CHAPTER FIVE

LOGARITHM

Introduction:

* In $\log_b N$, N is referred to as the number and b is referred to as the base.

* The logarithm of a positive number N to a given base b, is the power to which b must be raised so as to be equal to N.

* For example, if $\log_x y = k$, then $x^k = y$.

* If $\log_3 9 = 2 \Rightarrow 3^2 = 9$.

* Also, $\log_2 16 = 4 \Rightarrow 2^4 = 16$

* Since $\log_4 16 = 2$, then $4^2 = 16$

(Q1) Determine the value of x, given that

a) $\text{Log}_5 25 = x$ (b) $\log_2 4 = 4$

(c) $\log_2 32 = x$ (d) $\log_5 125 = x$

(e) $\log_5 625 = x$ (f) $\log_3 81 = x$.

Soln:

(a) Since $\text{Log}_5 25 = x$, then $5^x = 25 \Rightarrow 5^x = 5^2$

$\Rightarrow x = 2$.

(b) if $\log_2 4 = x$, then $2^x = 4$

$\Rightarrow 2^x = 2^2 \Rightarrow x = 2$.

(c) Since $\log_2 32 = x$, then $2^x = 32$

$\Rightarrow 2^x = 2^5 \Rightarrow x = 5.$

(d) Since $\log_5 125 = x$, then $5^x = 125 \Rightarrow 5^x = 5^3 \Rightarrow x = 3.$

(e) If $\log_5 625 = x$, then $5^x = 625 \Rightarrow 5^x = 5^3 \Rightarrow x = 3.$

(Q2) Determine the value of y if

(a) $\log_y 4 = 2$ (b) $\log_y 16 = 2$

(c) $\log_y 36 = 2$ (d) $\log_y 81 = 4$

(e) $\log_y 64 = 3$ (f) $\log_y 27 = 3.$

Soln:

Since $\log_y 4 = 2$, then $y^2 = 4$

$\Rightarrow y^2 = 2^{2 =} \Rightarrow y = 2.$

a). Since $\log_y 16 = 2$, then $y^2 = 16$

$\Rightarrow Y^2 = 4^2 \Rightarrow y = 4.$

a) Since $\log_y 36 = 2$, then $y^2 = 36$

$\Rightarrow y^2 = 6^2 \Rightarrow y = 6.$

(d) Since $\log_y 81 = 4$, then $y^4 = 81$

$\Rightarrow y^4 = 3^4 \Rightarrow y = 3.$

(e) Since $\log_y 64 = 3$, then $y^3 = 64$

$\Rightarrow y^3 = 4^3 \Rightarrow y = 4.$

(f) $\log_y 27 = 3$, then $y^3 = 27$

$\Rightarrow y^3 = 3^3 \Rightarrow y = 3.$

N/B: (1) If no base is written or indicated, then we are dealing in base 10.

* For example, $\log 10 = \log_{10} 10$ and $\log 8 = \log_{10} 8$.

* (2) If the value of the number and the base are the same, then the value of the log is 1.

* For example, $\log_{10}{}^{10} = 1$ and $\log_2{}^2 = 1$.

* Also $\log_5{}^5 = 1$ and $\log_4{}^4 = 1$.

Determination of values of logarithm:

*This can be done by either using a four figure table or a scientific calculator.

Using the four figure table:

*In this case, the decimal point must be after the first number.

* If this is not so, then it must be brought after the first number.

* If this point has to be moved or shifted once towards the left, then the character is 1.

* If it is moved twice, or by two steps, then the characteristic is two.

* If it is moved thrice or by three steps toward the left, then the characteristic becomes three and so on.

* If the decimal point is already after the first number, then there is no movement or shifting of this point, and the characteristic is zero.

(Q1) Determine the characteristic of each of these numbers:

(a) 2.45 (b)3.817

(c)24.5 (d)388.5

(e)24 (f)345

(g)2401 (h)73105

(i)4445.8 (j)3000.43

Soln:

(a) In 2.45, the characteristic is 0, since the point is already after the first number.

(b)Also in 3.817, the characteristic is zero.

(c) 24.5. In this case, the characteristic is 1, since the point has to be shifted one step to the left, in order to be after the first number.

(d) 388.5. In this case, the characteristic is 2, since the point must be shifted two steps left in order to appear after the first number.

(e)24 = 24.0. In this case, the characteristic is 1.

(f) 345 = 345.0. In this case, the characteristic is 2.

(g) 2401 = 2401.0. In this case, the characteristic is 3, since the point must be shifted three steps left, in order to appear after the first number.

(h) 73105 = 73105.0. In this case, the characteristic is 4.

(i) 4445.8. The characteristic is 3.

(j)3000.43. The characteristic is 3.

N/B:

- On the other hand, if the decimal point has to be moved once toward the right before it comes after the first number, then the characteristic is -1.

- If it is moved twice or two steps toward the right, then the characteristic is -2.

- If this movement towards the right is by three steps, then the characteristic is -3.

(Q2) Determine the characteristic of each of these numbers:

(a) 0.24 (b) 0.00789

(c) 0.0005 (c) 0.00085

Soln:

(a) 0.24. The characteristic is -1 in this case, since the point has to be moved one step to the right, in order to appear after the first number.

(b) 0.00789. The characteristic is - 3, since the point has to be shifted three steps to the right, in order to appear after the first number.

(c) 0.0005. The characteristic is - 4.

(d) 0.00085. The characteristic is - 4.

- In the determination of the values of the given logarithm using the table, the characteristic is first determined before the actual value of the log is determined from the table.

- In certain cases, what is referred to as differences may arise.

- The values of these differences which are found at the extreme right hand side of the table, must be added to the values of the logarithm to get our final value.

(Q3) Determine the value of each of the following:

(a) log 0.451 (b) log 0.2453

(c) log 0.245 (d) log 0.2453

(e) log 0.4569 (f) log 0.0171

(g) log 0.01719 (h) log 0.00028

(i) log 0.0002865 (j) log 0.03

(k) log 0.008 (l) log 0.0000895

(m) log 0.0000821368

Soln:

(a) In log 0.451, the characteristic is -1.

- We then determine the value of log 45 under 1, which is = 6542.

=> log 0.451 = -1.6542.

(b) In log 0.45, the characteristic is -1. Since log 0.45 = log 0.450, we determine the value of log 45 under 0 which = 6532.

The value of log 0.45 = - 1.6532.

(c) The characteristic of log 0.245 = -1.

Log 24 under 5 = 3892.

=> The value of log 0.245 = -1.3892.

(d) In Log 0.2453, the characteristic is -1.

- We then determine the value of log 24 under 5, and add to it the value of difference 3.

- Log 24 under 5 = 3892 and the value of the difference 3 in this case = 5.

3892 + 5 = 3897.

=> log 0.2453 = -1.3897.

(e) In log 0.4569, the characteristic = -1.

Log 45 under 6 = 6590 and the difference 9=9.

6590 + 9 = 6599.

Log 0.4569 =-1.6599.

(f) In log 0.0171, the characteristic = - 2.

Log 17 under 1 = 2330.

=> log 0.0171 = -2.2330.

