

# CHAPTER TWO

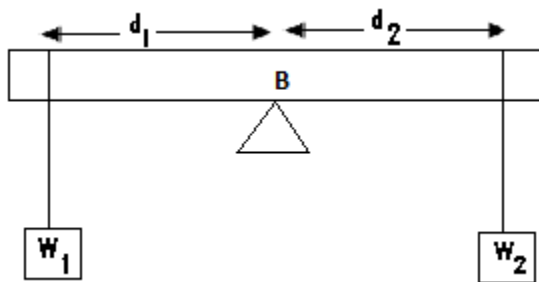
## TURNING FORCES

### Moment:

- The moment of a force about a point is the product of the force, and the perpendicular distance of its line of action from the point.
- While the clockwise moment causes an object to turn in the clockwise direction, that of the anticlockwise will cause it to turn in the anticlockwise direction.

### The principle of moments:

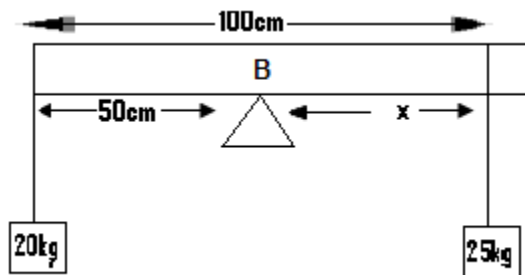
- When a body is in equilibrium, the sum of the clockwise moment about any point, is equal to the sum of the anticlockwise moment about the same point.



- Taking moment about the point B, the clockwise moment =  $W_2 \times d_2$  and the anticlockwise moment =  $W_1 \times d_1$ . - At equilibrium, the clockwise moment = the anticlockwise moment,  $\Rightarrow W_2 d_2 = W_1 d_1$ .

(Q1) A metre rule or a 100cm long ruler is pivoted at the 50cm mark, with a mass of 20g suspending from one of its ends. At what distance away from the pivot must a mass of 25g be suspended, in order to keep the rule in equilibrium.

Soln:



Let  $x$  = the distance of the 25kg mass from the pivot.

Taking moments about the point  $B \Rightarrow$  clockwise moment =  $25 \times x = 25x$ .

Anticlockwise moment =  $20 \times 50 = 1000$ .

At equilibrium, the clockwise moment = the anticlockwise moment,  $\Rightarrow 25x = 1000$

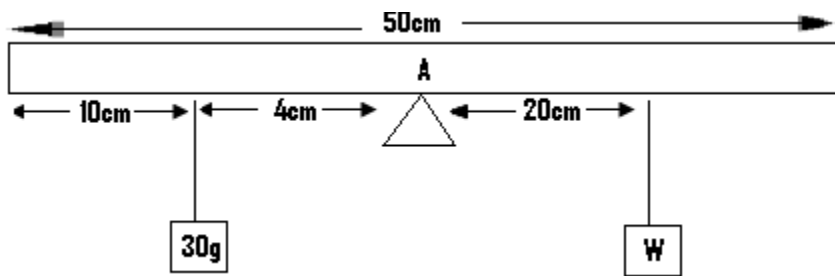
$$\Rightarrow x = \frac{1000}{25} \Rightarrow x = 40.$$

The 25g mass, must be suspended at a distance of 40cm away from the pivot.

(Q2) A half-metre rule (i.e. a 50cm long rule), has a 30g weight suspended 10cm away from one end, with the fulcum or the pivot being located 4cm away from the 30g

weight. Determine the weight which must be suspended 20cm away from the pivot in the opposite direction in order to keep the rule in equilibrium.

Soln:



Let  $W$  = the weight needed to keep the half metre rule in equilibrium.

Taking moment about the point A, clockwise moment =  $20 \times W = 20W$ .

The anticlockwise moment =  $30 \times 4 = 120$ .

At equilibrium, the sum of the clockwise moment = that of the anticlockwise moment.

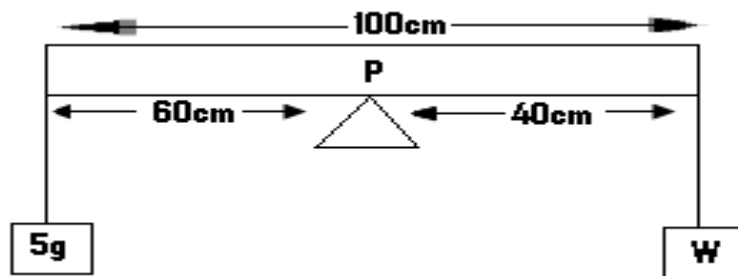
$$\Rightarrow 20W = 120 \Rightarrow W = \frac{120}{20} = 6.$$

$\Rightarrow$  The required weight = 6g.

(Q3) A metre rule is pivoted at the 60cm mark. A 5g weight is hanged at the 0cm mark (the zero centimeter mark). Find the weight which must be hanged at the 100cm mark in order to balance the rule.

N/B: Since the rule is 100cm long, the 0cm mark and the 100cm mark refer to the two ends of the rule.

Soln:



Let  $W$  = the required weight.

Taking moment about the point P,  $\Rightarrow$  clockwise moment =  $40 \times W = 40W$ .

The anticlockwise moment =  $5 \times 60 = 300$ .

At equilibrium, sum of clockwise moment = the sum of the anticlockwise moment.

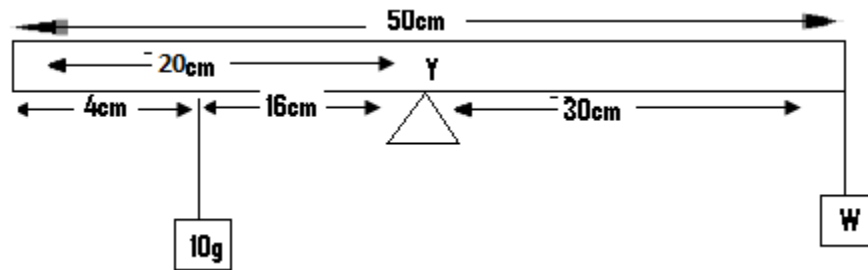
$$\Rightarrow 40W = 300 \Rightarrow W = \frac{300}{40}$$

$$\Rightarrow W = 7.5.$$

$\Rightarrow$  The required weight = 7.5g.

(Q4) A uniform-half meter rule is pivoted at the 20cm mark, and a weight of 10kg is hanged at the 4cm mark. Determine the weight which must be hanged at the other end of the rule in order to balance it.

Soln:



Let the required weight =  $W$ .

Taking moment about the point Y  $\Rightarrow$  the clockwise moment =  $30 \times W = 30W$ .

The anticlockwise moment =  $10 \times 16 = 160$ .

