CHAPTER NINE

GRAVITATION, CIRCULAR MOTION AND SIMPLE HARMONIC MOTION

The angular velocity (w): This is the rate of change of angular displacement.

The period (T): This is the time interval taken by a body to complete one revolution.

N/B: V = rw, where V = linear velocity and w = angular velocity.

(Q1) A body makes six complete revolutions in 4.0 seconds. If it moves in a circle of radius 25cm, calculate

(a) the angular velocity.

(b) the velocity.

Soln:

(a)

θ = the number of revolutions = 6.

t = time in seconds = 4 seconds.

w = $\frac{\theta}{t} = \frac{6}{4} = 1.5.$

(b) Radius = r = 25cm = 0.25m.

But V = wr = 1.5×0.25

= 0.4ms^{-1}

N/B: The angular velocity, w = $\frac{2\pi}{T}$.

Linear acceleration: This is the rate of change of linear velocity.

Angular acceleration: This is the rate of change of angular velocity.

- Now it can be proved that with reference to circular motion, $T^2 \propto r^3$ where T = the period and r = the radius.
- For this to be done, we consider the motion of a planet moving in a circle round the sun.
- If the mass of this planet is m, then the force acting on the planet = mrw^2, where r = the radius of the circle (the circular motion) and w = the angular velocity of the motion.
- But since w = $\frac{2\pi}{T}$, then the force acting on the planet = mrw^2 = $mr(\frac{2\pi}{T})^2$ = $mr(\frac{2^2\pi^2}{T^2}) = \frac{4mr\pi^2}{T^2}$
- But this force is equal to the force of attraction of the sun on the planet.
- Assuming the inverse square law holds, then the force acting on the planet = $\frac{km}{r^2}$, where k is a constant, we can say that $\frac{km}{r^2}$ = $\frac{4\pi^2 mr}{T^2}$

  => $T^2 \times km = 4\pi^2 mr \times r^2$

  => $T^2\ km = 4\pi^2 mr^3$

  => $T^2 = \frac{4\pi^2 mr^3}{km}$ => $T^2 = \frac{4\pi^2 r^3}{k}$

- But since π and k are constant, then it is clear that $T^2 \propto r^3$.

(Q2) What is the angular displacement in radians, when an object moves through 5m in a circular path of radius 2.5m?

Soln:

Distance = s = 5m.

Radius = r = 2.5m.

Angular displacement = $\theta = \frac{S}{r} = \frac{5}{2.5} = 2\ radians.$

(Q3) The rim of a wheel starts from rest, and after 20 seconds attains an angular velocity of 6.4 $revs^{-1}$.Determine the angular acceleration.

Soln:

Let W_0 = the original velocity and W_1 = the final velocity.

Since the body starts from rest => $w_0 = 0$. Also $w_1 = 6.4revs^{-1}$. Time = t = 20s.

Angular acceleration = a = $\frac{w_1 - w_0}{t} = \frac{6.4-0}{20}$
=> a = 2 $rads^{-2}$

N/B:

- From v = rw => w = $\frac{v}{r}$.
- The centripetal acceleration = $\frac{v^2}{r}$
- The centripetal force = $\frac{mv^2}{r}$
- Also the centripetal force = mrw^2.

(Q4) The linear velocity of a body moving round a circle of radius 5.0m is $6.5ms^{-2}$. Find

(i) its angular velocity.

(ii)the centripetal force.

Soln:

(I) V = $6.5ms^{-1}$, r = 5.0m.

w = $\frac{v}{r} = \frac{6.5}{5} = 1.3\ rads^{-1}$.

(II)The centripetal acceleration = $\frac{v^2}{r} = \frac{6.5^2}{5}$ = 8.5 $rads^{-2}$

(Q5) A body of mass 63kg moves round a circle of radius 9m. If the body moves with a velocity of $18kmh^1$, calculate the centripetal force required to keep the body in its circular motion.

N/B:

- Since the radius is given in metres, the velocity which is given in kilometres per hour, must be changed into metres per second.
- Since 1000m = 1km and 1 hour = $60 \times 60 = 3600\ seconds$,

=> 18km/h = $\frac{18 \times 1000}{3600}$
= $5ms^{-1}$.

Soln:

Mass = m = 63kg

r = 9m, v = $5ms^{-1}$.

If F_c = centripetal force, then $F_C = \frac{mv^2}{r} = \frac{63 \times 5^2}{9} = 175N$.

(Q6) A body of mass 5kg moves round a circle of radius 6.0m with a speed of $10.0ms^{-1}$. Calculate

(i) the angular velocity.

(ii) the centripetal force.

Soln:

(i) $v = 10.0ms^{-1}$, $r = 6.0m$.

The angular velocity $= \frac{v}{r}$

$= \frac{10.0}{6} = 1.67 rad\ s^{-1}$.

(ii)The centripetal force $= \frac{mv^2}{r} = \frac{5 \times 10^2}{6} = 83.3N$.

(Q7) A toy car of mass 2.0kg is made to move in a circular track of radius 10.0m. If the centripetal force acting is 800.0N, determine the angular velocity of the car.

Soln:

If F_c = the centripetal force, then $F_c = mrw^2 \Rightarrow w = \sqrt{\frac{F_c}{mr}}$

Since $F_c = 800N$, $m = 2kg$ and $r = 10m$, $\Rightarrow w = \sqrt{\frac{800}{2\times10}} = 20\ rads^{-1}$

(Q8) A car of mass 80kg moves in a circular track of radius 100m. If its velocity is $20ms^{-1}$, find the centripetal force which acts on the car.

Soln:

$m = 80kg$, $v = 20ms^{-1}$, $r = 100m$.

If F_c = The centripetal force, then $F_C = \frac{mv^2}{r} = \frac{80\times20^2}{100}$

$\Rightarrow F_C = 320N$.

