CHAPTER SIX

SIMULTANEOUS EQUATIONS

* In simultaneous equation, one may be given two equations, containing two unknown variables.
* To solve these equations simultaneously means that you must determine a set of values for these unknown variables, such that when these values are substituted into any of the two equations in turn, each will be satisfied.
* Different methods such as the elimination method, the substitution method or the graphical method can be applied.

(Q1) Solve the equations given simultaneously

$a + b = 10$

$a - b = 4$

N/B

(1) Let the first equation be equation (1) and the second one be equation (2).
(2) Ensure that the second letters or the unknown variables of each of the equations (i.e. the b in 'this case) are of the same value.
(3) Ensure also that one of the signs is positive while the other is negative.
(4) When all these conditions have been satisfied, the two equations are added up.

Solution

$a + b = 10$----equation (1)

$a - b = 4$------equation (2)

Since each b has the same value as the other one , and we have both the positive as well as the negative signs being available, we add them together.

i.e.

$a + b = 10$ ----------equation (1)

$+\underline{a - b = 4}$ -----------equation (2)

$\underline{2a \quad = 14}$

$\Rightarrow 2a = 14 \Rightarrow a = \frac{14}{2} = 7$

N/B: When positive b is added to negative b, we get 0 for which there is no need to indicate.

In order to find the value of b, substitute or put a = 7 into either equation (1) or equation (2).

Substituting a =7 into eqn. (1)

$\Rightarrow a + b = 10$

$\therefore 7 + b = 10$

$\Rightarrow b = 10 - 7 = 3 \Rightarrow b = 3.$

N/B : The values a = 7 and b = 3 when substituted into either equation (1) or equation (2) must satisfy or balance it.

i.e. a + b = 10 ----------eqn (1)

$\Rightarrow 7 + 3 = 10$

$\Rightarrow 10 = 10.$

Also a - b = 4-------------eqn (2)

$\Rightarrow 7 - 3 = 4$

$\Rightarrow 4 = 4.$

(Q2) Solve the following equations simultaneously

$x + y = 3$ and $x - y = -1$.

Soln

Let $x + y = 3$ …………….eqn (1)

And $x - y = -1$………… eqn (2)

Adding the two equations up

$\Rightarrow x + y = 3$

$+\underline{x - y = -1}$

$\underline{\ \ 2x \qquad = 2}$

$\therefore 2x = 2 \Rightarrow x = 2/_2 = 1.$
Substitute $x = 1$ into eqn (1) to find the value of y
i.e. $x + y = 3 \Rightarrow 1 + y = 3,$
$\Rightarrow y = 3 - 1 \Rightarrow y = 2.$

The values of x and y which satisfy simultaneously the two given equations are

$x = 1$ and $y = 2.$

N/B: The above method used is referred to as the elimination method.

The same question could have been solved, using the substitution method, which is illustrated next:

$x + y = 3$ …………….eqn (1)

$x - y = -1$………… eqn (2)

From eqn (1) , $x + y = 3$

$\Rightarrow x = 3 - y$. Substitute $x = 3 - y$ into eqn (2)

i.e $x - y = -1 \Rightarrow$

$(3 - y) - y = -1, \Rightarrow 3 - y - y = -1,$

$\therefore 3 - 2y = -1 \Rightarrow -2y = -1 - 3,$

$\Rightarrow -2y = -4 \Rightarrow \frac{-2y}{-2} = \frac{-4}{-2},$

$\Rightarrow y = 2.$

Substitute $y = 2$ into eqn (1) to find x,

i.e. $x + y = 3 \Rightarrow x + 2 = 3,$

$\Rightarrow x = 3 - 2$

$\Rightarrow x = 1$.

(Q3) Solve the following equations simultaneously:
$p + 2q = 12$
$p - q = 3$

Soln

Let $p + 2q = 12$ …………………eqn (1) and $p - q = 3$ ... ...eqn (2)

N/B: Considering these two equations, the values of q are not the same, or equal.

– In order to make them equal, 2 is used to multiply through eqn. 2 (i.e $p - q = 3$)

– Multiplying through eqn (2) by 2

$\Rightarrow 2 \times p - 2 \times q = 2 \times 3$

$\therefore 2p - 2q = 6$ …………………eqn (3) –
After multiplying through an equation with any number, it changes into another equation

–For this reason, eqn (2) changes into eqn (3) after using 2 to multiply through it.

– We now consider equation (1) and equation (3)

i.e.

$p + 2q = 12$ …………………eqn (1)

$2p - 2q = 6$ ... ...eqn (3)

Since each q has the same value as the other one, with both the negative and positive signs being present, we add them up.

i.e

$.p + 2q = 12$
$+ \quad 2p - 2q = 6$

3p =18

$3p = 18 \Rightarrow p = \frac{18}{3} = 6$.

Substitute $p = 6$ into eqn (2)

i.e $p - q = 3, \Rightarrow 6 - q = 3,$

$\therefore 6 - 3 = q \Rightarrow q = 3$.

The required answer is p = 6 and q = 3.

Method 2 (Substitution Method):

$p + 2q = 12$……………….eqn (1)

$p - q = 3$ …………………eqn (2)

From eqn (2) $p - q = 3 \Rightarrow p = 3 + q$.

Substitute $p = 3 + q$ into eqn (1) i.e $p + 2q = 12$

$\Rightarrow (3 + q) + 2q = 12,$

$\Rightarrow 3 + q + 2q = 12$

$\Rightarrow 3 + 3q = 12, \Rightarrow 3q = 12 - 3$

$\Rightarrow 3q = 9, \Rightarrow q = \frac{9}{3} = 3$

$\therefore$ q = 3.

Now substitute q = 3 into eqn (1) or eqn (2) to find p.

Using eqn (1) i.e $p + 2q = 12$

$\Rightarrow p + 2(3) = 12 \Rightarrow p + 6 = 12,$

$\Rightarrow$ p $= 12 - 6 \Rightarrow$ p $= 6$.

(Q 4) Find the values of x and y which satisfy the equations $2x + 4y = 6$

and $3x - y = 4$ simulteniously .

<u>Soln</u>

Let $2x + 4y = 6$ ………………. (1) and $3x - y = 4$ ……………..eqn (2)

Multiply eqn (2) by 4 $\Rightarrow 4 \times 3x - 4 \times y = 4 \times 4$

$\Rightarrow 12x - 4y = 16$………………..eqn (3)

Add eqn (1) and eqn (3)

$\Rightarrow 2x + 4y = 6$

$+\underline{12x - 4y = 16}$

$\underline{14x \qquad = 22}$

$14x = 22 \Rightarrow$ x $= \frac{22}{14} = 1.6$.

Substitute x = 1.6 into eqn (1) i.e. $2x + 4y = 6 \Rightarrow 2(1.6) + 4y = 6$

$\Rightarrow 3.2 + 4$y $= 6$

$\Rightarrow 4$y $= 6 - 3.2 = 2.8$

$\therefore y = \frac{2.8}{4} = 0.7$

(Q5) Find the values of p and w which satisfy these given equations simultaneously:

$5p + 2w = -3$

$6p - 2w = -8$

N/B: There is the presence of the positive as well as the negative sign and each w has the same value as the other .We therefore add them up straight away.

Soln

$5p + 2w = -3$

$\underline{+ \ 6p - 2w = -8}$
$\underline{11p \qquad = -11}$
$\therefore 11p = -11 \Rightarrow p = -\frac{11}{11} = -1$

$\therefore p = -1$. Substitute $p = -1$ into equation (1) i.e. $5p + 2w$ = -3

$\Rightarrow 5(-1) + 2w = -3$

$\Rightarrow -5 + 2w = -3 \Rightarrow 2w = -3 + 5,$

$\Rightarrow 2w = 2 \Rightarrow w = \frac{2}{2} = 1.$

Therefore the required values are $p = -1$ *and* $w = 1$

(Q6) Solve simultaneously the equations $2p - q = 2$ and $5p + 3q = 27$

Soln

Let $2p - q = 2$………………..eqn (1) and 5p + 3q = 27………………eqn (2)

N/B: We have 3q in eqn (2) and only q in eqn (1)

–In order to make them equal, multiply eqn (1) by 3

Soln

Eqn (1) $\times 3 \Rightarrow 3 \times 2p - 3 \times q = 3 \times 2,$

$\Rightarrow 6p - 3q = 6$………………………eqn (3)

Add eqn (2) and eqn (3)

$\Rightarrow 5p + 3q = 27$ … … … … … … … … …eqn (2)

+$\underline{6p - 3q = 6}$…………………..eqn (3)
$\underline{11p \qquad = 33}$

$\Rightarrow 11\text{p} = 33 \Rightarrow \text{p} = \frac{33}{11} = 3$

$\therefore$ p = 3. Substitute p = 3 into eqn (1) i.e. $2\text{p} - q = 2,$

$\Rightarrow 2(3) - q = 2 \Rightarrow 6 - \text{q} = 2,$

$\Rightarrow 6 - 2 = \text{q} \Rightarrow \text{q} = 4.$

(Q 7) Solve simultaneously the equations $\text{p} - \text{q} = 3$ and $\text{p} + 2\text{q} = 12.$

Soln

Let $\text{p} - \text{q} = 3$ ……………………….eqn (1) and

$\text{p} + 2\text{q} = 12$ ………………..eqn (2)

Multiply through eqn (1) using 2

$\Rightarrow 2 \times \text{p} - 2 \times \text{q} = 2 \times 3$

$\Rightarrow 2\text{p} - 2\text{q} = 6$ …………………………eqn (3)

Now adding eqn (2) and eqn (3) together

$\Rightarrow 2\text{p} - 2\text{q} = 6$

$\underline{\quad + \text{P} + 2\text{q} = 12}$

$\underline{\quad 3\text{p} \qquad = 18}$

3p = 18 $\Rightarrow \text{p} = \frac{18}{3} = 6.$

Substitute p = 6 into eqn (1) to find q i.e $p - q = 3 \Rightarrow 6 - \text{q} = 3,$

$\therefore 6 - 3 = q \Rightarrow 3 = \text{q}$

$\Rightarrow \text{q} = 3$.

(Q 8) Determine the set of values of $x \; and \; y$ which satisfy simultaneously these given equations:

$2x + 4y = 6$

$3x + 4y = 7$

N/B: Even though the value of the y in each of these two given equations is the same, all the two given signs are positive.