At equilibrium, sum of the clockwise moment = the sum of the anticlockwise moment

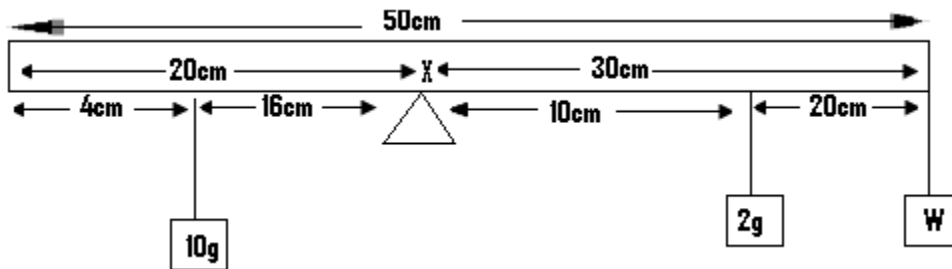
$$\Rightarrow 30W = 160 \Rightarrow W = \frac{160}{30}$$

$$\Rightarrow W = 5.3.$$

$\Rightarrow$  the required weight = 5.3g.

(Q5) A half – metre rule is pivoted at the 20cm mark, and a 10gf weight is hanged at the 4cm mark. If a weight of 2g is to be hung at the 30cm mark, find the weight which must be hanged at its end which is closer to the 2g weight, in order to keep the half metre rule in equilibrium.

Soln:



Let  $W$  = the required weight.

Taking moment about X,  $\Rightarrow$  clockwise moment  $(2 \times 10) + (30 \times W) = 20 + 30W$ .

The anticlockwise moment  $= 10 \times 16 = 160$ .

At equilibrium, the sum of the clockwise moment = the sum of the anticlockwise moment.

$$\Rightarrow 20 + 30W = 160$$

$$\Rightarrow 30W = 160 - 20 = 140,$$

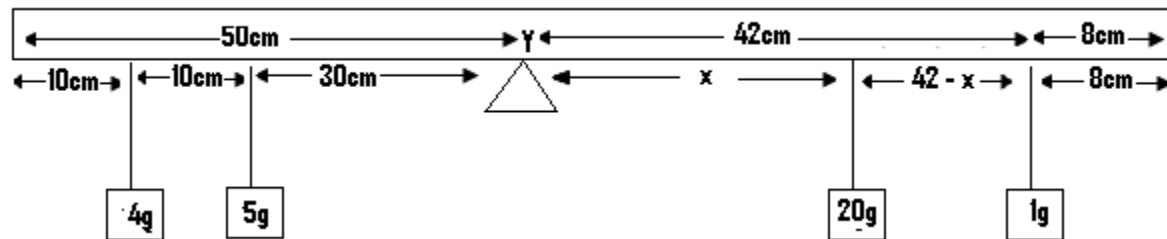
$$\Rightarrow W = \frac{140}{30} = 4.6$$

$\Rightarrow$  The required weight = 4.6g.

(Q6) A 100cm rule is pivoted at its mid point or the 50cm mark, and as such divided into two portions. With reference to the first portion, a 5gf weight is hanged 30cm away from the pivot. Another weight of mass 4gf also hangs at a point which is 40cm away

from the pivot. With respect to the other portion, a 1gf weight hangs 8cm away from the end. At what distance away from the pivot must we hang a 20gf weight in order to keep the rule in equilibrium.

Soln:



Let  $x$  = the distance of the 20g weight from the pivot.

Taking moment about the point Y  $\Rightarrow$  the sum of the clockwise moments =  $(20 \times x) + (1 \times 42) = 20x + 42$ .

The sum of the anticlockwise moment =  $\{4 \times (10 + 30)\} + \{(5 \times 30) = (4 \times 40) + (150) = 160 + 150 = 310$ .

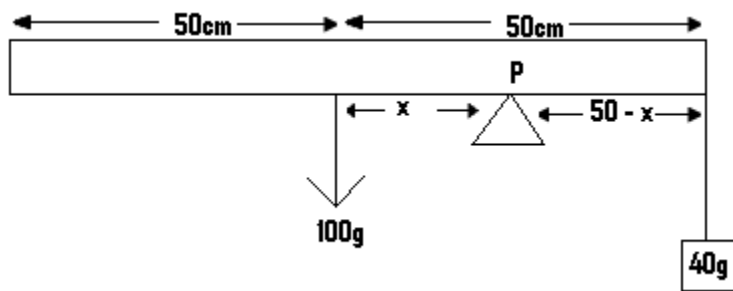
But at equilibrium, the sum of the clockwise moment = the sum of the anticlockwise moment  $\Rightarrow 20x + 42 = 310 \Rightarrow x = 13.4 \Rightarrow$  the 20g weight must be positioned 13.4cm away from the pivot.

N/B:

- The weight or mass of a metre rule or a half metre rule, always acts through its centre.
- In the case of a metre rule, the weight acts through the 50cm mark, and with the half metre rule, it acts through the 25cm mark.
- If the weight of the rule is given in a question, then it must be taken into consideration during calculation.

(Q1) A uniform metre rule has a mass of 100g, and a load of 40g hangs at one of its ends. At what position must the pivot be placed, in order to balance the rule.

Soln:



The weight of the rule which is 100g will act through the 50cm mark.

Taking moment about the point P => the clockwise moment

$$= 40 \times (50 - x) = 40(50 - x) = 2000 - 40x.$$

$$\text{The anticlockwise moment} = 100 \times x = 100x.$$



At equilibrium, sum of the clockwise moment = the sum of the anticlockwise moment

$$\Rightarrow 2000 - 40x = 100x,$$

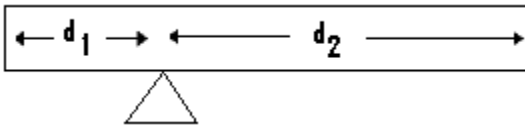
$$\Rightarrow 2000 = 100x + 40x,$$

$$\Rightarrow 2000 = 140x \Rightarrow x = \frac{2000}{140}$$

$$= 14\text{cm}.$$

$\Rightarrow$  the pivot must be placed at a distance of 14cm away from the centre of the rule.

N/B:



Consider the diagram drawn. Since the length  $d_2$  is greater than that of  $d_1$ ,  $d_2$  is referred to as the longer arm, while  $d_1$  is referred to as the shorter arm.

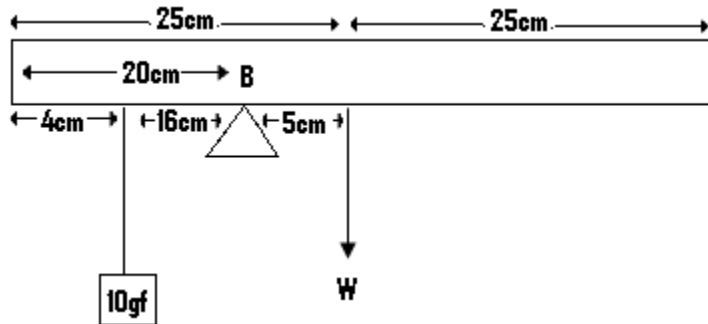
(Q2) A uniform half – metre rule which is freely pivoted at the 20cm mark, balances horizontally when a 10gf weight is hanged from the 4cm mark.