(Q9) An object of mass 50kg, moves at $5ms^{-1}$ round a circular path of radius 10m. Calculate

(a) the centripetal acceleration.

(b) the centripetal force needed to keep it in circular motion.

Soln:

(a) Centripetal acceleration $= \frac{v^2}{r} = \frac{5^2}{10} = \frac{25}{10} = 2.5ms^{-2}$.

(b)the centripetal force $= \frac{mv^2}{r} = \frac{50\times5^2}{10} = 125N$.

(Q10) A stone tied to a string is made to revolve in a horizontal circle of radius 4m, with an angular speed of 2 radians per second. With what tangential speed(velocity) will the stone move off the circle, if the string is cut.

Soln:

Tangential velocity = linear velocity, $\Rightarrow v = wr = 2 \times 4 = 8ms^{-1}$

(Q11) Find the force which is necessary to keep a mass of 0.8kg, revolving in a horizontal circular motion of radius 0.7m, with a period of 0.5 seconds.[Take $\pi^2 = 10m/s^2$].

Soln:

The needed force = F = $\frac{4mr\pi^2}{T^2} = \frac{4\times0.8\times0.7\times10}{0.5^2}$ = 90N.

(Q12) What accounts for the centripetal force in the following situations?

(i) the motion of the moon around the earth.

Soln.

It is the gravitational force of attraction which constitutes the centripetal force, when the moon moves in a circular orbit around the sun.

(ii) the motion of a bus around a round about.

Soln

The frictional force acting between the wheels and the road provides the centripetal force.

(iii) the motion of a car on a banked road.

Soln

The centripetal force is due to the horizontal reactions between the surfaces in contact.

(Q13) The motion of a body along a circular path or a circle of radius 2m is defined by the equation w = $\theta^2 - 2\theta$, where w $rads^{-1}$and θ rad are the angular velocity and displacement respectively. The instantaneous acceleration of the body for an angular displacement θ is $24ms^{-2}$. Show that θ= 3 rad.

Soln:

The angular velocity W = $\theta^2 - 2\theta$.

The radius (r) of the circle = 2m.

Instantaneous acceleration

(a) = $24ms^{-1}$

w = $\sqrt{\frac{a}{r}} = \sqrt{\frac{24}{2}} = 3.46$

=> w = 3.46 rad /s.

Since W = θ^2 - 2 θ => 3.46 = θ^2 - 2 θ,=>θ^2 - 2 θ - 3.46 = 0, which is a quadratic inθ.

Solving using the formula x = $\frac{-b \pm \sqrt{b^2-4ac}}{2a}$

=> $\theta = \frac{-2 \pm \sqrt{2^2-4(3.46)}}{2(1)}$

=> $\theta = \frac{-2 \pm 4.22}{2}$ => θ = 3.11

or θ = -1.11

Since $\theta = -1.11$ *is inadmissible, then* $\theta = 3.11 = 3$ rad.

N/B: The angle of inclination, or the angle that a rider must be inclined to the vertical or the angle of inclination of a track to the horizontal, to avoid sliding = θ, *where* $\tan\theta = \frac{V^2}{rg}$

(Q14) A bicycle rider traveling at a speed of $5ms^{-1}$, negotiates a curve of radius 5m. Calculate the angle at which both the rider and the bicycle must incline to the vertical so as to avoid sliding. [Take g = $10ms^{-2}$.]

Soln:

r = 5m, V = $5ms^{-1}$, g = $10ms^{-2}$

From tan $\theta = \frac{V^2}{r.g}$ => $\tan\theta$

$= \frac{5^2}{5 \times 10}$ => $\tan\theta = 0.5$,

=> θ = $\tan^{-1}$ 0.5, => θ = 26.6

N/B:

- If the reaction at the wheel = N, then Nsin θ = the centripetal force => Nsin $\theta = \frac{mV^2}{r}$
- The total reaction on the vehicle = $\frac{mg}{c0s\theta}$

(Q15)A racing car of mass 1000kg, moves round a banked track at a constant speed of 108km/h. Assuming that the total reaction at the wheel is normal to the track, and the horizontal radius to the track is 100m, calculate the angle of inclination of the truck to the horizontal, as well as the reaction at the wheel.[Take g = 10ms^{-2}].

Soln:

(a)v = 108kmh^{-1} = $\frac{108\times1000}{60\times60}$

= 30ms^{-1}.

r = 100m, m = 1000kg.

But tan$\theta = \frac{v^2}{rg} = \frac{30^2}{100\times10}$ = 0.9, =>θ = tan^{-1} 0.9,

=> $\theta = 42^0$.

(b) Let the reaction = N

=> Nsin $\theta = \frac{mv^2}{r}$

=> N $sin\ 42^0 = \frac{1000\times30^2}{100}$

=> N sin 42^0 = 9000 => N = $\frac{9000}{sin42^0}$

=> N = 13450N.

(Q16) A vehicle of mass 600kg travels along a banked circular truck of radius 50m. If the speed of the vehicle is 15ms^{-1}, calculate

(a) the angle of inclination of the track to the horizontal.

(b) the total reaction on the vehicle. [Take g = 10ms^{-2}].

Soln

(a)$V = 15ms^{-1}$, $r = 50m$, $m = 600kg$.

From $\tan\theta = \frac{v^2}{rg} \Rightarrow tan\ \theta = \frac{15^2}{50\times10}$

$= 0.45, \Rightarrow \tan\theta = 0.45,$

$\Rightarrow \theta = \tan^{-1} 0.45 \Rightarrow \theta = 24^0$.

(b) If N = the total reaction on the vehicle, then

$N = \frac{mg}{\cos\theta} = \frac{600\times10}{cos24^0}$

$\Rightarrow N = 6600N$

$= 6.6KN$.