– We must therefore change the positive sign of any of them, into the negative sign by using -1 to multiply through it .

soln

Let $2x + 4y = 6 \ldots\ldots\ldots\ldots\ldots\ldots\ldots eqn\ (1)$ and $3x + 4y = 7 \ldots\ldots\ldots\ldots\ldots\ldots\ldots eqn\ (2)$

Using -1 to multiply through eqn (2) $\Rightarrow -1 \times 3x + -1 \times 4y = -1 \times 7$

$\Rightarrow -3x - 4y = -7 \ldots\ldots\ldots\ldots\ldots\ldots\ldots . eqn\ (3)$

Add eqn (3) and eqn (1) together i.e.

$$\begin{array}{rrl} & 2x + 4y & = 6 \\ + & \underline{-3x - 4y} & \underline{= -7} \\ & \underline{-x } & \underline{= -1} \end{array}$$

$\Rightarrow - x = -1 \Rightarrow x = 1.$

Put $x = 1$ $into$ eqn (1) $i.e.$ $2x + 4y = 6 \Rightarrow 2(1) + 4y = 6$

$\Rightarrow 2 + 4y = 6 \Rightarrow 4y = 6 - 2$

$\therefore 4y = 4 \quad \Rightarrow \quad y = \frac{4}{4} = 1, \therefore y = 1$.

(Q9) Find the values of x and y which satisfy simultaneously the equations

$3x + 2y = -1$

$5x + 2y = -3$

<u>Soln</u>

Let $3x + 2y = -1 \ldots\ldots\ldots\ldots\ldots\ldots\ldots . eqn\ (1)$

and $5x + 2y = -3 \ldots\ldots\ldots\ldots\ldots\ldots\ldots .. eqn(2)$

Multiply through eqn (2) using -1

$\Rightarrow -1 \times 5x + -1 \times 2y = -1 \times -3$

$\Rightarrow -5x - 2y = 3 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.\, eqn\ (3)$

Adding eqn (3) and eqn (1)

$\Rightarrow\ 3x + 2y = -1$

+ $\underline{-5x - 2y = 3}$

$\underline{-2x \quad\quad = 2\ }$

$\Rightarrow -2\text{x} = 2 \Rightarrow x = \frac{2}{-2} = -1$

$\therefore x = -1.$

Substitute $x = -1\ into\ eqn\ (1)\quad i.e\ 3x + 2y = -1$

$\therefore 3(-1) + 2y = -1 \Rightarrow -3 + 2\text{y} = -1$

$\therefore 2y = -1 + 3 \Rightarrow 2\text{y} = 2 \Rightarrow \text{y} = \frac{2}{2} = 1$

$\therefore y = 1.$

(Q10) Determine the set of values of x and y which can satisfy the equations $3x + 4y = 13\ and\ x + 2y = 5$ simultaneously.

Soln

$3x + 4y = 13 \ldots\ldots\ldots\ldots\ldots\ldots eqn\ (1)$

$x + 2y = 5 \ldots\ldots\ldots\ldots\ldots\ldots\ldots..\, eqn\ (2)$

N/B: The + 2y in eqn (2) must first be changed into $-4y$.

To do so, multiply through eqn (2) with -2.

Eqn (2) $\times -2 \Rightarrow -2 \times x + -2 \times 2y = -2 \times 5$

$\Rightarrow -2x - 4y = -10 \ldots\ldots\ldots\ldots\ldots\ldots.\, eqn\ \ (3)$

Add eqn (1) and eqn (3)

$\Rightarrow \quad 3x + 4y = 13$

$\ \ + -2x - 4y = -10$

…………………

$\underline{x \qquad\qquad = 3}$

$\Rightarrow x = \mathbf{3}$. Put $x = \mathbf{3}$ into eqn (2) i.e. $x + 2y = 5$

$\Rightarrow 3 + 2y = 5, \Rightarrow 2y = 5 - 3 = 2,$

$\therefore 2y = 2 \Rightarrow y = \frac{2}{2} = \underline{1}$

The requred values are $x = 3$ and $y = 1$.

(Q11) Solve the equations $2a + 10b = 14$ and $a + 2b = 4$ $simultaneously$.

Soln

Let $2a + 10b = 14$ $\ldots\ldots\ldots\ldots\ldots\ldots\ldots.. eqn$ (1)

and $a + 2b = 4 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots. eqn$ (2)

The +2b in eqn (2) must be changed into $-10b$ ($since$ eqn (1) $contains$ $10b$).

$-To$ do $that$ $multiply$ eqn (2) by -5.

eqn (2) $\times -5 \Rightarrow -5 \times a + -5 \times 2b = -5 \times 4$

$\Rightarrow -5a - 10b = -20 \ldots\ldots\ldots\ldots\ldots\ldots.. eqn$ (3)

Adding eqn (1) and eqn (3) $\Rightarrow$

$2a + 10b = 14$

$\underline{+-5a - 10b = -20}$

$\underline{-3a \qquad = -6}$

$\therefore -3a = -6 \Rightarrow a = -\frac{-6}{-3} = 2$

$\Rightarrow a = 2$

Substitute a = 2 into eqn (2) i.e. a + 2b = 4 $\Rightarrow 2 + 2b = 4$

$\Rightarrow 2b = 4 - 2 = 2 \Rightarrow b = \frac{2}{2} = 1$

$\therefore$ a =2 and b = 1.

(Q12) Determine the values of x and y $which$ can $satisfy$ simultaneously each of these given equations:

$2x + 4y = 6$

$3x + 12y = 15$

<u>Soln</u>

Let $2x + 4y = 6$……………eqn (1)

and $3x + 12y = 15$…………..eqn (2)

Since eqn (2) contains + $12y$, the +$4y$ must be converted into $-12y$.

To do so, we multiply equation (1) by-3.

eqn $(1) \times -3 \Rightarrow -3 \times 2\,x + \ -3 \times 4y = -3 \times 6$

$\Rightarrow -6x - 12y = -18 \ldots\ldots\ldots\ldots\ldots.\, eqn$ (3)Add eqn (2) and eqn (3) i.e

$$\begin{array}{l} \quad 3x + 12y = 15 \\ \underline{+\ -6x - 12y = -18} \\ \underline{\quad -3x \qquad\quad = -3} \end{array}$$

$\Rightarrow -3x = -3 \Rightarrow x = \dfrac{-3}{-3} = 1$

$\therefore x = 1.$

$Put\ x = 1\ into\ eqn\ \ (1)\ \ i.e\ 2x + 4y = 6 \Rightarrow 2(1) + 4y = 6,$

$\Rightarrow 2 + 4\mathrm{y} = 6 \Rightarrow 4\mathrm{y} = 6 - 2,$

$\therefore 4y = 4 \Rightarrow y = \frac{4}{4} = 1.$

(Q13) Solve these equations simultaneously:

$x - 3y = -4$

$2x + 2y = 8$

N/B: The negative and positive signs are present, but there is no whole number which can multiply the 2y to give us 3y.

In order to make the two values of the y equal, Use the number attached to the y in the second equation i.e.2 to multiply the first equation. Also use the number attached to the y in the first equation i.e 3 to multiply the second equation.

Soln

Let $x - 3y = -4 \ldots\ldots\ldots\ldots\ldots\ldots. eqn \quad (1)$

$and\ 2x + 2y = 8 \ldots\ldots\ldots\ldots\ldots\ldots eqn \ (2)$

$eqn \ (1) \times 2 \Rightarrow 2 \times x - 2 \times 3y = 2 \times -4$

$\Rightarrow 2x - 6y = -8 \ldots\ldots\ldots\ldots\ldots\ldots. eqn\ (3)$

eqn $(2) \times 3 \Rightarrow 3 \times 2x + 3 \times 2y = 3 \times 8$

$\Rightarrow 6x + 6y = 24 \ldots\ldots\ldots\ldots\ldots\ldots.. eqn(4). Add\ eqn\ (3)\ and\ eqn\ \ (4)$

$i.e.\ 2x - 6y = -8$

$+\ \underline{6x + 6y = 24}$

$\underline{\quad 8x \quad\quad = 16}$

$\Rightarrow 8x = 16 \Rightarrow x = \frac{16}{8} = 2$

$\therefore x = 2.\ Put\ x = 2\ into\ eqn \ \ (1)\ i.e\ \ x - 3y = -4$

$\Rightarrow 2 - 3\text{y} = -4 \Rightarrow 2 + 4 = 3\text{y}$

$\Rightarrow 6 = 3\text{y} \Rightarrow 3\text{y} = 6 \ \Rightarrow \ \text{y} = \frac{6}{3} = 2.$

(Q14) Solve for the values of $x\ and\ y\ given\ that\ 2x + 5y = -1\ and\ x - 3y$

$= 5.$

Soln

Let $2x + 5y = -1 \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots eqn \quad (1)$

$and\ x - 3y = 5 \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots .. eqn\ \ (2)$

$eqn\ \ (1) \times 3 \Rightarrow 3 \times 2x + 3 \times 5y = 3 \times -1$

$\Rightarrow 6x + 15\text{y} = -3 \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \text{eqn}\ \ (3)$

$eqn\ \ (2) \times 5 \Rightarrow 5 \times x - 5 \times 3y = 5 \times 5$

$\Rightarrow 5x - 15y = 25 \ldots \ldots \ldots \ldots \ldots \ldots \ldots .. eqn\ (4)$

$Add\ eqn\ (3) and\ eqn\ \ (4)$

$i.e \quad 6x + 15y = -3$

$\underline{\quad + 5x - 15y = 25}$

$\underline{\quad 11x \qquad\qquad = 22}$

$11x = 22 \Rightarrow x = \frac{22}{11} = 2.$

$Put\ x = 2\ into\ eqn\ \ (2)\ i.e\ \ x - 3y = 5 \Rightarrow 2 - 3\text{y} = 5,$

$\Rightarrow -3\text{y} = 5 - 2 \Rightarrow -3\text{y} = 3,$

$\Rightarrow \text{y} = \frac{3}{-3} = -1.$

$\therefore x = 2 \quad and \quad y = \ -1.$

(Q15) Given that a + 2b =3 and

2a$-7b = -5, determine\ the\ values\ of\ \ a\ and \quad b$

Soln

$a\ +\ \ 2b\ \ = 3 \ldots \ldots \ldots \ldots \ldots \ldots . eqn \quad (1)$

$2a\ \ - 7b = -5 \ldots \ldots \ldots \ldots \ldots \ldots . eqn(2)$

$eqn\ \ (1) \times 7 \Rightarrow 7 \times \text{a} + 7 \times 2\text{b} = 7 \times 3$

$\Rightarrow 7\text{a} + 14\text{b} = 21 \ldots \ldots \ldots \ldots \ldots .. \text{eqn}(3)$

$eqn\ \ (2) \times 2 \Rightarrow 2 \times 2\text{a} - 2 \times 7\text{b} = -5 \times 2$

$\Rightarrow$ 4a $-$ 14b $= -10 \ldots \ldots \ldots \ldots \ldots$ eqn (4)

Add eqn (3) and eqn (4)

$i.e\ \ 7a + 14b = 21$

$\underline{\ \ + 4a - 14b = -10}$
$\underline{11a \qquad\qquad = 11}$

$\Rightarrow 11a = 11 \Rightarrow a = \frac{11}{11} = 1.$

$Put\ a = 1\ \ \ into\ \ eqn\ (1)\ i.e\ \ a + 2b = 3$

$\Rightarrow 1 + 2b = 3, \Rightarrow 2b = 3 - 1 = 2,$

$\Rightarrow b = 1.$

(Q16) Given that $x + 2y = 7\ and\ \ 2x + 3y$

$12\ .\quad Find\ the\ vlues\ of\ x\ \ and\ \ y,$ which can simultaneously satisfy these equations.