(g) In log 0.01719, the characteristic = -2.

log 17 under 1 = 2330 and its difference 9 = 22.

2330 + 22 = 2352.

=> log 0.01719 = -2.2352.

(h) In log 0.00028 which is the same as log 0.000280, the characteristic = - 4 and log 28 under 0 = 4472.

=> log o.ooo28 = - 4.4472.

(i) In log 0.0002865, the characteristic = - 4.

Log 28 under 6 = 4564 and its difference 5 = 8.

4564 + 8 = 4572.

Log 0.0002865 = - 4.4572.

(j) log 0.03 = log 0.0300 and the characteristic = -2.

log 30 under 0 = 4771.

=> log 0.03 = -2.4771.

(k) log 0.008 = log 0.00800 and the characteristic= -3.

Log 80 under zero = 9031.

=> log 0.008 = -3.9031.

(l) In log 0.0000895, the characteristic = -5.

log 89 under 5 = 9518.

=> log 0.0000895 = -5.9518.

(m) In log 0.0000821368, the characteristic = -5.

Log 82 under 1 = 9145 and the difference 3 = 2.

9145 + 2 = 9147.

=> Log 0.0000821364 = -5.9147.

N/B: In log 0.000821368, we only consider log 82 under 1 difference 3.

(Q4) Determine the value of the following:

(a) log 45.1 (b) log 4.51

(c) Log 488 (d) log 4.88

(e) log 4883 (f) log 20.1

(g) log 200.54 (h) log 3.216

(i) log 89668 (j) log 341.67

(k) log 453816 (l) log 4553. 29

Soln:

(a) In log 45.1, the characteristic = 1, and log 45 under 1 = 6542.

=> Log 45.1 = 1.6542.

(b) In 4.51, the characteristic = 0 and log 45 under 1 = 6542.

=> log 4.51 = 0.6542.

(c) In log 488, the characteristic = 2, and log 48 under 8 = 6884.

=> Log 488 = 2.6884.

(d) In log 4.88, the characteristic = 0 and log 48 under 8 = 6884.

=> log 4.88 = 0.6884.

(e) In log 4883, the characteristic = 3.

Log 48 under 8 = 6884 and its difference 3 = 3.

6884 + 3 = 6887.

=> log 4883 = 3.6887

(f) In log 20.1, the characteristic = 1 and log 20 under 1= 3032.

=> log 20.1 = 1.3032.

(g) In log 200.54, the characteristic = 2.

Log 20 under 0 = 3010 and the difference 5 = 11.

3010 + 11 = 3021.

Log 200.54 = 2.3021.

(h) In log 3.216, the characteristic = 0.

Log 32 under 1 = 5065 and the difference 6 = 8.

5065 + 8 = 5073

=> Log 3.216 = 0.5073.

(i) In Log 89668, the characteristic = 4.

Log 89 under 6 = 9523 and the difference 6 = 3.

9523 + 3 = 9526.

=> Log 89668 = 4.9526.

(j) In log 341.67, the characteristic = 2.

Log 34 under 1 = 5328 and the difference 6 = 8.

5328 + 8= 5336.

=> log 341.67 = 2.5336.

(k) Log 453816 = log 453816.0. The characteristic = 5.

Log 45 under 3 = 6561 and the difference 8 = 8.

6561 + 8 = 6569.

Log 453816 = 5.6569.

Determination of the value of logarithm using the calculator:

- If the number has a characteristic which is either zero or positive, then the calculator can be used to determine the value of the logarithm directly or straight away.
- For example, to determine log 1.675 or log 72.1, simply, press log followed by the number.
- If the number has a negative characteristic, we first write down this characteristic.
- We then press log followed by the number which comes after the decimal point and this will give us a number in the form of a decimal.
- The number after the decimal point is approximated and written after the characteristic.
- But a decimal point must first be brought after the characteristic.
- For example, to determine log 0.2715, we first write down the characteristic which is -1.
- The value of log 2715 according to the calculator is 3.434.
- The figure after the decimal point which can be approximated to 434 is then written after the -1.
- But first bring a decimal point after the characteristic.
- Therefore log 0.2715 = -1.434.
- To determine the value of log 0.04343, the characteristic is -2 and log 4343 = 3.637789829.
- The value after the decimal point can be approximated to 6378.
- Therefore log 0.04343 = -2.6378.

Determination of antilog:

- To determine the antilog of a given number which is always in the decimal form using a calculator, you must first press shift or drag followed by log and then the number which comes after the decimal point.

- For example, to get the antilog of 2.752, first press shift, followed by log, and then 0.752.

- This will give us 5.649.

- Also to get antilog -1.6723, press shift , followed by log and then 0.6723.

- This will give us4.7022.

- If log x = 0.275, then x = antilog 0.275 x 10^0, i.e. the number after the decimal point times 10 raised to the digit before the point.

- If log x = -3.214, then x = antilog 0.214 × 10^{-3}

- If log x = 2.823, then x = antilog 0.823 × 10^2.

- If log x = -2.430 , then x = antilog 0.430×10^{-2}.

-If log x = 5.720, then x = antilog 0.720 × 10^5

- If log x = -5.360, then x = antilog 0.360 × 10^{-5}

- If log x = 2.0371, then x = antilog 0.0371 x 10^2,

- If x = 0.0241, then x = antilog 0.0241 × 10^0 = 0.0241 × 1 = 0.0241.

N/B:

$\log_{10}(a \times b) = \log_{10} a + \log_{10} b$

=> log (a × b) = log a + log b

Example(1)

$\log_{10}(371 \times 4211) = \log_{10} 371 + \log_{10} 4211$

Example (2)

$\log(0.3741 \times 91.7) = \log 0.3741 + \log 91.7.$

Example (3)

$\log(0.721 \times 0.0043)$

$= \log 0.721 + \log 0.0043$

- Any number raised to the power zero = 1

$\Rightarrow 6^0 = 1$ and $2^0 = 1$.

(Q1) Determine value of 37.1×4481.

Soln:

Let $x = 37.1 \times 4481$

Taking log of both sides $\Rightarrow \log x = \log(37.1 \times 4481)$

$\Rightarrow \log x = \log 37.1 + \log 4481$

$\Rightarrow \log x = 1.5693 + 3.6514$

$\Rightarrow \log x = 5.2207$

$\Rightarrow x = \text{antilog } 0.2207 \times 10^5$

$\Rightarrow x = 1.66226 \times 10^5$

$\Rightarrow x = 166226.$

(Q2) Evaluate 12×20 by means of logarithm.

Soln:

Let $x = 12 \times 20$

=> log x = log (12 × 20)

=> log x= log 12 + log 20

=> log x = 1.079 + 1.301

=> log x = 2.380

x = antilog 0.380 × 10^2

= 2.399 × 100 = 239.9

=> x = 240

(Q3) By using logarithm, determine the value of 0.0713 × 2118.

Soln:

Let x = 0.0713 × 2118

Taking log of both sides

=> log x = log(0.0713 × 2118)

=> log x = log 0.0713 + log 2118

=> log x = -2.8531 + 3.3259

log x = 2.179

x = antilog 0.179 × 10^2

=> x = 1.5100 × 100 = 151.