<u>Newton`s law of universal gravitation and the gravitation constant.</u>

- Newton`s law of universal gravitation, states that the force of attraction between any two particles is directly proportional to the square of their distance apart.

- If M_1 and M_2 are the two masses and their separation is r, then from Newton`s law the force of attraction, F, between these two masses is proportional to $\frac{M_1M_2}{r^2}$

i.e. $f \propto \frac{M_1M_2}{r^2} \Rightarrow F = \frac{GM_1M_2}{r^2}$, where G is a universal constant referred to as the gravitational constant.
- This constant, G, can either be expressed in Nm^2kg^{-2} or $m^2kg^{-1}S^{-2}$, and it is defined as the attractive force between two units masses at a distance of 1M apart.
- G is described as universal because it has the same value for the mutual attraction of any pair of bodies in the universe.

(Q1) (a)State Newton`s law of gravitation and hence derive the equation relating the universal gravitational constant, G and the acceleration of free fall, g, at the surface of the earth. [Neglect the rotation of the earth].

(c) State four assumptions made in your derivative.

Soln;

(a)The law states that the force of attraction between any two particles, is directly proportional to the product of their masses, and inversely proportional to the square of their distance apart,

=> $f \propto \frac{M_1 M_2}{r^2}$, where M_1 and M_2 are the two masses and r is their distance apart.

- Now consider a body of mass m on the surface of the earth. Let M = the mass of the earth and R = the radius of the earth. Then with reference to Newton`s gravitational law, the force of attraction between the earth and the body is given by

$F = \frac{GmM}{R^2}$

But since F = ma = mg,

=> $mg = \frac{GmM}{R^2}$

=> $mg \times R^2 = GmM$,

=>$g = \frac{GmM}{mR^2}$ => $g = \frac{GM}{R^2}$, where G = the universal gravitational constant.

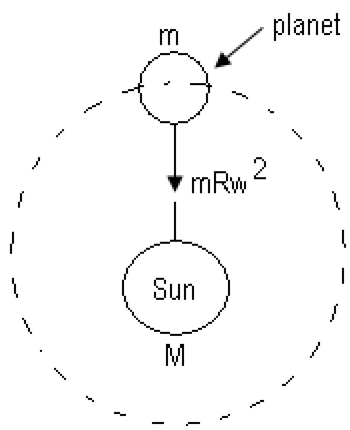
(b)The four assumptions made are:
(I) The mass of the earth is concentrated at the centre.

(II) The earth is spherical in shape.

(I) The earth does not rotate.
(II) The value of 'g' or the acceleration due to gravity, remains constant at every portion of the earth.

(Q2) If a planet moves around the sun in a circle of radius R, show that if one revolution is completed in time T, $R^3 \propto T^2$.

Soln:



Let m = the mass of the planet and M = the mass of the sun.

- If the planet of mass m, moves in a circle round the sun, then the force acting on it is given by mRw^2, where R is the radius of the circle and w is the angular velocity of the motion.
- The force which acts on the planet which is the centripetal force = mRw^2.
- Since $T = \frac{2\pi}{w} \Rightarrow w = \frac{2\pi}{T}$

  => the centripetal force = mRw^2

  $= mR\left(\frac{2\pi}{T}\right)^2 = mR\left(\frac{4\pi^2}{T^2}\right) = \frac{4mR\pi^2}{T^2}$
- From Newton`s law of universal gravitation, the force of attraction of the sun on the planet = $\frac{GMm}{R^2}$.
- Since this force of attraction = the centripetal force => $\frac{4\pi^2 mR}{T^2} = \frac{GMm}{R^2}$,

  => $4\pi^2 mRxR^2 = GMm \times T^2 \Rightarrow 4\pi^2 mR^3 = GMmT^2$

  Divide through using m => $\frac{4\pi^2 R^3 m}{m} = \frac{GMmT^2}{m}$,

  => $4\pi^2 R^3 = GMT^2$

  => $R^3 = \frac{GMT^2}{4\pi^2}$.

Since G, M, 4 and π are all constants => $\frac{GM}{4\pi^2}$ = a constant and let this constant = k.

Then from $R^3 = \frac{GMT^2}{4\pi^2} \Rightarrow R^3 = kT^2 \Rightarrow R^3 \propto T^2$

(Q3) Explain the terms gravitational field and the gravitational potential.

Soln:

- The gravitational field is the region in space, where gravitational force can be experienced.
- But gravitational potential is the work done, in bringing a unit mass from infinity to a point in the gravitational field.

(Q4) On Jupiter, it is predicted that g = 26NJkg^{-1}.

On the moon, g = 1.6Nkg^{-1} and on the earth`s surface g = 9.8 Nkg^{-1}.

(I)If a hammer weighs 8N on earth, what will be the mass of the hammer on

(a) the earth.

(b) Jupiter.

(II) Determine the weight of the hammer on

(a) the moon

(b) Jupiter.

Soln:

(a) Let W = weight of hammer, m = mass of hammer and g = acceleration due to gravity.

From W = mg => m = $\frac{W}{g}$.

But since the weight of hammer on earth = 8N,

=> mass of hammer on earth = m $= \frac{W}{g} = \frac{8}{10} = 0.8kg.$

(b) Since the mass of a body remains constant or the same throughout the whole universe, then the mass of the hammer on Jupiter = 0.8kg.

(ii)

(a) The weight of hammer on the moon = mass $\times$ the value of (acceleration due to gravity) on the moon = 0.8 $\times$ 1.6 = 1.3N.

