CHAPTRE EIGHT

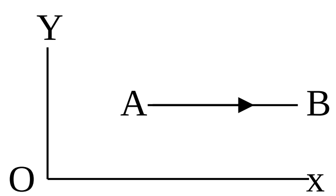
VECTORS

- A vector is a physical quantity which has both magnitude and direction.
- Example are
  a. A force of 20N acting North.
  b. A velocity of 5km/h East.

Types of vectors:

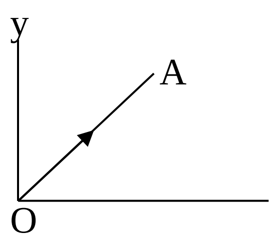
- In general the are two types and these are
  i. Free vector.
  ii. Position vector.

Free vector:



- A free vector is a vector which does not pass through any specific position.
- They are usually represented by small letters e.g $e.g\ \underset{\sim}{a}, \underset{\sim}{b}\ \ and\ \underset{\sim}{c}$

Position vector :



This is a vector which passes through the origin or a specified point.

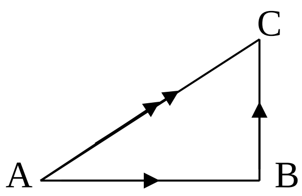
Vector notation:

- A vector may be represented by a line segment as shown next:

A $\longrightarrow$ B

- This given vector can be represented by $\overrightarrow{AB}, \overline{AB}, \underline{AB}, \widetilde{AB}, \underset{\sim}{AB}$.

The Triangle law:



According to the triangle law, $\overline{AC} = \overline{AB} + \overline{BC} \Longrightarrow \overline{AB} = \overline{AC} - \overline{BC}$ and $\overline{BC} = \overline{AC} - \overline{AB}$

The unit vector:

- This is a vector whose magnitude is one in the direction under consideration.
- The unit vector along a vector $\vec{a}$is written as $\hat{a}$
- Also the unit vector along a vector $\overrightarrow{b}$ is written as $\hat{b}$
- The unit vector along the vector $\overline{BC}$ is written as $\widehat{BC}$
- Consider the vector A $\longrightarrow$ B = 1
- The vector is written as $\overrightarrow{AB}$and its unit vector is written as $\widehat{AB}$.

Equal vectors:

- Two vectors are said to be equal if their magnitudes and directions are equal
- Example are $\overline{AB} = 50km/hE$ and $\overline{CD} = 50km/h\ E$.

The negative vector:

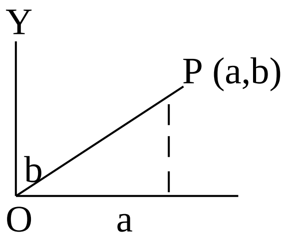
- The negative of the vector $\underset{\sim}{a}$ is written as –a
- If $-\underset{\sim}{a}$ is the negative vector of the vector $\underset{\sim}{a}$, $then$ $\underset{\sim}{a}+(-\underset{\sim}{a}) = \underset{\sim}{o}$.
- The vector $-\underset{\sim}{a}$ $is\ a\ vector$ of the same magnitude as $\underset{\sim}{a}$, but it is opposite in direction.
- It must be noted that $\overline{AB} + \overline{BA} = \underset{\sim}{o}$.
- Also if $\underset{\sim}{b} = \overrightarrow{CD}$, $then$ $-\underset{\sim}{b} = \overrightarrow{DC}$, and $\overline{CD} + \overline{DC} = \underset{\sim}{o}$.
- If we consider a vector $\overline{CD}$, then its negative vector is $\overline{DC}$.

The zero vector (null vector):

- This is a vector where magnitude is zero and its direction is undefined.
- It is represented by $\underset{\rightarrow}{0} = \binom{0}{0}$

Notation of the magnitude of a vectors:

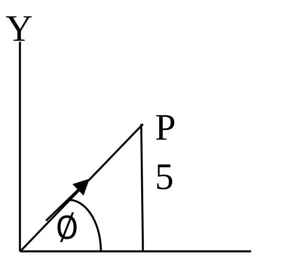
- If $\overline{AB}$ is a vector, then its magnitude is written as $|\overrightarrow{AB}|$
- Similarly the magnitude of the vector $\vec{b}$ is written as $|\vec{b}|$
- If $\overline{OP} = \binom{a}{b}, then\ its$ magnitude $=|\overline{OP}| = \sqrt{a^2 + b^2}$



Q1. i. If OP = $\binom{6}{5}, find\ the$ magnitude of $\overline{OP}$.

ii. Find $\emptyset$ the angle between $\overline{OP}$ and the x – axis

Soln.



i. $|\overrightarrow{OP}| = \sqrt{6^2 + 5^2} = \sqrt{61} = 7.8$

ii. $\tan \emptyset = {}^{5}/_{6} \Longrightarrow \tan \emptyset = 0.83 \Longrightarrow \emptyset = tan^{-1} 0.83 \Longrightarrow \emptyset = 40.$

Scalar multiplication of vector:

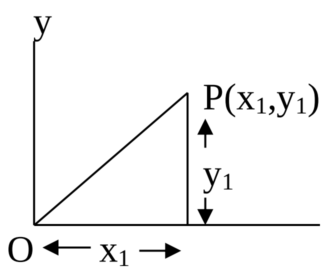
- If ^ is the scalar and $\overline{a}$ is the vector, then the scalar x the vector $= \hat{}\vec{a}$

- When a scalar multiplies a vector, the product is also a vector, and for this reason ^$\vec{a}$ is also a vector.
- The vector ^$\underset{\sim}{a}$ is parallel to $\underset{\sim}{a}$, and is in the same direction as $\underset{\sim}{a}$,but has ^ times the magnitude of $\underset{\sim}{a}$.
- For example the vectors $\vec{a}$ *and* $2\vec{a}$ have the same direction.

  i.e $|\vec{a}|$ $\longrightarrow$ $|2\vec{a}|$ $\longrightarrow$

- But the vectors $\vec{a}$ and and $-2\vec{a}$ are opposite in direction.
- ^$(\vec{a} + \vec{b}) =$ ^$\vec{a} +$ ^$\vec{b}$, e.g $6(\underset{\sim}{a}+\underset{\sim}{b}) = 6\underset{\sim}{a}+6\underset{\sim}{b}$
- Also (2 + 4) $\vec{a} = 2\vec{a} + 4\vec{a}$
- Finally $^{\wedge}{}_1(^{\wedge}{}_2\vec{a}) = {}^{\wedge}{}_1{}^{\wedge}{}_2\vec{a}$, e.g $3(2\vec{a}) = 6\vec{a}$

N/B:



- If P(x_1,y_1) is a point in the x – y plane, then the position vector of P relative to the origin, O is defined by $\overrightarrow{OP} = \binom{x_1}{y_1}$
- Also if A = (0,6), then $\overrightarrow{OA} = \binom{0}{6}$

Q2. Find the numbers m and n such that

M$\binom{3}{5} + n\binom{2}{1} = \binom{4}{9}$

Soln.

$$M\begin{pmatrix}3\\5\end{pmatrix} + n\begin{pmatrix}2\\1\end{pmatrix} = \begin{pmatrix}4\\9\end{pmatrix} \Rightarrow \begin{pmatrix}3m\\5m\end{pmatrix} + \begin{pmatrix}2n\\n\end{pmatrix} = \begin{matrix}4\\9\end{matrix}$$

$\Rightarrow 3m + 2n = 4 \ldots\ldots eqn(1).$

$5m + n = 9 \ldots\ldots.\ldots eqn(2)$

Solve eqns (1) and (2) simultaneously

$\Longrightarrow m = 2 \; and \; n = -1$

Q3. If mp + nq = $\binom{4}{3}, find \; m \; and \; n$ where m and n are scalar, given that p = $\binom{2}{3} \; and \; q = \binom{2}{5}$

Soln.

p = $\binom{2}{3} \; and \; q = \binom{2}{5} \; but \; mp + nq = \binom{4}{3}$

$\Longrightarrow m\binom{2}{3} + n\binom{2}{5} = \binom{4}{3} \Longrightarrow \binom{2m}{3m} + \binom{2n}{5n} = \binom{4}{3}$

$\Longrightarrow 2m + 2n = 4 - (1)$

$3m + 5n = 3 - (3)$

Solve eqns (1) and (2) simultaneously to get the values of m and n.