N/B: In adding together -2.8531 to 3.3259, first add together the numbers after the decimal points, i.e

0.8531 + 0.3259 = 1.179.

- The 179 is written down while the 1 is carried forward.

- Add the two numbers which come before the two decimal points i.e -2 + 3 = 1.

- Then add the 1 which was carried forward to this 1 to get 2 and bring a decimal point after the 2.

- The 179 is then brought after the 2 to get 2.179.

(Q4) By the means of logarithm, determine the value of 0.378 × 0.0175.

Soln.

Let x = 0.378 × 0.0175

=> log x = log (0.378 × 0.0175)

=> log x = log 0.378 + log 0.0175

=> log x = -1.5775 + (- 2.2430)

=>log x = -1.5775 – 2.2430

=> log x = -3.820

=> x = antilog o.820 $\times 10^{-3}$

= 6.6 $\times$ 10^{-3} = 6.6 x $\frac{1}{1000} = \frac{6.6}{1000}$

= 0.0066

* In adding -1.5775 to - 2.2430, first add the numbers after the decimals points. i.e, 0.5775 + 0.2430 = 0.820.

* Add the numbers which come before the decimal points. i.e, -1 + -2 = -3.

* The 0.820 or .820 is brought after the -3 to get -3.820.

N/B:

- Log $_a b^2$ = 2 log $_a b$
- Log $_a b^5$ = 5 log $_a b$
- Log $_a b^{1/2} = \frac{1}{2} \log_a b$
- Log $_a b^{1/3} = \frac{1}{3} \log_a b$

- Log 273^2 = 2 log 273
- Log 78.36^5 = 5 log 78.36.

(Q5) Evaluate $0.274^2 \times 117^3$ by using logarithm.

Soln.

Let x = $0.274^2 \times 117^3$

Taking log of both sides,

=> log x = log ($0.274^2 \times 117^3$)

=> log x = log 0.274^2 + log 117^3

=> log x = 2 log 0.274 + 3 log 117

=> log x = 2(-1.4378) + 3(2.0682)

=> log x = -2.8756 + 6.2046

=>Log x = 5.0802

=>x = antilog 0.0802×10^5

=> x = 1.202818×10^5

=> x = 120281.8 = 120282.

N/B:

(1) In the evaluation of 2(-1.4378), we first multiply the 0.4378 by the 2 to determine whether there will be a carry forward or not, i.e, 2×0.4378 = 0.8756.

- In this case, there is no carry forward.
- The -1 is then multiplied by the 2 to get -2.
- The .8756 is finally brought after the -2 to get -2.8756.
- Also, in adding -2.8756 to the 6.2046, you must first add the numbers after the decimal points, i.e, 0.8756 and 0.2046 to get the 1.0802.

- The 1 is carried forward.
- Add the numbers before the decimal points, i.e, -2 + 6 = 4.
- The 1 is then added to the 4 to get 5.
- Finally, the .0802 is brought after the 5 to get 5.0802.

(Q6) By means of logarithm, evaluate 0.0231×0.374^4.

Soln:

Let $x = 0.0231^3 \times 0.374^4$

$\Rightarrow \log x = \log(0.0231^3 \times 0.374^4)$

$\Rightarrow \log x = \log 0.0231^3 + \log 0.374^4$

$\Rightarrow \log x = 3 \log 0.0231 + 4 \log 0.374$

$\Rightarrow \log x = 3 (-2.3617) + 4 (-1.5729)$

$\Rightarrow \log x = -5.08510 + -2.2916$

$\Rightarrow \log x = - 6.3767$

$\Rightarrow x = \text{antilog } 0.3767 \times 10^{-6}$

$X = 0.28 \times 10^{-6}$

N/B:

* To multiply the 3 by -2.3617, the 0.3617 is multiplied by 3 to get 1.0851.

* While the .0851 is maintained, the 1 is carried forward.

* The -2 is then multiplied by the 3 to get -6, to which the 1 is finally added to get -5.

* The .0851 is brought after the -5 to get -5.0851.

* To multiply the -5.5729 by 4, we first multiply the 0.5729 by 4 to get 2.2916.

*While the 2 is carried forward, the .2916 is maintained.

* The -1 is then multiplied by 4 to get -4, to which the 2 is added to get -2.

*The .2916 is finally brought after the -2 to get -2.2916.

*To add -5.0851 to -2.2916, we first add 0.0851 to 0.2916 to get 0.3767.

*The -5 is then added to the -2 to get -7.

*Bring the .3767 after the -7 to get -7.3767.

(Q7) Use logarithm to find the value of $376^{\frac{1}{2}} \times 7132$.

Soln:

Let x = $376^{\frac{1}{2}} \times 7132$

=> log x = log ($376^{\frac{1}{2}} \times 7132$)

=> log x = log $376^{\frac{1}{2}}$ + log 7132

=> log x = $\frac{1}{2}$ log376 + log 7132

=> log x = 0.5(2.5752) + 3.8532

=> log x = 1.2876 + 3.8532

=>log x = 5.1408

=> x = antilog 0.1408 $\times$ 10^5

=> x = 1.382929 $\times$ 10^5

=> x = 138292.9

=> x = 138293 .

(Q8) By means of logarithm, evaluate $446^{\frac{1}{4}} \times 0.0248^{\frac{1}{2}}$

Soln:

Let x = $446^{\frac{1}{4}} \times 0.0248^{\frac{1}{2}}$

=> log x = log ($446^{\frac{1}{4}} \times 0.0248^{\frac{1}{2}}$)

=> log x = log $446^{\frac{1}{4}}$ + log $0.0248^{\frac{1}{2}}$

=> log x = $\frac{1}{4}$ log 446 + $\frac{1}{2}$ log 0.0248

=> log x = 0.25 log 446 + 0.5 log 0.0248

=> log x = 0.25(2.649) + 0.5(-2.3945)

=> log x = 0.6623 + -1.1973

=> log x = -1.8596

=> x = antilog 0.8596 $\times 10^{-1}$

X = 7.2 $\times 10^{-1}$ = 0.72 .

(Q9) By means of logarithm, evaluate $0.034^{\frac{1}{5}} \times 0.244^{\frac{1}{3}}$.

Soln:

Let x = $0.034^{\frac{1}{5}} \times 0.244^{\frac{1}{3}}$.

=> log x = log($0.034^{\frac{1}{5}} \times 0.244^{\frac{1}{3}}$)

=> log x = log $0.034^{\frac{1}{5}}$ + log $0.244^{\frac{1}{3}}$

=> log x = $\frac{1}{5}$ $log 0.034 + \frac{1}{3}$ log 0.244

=> log x = 0.2(-2.5315) + 0.33 (-1.3874)

=> log x = - 0.5063 + - 0.457842

=- 0.9641

=> log x = - 0.9641

=> x = antilog 0.9641 $\times$ 10^0

=> x = 0.11

N/B:

* To evaluate 0.2(-2.5315) and since 0.2 is a decimal, we multiply 0.2 by -2.5315 directly to get - 0.5063.