$x + 2y = 7 \ldots\ldots\ldots\ldots\ldots..eqn\ \ (1)$

$2x + 3y = 12$…………eqn (2)

N/B: Since all the two signs are the positive signs, one of them must be converted into the negative sign, and the values of y must be made equal.

-To do so, we rather use $-2\ to\ multiply$ eqn (2) instead of using 2, and use 3 to multiply eqn. (1) .

Soln $\qquad x + 2y =$

$7 \ldots\ldots\ldots\ldots\ldots.(1)\qquad 2x + 3y = 12 \ldots\ldots\ldots\ldots eqn\ \ (2)$

$eqn\ \ \ (2)\ \ \times -2 \Rightarrow -2 \times 2x + \ -2 \times 3y = -2 \times 12 - 4x \pm 6y = -24$

$$\Rightarrow -4x - 6y = -24 \ldots\ldots\ldots\ldots\ldots\ldots eqn\ \ (3).$$

$$eqn\ \ \ (1) \times 3 \Rightarrow 3 \times x + 3 \times 2y = 3 \times 7$$

$$\Rightarrow 3x + 6y = 21 \ldots\ldots\ldots\ldots\ldots\ldots.eqn\ \ (4)$$

$$Add\ eqn\ \ (3)\ and\ eqn\ \ (4)$$

$$i.e\ \ \ -4x - 6y\ \ = -24$$

$$\underline{3x + 6y = 21}$$
$$\underline{-x \qquad\quad = -3}$$
$$\Rightarrow -x = -3 \Rightarrow x = 3$$

Now substitute $x = 3\ \ \ into\ \ eqn\ \ \ (1)\ \ i.e\ \ x + 2y = 7$

$$\Rightarrow 3 + 2y = 7, \Rightarrow 2y = 7 - 3 = 4,$$

$$\therefore 2y = 4 \Rightarrow y = \frac{4}{2} = 2.$$

$$\therefore x = 3 \ \ and \ y = 2.$$

$(Q\ 17)\ \ Given\ that\ \ 2a\ +3b = 11\ \ and\ \ a + 5b$
$= 16, determine\ the\ values\ of\ a\ \ and\ \ b\ \ which\ \ simultaneously\ satisfy$
$each\ of\ the\ given\ equations.$

$$soln$$

$$Let\ 2a + 3b = 11 \dots \dots \dots \dots \dots .. eqn\ \ (1)$$

$$and\ \ a + 5b = 16 \dots \dots \dots \dots \dots \dots \dots eqn\ \ (2)$$

$$eqn\ \ (2) \times -3 \Rightarrow -3 \times a + \ -3 \times 5b = -3 \times 16,$$

$$\Rightarrow -3a + \ -15b = -48$$

$$\Rightarrow -3a - 15b = -48 \dots \dots \dots \dots \dots \dots .. eqn\ (3)$$

$$eqn\ (1) \times 5 \Rightarrow 5 \times 2a + 5 \times 3b = 5 \times 11,$$

$$\Rightarrow 10a + 15b = 55 \dots \dots \dots \dots \dots \dots . eqn\ \ (4)$$

$$Add\ eqn\ (3)\ and\ \ \ (4)$$

$$\Rightarrow \ -3a - 15b = -48$$

$$\frac{+\ \ 10a + 15b = 55}{7a \qquad\qquad = 7}$$

$$\therefore 7a = 7 \Rightarrow a = \frac{7}{7} = 1.$$

$$Put\ a = 1\ \ \ into\ \ eqn\ (2)\ i.e\ \ a + 5b = 16 \Rightarrow 1 + 5b = 16,$$

$$\Rightarrow 5b = 16 - 1 = 15, \Rightarrow b = \frac{15}{5} = 3. \qquad\qquad \therefore a = 1\ \ and\ b = 3.$$

$(Q18) Solve\ simultaneously\ \ x + 3y = -5$ and 2x+8y = -14.

N/B: Since the signs are all positive, use -3 to multiply through the second equation instead of using 3.

$$soln$$

$$Let\ x + 3y = -5 \ldots\ldots\ldots\ldots\ldots\ldots eqn\ (1)$$

$$and\ \ 2x + 8y = -14 \ldots\ldots\ldots\ldots\ldots eqn\ (2)$$

$$eqn\ \ (2)\ \times -3$$

$$\Rightarrow -\ 3 \times 2x +\ -3 \times 8y = -3 \times -14$$

$$\Rightarrow -\ 6x +\ -24y = 42$$

$$\Rightarrow -\ 6x - 24y = 42 \ldots\ldots\ldots\ldots\ldots. eqn\ \ (3)$$

$$eqn\ (1)\ \times 8 \Rightarrow 8 \times x + 8 \times 3y = 8 \times -5$$

$$\Rightarrow 8x + 24y = -40 \ldots\ldots\ldots\ldots\ldots\ldots eqn\ (4)$$

$$Add\ eqn\ :(3)\ and\ eqn\ \ (4)\ i.e$$

$$-6x - 24y = 42$$

$$\underline{+\ 8x + 24y = -40}$$

$$\underline{2x \qquad\quad = \ \ 2}$$

$$\Rightarrow 2x\ = 2 \Rightarrow x = \frac{2}{2} = 1$$

$$Put\ x = 1\ into\ eqn\ \ (1)$$

$$i.e\ x + 3y = -5 \Rightarrow 1 + 3\text{y} = -5$$

$$\Rightarrow 3\text{y} = -5 - 1 = -6,$$

$$3y = -6 \Rightarrow \text{y}\ = \frac{-6}{3} = -2$$

$$\therefore x = 1\ and\ y = -2$$

(Q19) Find the values of x and y which satisfy the equations given simultaneously

$$3x - 2y = 4$$

$$4x - 2y = 6$$

N/B: Both of the signs given are negative. Therefore one of them must be converted into the positive sign by using -1 to multiply through any of the given equations.

$$soln$$

$$3x - 2y = 4 \ldots\ldots\ldots\ldots\ldots\ldots.eqn\ (1)$$

$$4x - 2y = 6 \ldots\ldots\ldots\ldots\ldots\ldots..eqn\ \ (2)$$

$$eqn\ \ (1)\ \times -1 \quad gives\ us - 3x + 2y = -4 \ldots\ldots\ldots\ldots\ldots.eqn(3)$$

$$Now\ add\ eqn\ \ (2)\ and\ \ eqn\ \ (3)$$

$$\Rightarrow\ \ 4x - 2y = 6$$

$$+\ \underline{-3x + 2y = -4}$$

$$\underline{x \qquad\quad = 2}$$

$$\therefore x = 2$$

$$Put\ x = 2\ into\ eqn\ \ (1)\ i.e\ \ 3x - 2y = 4 \Rightarrow 3(2) - 2\mathrm{y} = 4$$

$$\Rightarrow 6 - 2\mathrm{y} = 4 \Rightarrow -2\mathrm{y} = 4 - 6$$

$$\Rightarrow -2\mathrm{y} = -2 \Rightarrow \mathrm{y} = \frac{-2}{-2} = 1$$

$$\therefore\ x = 2\ and\ y = 1.$$

(Q20) Given that 8w - 2k = 12 and 2w – 6k = 12, determine the values of w and k which simultaneously satisfy these given equations.

N/B:

$$8w - 2k = 12 \ldots\ldots\ldots\ldots\ldots..eqn\ (1)$$

$$2w - 6k = -8 \ldots\ldots\ldots\ldots\ldots..eqn\ (2)$$

Since eqn (2) contains - 6k, the -2k found in eqn (1) must be converted into 6k.

$To\ do\ this\ , eqn\ \ (1)\ is\ multiplied\ \ through\ using - 3.$

$$soln$$

$$8w - 2k = 12 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.eqn\ (1)$$

$$2w - 6k = -8 \ldots\ldots\ldots\ldots\ldots\ldots..eqn\ \ (2)$$

$$eqn\ \ (1) \times -3$$

$$\Rightarrow -3 \times 8\mathrm{w} - -3 \times 2\mathrm{k} = -3 \times 12$$

$$\Rightarrow -24\mathrm{w} - -6\mathrm{k} = -36$$

$$\Rightarrow -24w + 6k = -36$$

$$\Rightarrow -24w + 6k = -36 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{eqn } (3)$$

eqn (2) + eqn (3) $\Rightarrow$

$$\begin{array}{r} 2w - 6k = -8 \\ \underline{+\ -24w + 6k = -36} \\ \underline{-22w \qquad\quad = -44} \end{array}$$

$$\Rightarrow -22w = -44 \Rightarrow w = \frac{-44}{-22} = 2.$$

$$Put\ w = 2\ \ into\ \ eqn\ \ (1)$$

$$i.e\ \ 8w - 2k = 12 \Rightarrow 8(2) - 2k = 12,$$

$$\Rightarrow 16 - 2k = 12 \Rightarrow 16 - 12 = 2k,$$

$$\Rightarrow 4 = 2k \Rightarrow k = \frac{4}{2} = 2..$$

$$\therefore w = 2\ and\ k = 2.$$

(Q21) Solve simultaneously the equations $4x - 2y = 12\ and\ x - 3y = -2.$

$$soln$$

$$4x - 2y = 12 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots eqn\ (1)$$

$$x - 3y = -2 \ldots\ldots\ldots\ldots\ldots eqn\ (2)$$

$Multiply\ through\ eqn\ \ (2) using -2\ \ and\ eqn\ \ (1) using\ \ 3.$

$$eqn\ (2) \times\ -2\ given\ us -2x + 6y = 4 \ldots\ldots\ldots\ldots\ldots eqn\ (3)$$

$$eqn\ \ (1) \times 3\ gives\ us\ 12x - 6y = 36 \ldots\ldots\ldots\ldots\ldots\ldots\ldots eqn\ \ (4)$$

$$Add\ eqn\ \ (3) and\ eqn\ (4)$$

$$\begin{array}{r} -2x + 6y = 4 \\ \underline{+\ 12x - 6y = 36} \\ \underline{10x \qquad\quad = 40} \end{array}$$