(a) Draw a diagram to show all the forces which acts on the rule.

(b) Find the weight of the rule.

Soln:

(a)



Where  $W$  = the weight of the rule, which must act through its mid portion i.e. the 25cm mark.

(b) Taking moment about point B  $\Rightarrow$  clockwise moment =  $5 \times W = 5W$ .

Anticlockwise moment =  $10 \times 16 = 160$ .

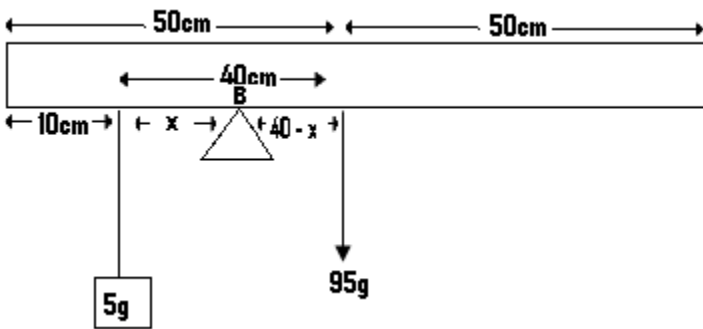
Since at equilibrium, sum of clockwise moment = sum of anticlockwise moment

$$\Rightarrow 5W = 160$$

$$\Rightarrow W = \frac{160}{5} = 32 \Rightarrow \text{the weight of the half meter rule} = 32\text{gf}.$$

(Q3) A wooden lath of length 100cm and mass 95g, balances on a knife edge when a 5g mass is hanged 10cm away from one of its ends. Determine the distance between the knife edge and the centre of the lath.

Soln:



The weight of the rule which is 95g will act through the mid point i.e the 50cm mark.

Let  $x$  = the distance between the 5g weight and the pivot,  $\Rightarrow$  the distance between the pivot and the centre of the lath =  $40 - x$ .

Taking moment about the point B  $\Rightarrow$  the clockwise moment =  $95(40 - x) = 3800 - 95x$ .

The anticlockwise moment =  $5 \times x = 5x$ .

But at equilibrium, the sum of the clockwise moment = the sum of the anticlockwise moment.

$$\Rightarrow 3800 - 95x = 5x.$$

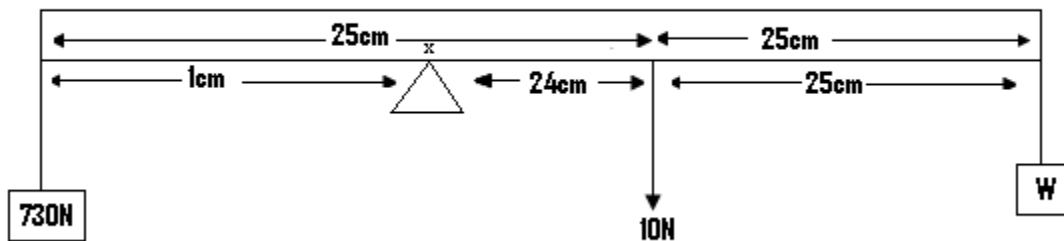
$$\Rightarrow 3800 = 5x + 95x = 100x,$$

$$\Rightarrow x = \frac{3800}{100} = 38\text{cm}.$$

But the distance between the centre of the lath and the knife edge =  $40 - x = 40 - 38 = 2\text{cm}$ .

(Q4) A simple weighing machine in the form of a uniform bar of length 50cm and of weight 10N. It is pivoted at 1cm from one of its ends. Find the weight which must be suspended at the end of the long arm, in order to balance a 730N weight, which has been suspended at the end of the short arm.

Soln:



Let  $W$  = the weight hanged at the end of the longer arm.

Taking moment about the point  $x \Rightarrow$  clockwise moment

$$= (24 \times 10) + (24 + 25) \times W = 240 + 49W.$$

$$\text{The anticlockwise moment} = 730 \times 1 = 730.$$

At equilibrium, sum of clockwise moment = that of the anticlockwise moment

$$\Rightarrow 240 + 49W = 730,$$

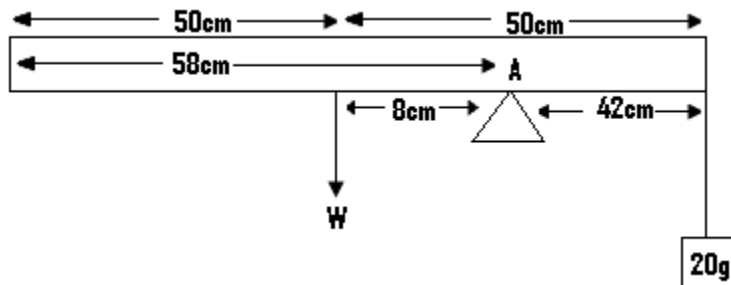
$$\Rightarrow 49W = 730 - 240 = 490,$$

$$\Rightarrow W = \frac{490}{49} = 10.$$

$\Rightarrow$  The required weight = 10g,

(Q5) A uniform metre rule has the knife edge placed at the 58cm mark, and balances when a 20gf weight is suspended from the short arm. Find the weight of the rule.

Soln:



Let  $W$  = the weight of the metre rule which must act through the 50cm mark

Taking moment about the point A  $\Rightarrow$  the clockwise moment =  $42 \times 20 = 840$ .

Anticlockwise moment =  $8 \times W = 8W$ .

At equilibrium, sum of clockwise moment = sum of anticlockwise moment  $\Rightarrow 8W = 840$

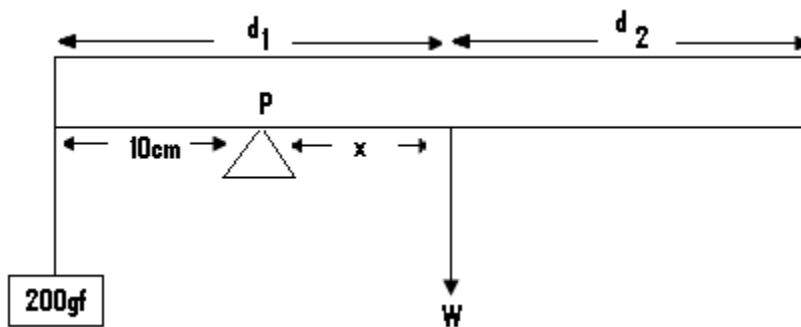
$$\Rightarrow W = \frac{840}{8}, \Rightarrow w = 105g.$$

(Q6) A uniform stick can be balanced on a knife edge, 10cm from one end when a weight of 200gf is hung from that end. When the knife edge is moved 5cm further from that end, the weight had to be moved to a point 8.75cm from the knife – edge to obtain a balance. Find the length of the stick as well as its weight.

