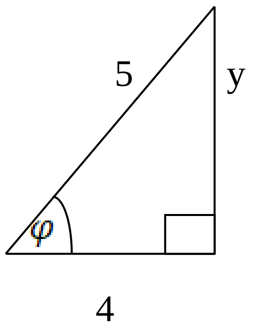
soln.



$$\cos\varphi = \frac{x}{r}$$



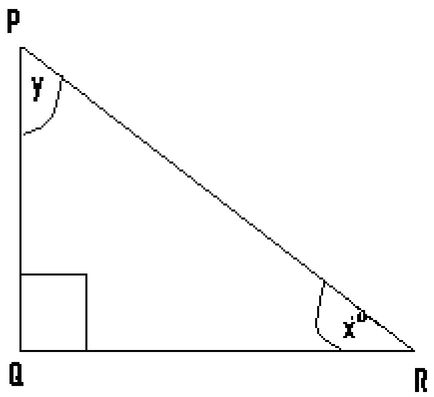
Since $\cos\varphi = \frac{4}{5} \Longrightarrow x = 4\ and\ r = 5.$



From Pythagoras theorem, $5^2 = y^2 + 4^2$,

$$\Longrightarrow 25 = y^2 + 16 \Longrightarrow 25 - 16 = y^2, \Longrightarrow 9 = y^2 \Longrightarrow y = \sqrt{9} = 3.$$

(Q9)



In the triangle PQR, $\cos x^0 = \frac{15}{17}$. Find tan y..

Soln.

Since cosx = $\frac{15}{17} \Rightarrow$ cos x = 0.88,

$\Rightarrow$ x = $cos^{-1}\ 0.88 \Rightarrow x = 42^0$.

Since the sum of angles within a triangle = 180^0, then x + y + 90 = 180^0,

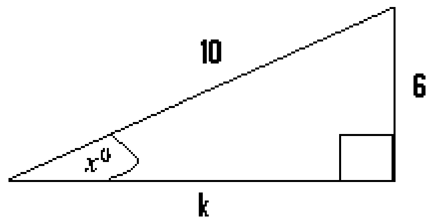
$\Rightarrow 42^0 + \ y + 90 = \ 180^0, \ \Rightarrow$ y = 180 – 90 – 42, $\Rightarrow$ y = 48^0.

Tan y^0 = tan 48^0 = 1.1.

(Q10) Given that sin x = 0.6 and $0^0 \leq x \leq 90^0$, find 1-tanx and leave your answer in the form $\frac{a}{b}$, where a,b $\in$ integers.

Soln.

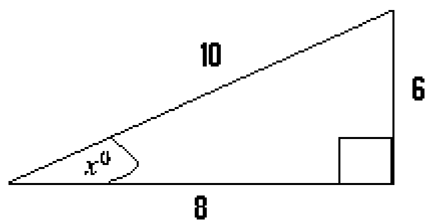
From sin x^0 = 0.6 $\Rightarrow$ sin $x^0 = \frac{6}{10}$.



From Pythagoras theorem, $10^2 = k^2 + 6^2 \Rightarrow 100 = \ k^2 + 36$,

$\Rightarrow 100 - 36 = \ k^2, \Rightarrow 64 = \ k^2$

$\Rightarrow k = \ \sqrt{64} \ = 8.$



From this second figure, tan x = $\frac{6}{8} = 0.75$,

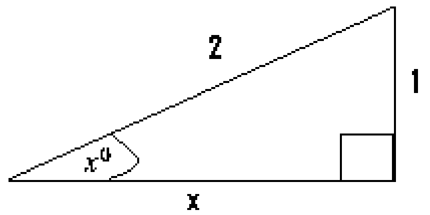
=>1 – tan x = 1 – 0.75,

= 0.25 = $\frac{25}{100} = \frac{1}{4}$ $, which\ is\ of\ the\ form\ \frac{a}{b}\ where\ a = 1\ and\ b = 4.$

(11

) If sin x = $\frac{1}{2}$, $where\ 0^0 \leq x \leq\ 90^0, evaluate\ \frac{\sin x.\cos x}{\cos x+\tan x}$.

Soln.



From Pythagoras theorem . $2^2 = x^2 + 1^2$,

$\Rightarrow 4 = \ x^2 + \ 1 \Rightarrow \ 4 - 1 = \ x^2, \Rightarrow \ x = \sqrt{3} = 1.75.$

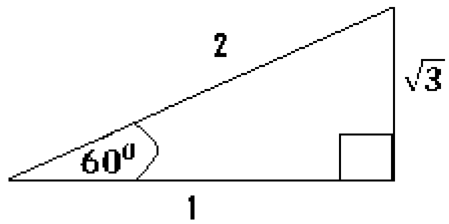
Sin x = $\frac{1}{2} = 0.5, \cos x\ = \frac{1.75}{2} = 0.9\ and$ tan x = $\frac{1}{1.75} = 0.6$

Now $\frac{sinx.cosx}{cosx+\tan x}$=$\frac{(0.5)(0.9)}{0.9+0.6} = \ \frac{0.45}{0.15} = 3.$

The special angles:

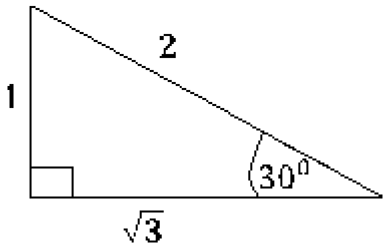
These angles are $30^0, 45^0\ and\ 60^0$. Their values can be had by using three different types of triangles.

(a)



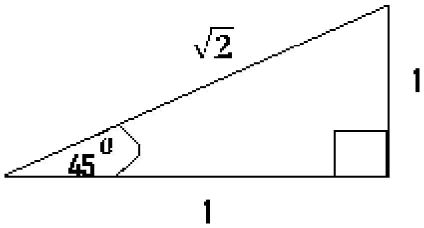
i) cos $60^0 = \frac{1}{2}$ii) sin 60^0 =$\frac{\sqrt{3}}{2}$

(b)



i) Cos $30^0 = \frac{\sqrt{3}}{2}$ ii) sin $30^0 = \frac{1}{2}$ $iii)$ tan $30^0 = \frac{1}{\sqrt{3}}$

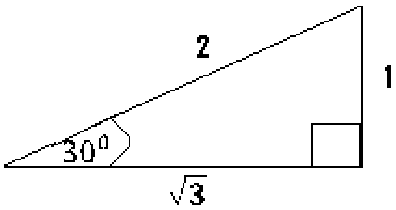
(c)

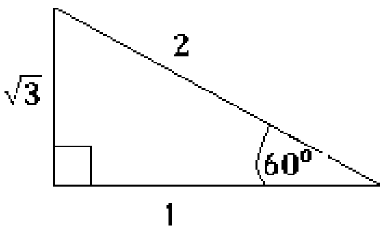


i) Cos $45^0 = \frac{1}{\sqrt{2}}$ ii) sin $45^0 = \frac{1}{\sqrt{2}}$ iii) tan $45^0 = \frac{1}{1} = 1$

(11) Without using tables or calculator, simplify $\frac{\sin 45^0 + \tan 30^0}{\tan 45 - \cos 60^0}$

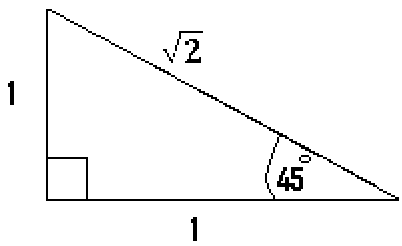
Soln.





tan $30^0 = \frac{1}{\sqrt{3}}$

cos $60^0 = \frac{1}{2}$



sin $45^0 = \frac{1}{\sqrt{2}}$ and cos $45^0 = \frac{1}{\sqrt{2}}$.

(i) $2 \tan 30^0 - 4\, cos\, 60^0$

$$= 2\left(\frac{1}{\sqrt{3}}\right) - 4\left(\frac{1}{2}\right)$$

$$= \frac{2}{\sqrt{3}} - \frac{4}{2} = \frac{2}{\sqrt{3}} - 2$$

$$= \frac{2}{\sqrt{3}} - \frac{2}{1} \quad = \quad \frac{2-2\sqrt{3}}{\sqrt{3}}$$

N/B: The L.C.M of $\sqrt{3}$ and 1 = $\sqrt{3} \times 1 = \sqrt{3}$.

(ii)$\frac{3sin45^0 + cos45^0}{2\tan 30^0}$

$$= \frac{3\left(\frac{1}{\sqrt{2}}\right) + \frac{1}{\sqrt{2}}}{2\left(\frac{1}{\sqrt{3}}\right)}$$

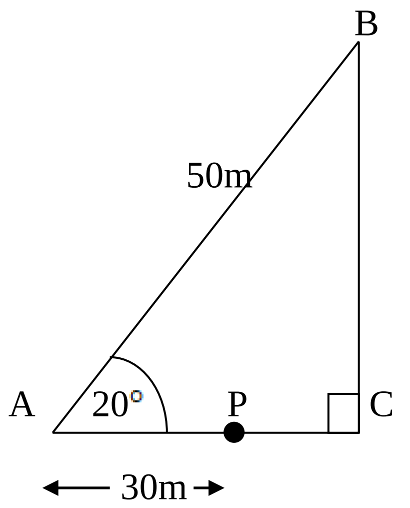
$$= \frac{\frac{3}{\sqrt{2}} + \frac{1}{\sqrt{2}}}{\frac{2}{\sqrt{3}}}$$, simplify the numerator

$$\Rightarrow \frac{3}{\sqrt{2}} + \frac{1}{\sqrt{2}}\frac{3\ +\ 1}{\sqrt{2}} = \frac{4}{\sqrt{2}} => \frac{\frac{3}{\sqrt{2}} + \frac{1}{\sqrt{2}}}{\frac{2}{\sqrt{3}}} = \frac{\frac{4}{\sqrt{2}}}{\frac{2}{\sqrt{3}}}$$

$$= \frac{4 \times \sqrt{3}}{2 \times \sqrt{2}} = \frac{4\sqrt{3}}{2\sqrt{2}}.$$

<u>General application of trigonometry:</u>

Q1.