* (If the number outside the bracket is a decimal, then we multiply it directly by the number inside the bracket.

* To evaluate also 0.33(-1.3874) and since 0.33 is a decimal, it is multiplied directly by -1.3874 to get -0.4578.

*To add -0.5063 to - 0.4578, we add 0.5063 to 0.4578, and bring the negative sign in front of our answer.

*Therefore -0.5063 + - 0.4578 = - 0.9641

*If log x= - 0.96, then x = antilog - 0.96 $\times$ 10^0.

(Q10) By means of logarithm, evaluate $(0.34 \times 1178)^2$

Hint:

$(0.34 \times 1178)^2 = 0.34^2 \times 1178^2$

Let $x = 0.34^2 \times 1178^2$

=> log x = log $(0.34^2 \times 1178^2)$

=> log x = log 0.34^2 + log 1178^2

=> log x = 2log 0.34 + 2log 1178.

(Q11) Determine the value of $(3415^2 \times 0.114)^3$ by using logarithm.

Hint:

$(3415^2 \times 0.114)^3 = 3415^6 \times 0.114^3$.

Let $x = 3415^6 \times 0.114^3$

Taking log of both sides

$\Rightarrow \log x = \log(3415^6 \times 0.114^3)$

$\Rightarrow \log x = \log 3415^6 + \log 0.114^3$

$\Rightarrow \log x = 6\log 3415 + 3 \log 0.114$

N/B: $(3415^2 \times 0.114)^3 = (3415^2 \times 0.114^1)^3 = 3415^6 \times 0.114^3$

(Q12) Determine the value of $(119.2^6 \times 44.3^3)^2$ by using logarithm.

Hint:

$(119.2^6 \times 44.3^3)^2 = (119.2^{12} \times 44.3^6)$

Let $x = 119.2^{12} \times 44.3^6$

$\Rightarrow \text{Log } x = \log(119.2^{12} \times 44.3^6)$

$\Rightarrow \log x = \log 119.2^{12} + \log 44.3^6$

$\Rightarrow \log x = 12 \log 119.2 + 6 \log 44.3$

N/B: $(119.2^6 \times 44.3^3)^2 = (119.2^{12} \times 44.3^6)$

(Q13) Find the value of or evaluate $(0.32^2 \times 442^6)^{\frac{1}{2}}$ by using logarithm.

Hint:

$(0.32^2 \times 442^6)^{\frac{1}{2}} = (0.32^{2\times\frac{1}{2}} \times 442^{6\times\frac{1}{2}})$

$= (0.32^1 \times 442^3) = (0.32 \times 442^3)$

Let $x = 0.32 \times 442^3$

$\Rightarrow \log x = \log (0.32 \times 442^3)$

$\Rightarrow \log x = \log 0.32 + \log 442^3$

(Q14) Determine by means of logarithm, the value of $(4.311^6 \times 0.00438)^{\frac{1}{3}}$

Hint:

$(4.311^6 \times 0.00438)^{\frac{1}{3}} = (4.311^6 \times 0.00438^1)^{\frac{1}{3}}$

$= (4.3116^2 \times 0.00438^{\frac{1}{3}})$

Let $x = 4.3116^2 \times 0.00438^{\frac{1}{3}}$

$\Rightarrow \log x = \log (4.3116^2 \times 0.00438^{\frac{1}{3}})$

$\Rightarrow \log x = \log 4.3116^2 + \log 0.00438^{\frac{1}{3}}$

$\Rightarrow \log x = 2 \log 4.3116 + \frac{1}{3} \log 0.00438$

N/B:

(1) $\sqrt[2]{a} = a^{\frac{1}{2}}$ i.e $\sqrt{a} = a^{\frac{1}{2}}$

(2) $\sqrt[3]{a} = a^{\frac{1}{3}}$

(3) $\sqrt[4]{a} = a^{\frac{1}{4}}$

(Q15) Use logarithm to evaluate $\sqrt{47.34^4 \times 0.1138}$

Hint:

$\sqrt{47.34^4 \times 0.1138}$=$(47.34^4 \times 0.1138^1)^{\frac{1}{2}} = (47.34^{4\times\frac{1}{2}} \times 0.1138^{1\times\frac{1}{2}})$

$= (47.34^2 \times 0.1138^{\frac{1}{2}})$

Let $x = 47.34^2 \times 0.1138^{\frac{1}{2}}$

=> $\log x = \log(47.34^2 \times 0.1138^{\frac{1}{2}})$

(Q16) Use logarithm to find the value of $\sqrt[3]{1391^6 \times 458^9}$

Hint:

$\sqrt[3]{1391^6 \times 458^9} = (1391^6 \times 458^9)^{\frac{1}{3}}$

$= (1391^{6\times\frac{1}{3}} \times 458^{9\times\frac{1}{3}}) = (1391^2 \times 458^3)$

Let $x = 1391^2 \times 458^3$

=> $\log x = \log (1391^2 \times \log 458^3)$

=> $\log x = \log 1391^2 + \log 458^3$

=> $\log x = 2 \log 1391 + 3 \log 458$

(Q17) Find the value of $(34.11^{\frac{1}{2}} \times 0.0034^{\frac{2}{3}})^2$ by using logarithm.

Hint:

$(34.11^{\frac{1}{2}} \times 0.0034^{\frac{2}{3}})^2 = (34.11^{\frac{1}{2}\times 2} \times 0.0034^{\frac{2}{3}\times 2})$

$= (34.11^1 \times 0.0034^{\frac{4}{3}}) = (34.11 \times 0.0034^{\frac{4}{3}})$

let $x = \log 34.11 \times 0.0034^{\frac{4}{3}}$

=> $\log x = \log(34.11 \times 0.0034^{\frac{4}{3}})$

=> log x = log 34.11 + $\frac{4}{3}$ log 0 . 0034

=> log x = log 34.11 + 1.33 log 0.0034.

N/B: $log_{10}\frac{a}{b} = log_{10}$a – $\log_{10}$b

=> log $\frac{a}{b}$ = log a – log b

- For example, $\log_{10}\frac{271}{3.11}$

= $\log_{10}$271 – $\log_{10}$3.11

Also log $\frac{92.14}{0.77} = log 92.14 - \log 0.77$

(Q1) By means of logarithm, evaluate $\frac{4348}{312}$

Soln:

Let x = $\frac{4348}{312}$

Taking log of both sides => log x = log ($\frac{4348}{312}$)

=> log x = log 4348 – log 312

=> log x = 3.6383 – 2.4941

=> log x = 1.1442

=> x = antilog 0.1442 $\times$ 10^1

=> x = 1.394 $\times$ 10 = 13.94 = 14.

(Q2) By means of logarithm, determine the value of ($\frac{34.8^2}{583}$).

Soln:

Let x = ($\frac{34.8^2}{583}$)

=> log x = log $(\frac{34.8^2}{583})$

=> log x = log 34.8^2 – log 583

=> log x = 2 log 34.8 – log 583

=> log x = 2(1.5416) – 2.7657

=> log x= 3.0832 – 2.7657

=> log x = 0.318

=> x = antilog 0.318 $\times$ 10^0

=> x = $1.98 \times 1 = 2$ approx.