Soln:

The first case will give rise to the next figure:

Figure (1):

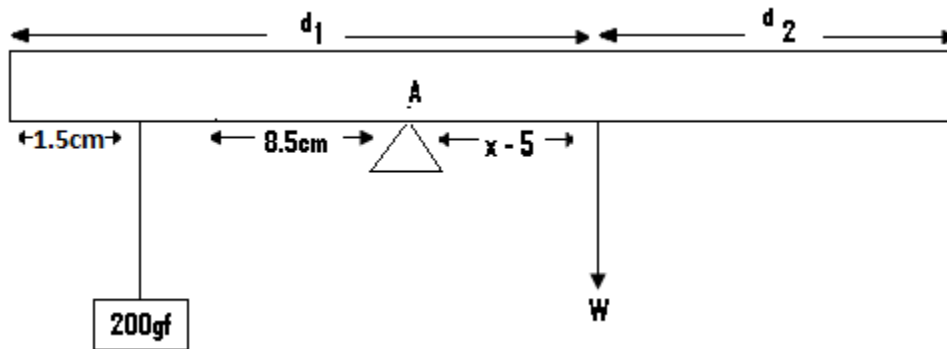


Where  $d_1 = d_2$  and  $W$  = the weight of the stick which acts through its centre.

In the second case when the knife edge was moved 5cm further from the end, the weight has to be moved to a point 8.75cm from the knife edge, in order to obtain balance.

This will give rise to this second figure:

Figure (2):



From figure (1), taking moment about the point P => clockwise moment =  $W \times x = wx$ .

Anticlockwise moment =  $200 \times 10 = 2000$ .

At equilibrium, sum of clockwise moments = sum of anticlockwise moment.

$$\Rightarrow wx = 2000 \text{ .....eqn (1)}$$

From figure (2), taking moment about the point A, clockwise moment =  $w(x - 5) = wx - 5W$ .

Anticlockwise moment =  $200 \times 8.75 = 1750$ .

At equilibrium, sum of clockwise moment = sum of anticlockwise moment

$$\Rightarrow wx - 5w = 1750 \text{ .....eqn (2)}.$$

Now solve eqn (1) and eqn (2) simultaneously.

$$\text{i.e } wx = 2000 \text{ ..... eqn (1)}$$

$$Wx - 5 \times 1750 \text{ ..eqn (2)}$$

Multiply through eqn (2) using -1 and this gives us

$$-wx + 5w = -1750 \text{ .....eqn (3)}$$

Solve eqn (1) and eqn (3) simultaneously by adding them together.

$$wx = 2000$$

$$\underline{-wx + 5w = -1750}$$

$$\underline{\hspace{1cm} 5w = 250}$$

$$\Rightarrow 5w = 250 \Rightarrow w = \frac{250}{5} = 50.$$

Therefore  $w = 50\text{g}$ .

$$\text{From } wx = 2000 \Rightarrow 50x = 2000$$

$$\Rightarrow x = \frac{2000}{50} = 40.$$

But from figure (1)  $d_1 =$  half the length of the stick  $= 10 + x = 10 + 40 = 50\text{cm}$

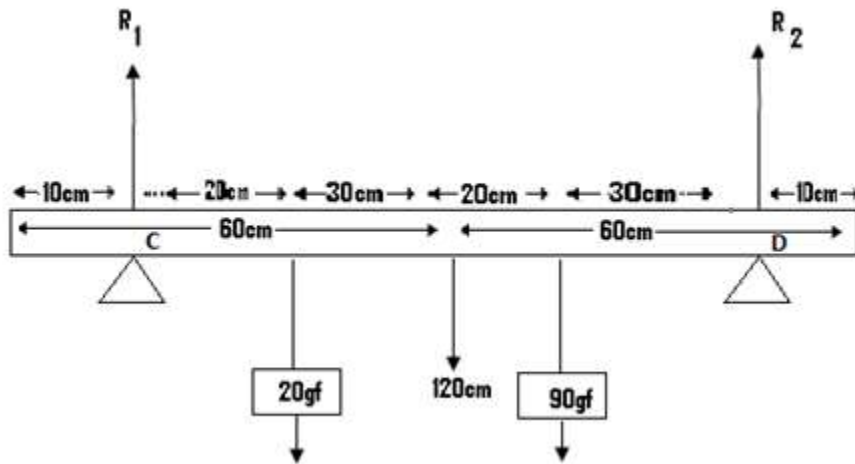
$\Rightarrow$  the total length of the stick  $= 2 \times 50 = 100\text{cm}$ .

(Q7) A uniform wooden lath, 120cm long and weighing 120gf, rests on two sharp edged supports C and D, placed 10cm from each end of the lath respectively. A 20gf weight hangs from a loop of thread 30cm from A and a 90gf weight hangs similarly 40cm from



B. Determine the reaction at the supports, if A and B refers to the ends of the wooden lath.

Soln:



Let  $R_1$  = the reaction at C and  $R_2$  = the reaction at D, total downwards forces

$$= 20 + 120 + 90 = 230gf.$$

Total upward reaction =  $R_1 + R_2$ .

But the total upward reaction = the total downward forces  $\Rightarrow R_1 + R_2 = 230$ .

$$\Rightarrow R_2 = 230 - R_1 \dots\dots\dots \text{eqn (1)}$$

Taking moments about the point D, clockwise moments

$$= (90 \times 30) + (120 \times 50) + (20 \times 80) = 10300gf.$$

Also, the anticlockwise moment =  $(R_2 \times 0) + (R_1 \times 100)$

$$= 100R_1.$$

But at equilibrium, sum of clockwise moment = sum of anticlockwise moment

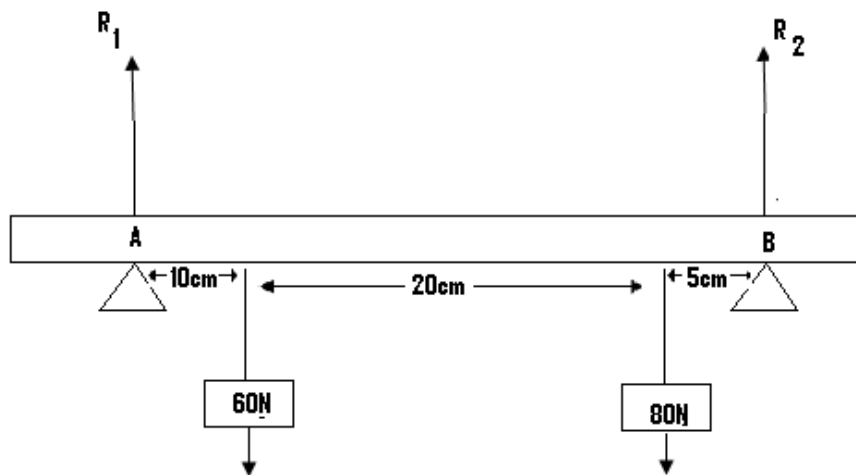
$$\Rightarrow 100R_1 = 10300.$$

$$\Rightarrow R_1 = \frac{10300}{100} = 103gf.$$

But from eqn(1)  $R_2 = 230 - R_1$

$$\Rightarrow R_2 = 230 - 103 = 127gf.$$

(Q8)



Determine the values of  $R_1$  and  $R_2$ .