Q4. If r = $\binom{3}{1} and \; s = \binom{-2}{1}$, evaluate 6(r + 25)

Soln.

Consider 6(r + 2s), solve what is inside the bracket first $\Longrightarrow r + 2s = \binom{3}{1} + \binom{-2}{1} = \binom{3}{1} + 2\binom{-4}{2} \Longrightarrow r + 2s = \binom{3+\overline{4}}{1+2} = \binom{-1}{3} \Longrightarrow 6(r + 2s) = 6\binom{-1}{3} = \binom{-6}{18}$

Q5. If p = $\binom{1}{2}, q = \binom{-2}{3} \; and \; r = \binom{1}{1}$, find 2p – q + r

Soln.

2p – q + r = $2\binom{1}{2} - \binom{-2}{3} + \binom{1}{1} = \binom{2}{4} - \binom{-2}{3} + \binom{1}{1} = \binom{2+2+1}{4-3+1} = \binom{5}{2} \Longrightarrow 2p - q + r = \binom{5}{2}$.

Q6. If the vector p = $\binom{2}{3}, q = \binom{2}{5} and \; r = \frac{1}{2}(q - p)$,

Find the vector r.

Soln.

r = $\frac{1}{2}(q-p) \Longrightarrow r = \frac{1}{2}\left\{\binom{2}{5}-\binom{2}{3}\right\} \Longrightarrow r = \frac{1}{2}\left\{\begin{matrix}2-2\\5-3\end{matrix}\right\} = \frac{1}{2}\left\{\begin{matrix}0\\2\end{matrix}\right\} = \left\{\begin{matrix}\frac{1}{2}(0)\\ \frac{1}{2}(2)\end{matrix}\right\} = \left\{\begin{matrix}0\\1\end{matrix}\right\} \Longrightarrow r = \left\{\begin{matrix}0\\1\end{matrix}\right\}$

N/B: Given the points A and B, then $\overrightarrow{AB} = B - A$.

Examples: If A = $\binom{5}{2}$ *and* $B = \binom{10}{6}$, *then* $\overrightarrow{AB} = B - A = \binom{10}{6} - \binom{5}{2} = \binom{10-5}{6-2} = \binom{5}{4}$

Also if C = $\binom{4}{2}$ *and* $D = \binom{6}{1}$, *then* $\overrightarrow{CD} = D - C = \binom{6}{1} - \binom{4}{2} = \binom{6-4}{1-2} = \binom{2}{-1} \Longrightarrow \overrightarrow{CD} = \binom{2}{-1}$

Q7. If A = (4, 5) and B = (6, 2), find $\overline{AB}$

Soln.

A = (4,5) $\Longrightarrow A = \binom{4}{5}$. Also B = (6, 2) $\Longrightarrow B = \binom{6}{2}$. $\overline{AB} = B - A = \binom{6}{2} - \binom{4}{5} = \binom{6-4}{2-5} = \binom{2}{-3} \Longrightarrow \overline{AB} = \binom{2}{-3}$.

N/B: If $\overline{AB} = \binom{4}{2} \Longrightarrow \overline{BA} = -\overline{AB} = -\binom{4}{2} = \binom{-4}{-2}$

Also if $\overrightarrow{CD} = \binom{-2}{5} \Longrightarrow \overrightarrow{DC} = -\overrightarrow{CD} = -\binom{-2}{5} = \binom{2}{-5}$

Q8. If A and B are the points (2, 1) and (1, 2) respectively, find $\overline{AB}$ *and* $\overline{BA}$

Soln.

A = $\binom{2}{1}$ *and* $B = \binom{1}{2} \Longrightarrow \overrightarrow{AB} = B - A = \binom{1}{2} - \binom{2}{1} = \binom{1-2}{2-1} = \binom{-1}{1}$

$\overline{BA} = -\overline{AB} = -\binom{-1}{1} = \binom{1}{-1}$.

Q9. Given A = $\binom{4}{1}$ *and* $B = \binom{-5}{4}$ *and the scalar as* 2,

evaluate i. $\overrightarrow{2A}$ ii. $2\overrightarrow{AB}$ iii. 2(A – B)

Soln.

i. A = $\binom{4}{1} \Longrightarrow 2A = 2\binom{4}{1} = \binom{8}{2}$

ii. A = $\binom{4}{1}$ $and\ B = \binom{-5}{4}$, then $\overline{AB} = B - A = \binom{-5}{4} - \binom{4}{1} = \binom{-5-4}{4-1} = \binom{-9}{3} \Longrightarrow \overline{AB} = \binom{-9}{3}$

Since $\overline{AB} = \binom{-9}{3}, then\ 2\overline{AB} = 2\binom{-9}{3} = \binom{-18}{6}$

iii. 2(A – B) = ? but A = $\binom{4}{1}$and B =$\binom{-5}{4}$

A – B = $\binom{4}{1} - \binom{-5}{4} = \binom{4+5}{1-4} = \binom{9}{-3}$

Since A – B =$\binom{9}{-3} \Longrightarrow 2(A-B) = 2\binom{9}{-3} = \binom{18}{-6}$

Q10. If A = (2, 4) and B = (4, 9), find$|\overline{AB}|$ie the magnitude of AB.

Soln.

A = $\binom{2}{4}$ $and\ B = \binom{4}{9} \Longrightarrow \overline{AB} = B - A = \binom{4}{9} - \binom{2}{4} = \binom{2}{5} \Longrightarrow \overline{AB} = \binom{2}{5}$. $|\overline{AB}|$= $\sqrt{2^2 + 5^2} = \sqrt{4 + 25} \Longrightarrow AB = \sqrt{29} = 5.4$

Q11. If A = (-5, 2) and B(-8 – -9),

i. Find the vector $\overrightarrow{BA}$

ii. Calculate the length of $\overline{BA}$

Soln.

i. A = $\binom{-5}{2}$ $and\ B\binom{-8}{-9} \Longrightarrow AB = B - A = \binom{-8}{-9} - \binom{-5}{2} = \binom{-8+5}{-9-2} = \binom{-3}{-11} \Longrightarrow \overline{AB} = \binom{-3}{-11}, but\ \overrightarrow{BA} = -AB \Longrightarrow \overline{BA} = \binom{3}{11}$

ii. The length of $\overline{BA}$ = the magnitude of $\overline{BA} \Longrightarrow length\ of\ \overline{BA} = \sqrt{3^2 + 11^2} = \sqrt{9 + 121} = \sqrt{130} = 11.4 \Longrightarrow the\ length\ of\ \overline{BA}$ =11.4

Q12. If C = (4, 1) and D = (2, 6),

a. find the vector $\overline{CD}$
b. calculate the length of $\overline{DC}$

Soln.

a. $\overline{CD} = D - C = \binom{2}{6} - \binom{4}{1} = \binom{2-4}{6-1} = \binom{-2}{5}. \overline{DC} = -\binom{-2}{5} = \binom{2}{-5} \Longrightarrow \overline{DC} = \binom{2}{-5}$.

b. The length of $\overline{DC} = \sqrt{2^2 + (-5)^2} = \sqrt{4 + 25} = \sqrt{29} = 5.4$

Q13. If C = (1, 3) and D = (2, 4) find $|\overline{CD}|$

Soln.