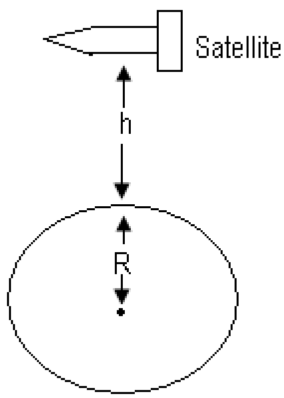
(b) The weight of hammer on Jupiter = mass × the value of (acceleration due to gravity) on Jupiter = 0.8 × 26 = 212N..

Geostationary Satellite:

- A geostationary satellite is one that circles the earth in a circular orbit, at a constant height above the equator, with a time of revolution equal to that taken by the earth, i.e with a period of rotation of 24 hours.

(Q1) Obtain an expression for the orbit velocity of a satellite, in terms of its height h above the surface of the earth.

Soln:



Let h = height of satellite above the surface of the earth.

Let F_c = the centripetal force between the earth and the satellite.

Let F_G= the gravitational force between the earth and the satellite.

- Then $F_c = \frac{mv^2}{(R+h)}$, where R+ h = the distance between the satellite and the centre of the earth, and m = the mass of the satellite.
- From Newton`s law of universal gravitation, $F_G = \frac{GMm}{(R+h)^2}$, where M = mass of the earth.
- But since $F_C = F_G$, => $\frac{mV^2}{R+h} = \frac{GMm}{(R+h)^2}$,

  => $mV^2(R+h)^2$= (R + h) GMm,

  MV^2(R+h)(R+h) = (R+h)GMm

Divide through using M(R+h) => $V^2(R + h) = GM$,

=> $V^2 = \frac{GM}{R+h}$ => $V = \sqrt{\frac{GM}{R+h}}$

N/B: The period T of a satellite in orbit is given by $T = \frac{2\pi R}{V}$, where R = the distance between the satellite and the centre of the earth.

V = the velocity of the satellite.

(Q2) The period T of a satellite in orbit is given by $T = \frac{2\pi R}{V}$.

If a satellite orbits at 50,000km above the earth with a velocity of $3.5kms^{-1}$, calculate its period in orbits. [Radius of the earth = 6400km].

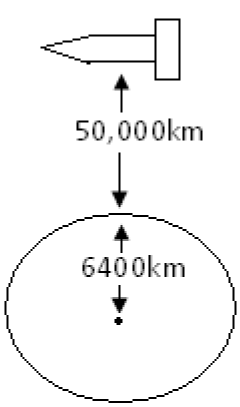
N/B: The velocity of the satellite = 3.5km/s. This is given in kilometres per second, and must be converted into kilometres per hour. Now V = 3.5km/s, and the number of seconds in an hour = 60×60 = 3600 seconds.

If 1 second = 3.5km

=> 3600 seconds = $\frac{3600 \times 3.5}{1}$ = 12600,

=> the velocity of the satellite = 12600km/h.

Soln:



R = 50,000 + 6400 = 56400KM.

V = 12600km/h.

But T = $\frac{2\pi R}{V}$ = $\frac{2\times3.14\times56400}{12600}$

= 28.

=> T = 28 hours.

N/B: For an orbiting satellite, $T^2 = \frac{4\pi^2 r^3}{gR^3}$, where T = the period of the satellite, r = the distance of the satellite`s orbit from the centre of the earth.

R = the radius of the earth.

(Q3) A relay satellite was placed in parking orbit, on the surface of the earth,

(a) how high was this satellite?

(c) determine the height of the satellite above the earth`s surface. [Take the radius of the earth = 6.4×10^6m and g = $9.8ms^{-2}$].

Soln:

(a) $T^2 = \frac{4\pi^2 r^3}{gR^3}$

where T = the period of the satellite.

r = the distance of the satellite`s orbit from the centre of the earth.

Since the satellite is in parking orbit => R = 6.4×10^6m.

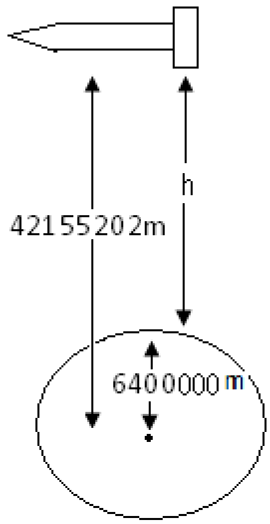
From $T^2 = \frac{4\pi^2 r^3}{gR^2}$ => $T^2\times gR^2 = 4\pi^2 r^3$

=> $r^3 = \frac{T^2 gR^2}{4\pi^2}$

But T = 24 hours = $(24 \times 3600)\ seconds = 86400\ seconds$.

$From\ r^3 = \frac{T^2 gR^2}{4\pi^2}$

=> $r^3 = \frac{(24\times3600)^2(9.8)(6.4\times10^6)^2}{4(3.14)^2}$ => r = 42155202 => the satellite was 42155202 M from the centre of the earth.



(b) If h = the height of the satellite above the ground => h = 42155202

- 6400000 = 35755202m

(Q4) A satellite of mass m orbits the earth of mass M at a distance, H from the centre of the earth. Show that the period T of the revolution of the satellite is given by the equation T = $2\pi\sqrt{\frac{H^3}{GM}}$, where G = the universal gravitational constant.

(b)Calculate H for a geostationary satellite.

[Mass of the earth = 6.0×10^{24}kg, G = 6.7×10^{-11} $NMkg^{-2}$].

Soln:

Let F_a = the attractive force between the satellite and the earth, => $F_a = \frac{GMm}{H^2}$.

Since this force F_a is provided by the centripetal force, F_c acting on the satellite, then

$F_a = F_c$.

