CHAPTER THREE

TRANSFORMATION

Introduction:

There are various types of transformation and the types to be considered are:

1. Translation .
2. Reflection.
3. Rotation.
4. Enlargement.

Translation:

- This is the types of transformation in which every point moves the same distance, and in the same direction.
- Under translation, the lengths of lines and the sizes of angles do not change
- This implies that if a figure undergoes translation, its size as well as its angles remain unchanged.
- If the point (x, y) is translated by the vector $\binom{a}{b}$, then

  $(x, y) \longrightarrow (x + a, y + b)$,

  ie (x, y) $\xrightarrow{\text{tranlation by vector} \binom{a}{b}}$ $(x + a, y + b)$.

  Example (1)

  If $(x, y)$is translated by the vector $\binom{1}{4}$, then

$(x, y) \longrightarrow (x + 1, y + 4)$.

Example (2)

If $(2,5)$ is translated by the vector $\binom{1}{3}$, then

$(2,5) \longrightarrow (2 + 1,5 + 3)$.

$(2,5) \longrightarrow (3,8)$.

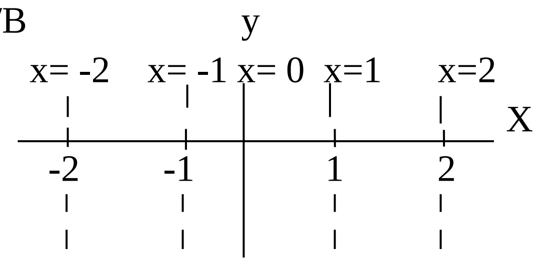
N/B: The point (3, 8) is called the image of the point (2, 5).

Reflection :

The reflection of a point or a figure can only be described, only when the position of the mirror line is well defined or known.

Under this type of transformation, the sizes of angles as well as the lengths of lines remain unchanged.

N/B



- The graph whose equation is x = 1, is a straight lines which is perpendicular to the x-axis, and passes through the point 1 on the x-axis.
- Also the line x = -2 passes through the point -2 on the x-axis.
- The y axis is also the same as the line x = 0

Types of reflections:

There are various types of reflections, and those to be considered are:

1. **Reflection in the y-axis or line x = 0:**

- For such a reflection,

  $(x, y) \longrightarrow (-x, y)$.

Example (1).

P (2, 5) $\xrightarrow{\text{reflection in the y- axis}}$ P_1(-2,5).

Example (2)

If the Q(3, 8) undergoes a reflection in the line x = 0, then for its image Q_1,

$Q(3,8) \longrightarrow Q_1(-3,8)$.

2. **Reflection in the line y = b:**

   For such a reflection, (x, y) $\longrightarrow$ (x, 2b – y).

Example (1)

If the point (2,4) undergoes a reflection in the line

y = 3, then

(2, 4) $\xrightarrow{\text{reflection in line y = 3}}$ {2, 2(3) -4}

(2, 4) $\longrightarrow$ (2, 6 – 4)

(2, 4) $\longrightarrow$ (2, 2).

Example (2)

If the point Q(2,3) undergoes a reflection in the line y = 5, then for its image Q_1,

$(x, y) \xrightarrow{\text{reflection in the line } y = b} (x, 2b - y)$,

$\Rightarrow Q(2,3) \xrightarrow{reflection\ in\ line\ y = 5} Q_1\{2, 2(5) -3\}$

$Q(2, 3) \longrightarrow Q_1(2,10 - 3)$

$Q(2, 3) \longrightarrow Q_1(2,7)$.

N/B: In this case, (x, y) = (2, 3) and y = b

becomes equal to y = 5.

$Therefore\ x = 2, y = 3\ and\ b = 5.$

There values are the substituted into the formula

$(x, y) \xrightarrow{\text{reflection in line } y = b} (x, 2b - y)$.

3. Reflection in the x-axis or the line y = 0:

- For such a reflection,

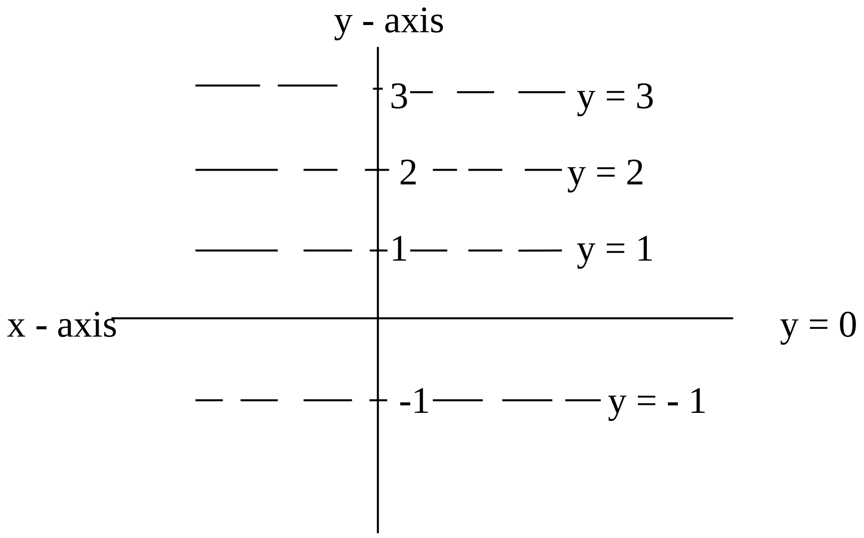
  $(x, y) \xrightarrow{\text{reflection in x-axis}} (x, -y)$

  Example (1)

If P(4, 3) undergoes a reflection in the x-axis, then its image P_1 is given by
$P(4,3) \xrightarrow{\text{reflection in x-axis}} P_1(4,-3)$.

Example (2)

If the point A(-3,-4) undergoes a reflection in the x-axis, then its image A_1, is given by $A(-3, -4) \xrightarrow{\text{reflection in a-axis}} A_1(-3, 4)$.



— —— - - -2 — — — y = - 2

- The line graph whose equation is y = 3, is a straight line which is perpendicular to the y-axis, and passes through the point 3 on they-axis.

- Also the line y = -2, passes through the point -2 on the y - axis.

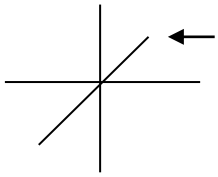
- Lastly the x- axis is the same as the line y = 0.

4. **Reflection in line y = x, or the line y-x = 0, or line –y = -x:**
- The line y = x is the same as the line y-x = 0, or the line –y = - x
- For such a reflection,
  $(x,y) \xrightarrow{\text{reflection in line } y = x} (y, x)$.

Example: if the point P(3, 5) undergoes a reflection in the line y = x, then for its image P_1,

$P(3,5) \xrightarrow{\text{reflection in line } y = x} P_1(5, 3)$.

The line y = x is shown next:



5. **Refle.ction in the line y = -x or the line y + x = 0 or the line – y = x:**
- The line y = -x is the same as the line y + x = 0, or the line -y = x.
- For such a reflection,
  $(x, y) \xrightarrow{\text{reflection in line } y = -x} (-y,-x)$.

Example (1)

If the point B(2, 5) undergoes a reflection in the line y = -x, then for its image B_1,

$B(2,5) \xrightarrow{\text{reflection in line } y = -x} B_1(-5, -2)$.

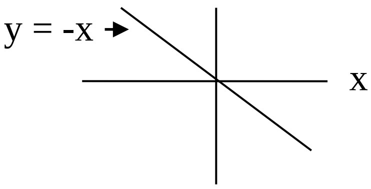
Example (2)

If the point C(-3, -2) undergoes a reflection in the line
y + x = 0, then for its image

$C(-3, -2) \xrightarrow{\text{reflection in line } y + x = 0} C_1(2, 3)$.

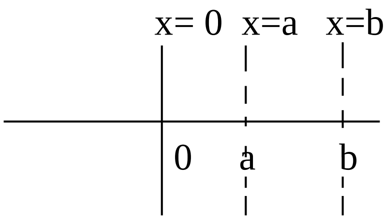
- Next is a diagrammatic representation of the line
  y = - x

y



N/B: The line x = a is a straight which is perpendicular to the x-axis, and passes through the point a, on the x-axis.

Also the line x = b is perpendicular to the x-axis, and passes through the point b on the x-axis.



6. **<u>Reflection in the line x = a:</u>**
   If the point (x, y) undergoes a reflection in the line x = a, then
   $(x, y) \xrightarrow{\text{reflection in line } x = a} (2a - x, y)$.

Example (1)

If the point (4, 3) undergoes a reflection in the line x = 5, then (x, y) = (4, 3) and x = a becomes equal to x = 5. Therefore x = 4, y = 3 and a = 5
From $(x, y) \xrightarrow{\text{reflection in line } x = a} (2a\text{-}x, y)$.
$(4, 3) \xrightarrow{\text{reflection in line } x = 5} \{2(5)\text{-}4, 3\}$.
$(4, 3) \longrightarrow (10 - 4, 3)$
$(4, 3) \longrightarrow (6, 3)$.

Example (2)

If the point p(-3, 4) undergoes a reflection in the line

y = 8, then for its image $P_{1,}$

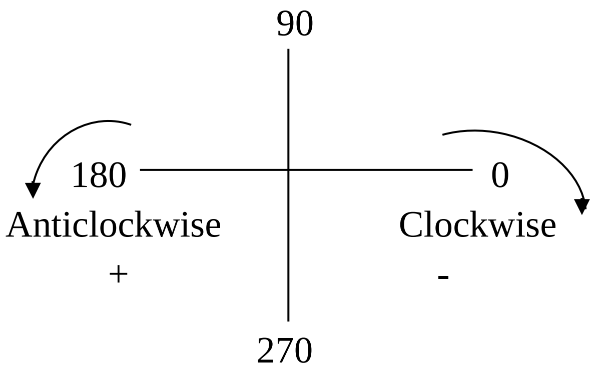
$P(-3, 4) \xrightarrow{\text{reflection in line } y = 8} P_1\{2(8) - (-3), 4\}$

$P(-3, 4) \longrightarrow P_1(16 + 3, 4)$.

$P(-3, 4) \longrightarrow P_1(19, 4)$.

<u>Rotation :</u>

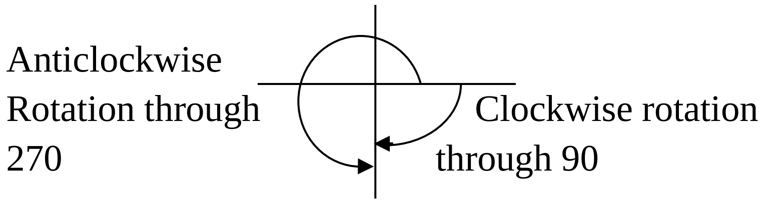
- This is measured in degrees and from the x-axis.
- It is either measured in a clockwise or an anticlockwise direction.
- Rotation in the clockwise direction is negative rotation, and that in the anticlockwise direction is positive rotation.



The different types of rotation to be considered are:

1. **Clockwise rotation of 90 or rotation through -90°:**

- This type of rotation is the same as an anticlockwise rotation through 270°or rotation through 270°.



From the sketch made, it can be seen that from the same starting point, a clockwise rotation through 90°, and an anticlockwise rotation through 270°, all meet on the same line or at the same point.

For this reason the two are the same. For a clockwise rotation through 90° or an anticlockwise rotation through 270°, about the origin

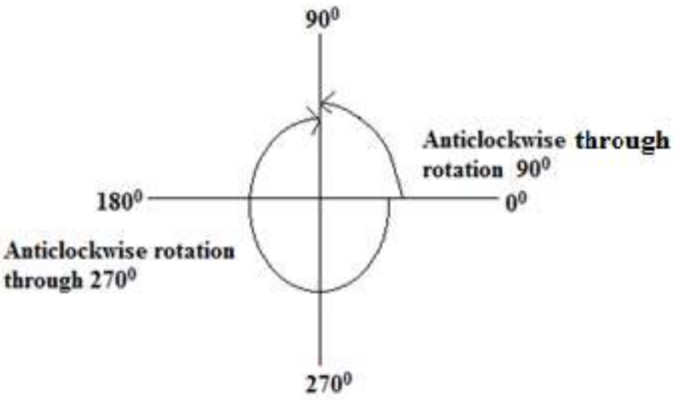
$(x, y) \longrightarrow (y, -x)$.

The following rotations are all the same, and as such the formula given must be used:

a) $(x, y) \xrightarrow{\text{Clockwise rotation through } 90^\circ \text{ about the origin}} (y, -x)$.
b) $(x, y) \xrightarrow{\text{Rotation through } -90^\circ \text{ about the origin}} (y, -x)$.
c) $(x, y) \xrightarrow{\text{Anticlockwise rotation through } 270^\circ} (y, -x)$.
d) $(x, y) \xrightarrow{\text{Rotation through } 270^\circ} (y, x)$.

2. **Anticlockwise rotation through 90° or rotation 90°**

- This types of rotation is the same as clockwise rotation through 270° or rotation through -270°.



From the diagram drawn, it can be seen that an anticlockwise rotation through 90, and a clockwise rotation through 270, originating from the same starting point or line, all end at the same starting point or line.

For this reason, they are the same. If the point (x, y) undergoes an anticlockwise rotation through 90°, or clockwise rotation through 270°, then

$(x, y) \longrightarrow (-y, x)$.

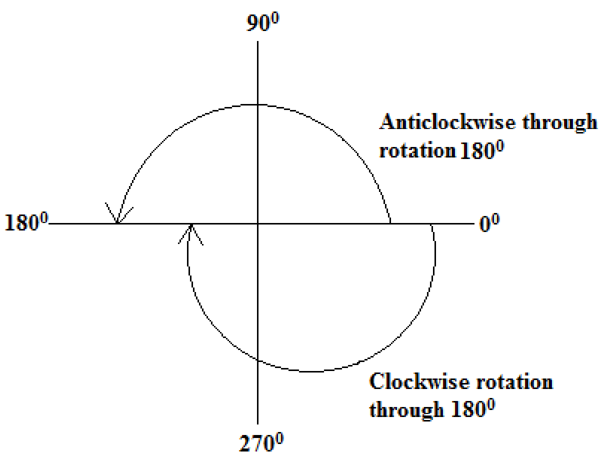
The following transformations are the same, and questions based on them must be solved using the given formula:

a. $(x, y) \xrightarrow{\text{Anticlockwise rotation through } 90^\circ \text{ about the origin}} (-y, x)$.
b. $(x, y) \xrightarrow{\text{rotation through } 90^\circ} (-y, x)$
c. $(x, y) \xrightarrow{\text{clockwise rotation through } 270^\circ} (-y, x)$
d. $(x, y) \xrightarrow{\text{rotation through } -270^\circ} (-y, x)$

Anticlockwise rotation through 90 or a clockwise rotation through 270 is known as or referred to as quarter turn.