$\vec{C} = \binom{1}{3} and\ \vec{D} = \binom{2}{4} \Longrightarrow \overrightarrow{CD} = \vec{D} - \vec{C} = \binom{2}{4} - \binom{1}{3} = \binom{1}{1} \therefore \overline{CD} = \binom{1}{1} \Longrightarrow |\overline{CD}| = \sqrt{1+1} = \sqrt{2} = 1.4$

Q14. If $\vec{p} = \binom{2}{1}$ $and\ \vec{q} = \binom{-3}{2}, evalute$

i. $|\vec{P} + \vec{q}|$ ii. $|\overrightarrow{pq}|$

Soln.

i. $\vec{P} + \vec{q} = \binom{2}{1} + \binom{-3}{2} = \binom{-1}{3}, \therefore \vec{p} + \vec{q} = \binom{-1}{3} \Longrightarrow |\vec{p} + \vec{q}| = \sqrt{(-1)^2 + 3^2} = \sqrt{1+9} = \sqrt{10} = 3.2$

ii. $\overrightarrow{pq} = q - p = \binom{-3}{2} - \binom{2}{1} = \binom{-5}{1} \Longrightarrow \overrightarrow{pq} = \binom{-5}{1}$.

$|pq| = \sqrt{(-5)^2 + 1^2} = \sqrt{25+1} = \sqrt{26} = 5.1$

Q15. If Q is the point (2,4) and $\overrightarrow{QR} = \binom{1}{3}$, find the coordinates of R.

Soln.

Q = (2, 4) and $\overrightarrow{QR} = \binom{1}{3}$, then the coordinates of R

= (2+1, 4+3) = (3, 7)

The coordinates of R = (3, 7)

Q16. If $z = (1{,}2)\, and\ \overline{zy} = \binom{-1}{3}$, find the coordinates of y.

Soln.

Since z = (1,2) and $\overline{zy} = \binom{-1}{3}$, then the coordinates of

y = $(1 + \bar{1}, 2 + 3) = (0{,}5) \Longrightarrow the\ coordinates\ of\ y = (0{,}5)\ or\ \binom{0}{5}$.

Q17. If A = (1, 5) and $\overline{BA} = \binom{-2}{-3}$, find the coordinates of B.

N/B: Since the point given is A and the vector given is $\overline{BA}$, then $\overline{BA}$ must first be changed into $\overline{AB}$

Soln.

Since A is given as (1, 5), we must find$\overrightarrow{AB}, but\ since \overline{BA} = \binom{-2}{-3}$, then $\overrightarrow{AB} = -\overline{BA} \Longrightarrow \overline{AB} = -\binom{-2}{-3} \Longrightarrow \overline{AB} = \binom{2}{3}$.

Now since A = (1, 5) and$\overline{AB} = \binom{2}{3}$, then

B = (1+2, 5+3)$\therefore B = (3, 8)$.

Q18. If Q = (4, 1) and $\overline{RQ} = \binom{2}{-3}, find\ the$ coordinates of R.

Soln.

Since Q = (4, 1) and $\overline{RQ} = \binom{2}{-3}$, we must first find $\overline{QR}$.

$$\overline{QR} = -\overline{RQ} \Longrightarrow \overline{QR} = -\begin{pmatrix} 2 \\ -3 \end{pmatrix} \Longrightarrow \overline{QR} = \begin{pmatrix} -2 \\ 3 \end{pmatrix}$$

Now Q = (4, 1) and $\overline{QR} = \binom{-2}{3}, then\ the$

$coordinates\ of\ R = (-2 + 4{,}1 + 3) = (2{,}4)$

Q19. If C = $\binom{1}{3}\ and\ \overline{DC} = \binom{-1}{2}, find\ the$

$coordinates\ of\ D$.

Soln.

$$C = \begin{pmatrix} 1 \\ 3 \end{pmatrix} \Longrightarrow C = (1,3). Since\ \overline{DC} = \begin{pmatrix} -1 \\ 2 \end{pmatrix} \Longrightarrow \overline{CD} = -\overline{DC} \Longrightarrow \overline{CD} = -\begin{pmatrix} -1 \\ 2 \end{pmatrix}$$
$$= \begin{pmatrix} 1 \\ -2 \end{pmatrix}$$

Now C = $\binom{1}{3}$ $and \overline{CD} = \binom{1}{-2} \Longrightarrow the\ coordinates\ of\ D$

$$= \left(1 + 1, 3 + \bar{2}\right) = (2,1)$$

Q20. If A = (1,2),$\overline{AB} = \binom{3}{4}$ $and\ \overline{AC} = \binom{5}{-3}, find\ the\ coordinates\ of\ B\ and\ C.$

Soln.

Since A = (1, 2) and $\overline{AB} = \binom{3}{4}, then\ the\ coordinates\ of\ B = (1 + 3, 2 + 4) =$ $(4, 6)$

Also Since A = (1, 2) and $\overline{AC} = \binom{5}{-3} then\ the\ coordinates\ of\ C =$ $\left(1 + 5, 2 + \bar{3}\right) = (6, -1)$

Q21. Given B(4,2), $\overline{BC} = \binom{-1}{-5} and\ \overline{BD} = (1,3),$

determine the coordinates of C and D.

Soln.

Since B = (4, 2) and $\overline{BC} = \binom{-1}{-5} \Longrightarrow the\ coordinates$
$$of\ C = \left(4 + \bar{1}, 2 + \bar{5}\right) = (4 - 1, 2 - 5) = (3, -3). B = (4,2) and\ BD = (1,3)$$
$$\Longrightarrow coordinatesof\ D = (4 + 1, 2 + 3) = (5,5)$$

Q22. If A is the point (2, 3), $\overline{BA} = \binom{2}{-3}$ $and\ \overline{CA} =$ $\binom{-1}{-5}, determine\ the\ coordinates\ of\ B\ and\ C$

N/B: Since the point given is point A, then $\overline{BA}$ must be changed into $\overline{AB}$. Also $\overline{CA}$ must be changed into $\overline{AC}$.

Soln.

$\overline{BA} = \binom{2}{-3} \Longrightarrow \overline{AB} = -\overline{BA} = -\binom{2}{-3} \Longrightarrow \overline{AB} = \binom{-2}{3}$.