N/B: Since in this case, the forces are given in newtons, all the measurement or values given in centimeters must be converted into metres.

$$10cm = 0.1m, 20cm = 0.2m \text{ and } 5cm = 0.05m.$$

Soln:

Let  $R_1$  = the reaction at A and  $R_2$  = the reaction at B.

$$\text{Downward forces} = 60 + 80 = 140N.$$

$$\text{Total upward reaction} = R_1 + R_2.$$

$$\text{But the total upward reaction} = \text{the downward forces}, \Rightarrow R_1 + R_2 = 140$$

$$\Rightarrow R_2 = 140 - R_1.$$

Taking moment about A  $\Rightarrow$  the clockwise moment

$$= (80 \times 30cm) + (60 \times 10cm) = 3000,$$

$$\Rightarrow (80 \times 0.3m) + (60 \times 0.1m) = 24 + 6 = 30.$$

The anticlockwise moment =  $(R_1 \times 0) + (R_2 \times 35\text{cm})$

$$= (R_1 \times 0) + (R_2 \times 0.35m)$$

$$= 0.35R_2.$$

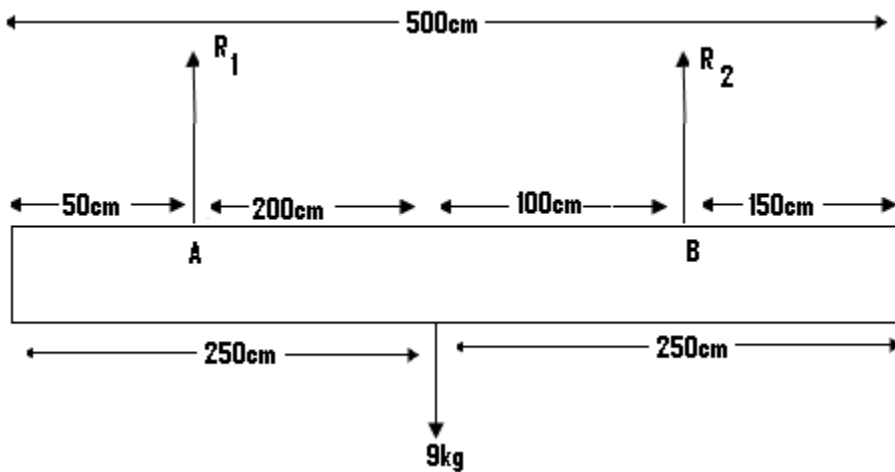
Since sum of clockwise moment = the sum of the anticlockwise moment

$$\Rightarrow 0.35R_2 = 30, \Rightarrow R_2 = 86\text{N}.$$

$$\text{But } R_1 + R_2 = 140, \Rightarrow R_1 = 140 - R_2 = 140 - 86 = 54\text{N}.$$

(Q9) A uniform metal tube of length 5m and mass 9kg, is suspended horizontally by two vertical wires, attached at 50cm and 150cm respectively from the ends of the tube. Find the tension in each of the wires.

Soln:



The length of the tube = 5m = the 500cm. Since the mass of the rule is given in kg, all values given in centimeter must be changed into metres.

The weight of the tube will act through its mid point or the 250cm mark.

Let  $R_1$  and  $R_2$  be the tensions in the wires.

Total downward forces = 9kgf.

The total tension or reaction =  $R_1 + R_2$ .

At equilibrium, the total tension = the downward forces or force

$$\Rightarrow R_1 + R_2 = 9 \Rightarrow R_2 = 9 - R_1.$$

Taking moments about the point A  $\Rightarrow$  clockwise moment

$$= 9 \times 200\text{cm} = 9 \times 2\text{m} = 18$$

The anticlockwise moment =  $(R_1 \times 0) + R_2 \times 300\text{cm} = R_2 \times 3\text{m} = 3R_2$

Since sum of clockwise moment = sum of anticlockwise moment

$$\Rightarrow 18 = 3R_2 \Rightarrow R_2 = \frac{18}{3} = 6 \Rightarrow R_2 = 6\text{kgf.}$$

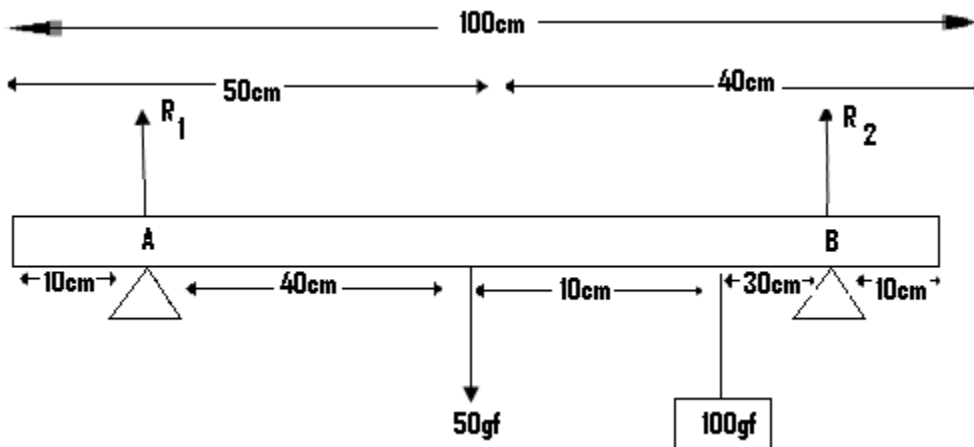
$$\text{From } R_1 + R_2 = 9 \Rightarrow R_1 + 6 = 9,$$

$$\Rightarrow R_1 = 9 - 6 = 3,$$

$$\Rightarrow R_1 = 3\text{kgf.}$$

(Q10) A uniform rod 1m long and weighing 50gf is supported horizontally on two knife edges, placed 10cm from its ends. What will be the reactions at these supports, when a 100gf weight is suspended 10cm from the mid – point of the rod.