$\Rightarrow 10x = 40 \Rightarrow x = \frac{40}{10} = 4.$

$$Put\ x = 4\quad into\ eqn\ \ (2)$$

$$i.e.\ \ x - 3y = -2$$

$$\Rightarrow 4 - 3\mathrm{y} = -2$$

$$\Rightarrow -3\mathrm{y} = -2 - 4 = -6$$

$$\Rightarrow \mathrm{y} = \frac{-6}{-3} = 2$$

$$\therefore x = 4\ and\ \ y = 2.$$

(Q22) Solve the following equations simultaneously: $-5x = 7 + 3y$ and $4x - 7y = -15.$

$soln$

$-5x = 7 + 3y\ can\ be\ rewritten\ as -5x - 3y = 7 \ldots\ldots\ldots\ldots.\ eqn\ \ (1)$

$and\ \ 4x - 7y = -15 \ldots\ldots\ldots\ldots\ldots\ldots..\ eqn\ (2).$

$Multiply\ \ eqn\ \ (1)\ by\ -7\ and\ \ eqn\ \ (2)\ by\ \ 3.$

$eqn\ (1) \times -7\ gives\ us\ 35x + 21y = -49 \ldots\ldots\ldots\ldots\ldots\ldots.\ eqn\ \ (3)$

$eqn\ (2) \times 3\ gives\ us\ \ 12x - 21y = -45 \ldots\ldots\ldots\ldots\ldots\ldots..\ eqn\ (4).$

$Solving\ eqn\ \ (3)\ and\ eqn\ (4)\ simultaneously\ by\ adding\ them\ together$

$$\Rightarrow\ \ 35x + 21y = -49$$

$$\underline{+\ 12x - 21y = -45}$$

$$\underline{47x \qquad\quad = -94}$$

$$\Rightarrow 47x = -94 \Rightarrow x\ \ = \tfrac{-94}{47} \Rightarrow x = -2.$$

Putting $x = -2\ into\ \ eqn\ \ (1)\ i.e. -5x - 3y = 7$

$$\Rightarrow -5(-2) - 3\mathrm{y} = 7 \Rightarrow 10 - 3\mathrm{y} = 7$$

$$\Rightarrow -3\mathrm{y} = 7 - 10 = -3$$

$$\therefore y = \tfrac{-3}{-3} = 1.$$

(Q23) Solve the equations given below simultaneously:

$x = 3y - 4$

$5x - 2y = 32$

$soln$

$The\ equation\ x = 3y - 4\ can\ be\ rewritten\ \ as\ x - 3y = -4$.

$Let\ x - 3y = -4 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.eqn\ (1)$

$and\ \ 5x - 2y = 32 \ldots\ldots\ldots\ldots\ldots\ldots\ldots.eqn\ (2)$

$Multiply\ eqn\ \ (1)\ by\ \ -2\ \ and\ eqn\ \ (2) by\ \ 3.$

$eqn\ (1) \times -2\ \ gives\ us\ \ -2x + 6y = 8 \ldots\ldots\ldots\ldots\ldots\ldots eqn\ \ (3)$

$eqn\ \ (2) \times 3\ gives\ us\ \ 15x - 6y = 96 \ldots\ldots\ldots\ldots\ldots\ldots\ldots eqn\ (4)$

$Now\ \ solve\ \ eqn\ (3)\ and\ eqn(4)\ simultaneously\ \ i.e$

$-2x + 6y = 8$

$\underline{+\,15x - 6y = 96}$

$\underline{13x \qquad\quad = 104}$

$\therefore 13x = 104 \Rightarrow \mathrm{x} = \dfrac{104}{13}$

$\Rightarrow x = 8.$

$Put\ x = 8\ into\ eqn\ \ (1) i.e.\ \ x - 3y = -4 \Rightarrow 8 - 3\mathrm{y} = -4$

$\Rightarrow -3\mathrm{y} = -4 - 8 \Rightarrow -3\mathrm{y} = -12$

$\Rightarrow \mathrm{y} = \dfrac{-12}{-3} = 4$

<u>Method (2) Substitution method:</u>

$Let\ x = 3y - 4 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots eqn\ \ (1)$

$and\ \ 5x - 2y = 32 \ldots\ldots\ldots\ldots\ldots\ldots..eqn\ (2)$

$Substitute\ x = 3y - 4\ \ into\ eqn\ \ (2)\ \ i.e.$ 5x – 2y = 32

$\Rightarrow 5(3\mathrm{y} - 4) - 2\mathrm{y} = 32$

$$\Rightarrow 15\text{y} - 20 - 2\text{y} = 32,$$

$$\Rightarrow 15\text{y} - 2\text{y} = 32 + 20$$

$$\therefore 13y = 52,$$

$$\Rightarrow \text{y} = \frac{52}{13} = 4.$$

$$Put\ y = 4\ into\ \ eqn\ \ (1)\ i.e\ \ x = 3y - 4 \Rightarrow x = 3(4) - 4,$$

$$\Rightarrow x = 12 - 4 \Rightarrow x = 8.$$

The story problem form of simultaneous equation:

- Simultaneous equations may be presented in an indirect form, such as the story form.
- A brief story may be presented and from this, two equations can be had through mathematical analysis, with two unknowns and then solved simultaneously.
- (Q1) A group of six ladies and seven gentlemen paid tax totaling ¢127. Another group of eight ladies and ten gentlemen paid tax totaling ¢ 170. If the tax paid by a male is fixed, and that paid by a female is also of a fixed value, determine the tax paid by

(a) a single lady. (b) a single gentleman.

Soln

Let $l =$ the tax paid by each lady.

Let g = the tax paid by each gentleman.

Since 6 ladies and 7 gentlemen paid ¢127 $\Rightarrow$

$6l + 7\text{g} = 127 \dots\dots\dots\dots\dots\dots.\, eqn\ \ (1)$

Also since 8 ladies and 10 gentlemen paid a total tax of ¢170

$\Rightarrow 8\,l + 10\text{g} = ¢170 \dots\dots\dots\dots\dots\dots\dots\dots.\, \text{eqn}\ \ (2)$

Consider now eqn (1) and eqn (2)

$6\,l + 7\text{g} = 127 \dots\dots\dots\dots\dots\dots\dots.\, \text{eqn}\ \ (1)$

$8\ \ l\ + 10\text{g} = 170 \dots\dots\dots\dots\dots\dots\dots\dots\dots.\, \text{eqn}\ \ (2)$

$eqn\ \ (1)\ \times 10\ gives\ us\ \ 60\ \ l + 70\ \text{g} = 1270 \dots\dots\dots\dots\dots\dots\dots..\, \text{eqn}\ \ \ (3)$

$eqn\ \ (2)\ \times -7\ \ gives\ \ us\ \ -56\,l - 70\text{g} = -1190 \dots\dots\dots\dots\dots\dots\, \text{eqn}\ \ (4)$

Add eqn (3) *and eqn*(4)

$$\begin{array}{rcl} 60\ \ l + 70\text{g} & = & 1270 \\ \underline{+\ -56\,l - 70\ \text{g}} & \underline{=} & \underline{-1190} \\ \underline{4\,l} & \underline{=} & \underline{80} \end{array}$$

$\therefore 4\,l = 80 \Rightarrow \text{l} = \dfrac{80}{4} = 20$

$\Rightarrow$ tax paid by a lady = l = ¢20.

Substitute $l = 20$ *into eqn* (1)

$i.e\ 6\,l + 7\text{g} = 127$

$\Rightarrow 6(20) + 7\text{g} = 127$

$\therefore 120 + 7\text{g} = 127$

$\Rightarrow 7\text{g} = 127 - 120 = 7$

$\Rightarrow 7\text{g} = 7 \Rightarrow \text{g} = \frac{7}{7} = 1$

$\therefore Tax\ paid\ by\ each\ gentleman\ = \text{g} = ¢1.$

(Q2) A family of 3 adults and 2 children paid ¢ 28 as taxi fare. Another family of an adult and 9 children paid ¢ 26 as taxi fare.

(a) Determine the fare per child and that per adult.
(b) How much will be paid by two children and two adults.
(c) Find how much a group of 5 adults and 3 children will pay, given that the fare per child is the same, and that for an adult is also the same.

Soln

Let y = the fare paid by an adult, and let x = that paid by each child. In the first case, 3 adults and 2 children paid ¢28S.

Fare paid by each adult = y .

$\Rightarrow$ Fare paid by 3 children $= 3\text{y}$.

Fare paid by each child = x.

$\Rightarrow$The fare paid by 2 children $= 2x$.

Since the total fare paid by the three adults and the two children =¢28

$\Rightarrow 3\text{y} + 2x = 28 \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots eqn\ (1)$

In the second case, an adult and 9 children paid ¢26.

Fare paid by the adult = y and the fare paid by 9 children= 9 x.

Since the adult and the 9 children paid ¢26

$\Rightarrow$ y + 9$x = 26 \ldots\ldots\ldots\ldots\ldots\ldots..eqn\ (2)$

$Now\ consider\ eqn\ (1)\ and\ \ eqn\ (2)$

$3y + 2x = 28 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.eqn\ (1)$

$y + 9x = 26 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots eqn\ (2)$

$Multiply\ eqn\ (1)\ by\ \ 9\ and\ \ this\ gives\ us$

$27y + 18x = 252 \ldots\ldots\ldots\ldots\ldots\ldots\ldots eqn\ (3)$

$Multiply\ eqn\ \ (2)\ by - 2\ to\ give\ us\ \ - 2y - 18x = -52 \ldots\ldots\ldots.eqn\ \ \ (4)$

$Now\ solve\ \ eqn\ (3)\ and\ eqn\ (4)\ \ simutaneously\ .$

$$
\begin{array}{rl}
 & 27y + 18x = 252 \\
+ & -2y - 18x = -52 \\
\hline
 & 25y \qquad\quad = 200 \\
\hline
\end{array}
$$

$\therefore\ 25y = 200 \Rightarrow \mathrm{y} = \dfrac{200}{25}$

$\Rightarrow \mathrm{y} = 8$

$Put\ \ y = 8\ into\ \ eqn\ \ (2)\ \ \ i.e\ .\ \ y + 9x = 26$

$\Rightarrow 8 + 9x = 26$

$\Rightarrow 9x = 26 - 8 = 18$

$\therefore\ 9x = 18 \Rightarrow x = \frac{18}{9} = 2.$

(a) Fare paid per child $= x =$ ¢2 $and\ fare\ per\ adult = y =$ ¢8.