Also $\overline{AC} = -\overline{CA} = -\binom{-1}{-5} = \binom{1}{5}, \therefore \overline{AC} = \binom{1}{5}$

Now since A = (2, 3) and $\overline{AB} = \binom{-2}{3}$, then the coordinates of $B =$ $(2 + \overline{2}, 3 + 3) = (0{,}6)$

Also since A = (2,3) and $\overline{AC} = \binom{1}{5}$, *then the* coordinates of C = (2+1, 3+5)= (3, 8)

Q23. The point C is given as (4, 1), $\overline{CD} = \binom{1}{2}$ *and* $\overline{DE} = \binom{3}{5}$, *find the coordinates of D and E*.

Soln.

Since C = (4, 1) and $\overline{CD} = \binom{1}{2}$, *the the coordinates of*

$D = (4 + 1{,}1 + 2) \Longrightarrow D(5{,}3)$

Now D = (5, 3) and $\overline{DE} = \binom{3}{5} \Longrightarrow$ *the coordinates of E*

$= (5 + 3{,}3 + 5) \Longrightarrow E = (8{,}8)$

Q24. If the point A is given as $\binom{3}{4}$ *and* $\overrightarrow{AB} = \binom{-2}{1}$ *and* $\overrightarrow{BC} = \binom{5}{1}$, *find the coordinates of B*

and C.

Soln.

$A = \binom{3}{4}$*and* $\overline{AB} = \binom{-2}{1} \Longrightarrow$the coordinates of B = (3+$\overline{2}, 4 + 1) = (1{,}5)$.

Now B = (1, 5) and $\overline{BC} = \binom{5}{1} \Longrightarrow$ *the coordinates of C*

= (1+5, 5+1) = (6, 6)

Q25. Given A(4, 1), $\overline{BA} = \binom{1}{6}$ *and* $\overline{BC} = \binom{2}{2}$, *find* the coordinates of B and C.

N/B: The point given is point A and the vector given is $\overline{BA}$.

First find $\overline{AB}$.

Soln.

$\overline{BA} = \begin{pmatrix}1\\6\end{pmatrix}$. $\overline{AB} = -\overline{BA} = -\begin{pmatrix}1\\6\end{pmatrix} \Longrightarrow \overline{AB} = \begin{pmatrix}-1\\-6\end{pmatrix}$

Now A = (4, 1) and $\overline{AB} = \begin{pmatrix}-1\\-6\end{pmatrix} \Longrightarrow$ *the coordinates of B*

$= \left(4+\bar{1}, 1+\bar{6}\right) = (3, -5)$

Now B = (3, -5) and $\overline{BC} = \begin{pmatrix}2\\2\end{pmatrix} \Longrightarrow$ *the coordinates of C*

$= (3+2, -5+2) = (5, -3)$

Q26. B is given as the point (4, 8), $\overline{CD} = \begin{pmatrix}1\\1\end{pmatrix}$ *and* $\overline{BC} = \begin{pmatrix}1\\7\end{pmatrix}$, *find the coordinates of C and D.*

Soln.

Since B = (4, 8) and $\overline{BC} = \begin{pmatrix}1\\7\end{pmatrix} \Longrightarrow$ *the coordinates of C*

$= (4+1, 8+7) = (5, 15) \Longrightarrow C(5,15)$

Now since C = (5, 15) and CD = $\begin{pmatrix}1\\1\end{pmatrix}$, *then the coordinatesof D* $= (5+1, 15+1) = (6, 16) \Longrightarrow D(6, 16)$

Q27. If A = (1, 3), $\overline{CB} = \begin{pmatrix}5\\5\end{pmatrix}$ *and* $\overline{BA} = \begin{pmatrix}1\\2\end{pmatrix}$, find the coordinates of B and C.

Soln.

$\overline{BA} = \begin{pmatrix}1\\2\end{pmatrix}$ *but* $\overline{AB} = -\overline{BA} = -\begin{pmatrix}1\\2\end{pmatrix} = \begin{pmatrix}-1\\-2\end{pmatrix}$s

Now A = (1, 3) and $\overline{AB} = \begin{pmatrix}-1\\-2\end{pmatrix} \Longrightarrow$ *the coordinates*

of $B = \left(1+\bar{1}, 3+\bar{2}\right) = (0,1) \Longrightarrow B = (0,1)$

Since CB = $\begin{pmatrix}5\\5\end{pmatrix} \Longrightarrow \overline{BC} = -\overline{CB} = -\begin{pmatrix}5\\5\end{pmatrix} \Longrightarrow \overline{BC} = \begin{pmatrix}-5\\-5\end{pmatrix}$.

Now B(0, 1) and $\overline{BC} = \begin{pmatrix}-5\\-5\end{pmatrix} \Longrightarrow$ *the coordinates of*

$C = (-5+0, -5+1) = (-5,-4) \Longrightarrow C(-5,-4)$.

Q28. P is the point (4, 1) and Q is (-3, 2). If $\overline{PS} = \binom{-1}{2}$ *and* $\overrightarrow{QT} = \binom{-2}{-3}$,

i. find the coordinates of S and T.
ii. find also $\overrightarrow{ST}$.

Soln.

i. Since P = (4, 1) and $\overline{PS} = \binom{-1}{2}$, *then the coordinates of* $S = (4+\overline{1}, 1+2) = (3,3) \Longrightarrow S(3,3)$. *Also since* Q

$$= (-3,2) \text{ and } \overrightarrow{QT} = \begin{pmatrix} -2 \\ -3 \end{pmatrix} \Longrightarrow \text{the coordinates of } T$$

$$= (-3+\overline{2}, 2+\overline{3}) = (-5,-1)$$

ii. $\overrightarrow{ST} = T - S = \binom{-5}{-1} - \binom{3}{3} = \binom{-5-3}{-1-3} = \binom{-8}{-4}$

Q29. If A = (4, 3) and B = (1, 1), $\overline{CA} = \binom{2}{1}$ *and* $\overline{DB} = \binom{-3}{-4}$, *find* $\overline{CD}$.

N/B: Before we can find $\overline{CD}$, we must first determine the coordinates of C and D.

Soln.

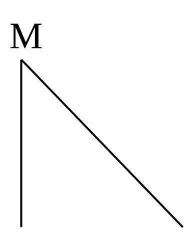
$\overline{AC} = -\overline{CA} = -\begin{pmatrix} 2 \\ 1 \end{pmatrix} = \begin{pmatrix} -2 \\ -1 \end{pmatrix} \therefore \overline{AC} = \begin{pmatrix} -2 \\ -1 \end{pmatrix}$. *since* $A = (4,3)$ *and* $\overline{AC} = \begin{pmatrix} -2 \\ -1 \end{pmatrix}$,

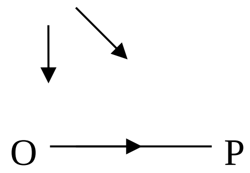
then the coordinates of $C = (-2+4, -1+3) \Longrightarrow C(2,2) = \begin{pmatrix} 2 \\ 2 \end{pmatrix}$

Also $\overline{BD} = -\overline{DB} = -\binom{-3}{-4} = \binom{3}{4} \Longrightarrow \overline{BD} = \binom{3}{4}$. *Since* $B = (1,1) \Longrightarrow$ *the coordinates of* $D = (1+3, 1+4) = (4,5)$. $\overline{CD} = D - C = \binom{4}{5} - \binom{2}{2} = \binom{2}{3}$

N/B:

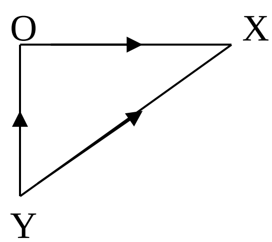
i.