Clockwise rotation through 180° or rotation through -180°

This type of rotation through which is referred to as half turn, is the same as an anticlockwise rotation through 180° or rotation through 180°



For this type of rotation $(x, y) \longrightarrow (-x, -y)$.

Example (1)

The image P_1 of the point P(2, 5), which undergoes a clockwise rotation of 180 about the origin, is given by $P(2,5) \longrightarrow P_1(-2, -5)$.

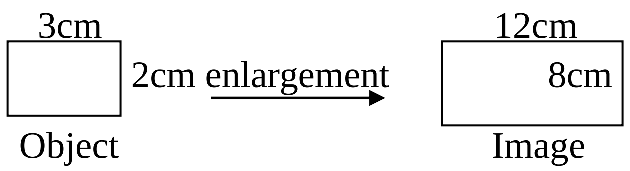
Example (2)

The image C_1 of the point C (-4, 7), after an anticlockwise rotation through 180°, is given by $C(-4, 7) \longrightarrow C_1(4, -7)$

Enlargement/ reduction:

- An important issue in connection with enlargement or reduction, is the scale factor (S.F).
- This scale factor tells us the number of times each length of the object has to be increased or decreased, so as to get the image.
- If the scale factor is greater than 1, then the size of the image is larger than that of the object.
- But if the scale factor is less than 1 or is a fraction, then the size of the image is smaller than that of the object.
- For example if the scale factor is 4, then the implication is that each length of the object had to be increased four times, in order to get the lengths of the image.
- On the other hand if the scale factor is $\frac{1}{4}$, then each length of the object had to be decreased to one quarter of its value, in order to get the lengths of the image.
- $S.F = \dfrac{\text{One length of the image}}{\text{Corresponding length of the object}}$

Example (1)



S.F $= \frac{12}{3} = 4 \; or \; S.F = \frac{8}{2} = 4.$

Example 2

9cm		3cm	
Object	6cm enlargement $\longrightarrow$	image	2cm

S.F $=\frac{3}{9} = \frac{1}{3} \; or \; S.F = \frac{2}{6} = \frac{1}{3}$

S.F$= \frac{3}{9} = \frac{1}{3} \; or \; S.F = \frac{2}{6} = \frac{1}{3}.$

- The type of enlargement in which the scale factor is a fraction, is referred to as a reduction.

- If the point (x, y) is enlarged with a scale factor of K, then

(x, y) enlargement with $\xrightarrow{\text{S. F. K}}$ (kx, ky).

Example (1)

If (x, y) is enlarged with scale factor 2, then

(x, y) $\longrightarrow$ (2x, 2y).

Example (2)

If (2, 5) is enlarged with scale factor 4, then

(2, 5) $\longrightarrow$ {4(2), 4(5)}

(2,5) $\longrightarrow$ (8, 20).

Summary

A summary of formulae, which must be kept in memory are presented next.

1. (x, y) translation by vector $\binom{a}{b}$ $\longrightarrow$ (x + a, y + b).
2. (x, y) reflection in the $\xrightarrow{\text{y – axis}}$ (-x , y).
3. (x, y) $\xrightarrow{\text{reflection in line y = b}}$ (x, 2b – y).
4. (x, y) reflection in the $\xrightarrow{\text{x-axis}}$ (x, -y)
5. (x, y) $\xrightarrow{\text{reflection in line y = x}}$ (y, x).
6. (x, y) $\xrightarrow{\text{reflection in line y = -x}}$ (-y, -x)
7. (x, y) $\xrightarrow{\text{reflection in line x = a}}$ (2a – x, y)
8. Clockwise or anticlockwise rotation through 180 about the origin

   (x, y) $\longrightarrow$ (-x, y).
9. Clockwise rotation through 90 or anticlockwise rotation through 270, about the origin.

   (x, y) $\longrightarrow$ (y, -x).
10. Anticlockwise rotation through 90 or clockwise rotation through 270, about the origin

    (x, y) $\longrightarrow$ (-y, x).

Q1. i. Using a scale of 2cm to 2units on each axis, draw the line ox and oy for
$-6 \leq x \leq 10$ *and* $-5 \leq y \leq 6$

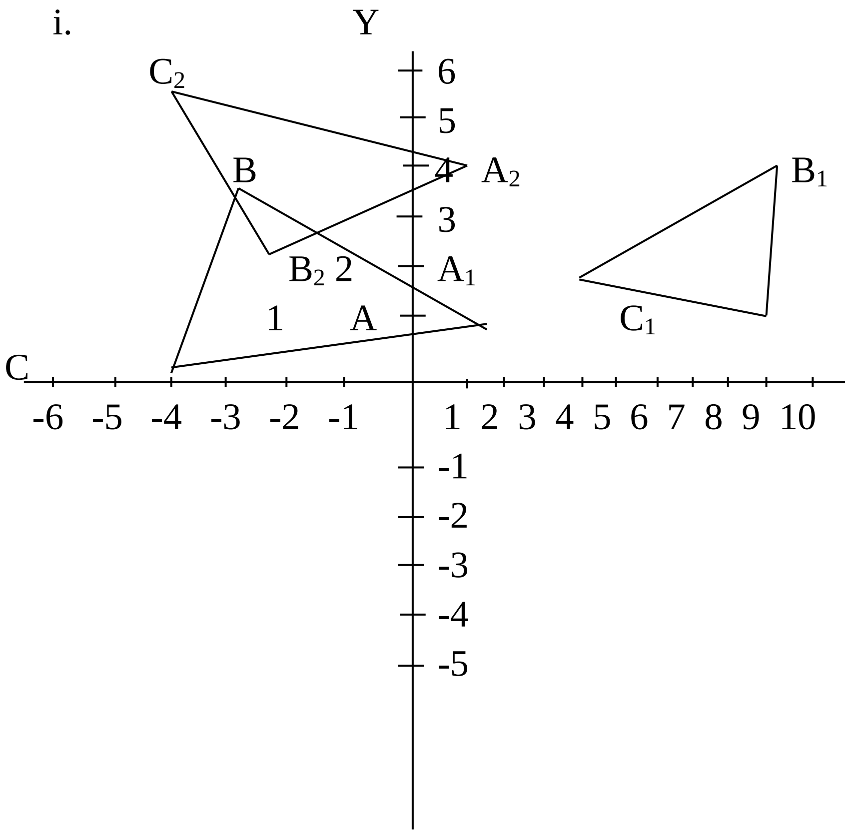
i. Draw triangle ABC with vertices A(1,2), B(-3,4) and C(-4,1).

ii. On the same axes, draw the image $A_1B_1C_1$ of ABC, after a reflection in the line x = 3.

iii. Draw the image $A_2B_2C_2$ of ABC under a reflection in the line y = 3..

Soln

i.



$(x, y) \xrightarrow{\text{reflection in line } x = a} (2a - x, y)$

$\Rightarrow (x, y) \xrightarrow{\text{reflection in line } x = 3} \{2(3) - x , y\}$

$\Rightarrow (x, y) \longrightarrow (6 - x, y)$.

$A \longrightarrow A_1, \Rightarrow A(1, 2) \xrightarrow{\text{reflection in line } x = 3} A_1(6 - 1,2)$,

$\Rightarrow A(1, 2) \longrightarrow A_1(5,2)$

$B \longrightarrow B_1$ *and* $(x, y) \longrightarrow (6 - x, y)$.

$\therefore B(-3, 4) \xrightarrow{\text{reflection in line } x = 3} B_1(6 - \bar{3}, 4)$,

$\Rightarrow B(-3, 4) \longrightarrow B_1(6 + 3,4)$,

$\Rightarrow B(-3, 4) \longrightarrow B_1(9, 4)$.

$C \longrightarrow C_1 \Rightarrow C(-4, 1) \longrightarrow C_1(6-x, y)$,

$\Rightarrow C(-4, 1) \longrightarrow C_1(6 - \bar{4}, 1)$,

$\Rightarrow C(-4, 1) \longrightarrow C_1(6+4,1) \Rightarrow C(-4, 1) \longrightarrow C_1(10,1)$.

To get $\Delta A_1B_1C_1$, we plot the point A_1,B_1 and C_1 and join one to the other.

ii. $(x, y) \xrightarrow{\text{reflection in line } y = b} (x, 2b - y)$,

$\Longrightarrow$ (x, y) $\xrightarrow{\text{reflection in line } y = 3}$ (x, 2(3) – y),

$\Longrightarrow$ (x, y) $\longrightarrow$ (x, 6 – y).

$A \longrightarrow A_2 \Longrightarrow$ A (1,2) $\longrightarrow$ A_2(1, 6 – 2),

$=> A(1,2) \longrightarrow A_2(1,4)$.Since B $\longrightarrow$ B_2

and (x, y) $\longrightarrow$ (x, 6 – y),

B(-3, 4) $\longrightarrow$ B_2(-3, 6 – 4),

=>B(-3, 4) $\longrightarrow$ B_2(-3, 2).

C(- 4, 1) $\longrightarrow$ C_2(- 4, 6 – 1),

=> C(- 4, 1) $\longrightarrow$ C_2(- s4, 5).

We finally draw $\Delta A_2B_2C_2$ using the points A_2, B_2 and C_2.

Q2. i. Using a scale of 2cm to 2units on each axis, draw on a sheet of graph paper two perpendicular axes ox and oy, for the interval

$-10 \le x \le 10 \; and - 10 \le y \le 10$.

ii. Draw ΔPOR with vertices P(2, 8), Q(5, 8) and R(2, 4).

iii Draw the image $\Delta P_1 Q_1 R_1$of ΔPQR under a reflection in the line y = 2, where

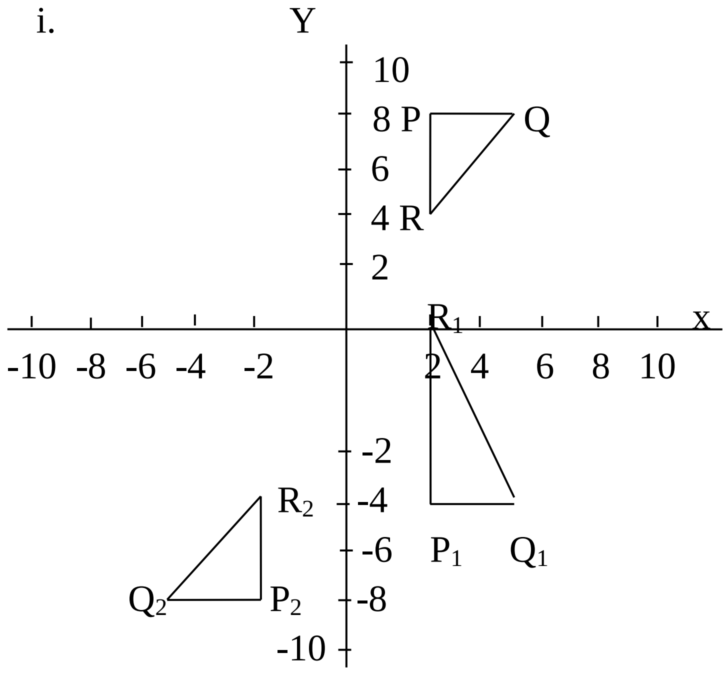
P $\longrightarrow$ P_1, Q $\longrightarrow$ Q_1 and R→R_1.

iii. Draw also the image $\Delta P_2Q_2R_2 \; of \; \Delta PQR$ after a half turn about the origin o,

where P $\longrightarrow$ P_2 ,Q $\longrightarrow$ Q_2 and R $\longrightarrow$ R_2

Soln.

i.



ii. $(x, y) \xrightarrow{\text{reflection in line } y = b} (x, 2b - y)$

$(x, y) \xrightarrow{\text{reflection in line } y = 2} \{x, 2(2) - y\}$

$(x, y) \longrightarrow (x, 4 - y)$.

Since $P \longrightarrow P_1 \Rightarrow P(2,8) \longrightarrow P_1(2, 4 - 8)$,

$\Rightarrow P(2,8) \longrightarrow P_1(2, -4)$.

Since $Q \longrightarrow Q_1$ and $(x, y) \longrightarrow (x, 4 - y)$,

$\Rightarrow Q(5, 8) \longrightarrow Q_1(5, 4 - 8)$,

$\Rightarrow Q(5, 8) \rightarrow Q_1(5, -4)$.

Lastly $R \longrightarrow R_1$ and $(x, y) \longrightarrow (x, 4 - y)$, =>

$R(2, 4) \longrightarrow R_1(2, 4 - 4), \Rightarrow R(2,4) \longrightarrow R_1(2,0)$.

Using P_1,Q_1 and R_1, we draw $\Delta P_1 Q_1 R_1$.

iii. For a half turn about the origin $(x, y) \longrightarrow (-x, -y)$

$P \longrightarrow P_2 \Rightarrow P(2, 8) \longrightarrow P_2(-2, -8)$..

$Q \longrightarrow Q_2 \Rightarrow Q(5,8) \longrightarrow Q_2(-5, -8)$.

$R \longrightarrow R_2 \Rightarrow R(2, 4) \longrightarrow R_2(-2, -4)$.

Q3. Using a scale of 2cm to 1unit on both axes, draw the x and the y axes for $0 \leq x \leq 8 \text{ and } -6 \leq y \leq 6$.

i. Plot A(3, 1), B(1, 1) and C(1, 5).

ii. Find the equation of line AC.

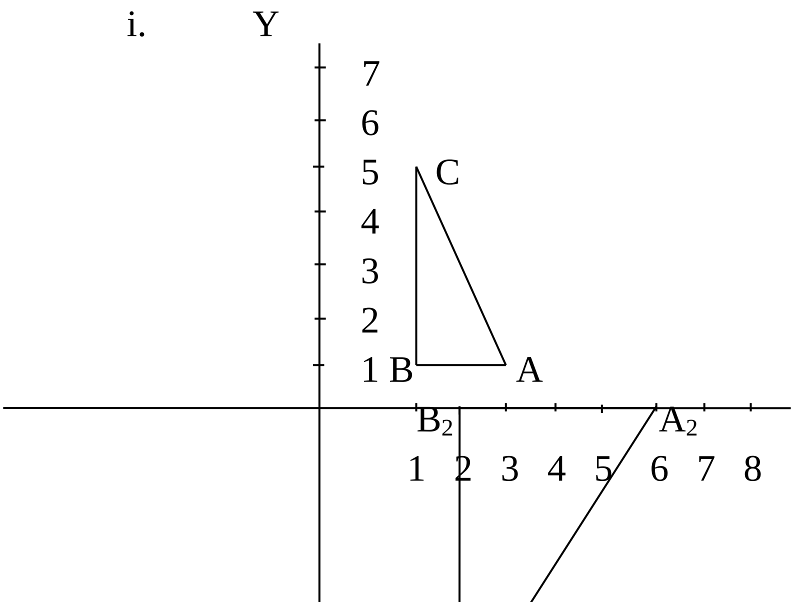
iii. Draw $\Delta A_1 B_1 C_1$ where $A \longrightarrow A_1$, $B \longrightarrow B_1$ and $C \longrightarrow C_1$.