Soln:



Let the reactions at A and B =  $R_1$  and  $R_2$ .

$$\Rightarrow \text{Total reaction} = R_1 + R_2.$$

$$\text{Downward forces} = 50 + 100 = 150\text{ gf.}$$

$$\text{But since downward forces} = \text{total reaction} \Rightarrow R_1 + R_2 = 15 \Rightarrow R_2 = 15 - R_1.$$

$$\text{Taking moment about A,} \Rightarrow \text{clockwise moment} = (50 \times 40) + (100 \times 50) = 7000.$$

$$\text{Anticlockwise moment} = (R_1 \times 0) + (R_2 \times 80) = 80R_2$$

But at equilibrium, the sum of the clockwise moment = the sum of the anticlockwise moment.

$$\Rightarrow 80R_2 = 7000 \Rightarrow R_2 = \frac{7000}{80},$$

$$\Rightarrow R_2 = 87.5\text{gf.}$$

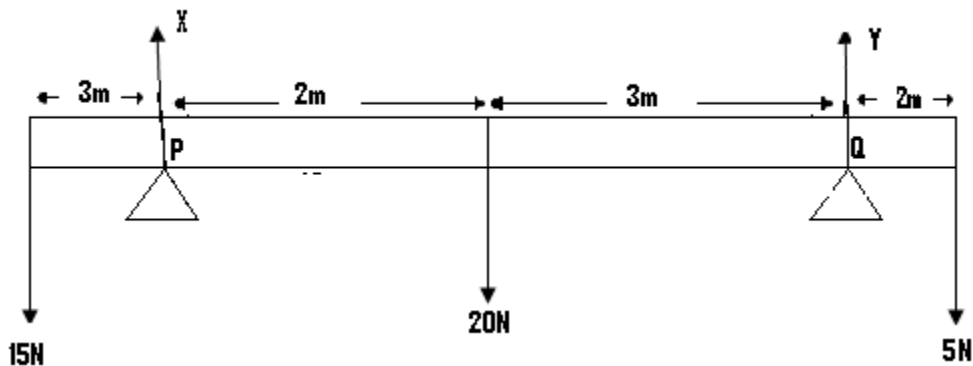
$$\text{From } R_1 + R_2 = 150 \Rightarrow R_1 = 150 - 87.5$$

$$\Rightarrow R_1 = 62.5\text{gf.}$$

N/B:

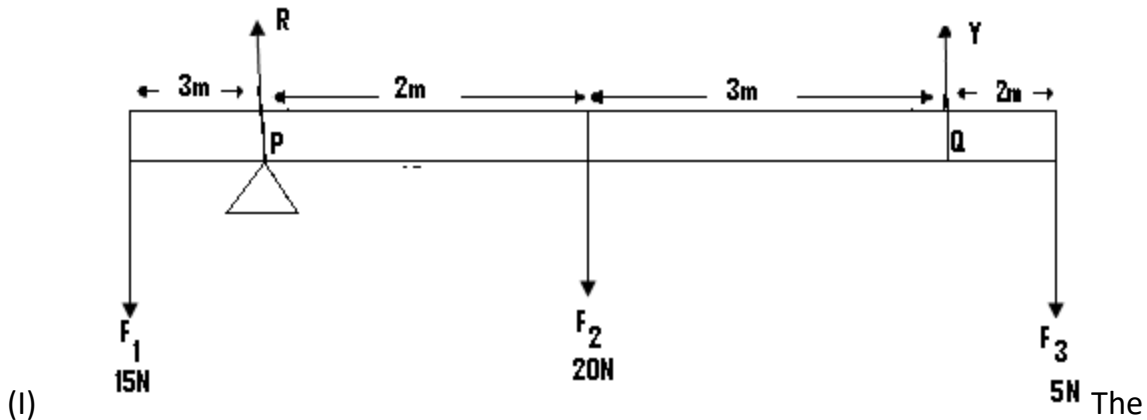
Another method which can be used in solving questions on moments are illustrated in the given examples.

(Q1) A uniform bar with weight 20N rests on supports P and Q. If weights are suspended as shown in the given diagram, calculate the reactions at P and Q.



Soln:

Eliminating one of the supports, the given diagram becomes as shown next:



But since sum of clockwise moment = sum of the anticlockwise moment =>  $45 + 5Y = 75$ ,

$$\Rightarrow 5Y = 75 - 45 = 30,$$

$$\Rightarrow Y = \frac{30}{5} = 6N \Rightarrow Y = 6N.$$

The sum of the upward forces =  $R + Y = R + 6$ .

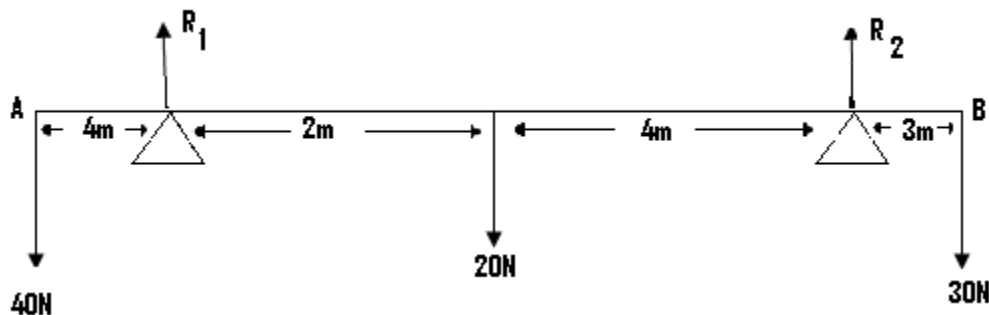
The sum of the downward forces =  $15 + 20 + 5 = 40$ .

Since the sum of the upward forces = the sum of the downward forces, then  $R + 6 = 40$ ,

$$\Rightarrow R = 40 - 6 = 34,$$

$$\Rightarrow R = 34N.$$

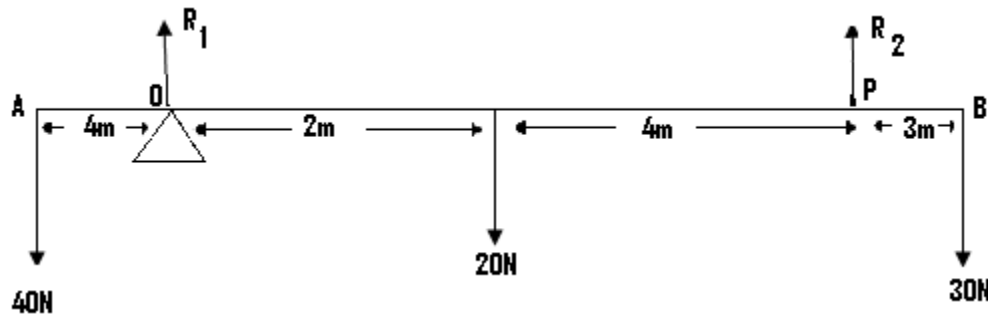
(Q2) Consider the figure given:



Determine the values of  $R_1$  and  $R_2$ .

N/B:  $R_1$  which also causes anticlockwise moment can be neglected, since moment is taken about the point O, for which the moment of  $R_1$  will be zero.