In the figure drawn, moving from M to O, and then from O to P is the same as moving from M directly to P, since in both cases we end at the same point, which is P. $\Longrightarrow \overline{MO} + \overline{OP} = \overline{MP}$

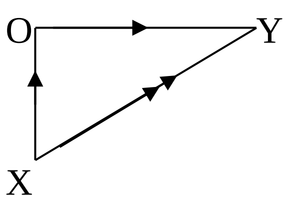
ii.



In the given figure $\overline{YX} = \overline{YO} + OX$

Q1. If $\overline{xo} = \begin{pmatrix} 4 \\ 3 \end{pmatrix}$ $and\ O\overline{y} = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$find|xy|.

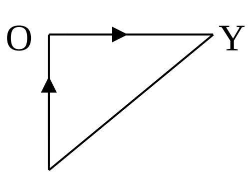
Soln.

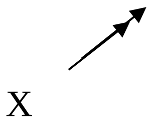


$\overline{xy} = \overline{xo} + \overline{oy} \Longrightarrow xy = \begin{pmatrix} 4 \\ 3 \end{pmatrix} + \begin{pmatrix} 2 \\ -1 \end{pmatrix} = \begin{pmatrix} 6 \\ 2 \end{pmatrix} \Longrightarrow |xy| = \sqrt{6^2 + 2^2} \Longrightarrow |xy| = \sqrt{36 + 4} = \sqrt{40} = 6.3$

Q2. If $\overline{ox} = \begin{pmatrix} -2 \\ 3 \end{pmatrix}$ $and\ \overline{oy} = \begin{pmatrix} 4 \\ 1 \end{pmatrix}, find|xy|$

Soln.





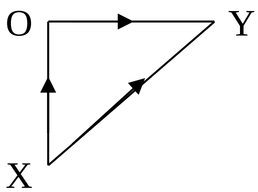
From the diagram, $\overline{xy} = \overline{xo} + \overline{oy}$

Since $\overline{ox} = \binom{-2}{3} \Longrightarrow \overline{xo} = -\overline{ox} = -\binom{-2}{3} = \binom{2}{-3}$.

$Since\ \overline{xy} = \overline{xo} + \overline{oy} \Longrightarrow \overline{xy} = \binom{2}{-3} + \binom{4}{1} = \binom{6}{-2} \Longrightarrow |xy| = \sqrt{6^2 + (-2)^2} = \sqrt{36+4} = \sqrt{40} = 6.3$

Q3. Given that $\overline{ox} = \binom{-2}{4}$ and $\overline{yo} = \binom{-1}{3}$. $find|xy|$.

Soln.



$\overline{xy} = \overline{xo} + \overline{oy}$

$$Since\ \overline{ox} = \begin{pmatrix}-2\\4\end{pmatrix} \Longrightarrow \overline{xo} = -\overline{ox} = -\begin{pmatrix}-2\\4\end{pmatrix} = \begin{pmatrix}2\\-4\end{pmatrix} \Longrightarrow \overline{xo}$$
$$= \begin{pmatrix}2\\-4\end{pmatrix}\ Also\ since\ \overline{yo} = \begin{pmatrix}-1\\3\end{pmatrix} \Longrightarrow \overline{oy} = -\overline{yo} = -\begin{pmatrix}-1\\3\end{pmatrix} = \begin{pmatrix}1\\-3\end{pmatrix}$$
$$\Longrightarrow \overline{oy} = \begin{pmatrix}1\\-3\end{pmatrix}$$

$\overline{xy} = \overline{xo} + \overline{oy} \Longrightarrow \overline{xy} = \binom{2}{-4} + \binom{1}{-3} = \binom{2+1}{-4+\overline{3}} = \binom{3}{-7}. |xy| = \sqrt{3^2 + (-7)^2}$ => $|xy| = \sqrt{9+49} = \sqrt{58} = 7.6$

The inverse of a vector or the negative vector.

- If $\overline{AB} = \binom{x}{y}$ $then\ \overline{BA} = -\overline{AB}\ and - \overline{AB} = \binom{-x}{-y}$
- $\overline{AB} = \binom{-x}{-y}$ $is\ called\ the\ inverse\ or\ the\ negative\ vector\ of\ \overline{AB}$
- A vector and its inverse have the same magnitude, but have opposite direction

- For example if $\overline{PQ} = \binom{-3}{-4}$, $then\ its\ inverse$

  $or\ negative\ which\ is\ \overline{QP} = -\begin{pmatrix}-3\\-4\end{pmatrix} = \begin{pmatrix}3\\4\end{pmatrix}$
- Also if $\overline{AB} = \binom{1}{-3}$, $then\ its\ inverse$

  $or\ negative, which\ is\ \overline{BA} = -\begin{pmatrix}1\\-3\end{pmatrix} = \begin{pmatrix}-1\\3\end{pmatrix}$

<u>The direction of a vector:</u>

- This is the angle φ,which the vector makes with the x-axis
- If P(x, y) and Q(x_2, y_2), then the direction of $\overline{PQ}$ is given by $\tan\varphi = \frac{y_2-y_1}{x_2-x_1}$

Q1. Given A(5, 4) and B(3, 1), find the direction of $\overline{AB}$

Soln.

Let (x_1, y_1) = (5, 4)and (x_2, y_2) = (3,1) $\Longrightarrow x_1 = 5, y_1 = 4, x_2 = 3\ and\ y_2 = 1.$
$\tan\theta$=$\frac{y_2 - y_1}{x_2 - x_1} = \frac{1 - 4}{3 - 5} = \frac{-3}{2} = -1.5.$

$\tan\varphi = -1.5 \Longrightarrow \varphi = tan^{-1} - 1.5 \Longrightarrow \varphi = -56.$

Q2.

a. Find the magnitude and the direction of the displacement vector $\overline{AB}, where$ A and B are the points (2, 1) and (8, 9) respectively.
b. Determine the magnitude of the vector $\overline{BA}$.

Soln.

a. $\overline{A} = \binom{2}{1}\ and\ \overline{B} = \binom{8}{9}. But\ \overline{AB} = \overline{B} - \overline{A} = \binom{8}{9} - \binom{2}{1} = \binom{6}{8}. Since\ \overline{AB} = \binom{6}{8} \Longrightarrow |\overline{AB}| = \sqrt{6^2 + 8^2} = \sqrt{100} = 10$

   Let (2, 1) = (x_1, y_1) and (8, 9) = $(x_2, y_2), then \tan\varphi = \frac{y_2-y_1}{x_2-x_1} = \frac{9-1}{8-2} = \frac{8}{6} = 1.33 \Longrightarrow \varphi = tan^{-1}1.33 = 53°$
b. $\overline{A} = \binom{2}{1}\ and\ \overline{B} = \binom{8}{9}\overline{BA} = \overline{A} - \overline{B} = \binom{2}{1} - \binom{8}{9} = \binom{-6}{-8}$

   $Since\ \overline{BA} = \binom{-6}{-8} \Longrightarrow |\overline{BA}| = \sqrt{(-6)^2 + (-8)^2} = \sqrt{100} = 10$

Parallel vectors:

- Two vectors are said to be parallel vectors, if one is the scalar multiplication of the other.
- Consider the vectors $\overline{A} = \binom{1}{3}$ and $\overline{B} = \binom{2}{6}$. These are parallel vectors, since one is the scalar multiple of the other, i.e 2 x $\overline{A} = \overline{B}$ or $2\binom{1}{3} = \binom{2}{6}$, $where\ 2\ is\ the\ scalar.$
- If the scalar is positive or a positive number, as in the example just given, then the two given vectors are in the same direction.
- But if the scalar is negative, then the two vectors are in the opposite direction
- Also the vectors $\overline{C} = \binom{3}{5}$ and $\overline{D} = \binom{9}{15}$ are parallel vectors, since one is the scalar multiple of the other i.e 3 x C = $\overline{D}$ or $3 \times \binom{3}{5} = \binom{9}{15}$.
- In this case, the scalar is 3 and since it is positive, then the two vectors are in the same direction.
- Now consider $\overline{A} = \binom{4}{5}$ and $\overline{B} = \binom{-16}{-20}$. These are parallel vectors, since one is the scalar multiple of the other i.e $-4 \times \overline{A} = \overline{B}$ or $-4\binom{4}{5} = \binom{-16}{-20}$.
- In this case, since the scalar is negative i.e – 4, then the two given vectors are in the opposite direction, eventhough they are parallel.

Q1. Determine whether the vector $\overline{B} = \binom{1}{-2}$ and $\overline{C} = \binom{-3}{6}$are parallel to each other, and determine whether they are in the same or opposite in direction.

Soln.

$$\overline{B} = \begin{pmatrix}1\\-2\end{pmatrix} and\ \overline{C} = \begin{pmatrix}-3\\6\end{pmatrix}\ But - 3 \times \overline{B} = \overline{C}\ i.e - 3\begin{pmatrix}1\\-2\end{pmatrix} = \begin{pmatrix}-3\\6\end{pmatrix} i.e$$

One is a scalar multiple of the other $\Longrightarrow$ they are parallel vectors. Since the scalar is negative or a negative number i.e -3, then the two vectors are opposite in direction.

Q2. Determine whether the vectors $\overline{x} = \binom{8}{10}$ and $\overline{y} = \binom{4}{5}$ are parallel to each other, and determine also whether they are in the same direction

Soln.

$\overline{x} = \begin{pmatrix} 8 \\ 10 \end{pmatrix}$ $and\ \overline{y} = \begin{pmatrix} 4 \\ 5 \end{pmatrix}. 2 \times \overline{y} = \overline{x}\ \ i.e\ 2\begin{pmatrix} 4 \\ 5 \end{pmatrix} = \begin{pmatrix} 8 \\ 10 \end{pmatrix}$

Since one of the vectors is a scalar multiple of the other, the two given vectors are parallel. Since the scalar = 2 which is positive $\Longrightarrow$ *the* two given vectors are in the same direction.

Determination whether two vectors are parallel – method two:

Let $\vec{A} = \binom{a}{b}\ and\ \vec{B} = \binom{c}{d}$ if ad – bc = o, then the two vectors are parallel.

Q1. Show that the vectors$\vec{A} = \binom{4}{2}\ and\ \vec{B} = \binom{8}{4}$ are parallel vectors.

Soln.

If $\vec{A}$ is parallel to $\vec{B}$, then (4 x 4) – (2 x 8) = 0 i.e if the left hand side is equal to zero, then they are parallel.

Now L.H.S = (4 x 4) – (2 x 8) = 16 – 16 = 0$\Longrightarrow$the two vectors are parallel

Q2. Determine whether or not the vectors $\vec{M} = \binom{-2}{-3}\ and\ \vec{N}\ are\ parallel$ vectors.
If $\vec{M}$ and $\vec{N}$ are parallel vectors, then

(-2 x 3) – (-3 x 2) = 0

L.H.S = (-6) – (-6) = -6 + 6 = 0

Since L.H.S = 0 $\Longrightarrow$ the two vectors are parallel vectors.

Q3. Determine whether or not $\vec{M} = \binom{4}{1}\ and\ \vec{N} = \binom{8}{6}$ are parallel vectors.

Soln.

$If\ \vec{M} = \binom{4}{1}\ and\ \vec{N} = \binom{8}{6}$ are parallel vectors, then

(4 x 6) – (1 x 8) = 0

L.H.S = 24 – 8 = 16.

Since the L.H.S $\neq o$ i.e not equal to zero, then the two vectors are not parallel.

Q4. Find the value of x, such that the vector $\overline{A}\binom{4}{9}$will be parallel to the vector $\overline{B}\binom{8}{x}$.

Soln.

$\overline{A} = \binom{4}{9}$ and $\overline{B} = \binom{8}{x}$. For them to be parallel, then $(4 \times x) - (9 \times 8) = 0 \Longrightarrow 4x - 72 = 0 \Longrightarrow 4x = 0 + 72 \Longrightarrow 4x = 72 \Longrightarrow x = \frac{72}{4} = 18 \Longrightarrow x = 18.$

Q5. Given $\vec{C}\binom{8}{x}$ and $\vec{D}\binom{-4}{-3},$ $determine$ the value of x such that the two vectors become parallel.

Soln.

$\vec{C}\binom{8}{x}$ and $\vec{D}\binom{-4}{-3}$are given. If they are parallel, then $(8 \times \overline{3}) - (-4 \times x) = 0 \Longrightarrow (-24) - (-4x) = 0 \Longrightarrow -24 + 4x = 0 \Longrightarrow 4x = 24 \Longrightarrow x = \frac{24}{4} = 6 \Longrightarrow x = 6$

Q6. Given $\overline{A} = \binom{y}{2}$ and $\overline{B} = \binom{9}{6}$, find the value of y such that the two vectors are parallel.

Soln.

$\overline{A} = \binom{y}{2}$ and $\overline{B} = \binom{9}{6}$. For them to be parallel, then

(y x 6) – (2 x 9) = 0 $\Longrightarrow 6y - 18 = 0 \Longrightarrow 6y = 18 \Longrightarrow y = \frac{18}{6} = 3$

Perpendicular vectors:

Consider the vectors $\overline{A} = \binom{a}{b}$ and $\overline{B} = \binom{c}{d}$. if these two vectors are perpendicular, then ac + db = 0

Q1. Show that the vectors $\overline{A}\binom{4}{-2}$ and $\overline{B}\binom{2}{4}$are perpendicular

Soln.

For $\vec{A}\binom{4}{-2}$ *and* $\vec{B}\binom{2}{4}$ to be perpendicular, then

(4 x 2) + (-2 x 4) = 0 i.e the L.H.S must be equal to zero.

L.H.S = $8\ +\overline{8} = 8 - 8 = 0$

Since L.H.S = 0 ⟹ the two vectors are perpendicular.

Q2. Determine whether or not $\overline{B}\binom{8}{2}$ *and* $\overline{C}\binom{4}{5}$ are perpendicular vectors.

Soln.

Given $\overline{B}\binom{8}{2}$ *and* $\overline{C}\binom{4}{5}$. If these two vectors are perpendicular, then (8 x 4) + (2 x 5) = 0 i.e

The L.H.S must be equal to zero. L.H.S 32 + 10 = 42.

Since L.H.S ≠0, then the two vectors are not perpendicular.

Q3. Given $\overline{A} = \binom{4}{2}$ *and* $\overline{B} = \binom{-3}{6}$, *determine* whether these two given vectors are perpendicular vectors.

Soln.