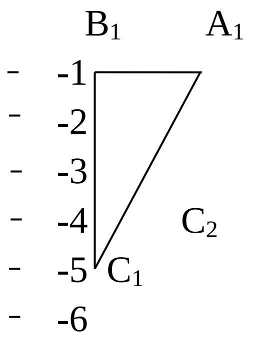
iv. Draw $\Delta A_2 B_2 C_2$ which is the image of ΔABC under the mapping $\begin{pmatrix} x \\ y \end{pmatrix} \longrightarrow \begin{pmatrix} 2x \\ 1 - y \end{pmatrix}$, where $A \longrightarrow A_2$, $B \longrightarrow B_2$ and $C \longrightarrow C_2$.

.

. Soln.

i.





ii. Line AC passes through the points

(3 1) and (1, 5). Let $(x_1, y_1) = (3, 1)$ and $(x_2, y_2) = (1, 5) \Longrightarrow x_1 = 3, y_1 = 1$, $x_2 = 1$ and $y_2 = 5$.

Using $y - y_1 =$

$\frac{y_2 - y_1}{x_2 - x_1}(x - x_1) \Rightarrow y - 1 = \frac{5 - 1}{1 - 3}(x - x_1), \Rightarrow y - 1 = \frac{4}{-2}(x - x_1), \Rightarrow$

$y - 1 = -2(x - 3) \Rightarrow y - 1 = -2x + 6 \Rightarrow y = -2x + 6 + 1 \Rightarrow y = -2x + 7$

iii. For a reflection in the x – axis, $(x, y) \rightarrow (x, -y)$

Since $A \rightarrow A_1 \Longrightarrow A(3,1) \longrightarrow A_1(3, -1)$

Since $B \longrightarrow B_1 \Longrightarrow B(1, 1) \longrightarrow B_1(1, -1)$

Since $C \longrightarrow C_1 \Longrightarrow C(1,5) \longrightarrow C_1(1, -5)$

Using $A_1(3, -1)$, $B_1(1, -1)$ and $C_1(1, -5)$, we draw $\Delta A_1 B_1 C_1$

iv. $\begin{pmatrix} x \\ y \end{pmatrix} \dashrightarrow \begin{pmatrix} 2x \\ 1 - y \end{pmatrix} \Rightarrow (x, y) \longrightarrow (2x, 1 - y)$.

Since $A \longrightarrow A_2$, then $A(3, 1) \longrightarrow A_2\ \{2(3), 1 - 1\}$,

$\Longrightarrow A(3, 1) \dashrightarrow A_2 6, 0)$.

Since $B \rightarrow B_2$, then $B(1, 1) \longrightarrow B_2\{2(1), 1 - 1\}$,

$\Longrightarrow B(1,1) \longrightarrow B_2(2, 0)$.

Since $C \longrightarrow C_2$, then $C(1, 5) \longrightarrow C_2\{2(1), 1 - 5)\}, \Longrightarrow$

$C_2(1,5) \longrightarrow C_2(2, -4)$.

Using $A_2(6, 0)$, B_2 (2, 0) and $C_2(2, -4)$, we draw $\Delta A_2 B_2 C_2$.

Q4. Draw on a sheet of graph paper two perpendicular axes, ox and oy for $-10 \le x \le 10$ *and* $-10 \le y \le 10$, using a scale of 2cm to 2units on both axes.

Given the point A(1, 2) and the vector $\overrightarrow{AB} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}, \overrightarrow{BC} = \begin{pmatrix} -1 \\ 3 \end{pmatrix}$ *and* $\overrightarrow{CD} = \begin{pmatrix} -2 \\ -2 \end{pmatrix}$,

draw on the same graph

a. the quadrilateral ABCD.
b. the image $A_1B_1C_1D_1$ of ABCD under a clockwise rotation of 90 about the origin (0, 0), Where $A \longrightarrow A_1$, $B \longrightarrow B_1$, $C \longrightarrow C_1$ and $D \longrightarrow D_1$.

c. The image $A_2B_2C_2D_2$ of $A_1B_1C_1D_1$ under a reflection in the line y = x, where $A_1 \longrightarrow A_2$, $B_1 \longrightarrow B_2$, $C_1 \longrightarrow C_2$ and $D_1 \longrightarrow D_2$

d. Determine the length A_2C_2.

Soln.

a. Given A(1, 2), and $\overline{AB} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$

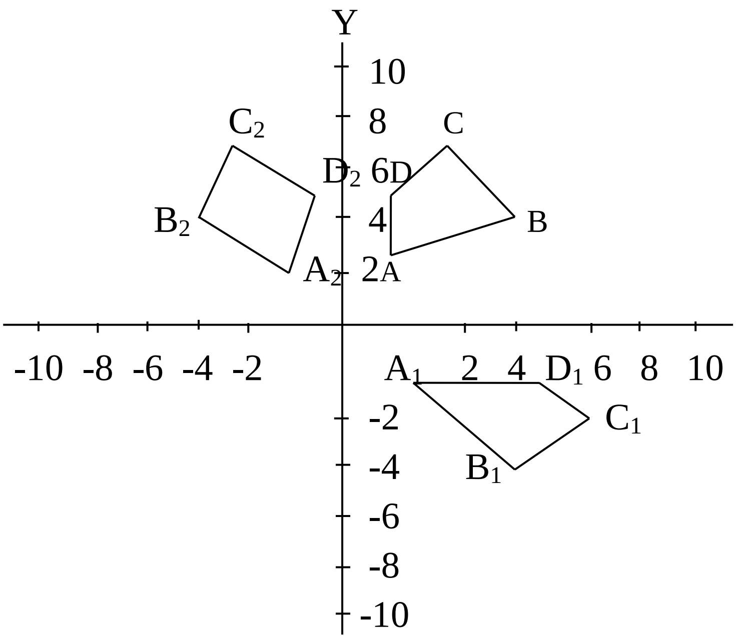
$\Rightarrow B = (1+3, 2+2) \Rightarrow B(4,4)$.

Now B(4, 4) and

$\overline{BC} = \begin{pmatrix} -1 \\ 3 \end{pmatrix} \Rightarrow C(4+\bar{1}, 4+3) \Rightarrow C(3,7). C(3,7) and\, \overline{CD} = \begin{pmatrix} -2 \\ -2 \end{pmatrix} \Rightarrow$

$D(3+\bar{2}, 7+\bar{2}) \Rightarrow D(1,5)$.

At this stage, since we now know the coordinates of A,B,C and D, we can now draw the quadrilateral ABCD



b. Under a clockwise rotation of 90 about the origin (0,0),

c. $(x, y) \longrightarrow (y, -x)$. Since $A \longrightarrow A_1$, then $A(1, 2) \rightarrow A_1(2, -1)$

Since $B \longrightarrow B_1$, then $B(4, 4) \longrightarrow B_1(4, -4)$. Also since $C \longrightarrow C_1$ then $C(3, 7) \longrightarrow C_1(7, -3)$.

Lastly since $D \rightarrow D_1$, then $D(1, 5) \longrightarrow D_1(5,-1)$. With the coordinate of $A_1B_1C_1$ and D_1 known, we can now draw quadrilateral $A_1B_1C_1D_1$.

d. Under a reflection in the line y = x, $(x, y) \longrightarrow (y, ,x)$

Since $A_1 \longrightarrow A_2$, then $A_1(2,-1) \longrightarrow A_2(-1,2)$. Since $B_1 \longrightarrow B_2$, then $B_1(4,- 4) \longrightarrow B_2(- 4, 4)$.

Also $C_1 \rightarrow C_2$, => $C_1(7,-3) \longrightarrow C_2(-3,7)$.

Lastly $D_1 \longrightarrow D_2$, =>$D_1(5,-1) \rightarrow D_2(-1,5)$..

We then draw the quadrilateral $A_2B_2C_2D_2$.

N/B: If $A_1 \longrightarrow A_2$, then the coordinates of A_1 must be used but not those of A.

e. $A_2 = (-1, 2)$ and $C_2 = (-3, 7)$. Let $(x_1, y_1) = (-1, 2)$ and $(x_2, y_2) = (-3, 7)$, $\Rightarrow x_1 = -1$, $y_1 = 2$, $x2 = -3$ and $y_2 = 7$. If l = the length of A_2C_2

$\Rightarrow l = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \Rightarrow l = \sqrt{(-3+1)^2 + 5^2} = \sqrt{(-2)^2 + 5^2} = \sqrt{4 + 25} = \sqrt{29} = 5.4$

Q5. Draw on the graph sheet, using a scale of 2cm to 2units, two perpendicular axes ox and oy for $-10 \leq x \leq 10 \; and -10 \leq y \leq 10$

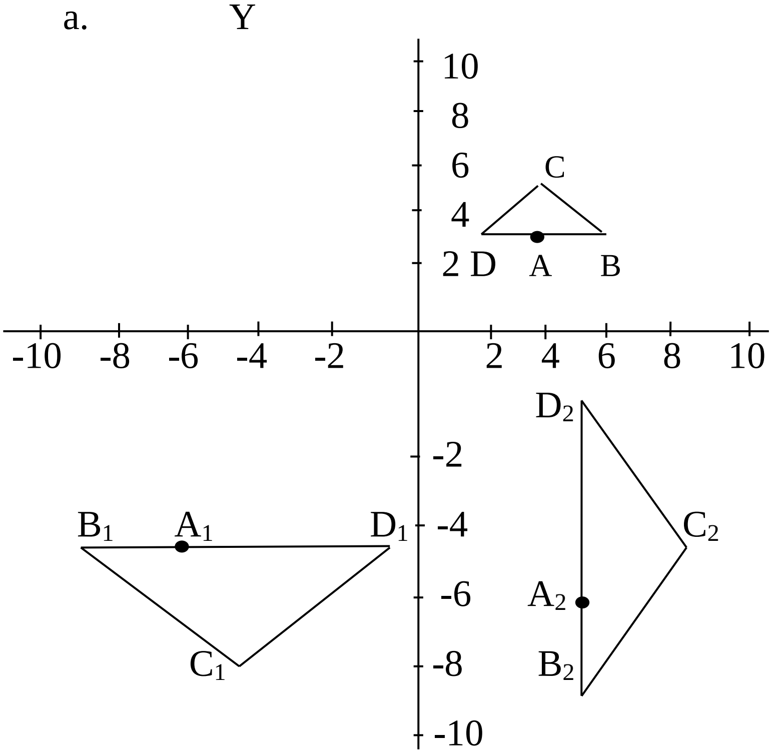
a. Plot the points A(4,3), B(6, 3), C(3,5) and D(1,3).

b. Draw the image $A_1B_1C_1D_1$ of ABCD under an enlargement from the origin (0, 0) with a scale factor of $\frac{-3}{2}$, where $A \longrightarrow A_1$, $B \longrightarrow B_1$, $C \longrightarrow C_1$ and $D \longrightarrow D_1$.

c. Draw the image $A_2B_2C_2D_2$ of $A_1B_1C_1D_1$ under an anticlockwise rotation of 90 about origin 0(0,0), where $A_1 \longrightarrow A_2$, $B_1 \longrightarrow B_2$, $C_1 \rightarrow C_2$ and $D_1 \longrightarrow D_2$..

Soln.

a.



b. If (x,y) undergoes an enlargement with scale factor k, then

$(x, y) \longrightarrow (kx, ky)$,

$\Rightarrow$ if (x, y) undergoes an enlargement with scale factor $\frac{-3}{2}$, then

. $(x, y) \longrightarrow (\frac{-3}{2}x, \frac{-3}{2}y)$.

Since $A \longrightarrow A_1$, then $A(4, 3) \longrightarrow$ $\longrightarrow$
$A_1\{$

$\longrightarrow$

$\frac{-3}{2}(4), \frac{-3}{2}(3)\}, \Rightarrow A(4,3) \Rightarrow A_1\left(\frac{-12}{2}, \frac{-9}{2}\right), =>$
$A(4,3) \qquad A_1(-6, -4.5$
$)$.

Since B $\longrightarrow$ B_1, then B(6, 3) $\longrightarrow$ B_1

$\left(\frac{-3}{2}(6), \frac{-3}{2}(3)\right), \Rightarrow B(6,3) -\!\!\rightarrow B_1\left(\frac{-18}{2}, \frac{-9}{2}\right), \Rightarrow B(6,3) -$
$B_1(-9, -4.5)$.