But $F_C = \frac{mv^2}{H}$, *and since* $V = wH$ => $F_C = \frac{m(wH)^2}{H}$

=> $F_C = \frac{m \times W^2 H^2}{H}$

=> $F_C = mw^2H$

But since $F_C = F_a$

=> mw^{2}H = $\frac{GMm}{H^2}$

=> mw^{2}H × H^2 = GMm

=> mw^{2}H^3 = GMm

=> $\frac{mw^2H^3}{m} = \frac{GMm}{m}$

=> w^{2}H^3 = GM => w^2 = $\frac{GM}{H^3}$

=> w = $\sqrt{\frac{GM}{H^3}}$

(b) For a geostationary satellite, the period T = 24 hours = 24 × 3600 = 86400 seconds.

But T = $^{2\pi}\sqrt{\frac{H^3}{GM}}$

Square both sides

=> T^2 = 2^{2}π^2 ($\sqrt{\frac{H^3}{GM}}$)^2

=> T^2 = 4 π^2 ($\frac{H^3}{GM}$)

=> T^2 = $\frac{4\pi^2 H^3}{GM}$

=> T^{2}GM = 4π$^2H^3$ => H^3 = $\frac{T^2GM}{4\pi^2}$

=> H = $\sqrt[3]{\frac{T^2GM}{4\pi^2}}$.

Simple Harmonic Motion:

This is a kind of oscillatory motion, in which the acceleration of the body is always directed towards an equilibrium point, and varies directly as its distance from this fixed point.

S.H.M. Formulae:

(1) Acceleration = $\frac{d^2x}{dt^2} = -w^2x$

(2) Velocity = $\pm w\sqrt{a^2 - x^2}$, where a = the amplitude or the maximum displacement of the body from the equilibrium position, and x = the distance of the body from the equilibrium position or the centre.

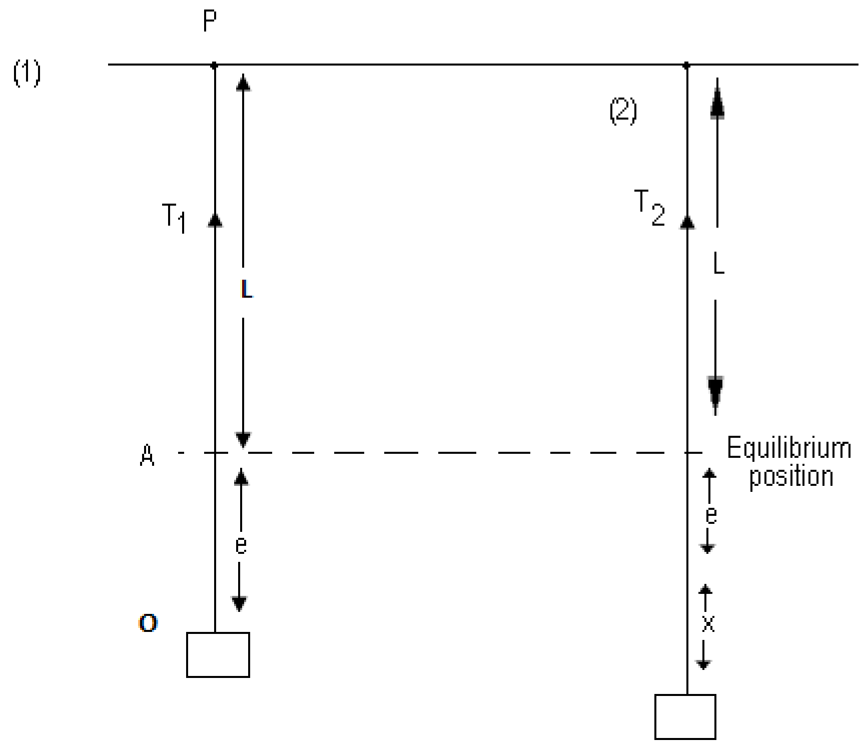
(3) Period = T = $\frac{2\pi}{w}$.

Also, the period = T= $2\pi\sqrt{\frac{m}{k}}$, where m = the suspended mass and k = the spring constant.

(1) The energy of the given system = E = $\frac{1}{2} \times k \times A^2$, where A = the amplitude in metres.
(2) The energy of S.H.M = $\frac{1}{2}mw^2a^2$, where m = the mass of the body and a = maximum displacement.
(3) The maximum velocity = wa and the maximum acceleration = aw^2, where a = the maximum displacement or the amplitude.
(4) The displacement = x = a sin wt, if displacement is measured from the equilibrium position.
(5) Also the displacement = x = a cos wt, when measurement is taken from the extreme end.

N/B:

- The amplitude, a, of the motion is the maximum distance on either side of the centre of oscillation.
- The frequency = f = $\frac{1}{T}$, where T = the period.



- According to Hooke`s law, when a mass or weight is suspended from the end of a spring or an elastic thread, the extension of the spring is proportional to the weight, provided the elastic limit is not exceeded.
- The tension T in the spring is proportional to the extension e => $T \propto e$ => T = ke, where k is a constant which can be referred to as the elasticity constant, the spring constant or the force constant.
- The unit of this force constant is Nm^{-1}.
- Figure (1) shows a spring or an elastic thread PA of length l, which is suspended from a point P.
- When a mass m is hanged on it, the spring stretches to the point 0 by a length e, and mg = ke.
- Since the tension T_1 = mg, then T_1 = mg = ke => T_1 = k.e.
- Now If the spring is pulled further by a length x as shown in figure (2), then $T_2 = k(e + x)$.