$\overline{A} = \binom{4}{2}$ *and* $\overline{B} = \binom{-3}{6}$. For these two vectors to be perpendicular, then $\left(4 \times \overline{3}\right) + (2 \times 6) = 0$

L.H.S = (-12) + (12) = 0. Since L.H.S = 0, then the two vectors are perpendicular.

Q4. Are the vectors $\overline{B} = \binom{-5}{-2}$*and* $\overline{D} = \binom{2}{-3}$ *perpendicualr vectors*?

Soln.

$\overline{B} = \binom{-5}{-2}$*and* $\overline{D} = \binom{2}{-3}$. If these two vectors are perpendicular, then $(-5 \times 2) + \left(-2 \times \overline{3}\right) = 0$

L.H.S = (-10) + (6) = - 4

Since L.H.S $\neq 0$, then the two given vectors are not perpendicular

Q5. Find the value of x such that the vectors

$\vec{A} = \binom{9}{2}$ and $\vec{B} = \binom{x}{-18}$ $will\ be$ perpendicular to each other.

Soln.

$\vec{A} = \binom{9}{2}$ and $\vec{B} = \binom{x}{-18}$. If these two vectors are to be perpendicular, then
$(9 \times x) + (2 \times \overline{18}) = 0 \Longrightarrow 9x + (-36) = 0 \Longrightarrow 9x - 36 = 0 \Longrightarrow 9x = 36 \Longrightarrow$
$x = \frac{36}{9}$

$= 4 \Longrightarrow x = 4$

Q6. If $\overline{C} = \binom{-2}{y}$ and $\overline{D} = \binom{-8}{4}$ are two vectors, determine the value of y, if these two vectors are perpendicular.

Soln.

If $\overline{C} = \binom{-2}{y}$ and $\overline{D} = \binom{-8}{4}$ are perpendicular, then $(-2 \times \overline{8}) + (y \times 4) = 0 \Longrightarrow$
$16 + 4y = 0 \Longrightarrow 4y = -16 \Longrightarrow y = \frac{-16}{4} = -4 \Longrightarrow y = -$

Q7. Find the values of x and y such that $\binom{x+3}{2} - \binom{y}{x+y} = \binom{2}{-1}$.

Soln.

$$\begin{pmatrix} x+3 \\ 2 \end{pmatrix} - \begin{pmatrix} y \\ x+y \end{pmatrix} = \begin{pmatrix} 2 \\ -1 \end{pmatrix} \Longrightarrow \begin{pmatrix} x+3-y \\ 2-x-y \end{pmatrix} = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$$

Equating corresponding component $\Longrightarrow x + 3 - y = 2$

$\Longrightarrow x - y = 2 - 3 \Longrightarrow x - y = -1 \ldots .\ eqn(1)$

Also $2 - x - y = -1 \Longrightarrow -x - y = -1 - 2 \Longrightarrow -x - y = -3 \ldots eqn(2)$

Solve eqn (1) and eqn (2) simultaneously $\Longrightarrow x = 1\ and\ y = 2$

Q8. If p = $\binom{2}{-2}$ and $q = \binom{3}{4}$, $find\ r\ such\ that\ \frac{1}{2}p - q - r = \binom{0}{0}$

Soln.

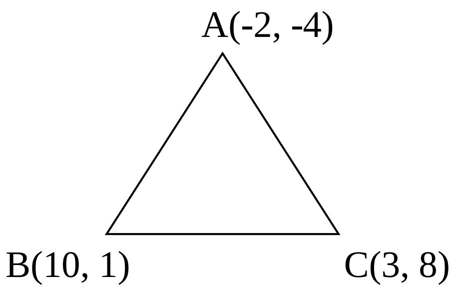
$$\frac{1}{2}p - q + r = \binom{0}{0} \Longrightarrow \frac{1}{2}\binom{2}{-2} - \binom{3}{4} + r = \binom{0}{0} \Longrightarrow \binom{1}{-1} - \binom{3}{4} + r = \binom{0}{0}$$
$$\Longrightarrow \binom{1-3}{-1-4} + r = \binom{0}{0} \Longrightarrow \binom{-2}{-5} + r = \binom{0}{0} \Longrightarrow r = \binom{0+2}{0+5} \Longrightarrow r = \binom{2}{5}$$

Q9. Triangle ABC has vertices A(-2, -4), B(10, 1) and C(3, 8).

i. Find the length of the side AB
ii. Show that the triangle is isosceles

Soln.

i.



The length of $\overline{AB}$ is the same as the magnitude of $\overline{AB}$.

A = $\binom{-2}{-4}$ *and* $B\binom{10}{1}$

$$\overline{AB} = B - A = \binom{10}{1} - \binom{-2}{-4} = \binom{12}{5}$$

Since $\overline{AB} = \binom{12}{5}, \Longrightarrow \left|\overline{AB}\right| = \sqrt{12^2 + 5^2} = \sqrt{169} = 13$

Also C = (3, 8) = $\binom{3}{8}$ $and\ A = \binom{-2}{-4}$

$\overline{AC} = C - A = \binom{3}{8} - \binom{-2}{-4} = \binom{5}{12} \left|\overline{AC}\right| = \sqrt{5^2 + 12^2} = \sqrt{169} = 13 \therefore AC = 13$

ii. $B = \binom{10}{1}\ and\ C = \binom{3}{8}$

$$\overline{BC} = C - B = \binom{3}{8} - \binom{10}{1} = \binom{-7}{7}$$

$$\left|\overline{BC}\right| = \sqrt{(-7)^2 + 7^2} = \sqrt{49 + 49} = \sqrt{98} = 9.9$$

Now for the given triangle, $\left|\overline{BC}\right| = 9.9, \left|\overline{AC}\right| = 13\ and\ \overline{AB} = 13$. Since two lengths of the given triangle are equal, (ie $\left|\overline{AC}\right| = 13\ and\ \left|\overline{AB}\right| = 13$), then it is an isosceles triangle

__Questions:__

Q1. Find the values of K and M such that

$$K\begin{pmatrix}2\\3\end{pmatrix}+M\begin{pmatrix}4\\5\end{pmatrix}=\begin{pmatrix}16\\21\end{pmatrix}$$

Ans: K = 2 and M = 3

Q2. Determine the values of Q and R such that

$$Q\begin{pmatrix}1\\3\end{pmatrix}+R\begin{pmatrix}2\\5\end{pmatrix}=\begin{pmatrix}5\\12\end{pmatrix}$$

Ans: Q = -1 and R = 3

Q3. Given that $x\binom{2}{3}-Y\binom{3}{1}=\binom{2}{10}, find\ the\ valuesof\ x$ and y.
Ans: x = 4 and y = 2

Q4. Given A (6, 4) and B(3, 2), evaluate i. $\overline{AB}$ ii. $\overline{BA}$

Ans: i. $\overline{AB}=\binom{-3}{-2}$ ii. $\binom{3}{2}=\overline{BA}$

Q5. If $x=\binom{-3}{4}\ and\ y=\binom{6}{2}, evaluate$

i. $\overline{xy}$ Ans: $\binom{9}{-2}$
ii. $The\ magnitude\ of\ \overline{xy}$ Ans: 9.2
iii. $\overline{yx}$ Ans: $\binom{-9}{2}$

Q6. Given that x = (2, 4) and y = (4, 9), determine the length of $\overline{xy}$
Ans: 5.4

Q7. Given that $\overline{x}=\binom{3}{4}\ and\ \overline{y}=\binom{-2}{1}$,evaluate

a. $3\overline{xy}$ Ans: $\binom{-15}{-9}$

b. $4(\overline{x}-\overline{y})$Ans: $\binom{20}{12}$
c. $|\overline{xy}|$ Ans: 5.8
d. $|\overline{x}+\overline{y}|$Ans: 5.1
e. $|\overline{x}-\overline{y}|$ Ans: 5.8

Q8. Given $P(2{,}4) and\ \overline{PQ}=\binom{3}{6}, determine$ the coordinates of Q.
Ans: (5, 10)

Q9. Given $P(3{,}6) and\ \overline{QP}=\binom{-1}{-2}$, determine the coordinates of Q.
Ans: (4, 8).