C

C1

$\Rightarrow C(3,5) \longrightarrow C_1\left(\frac{-3}{2}(3), \frac{-3}{2}(5)\right), \Rightarrow C(3,5) \longrightarrow \ C_1\left(\frac{-9}{2}, -\frac{15}{2}\right), \Rightarrow$
$C(3,5) \longrightarrow C_1(-4.5, -7.5)$

Lastly

D

$\longrightarrow D_1 \Rightarrow D(1,3) \longrightarrow D_1\left(\frac{-3}{2}(1), \frac{-3}{2}(3)\right), \Rightarrow D(1,3) \longrightarrow D_1\left(\frac{-3}{2}, \frac{-9}{2}\right), \Rightarrow$
$D(1,3) \longrightarrow D_1(-1.5, -4.5)$

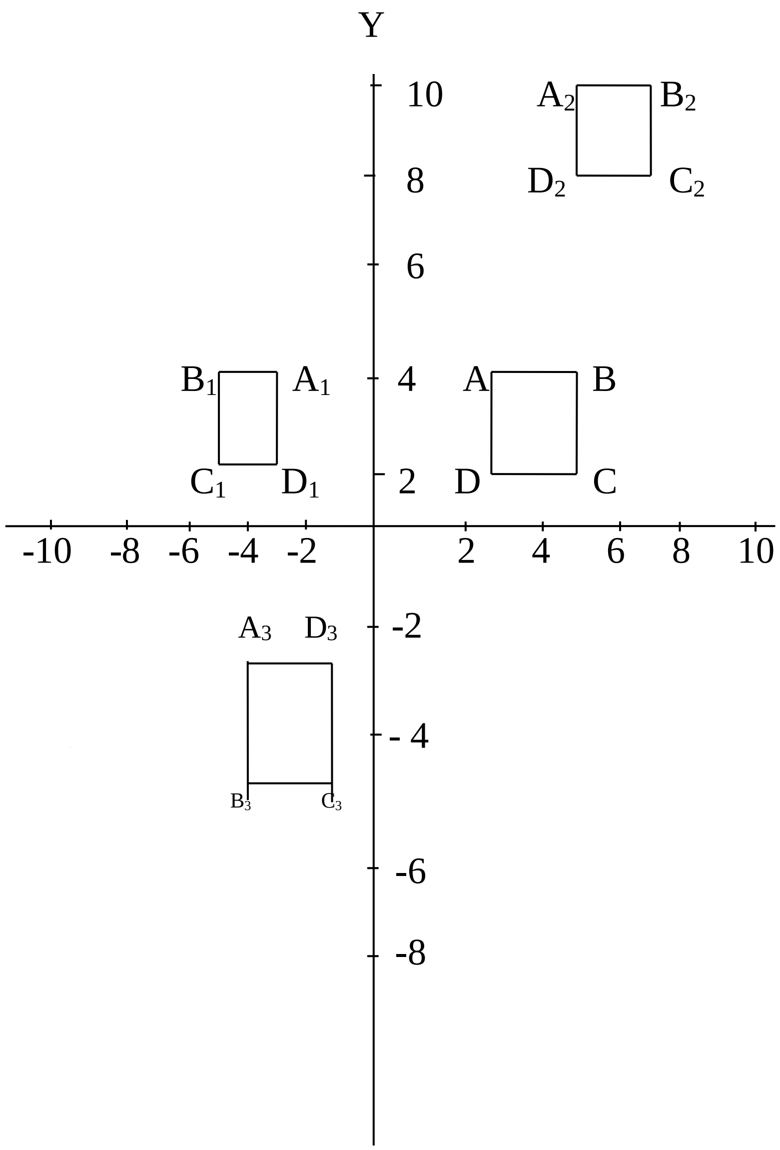
c. For an anticlockwise rotation of 90 about the origin 0(0, 0), (x, y) (-y, x)'
Since $A_1 \longrightarrow A_2, then\ A_1(-6, -4.5) \longrightarrow A_2(4.5, -6)$.
$B_1 \longrightarrow B_2 \Rightarrow B_1(-9, -4.5) \longrightarrow B_2(4.5, -9).Since$
$C_1 \longrightarrow C_2 \Rightarrow C_1(-4.5, -7.5) \longrightarrow C_2(7.5, -4.5)$.
$lastly\ D_1 \longrightarrow D_2 \Rightarrow D_1(-1.5, -4.5) \longrightarrow D_2(4.5, -1.5)$.

We then draw figure $A_2B_2C_2D_2$.

Q6. Using a scale of 1cm to 1 unit on each axis, draw the x and y axes.

i. Draw quadrilateral ABCD whose vertices are A(3, 4), B(5,4), C(5, 2) and D(3, 2).
ii. Draw the quadrilateral $A_1B_1C_1D_1$ which is the image of ABCD, after a reflection in the y-axis, where A$\longrightarrow A_1, B \longrightarrow B_1, C \longrightarrow C_1$ $and\ D \longrightarrow D_1$.
iii. Draw the image $A_2B_2C_2D_2$ of ABCD after a translation by the vector $\binom{2}{6}$.
iv. Draw quadrilateral $A_3B_3C_3D_3$ which is the image of $A_1B_1C_1D_1$, after a rotation through 270 about the origin in the clockwise direction.

Soln



(ii) For a reflection in the y- axis, (x,y) $\longrightarrow$ (-x,y). Since

A $\longrightarrow A_1, then\ A(3,4) \longrightarrow$ A(-3,4).

B $\longrightarrow B_1 \Rightarrow$ B$(5,\ 4) \longrightarrow B_1(-5,\ 4)$. Since C $\longrightarrow C_1$, then

C(5 ,2) $\longrightarrow C_1(-5,\ 2)$. Lastly D $\longrightarrow D_1$

$\Rightarrow$ D (3,2) $\longrightarrow D_1(-3,2)$.

We then draw quadrilateral A_1, B_1, C_1, D_1.

(i) If (x, y) undergoes a translational by the vector $\binom{a}{b}$, then (x, y) $\longrightarrow$ (x + a, y+ b).

$\Rightarrow$ If (x, y) undergoes a translation by the vector$\binom{2}{6}$, then

(x, y) $\longrightarrow$ (x + 2, y + 6).

Since $A_2\ B_2\ C_2 D_2$ *is the image of ABCD, then*

$A \longrightarrow A_2, B \longrightarrow B_2$, C $\longrightarrow C_2$ *and* $D \longrightarrow D_2$.

Since A $\longrightarrow A_2$, *then* A(3, 4) $\longrightarrow A_2(3+2, 4+6)$,

$\Rightarrow$A(3, 4) $\longrightarrow A_2(5,10)$'

B $\longrightarrow B_2 \Rightarrow$ B(5, 4) $\longrightarrow B_2(5+2, 4+6)$,

$\Rightarrow$B(5, 4) $\longrightarrow B_2\ (7,10)$..

C $\longrightarrow C_2 \Rightarrow$C(5, 2) $\longrightarrow C_2(5+2, 2+6)$,

$\Rightarrow$C(5,2) $\longrightarrow C_2(7,\ 8)$.

Lastly D $\longrightarrow D_2 \Rightarrow$ D(3, 2) $\longrightarrow D_2(3+2, 2+6)$

$\Rightarrow$D(3, 2) $\longrightarrow D_2$(5, 8).

With these coordinates, draw $A_2\ B_2\ C_2 D_2$.

(iv) Under a clockwise rotation through 270^0,

(x, y) $\longrightarrow$ (-y, x).

Since $A_3\ B_3 C_3 D_3$ is the image of $A_1 B_1 C_1 D_1$, then

$A_1 \longrightarrow A_2$, $B_1 \longrightarrow B_3$, $C_1 \longrightarrow C_3$ *and* $D_1 \longrightarrow D_3$.

Since $A_1 \longrightarrow A_3$, then$A_1(-3,4) \longrightarrow A_1(-4,-3)$.

Since $B_1 \longrightarrow B_3$, then $B_1(-5,4) \longrightarrow B_3(-4,-5)$.

Since $C_1 \longrightarrow C_3$, then $C_1(-5,2) \longrightarrow C_3(-2,-5)$.

Lastly Since $D_1 \longrightarrow D_3$, then D_1(-3, 2) $\longrightarrow D_3$(-2,-3)

Q7)(i). Using a scale of 1cm to 1 unit on each axis, draw the quadrilateral whose vertices are A(-5, 3), B(0, 3), C(-1,2) and D(-4,2).

(ii) Construct $A_1 B_1 C_1 D_1$ which is a reflection of $A_1 B_1 C_1 D_1$ the line y – x= 0, where A $\rightarrow A_1$, B $\rightarrow B_1$, C$\rightarrow C_1$ and D $\rightarrow D_1$.

(iii)Draw $A_2\ B_2\ C_2 D_2$ which is the image of$A_1 B_1 C_1 D_1$ after a reflection in the line x= 5, where$A \longrightarrow A_1, B_1 \longrightarrow B_2, C_1 \longrightarrow C_2$ *and*

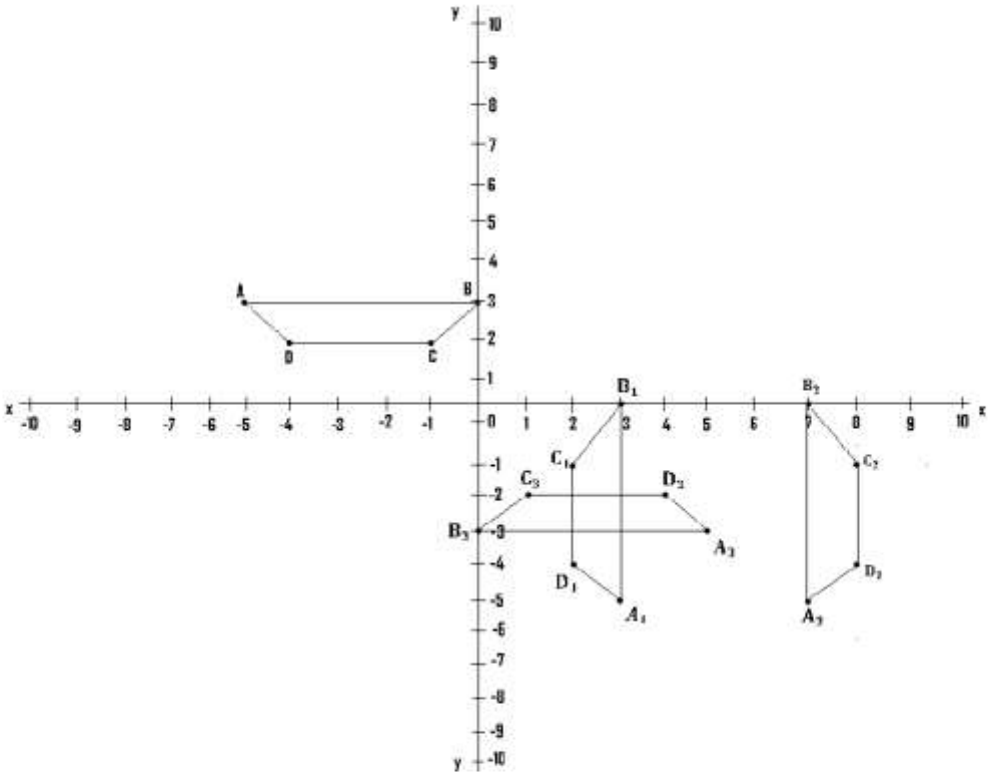
$D_1 \longrightarrow D_2$..

(iv) Draw $A_3 B_3 C_3 D_3$ which is the image of ABCD after a half turn, where A$\longrightarrow$ $A_3,$ $\longrightarrow$ B $B_3,$ $\longrightarrow$ C C_3 and $\longrightarrow$ D D_3..

(v) Find the gradient of the line $B_1 C_1$..

(vi) Find the equation of the line$B_1 C_1$.

(vii) Determine also the equation of line AD.



(ii) Line y – x =0 is the same as the line y = x. If (x, y) is reflected in the line y = x, (x , y)$\longrightarrow$ (y, x).

Since A$\longrightarrow A_1$, then A(-5, 3) $\longrightarrow A_1(3,-5)$.

Since B$\longrightarrow B_1$, then B(0, 3) $\longrightarrow B_1(3,0)$.

Also C $\longrightarrow$ $C_1 \Rightarrow$ C(-1, 2) $\longrightarrow$ $C_1(2,-1)$.

Lastly D $\rightarrow D_1 \Rightarrow$ D(-4, 2) $\longrightarrow D_1(2,-4.)$

We then draw $A_1 B_1 C_1 D_1$.

(iii)

$(x, y) \xrightarrow{\text{Reflection in line } x = a} (2a - x, y)$

$\Rightarrow (x, y) \xrightarrow{\text{Reflection in line } x = 5} (2(5) - x, y)$

$\Rightarrow$**(x, y)** $\longrightarrow$ **(10 – x, y).**

$A_1 \longrightarrow A_2 \Rightarrow A_1(3, -5) \longrightarrow A_2(10-3, -5),$

$\Rightarrow A_1(3, 5) \longrightarrow A_2(7, -5),$

$B_1 \longrightarrow B_2 \Rightarrow \mathrm{B}_1(3, 0) \longrightarrow B_2(10-3, 0),$

$\Rightarrow B_1(3, 0) \longrightarrow B_2(7, 0).$

$C_1 \longrightarrow C_2 \Rightarrow C_1(2, -1) \longrightarrow C_2(10 - 2, -1), \Rightarrow C_1(2, -1) \longrightarrow C_2(8, -1)$

.

$D_1 \longrightarrow D_2 \Rightarrow D_1(2, -4) \longrightarrow D_2$(10 - 2, -4),

$\Rightarrow D_1(2, -4) \longrightarrow D_2(8, -4)$. .

We then draw $A_2B_2C_2D_2$.

(iv) If (x, y) undergoes a half turn, then

(x,y) $\longrightarrow$ (-x,-y).

Since A $\longrightarrow A_3$, then A(-5, 3) $\longrightarrow A_3$ {-(-5), -3},

A(-5, 3) $\longrightarrow A_3(5, -3)$.

Also B $\longrightarrow B_3 \Rightarrow$ B(0, 3) $\longrightarrow B_3(0, -3)$.

C $\longrightarrow C_3 \Rightarrow$ C(-1, 2) $\longrightarrow C_3(1, -2)$.

Lastly since D $\longrightarrow D_3$, then D(-4, -2) $\longrightarrow D_3(4, -2)$

We now draw $A_3B_3C_3D_3$.

(v) The line B_1C_1 has coordinatedes $B_1(3,0)$ and C_1(2, -1).

Let (x_1, y_1) = (3, 0) and (x_2, y_2) = (2, -1) $\Rightarrow x_1 = 3$

$y_1 = 0,\ x_2 = 2\ and\ y_2 = 1$.The gradient of $B_1C_1 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-1-0}{2-3} = \frac{-1}{-1} = 1.$

(vi) The line B_1C_1 has a gradient of 1 and passes through the point $C_1(2,-1). Let\ x_1$= 2 and $y_1 = -1$.

Using y - y_1= m(x - x_1) ⇒ y –(1) = 1(x – 2),

⇒y - 1 = -x - 2 ⇒ y = x - 2 + 1

⇒ y = x + 1.

(vii) The line AD passes through the points A(-5, 3) and D(-4, 2). Let $(x_1, y_1) = (-5, 3)$ *and let* (x_2, y_2)

= (- 4, 2) ⇒ x_1=-5, $y_1 = 3$, $x_2 = -4$ *and* $y_2 = 2$.

Now y-y_1 = $\frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$,

⇒ y – 3 = $\frac{2 - 3}{-4 - -5}\{x - (-5)\}$,

⇒ y -3 = $\frac{-1}{-4+5}(x + 5)$,

⇒ y – 3 = $\frac{-1}{1}(x + 5)$⇒$y - 3 = -1(x + 5)$,

⇒ $y - 3 = -x - 5$,⇒y=-x – 5+3, ⇒ y = -x – 2.

(Q8) Find the image of the position vector $\binom{4}{3}$

Under the translation$\binom{-2}{1}$..

Soln.

$\binom{4}{3} = \binom{-2}{1}$.

$\therefore (4,3) \xrightarrow{\text{translation by } \binom{-2}{1}} (4+-2, 3+1)$

⇒ (4, 3) ⟶ (4- 2, 3 + 1)

(4, 3) ⟶ (2,4). =>the required image = (2, 4) or $\binom{2}{4}$.