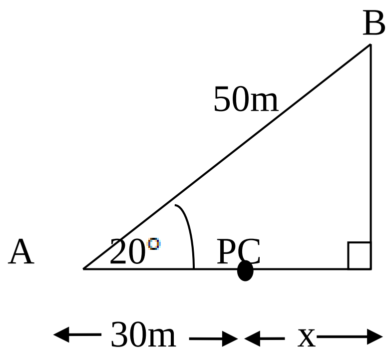


In the figure given AB = 50m and AP = 30m

(a) Calculate the length Pc.

(b) Find BC

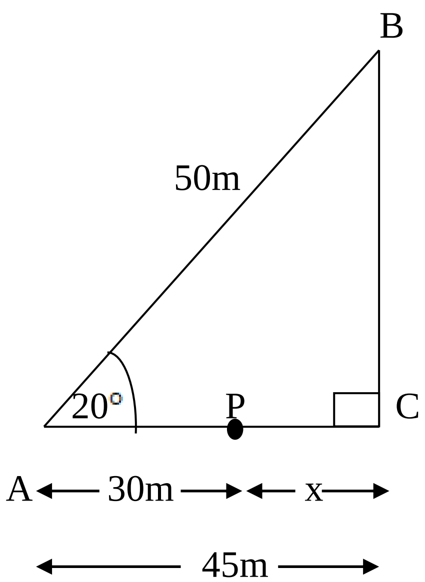
Soln.



Let PC = x. We must first find AC.

AC = 50cos 20°

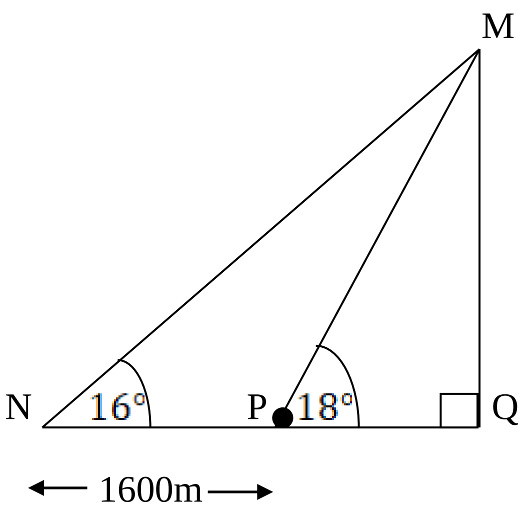
AC = 50 x 0.9 = 45m.



$PC \; = \; x = 45 - 30 \Longrightarrow PC = 15m$

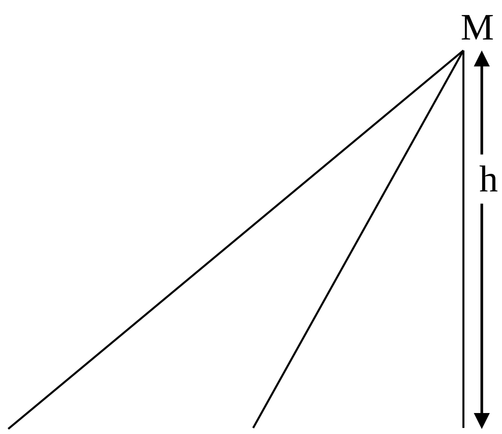
b. $\tan 20° = \frac{BC}{AC}$, $\Rightarrow \tan 20 = \frac{BC}{45} \Longrightarrow 45\tan 20° = BC, \Longrightarrow 45 \times 0.36 = BC, \Longrightarrow 16.20m = BC$

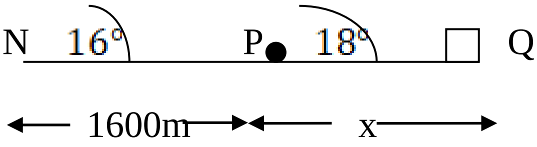
Q2.



A surveyor at sea level observes the angle of elevation, M, of the top of a mountain, from two points N and P west of the mountain, and had them to be $16°$ and $18°$ respectively as shown in the diagram. If NP = 1600m and the base of the mountain Q is vertically below M, calculate the height of the mountain.

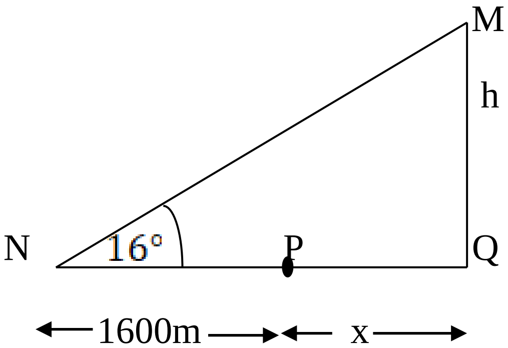
Soln.





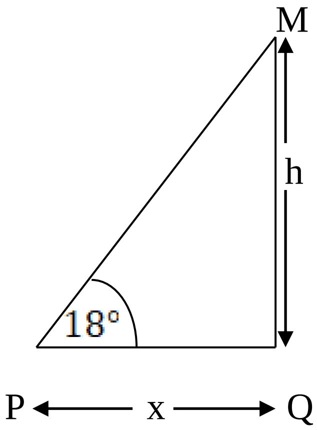
Let PQ = x and h = the height of the mountain

N/B: Two separate diagrams can be obtained from the figure drawn. With respect to the angle of 16°, we shall obtain this diagram.



$\tan 16° = \frac{h}{1600 + x} \Longrightarrow 0.3 = \frac{h}{1600 + x}$

$\therefore h = 0.3(1600 + x), \Longrightarrow h = 480 + 0.3x \ldots \ldots eqn\ (1)$



With respect to angle 18°, the above diagram can be had and

$18° = \frac{h}{x} \Longrightarrow 0.32 = \frac{h}{x}, => h = 0.32x \ldots \ldots \ldots eqn\ (2)$

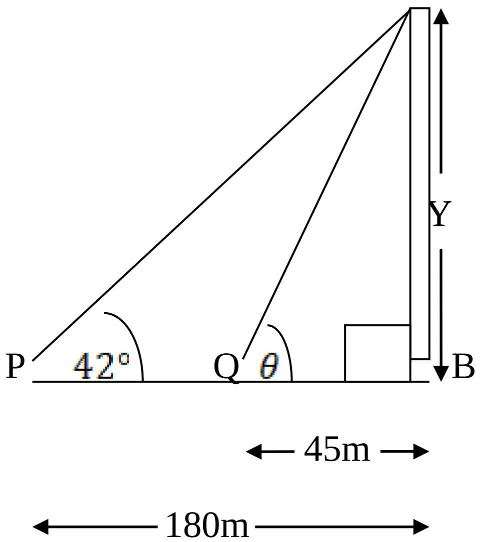
Substitute h = 0.32x into eqn.(1)

$$i.e\ h = 480 + 0.3x \Longrightarrow 0.32x = 480 + 0.3x, \Longrightarrow 0.32x - 0.3x = 480 \Longrightarrow 0.02x = 480, \Longrightarrow x = \frac{480}{0.02} = \frac{48000}{2}, \Longrightarrow x = 24000.$$

From eqn.(2) h = 0.32$x \Longrightarrow h = 0.32(24000) = 7680m.$

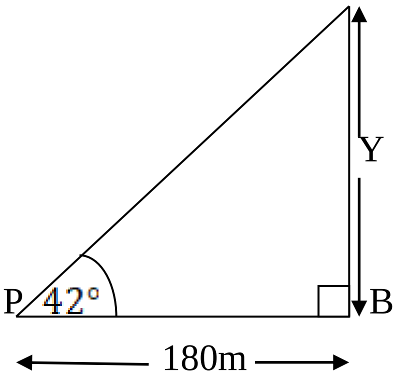
Q3. The angle of elevation of the top of a flag pole is 42° from a point P which is 180m from the foot (B). A point Q is on the same horizontal line BP such that BQ = 45m. Calculate correct to one decimal place, the angle of elevation of the top of the flag pole from Q.

Soln.



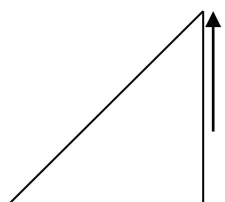
Let θ = the angle of elevation of the top of the pole from Q. Separate the diagram above into two parts.

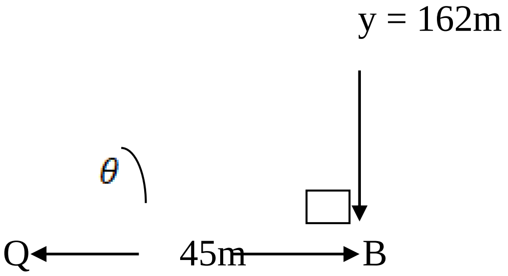
1.



Let y = the height of the flagpole. Then tan $42^\circ = \frac{y}{180} \Rightarrow 180^\circ \tan 42^\circ = y, \Rightarrow 180 \times 0.90 = y, \Rightarrow y = 162, \Rightarrow$ the height of the flag pole = 162m.