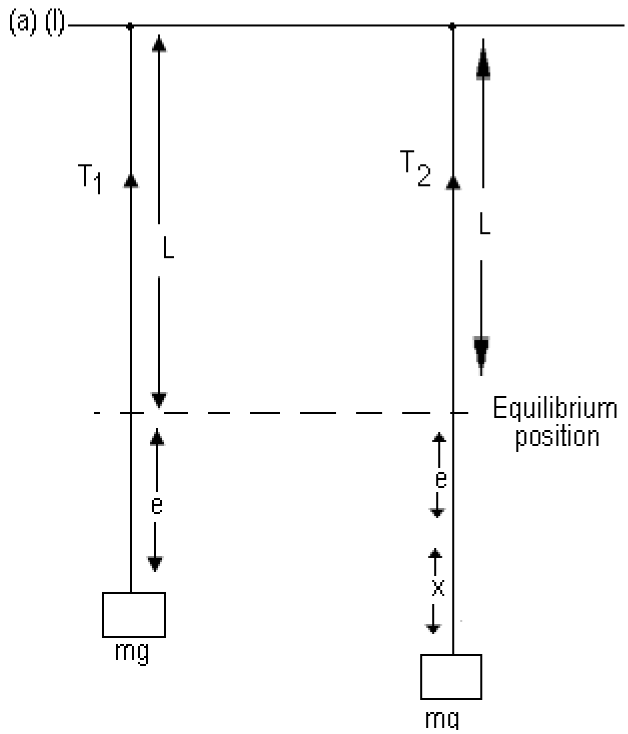
(Q1)(a)

(I) Prove that a mass m, suspended from a fixed point by a new helical spring, which obeys Hooke`s law, undergoes simple harmonic motion when it is displaced vertically from its equilibrium position.

(II) Find an expression for the period T.

(b) If m = 0.1kg and the extension of the spring is 4cm when the spring is in its equilibrium position, determine the period T, for the oscillations.

Soln:



- In the first case, when a mass of m was hanged from the spring, the extension produced was e, => T_1 = ke, where k = the force constant of the spring.
- If the extension is increased by a length x as shown in the second figure, then

  T_2 = K(e + x).

- The resultant force = T_2 – T_1 = K(e + x) – ke = ke + kx – ke

  = ke – ke + kx = kx.

- Since resultant force = ma =>ma = kx, => a =

  $\frac{-k}{m}x$ $(i.e. a\ negative\ sign\ must\ always\ be\ brought\ before\ the\ value\ of\ the\ acceleration)$

- Since this is of the form a = $-w^2x$, then the mass performs S.H.M.

  (III) Comparing a = $\frac{-k}{m}x$ with a = $-w^2x$, => $w^2 = \frac{k}{m}$

=> w = $\sqrt{\frac{k}{m}}$, where w = the angular velocity.

The period T = $\frac{2\pi}{w} = \frac{2\pi}{\sqrt{\frac{k}{m}}}$

=> T = $2\pi\sqrt{\frac{m}{k}}$.

(b) The spring constant = k = $\frac{force}{extension} = \frac{0.1\times 10N}{0.04m} = 25Nm^{-1}$

[since the extension = 4cm = 0.04m].

Substituting these values into T = $2\pi\sqrt{\frac{m}{k}}$ => T = $2\pi\sqrt{\frac{0.1}{25}}$

= $2(3.14)\sqrt{\frac{0.1}{25}}$ = 0.4 seconds.

N/B: -Since the units of the spring constant is Nm^{-1}, then the mass = 0.1kg must be converted into Newtons or force by multiplying by 10, when determining the spring constant.

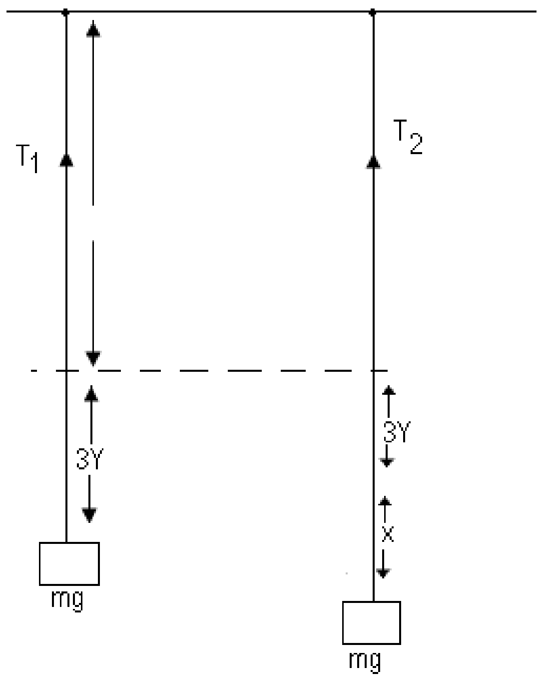
Also the extension which is given in centimeters must be converted into metres by dividing by 100, as already indicated.

(Q2) An elastic thread is hanged from a roof and a weight of mass m hangs from its other end. The extension produced is 3Y. If the weight is pulled further down through a distance of length x, and then released to go,

(a) Show that the motion is S.H.M and determine the period as well as the frequency.

(b) If the force constant of the spring is mg, what will be the period?

Soln:



(a)

- Since the extension was 3Y when the mass m was hanged on it, => $T_1 = k(3Y)$.
- If the mass is further pulled down through a distance x, then the extension = 3Y + x

  and $T_2 = k(3Y + x)$.
- The resultant force = $T_2 - T_1$

  = K (3Y + x) – k(3Y) = 3KY + KX – 3KY = KX.
- Since force = ma, then ma = kx => a = $\frac{-k}{m}x$, and since this is of the form a = - w^2x,

  then the motion is S.H.M.
- Now by comparing a = $\frac{-k}{m}x$ with a = - w^2x

=> $w^2 = \frac{k}{m}$ => w = $\sqrt{\frac{k}{m}}$.

- The period T = $\frac{2\pi}{w} = \frac{2\pi}{\sqrt{\frac{k}{m}}}$

= $2\pi\sqrt{\frac{m}{k}}$.

Frequency = $\frac{1}{T}$ = $\frac{1}{2\pi\sqrt{\frac{m}{k}}}$

N/B: From ma = kx => a = $\frac{k}{m}$ => a = $\frac{-k}{m}$. As already stated, ensure the negative sign is always around and if not, then bring it there.