(b) If A(2,3) is reflected in the x – axis, find the image

$A^1$of A.

Soln.

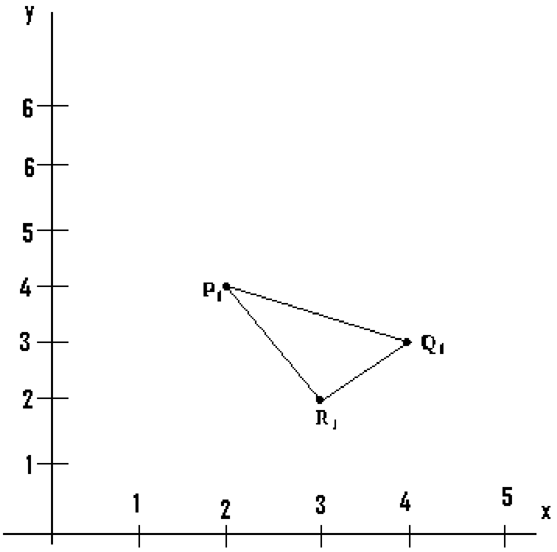
For a reflection in the x – axis,

(x, y) ⟶ (x, - y).

$\Rightarrow$ A(2, 3) $\longrightarrow A_1(2, -3)$.

$\therefore$ The image of $A_1(2, -3)$.

(c)



In the given diagram $P_1 Q_1 R_1$ is the image of $\triangle$ P Q R, under a translation by the vector $\binom{-2}{4}$.Draw using the scale on the graph indicating all coordinates.

(i) $\triangle P_1 Q_1 R_1$.

(ii) $\triangle$ P Q R before it was transformed.

(iii) The image $\triangle P_{II} Q_{II} R_{II}$ of $\triangle P_1 Q_1 R_1$ under a rotation about the origin through 90^0clockwise.

Soln.

From the diagram given, P_1 has coordinates (2, 4), Q_1 has coordinates (2, 3) and R_1 *has* coordinates (3, 2).

We then draw the given figure, using the same scale as that given in the diagram and also using the points $P_1(2,4), Q_1$(4, 3) and$R_1(3, \ 2)$.

This must be done on a graph paper.

N/B: Since this transformation took place with the help of a vector, then it is a translation and the given vector is the translation vector.

(ii) Let (x, y) be the original coordinates of P before it was transformed by the vector $\binom{-2}{4}$ into $P_1\,(2, 4)$.Then

$$P(x, y) \xrightarrow{\text{translation by } \binom{-2}{4}} P_1\ (2, 4)$$

$\Rightarrow x + \ -2 = 2 \ \Rightarrow\ x - 2 = 2,$

$\Rightarrow x = 2 + 2 \ \Rightarrow\ x = 4$

Also $y + 4 = 4 \Rightarrow y = 4 - 4 = 0.$

$\therefore$ P(4, 0) i.e P has coordinates (4, 0).

Let (x_1, y_1) *be the coordinates of Q before it was* transformed by $\binom{-2}{4}$ *into* Q_1, *then*

$$Q(x_1, y_1) \xrightarrow{\text{translation by } \binom{-2}{4}} Q_1\ (4, 3)$$ $\Rightarrow x_1 + \bar{2} = 4,\ \Rightarrow x_1 - 2 = 4,$

$\Rightarrow x_1 = 4 + 2 = 6.$

Also $y_1 + \ 4 = 3 \Rightarrow y_1 = 3 - 4 = -1$

$\therefore$ Q(6, 1) $\Rightarrow$ the coordinates of Q are (6, -1).

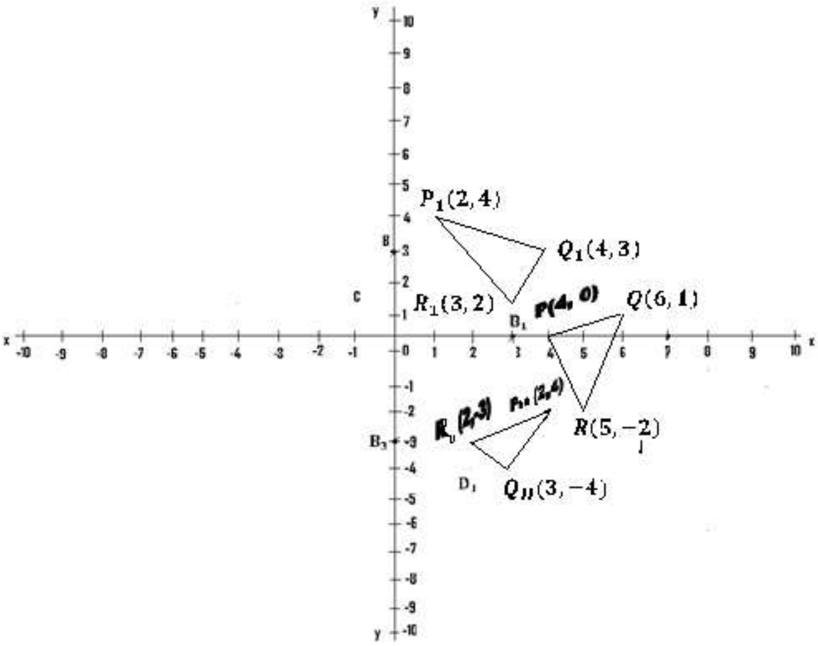
Let (x_2, y_2) be the coordinates of R before it underwent the translation by $\binom{-2}{4}$ to become R_1.Then $R(x_2, y_2) \xrightarrow{\text{translation by } \binom{-2}{4}} R_1\,(3, 2)$

$\Rightarrow x_2 + \ -2 = 3, \Rightarrow x_2 - 2 = 3, \Rightarrow x_2 = 3 + 2 = 5.$

Also $y_2 + 4 = 2, \Rightarrow y_2 = 2 - 4,\ \Rightarrow y_2 = -2.$

$\therefore$R(5, -2) i.e the coordinates of R are (5, -2).

We then draw $\triangle$PQR using these determined coordinates on the graph paper.



(iii) If (x, y) undergoes a rotation through 90^0 clockwise, about the origin, then

$(x, y) \longrightarrow (y, -x)$.

$P_1 \longrightarrow P_{II} \Rightarrow P_1(2,4) \longrightarrow P_{II}(4, -2)$.

$Q_1 \longrightarrow Q_{II} \Rightarrow Q_1(4, 3) \longrightarrow Q_{II}(3, -4)$.

$R_1 \longrightarrow R_{II} \Rightarrow R_1(3, 2) \longrightarrow R_{II}(2, -3)$.

Q9) Using a scale of 2cm to 2 units on both axes, draw on a sheet of graph paper two perpendicular axes ox and oy for $-10 \leq x \leq 10$ and $-12 \leq y \leq 12$. Draw

(i) on the same graph sheet the $\triangle PQR$ with P(4, 8), $\overrightarrow{QR} = \binom{-2}{2}$ and $\overrightarrow{RP} = \binom{2}{4}$.

(ii) $\triangle P_1Q_1R_1$ which is the image of PQR under a reflection in the line y = -2, where $P \longrightarrow P_I$, $Q \longrightarrow Q_I$ and $R \longrightarrow R_1$.

(iii) the image $\triangle P_2Q_2R_2$ of $\triangle PQR$ under a translation by the vector $\binom{-8}{2}$, where $P \longrightarrow P_2$, $Q \longrightarrow Q_2$ and $R \longrightarrow R_2$.

(iv) the image $\triangle P_3Q_3R_3$ of $\triangle PQR$ under a rotation through 180^0 about the origin.

Soln.

(i) We are given P(4, 8) and $\overline{RP} = \binom{2}{4}$.

From $\overrightarrow{RP} = \binom{2}{4} \Rightarrow \overline{PR} = -\binom{2}{4}, \Rightarrow \overrightarrow{PR} = \binom{-2}{-4}$.

Now P(4, 8) and $\overrightarrow{PR} = \binom{-2}{-4}, \Rightarrow$ R(4 + - 2, 8+ - 4),

$\Rightarrow$ R(2, 4) $\Rightarrow$ the coordinates of R = (2, 4).

Now R = (2, 4) and we are also given that $\overrightarrow{QR} = \binom{-2}{2}$.

From $\overrightarrow{QR} = \binom{-2}{2} \Rightarrow \overrightarrow{RQ} = -\binom{-2}{2}$, => $\overrightarrow{RQ} = -\binom{-2}{2} = \binom{2}{-2}$.

Since R= (2, 4) and $\overrightarrow{RQ} = \binom{2}{-2}$

$\Rightarrow$ Q(2+2, 4+-2) => Q(4, 2) => Q has coordinates (4,2)

With the help of or using P(4, 8), R(2, 4) and Q(4, 2) we draw PQR△ .

(ii) $(x, y) \xrightarrow{\text{Reflection in the line } y = b} (x, 2b - y)$

$\Rightarrow$ $(x, y) \xrightarrow{\text{reflection in the line } y = -2,}$ {(x, 2(-2) – }

$\Rightarrow (x, y) \longrightarrow (x, -4-y)$.

$P \longrightarrow P_1$, $\Rightarrow P(4,8) \longrightarrow P_1(4, -4-8)$,

$\Rightarrow p(4, 8) \longrightarrow P_1(4,-12)$.

$R \rightarrow R_1$, $\Rightarrow R(2, 4) \longrightarrow R_1(2, -4-4)$,

$\Rightarrow R(2, 4) \longrightarrow R_1(2, -8)$.

$Q \rightarrow Q_1 \Rightarrow Q(4, 2) \longrightarrow Q_1(4, -4-2), \Rightarrow Q(4, 2) \rightarrow (4,-6)$.

Now using $P_1(4,-12)$, $R_1(2,-8)$ and $Q_1(4,-6)$, we draw $\triangle\ P_1R_1Q_1$.

iii.) $(x, y) \xrightarrow{\text{translation by the vector } \binom{a}{b}} (x+a, y+b)$,

$\Rightarrow (x, y) \xrightarrow{\text{translation by the vector } \binom{-8}{2}} (x+-8, y+2)$,

$\Rightarrow (x,y) \longrightarrow (x-8, y+2)$

$P \longrightarrow P_2 \Rightarrow P(4, 8) \longrightarrow P_2(4-8, 8+2)$,

$\Rightarrow P(4, 8) \longrightarrow (-4, 10)$.

$R \rightarrow R_2$, $\Rightarrow R(2, 4) \longrightarrow R_2(2-8, 4+2)$,

$\Rightarrow R(2,4) \longrightarrow R_2(-6, 6)$.

$Q \longrightarrow Q_2 \Rightarrow Q(4, 10) \longrightarrow Q_2(4-8, 10+2)$,

$\Rightarrow$ Q(4, 10) $\longrightarrow$ Q_2 (- 4, 12).

(iv) If (x, y) undergoes a rotation through 180^0 about the origin, then (x, y). $\longrightarrow$ (-x, -y).

Since $\triangle P_3Q_3R_3$ is the image of $\triangle$PQR, then P $\rightarrow P_3$,

Q$\rightarrow Q_3$ and R$\rightarrow R_3$.

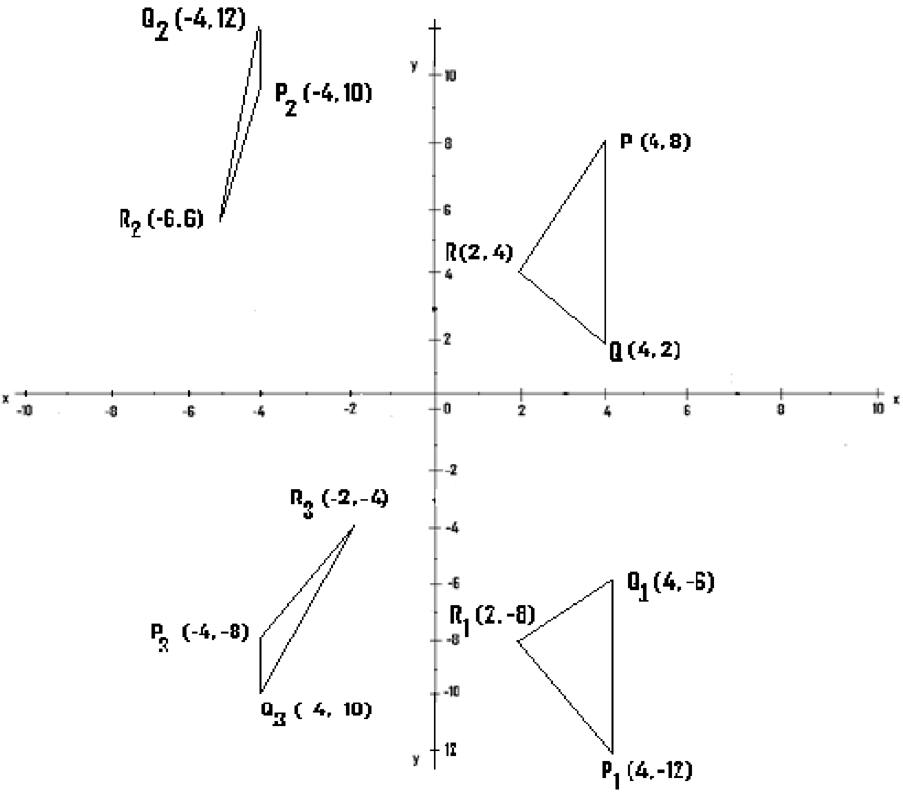
P$\rightarrow P_3 \Rightarrow$ P(4,8) $\longrightarrow$ P_3(- 4, - 8).

Q $\rightarrow Q_3 \Rightarrow$ Q(4,10) $\longrightarrow$ Q_3(-4, -10).

R$\rightarrow R_3 \Rightarrow$ R(2, 4) $\longrightarrow$ R_3(-2, - 4).

Using P_3(- 4, - 8), Q_3(- 4, -10) and R_3 (-2, - 4), we draw

$\triangle P_3Q_3R_3$.



Q10) A triangle has vertices A(1, 1), B(2, 4) and C(5, 8). This triangle undergoes transformation involving anticlockwise rotation through 90^0 about the origin, followed by a translation such that

A$\longrightarrow A^1$, $B \longrightarrow B^1$ and C$\longrightarrow C^1$. If the final positions are A^1(2, -1), B^1(-1, 0) and C^1(-5, 3), find the vector of translation.

N/B: (1) For an anticlockwise rotation through 90^0 about the origin, (x, y)$\longrightarrow$(-y, x).

(2) Consider the vertex A(1, 1). Such a vertex will first undergo an anticlockwise rotation through $90^0$to get an image. This image is then translated by this vector of translation to get the final image A^1(2, 4).