2.)



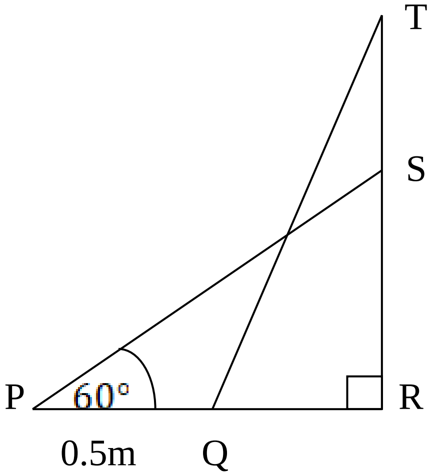


Tan $\theta = \frac{y}{45} \Rightarrow \tan\theta = \frac{162}{45}$,

$\Rightarrow \tan\theta = 3.6$, $\therefore \theta = tan^{-1}\, 3.6 = 74^\circ$

$\therefore$ The angle of elevation of the top of the tower from Q = 74°

Q4.



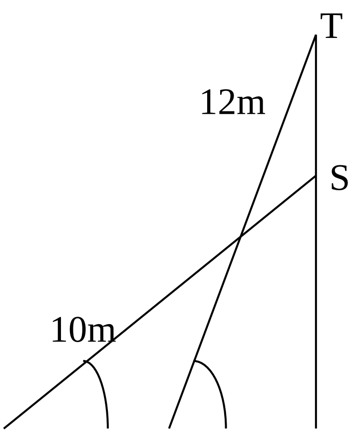
In the diagram, PS and QT are two ladders 10m and 12m long respectively, placed against a vertical wall TR. PS makes an angle of 60° with the horizontal and PQ = 0.5m.

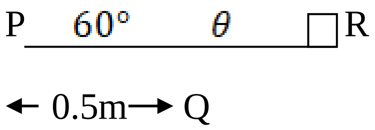
Calculate correct to two significant figures

i. the angle which QT makes with the horizontal.

ii. the height of point T above the horizontal .

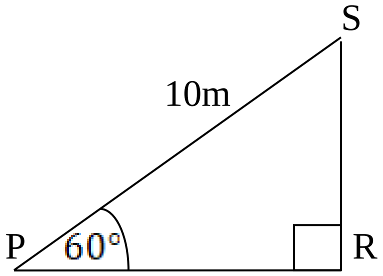
Soln.





Let θ = the angle QT makes with the horizontal. Separate the above diagram into two portions.

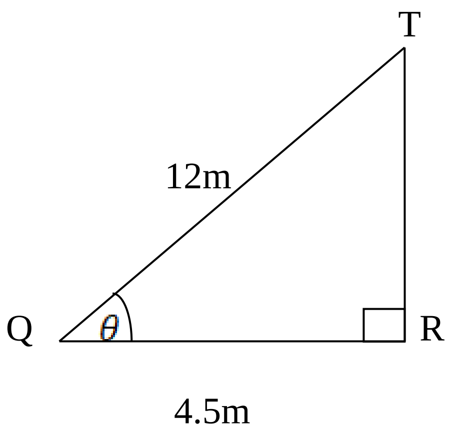
1.



PR = 10 cos $60^\circ = 10 \times 0.5 = 5m. Since\ PR = 5m, \Longrightarrow PQ + QR = 5, \Longrightarrow QR = 5 - PQ \Longrightarrow QR = 5 - 0.5 = 4.5m$

2.

i.



$\cos\theta = \frac{4.5}{12} \Longrightarrow \cos\theta = 0.38, \Rightarrow \theta = \cos^{-1}0.38 = 68^0, \Longrightarrow QT$

makes an angle of 68° with the horizontal.

ii. The height of point T above the horizontal = the length TR. Tan $\theta = \frac{TR}{4.5}$, but $\theta = 68^\circ \Longrightarrow \tan 68^\circ = \frac{TR}{4.5}, \Longrightarrow TR = 4.5 \times \tan 68 = 4.5 \times 2.5 = 11.3m, \Longrightarrow$the height of point T above the horizontal = 11.3m.

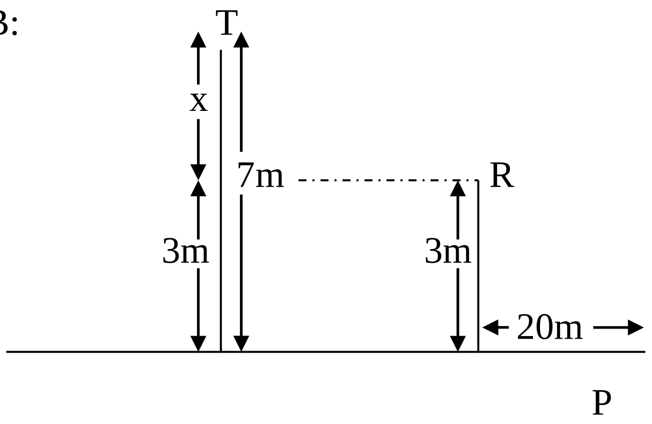
Q5. Two vertical poles, 3m and 7m long are on the same straight line with a point P on the ground. The shorter pole is 20m from P and is between P and the longer pole. The angle of elevation of the top T of the longer pole from the top R of the shorter one is 30. Calculate

i. $|TR|$

ii. the horizontal distance from P to the longer pole .

iii. the angle of elevation of T from P, correct to the nearest degree.

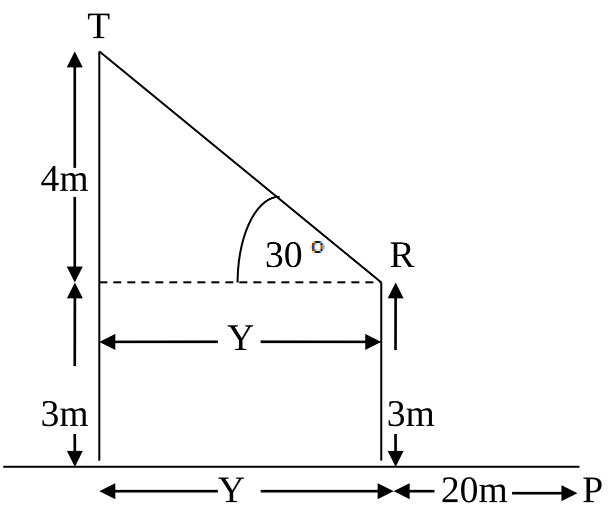
N/B:



The distance x shown in the diagram = 7 – 3 = 4m .

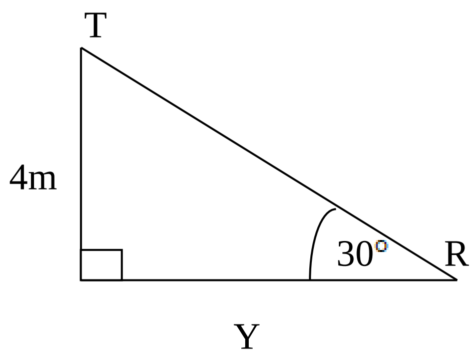
T = the top part of the 7m pole and R = the top part of the 3m pole.

Soln.



The angle marked 30 ° = the angle of elevation of the top T of the longer pole from the top R of the shorter pole.

Also let Y = the distance between the foot of the shorter pole, and that of the longer pole. From this diagram, we can extract the next diagram

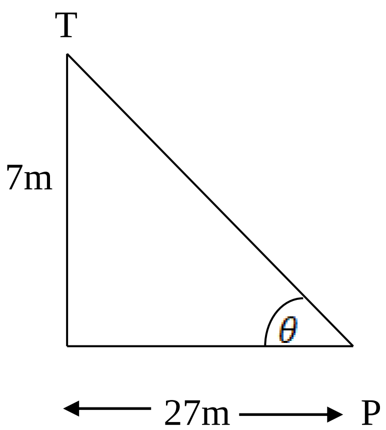


i. $\sin 30° = \frac{4}{TR} \Longrightarrow TR \sin 30° = 4, \Longrightarrow TR = \frac{4}{\sin 30} = \frac{4}{0.5} = 8 \Longrightarrow TR = 8m$

ii. From the diagram, $\cos 30° = \frac{Y}{TR} \Longrightarrow 0.87 = \frac{Y}{8}, \Longrightarrow Y = 0.87 \times 8 = 7m$

The horizontal distance from P to the longer pole = y + 20 = 7 + 20 = 27m.

iii.

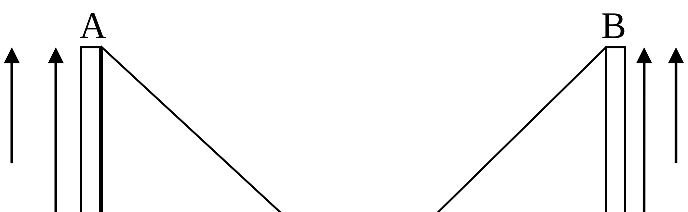


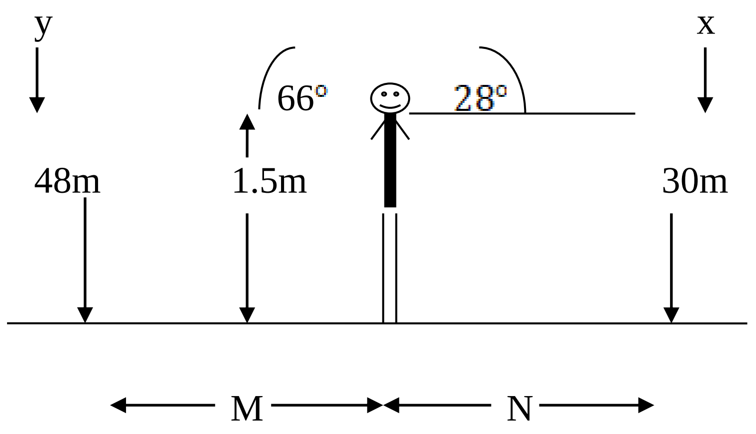
Let θ = the angle of elevation of T, the top of the longer pole from P.