Q10. Given A(3, 2), $\overline{AB}=\binom{1}{5}$ and $\overline{AC}=\binom{4}{6}$, determine the coordinates of

a. the point B Ans: (4,7)
b. The point C Ans: (7,8)

Q11. Given A(2, 1), $\overline{BA}=\binom{-2}{3}$ $and\ CA=\binom{4}{1}$, determine the coordinates of

a. the point B Ans: $\binom{4}{-2}$
b. the point C Ans: $\binom{-2}{0}$

Q12. Given x (-2, 1), $\overline{yx}=\binom{-3}{2}$ and $\overline{xz}=(-3,-4)$, determine the coordinates of

a. the point Y. Ans: $\binom{1}{-1}$
b. the point Z Ans: $\binom{-5}{-3}$

Q13. Given C(2, 3), $\overrightarrow{\overline{CD}}=\binom{2}{1}$ and $\overrightarrow{\overline{DE}}=\binom{5}{4}, find\ the$ coordinates of

i. point D. Ans: (4, 4)
ii. point E. Ans: (9, 8)

Q14. Given A(3,2), $\overline{BA}=\binom{4}{-5}$ and $\overline{BC}=\binom{1}{2}$, determine the coordiantes of

a. point B. Ans: $\binom{-1}{7}$
b. point C Ans:$\binom{0}{9}$

Q15. Given A(4,2), $\overline{CB} = \binom{1}{3}$ $and\ \overrightarrow{BA} = \binom{5}{6}$, determine the coordinates of

a. the point B. Ans: (-1, -4)
b. the point C Ans: (-2, -7)

Q16. Given x(2, 4), y =(3,6), $\overline{xp} = \binom{2}{1}$ $and\ \overline{yz} = \binom{1}{1}$. Determine the coordinate of

i. Point P Ans: (4,5)
ii. Point Z Ans:(4, 7)

Q17. Given x(2, 3), y(4, 1), $\overrightarrow{Bx} = \binom{-2}{-1}$ $and\ \overrightarrow{ym} = \binom{-6}{5}$,

find the coordinates of

a. the point B. Ans: (4, 4)
b. the point M. Ans (-2, 6)

Q18. If $\overline{PQ} = \binom{1}{2}$ $and\ \overline{QR} = \binom{2}{5}, find$

i. $\overline{PR}$. Ans: $\binom{3}{7}$ ii. $|\overline{PR}|$ Ans: 7.6

Q19. If $\overline{PQ} = \binom{1}{2}$ $and\ \overline{RQ} = \binom{3}{4}$ find

i. $|\overline{PR}|$ $Ans: \binom{-2}{-2}$ ii. $\overline{PR}$ $Ans: 2.8$

Q20. Given that $\overrightarrow{QO} = \binom{-3}{4}$ $and\ \overrightarrow{OP} = \binom{-1}{2}, find$

i. $\overrightarrow{QP}$ $Ans: \binom{-4}{6}$
ii. $|\overrightarrow{QP}|$ $Ans: 7.2$

Q21. Given that $\overline{xY} = \binom{2}{1}, \overline{OY} = \binom{4}{3}, find\ \overline{xO}$.

Ans: $\binom{-2}{-2}$

Q22. If M $= \binom{2}{1}$ $and\ k = \binom{5}{3}$, evaluate

i. $4(3M + 3k)$ Ans: $\binom{64}{36}$

ii. $2(2M - k)$ Ans: $\binom{-2}{-2}$

Q23. If $P = \binom{2}{3}, q = \binom{-2}{5}$ *and* $r = \binom{5}{1}$, *evaluate* $3p - 2q + r$. *Ans*: $\binom{15}{0}$

Q24. Given $P = \binom{1}{2}, q = \binom{-2}{-3}$ *and* $r = \frac{1}{3}(2p + q)$,

evaluate r. *Ans*: $r = \begin{pmatrix} 0 \\ 1/3 \end{pmatrix}$

Q25. Find the direction of the displacement vector $\overrightarrow{AB}$, where A and B are the points (8, 4) and (6, 2) respectively. Ans:45°

Q26. Find the direction of the displacement vector $\overline{AB}$, where A and B are the points (2, -4) and (-6, -10) respectively. Ans: 37°

Q27. Determine whether or not these pairs of vectors and parallel.

a. $\overline{A} = \binom{2}{4}$ *and* $\overline{B} = \binom{6}{12}$
   Ans: *They are parallel*
b. $\overline{x} = \binom{3}{1}$ *and* $\overline{Y} = \binom{9}{3}$
   Ans: They are parallel
c. $\overline{x} = \binom{4}{3}$ *and* $\overline{Y} = \binom{8}{9}$
   Ans: They are not parallel
d. $\overline{A} = \binom{1}{2}$ *and* $\overline{B} = \binom{5}{10}$
   Ans : They are parallel

Q28. Given that the vector $\overline{A} = \binom{x}{2}$ *and* $\overline{B} = \binom{16}{8}$ are parallel vectors, determine the value of x. Ans: 4

Q29. If $\overline{P} = \binom{5}{3}$ *and* $\overline{Q} = \binom{25}{y}$, find the value of y, so that P and Q become parallel vectors. Ans: 15

Q30. Determine whether or not the following pairs of vectors are perpendiculars

a. $\overline{x} = \begin{pmatrix} 8 \\ 2 \end{pmatrix}$ $and\ \overline{Y} = \begin{pmatrix} 8 \\ 10 \end{pmatrix}$

   Ans: They are not perpendicular

b. $\overline{x} = \begin{pmatrix} 12 \\ 6 \end{pmatrix}$ $and\ \overline{Y} = \begin{pmatrix} -3 \\ 6 \end{pmatrix}$

   Ans: They are perpendicular

c. $\overline{A} = \begin{pmatrix} 4 \\ -18 \end{pmatrix}$ $and\ \overline{B} = \begin{pmatrix} 36 \\ 8 \end{pmatrix}$

   Ans: They are perpendicualr

d. $\overline{A} = \begin{pmatrix} 4 \\ 2 \end{pmatrix}$ $and\ \overline{B} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$

   Ans: They are not perpendicular

Q31. Given that $\overline{C}\begin{pmatrix} -8 \\ 4 \end{pmatrix}$ $and\ \overline{D}\begin{pmatrix} x \\ -12 \end{pmatrix}$ are two perpendicular vector, find x.

Ans: -6

Q32. Find the values of x and y such that $\begin{pmatrix} 3x+1 \\ 4 \end{pmatrix} + \begin{pmatrix} y \\ x-y \end{pmatrix} = \begin{pmatrix} 6 \\ 7 \end{pmatrix}$

Ans: x = 2 and y = -1