Soln.

If A(1, 1) undergoes an anticlockwise rotation through 90^0, then A(1, 1) $\longrightarrow$ (-1, 1).

After such a rotation to get the point (-1, 1), this point or image is then translated by a certain vector to get the image A^1(2, -1).

If the vector of translation is $\binom{a}{b}$, then

(-1, 1) $\xrightarrow{\text{translation by vector } \binom{a}{b}}$ A^1(2, -1),

$\Rightarrow$ -1+ a =2,$\Rightarrow$ a = 2+1 = 3,

and 1+ b = -1,$\Rightarrow$ b = -1-1= -2.

The vector of translation = $\binom{a}{b}$ = $\binom{3}{-2}$.

Anticlockwise rotation of 90^0about the point (a, b):

-This type of rotation does not occur about the origin or the point (0, 0).

-Assuming the rotation occurs about the point (a, b), then

$$\binom{x}{y} \xrightarrow{\text{anticlockwise rotation of 90 about the point (a, b)}} \binom{-y + a - b}{x - a + b}$$

(Q1)(a) Using a scale of 2cm to 2 units on both axes, draw on a sheet of graph paper two perpendicular axes ox and oy for the interval -10 $\leq$ x $\leq$ 10 and -10 $\leq$ y $\leq$ 10.

(b) Draw indicating the coordinates of all vertices

(i)$\triangle$ABC with vertices A(4, 2), B(0, 2) and C(0, 8).

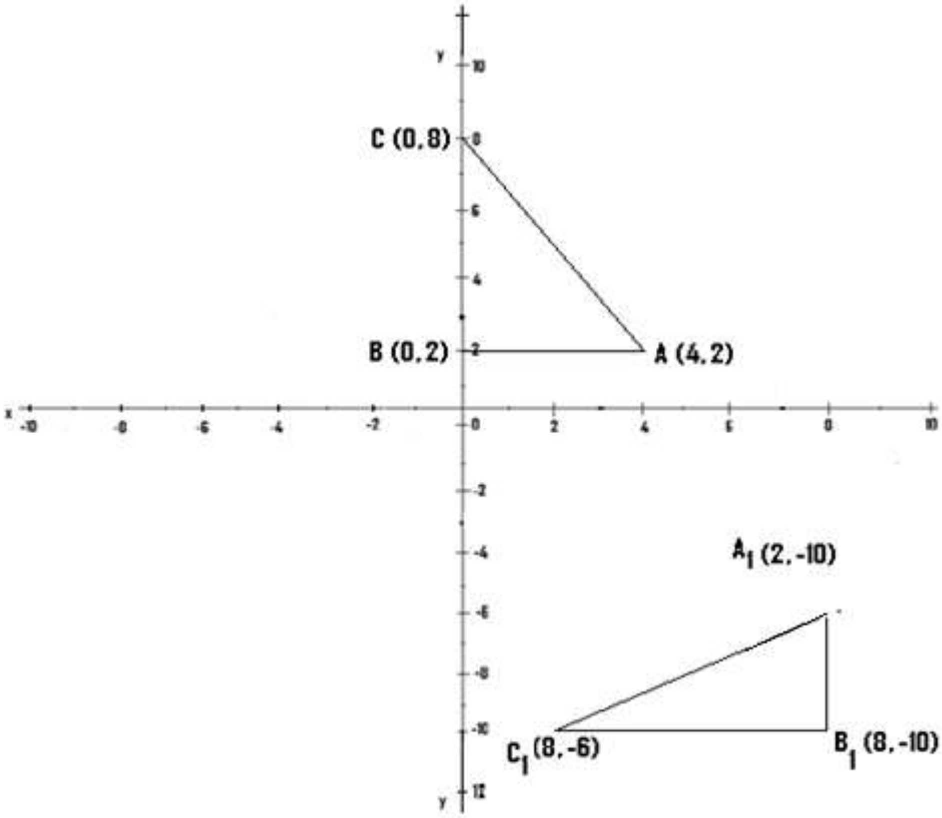
$\triangle$ $\triangle$

(ii) the image $A_1B_1C_1$ of ABC under an anticlockwise rotation through 90^0 about the point (10, 0), where A$\longrightarrow A_1$, B$\longrightarrow B_1$ and C$\longrightarrow C_1$,

N/B: An anticlockwise rotation of 90^0about the point (10, 0) is always centered at the point (10, 0).

Soln.

(a)



(b) $\binom{x}{y} \xrightarrow{\text{Anticlockwise rotation of } 90^0 \text{about the point (a,b)}} \binom{-y+a-b}{x-a+b}$

$\Rightarrow \binom{x}{y} \xrightarrow{\text{Anticlockwise rotation of } 90^0 \text{ about the point (10, 0)}} \binom{-y+10-0}{x-10-0}$

Since a = 10 and b = 0

$$\binom{x}{y} \longrightarrow \binom{-y+10}{x-10}$$

$A\binom{4}{2} \longrightarrow A_1\binom{-2+10}{4-10}$,

$\Rightarrow A\binom{4}{2} \longrightarrow A_1\binom{8}{-6}$.

Since B $\longrightarrow B_1$, then

B$\binom{0}{2} \longrightarrow B_1\binom{-2+10}{0\ -10}$, => B$\binom{0}{2} \longrightarrow B_1\binom{8}{-10}$.

Lastly since C $\longrightarrow C_1$, then

C(0, 8) $\longrightarrow C_1\binom{-8+10}{0\ -10}$,

C(0, 8) $\longrightarrow C_1\binom{2}{-10}$..

Q2)(a) Using a scale of 2cm to 2 units on each axis, draw on a sheet of graph paper, two perpendicular axes ox and oy for the interval $-10 \leq x \leq 10$ and $-10 \leq y \leq 10$.

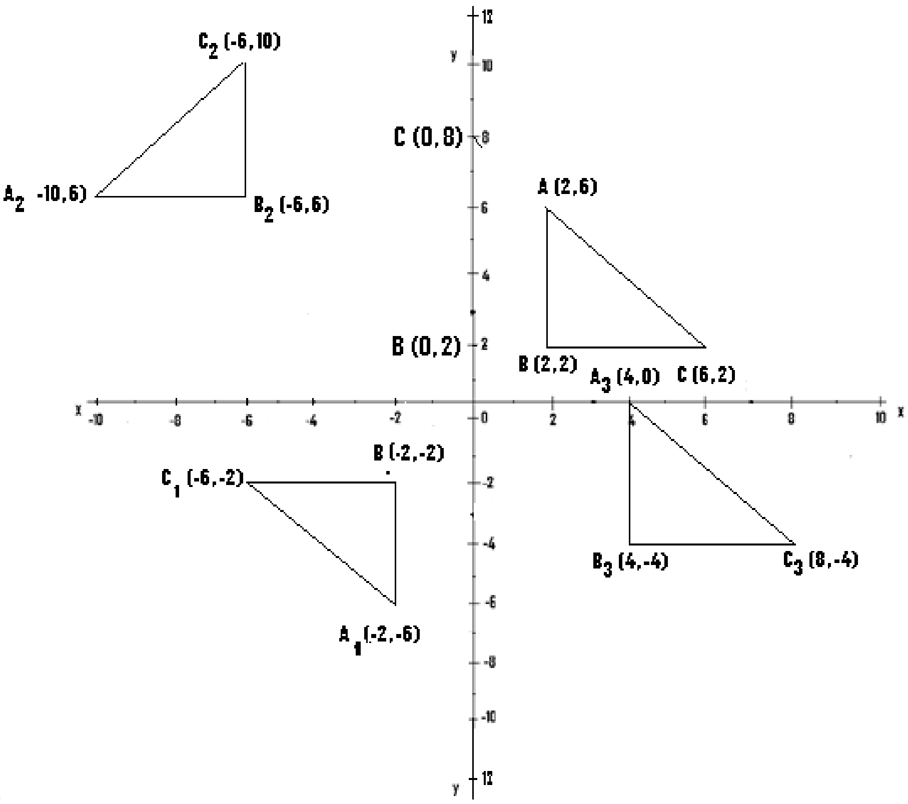
(b) Draw labeling all the vertices clearly together with their coordinates.

(i) $\triangle$ ABC with vertices A(2, 6), B(2, 2) and C(6, 2).

(ii) the image $\triangle A_1B_1C_1$ of $\triangle$ABC under an enlargement with a scale factor of -1, about the origin, where A $\longrightarrow A_1$, B $\longrightarrow B_1$ and C $\longrightarrow C_1$.

(iii) the image $\triangle A_2B_2C_2$ of $\triangle$ABC under an anticlockwise rotation of 90^0 about the point (-2, 2), where A $\longrightarrow A_2$, B $\longrightarrow B_2$ and C $\longrightarrow C_2$.

(iv) the image triangle $A_3B_3C_3$ of triangle ABC under a translation by the vector $\binom{2}{-6}$, where A $\longrightarrow A_3$,B $\longrightarrow B_3$ and C $\rightarrow C_3$.



s

(b) (ii) If the point (x, y) is enlarged with a scale factor of -1, then (x, y) .
. $\longrightarrow$ (-x, -y).

Since A $\longrightarrow A_1$, then A(2, 6) $\longrightarrow A_1$(-2, - 6).

Since B$\rightarrow B_1$, then B(2, 2) $\rightarrow B_1$(-2, -2).

Lastly since C $\rightarrow C_1$, then C(6,2) $\longrightarrow C_1$(- 6,-2).

(iii)

$$\begin{pmatrix} x \\ y \end{pmatrix} \xrightarrow{\text{Anticlockwise rotation of } 90^\circ \text{ about } (a,b)} \begin{pmatrix} -y+a-b \\ x-a+b \end{pmatrix} \Rightarrow$$

$$\begin{pmatrix} x \\ y \end{pmatrix} \xrightarrow{\text{Anticlockwise rotation of } 90^\circ \text{ about } (-2,2)} \begin{pmatrix} -y+(-2)-2 \\ x-(-2)+2 \end{pmatrix}$$

Since a = -2 and b = 2.

$$\Rightarrow \begin{pmatrix} x \\ y \end{pmatrix} \longrightarrow \begin{pmatrix} -y-2-2 \\ x+2+2 \end{pmatrix} \Rightarrow \begin{pmatrix} x \\ y \end{pmatrix} \longrightarrow \begin{pmatrix} -y-4 \\ x+4 \end{pmatrix}.$$

Since A$\rightarrow A_2$, then A(2,6) $\longrightarrow A_2\begin{pmatrix} -6-4 \\ 2+4 \end{pmatrix}$,

$\Rightarrow$A(2, 6) $\longrightarrow A_2\begin{pmatrix} -10 \\ 2 \end{pmatrix}$.

Since B $\rightarrow B_1$, then B(2, 2) $\rightarrow B_2\binom{-2-4}{2+4}$,

$\Rightarrow$ B(2, 2) $\longrightarrow B_2\binom{-6}{6}$.

Lastly since C $\longrightarrow C_1$, then C(6, 2) $\rightarrow C_2\binom{-2-4}{6+4}$,

$\Rightarrow$ C,(6, 2) $\longrightarrow C_2\binom{-,6}{10}$.

(iv) If (x, y) is translated by the vector$\binom{a}{b}$, then

$\binom{x}{y} \longrightarrow$ (x + a, y + b).

$\therefore$ If (x, y) is translated by the vector $\binom{2}{-6}$, then

(x, y) $\longrightarrow$ (x+2, y+6) , $\Rightarrow$ (x, y) $\longrightarrow$ (x+2,y - 6).

Since A $\longrightarrow A_3 \Rightarrow$ A(2, 6) $\longrightarrow A_3$(2+2, 6 – 6),

$\Rightarrow$ A(2, 6) $\longrightarrow A_3$(4, 0) .Since B $\longrightarrow B_3 \Rightarrow$ B(2, 2) $\longrightarrow B_3$(2+2, 2 - 6),

$\Rightarrow$ B (2, 2) $\longrightarrow B_3$(4, - 4).

Lastly since C $\longrightarrow C_3 \Rightarrow$ C (6, 2) $\rightarrow C_3$(6+2, 2- 6),

$\Rightarrow$ C(6, 2) $\longrightarrow C_3$(8, - 4).

(Q3) After a translation, the point P$\binom{8}{2}$ became

$P_1\binom{12}{7}$. Find the vector of translation.

Soln.

P$\binom{8}{2}$ $\longrightarrow P_1\binom{12}{7}$. It can be seen that 4 was added to the 8 or the x component to get the 12, and 5 was added to the 2 or the y component to get the 7.

$\Rightarrow$ The vector of translation = $\binom{x}{y} \longrightarrow \binom{x\ +\ 4}{y\ +\ 5}$..

(Q4) After undergoing a translation, the point X$\binom{10}{2}$

became $X_1\binom{8}{6}$. Find the vector of translation.

Soln.

X$\binom{10}{2}$ $\longrightarrow X_1\binom{8}{6}$. From this, it can be seen that 2 was subtracted from the 10 or the x component to get the 8, and 4 was added to the 2 or the y component to get the 6.

$\Rightarrow The\ vector\ of\ translation\ is\ \binom{x}{y} \longrightarrow \binom{x-2}{y+4}$..

(Q5) A triangle underwent a translation such that a vertex R$\binom{6}{5}$ became $R_1\binom{4}{1}$. Find the vector of translation.

Soln.

R$\binom{6}{5} \longrightarrow R_1\binom{4}{1}$. From this, it can be seen that 2 was subtracted from the 6 or the x component to get 4, and 4 was subtracted from the 5 or the y component to get the 1.

$\Rightarrow$The vector of translation is given by $\binom{x}{y} \longrightarrow \binom{x-2}{y-4}$.....

(Q6) After being subjected to a translation, the point $\binom{-4}{8}$ *became* $\binom{-2}{10}$. Determine the vector of the translation.

Soln.

$\binom{-4}{8} \longrightarrow \binom{-2}{10}$. From this, it can be seen that 2 was added to the - 4 or the x component to get the -2, and 2 was also added to the 8 or the y component to get the 10.

The vector of the translation is $\binom{x}{y} \longrightarrow \binom{x+2}{y+2}$.

(Q7) (a) Using a scale of 1cm to 1 unit on each axis, draw the two perpendicular axes ox and oy for the interval -10 ≤ x ≤ 10 and -10 ≤ y ≤ 10.

(b) Draw$\triangle$ ABC with vertices A(2, 2), B(6, 6) and C(8, 4).