Then $\tan \theta = \frac{7}{27} = 0.26 \Longrightarrow \theta = tan^{-1} 0.26 = 14.6°$

Q6. Two towers A and B are 48m and 30m high respectively. Tower A lies to the west and B to the east of a man 1.5m tall. From the man's eyes level, the angle of elevation of the top of A and B are 66° and 28° respectively. Calculate the distance between A and B.

Soln.

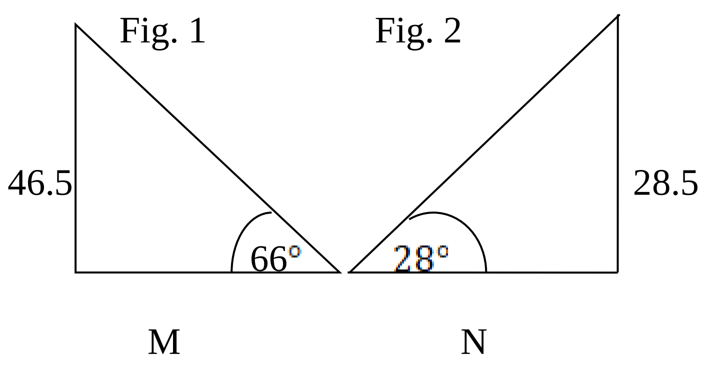




$y = 48 - 1.5 = 46.5m$

$x = 30 - 1.5 = 28.5m$

Let M = distance between the point on the ground where the man is standing and the foot of tower A. Let N = distance between the point on the ground where the man is standing and the foot of tower B



The distance between the two towers = M + N

From fig. (1), tan $66^\circ = \frac{46.5}{M} \Longrightarrow 2.25 = \frac{46.5}{M}$

$$\Longrightarrow 2.25M = 46.5, \therefore M = \frac{46.5}{2.25} = 20.7 \Longrightarrow M = 20.7m.$$

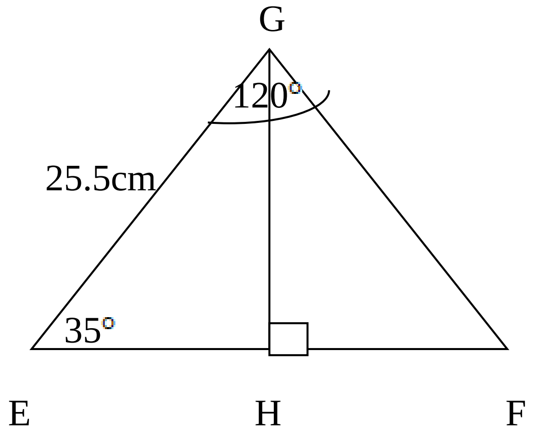
From fig. (2), tan $28^\circ = \frac{28.5}{N} \Longrightarrow 0.53 = \frac{28.5}{N}$,

$\therefore N = \frac{28.5}{0.53} = 54 \Rightarrow N = 54m.$

. Distance between the two towers =

M + N = 20.7 + 54 = 74.7m

Q7.



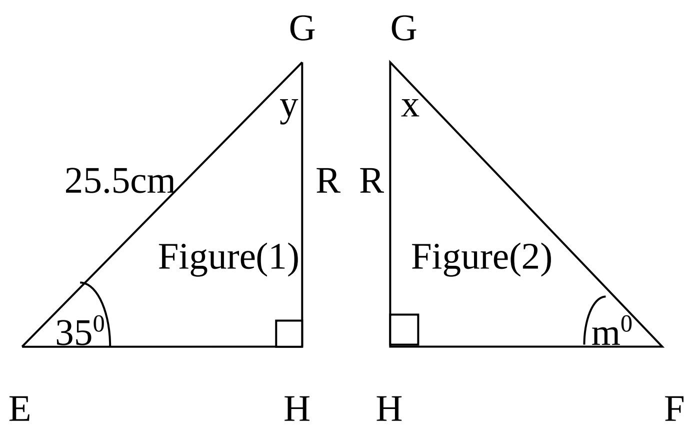
Not drawn to scale. In the figure, angle GEH = 35°, angle EGF = 120°, |EG| = 25.5cm and GH is perpendicular to $\overline{EF}$.

Calculate $|\overline{EF}|$

N/B: Since GH is perpendicular to EF, $\Longrightarrow < GHE = 90°$

Separate the diagram into two parts.

Soln



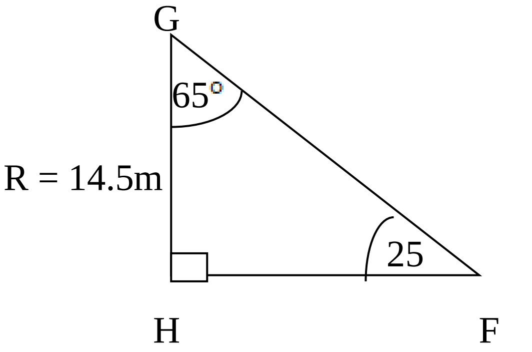
Considering figure(1)

1. Since the sum of angles within a triangle = 180°,
   $\Rightarrow 35 + 90 + y = 180, \Rightarrow 125 + y = 180 \Rightarrow y = 180 - 125 = 55°.$
2. EH = 25.5 cos 35° $= 25.5 \times 0.82 = 21m.$
3. R = 25.5 sin 35° $= 25.5 \times 0.57 = 14.5m$

Considering figure (2)

1. Since y = 55° $and\ y + x = 120°, \Rightarrow 55 + x = 120 \Rightarrow x = 120 - 55 = 65°.$

2. Since the sum of angles within a triangle = $180°, \Rightarrow 65° + 90 + m° = 180°, \Rightarrow 155 + m = 180 \Rightarrow m = 180 - 155 = 25°$



Tan 25° $= \frac{14.5}{HF} \Rightarrow HF \tan 25 = 14.5, \Rightarrow HF = \frac{14.5}{\tan 25} = \frac{14.5}{0.47} = 31m$

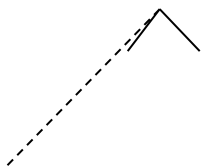
EF = EH + HF = 21 + 31 = 52m.

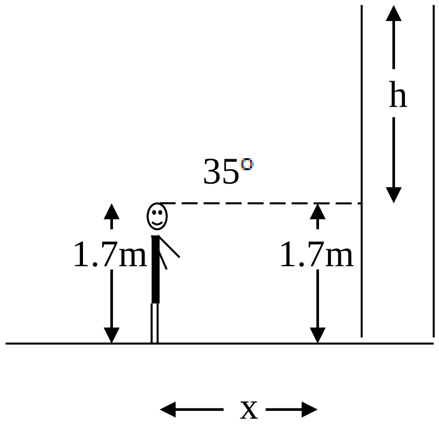
Q8. A man 1.7m tall observes the angle of elevation of the top of a tower to be 35°. He moves 50m away from the tower and observes the angle of elevation to be 28°. How far above the ground is the tip of the tower.

Soln.

First case:

In the first case the man observed the angle of elevation of the top of the tower to be 35°. Let x = his distance away from the tower.

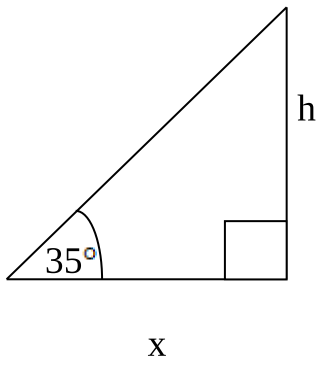




The height of the top of the tower from the ground

= 1.7 + h.

From this diagram, the next diagram is had.

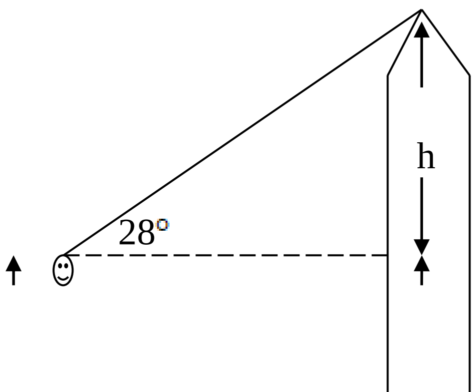


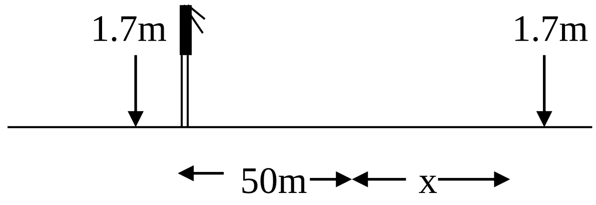
tan 35° $= \frac{h}{x} \Longrightarrow h = x \tan 35,$

$\Longrightarrow h = x \times 0.7 = 0.7x \ldots\ldots..eqn(1)$

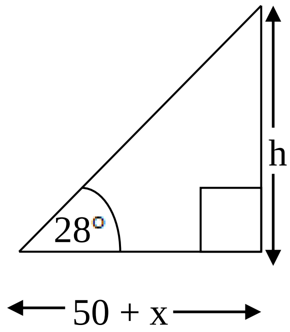
<u>Second case:</u>

In the second case, when he moved 50m away from the tower, the angle of elevation becomes 28°.