(Q3) Determine the value of $\frac{341^2}{82.5^3}$ by means of logarithm.

Soln:

Let x = $\frac{341^2}{82.5^3}$

=> Log x = log $(\frac{341^2}{82.5^3})$

=> log x = log 341^2 – log 82.5^3

=> log x = 2 log 314 – 3 log 82.5

=> log x = 2(2.5328) – 3(1.9165)

=> log x = 5.0656 – 5.7495

=> log x = -1.3161

=> x = antilog 0.3161 $\times$ 10^{-1}

=> x = 2.1 $\times$ 10^{-1} = 0.21 .

(Q4) By means of logarithm evaluate $(\frac{34.5^2}{82.3})^3$

Hint:

$(\frac{34.5^2}{82.3})^3 = (\frac{34.5^2}{82.3^1})^3$

$= (\frac{34.5^{2\times3}}{82.3^{1\times3}}) = (\frac{34.5^6}{82.3^3})$

Let x = $\frac{34.5^6}{82.3^3}$

=> log x = log $(\frac{34.5^6}{82.3^3})$

=> log x = log $(\frac{34.5^6}{82.3^3})$

=> log x = log 34.5^6 – log 82.3^3

(Q5) Evaluate $(\frac{42.5^2}{2.17^3})^2$ using logarithm.

Hint:$(\frac{42.5^2}{2.17^3})^2 = \frac{42.5^{2\times2}}{2.17^{3\times2}}$

= $\frac{42.5^4}{2.17^6}$

Let x =$\frac{42.5^4}{2.17^6}$ => log x = log42.5^4 – log2.17^6

=> log x = 4log 42.5 – 6log2.17

(Q6) Evaluate $(\frac{34.1^4}{22.6^2})^{\frac{1}{2}}$ using logarithm.

Hint:

$(\frac{34.1^4}{22.6^2})^{\frac{1}{2}} = (\frac{34.1^{4\times\frac{1}{2}}}{22.6^{2\times\frac{1}{2}}})$

= $\frac{34.1^2}{22.6^1} = \frac{34.1^2}{22.6}$

Let x = $\frac{34.1^2}{22.6}$

(Q7) Determine by means of logarithm the value of $\sqrt{\frac{66.8}{44.2}}$

Hint:

$\sqrt{\frac{66.8}{44.2}} = (\frac{66.8}{44.2})^{\frac{1}{2}}$

$= (\frac{66.8^1}{44.2^1})^{\frac{1}{2}} = \frac{66.8^{1\times\frac{1}{2}}}{44.2^{1\times\frac{1}{2}}}$

$= \frac{66.8^{\frac{1}{2}}}{44.2^{\frac{1}{2}}}$

Let $x = \frac{66.8^{\frac{1}{2}}}{44.2^{\frac{1}{2}}}$

=> $\log x = \frac{1}{2}\log 66.8 - \frac{1}{2}\log 44.2$

(Q8) By using logarithm, evaluate $(\frac{2.88^6}{1.21^2})^{\frac{1}{3}}$

Hint:

$(\frac{2.88^6}{1.21^2})^{\frac{1}{3}} = \frac{2.88^{6\times\frac{1}{3}}}{1.21^{2\times\frac{1}{3}}} = \frac{2.88^2}{1.21^{\frac{2}{3}}}$

Let $x = \frac{2.88^2}{1.21^{\frac{2}{3}}}$

=> $\log x = 2 \log 2.88 - \frac{2}{3}\log 1.21$

(Q9) Determine the value of $\sqrt[3]{\frac{442}{0.131^2}}$ *with the aid of logarithm.*

Hint:

$\sqrt[3]{\frac{442}{0.131^6}} = (\frac{442}{0.131^6})^{\frac{1}{3}}$

$=\left(\frac{442^1}{0.131^6}\right)^{\frac{1}{3}} = \frac{442^{1\times\frac{1}{3}}}{0.131^{6\times\frac{1}{3}}}$

$= \frac{442^{\frac{1}{3}}}{0.131^2}$

Let x = $\frac{442^{\frac{1}{3}}}{0.131^2}$

=> log x = $\frac{1}{3}\log 442 - 2\log 0.131.$

(Q10) Determine by means of logarithm, the value of $\sqrt[4]{\frac{324.1^8}{24.5^4}}$

Hint:

$\sqrt[4]{\frac{324.1^3}{24.5^4}} = \left(\frac{324.1^8}{245.5^4}\right)^{\frac{1}{4}}$

$= \frac{324.1^{8\ x\frac{1}{8}}}{24.5^{4\ x\frac{1}{4}}} = \frac{324.1^2}{24.5}$

Let x = $\frac{324.1^2}{24.5}$

=> log x = 2log 324.1 – log 24.5

(Q11) With the help of logarithm, find the value of $\frac{0.024^3}{0.512^2}$

Hint: Let x = $\frac{0.024^3}{0.512^2}$ => log x = 3log0.024 – 2log0.512

N/B: (a)(1) 2-3 = - 1 (2) 5 -7 = -2

(3) -2 -3 = -5 (4) -5 -7 = -12

(5) -2 - (-3) = -2 + 3 = 1 (6) - 5 - (-7) = -5 + 7 = 2

- To evaluate 2.6072 - (-2.5378), we first subtract 5378 from 6072 to get 694.

- The -2 is then subtracted from the 2, i.e 2 - (-2) = 2 + 2 = 4 and a decimal point brought after the 4.

- The 694 is then brought after the 4 to get 4.694.

(2) - To evaluate 2.6072 - 3.5372, we first remove the 5372 from 6072 to get 700.

- With respect to the numbers which come before the decimal points, he 3 is subtracted from the 2 to get -1 and a decimal point is brought after the -1.

- The 700 is then brought after the -1 to get -1.700.

(3) - To evaluate 2.6077 - 3.8000, the 8000 must be removed from 6077.

- But since this cannot be, 1 is borrowed from the 2 and brought before the 6 to get 16077.

-We therefore subtract the 8000 from 16077 to get 8077.

- After the removal of the 1, the value 2.6077 becomes 1.6077.

- With respect to the numbers before the decimal points, 3 is removed from the 1 to get -2 and a decimal point is brought after the -2.

- The 8077 is then brought after the -2 to get -2.8077.

(4) To evaluate 2.6077 - (-3.8000), 1 is borrowed from the 2 and brought before the 6 to give us 16077.

- Remove 8000 from 16077 to get 8077.

- After the removal of the 1, the value of 2.6077 becomes 1.6077 and consider 1.6077 and -3.8000.

- Now considering the numbers before the decimal point, -3 is removed from the 1 to get 4 (i.e 1- (-3) = 1 + 3 = 4) and a decimal point is brought after the 4.

- Bring the 8077 after the 4 to get 4.8077.

(5) To evaluate 0.271 - (-3.461), 1 is removed from the 0 and this is brought before the 2 to give us 1271.

- The 461 is removed from the 1271 to give us 810.

- After the removal of the 1 from the 0, the value of 0.271 now becomes -1.271, since 0 - 1= -1.

- Consider -1.271 and -3.461.