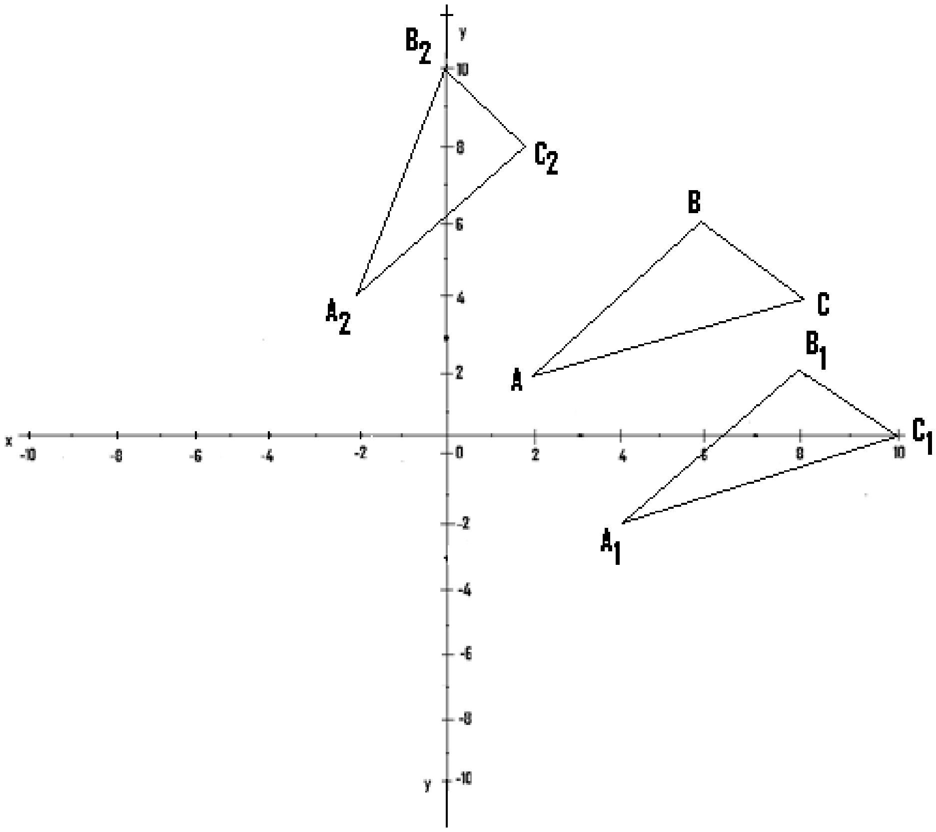
(C) Draw $\triangle A_1B_1C_1$ which is the image of $\triangle$ABC, after it underwent a translation, in which the point B(6, 6) became $B_1\binom{8}{2}$.).

(d) Draw $\triangle A_2B_2C_2$ which is the image of $\triangle A_1B_1C_1$,

as a result of a reflection in the line $\frac{1}{2}x = \frac{1}{2}y$, where

$A_1 \longrightarrow A_2$,, B_1 B_2 and C_1 C_2.

Soln.



(C) Since after the translation the point B(6, 6) became B_1(8, 2), then the vector of translation is given by $\binom{x}{y} \longrightarrow \binom{x+2}{y-4}$.. Using this translation B $\longrightarrow B_1(8,2)$.

Since A $\longrightarrow A_1$, $\Rightarrow$ A$\binom{2}{2} \rightarrow A_1\binom{2+2}{2-4}$,

$\Rightarrow$A$\binom{2}{2} \rightarrow A_1\binom{4}{-2}$.

Since C$\longrightarrow C_1 \Rightarrow$ C$\binom{8}{4} \rightarrow C_1\binom{8+2}{4-4}$,

$\Rightarrow$ C $\binom{8}{4} \longrightarrow C_1\binom{10}{0}$.

Using A_1(4, -2), B_1(8, 2) and C_1(10, 0), we draw $\triangle A_1B_1C_1$.

(d) Consider the line $\frac{1}{2}x = \frac{1}{2}y$.Multiply through using 2

$\Rightarrow 2 \times \frac{1}{2}x = 2\times \frac{1}{2}y \Rightarrow$ x = y. Therefore the reflection in the line $\frac{1}{2}x = \frac{1}{2}y$, is the same as the reflection in the line y = x, and for such a reflection, (x, y)$\longrightarrow$ (y, x).

Since$A_1 \longrightarrow A_2$, then A_1(4, -2) $\rightarrow A_2$(-2, 4).

Also since $B_1 \longrightarrow B_2$, then $B_1(8, 2) \rightarrow B_2(2, 8)$.

Lastly since $C_1 \longrightarrow C_2$, then $C_1(10, 0)$ $C_2(0, 10)$.

Using $A_2(-2, 4)$, $B_2(2, 8)$ and $C_2(0, 10)$, we draw

$\Delta_2 B_2 C_2$..

(Q8) (a) Using a scale of 2 cm to 2 units on each axis, draw the ox and the oy axes for the interval $-12 \leq x \leq 10$ and $-12 \leq y \leq 10$. Draw

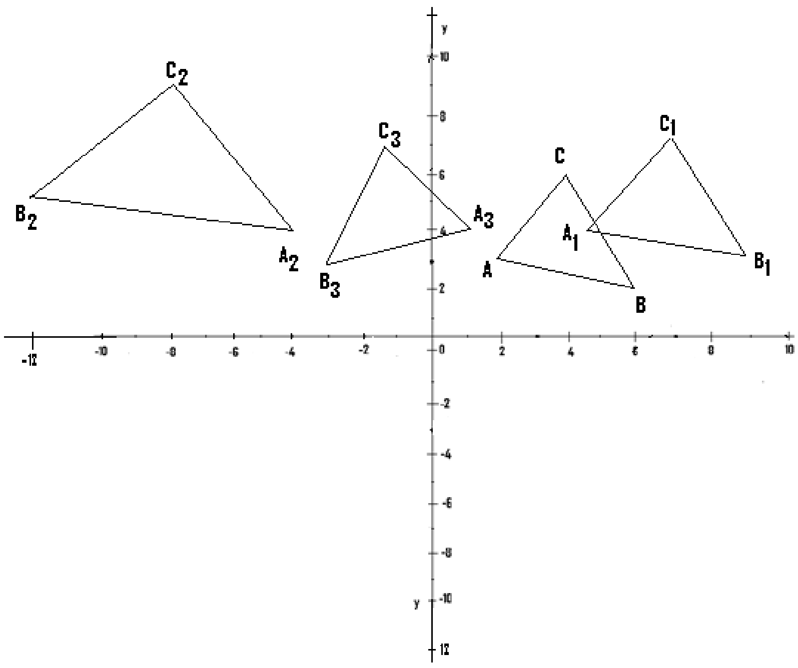
a). triangle ABC with vertices A (2, 3), B(6, 2) and C(4, 6).

(b) the image triangle $A_1B_1C_1$ of triangle ABC, if it underwent a translation such that A(2, 3) $\rightarrow$ A_1 $\binom{5}{4}$, B $\rightarrow$ B_1 and C $\rightarrow$ C_1.

(c) the image triangle $A_2B_2C_2$ of triangle ABC under the mapping $\binom{x}{y} \longrightarrow \binom{-2x}{1+y}$, where A $\longrightarrow A_2$, B $\longrightarrow B_2$ and C $\longrightarrow C_2$.

(d) the image triangle$A_3B_3C_3$ of triangle$A_1B_1C_1$ under a reflection in the line x – 3 = 0, where $A_1 \longrightarrow A_3$, $B_1 \longrightarrow B_3$ and $C_1 \longrightarrow C_3$.

Soln.



(c) We must first determine the translation vector. But A$\binom{2}{3} \longrightarrow A_1\binom{5}{4}$. From this, it can be seen that 3 was added to the 2 to get the 5 and 1 was added to the 3 to get the 4.The vector of translation is therefore $\binom{x}{y} \longrightarrow \binom{x+3}{y+1}$.

Since B $\longrightarrow B_1 \Rightarrow$ B$\binom{6}{2} \longrightarrow B_1\binom{6+3}{2+1}, \Rightarrow$ B $\binom{6}{2} \longrightarrow B_1\binom{9}{3}$.

Also since C$\longrightarrow C_1 \Rightarrow$ C$\binom{4}{6} \longrightarrow C_1\binom{4+3}{6+1}$,

$\Rightarrow$ C (4,6) $\rightarrow C_1\binom{7}{7}$.

(d) The given mapping is $\binom{x}{y} \longrightarrow \binom{-2x}{1+y}$.

Since A $\longrightarrow A_2 \Rightarrow$ A$\binom{2}{3} \longrightarrow A_2\binom{-2(2)}{1+3}$,

$\Rightarrow$ A$\binom{2}{3} \longrightarrow A_2\binom{-4}{4}$.

B $\longrightarrow B_2 \Rightarrow$B$\binom{6}{2} \longrightarrow B_2\binom{-2(6)}{1+2}$,

$\Rightarrow$B $\binom{6}{2} \longrightarrow B_2\binom{-12}{3}$.

C$\longrightarrow C_2 \Rightarrow$ C$\binom{4}{6} \longrightarrow C_2\binom{-2(4)}{1+6}$,

$\Rightarrow$ C $\binom{4}{6} \longrightarrow C_2\binom{-8}{7}$.

(e) A reflection in the line x – 3 = 0 $\Rightarrow$ a reflection in line x = 3.

(x,y) $\xrightarrow{\text{reflection in line x = a}}$ (2a – x, y)

(x,y) $\xrightarrow{\text{reflection in line x = 3}}$ {2(3) – x, y}

(x,y) $\xrightarrow{\text{reflection in line x = 3}}$ (6 – x, y).

Since $A_1 \longrightarrow A_3$, $\Rightarrow A_1$(5,4) $\longrightarrow A_3$(6-5, 4),

$\Rightarrow A_1$(5,4) $\longrightarrow A_3$(1, 4).

Since $B_1 \longrightarrow B_3$,$\Rightarrow B_1$(9, 3) $\longrightarrow B_3$(6-9, 3),

$\Rightarrow B_1$(9, 3) $\longrightarrow B_3$(-3, 3) .

Lastly since $C_1 \Longrightarrow C_3$, $\Rightarrow C_1$(7, 7)$\Longrightarrow$ C_3(6-7, 7),

$\Rightarrow C_1$(7, 7) $\Longrightarrow C_3$(-1, 7).

<u>Questions:</u>

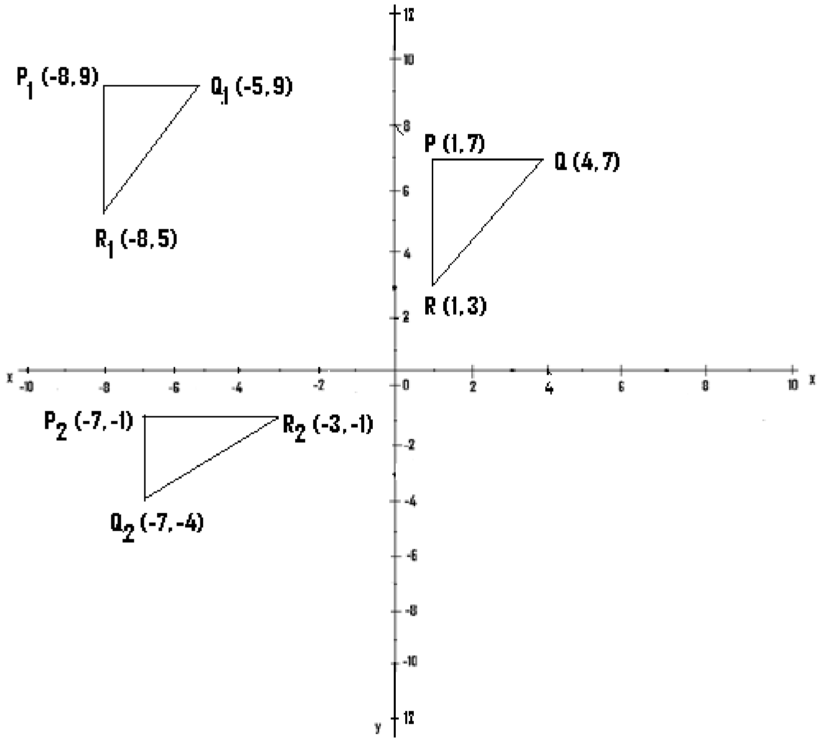
Q1)(a) Using an appropriate scale, draw on a graph paper $\triangle$PQR with vertices P(1, 7), Q(4, 7) and R(1, 3).

(b) Draw $\triangle P_1Q_1R_1$ which is the image of $\triangle$PQR after a translation by the vector $\binom{-9}{2}$, where P $\longrightarrow P_1$, Q$\longrightarrow Q_1$ and R$\longrightarrow R_1$.

(C) Draw also the image $\triangle P_2Q_2R_2$ of $\triangle$PQR, after a reflection in the line y = -x, where P $\longrightarrow P_2$, Q$\longrightarrow Q_2$ and R$\longrightarrow R_2$.

$\longrightarrow$ $\longrightarrow$ $\longrightarrow$

Ans: $\triangle$



(Q2)(a) Using a scale of 2cm to 2units on both axes, draw $\triangle$ABC with the vertices A(4, 4), B(0, 7) and C(3, -6).

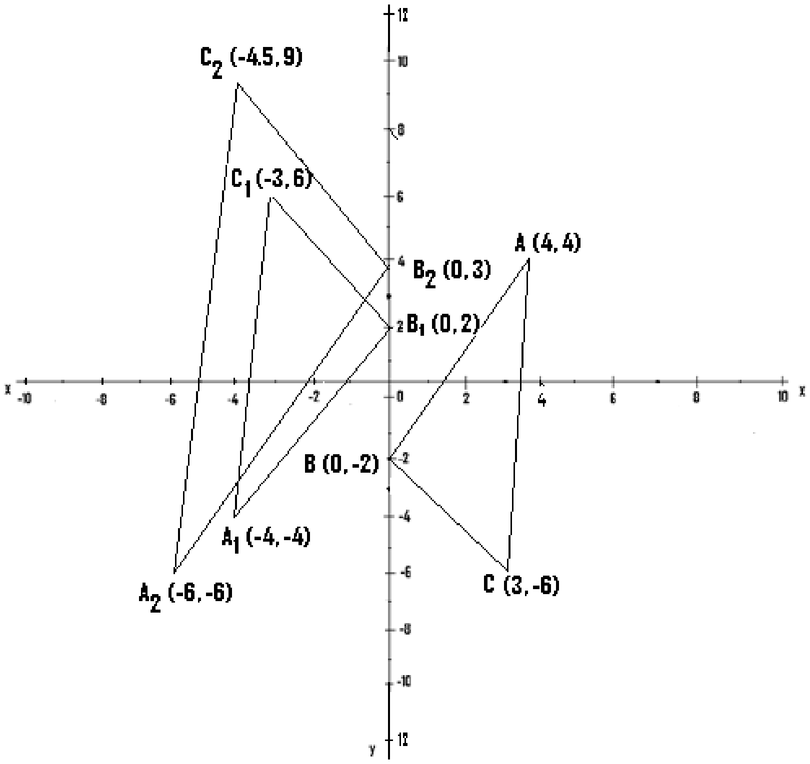
(b) Draw the image $\triangle A_1B_1C_1$ of $\triangle$ABC, after an anticlockwise rotation through 180^0 about the origin, where A $\to A_1$, B $\to B_1$ and C $\to C_1$.