From this diagram, we get the next diagram.



$\tan 28° = \frac{h}{50+x} \Rightarrow h = \tan 28\,(50+x)$,

$\Rightarrow h = \tan 28(50+x) \Rightarrow h = 0.53(50+x)$,

$\Rightarrow h = 265 + 0.53x \ldots\ldots eqn\ (2)$.

$Since\ from\ eqn\ (1), h\ =\ 0.7x\ and\ from\ eqn\ (2), h$
$=\ 265\ +\ 0.53x, then\ 0.7x\ =\ 265\ +\ 0.53x$

$\Rightarrow 0.7x - 0.53x = 265, \Rightarrow 0.17x = 265, \Rightarrow x = \frac{265}{0.17} = 1558.$

Since from eqn(1), h = $0.7x \Rightarrow h = 0.7(1558) = 1091m.$

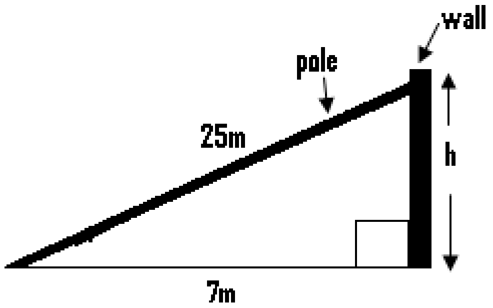
The height of the top of the tower from the ground = 1.7 + h = 1091+1.7 = 1092.7=1093m.

Q9. A pole 25m long is placed against a vertical wall such that its lower end from the foot of the wall is on the same horizontal ground. If the upper end of the pole is pushed down by 2m, calculate correct to 2 significant figures

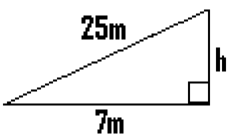
(a) How much further away from the wall the lower end will move.
(b) the angle the pole now makes with the horizontal ground.

Soln.

(a)



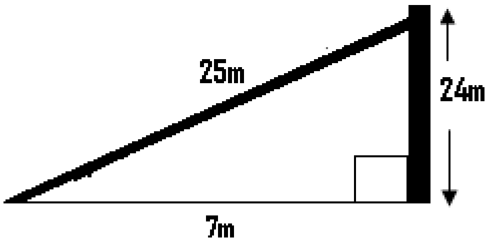
Let h = the height of the point where the pole touches the wall.



From Pythagoras theorem, $25^2 = 7^2 + h^2 \Rightarrow h^2 = 25^2 - 7^2$,

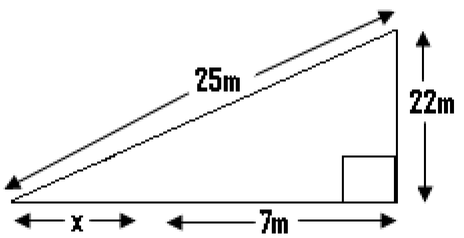
$\Rightarrow h^2 = 625 - 49$,$\Rightarrow h^2 = \sqrt{576}$

$\Rightarrow h = 24m.$

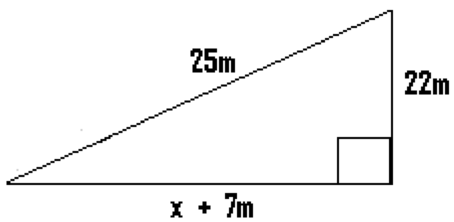


If the upper part of the pole is pushed down by 2m, then the new point at which it now touches the wall above the ground = 24 – 2 = 22m.

This is indicated in the next figure:

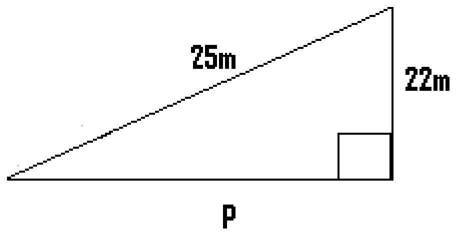


Let x = the further distance moved from the wall, by the lower end of the pole.



Let the length represented by x + 7m = p

i.e



From Pythagoras theorem, $25^2 = p^2 + 22^2$,

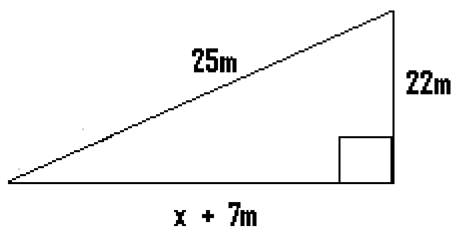
$\Rightarrow 25^2 - 22^2 = p^2, \Rightarrow 625 - 484 = p^2$,

$\Rightarrow 141 = p^2 \Rightarrow\ p = \sqrt{141} = 12m\ approx.$

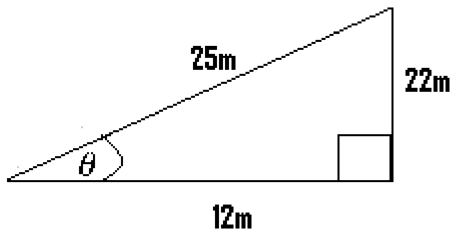
Since x + 7 = p, $\Rightarrow$ x + 7 = 12,

$\Rightarrow$ x = 12 – 7 = 5, $\Rightarrow$ the further distance away from the wall moved by the lower part of the pole = 5m approx.

(b) Consider the figure



Since x = 5m, then this figure becomes as shown next :



Letθ $= the\ angle\ the\ pole\ now\ makes\ with\ the\ horizontal.$

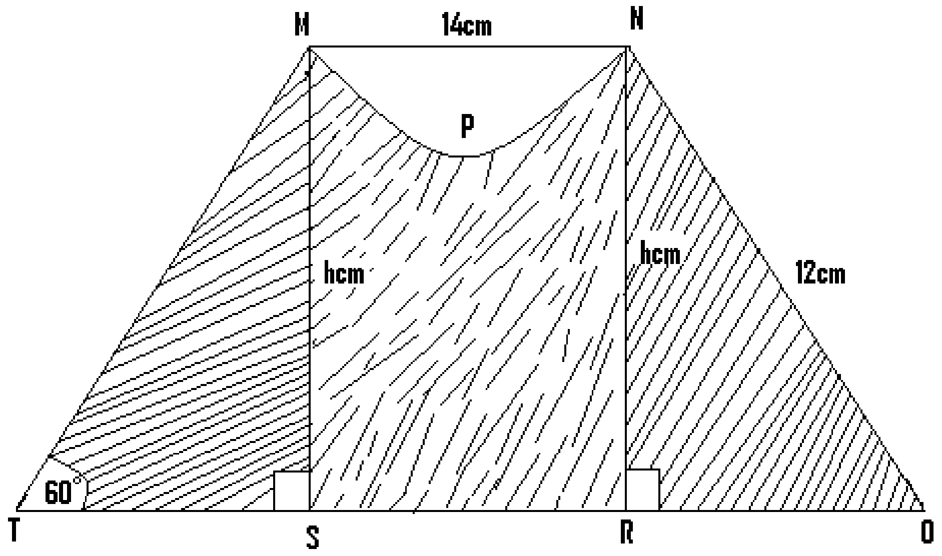
Tan $\theta = \frac{22}{12}$

$\Rightarrow \tan\theta = 1.8,$

$\Rightarrow \theta = tan^{-1}1.8,$

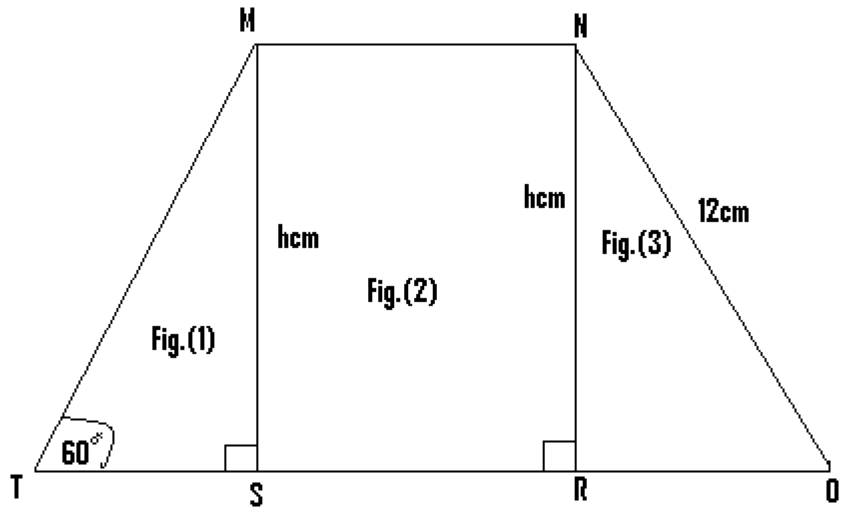
$\Rightarrow \theta = 61^0 approx.$

(Q10)



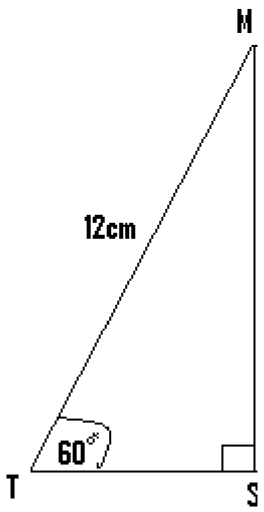
The diagram shows a trapezium MNOT in which, MN =14CM, <MTO=60^0 *and* |MT| = |NO| =12cm. If the semi- circle MPN is removed from the trapezium, calculate correct to the cm^2*, the area of the remaining portion. Take* $\pi = \frac{22}{7}$.