(b) Since w = $\sqrt{\frac{k}{m}}$, $\ and\ if\ the\ spring\ constant\ k = mg, then\ w = \ \sqrt{\frac{mg}{m}}$

=> w = $\sqrt{g}$.

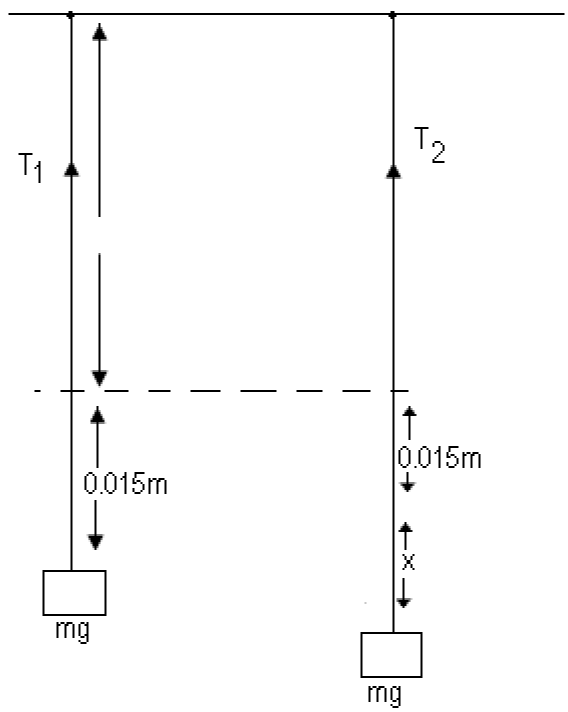
The period T = $\frac{2\pi}{w} = \frac{2\pi}{\sqrt{g}}$

= $\frac{2\pi}{\sqrt{\frac{g}{1}}} = 2\pi\sqrt{\frac{1}{g}}$.

(Q3) A body is supported by a helical spring and causes a stretch of 1.5cm in the spring. If the mass is now set in vertical oscillation of small amplitude, determine the periodic time of the oscillation.

N/B: First the 1.5cm must be converted into metres => 1.5cm = 0.015m.

Soln:



- In the first instance, the mass m stretched the spring by a length 0.015m before equilibrium was attained

=> $mg = T_1 = ke$,

=> $mg = T_1 = k(0.015)$

=> $mg = k(0.015)$ ………………..eqn (1)

=> $\frac{k}{m} = \frac{g}{0.015} \ldots \ldots \ldots \ldots \ldots \ldots \ldots eqn\ (2)$

- In the second case, the mass is assumed to have been pulled further through a distance of x, => the extension = 0.015 + x and $T_2 = K(X + 0.015)$.

The resultant force = $T_2 - T_1$

= K(X + 0.015) – K(0.015)

= KX + 0.015K – 0.015K = KX.

Introducing the negative sign => Resultant force = - kx.

Since force = ma => ma = - kx,

=> a = $\frac{-k}{m}x$.

But since from eqn (2), $\frac{k}{m} = \frac{g}{0.015}$

=> a = $-\frac{k}{m}x = -\frac{g}{0.015}$x.

$=> a = -\frac{g}{0.015}$x.

Comparing a = $\frac{-g}{0.015}$x with a = - w^2x => $w^2 = \frac{g}{0.015}$

=> w = $\sqrt{\frac{g}{0.015}}$

Now T = $\frac{2\pi}{w}$ $=> T = \frac{2\pi}{\sqrt{\frac{g}{0.015}}}$

=> T = $2\pi\sqrt{\frac{0.015}{g}}$ => T = $2(3.14)\sqrt{\frac{0.015}{9.9}}$

= 0.26 seconds.

(Q4) A particle moving with simple harmonic motion has a maximum displacement of 30cm and an angular velocity of 1.02 $rads^{-1}$. Calculate

(a) the maximum velocity.

(b) the maximum displacement of the particle.

(c) the speed and the acceleration of the particle, when it is 20cm from the centre of oscillation.

Soln:

Maximum displacement = amplitude = a = 30cm = 0.3m.

w = 1.02 $rads^{-1}$.

(a) The maximum velocity = V_{max} = wa = 1.02×0.3

= 0.31ms^{-1}.

(b) The maximum acceleration = $aw^2 = 0.3 \times 1.02^2$

= 0.312ms^{-2}

(c) When the particle is 20cm from the centre => x = 20cm = 0.2m. Also a = 0.3m and w = 1.02 $rads^{-1}$.

The required velocity or speed = V = $w\sqrt{a^2 - x^2}$

= $1.02 \times \sqrt{0.3^2 - 0.2^2} = 0.23$

=> $V = 0.23ms^{-1}$.

The acceleration = w^2x

= $1.02^2 \times 0.2 = 0.208ms^{-2}$.

(Q5) A body of mass 0.4kg , performs simple harmonic motion with a period of 0.04 seconds and an amplitude of 0.01m. Find the total energy of the motion.

Soln:

m = mass = 0.4kg.

Since the amplitude = 0.01m

=> a = 0.01m.

The period T = 0.04s.

$w = \frac{2\pi}{T} = \frac{2(3.14)}{0.04} = 157.$

=> $W = 157\ rads^{-2}$.

The energy stored = $\frac{1}{2}mw^2a^2$

$= \frac{1}{2} \times 0.4 \times 157^2 \times 0.01^2 = 0.5J.$

(Q6) A 10kg mass is suspended from the end of a spring and then released. If the spring oscillates in simple harmonic motion with a period of 0.5s, calculate the energy given to the 10kg mass, if the amplitude of the oscillation is 2cm.[Assume $\pi^2 = 10$].