. Considering now the numbers before the decimal points ,-3 is removed from -1 to give us 2 (i.e -1 - (-3) = -1 + 3 = 2) and a decimal point is brought after the 2.

- The 810 is brought after the 2 to give us 2.810.

(6) To evaluate -2.6211 - (-3.1100), we subtract 1100 from 6211 to get 5111.

- Considering the numbers before the decimal points, the -3 is subtracted from the -2 to get 1 (i.e -2 - (-3) = -2 + 3 = 1), and a decimal point brought after the 1.

- The 5111 is then brought after the 1 to get 1.5111.

(7) To evaluate -6.5211 - (-3.8211).

- Since 8211 cannot be removed from 5211, 1 is removed from the -6 and brought before the 5 to give us 15211.

- The removal of the 8211 from the 15211 gives us 7000.

- After the removal of the 1, the -6.5211 becomes – 7.5211

- Consider -7.5211 and -3.8211.

Removal of -3 from -7 gives us - 4, (since -7 - (-3) = -7 + 3 = - 4) and bring a point after the - 4.

- Bringing the 7000 after the - 4 gives us 4.7000

(8) To evaluate - 4.1200 -3.1000, the 1000 is removed from 1200 to give us 200.

- The 3 is removed from the - 4 to give us -7 (i.e - 4 -3 = -7) and a point is brought after the -7.

- The 200 is finally brought after the -7 to give us -7.200.

(9) - To evaluate 0.2711 - 5.4431, 4431 cannot be removed from 2711.

- We therefore borrow or remove 1 from the 0, and this is brought before the 2 to give us 12711.

- The 4431 is removed from 12711 to give us 8280.

- After the removal of the 1 from the 0, the value of 0.2711 becomes -1.2711 (since 0-1=-1).

- Consider now -1.2711 and 5.4431.

- Considering the numbers before the decimal points 5 is subtracted from -1 to get -6 and a decimal point is brought after the -6.

- The 8280 is brought after the - 6 to get -6.8280.

(10) To evaluate 0.5311 - (-6.2211), 2211 is subtracted from 5311 to get 3100.

-Considering the numbers before the decimal points, we subtract the -6 from the 0 to get 6 (i.e 0 - (-6) = 0 + 6 = 6) and a decimal point is brought after the 6.

- The 3100 is finally brought after the 6 to get 6.3100.

(Q11) To evaluate 0.6958 - 3.8952, 1 is removed from the 0 and brought before the 6 to give us 16958.

- The 8952 is then removed from 16958 to get 8006.

- After the removal of the 1 from the 0, the value of the 0.6958 becomes -1.6958 (since 0 -1 =-1).

Consider now -1.6958 – 3.8952

- Considering now the numbers before the decimal points, the 3 is subtracted from the -1 to get -4, and a decimal point is brought after the - 4.

- The 8006 is finally brought after the - 4 to get - 4.8006.

N/B: It is advisable for these types of calculation to be done as shown, with little use of the calculator.

- It is also good to do them partly manually or without a calculator, and partly with a calculator.

- It can also be done totally manually.

N/B:

- In the calculation of logarithm, logs of numbers which are multiplying are added together.
- And logs of numbers which are dividing are subtracted.
- The following must be well noted.

(1) Log ($\frac{a\times b}{c}$) = log a + log b – log c.

- In this case, a and b are multiplying while c is dividing.
- Log_{10} ($\frac{a\times b}{c}$) is the same as log ($\frac{a\times b}{c}$), and Log_{10} ($\frac{a\times b}{c}$) = log_{10}a + log_{10} b – log_{10}c.

  For example, log ($\frac{271\times 0.66}{44.8}$)

  =Log 271 + log 0.66 – log 44.8.

(1) Log ($\frac{a\times b}{c\times d}$) = log a + log b – log c – log d.

- In this case, while a and b are multiplying, c and d are dividing.
- Log($\frac{a\times b}{c\times d}$) is the same as log_{10}($\frac{a\times b}{c\times d}$).
- And log_{10}($\frac{a\times b}{c\times d}$) =$log_{10}$ a + log_{10} b – log_{10} c – log_{10} d.
- For example log ($\frac{227\times 43.1}{0.11\times 33.8}$)

  = log 227 + log 43.1 – log 0.11 – log 33.8

(2) Log ($\frac{a\times b}{c}$) = log a + log b – log c.

- In this case, while a and b are multiplying, c is dividing.
- For example, log ($\frac{0.722\times 44.6}{11.2}$)

= log 0.722 + log 44.6 – log 11.2

3) Log ($\frac{a\times b\times c}{d}$) = log a + log b + log c – log d.

In this case, while a, b and c are multiplying, d is dividing.

For example, log($\frac{78.1\times113\times0.35}{44.6}$)

= log 78.1 + log 113 + log 0.35 – log 44.6

3) Log ($\frac{a\times b\times c}{d\times e}$) = log a + log b + log c – log d – log e.

In this case, while a, b and c are multiplying, whiled and e are dividing.

For example, log ($\frac{22.1\times0.88\times98}{0.03\times48}$)

= log 22.1 + log 0.88 + log 98 – log 0.03 – log 48.

3) Log ($\frac{a}{b\times c}$)= log a – log b – log c.

In this case, b and c are dividing.

For example, log ($\frac{2711}{0.32\times47}$)

= log 2711 – log 0.32 – log 47.

(Q1) By means of logarithm, evaluate $\frac{0.1578\times31.46}{7852}$

Soln:

Let x = $\frac{0.1578\times31.46}{7852}$

Taking log of both sides =>

Log x = log ($\frac{0.1578\times31.46}{7852}$)

=> log x = log 0.1578 + log 31.46 – log 7852

=> log x = -1.1981 + 1.4977 – 3.8950

=> log x = 0.6958 – 3.8950

=> log x = - 4.8008

=> x = antilog 0.8008 $\times$ 10^{-4}

=> x = 6.3 $\times$ 10^{-4} = 0.00063

(Q2) By using logarithm, determine the value of $\frac{6.75 \times 0.751^2}{3.41^3}$

Hint:

Let x = $\frac{6.75 \times 0.751^2}{3.41^3}$

=> log x = log ($\frac{6.75 \times 0.751^2}{3.41^3}$)

=> log x = log 6.75 + log 0.751^2 $-$ log 3.41^3

=> log x = log 6.75 + 2 log 0.751 – 3 log 3.41

(Q3) With the help of logarithm, determine the appropriate value of
($\frac{66.81 \times 0.024^2}{0.334^3}$)^2

Hint:

($\frac{66.81 \times 0.024^2}{0.334^3}$)^2 = $\frac{66.81^2 \times 0.024^4}{0.334^6}$

Let x = $\frac{66.81^2 \times 0.024^4}{0.334^6}$

=> log x = log ($\frac{66.81^2 \times 0.024^4}{0.334^6}$)

=> log x = log 66.81^2 + log 0.024^4 – log 0.334^6

=> log x = 2 log 66.81 + 4 log 0.024 – 6 log 0.334

(Q4) Use logarithm to determine the value of $\frac{\sqrt{781} \times 0.045^2}{\sqrt[3]{625}}$

Hint:

$\frac{\sqrt{781} \times 0.045^2}{\sqrt[3]{625}} = \frac{781^{\frac{1}{2}} \times 0.045^2}{625^{\frac{1}{3}}}$

Let x = $\frac{781^{\frac{1}{2}} \times 0.045^2}{625^{\frac{1}{3}}}$

=> log x = log ($\frac{781^{\frac{1}{2}} \times 0.045^2}{625^{\frac{1}{3}}}$)

=> log x = log $781^{\frac{1}{2}} + \log 0.045^2 - \log 625^{\frac{1}{3}}$

=> log x = $\frac{1}{2}$ log 781 + 2 log 0.045 - $\frac{1}{3}$ log 625

=> log x = 0.5 log 781 + 2 log 0.045 -0.33 log 625

N/B: Every number is raised to the power 1. Therefore 8 = 8^1, 20 = 20^1 and 388 = 388^1.