Soln



The area of the remaining portion = area of trapezium – area of the semi-circle = area of fig.(1) + area of fig.(2) + area of fig.(3) – area of the semi-circle.

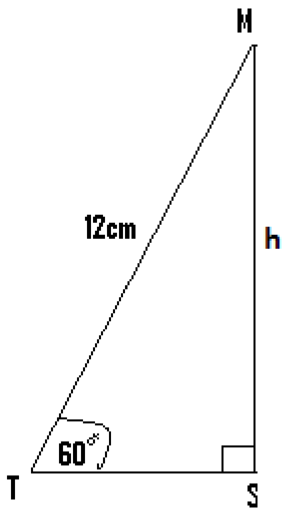
Consider Fig.(1) i.e



TS= 12 COS 60

$= 12\times 0.5 = 6cm.$

The figure then becomes as shown next



From Pythagoras theorem, $12^2 = 6^6 + h^2 \Rightarrow \mathrm{h}^2 = 12^2 - 6^2,$

$\Rightarrow \mathrm{h}^2 = 144 - 36,$

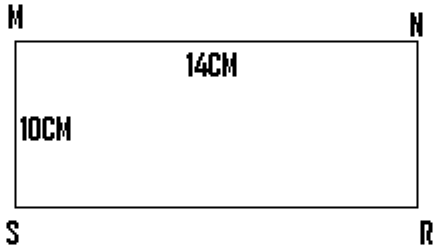
$\Rightarrow \mathrm{h}^2 = \sqrt{108} = 10,$

$\Rightarrow h = 10cm\ approx.$

The area of fig.(1) $= \frac{b\times h}{2}$

$= \frac{6\times 10}{2} = 30cm^2$

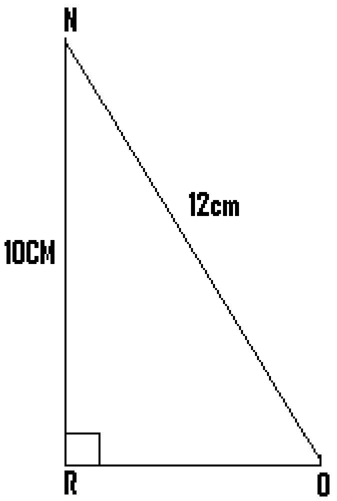
Consider fig.(2) i.e



The area of fig.(2) $= L\times B$

$= 14 \times 10 = 140cm^2$.

Consider fig.(3) i.3



From Pythagoras theorem, $12^2 = 10^2 + RO^2$,

$\Rightarrow RO^2 = 12^2 - 10^2$,

$\Rightarrow RO^2 = 144 - 100$,

$\Rightarrow RO^2 = 44 \Rightarrow RO = \sqrt{44}$

$= 6.6$cm approx.

Area of fig.(2) or the given triangle $= \frac{b \times h}{2} = \frac{6.6 \times 10}{2}$

$= 33cm^2$.

Area of the semi- circle

$= \frac{\pi r^2}{2} = \frac{22}{7} \times \frac{7^2}{2} = 77cm^2$.

Area of the remaining portion = area of fig.(1) + area of fig.(2) + area of fig(3) – area of the semi- circle = 30 + 140 + 33 – 77 = $126cm^2$.

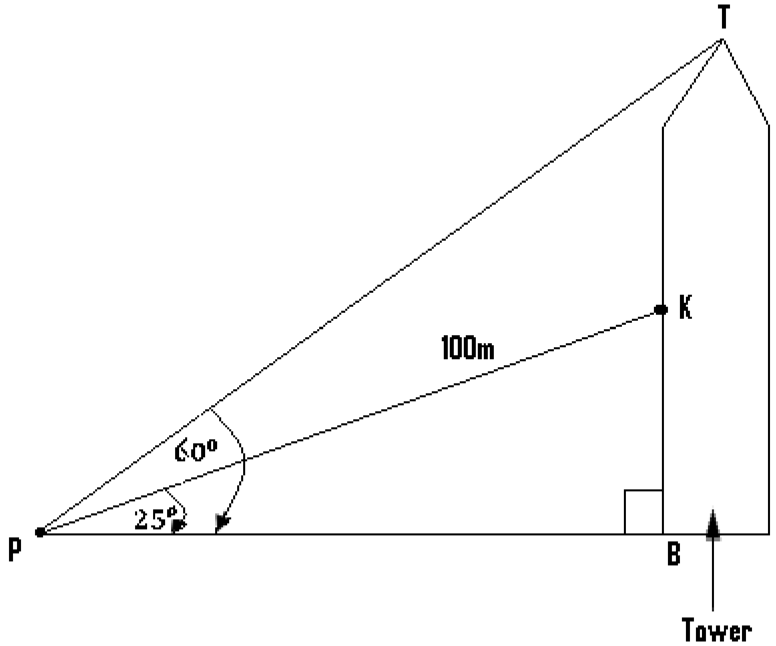
(Q11) K is a point on the side of a tower while T is its top. The angle of elevation from a point P on the same level ground as the foot of the tower, of K and T are $20^0 and\ 60^0$ respectively. If$|PK|$=100m, find

(a) the distance between P and the foot of the tower.

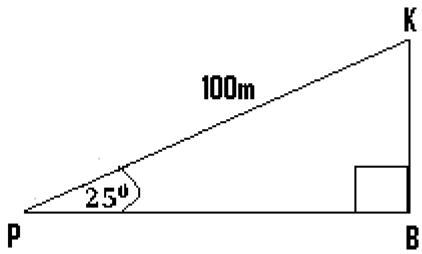
(b) the height of the tower

(c)$|KT|$

Soln.



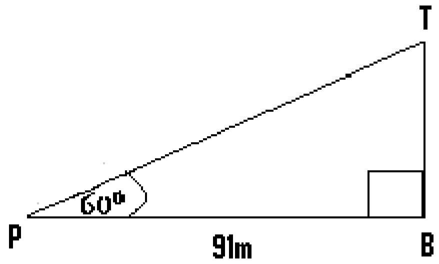
Let B = the point where the foot of the tower is located.
From this diagram, two diagrams can be drawn and the first one is shown next:



PB=100cos

$\mathbf{25^0} = \mathbf{100} \times \mathbf{0.91} = \mathbf{91}, \Rightarrow$ the distance between P and the foot of the tower = PB = 91m.

(b)The second diagram which can be drawn from the first diagram is shown next:



The height of the tower = TB.

From tan $60^0 = \frac{TB}{91} \Rightarrow TB = 91 \times tan60^0$

$= 91 \times 1.7 = 155,$

$\Rightarrow the\ height\ of\ the\ tower = 155m\ approx.$

$(c) From\ the\ first\ diagram\ drawn,$ $|KT| =$ TB – KB

$= 115m - KB$

$From\ fig.(1)\frac{KB}{PB} = tan25^0,$

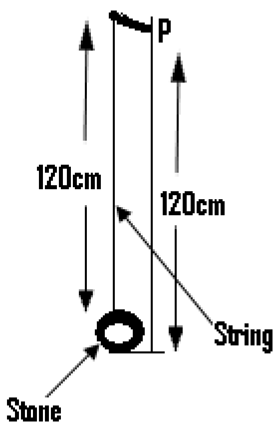
$\Rightarrow \frac{KB}{91} = 0.47 \Rightarrow KB = 91 \times 0.47 = 43.$

$\therefore$Magnitude of KT = 115 – 43 = 72m.

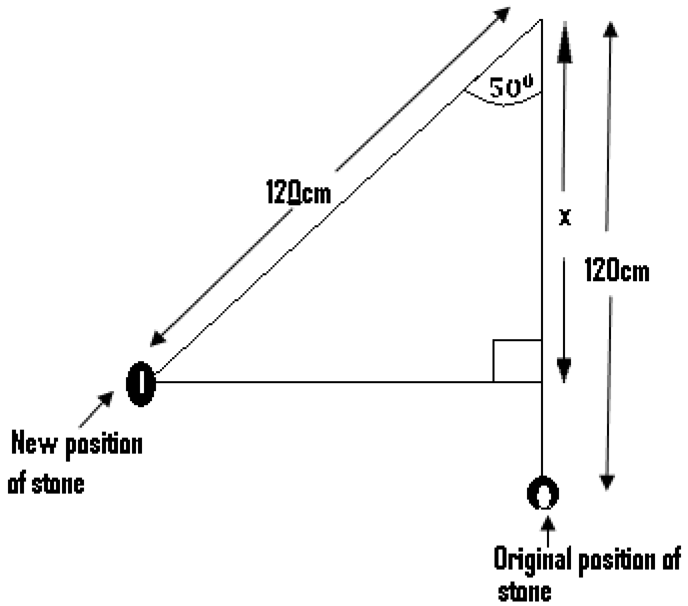
(Q12) A small stone is tied to a point P vertically above it by an inelastic string 102cm long. If the string is moved such that it is inclined at an angle of 50^0 to the vertical, how high does the stone rise?

Soln.

<u>Initial situation:</u>



Situation when the string in inclined at angle 50^0 to the vertical:



The level or height by which the stone rises = 120 – x.

From the figure drawn, x = 120 cos 50^0

$= 120\times 0.8 = 96$cm.

The length by which the stone rises = 120 - 96 = 24cm.

(Q13) If 8 sin x + 2 = 5, find x

Soln.