(b) The fare paid by 2 children $= 2x = 2(2) =$ ¢4.

$Fare\ paid\ by\ 2\ \ adults = \ \ 2y = 2(8) =$ ¢16.

$\Rightarrow$ The fare paid by two children and two adults $=$ ¢ $4 +$ ¢$16 =$ ¢20.

(c) $Fare\ paid\ by\ 5\ adults\ = 5y\ = 5(8) =$ ¢40,

$and\ the\ fare\ paid\ by\ 3\ children\ = 3x = 3(2) =$ ¢6.

$\therefore Fare\ paid\ by\ 5\ adults\ and\ three\ children =$ ¢$40 +$ ¢$6 =$ ¢46.

(Q3) The sum of the ages of Akin and Dop is 35 years, and the sum of twice Akin's age and 3 times Dop's age is 89. Find their present ages.

Soln

Let $x = Akin's\ age\ and\ y = Dop's\ age$.

$Since\ the\ sum\ of\ their\ ages = 35 years \Rightarrow\ x + y = 35 \ldots\ldots\ldots\ldots\ldots eqn\ \ (1)$

$Also\ since\ the\ sum\ of\ twice\ Akins\ age\ and\ thrice\ Dop's\ age\ is\ \ 89\ years \Rightarrow 2x + 3y = 89 \ldots\ldots\ldots\ldots\ldots eqn\ (2)$

$Solve\ eqn\ (1)\ and\ eqn\ \ (2)\ simultaneously\ and\ you\ will\ get\ x = 16\ \ and$

$y = 19.$

$\Rightarrow$ Akin`s present age $= 16$ years and Dop′s present age $= 19$ years .

(Q4) A man weighed 2 cups of rice together with 4 cups of wheat, and their total weight was 14g .He then weighed 3 cups of rice together with a cup of wheat, and their total weight was 11g.

(a) What is the weight of each cup of rice and each cup of wheat.

(b) Determine the total weight of 2 cups of rice and 3 cups of wheat.

Soln

Let r = the weight of each cup of rice, and let w = the weight of each cup of wheat. .

In the first case, he weighed 2 cups of rice together with 4 cups of wheat, and had a total weight of 14g, $\Rightarrow 2\mathrm{r} + 4\mathrm{w} = 14 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \mathrm{eqn}\ (1)$

In the second case , he weighed 3 cups of rice together with a cup of wheat and the total weight was 11g $\Rightarrow 3\mathrm{r} + \mathrm{w} = 11 \ldots\ldots\ldots\ldots\ldots\ldots\ldots.. \mathrm{eqn}\ (2)$

Consider eqn (1) and eqn (2)

2r +4w =14……………………….eqn (1)

3r + w = 11……………………….eqn (2)

Eqn (2) $\times -4\ gives -12r - 4w = -44 \ldots\ldots\ldots\ldots. eqn\ (3)$

Now add eqn (3) and eqn (1)

$$
\begin{array}{l}
\quad 2r + 4w = 14 \\
\underline{+\ -12r - 4w = -44} \\
\underline{-10r \qquad\quad = -30}
\end{array}
$$

$$\therefore -10r = -30 \Rightarrow \mathrm{r} = \frac{-30}{-10} = 3,$$

$\Rightarrow \mathrm{r} = 3.$

$Put\ r = 3\ into\ \ eqn\ \ (1)$

$i.e\ \ 2r + 4w = 14$

$\Rightarrow 2(3) + 4w = 14$

$\Rightarrow 6 + 4\mathrm{w} = 14$

$\Rightarrow\ 4w = 14 - 6 = 8$

$\Rightarrow \mathrm{w} = \frac{8}{4} = 2.$

$(a)\ \ The\ weight\ of\ each\ cup\ of\ rice = r = 3\text{g},$

and the weight of each cup of wheat = w = 2g.

$(b)\ \ Weight\ \ of\ 2\ cups\ of\ rice = 2r = 2(3) = 6g.$

$Weight\ of\ \ 3\ cups\ \ of\ \ wheat\ = 3w = 3(2) = 6\text{g}.$

$\Rightarrow$ The total weight of 2 cups of rice and 2 cups of wheat

$= 6 + 6 = 12\text{g}.$

(Q5) The total cost of 60 apples and 100 eggs is ¢ 108,000. If the cost of 72 apples is the same as that of 12 eggs, determine the cost of 101 apples and 20 eggs.

Soln

Let x = the cost of an apple $\Rightarrow$ the total cost of the 60 apples = 60 x.

Let y = the cost of an egg, $\Rightarrow$ the cost of the 100 eggs = 100y.

Since the total cost of the 60 apples and the 100 eggs is ¢108,000 $\Rightarrow$ 60 x+100y = 108,000……………………..eqn (1)

Cost of the 72 apples = 72 x, and the cost of the 12 eggs =12y.

Since the cost of the 72 apples and the 12 eggs are equal, then 72 x = 12y $\Rightarrow$ y = $\frac{72x}{12}$ $\Rightarrow$ y = 6x ……………………………….eqn (2)

Substitute y = 6 x into eqn (1)

i.e. 60 x + 100y =108,000

$\Rightarrow$ 60 x + 100(6 x) = 108,000

$\Rightarrow 60\,x + 6\,00x = 108{,}000$,

$\Rightarrow 660\,x = 108{,}000$,

$\Rightarrow x = \frac{108{,}000}{660} = 163.6$,

$\Rightarrow$ the cost of each apple =¢163.6.

Substitute x =163.6 into eqn (2) i.e. y = 6 $x \Rightarrow$ y = 6(163.6)

$\Rightarrow$ y = 981.6.

$\Rightarrow$ cost of each egg = ¢981.6.

The cost of the 10 apples and the 20 eggs = 10 x + 20y

=10(163.6) + 20 (981.6)

=1636 +19632 = ¢21,268.

Simultaneous equations involving indices:

- It is a possibility to be given questions based on simultaneous equations, which involve indices or exponential equations.

(Q1) Solve simultaneously the equations given next:

$3^m \times 3^{n\,=}\,243$

$3^m \div 3^{2n} = 9$

Soln

Let $3^m \times 3^n = 243$…………………………….eqn (1).

And $3^m \div 3^{2n} = 9$…………………………..eqn (2).

From eqn (1) $3^m \times 3^n = 243$.

But $243 = 3\times3\times3\times3\times3=3^5$, and $3^m\times3^n = 3^{m+n}$

$\Rightarrow 3^{m+n} = 3^5 \Rightarrow m + n = 5$.

Let $m + n = 5$ ……………………………….eqn (3)

From eqn (2) $3^m \div 3^{2n} = 9$

But $9 = 3\times3 = 3^2$ and $3^m \div 3^{2n} = 3^{m-2n}$

$\Rightarrow 3^{m-2n} = 3^2$, $\Rightarrow m - 2n = 2$.

Let $m - 2n = 2$ …………………………….. eqn (4)

Solve eqn (3) and eqn (4) simultaneously

i.e $m + n = 5$ ……………………………..eqn (3)

$m - n = 2$ ……………………………..eqn (4)

Eqn. (3) $\times$ (2) $\Rightarrow 2m + 2n = 10$ …………... eqn (5)

Add eqn (4) and eqn (5)

i.e $m - 2n = 2$

$+ \underline{2m + 2n = 10}$

$\underline{3m \quad = 12}$

From $3m = 12 \Rightarrow m = \frac{12}{3} = 4$. Substitute $m = 4$ into eqn (3)

i.e $m + n = 5 \Rightarrow 4 + n = 5, \Rightarrow n = 5 - 4 = 1$.

(Q2) Solve the equations given simultaneously:

$2^x \times 2^{2y} = 32$

$2^{6x} \div 2^{4y} = 4.$

<u>Soln.</u>

Let $2^x \times 2^{2y} = 32$………………………… eqn (1)

And let $2^{6x} \div 2^{4y} = 4$ …………………… eqn (2)

From eqn (1) $2^x \times 2^{2y} = 32$

$\Rightarrow 2^{x+2y} = 32$, but $32 = 2^5$

$\Rightarrow 2^{x+2y} = 2^5$

$\Rightarrow 2^{x+2y} = 2^5 \Rightarrow x + 2y = 5$………………… eqn (3)

From eqn (2) $2^{6x} \div 2^{4y} = 4$

$\Rightarrow 2^{6x-4y} = 4, \Rightarrow 2^{6x-4y} = 2^2$

$\Rightarrow$ 6x – 4y = 2……………………………. eqn (4)

Now consider eqn (3) and eqn (4)

i.e x + 2y = 5…………………………… eqn (3)

6x – 4y = 2…………………………... eqn (4)

Eqn. (3) × 2 gives us

2x + 4y = 10……………………………..eqn (5)

Now solve eqn (4) and eqn (5) simultaneously by adding them together.

$6x - 4y = 2$

$+ \underline{2x + 4y = 10}$

$\underline{\quad 8x \qquad = 12}$

$8x = 12 \Rightarrow x = \frac{12}{8} = 1.5.$

Put x = 1.5 into eqn (3)

i.e $x + 2y = 5 \Rightarrow 1.5 + 2y = 5,$

$\therefore 2y = 5 - 1.5, \Rightarrow 2y = 3.5$

$\Rightarrow y = \frac{3.5}{2} = 1.7.$

(Q3) Solve simultaneously the following equations:

$2^{x-y} = 8$

$2^{3x-y} = 128.$

Soln.

$8 = 2^3$ and $128 = 2^7$

From $2^{x-y} = 8 \Rightarrow 2^{x-y} = 2^3$

$\Rightarrow x - y = 3$……………………………… eqn (1)

From $2^{3x-y} = 128 \Rightarrow 2^{3x-y} = 2^7$

$\Rightarrow 3x - y = 7$…………………………… eqn (2)

Consider eqn (1) and eqn (2)

$x - y = 3$…………………………… eqn (1)

$3x - y = 7$…………………………... eqn (2)

Eqn. (1) $\times\ -1$ gives us

$-x + y = -3$………………………... eqn (3)

Add eqn (3) and eqn (2)

i.e $-x + y = -3$

$+\ \underline{3x - y = 7}$

$\underline{\ \ 2x \qquad = 4}$

$\Rightarrow 2x = 4 \Rightarrow x = \frac{4}{2} = 2.$

Put $x = 2$ into eqn (1)

i.e $x - y = 3 \Rightarrow 2 - y = 3,$

$\therefore \quad -y = 3 - 2 \Rightarrow -y = 1$

$\therefore \quad y = -1.$

(Q4) Solve simultaneously

$3^{2x-y} = 81$

$2^{x+2} = 32.$

<u>Soln.</u>

Let $3^{2x-y} = 81$……………………….. eqn (1)

And $2^{x-y} = 32$……………………… eqn (2)

From eqn (1) $3^{2x-y} = 81$ and since $81 = 3^4$

$\Rightarrow 3^{2x-y} = 3^4$

$\Rightarrow 2x - y = 4$

Let $2x - y = 4$……………………… eqn (3)

From eqn (2) $2^{x+y} = 32$ and since $32 = 2^5$

$\Rightarrow 2^{x+y} = 2^5$

$\Rightarrow x + y = 5$

Let $x + y = 5$……………………... eqn (4)

Now solve eqn (3) and eqn (4) simultaneously

$$
\begin{array}{rl}
\text{i.e} & 2x - y = 4 \\
+ & \underline{x + y = 5} \\
& \underline{3x \quad = 9}
\end{array}
$$

From $3x = 9 \Rightarrow x = \frac{9}{3} = 3.$

Put $x = 3$ into eqn (4)

i.e $x + y = 5 \Rightarrow 3 + y = 5,$

$\Rightarrow y = 5 - 3 = 2.$

$\therefore y = 2.$

(Q5) Find the values of x and y which satisfy simultaneously the equations given next:

$5^{2x+y} = 625$

$4^{x+y} = 64.$

Soln.