Soln:

The energy given = E = $\frac{1}{2} \times K \times A^2$, where A = amplitude and K = the spring constant.

For S.H.M, the period T = $2\pi\sqrt{\frac{m}{k}}$, $where\ m = the\ suspended\ mass\ and\ k = the$ spring constant.

From T = $2\pi\sqrt{\frac{m}{k}}$, squaring both sides =>

$T^2 = 2^2\pi^2 \left(\sqrt{\frac{m}{k}}\right)^2$,

=> $T^2 = 4\pi^2 \frac{m}{k} = \frac{4\pi^2 m}{k}$,

=> $T^2 = \frac{4\pi^2 m}{k}$ => $T^2 k = 4\pi^2 m$,

=> $k = \frac{4\pi^2 m}{T^2} = \frac{4 \times 10 \times 10}{0.5^2}$

=> K = 1600.

Amplitude = a = 2cm = 0.02m.

The energy given = E = $\frac{1}{2} \times 1600 \times 0.02^2$ = 0.3J.

Questions:

(1) Differentiate between angular velocity and angular acceleration.

(2) A spinning wheel makes 5 revolutions within 6 seconds. If it moves in a circle of radius 0.2m, determine

(a) the angular velocity.

Ans: 0.8 $rads^{-1}$.

(b) the linear velocity

Ans:0.16ms^{-1}

(3) A stone tied to the end of a rope is swinged, so as to move through 10m in a circular path of diameter 400cm. Calculate the angular displacement in radians.

Ans: 5 radians.

(4) A body moves in a circular motion of diameter 20cm, with an angular velocity of 1.5$rads^{-1}$. Find

(a) its linear velocity. Ans: 15ms^{-1}

(b) its centripetal acceleration. Ans: 22.5$rad.s^{-2}$.

(5) An object of mass 60kg moves in a circular motion of radius 900cm. If its velocity is 7ms^{-2}, determine the amount of centripetal force which is needed in order to keep it in the circular motion. Ans: 327N.

(6) A piece of iron whose mass is given as 70kg, performs a circular motion with a velocity of $20kmh^{-1}$. If the amount of centripetal force needed to keep it in the circular motion is 80N, find the radius of this circular motion. Ans: 27m.

(1) A body performing a circular motion, needed 900N of centripetal force. If its angular velocity was $24rads^{-1}$ and the radius of the circular motion was 8.2m, determine the mass of this body. Ans: 0.2kg.

(2) The tangential velocity of a circular motion moving body is $15ms^{-1}$. Given the radius of this circular motion to be 6m, find the angular speed. Ans: $2.5rad.S^{-1}$.

(3) A body of mass 2kg is expected to revolve in a horizontal circular motion of diameter 0.8m, with a period of 0.2 seconds. Determine the needed force.

[Take $\pi^2 = 10$] .Ans: 800N.

(4) With the aid of Newton`s law of universal gravitation, prove that $g = \frac{GM}{R^2}$, where R = the radius of the earth, M = the mass of the earth, G = the universal gravitational constant and g = the acceleration due to gravity.

(5) (a) What is the difference between gravitational field and gravitational potential?

(b) What do you understand by a geostationary satellite?

(12) A satellite whose velocity is 13000km/h, has an orbit which is located 40,000km above the earths surface. If the radius of the earth is 6400km, determine the period of this satellite. Ans: 22.4 hours.

(13) A suspended 5kg mass performs S.H.M with a period of 0.2 seconds. If the amplitude of the oscillation concerned is 0.06m, determine the energy given to the 5kg mass. [Take π^2=10]. Ans: 9J

(14) A particle which is performing S.H.M, is suspended by a spring whose spring constant is $200Nm^{-1}$. If the maximum displacement of the particle from the equilibrium position is 80cm, determine the amount of energy given to the particle.

Ans: 80J.

(15) A particle undergoing S.H.M has an amplitude of 60cm. Determine its angular velocity as well as its velocity when it is 0.2m from the centre, given that the maximum acceleration of the particle is $2.4ms^{-2}$.

Ans: Angular Velocity = $2rads^{-1}$

Velocity = $1.13ms^{-1}$.

(16) A body of mass 0.2kg, performs S.H.M with a period of 0.03 seconds, and an amplitude of 0.01m.

(a) Determine the total energy of the motion. Ans: 0.44J.

(b) Find the maximum acceleration. Ans: $437ms^{-2}$.

(c) What will be the velocity of the body when it is 0.8cm away from the centre.

Ans: 0.008m/s.

(17) One end of an elastic string whose force factor is k, is attached to a point and the other end attached to a mass M, so that the extension produced is e. If the string is pulled downwards through a distance of 2y, from the equilibrium position and then let to go, (a) prove that it performs simple harmonic motion.

Ans: since the acceleration a = $\frac{-2k}{m}y$ which is of the form a = -w^{2}x => the motion is S.H.M.

(b) Find the angular velocity.

Ans: w = $\sqrt{\frac{2k}{m}}$

(c) What is the period?

Ans: $2\pi\sqrt{\frac{m}{2k}}$.

(d) What is the frequency?

Ans: $\frac{1}{2\pi\sqrt{\frac{m}{2k}}}$.

(18) One end of an elastic thread hangs from a roof, and a weight m hangs from the other end. The extension produced is 4Y. If the string is pulled further down through a distance of 2Y, and then released to go,

(a) Show that its motion is S.H.M.

Ans:

(a) By solving the question, the acceleration = a = $\frac{-2\,k}{m}\,Y$, which is of the form a = $-w^2x$,

where $w^2 = \frac{2k}{m}$

(b) Find the period.

Ans: T = $2\pi\sqrt{\frac{m}{2k}}$.

(c) Given that the force or the spring constant = mg, determine the period.

Ans: T = $2\pi\sqrt{\frac{1}{2g}}$.