$8\sin x + 2 = 5 \Longrightarrow 8\sin x = 5 - 2 = 3,$

$\therefore 8\sin x = 3 \Longrightarrow \sin x = \frac{3}{8}, \Longrightarrow \sin x = 0.375 \Longrightarrow x = sin^{-1}0.375, \Longrightarrow x = 22^{\circ}$

Q14. If $14\cos\theta - 3 = 8, find\ the\ value\ of\ \theta.$

Soln.

$14\cos\theta - 3 = 8 \Longrightarrow 14\cos\theta = 8 + 3 \Longrightarrow 14\cos\theta = 11,$

$\Longrightarrow \cos\theta = \frac{11}{14}, \Longrightarrow \cos\theta = 0.79 \Longrightarrow \theta = cos^{-1}0.79$

$\Longrightarrow \theta = 38°$

(Q15). If 2tan $\theta + 1 = 4, find\ the\ value\ of\ \theta.$

Soln.

$2\tan\theta + 1 = 4, \Longrightarrow 2\tan\theta = 4 - 1 = 3.$

$Frum\ 2\tan\theta = 3 \Longrightarrow \tan\theta = \frac{3}{2}, \Longrightarrow \tan\theta = 1.5,$

$\Longrightarrow \theta = tan^{-1}1.5 \Longrightarrow \theta = 56°$

Graphs of trigonometrical functions:

Q1a. Plot the graph of the relation y

$= 2\cos\theta + 1\ for\ 0° \leq \theta \leq 360°, at\ 45°$ interval.

b. Using it, find the truth set of $2\cos\theta + 1 = 1.5$

N/B: We first construct a table.

Since we are to use values of $\theta\ from\ 0°\ up\ to\ 360°, at$

$45°$ interval, then the values of $\theta\ to\ be\ used\ are\ 0°, 45°, 90, 135 \ldots \ldots \ldots \ldots$

a. Soln.

θ	0°	45°	90	135°	180	225°	270°	315	360°
$\cos\theta$	1	0.71	0	-0.71	-1	-0.71	0	0.71	1
$2\cos\theta$	2	1.42	0	-1.42	-2	-1.42	0	1.42	2
Y=2cos $\theta + 1$	3	2.42	1	-0.4	-1	-0.42	1	2.42	3

N/B: 1. If $\theta = 45° \Longrightarrow \cos\theta = 0.71,$

$\Longrightarrow 2\cos\theta = 2(0.71) = 1.42, \Longrightarrow 2\cos\theta + 1 = 1.42 + 1 = 2.42,$

$\therefore y = 2\cos\theta + 1 = 2.42.$

2. If $\theta = 225° \Longrightarrow \cos\theta = -0.71,$

$\Longrightarrow 2\cos\theta = 2(-0.71) = -1.42,$

$\Longrightarrow 2\cos\theta + 1 = -1.42 + 1 = -0.42.$

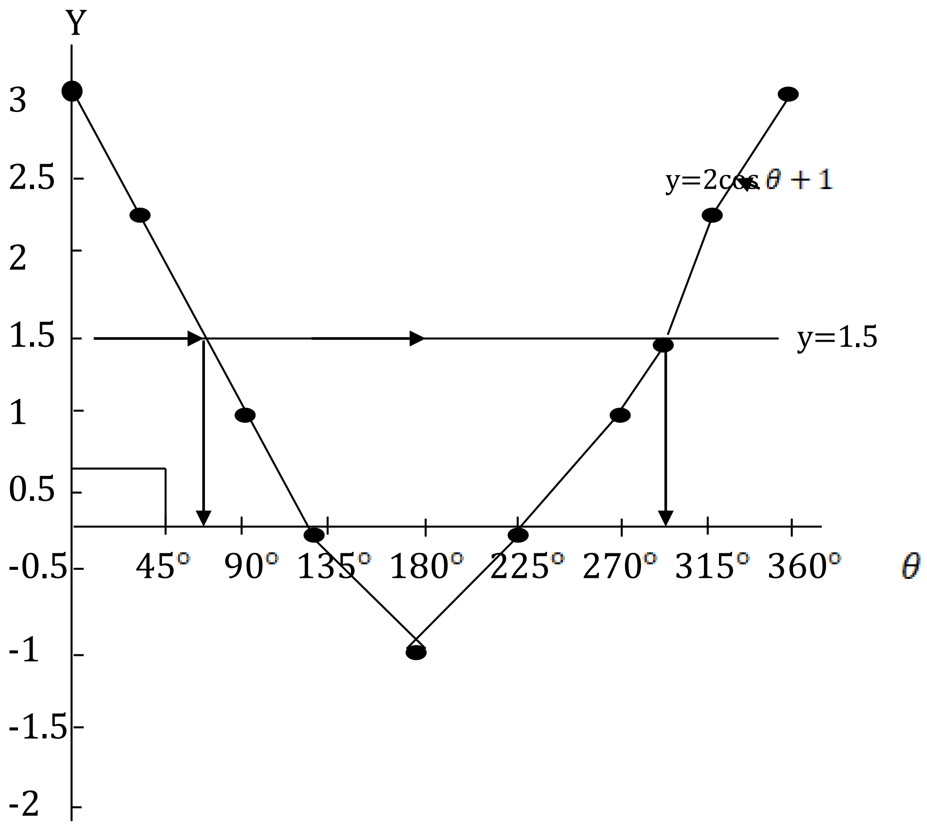
$\therefore y = 2\,cos\,\theta + 1 = -0.42$

3. if $\theta = 360° \Longrightarrow cos\,\theta = 1,$

$\Longrightarrow 2\cos\theta = 2(1) = 2,$

$\Longrightarrow 2\cos\theta + 1 = 2 + 1 = 3.$

$\therefore y = 2cos\theta + 1 = 3.$



b. From $2\cos\theta + 1 = 1.5 \Longrightarrow y = 2\,cos\,\theta + 1 = 1, \Longrightarrow y = 1.5.$

We therefore draw the line y = 1.5 and where it cuts the graph or curve, the corresponding values of θ are determined.

Required truth set = $\{\theta : \theta = 70° \ or \ \theta = 290°\} approx.$

Q2a. Plot the graph of the relation y = 3 sinθ – cos θ $for \ 0 \leq \theta \leq 180, at \ 30° \ interval.$

c. Using your graph solve the equation 3 sin $\theta - cos \ \theta = 2$.

soln.

θ	0°	30°	60°	90°	120°	150°	180°	210	240°
sin θ	0	0.5	0.9	1	0.9	0.5	0	-0.5	-0.9
3sin θ	0	1.5	2.7	3	2.7	1.5	0	-1.5	-2.7
cos θ	1	0.9	0.5	0	-0.5	0.9	-1	-0.9	-0.5
Y=3sin θ – cosθ	-1	0.6	2.2	3	3.2	2.4	1	-0.6	-2.2

N/B: 1. If $\theta = 30°$, sin θ = sin 30 = 0.5,

$\Rightarrow 3 \sin \theta = 3(0.5) = 1.5. Also \cos \theta = \cos 30 = 0.9.$

$Frm \ y = 3sin \ \theta - cos \ \theta \Rightarrow y = 1.5 - 0.9, \Rightarrow y = 0.6.$

2. If $\theta = 120°, sin \ \theta = \sin 120° = 0.9,$

$\Rightarrow 3sin \ \theta = 3(0.9) = 2.7.$

$Also \cos \theta = \cos 120° = -0.5.$

From $y = 3sin\theta - cos \ \theta \Rightarrow y = 2.7 - (-0.5),$

$\Rightarrow y = 2.7 + 0.5 \Rightarrow y = 3.2.$

3. If $\theta = 210, \sin \theta = \sin 210 = -0.5,$

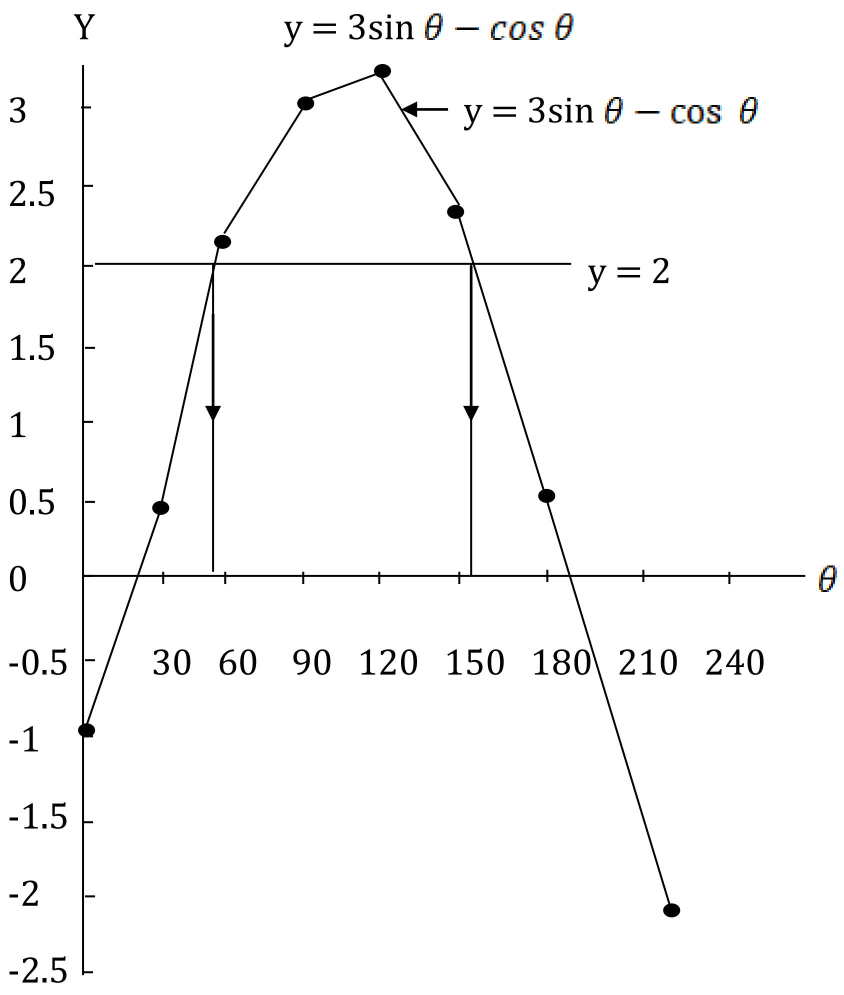
$\Rightarrow 3 \ \sin 120^0 = 3(-0.5) = -1.5.$

$Also \ cos \ \theta = \cos 210 = -0.9.$

$From\ y = 3\sin\theta - \cos\theta \Rightarrow y = -1.5 - (-0.9) = -1.5 + 0.9,$

$\Rightarrow y = -0.6.$

N/B: In the table constructed, the values have been approximated. E.g sin 120 = 0.866 has been approximated to 0.9.



b. From 3sin $\theta - \cos\theta = 2 \Rightarrow y = 3\sin\theta - \cos\ \theta = 2,$

$\Rightarrow y = 2$. We therefore draw the line y = 2.