Let $5^{2x+y} = 625$.................................eqn (1).

And $4^{x+y} = 64$.................................. eqn (2).

From eqn (1) $5^{2x-y} = 625$ and since 625 = 5 x 5 x 5 x 5 = 5^4

$\Rightarrow 5^{2x-y} = 5^4 \Rightarrow 2x - y = 4,$

Let 2x – y = 4..................................eqn (3)

From eqn (2) $4^{x+y} = 64$ and since 64 = 4^3

$\Rightarrow 4^{x+y} = 4^3$

=> x + y = 3

Let x + y = 3............................... eqn (4)

Now solve eqn (3) and eqn (4) simultaneously

i.e 2x – y = 4

+ x + y = 3

3x = 7

From 3x = 7 $\Rightarrow$ x = $\frac{7}{3} = 2.3$.

$\therefore$ x = 2.3.

From eqn (4), x + y = 3 => 2.3 + y = 3 => y = 0.7.

(Q6) Solve the given equations simultaneously.

2x + y = 4

$2^{x+y} = 8$

<u>Soln.</u>

Let 2x + y = 4…………………………………eqn (1)

and $2^{x+y} = 8$…………………………………eqn (2)

From eqn (2) $2^{x+y} = 8$

$\Rightarrow 2^{x+y} = 2^3$

=> x + y = 3………………………..eqn (3)

NB: 8 =$2 \times 2 \times 2 = 2^3$.

Now consider eqn (1) and eqn (3)

i.e 2x + y = 4……………….eqn (1)

x + y = 3………………..eqn (3)

eqn (3) $\times$ -1 $\Rightarrow$ -x –y = -3…………..eqn (4)

Solve eqn (1) and eqn (4) simultaneously

i.e $\quad 2x + y = 4$

$\quad + \underline{-x - y = -3}$

$\underline{\quad x \quad = 1}$

Put x = 1 into eqn (1)

i.e $\quad 2x + y = 4 \quad \Rightarrow 2(1) + y = 4,$

$\Rightarrow \quad 2 + y = 4 \quad \Rightarrow y = 4 - 2 = 2,$

$\therefore\ y = 2.$

(Q7) Solve the following equations simultaneously:

$2^{2x-1} = 16^{1+2y}$

$\left(\frac{1}{2}\right)^{3x} = \left(\frac{1}{16}\right)^{y-1}$

<u>Soln.</u>

$16 = 2^4 = 16^{1+2y} = 2^{4(1 + 2y)}$, and since $2^{2x - 1} = 16^{1 + 2y}$

$\Rightarrow 2^{2x-1} = 2^{4(1+2y)}$

$\Rightarrow 2x - 1 = 4(1 + 2y)$

$\Rightarrow 2x - 1 = 4 + 8y$

$\therefore 2x - 8y = 5$……………………………eqn (1)

Consider $\quad \left(\frac{1}{2}\right)^{3x} = \left(\frac{1}{16}\right)^{y-1}$

Since $\frac{1}{16} = \left(\frac{1}{2}\right)^4,$

then from $\left(\frac{1}{2}\right)^{3x} = \left(\frac{1}{16}\right)^{y-1}$

$\Rightarrow \left(\frac{1}{2}\right)^{3x} = \left(\frac{1}{2}\right)^{4(y-1)}$

$\Rightarrow$ 3x = 4(y – 1) $\Rightarrow$ 3x = 4y – 4

$\Rightarrow$ 3x – 4y = - 4……………………………eqn (2)

Now solve eqn (1) and eqn (2) simultaneously

i.e 2x – 8y = 5…………………….eqn (1)

 3x – 4y = -4…………………eqn (2)

Eqn (2) × -2

$\Rightarrow$ - 6x + 8y = 8……………………..eqn (3)

Add eqn (1) and eqn (3)

i.e 2x – 8y = 5…………………eqn (1)

 + -6x + 8y = 8……………………..eqn (3)

 -4x = 13

From -4x = 13 $\Rightarrow$ x = $\frac{13}{-4}$

$\Rightarrow$ x = -3.3

Put x = -3.3 into eqn (1)

i.e 2x - 8y = 5

$\Rightarrow$ 2(-3.3) – 8y = 5,

$\Rightarrow$ - 6.6 – 8y = 5

$\Rightarrow$ - 6.6 – 5 = 8y,

$\Rightarrow$ -11.6 = 8y $\Rightarrow$ y = $\frac{-11.6}{8}$

$\Rightarrow$ y = -1.45.

(Q8) Solve the equations given simultaneously.

$3^{x+1} = 27^{y-1}$

$\left(\frac{1}{3}\right)^{2x} = \left(\frac{1}{9}\right)^{y+2}$

<u>Soln.</u>

27 = 3^3 and $\left(\frac{1}{9}\right) = \left(\frac{1}{3}\right)^2$

From $3^{x+2} = 27^{y-1}$

$\Rightarrow 3^{x+1} = 3^{3(y-1)}$

$\Rightarrow$ x + 1 = 3(y -1) $\Rightarrow$ x + 1 = 3y – 3,

$\Rightarrow$ x – 3y = -3 – 1

$\therefore$ x – 3y = - 4…………………………..eqn (1)

Now $\left(\frac{1}{3}\right)^{2x} = \left(\frac{1}{9}\right)^{y+2}$

$\Rightarrow \left(\frac{1}{3}\right)^{2x} = \left(\frac{1}{3}\right)^{2(y+2)}$

$\Rightarrow$ 2x = 2(y + 2) $\Rightarrow$ 2x = 2y + 4,

$\therefore$ 2x – 2y = 4…………………………..eqn (2)

You then continue by solving eqn (1) and eqn (2) simultaneously.

<u>Simultaneous equations involving fractions:</u>

Sometimes one may be given equations to solve simultaneously, in which fractions are involved.

At times one of the equations may be associated with fractions, while the other may not.

Sometimes, both equations may involve fractions,

To solve such questions, those in the fractional form must be converted into linear or non fractional forms, by multiplying through them with the appropriate number, which will remove these fractions.

(Q1) Solve the equations given below simultaneously:

$$\frac{x}{2}+\frac{2y}{4}=1$$

$$-2x-y=-6$$

$$soln$$

Use 4 to multiply through the first equation.

Consider $\frac{x}{2}+\frac{2y}{4}=1$

$\Rightarrow 4\times\frac{x}{2}+\frac{4\times 2y}{4}=4\times 1$

$\Rightarrow 2x+2y=4$

Let $2x+2y=4 \ldots\ldots\ldots\ldots eqn\ (1)$

And $-2x-y=-6 \ldots\ldots\ldots\ldots.. eqn\ (2)$

Eqn (2) × 2 gives us

$-4x-2y=-12 \ldots\ldots\ldots\ldots eqn\ (3)$

Add eqn (1) and eqn (3)

i.e. $2x+2y=4$

\+ - $\underline{4x-2y=-12}$

$\underline{\quad -2x \qquad =-8}$

$\therefore x=\frac{-8}{-2}=4.$

$Substitute\ x=4\ into\ eqn\ \ (2)$

$i.e - 2x - y = -6 \;\; \Rightarrow -2(4) - y = -6,$

$\Rightarrow -8 - y = -6 \Rightarrow -8 + 6 = y,$

$\Rightarrow y = -2.$

(2) Solve the given equations simultaneously:

$$\frac{2x}{5} - \frac{3y}{2} = 1$$

$x + 3y = 16$

Soln

Consider $\frac{2x}{5} - \frac{3y}{2} = 1$

Multiply through using 10

$$10 \times \frac{2x}{5} - 10 \times \frac{3y}{2} = 10 \times 1,$$

$\Rightarrow 4x - 15y = 10$

Let $4x - 15y = 10$……………..eqn (1)

And $x \;+ 3y = 16 \ldots \ldots \ldots \ldots \ldots .. eqn\ (2)$

Multiply eqn (2) by 5 and this gives us

$5x + 15y = 80$ …………eqn (3)

Now solve eqn (1) and eqn (3) simultaneously

i.e $4x - 15 = \; 10$
$\underline{+5x + 15y = 80}$

$\underline{\;9x \qquad\qquad = 90}$

$\therefore 9x = 90 \Rightarrow \mathrm{x} = \frac{90}{9} = 10.$

Substitute $x = 10\ into\ eqn\ (1)$

i.e $4x - 15y = 10 \Rightarrow 4(10) - 15\text{y} = 10$,

$\therefore -15y = 10 - 40 \Rightarrow -15\text{y} = -30$,

$\Rightarrow$ y = $\frac{-30}{-15} \Rightarrow \text{y} = 2$.