Soln:



The forces which will cause clockwise movement = 20N and 30N.

The forces which will cause movement in the anticlockwise direction = 40N and  $R_2$ .

Taking moment about the point O  $\Rightarrow$  clockwise moment

$$= (20 \times 2) + (30 \times 9) = 40 + 270 = 310.$$

$$\text{The anticlockwise moment} = (40 \times 4) + (R_2 \times 6) = 160 + 6R_2.$$

At equilibrium, sum of clockwise moment = the sum of anticlockwise moment

$$\Rightarrow 160 + 6R_2 = 310,$$

$$\Rightarrow 6R_2 = 310 - 160 = 150,$$

$$\Rightarrow R_2 = \frac{150}{6} = 25.$$

The sum of the downward forces =  $40 + 20 + 30 = 90\text{N}$ .

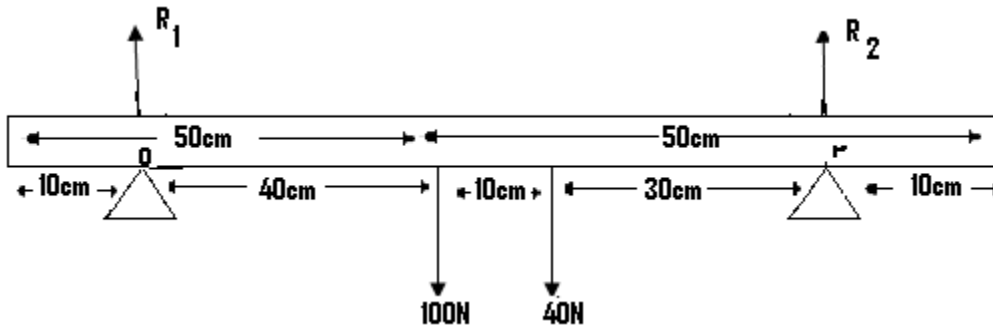
Since the sum of upward forces = that of downward forces  $\Rightarrow R_1 + R_2 = 90$ ,  $\Rightarrow R_1 + 25 = 90$ ,

$$\Rightarrow R_1 = 90 - 25 = 65\text{N}.$$



(Q3) A 1m uniform rod of weight 100N, is supported horizontally on two knife edges O and P, placed at 10cm from each of its ends. Determine the reactions at the supports, when a 40N weight is suspended 10cm from the mid point of the rod.

Soln:



(1) The forces which will cause clockwise moment = 100N and 40N.

(2) Those that will cause anticlockwise moment =  $R_1$  and  $R_2$ .

Taking moment about the point O  $\Rightarrow$  clockwise moment =  
 $(100 \times 40) + (40 \times 50) = 6000$ .

The anticlockwise moment =  $(R_1 \times 0) + (R_2 \times 80) = 80R_2$ .

Since sum of clockwise moment = sum of anticlockwise moment  $\Rightarrow 80R_2 = 6000$

$$\Rightarrow R_2 = \frac{6000}{80} = 75, \Rightarrow R_2 = 75\text{N}.$$

Also since downward forces = upward forces  $\Rightarrow R_1 + 75 = 140, \Rightarrow R_1 = 140 - 75 = 65\text{N}$ .

N/B:

In case the uniform object is resting or balanced only on one support, take moment about the point of support and proceed further.

(Q1) A uniform rod APC, 10m long and of mass 15kg, is supported in a horizontal position on a knife pivot at the point P. If masses 20kg and 25kg are supported from the ends, A and C respectively, calculate

(i) the magnitude of the reaction at P.

(ii) the distance AP.

[Take  $g = 10\text{ms}^{-2}$ ]

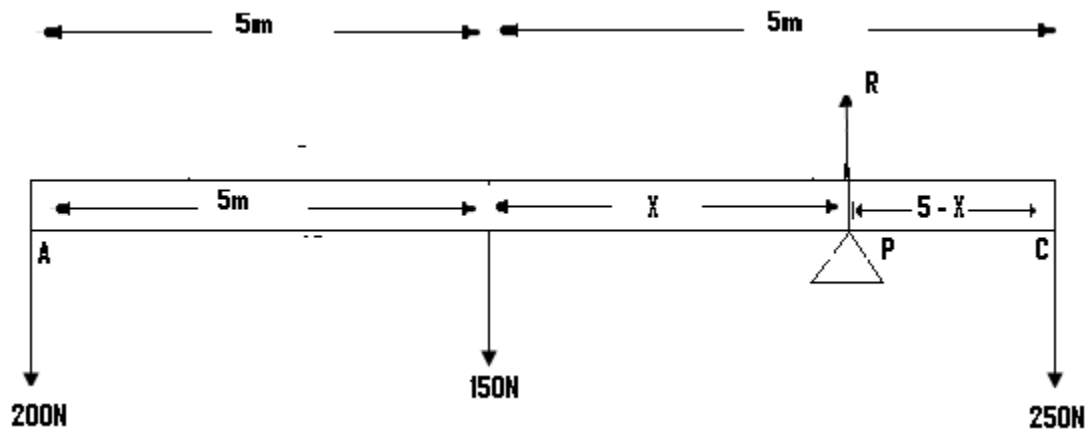
N/B: Convert all the given weights into newtons,

$$\Rightarrow 20\text{kg} = 200\text{N}, 15\text{kg} = 150\text{N} \text{ and } 25\text{kg} = 250\text{N}.$$

- The weight of the rod also acts through its mid portion.

Soln:

(a)



Let  $R$  = the reaction at the point  $P$ .

Let  $x$  = the distance of the pivot from the centre of the rod,  $\Rightarrow$  the distance of the pivot from the point  $C = 5 - x$ .

- (I) Upward force =  $R$ .
- (II) Downward forces =  $200 + 150 + 250 = 600\text{N}$ .

Since the upward force or forces = the downward forces  $\Rightarrow R = 600\text{N}$ .

- (III) Taking moment about the point  $P \Rightarrow$  sum of clockwise moment =  $250(5 - x)$   
 $= 1250 - 250x$ .  
 The sum of the anticlockwise moment =  $(150 \times x) + 200(5 + x)$   
 $= 150x + 1000 + 200x = 350x + 1000$ .

But since sum of clockwise moment = sum of anticlockwise moment

$$\Rightarrow 1250 - 250x = 350x + 1000.$$

$$\Rightarrow 1250 - 1000 = 350x + 250x,$$

$$\Rightarrow 250 = 600x \Rightarrow x = \frac{250}{600} = 0.42\text{m}.$$

The distance  $AP = 5\text{m} + x = 5\text{m} + 0.42 = 5.42\text{m}$ .