Truth set = { $\theta : \theta = 53^\circ\ or\ \theta = 155^\circ$} approx.

Q3a. Plot the graph of y – 2tan
$\theta = 3\sin\theta, for\ 20^\circ \leq \theta \leq 260^\circ, using\ an\ interval\ of\ 20^\circ.$

b. Use it to determine the truth set of the equation 3sin $\theta + 2\tan\theta = -6$.

N/B: First make y the subject.: From y – 2tan $\theta = 3\sin\theta$

=> y = $3\sin\theta + 2\,tan\,\theta$.

Soln.

$$y = 3\ sin\ \theta + 2\ tan\ \theta$$

θ	20°	40°	60°	80°	100°	120°	140°	160°	180°	200°	220°	240°	260°
3sin θ	1.02	1.93	2.59	2.95	2.95	2.59	1.92	1.02	0	-1.02	-1.92	-2.6	-2.94
2tan θ	0.72	1.67	3.46	11.34	-11.34	-3.46	-1.68	-0.72	0	0.73	1.68	3.46	11.34
y=3 sin $\theta + 2$ $tan\theta$	1.74	3.6	6.05	14.29	-8.4	-0.87	0.24	0.3	0	-0.29	-0.24	0.86	8.4

N/B: 1. If $\theta = 20°, 3\ sin\ \theta = 3\ sin\ 20° = 3(0.34) = 1.02$.

$2\ tan\ \theta = 2\tan 20° = 2(0.36) = 0.72$.

y = $3\sin\theta + 2\tan\theta \Rightarrow y = 1.02 + 0.72$, =>

$y = 1.74$.

2. If $\theta = 80°, 3\ sin\ \theta = 3\sin 80° = 2.95$.

2tan $\theta = 2\tan 80° = 2(5.67) = 11.34$

$y = 3\ sin\ \theta + 2\tan\theta = 2.95 + 11.34$,

$\Rightarrow y = 14.29$

3. If $\theta = 140, 3\sin\theta = 3\sin 140° = 3(0.64) = 1.92$.

2tan $\theta = 2\tan 140 = 2(-0.84) = -1.68$.

$y = 3\ sin\ \theta + 2\tan\theta \Rightarrow y = 1.92 + (-1.68), \Rightarrow y = 1.92 - 1.68 = 0.24$

4. If $\theta = 160, 3\sin\theta = 3\sin 160 = 3(0.34) = 1.02$.

$2\ tan\ \theta = 2\tan 160 = 2(-0.36) = -0.72$.

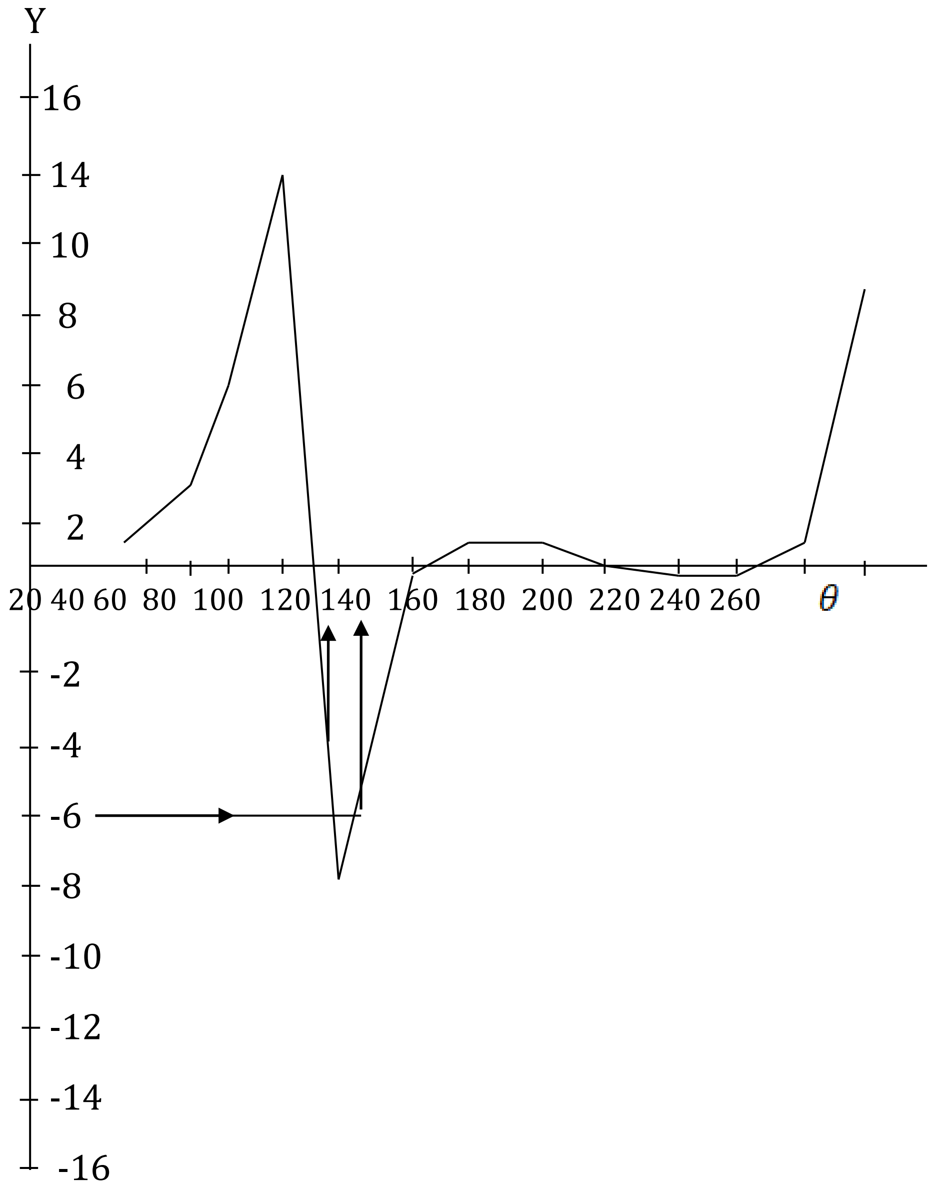
$y = 3\ sin\ \theta + 2\ tan\ \theta = 1.02 + (-0.72) = 1.02 - 0.72 = 0.3.$

5. If $\theta = 220, 3\ sin\ \theta = 3\ sin\ 220 = 3(-0.64) = -1.92.$

$2\ tan\ \theta = 2 \tan 220 = 2(0.84) = 1.68.$

$y = 3\ sin\ \theta + 2\ tan\ \theta \Rightarrow y = -1.92 + 1.68, \Rightarrow y = -0.24$
6. If $\theta = 260, 3\ sin\ \theta = 3 \sin 260 = 3(-0.98)$
$= -2.94$

$2\ tan\ \theta = 2 \tan 260 = 2(5.67) = 11.34.$
$y = 3\ sin\ \theta + 2\ tan\ \theta = -2.94 + 11.34, \Rightarrow y = 8.4.$



b. Truth set = { $\theta : \theta = 90° \ or \ \theta = 110°$} approx.,

Since 3sin $\theta + 2 \tan \theta = -6 \Rightarrow y = 3sin\,\theta + 2 \tan \theta = -6$,,

$\Longrightarrow y = -6$

Draw the line y = - 6 and at the points where it cuts the graph, determine the corresponding x components.

Q4. Plot the graph of the relation y = 3cos $\theta - 4\tan\theta, for$

$0 \leq x \leq 360° \; at \; 40° \; interval.$

θ	0°	40°	80°	120°	160°	200°	240°	280°	320°	360°
3cosθ	3	2.29	0.52	-1.5	-2.82	-2.82	-1.5	0.52	2.3	3
4 tan θ	0	3.36	22.7	-6.93	-1.46	1.46	6.93	-2.27	-3.36	0
y=3cos $\theta - 4tan\theta$	3	-1.07	-22.2	5.43	-1.38	-4.3	-8.43	2.8	5.66	3

N/B:1. If $\theta = 40°$,

3 cos $\theta = 3\;cos\;40^0 = 2.29.$

4 tan $\theta = 4\;tan\;40^0 = 3.36.$

3 cos θ – 4 tan θ= 2.29 – 3.36 = -1.07.