(Q3) Determine the values of $a\ and\ b\ which\ satisfy\ these\ given$

$equations\ simultaneously$:

$$\frac{3a}{5} - \frac{4b}{3} = 8$$

$$\frac{4a}{10} - \frac{14b}{3} = -6$$

Soln

Consider the equation

$$\frac{3a}{5} - \frac{4b}{3} = 8$$

Multiply through using 15

i.e. $15\ \times \frac{3a}{5} -\ 15 \times \frac{4b}{3} = 15\ \times 8$

$\Rightarrow 9a - 20b = 120 \ldots \ldots \ldots \ldots .. eqn\ (1)$

Now consider the second equation

Multiply through using 30

i.e $30\ \times \frac{4a}{10} -\ 30\ \times \frac{14b}{3} = 30\ \times -6$

$\Rightarrow 12a -\ \ 140b = -180 \ldots \ldots .. eqn\ (2)$

$Now\ consider\ eqn(1)\ \ and\ eqn\ (2)$

i.e $9a - 20b = 120 \ldots \ldots \ldots \ldots .. eqn\ (1)$

$12a - 140b =\ -180 \ldots \ldots \ldots .. eqn\ (2)$

Multiply eqn (1) by -7

$\Rightarrow -63a + 140b = -840 \ldots\ldots\ldots$ eqn (3)

Solve eqn (2) *and eqn* (3) *simultaneously*

$i.e. \quad 12a - 140b = -180$

$$\begin{array}{rl} & 12a - 140b = -180 \\ + & \underline{-63a + 140b = -840} \\ & \underline{-51a \qquad\quad = -1020} \end{array}$$

$\therefore \ a = \frac{-1020}{-51} = 20.$

Put a = 20 into eqn (1) , i.e $9a - 20b = 120$

$\Rightarrow 9\,(20) - 20\,b = 120, \Rightarrow 180 - 120 = 20\,b,$

$=> 60 = 20b \Rightarrow b = \frac{60}{20} = 3.$

(Q 4) Determine the values of x *and* y, *which satisfy the given equations simultaneously*:

$$\frac{2x}{3} + \frac{4y}{6} = 14$$

$$\frac{4x}{5} + \frac{2y}{3} = 16$$

Consider the first equation, i.e .$\frac{2x}{3} + \frac{4y}{6} = 14.$

Multiply through using 6

i.e $6 \times \frac{2x}{3} + 6 \times \frac{4y}{6} = 6 \times 14,$

$\Rightarrow 4x + 4y = 84 \ldots\ldots\ldots\ldots eqn\ (1)$

Now consider the second equation, i.e $\frac{4x}{5} + \frac{2}{3}\,y = 16$

Multiply through using 15

i.e $15 \times \frac{4x}{5} + 15 \times \frac{2y}{3} = 15 \times 16$,

$\Rightarrow 12x + 10y = 240 \ldots\ldots\ldots..eqn(2)$

Consider now eqn (1) and eqn (2)

i.e 4x. $+4y = 84 \ldots\ldots\ldots\ldots.eqn\ (1)$

$12x + 10y = 240 \ldots\ldots\ldots..eqn\ (2)$

Eqn (1) $\times -10\ gives\ us$

$-40x - 40y = -840 \ldots\ldots.eqn\ (3)$

$Eqn\ (2)\ \times 4\ gives\ us$

$48x + 40y = 960 \ldots\ldots\ldots\ldots.eqn\ (4)$

Add eqn (3) and eqn (4)

$$
\begin{array}{rl}
 & -40x - 40y = -840 \\
+ & \underline{48x + 40y = 960} \\
 & 8x \qquad\quad = 120
\end{array}
$$

$\Rightarrow x = \frac{120}{8} = 15 \ \Rightarrow x = 15.$

Substitute $x = 15\ into\ eqn\ (1)$

i.e $4x + 4y = 84 \Rightarrow 4(15) + 4y = 84$,

$\therefore 60 + 4y = 84 \ \Rightarrow 4y = 84 - 60 = 24$,

$\therefore$ 4y = 24 $\Rightarrow$ y = $\frac{24}{4} = 6$,

$\therefore y = 6.$

(Q5) Half a certain number was added to one third of another number and our answer was 2. But when twice the first number was added to four times the second number, our answer was 16 .Find these numbers.

Soln

Let a = the first number and let b = the second number in the first case, and since half the first number was added to one third the second number and our answer was 2,

$\Rightarrow \frac{1}{2}\ a + \frac{1}{3}\ b = 2 \Rightarrow \frac{a}{2} + \frac{b}{3} = 2$

Multiply through using 6

$\Rightarrow 6 \times \frac{a}{2} + \ 6\ \times \frac{b}{3} = 6 \times 2$

$\Rightarrow 3a + 2b = 12$

Let 3a + 2b = 12 …………….eqn (1).

In the second case, twice the first number was added to four times the second number and our answer was 16.

$\Rightarrow$ 2a +4b = 16 and let

2a + 4b = 16 …………eqn (2) .

Consider eqn (1) and eqn (2) .

3a + 2b = 12 ………..eqn (1)

2a + 4b = 16…………eqn (2)

Multiply eqn (1) by -2

$\Rightarrow -6a - 4b = \ -24 \ldots\ldots\ldots eqn\ (3)$

Solve now eqn (2) and eqn (3) simultaneously

i.e.$-6a - 4b = -24$

$+\ \underline{2a\ \ + 4b\ \ = 16}$

$\underline{\ \ -4a \qquad = -8}$

$-4a = -8 \Rightarrow$ a = $\dfrac{-8}{-4} = 2 \Rightarrow$ a $= 2.$

Put a = 2 in eqn (2)

$\Rightarrow 2(2) + 4b = 16 \Rightarrow 4 + 4\text{b} = 16$,

=> 4b = $16 - 4 = 12 \Rightarrow 4\text{b} = 12,$

$\Rightarrow$ b = $\frac{12}{4} = 3, \Rightarrow$ the second number is 3

and the first one is 2.

(Q6) Solve simultaneously:

$$\frac{3x}{5} + \frac{4y}{2} = 3$$

$$2 + \frac{4y}{2} - \frac{3x}{6} = 0$$

Hint:

Consider $\frac{3x}{5} + \frac{4y}{2} = 3$

Multiply through using 10

$\Rightarrow 10 \times \frac{3x}{5} + 10 \times \frac{4y}{2} = 10 \times 3,$

$\Rightarrow 6x + 20y = 30 \ldots\ldots\ldots. eqn\ (1)$

Consider $2 + \frac{4y}{2} - \frac{3x}{6} = 0$

$\Rightarrow \frac{4y}{2} - \frac{3x}{6} = 0 - 2,$

$\Rightarrow \frac{4y}{2} - \frac{3x}{6} = -2.$

Multiply through using 6

$\Rightarrow 6\times \frac{4y}{2} - 6 \times \frac{3x}{6} = 6 \times -2,$

$\Rightarrow 12y - 3x = -12$

$\Rightarrow -3x + 12y = -12 \dots\dots\dots\dots eqn\ (2)$

$\Rightarrow$ Continue by solving eqn (1) and eqn (2) simultaneously.

(Q7) Solve the simultaneous equations:

$\frac{1}{x} + \frac{1}{y} = 2$

$\frac{2}{x} + \frac{1}{y} = 3$

<u>*S*oln.</u>

Let $\frac{1}{x} + \frac{1}{y} = 2$…………………………eqn (1)

and $\frac{2}{x} + \frac{1}{y} = 3$……………………….....eqn (2)

Consider equation (1), since *x*x y = xy, we use xy to multiply through it.

$\Rightarrow \frac{1}{x} \times xy + \frac{1}{y} \times xy = 2 \times xy,$

$\Rightarrow$ y + x = 2xy…………………………………eqn (3)

Consider eqn (2), since the denominators are also x and y, we use xy to multiply through it since x$\times y = xy$.

i.e $\frac{2}{x} \times xy + \frac{1}{y} \times xy = 3xxy$

$\Rightarrow$ 2y + x = 3xy……………………………..eqn (4)

Now Consider eqn (3) and eqn (4)

i.e y + x = 2xy…………………………eqn (3)

2y + x = 3xy………………………..eqn (4)

Multiply through eqn (3) using -1 and this gives

-y – x = -2xy……………………eqn (5)

Solve eqn (4) and eqn (5) simultaneously by adding them up,

i.e 2y + x = 3xy

+ -y – x = -2xy

1y = 1xy

∴ From 1y = 1xy, divide through using y

$\Rightarrow \frac{1y}{y} = \frac{1xy}{y}$

$\Rightarrow 1 = x$ or $x = 1$.

Put x = 1 into eqn (1)

i.e $\frac{1}{x} + \frac{1}{y} = 2$

$\Rightarrow \frac{1}{1} + \frac{1}{y} = 2$, $\Rightarrow 1 + \frac{1}{y} = 2$

$\Rightarrow \frac{1}{y} = 2 - 1$, $\Rightarrow \frac{1}{y} = 1$

$\Rightarrow y \times 1 = 1 \Rightarrow y = 1$.

(Q8) Determine the values of a and b, which simultaneously satisfy the equations:

$\frac{1}{a} + \frac{2}{b} = 3$

$\frac{4}{2a} - \frac{1}{b} = 1.$

<u>Soln.</u>

Let $\frac{1}{a} + \frac{2}{b} = 3$…………………………………eqn (1) and $\frac{4}{2a} - \frac{1}{b} = 1$ ….……………………………eqn (2)

Consider equation (1) and since a$\times b = ab$, use ab to multiply through,

i.e $\frac{1}{a} \times ab + \frac{2}{b} \times ab = 3 \times ab$,

$\Rightarrow$ b + 2a = 3ab………………………………eqn (3).

Consider eqn (2) and since 2a$\times b = 2ab$, use 2ab to multiply through,

i.e $\frac{4}{2a} \times 2ab - \frac{1}{b} \times 2ab = 1 \times 2ab$,

$\Rightarrow$ 4b – 2a = 2ab……………………….……eqn (4).

Solve eqn (3) and eqn (4) simultaneously

$$\begin{array}{rl} & b + 2a = 3ab \\ + & \underline{4b - 2a = 2ab} \\ & \underline{5b \quad\quad = 5ab} \end{array}$$

$\Rightarrow 1\times 5b = 1 \times 5ab$

$\Rightarrow 1 = \frac{5ab}{5b}, \Rightarrow 1 = a.$

$\Rightarrow$ a = 1

(Q9) Solve the following equations simultaneously:
$\frac{4}{2x} + \frac{1}{y} = 3$ and $\frac{6}{3x} - \frac{1}{y} = 1$.

<u>Soln.</u>

Let $\frac{4}{2x} + \frac{1}{y} = 3$…………………………………eqn (1)

and $\frac{6}{3x} - \frac{1}{y} = 1 \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots .$ eqn (2)

Consider eqn (1), since 2x $\times\, y = 2xy$, use 2xy to multiply through.

i.e $\frac{4}{2x} \times 2xy + \frac{1}{y} \times 2xy = 3 \times 2xy,$

=> 4y + 2x = 6xy……………………………eqn (3)

Consider eqn (2), since $3x \times y = 3xy,$ use 3xy to multiply through.

i.e $\frac{6}{3x} \times 3xy - \frac{1}{y} \times 3xy = 1 \times 3xy,$

6y – 3x = 3xy…………………………………eqn (4)

Now consider eqn (3) and eqn (4)

i.e 4y + 2x = 6xy……………………………eqn (3)

6y – 3x = 3xy………………………….…eqn (4)

Eqn (3) × 3 ⇒ 12y + 6x = 18xy……………...eqn (5)

Eqn (4) × 2 ⇒ 12y – 6x = 6xy………………..eqn (6)

Add eqn (5) to eqn (6)

i.e 12+ 6x = 18xy

+ 12y – 6x = 6xy

24y = 24xy

$\Rightarrow 1\times 24y = 1 \times 24xy.$

Divide through using 24

$\Rightarrow \frac{1\times 24y}{24} = \frac{1\times 24xy}{24}$

$\Rightarrow 1\times y = 1 \times xy.$

Divide through using y

$\Rightarrow \frac{1\times y}{y} = \frac{1\times xy}{y}, \Rightarrow 1 = 1 \times x$

⇒ 1 = x or x = 1.