(Q2) A metre rule is found to balance at the 48cm mark. When a mass of 60g is suspended at the 6cm mark, the balance point is found to be 30cm. Calculate

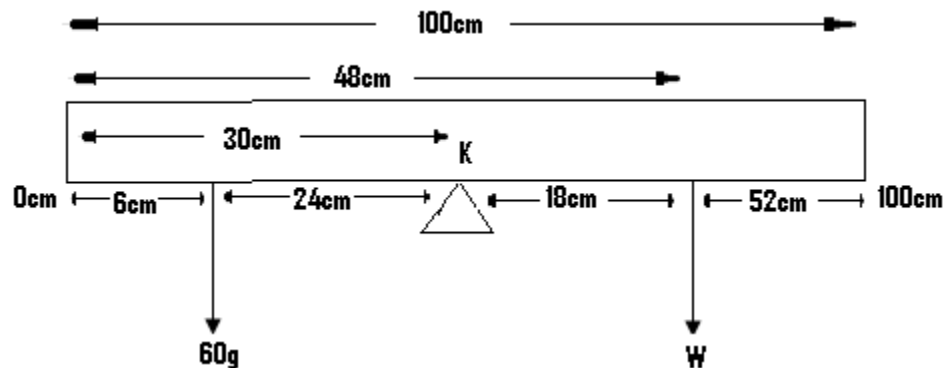
- (I) the mass of the metre rule.
- (II) the new position where the rule must be pivoted, if the 60g mass is now moved to the 13cm mark.

N/B:

- (I) The position where an object such as a metre rule can be made to balance, is at the point where the weight of the body acts.
- (II) It is only when a rule is uniform that its weight acts through its mid point.
- (III) If it is not uniform, then the weight will not act through its centre or mid point.
- (IV) In the given question, since the rule is not said to be uniform, then its weight will not act through the mid portion or the 50cm mark.
- (V) Because the rule balances at the 48cm mark,  $\Rightarrow$  the weight of the rule acts through the 48cm mark.

Soln:

(a)



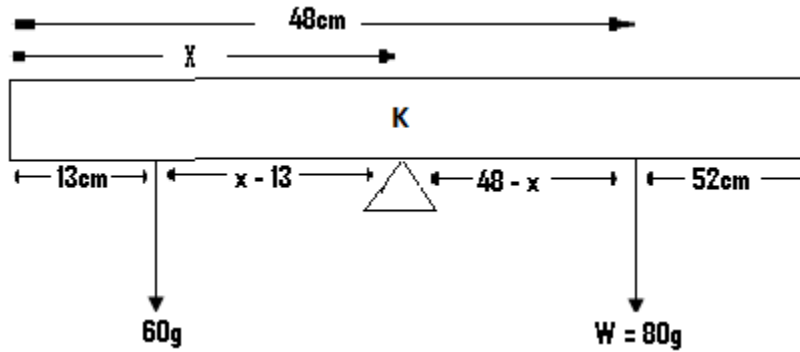
Taking moment about the point K  $\Rightarrow$  clockwise moment =  $18 \times W = 18W$ .

Anticlockwise moment =  $60 \times 24 = 1440$ .

Since the sum of clockwise moment = the sum of anticlockwise  $\Rightarrow 18W =$

$1440 \Rightarrow W = \frac{1440}{18} = 80g$ ,  $\Rightarrow$  the weight of the metre rule = 80g.

(b)



Let  $x$  = the distance of the new balance point from the zero mark end of the rule.

Taking moment about the point K  $\Rightarrow$  clockwise moment  $= 80(48 - x) = 3840 - 80x$ .

Anticlockwise moment  $= 60(x - 13) = 60x - 780$ .

At equilibrium sum of clockwise moment = the sum of the anticlockwise moment

$$\Rightarrow 60x - 780 = 3840 - 80x$$

$$\Rightarrow 60x + 80x = 3840 + 780,$$

$$\Rightarrow 140x = 4620$$

$$\Rightarrow x = \frac{4620}{140} = 33m.$$

### Questions:

- (1) A uniform metre rule is pivoted at the 60cm mark, and a weight of 20g hangs at the 15cm mark. Another weight of 40g hangs at the 30cm mark. Determine the weight which must be hanged at the other end of the rule, in order to keep it in equilibrium.  
Ans: 52.5g.
- (2) A uniform-half metre rule is pivoted at the 20cm mark. A 25g weight hangs at the zero centimetre mark. Between this and the pivot can be found a 10g weight, positioned xcm away from the pivot. 10cm away from the other end hangs a 30g weight and 5cm away from this weight, when moving towards the pivot is a 2g weight. If the system is expected to be in equilibrium, what must be the value of x.  
Ans: 13cm.
- (3) A ruler is pivoted at the 28cm mark and a 10g mass hangs at the 4cm mark, and between this and the pivot hangs a 15g mass. From the other end hangs a 30g mass, positioned 11cm away from the pivot and between this and the pivot hangs a 12g mass. Given that the distance between the 15g mass and the pivot is twice that between the 12g mass and the pivot, determine the actual distance between  
(a) the pivot and the 15g mass.   Ans: 10cm.  
(b) the pivot and the 12g mass.   Ans: 5cm.
- (4) A uniform metre rule of weight 90gf is pivoted at the 30cm mark. Moving away from the centre in the direction of the pivot, a 15g mass is positioned 5cm away from the pivot, and a 10gf load hangs at the 90cm mark. Determine the weight which must be hanged at the other end, in order to keep the rule in equilibrium.  
Ans: 83gf.
- (5) An almost uniform half metre rule of mass 17g, was balanced at the 20cm mark. It was then pivoted at the 15cm mark and weights of 5g and 10g hangs 10cm and 20cm respectively from the 50cm mark. If a 4g weight is hanged at the zero centimetre mark, determine the load which was hanged 5cm away from this end of the rule in order to obtain equilibrium.  
Ans: 30g.
- (6) A uniform plastic tube has a length of 6m and a mass of 12kg. It is supported in the horizontal position by two vertical cords at the 100cm mark as well as the 400cm mark. Determine the tensions in these cords.  
Ans: 8N and 4N.
- (7) A uniform 100cm rule has a mass of 50g, and is pivoted 30cm away from the zero centimetre mark. A body of mass 40g, hangs 15cm away from this end, and 10cm away from the the other end hangs a 2g mass. If the rule is to be made to be in equilibrium, find the mass which must be hanged 5cm away from the zero centimetre end of the rule.  
Ans: 21.6g.

(8) A uniform wooden lath which is 140cm long, has a weight of 90gf and rests on two sharp edged supports P and M. P is located 20cm away from the zero centimetre mark end of the lath, while M is located at the 130cm mark. Moving towards the mid portion of the rule, a 50gf weight hangs 10cm away from P, while a 60g weight hangs 30cm away from M. If the reactions at P and M are respectively  $R_1$  and  $R_2$ , determine the values of  $R_1$  and  $R_2$ .

Ans:  $R_1 = 111\text{N}$  and  $R_2 = 89\text{N}$ .