(Q5) Use logarithm to evaluate $\left(\frac{0.286^2 \times 0.445^4}{0.998} \right)^{\frac{1}{2}}$

Hint:

$\left(\frac{0.286^2 \times 0.445^4}{0.998} \right)^{\frac{1}{2}} = \left(\frac{0.286^{2 \times \frac{1}{2}} \times 0.445^{4 \text{ x } \frac{1}{2}}}{0.998^{1 \times \frac{1}{2}}} \right)$

$= \left(\frac{0.286 \times 0.445^2}{0.998^{\frac{1}{2}}} \right)$

Let x = $\left(\frac{0.286 \times 0.445^2}{0.998^{\frac{1}{2}}} \right)$

(Q6) Using logarithm, determine the value of $\frac{4118 \times 0.021}{293 \times 6.86}$

Soln:

Let x = $\frac{4118 \times 0.021}{293 \times 6.86}$

=> log x = log($\frac{4118 \times 0.021}{293 \times 6.86}$)

=> log x = log 4118 + log 0.021 – log 293 – log 6.86

=> log x = 3.615 + (-2.3222) – 2.4669 – 0.8363.

=> log x = 1.9372 – 2.4669 – 0.8363

=> log x = -1.4703 – 0.8363

=> log x = -2.6340

=> x = antilog 0.6340 $\times 10^{-2}$

=> x = 4.31 $\times 10^{-2}$ = 0.0431.

(Q7) Use logarithm to evaluate $\frac{74.54 \times 0.125^2}{11.6^3 \times 6.86}$

Hint:

Let x = $\frac{74.54 \times 0.125^2}{11.6^3 \times 6.86}$

=> log x = log ($\frac{74.54 \times 0.125^2}{11.6^3 \times 6.86}$)

=> log x = log 74.54 + log 0.125^2 - log 11.6^3 − log 6.86

=> log x = log 74.54 + 2 log 0.125 – 3 log 11.6 – log 6.86

(Q8) By means of logarithm, evaluate $\left(\frac{361^3 \times 0.116^2}{45 \times 0.54}\right)^2$

Hint:

$\left(\frac{361^3 \times 0.116^2}{45 \times 0.54}\right)^2 = \left(\frac{361^6 \times 0.116^4}{45^2 \times 0.54^2}\right)$

Let x = $\frac{361^6 \times 0.116^4}{45^2 \times 0.54^2}$

=> log x = log($\frac{361^6 \times 0.116^4}{45^2 \times 0.54^2}$)

=> log x = log $361^6 + \log 0.116^4 - \log 45^2 - \log 0.54^2$

=> log x = 6 log 361 + 4 log 0.116 – 2 log 45 – 2log 0.54.

Further application of logarithm:

Logarithm has further applications, and this is indicated in the next few questions.

(Q1) By means of logarithm, determine the value of 27^3.

Soln:

Let x = 27^3

Taking log of both sides =>

Log x = log 27^3

=> log x = 3 log 27,

=> log x = 3(1.43)

=> log x = 4.29,

=> x = antilog 0.29 $\times$ 10^4

=> x = 1.9498 $\times$ 10^4

=> x = 19498.

(Q2) Determine the value of $2.5^{0.34}$

Soln:

Let $x = 2.5^{0.34}$

Taking log of both sides

=> $\log x = \log 2.5^{0.34}$

=> $\log x = 0.34 \log 2.5$,

=> $\log x = 0.34(0.398)$

=> $\log x = 0.14$,

=> $x = \text{antilog } 0.14 \times 10^0$

=> $x = 1.38 \times 1 = 1.38$.

(Q3)Given that $2^x = 5$, determine the value of x.

Soln:

$2^x = 5$

Taking log of both sides,

=> $\log 2^x = \log 5$

=> $x \log 2 = \log 5$

Divide through using log 2

=> $\frac{x \log 2}{\log 2} = \frac{\log 5}{\log 2}$

=> $x = \frac{\log 5}{\log 2} = \frac{0.699}{0.301}$

=> x = 2.3

(Q4) If $25^n = 78.9$, find the value of n.

Soln:

$25^n = 78.9$

Taking log of both sides

=> log 25^n = log 78.9,

=> n log 25 = log 78.9.

Divide through using log 25

$=> \frac{\text{n log 25}}{\log 25} = \frac{\log 78.9}{\log 25} => n = \frac{1.897}{1.3979}$, => n = 1.4.

$(Q5)$ Given that $2^{x-1} = 5$, find the value of x.

Soln:

$2^{x-1} = 5$

Taking log of both sides

=> log 2^{x-1} = log 5,

=> (x-1) log 2 = log 5.

Divide through using log 2

$\Rightarrow \frac{(x-1)\log 2}{\log 2} = \frac{\log 5}{\log 2}$

$\Rightarrow x - 1 = \frac{\log 5}{\log 2} = \frac{0.699}{0.301}$,

=> x-1 = 2.3

=> x = 2.3 + 1 = 3.3.

(Q6) Given that $25^{2n+3} = 625$, find the value of x.

Soln:

$25^{2n+3} = 625$

Taking log of both sides

=> $\log 25^{2n+3} = \log 625$,

=> (2n +3) log 25 = log 625.

Divide through using log 25

$\Rightarrow \frac{(2n+3)\log 25}{\log 25} = \frac{\log 625}{\log 25}$

=> 2n + 3 = $\frac{2.7959}{1.3979}$,

=> 2n + 3 = 2 => 2n = 2 – 3 = -1

=> 2n = -1, => n = $\frac{-1}{2} = -0.5$.

(Q7) By means of logarithm, determine the square root of 72.46.

Soln:

Square root of 72.46 = $\sqrt{72.46}$

= $(72.46)^{\frac{1}{2}}$

Let x = $(72.46)^{\frac{1}{2}}$

=> Log x = $\frac{1}{2} log 72.46$

=> log x = 0.5 log 72.46,

=> log x = 0.5(1.860)

=> log x = 0.930,

=> x = antilog 0.930 $\times$ 10^0

=> x = 8.5 $\times$ 10^0 = 8.5 x 1 = 8.5.