(c) Draw the image $\triangle A_2B_2C_2$ of $\triangle A_1B_1C_1$ after an enlargement with a scale factor of of 1.5, where $B_1 \to B_2$, $C_1 \to C_2$. and so on

(d) Find the slope of the line A_1B_1.

Ans:

N/B: (-4, -4), B_1(0, 2) and C_1(-3, 6)
A_2(-6, -6), B_2(0,3) and C_2(-4.5, 9).



(Q3)(i) Using a scale of 2cm to 2 units on each axis, draw on a sheet of graph paper two perpendicular axes ox and oy for the interval

$-10 \leq x \leq 10$ and $-10 \leq y \leq 10$.

(ii) Draw quadrilateral ABCD with the vertices

A(3, 6), B(7, 6), C(9, 2) and D(2, 2).

(iii) Draw quadrilateral $A_1B_1C_1D_1$ which is the image of quadrilateral ABCD after a clockwise rotation through 90^0, about the origin where A $\longrightarrow A_1$, B $\longrightarrow B_1$,

C $\longrightarrow C_1$ and D $\longrightarrow D_1$.

(iv) Draw quadrilateral $A_2B_2C_2D_2$ which is the image of quadrilateral ABCD after a reflection in the line Y = -x, where A $\longrightarrow A_2$, B $\longrightarrow B_2$, C $\longrightarrow C_2$ and

D $\longrightarrow D_2$.

(v) Lastly draw quadrilateral $A_3B_3C_3D_3$ which is the image of quadrilateral $A_2B_2C_2D_2$, after a reflection in the line y = 0, where $A_2 \rightarrow A_3$, $B_2 \rightarrow B_3$, $C_2 \rightarrow C_3$ and $D_2 \rightarrow D_3$.

$A_2 \longrightarrow A_3$, $B_2 \longrightarrow B_3 C_2 \longrightarrow C_3$ and D_2 ---> D_3.

(v) Determine

(a) the slope of the line AD. Ans: 4

(b) the equation of line BC.

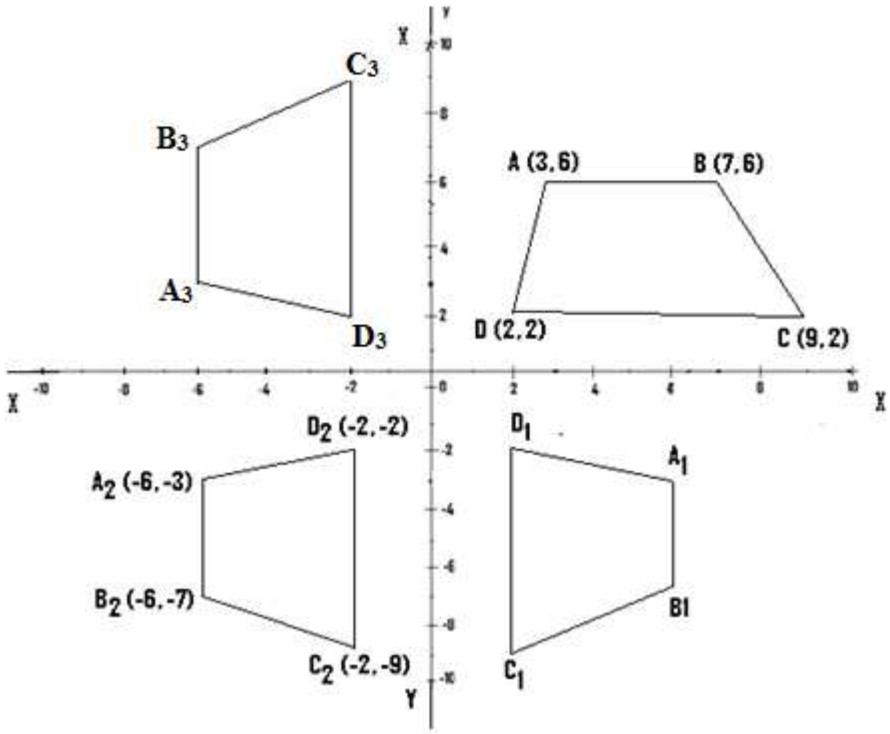
Ans: y = -2x + 20.

Ans:

N/B: $A_1(6_1, -3)$, $B_1(6, -7)$, $C_1(2, -9)$ and $D_1(2, -2)$

$A_2(-6, -3)$, $B_2(-6, -7)$, $C_2(-2, -9)$ and $D_2(-2, -2)$

$A_3(-6, 3)$, $B_3(-6, 7)$, $C_3(-2, 9)$ and $D_3(-2, 2)$



(4) (a) Using a scale of 1cm to 1 unit on each axis, draw the ox and oy axes for the interval $-5 \leq x \leq 8$and $-10 \leq y \leq 10$. Draw

(b) the quadrilateral ABCD whose vertices are A(3, 4) B(5, 4), C(5, 2) and D(3, 2).

(c) draw the image $A_1B_1C_1D_1$ of ABCD after a reflection in the line x = 1, where A $\rightarrow A_1$,B$\rightarrow B_1$, C$\rightarrow C_1$ and D $\rightarrow D_1$.

(d) the image $A_2B_2C_2D_2$ of ABCD after a clockwise rotation through 90^0 about the the origin, where

A $\rightarrow A_2$, B $\rightarrow$ $B_2, C \rightarrow C_2$ and D$\rightarrow D_2$.

(e) the image$A_3B_3C_3D_3$ of $A_1B_1C_1D_1$ after an enlargement with a scale factor of -2, where $A_1 \Rightarrow A_3$, $B_1 \rightarrow$ B_3, $C_1 \rightarrow C_3$ and D_1 $\rightarrow$ D_3.

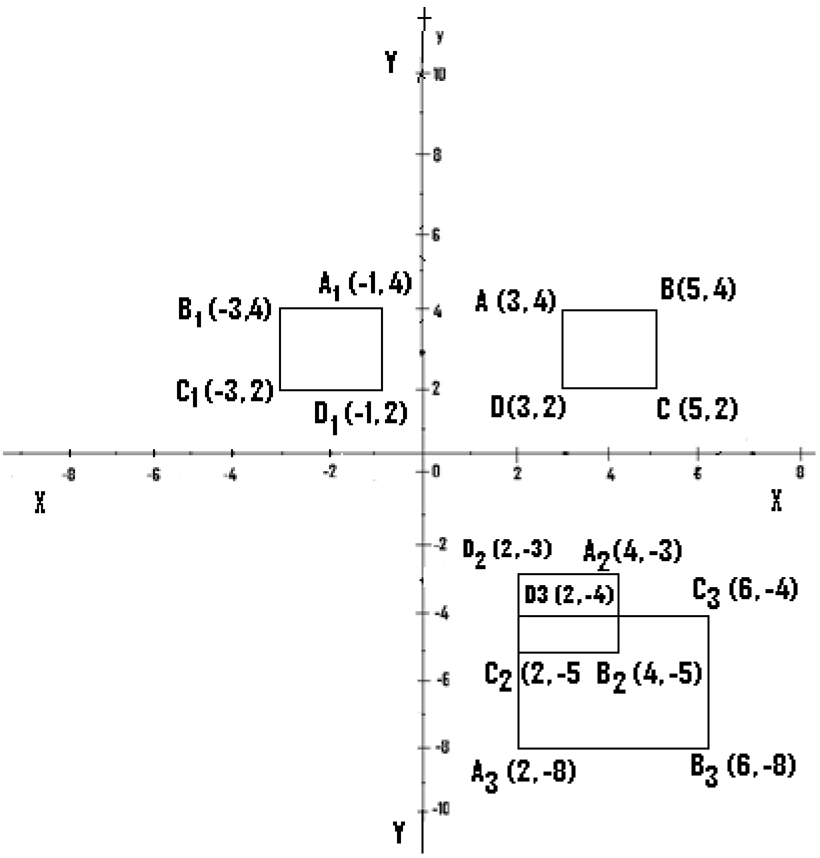
(f) Determine the length of line AD.

Ans:

N/B: $A_1(-1, 4)$ $B_1(-3, 4)$, $C_1(-3, 2)$ and $D_1(-1, 2)$

$A_2(4, -3)$, $B_2(4, -5)$, $C_2(2, -5)$ and $D_2(2, -3)$

$A_3(2, -8)$, $B_3(6, -8)$, $C_3(6, -4)$ and $D_3(2, -4)$.



(Q5)(a) Using a scale of 2cm to 2 units on both axes, draw on a sheet of graph paper two perpendicualar axes ox and oy for the interval $-10 \leq x \leq 10$ and $-10 \leq y \leq 10$.

(b) Draw indicating the coordinates of all the vertices

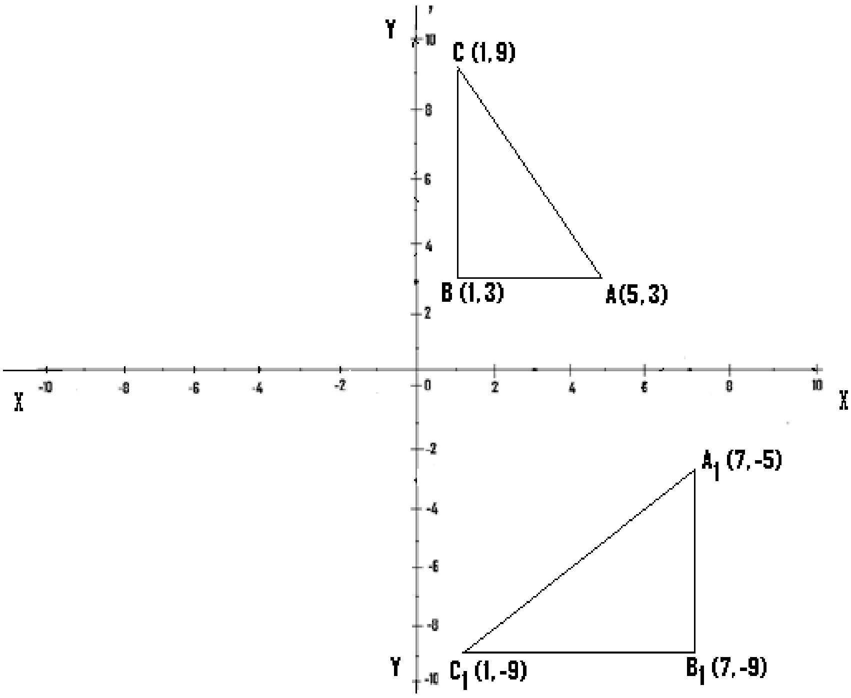
(i) $\triangle$ABC with vertices A(5, 3), B(1, 3) and C(1, 9).

(ii) the image $\triangle A_1B_1C_1$ of $\triangle$ABC under an anticlockwise rotation through 90^0 about the point (10, 0), where A $\longrightarrow A_1$, B $\longrightarrow B_1$ and

C $\longrightarrow C_1$.

Ans:

N/B: $A_1(7, -5)$, $B_1(7, -9)$ and $C_1(1, -9)$

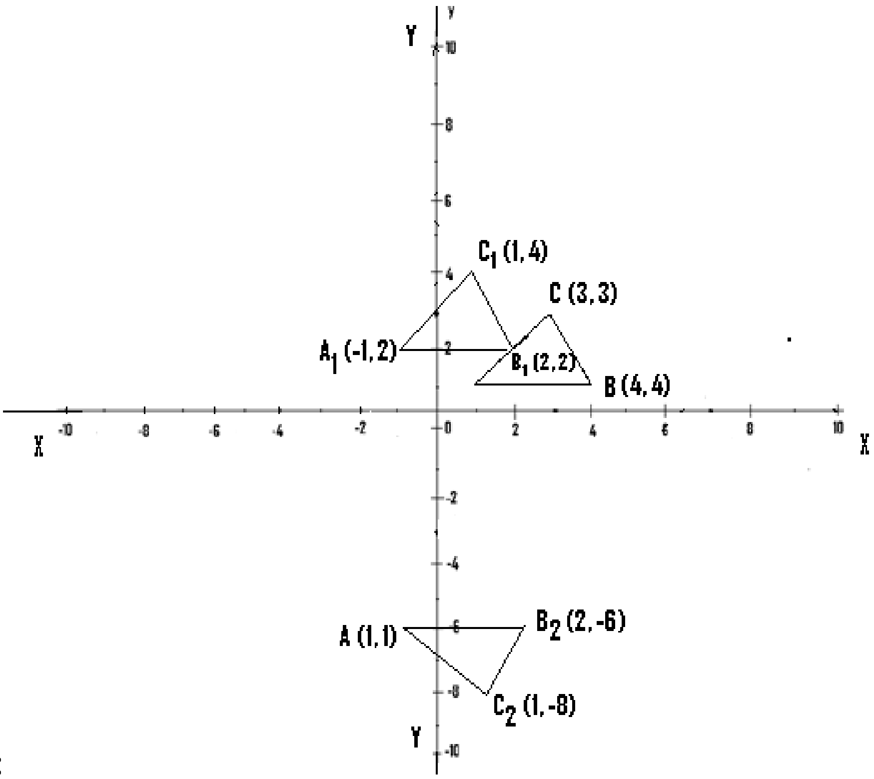


(Q6) By using an appropriate scale, draw

(i) $\triangle ABC$ with vertices A(1, 1), B(4, 1) and C(3, 3).

(ii) the image $\triangle A_1B_1C_1$ of $\triangle ABC$ after a translation in which B(4, 1) $\longrightarrow B_1$(2, 2), A $\longrightarrow A_1$ and C $\longrightarrow C_1$.

(iii) the image $\triangle A_2B_2C_2$ of $\triangle A_1B_1C_1$ after a reflection in the line y + 2 = 0, where $A_1 \longrightarrow A_2$, $B_1 \longrightarrow B_2$ and $C_1 \longrightarrow C_2$.



Ans:

N/B: A_1(-1, 2), B_1(2, 2) and C_1(1, 4).

$A_2(-1, -6)$, $B_2(2, -6)$ and $C_2(1, -8)$.