Substitute x = 1 into eqn (1)

i.e $\frac{4}{2x} + \frac{1}{y} = 3$

$\Rightarrow \frac{4}{2(1)} + \frac{1}{y} = 3,$

$\Rightarrow 2 + \frac{1}{y} = 3 \quad \Rightarrow \frac{1}{y} = 3 - 2,$

$\Rightarrow \frac{1}{y} = 1 \Rightarrow \mathrm{y} \times 1 = 1$

$\Rightarrow$ y = 1.

NB: If the two given variables such as the x and the y in the two given equations are not in line with each other, then they must be brought in line with each other. Also, xy = yx and mn = nm.

(Q10) Solve the following equations simultaneously: .

$\frac{1}{x} + \frac{1}{y} = 5$

$\frac{1}{y} - \frac{1}{x} = 1.$

$$Hint: Let\ \frac{1}{x} + \frac{1}{y} = 5 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots. \text{eqn (1)}$$

and $\frac{1}{y} - \frac{1}{x} = 1 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{eqn (2)}.$

Eqn (1) $\times\, xy$

$\Rightarrow \frac{1}{\mathrm{x}} \times \mathrm{xy} + \frac{1}{\mathrm{y}} \times \mathrm{xy} = 5 \times \mathrm{xy}$

$\Rightarrow \mathrm{y} + \mathrm{x} = 5\mathrm{xy} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots. \text{eqn (3)}$

Eqn (2) $\times\, xy$

$\Rightarrow \frac{1}{\mathrm{y}} \times \mathrm{xy} - \frac{1}{\mathrm{x}} \times \mathrm{xy} = 1 \times \mathrm{xy}$

$\Rightarrow x - y = xy \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$ eqn (4)

For the x and the y variables in eqn (3) eqn (4) to be in line with each other, rewrite eqn (3) i.e x + y = 5xy as y + x = xy.

Finally solve eqn (3) and eqn (4) simultaneously.

i.e   x + y = 5xy……………………………eqn (3)

x - y = xy……………………………..eqn (4).

<u>Simultaneous equations which give rise to quadratic equations:</u>

There are certain types of simultaneous equations, which give rise to quadratic equations, which need to be solved.

-Such quadratic equations can be solved by the completing of the square method, or by the use of the formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

(Q1) Find the values of x *and* y *which simultaneously satisfy the equations*:

$xy = 2$

$2x + y = 4$

Soln

Let $xy = 2 \ldots\ldots\ldots\ldots\ldots. eqn\ (1)$

and $2x + y = 4 \ldots\ldots\ldots\ldots eqn\ (2)$

From eqn (1) $xy = 2$

Dividing through using y

$\Rightarrow \frac{xy}{y} = \frac{2}{y} \Rightarrow x = \frac{2}{y}$.

Substitute $x = \frac{2}{y}$ $into\ eqn$ (2)

i.e $2x + y = 4 \Rightarrow 2\left(\frac{2}{y}\right) + y = 4$

$\Rightarrow 2 \times \frac{2}{y} + y = 4$

$\Rightarrow \frac{4}{y} + y = 4.$

Multiply through using y

$\Rightarrow y \times \frac{4}{y} + y \times y = y \times 4$

$\Rightarrow 4 + y^2 = 4y \Rightarrow 4 + y^2 - 4y = 0$

$\Rightarrow y^2 - 4y + 4 = 0$

$\Rightarrow 1y^2 - 4y + 4 = 0$

Which is a quadratic in y.

Now $1y^2 - 4y + 4 = 0$ and $ax^2 + bx + c = 0.$

By comparism,

$a = 1, b = -4$ and $c = 4$

Now $y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\Rightarrow y = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(4)}}{2(1)}$

$\Rightarrow y = \frac{4 \pm \sqrt{16 - 16}}{2}$

$\Rightarrow y = \frac{4 \pm \sqrt{0}}{2} \Rightarrow y = \frac{4 \pm 0}{2} = \frac{4}{2}$

$\Rightarrow y = 2$

Substitute $y = 2$ $into\ eqn$ (2)

i.e $2x + y = 4 \Rightarrow 2x + 2 = 4$

$\Rightarrow 2x = 4 - 2 = 2$

$\Rightarrow 2x = 2 \Rightarrow x = \frac{2}{2} = 1$

$\Rightarrow x = 1.$

(Q2) Solve the following equations simultaneously:

$2a + b = 5$

ab =3

soln

Let 2a + b = 5 …………….eqn (1)

and $ab = 3 \ldots \ldots \ldots \ldots \ldots \ldots \ldots eqn(2)$

From eqn (2) ab = 3 $\Rightarrow$ a = $\frac{3}{b}$.

Substitute a = $\frac{3}{b}$ $into\ \ eqn\ (1)$

i.e 2a +b = 5 $\Rightarrow 2\left(\frac{3}{b}\right) + b = 5, \Rightarrow 2 \times \frac{3}{b} + b = 5 \Rightarrow \frac{6}{b} + b = 5.$

Multiply through using b

$\Rightarrow b \times \frac{6}{b} + b \times b = b \times 5$

$\Rightarrow 6 + b^2 = 5b,$

$\Rightarrow 6 + b^2 - 5b = 0 \Rightarrow b^2 - 5b + 6 = 0,$

which is a quadratic in b.

Now $1b^2 - 5b + 6 = 0$ and $ax^2 + bx + c = 0$.

By comparism, a = 1, b = -5 and $c = 6$.

Questions

(Q1) Solve the following equations simultaneously:

(a) $a + 2b = 4$

$a - b = 1$

$Ans: a = 2$ and $b = 1$.

$(b)\ 3a - 6b = -6$

$2a + b = 6$

$Ans: a = 2$ and $b = 2$.

(c) $3x - y = 7$

$2x + 2y = 10$

$Ans: x = 3$ and $y = 2$

(d) $4x - 2y = -8$

$6x - 4y = -10$

Ans: x = -1 and y = 2.

(e) 3a + 2b =11

a + 4b =7

Ans: a = 3 and b = 1.

(f) $3x - 3y = 3$

$4x - 2y = 6$

$Ans: x = 2 \ and \ y = 1.$

(g) $5x + 7y = 27$

$3x + 2y = 14$

$Ans: x = 4 \ \ and \ y = 1$

(h) $3a = 15 - 2b$

$a + 5b = 18$

$Ans: a = 3 \ \ and \ b = 3$

(i) $-2x = 2 - 4y$
$y - x = -1$
$Ans: x = 2 \ and \ y = 1$

(j) $y = 2x$

$x + 2y = 10$

$Ans: x = 2 \ and \ y = 4$

(k) $3x - 4y = 20$

$2x + 3y = 2$

$Ans: \ x = 4 \ and \ y = -2$

(L) 2a + b = 14

$$\frac{1}{2}a - \frac{1}{6}b = 1$$

$Ans: a = 4 \ \ and \ \ b = 6.$

(m) $2x + b = 16$

$$\frac{1}{4}x + \frac{1}{2}b = 5$$

$Ans: \ x = 4 \ and \ b = 8.$

(N) 3a + 2b = 34

$$\frac{a}{3} - \frac{b}{2} = -2$$

$Ans: a = 6 \ \ and \ b = 8.$

(o) $\frac{1}{2} \ x + \frac{1}{3}y = 4$

$$\frac{1}{4}x + \frac{2}{3}y = 5$$

$Ans: x = 4 \ and \ y = 6.$

(P) $\frac{2}{5}x - \frac{1}{3}y = 2$

$$\frac{1}{2}x + \frac{1}{6}y = 6$$

$Ans: x = 10 \ \ and \ y = 6.$

(Q2) Esi bought 2 apples and 3 pineapples for ¢16. She later went to buy an apple and 4 pineapples for ¢18. If the price for an apple is the same and that for a pineapple is also the same, determine the cost of

(a) each apple . Ans¢2
(b) each pineapple . Ans ¢4
(c) five apples and two pineapples Ans ¢18

(Q3) A bucket containing 3 mangoes and 4 oranges had a total weight of 7g. Another bucket containing 6 mangoes and 2 oranges had a total weight of 8g .If the weight of a mango is fixed, and that of an orange is also fixed, find the weight of

(a) each mango Ans: 1g
(b) each orange Ans : 1g
(c) three mangoes and five oranges. Ans 8g

(Q4) A group of four men and two ladies paid ¢ 10 as lorry fare.

Another group of three gentlemen and three ladies paid ¢9 as lorry fare.

Determine the fare for

(a) each man. Ans: ¢2
(b) each woman Ans:¢1
(c) Determine the fare paid by 3 men and 5 women . Ans :¢11

(Q5) Solve simultaneously the following equations:

$3^{3x} \times 3^{y} = 81$

$3^{2x} \div 3^{y} = 9$

Ans: $x = 1.2 \; and \; y = 0.4$

(Q6) $If \; 2^{a-b} = 16 \; and \; 2^{2a-b} = 64 \;, determine \; the \; values \; of \; a \; and \; b$

Ans: a = 2 and b = -2

(Q7) Solve the following equations simultaneously:

$4^{x+2} = 16^{y+2}$

$\left(\frac{1}{3}\right)^{2x} = \left(\frac{1}{27}\right)^{y+2}$

Ans :$x = 6 \quad and \quad y = 2$

(Q8) Determine the values of $x \;\; and \; y \; which \; satisfy \; simutaneoulsy$

$these\ equations$:

$5^{2x-1} = 25^{y+5}$

$\left(\frac{1}{2}\right)^{3x} = \left(\frac{1}{64}\right)^{y+1}$

Ans: $x = 9\ and\ y = 3.5$

(Q9) Solve the following equations simultaneously:

a + 2b = 4

ab = 2

Ans: a = 2 and b = 1

(Q10) Given that $2xy = 12\ and\ 2x - y = 4, detemine\ the\ values\ of\ x\ and\ y$

$which\ satisfy\ the\ given\ equations\ simultaneously$:

Ans: $x = 3\ \ and\ y = 2$

: .