(Q8) Use logarithm to determine $\sqrt[3]{0.112}$

Hint:

$\sqrt[3]{0.112}$ = $(0.112)^{\frac{1}{3}}$

Let x = $0.112^{\frac{1}{3}}$

=> log x = log $0.112^{\frac{1}{3}}$

=> log x = $\frac{1}{3}$log 0.112.

(Q9) Determine the value of $\log_2 7$.

Soln:

Let $\log_2 7 = x$

=> $2^x = 7$

Taking log of both sides

=> log 2^x = log 7 => x log 2 = log 7

Divide through using log 2

$\Rightarrow \frac{x \log 2}{\log 2} = \frac{log 7}{\log 2}$

$\Rightarrow x = \frac{log 7}{\log 2} = \frac{0.8451}{0.3010} = 2.8.$

N/B:

- log 1 = 0
- Also $\log_{10}10 = 1$, $\log_2 2 = 1$ and $\log_4 4 = 1$.

(Q1) Simplify each of the following:

(a) Log 1000 + log 1 + log 100

Soln:

Log 1000 + log 1 + log 100

$= \log 10^3 + 0 + \log 10^2$

= 3 log 10 + 0 + 2 log 10

= 3(1) + 2(1) = 3 + 2 = 5

(b) $\text{Log}_2 16 + \log_2 8 + \log_2 2$.

Soln:

$\text{Log}_2 16 + \log_2 8 + \log_2 2$

$= \log_2 2^4 + \log_2 2^3 + 1$

$= 4\log_2 2 + 3\log_2 2 + 1,$

$= 4(1) + 3(1) + 1 = 4 + 3 + 1 = 8.$

(c) $3\log_{10} 3 - \log_{10} 27$

Soln:

$3\log_{10} 3 - \log_{10} 27$

$= \log_{10} 3^3 - \log_{10} 27,$

$= \log_{10} 27 - \log_{10} 27 = 0.$

(d) $\frac{\log 729}{\log 9}$

N/B: $729 = 3^6$ and $9 = 3^2$

Soln:

$\frac{\log 729}{\log 9} = \frac{\log 3^6}{log 3^2} = \frac{6 \log 3}{2 \log 3}$

$= \frac{6}{2} = 3.$

(e) $\text{Log}_{10} 81 \div log_{10} 9$

Soln:

$\text{Log}_{10} 81 \div log_{10} 9 = \frac{log_{10} 81}{log_{10} 9},$

$= \frac{log_{10} 9^2}{log_{10} 9} = \frac{2\ l0g_{10}9}{log_{10}9} = 2.$

(f) $\frac{log_{10} 125}{log_{10} 25}$

Soln

$$\frac{\log 125}{\log 25} = \frac{log 5^3}{log 5^2} = \frac{3 \log 5}{2 \log 5} = \frac{3}{2}$$

= 1.5 .

g) $\log_3 14 - \log_3 7$

Soln:

$\log_3 14 - \log_3 7 = \log_3 (\frac{14}{7})$

$= \log_3 2.$

(h) $\text{Log}_{10} 1 - \log_{10} 3^2$

Soln:

$\text{Log}_{10} 1 - \log_{10} 3^2 = \log_{10} 1 - \log_{10} 9$

$= \log_{10} (\frac{1}{9})$

(i) $\text{Log} \frac{b\sqrt{c}}{\sqrt[3]{c}}$

Soln:

Log $\frac{b\sqrt{a}}{\sqrt[3]{c}}$ = log $\frac{b\times a^{1/2}}{c^{1/3}}$

= log b + log $a^{1/2} - \log c^{1/3}$,

= log b + $\frac{1}{2}\log a - \frac{1}{3}\log c$.

(j) 3 log 2 + 2 log3 – 2 log 6.

Soln:

3 log 2 + 2 log 3 – 2 log 6

= log 2^3 + log3^2 – log 6^2

= log 8 + log 9 – log 36,

= log ($\frac{8\times 9}{36}$) $= \log 2.$.

(Q2) Given that x log 4 = log 16, find the value of x.

Soln:

x log 4 = log 16

=> x log 4 = $\log 4^2$

=> x log 4 = 2 log 4.

Divide through using log 4

=> $\frac{x\ log4}{log4} = \frac{2 \log 4}{\log 4}$

=> x = 2.

N/B: Given that xlog 5 = log125, then xlog5 = $\log 5^3$

=> xlog5 = 3log5.

Divide through using log5

=> $\frac{x \log 5}{\log 5} = \frac{3\ log5}{log5}$

=> x = 3.

(Q3 Express $\log_{10} \frac{a^2 b^3}{100\sqrt{c}}$ in terms of $\log_{10}$a, $\log_{10}$b and $\log_{10}$c.

Soln:

$\log_{10}\frac{a^2b^3}{100\sqrt{c}} = \log_{10}\frac{a^2b^3}{100c^{\frac{1}{2}}}$

$= \log_{10}a^2 + \log_{10} b^3 - \log_{10}100 - \log_{10} c^{\frac{1}{2}} = \log_{10}a^2 + \log_{10}b^3 - \log_{10}10^2 - \log_{10}c^{\frac{1}{2}}$

$= 2\log_{10} a + 3\log_{10} b - 2\log_{10}10 - \frac{1}{2}\log_{10}c$

$= 2\log_{10}a + 3\log_{10} b - 2(1) - \frac{1}{2}\log_{10} c$

$= 2\log_{10}a + 3\log_{10}b - 2 - \frac{1}{2}\log_{10}c$

(Q4) Express each of the following as a single number or as a single logarithm:

a) Log 2 + log 3.

Soln:

Log 2 + log 3 = log(2×3) = log 6.

a) Log 18 – log 9.

Soln:

Log 18 – log 9 = $\log(\frac{18}{9})$

= log 2.

a) Log x + logy – log z.

Soln:

Log x + log y – log z = log ($\frac{x \times y}{z}$)

= log ($\frac{x.y}{z}$)

(a) 2 log a – log b

Soln:

2 log a – log b = log a^2 – log b

= log ($\frac{a^2}{b}$).

e) 2 $\log_b$ x + 3 $\log_b$ y.

Soln:

2 $\log_b$ x + 3 $\log_b$y = $\log_b x^2$ + $\log_b y^3$

= $\log_b(x^2.y^3)$

c) Log_bx – 3 $\log_b$ y + $\log_b$ z

Soln:

$\text{Log}_b x - 3 \log_b y + \log_b z$

$= \log_b x - \log_b y^3 + \log_b Z$

$= \log_b x + \log_b z - \log_b \mathbf{y^3}$

$= \log_b \left(\frac{x \text{ x } z}{y^3}\right)$

$= \log_b \left(\frac{x \,.z}{y^3}\right)$

g) Log 3 + 3 log 2 – 3 log 4

Soln:

Log 3 + 3 log 2 – 3log 4

$= \log 3 + \log 2^3 - \log 4^3$

= log 3 + log 8 – log 64,

$= \log \left(\frac{3\times 8}{64}\right) = \log\left(\frac{3}{8}\right) = log\frac{3}{8}.$

Questions:

(1) Determine the value of x, given that

a) $\text{Log}_x\ 64 = 3$

Ans: x = 4

b) $\text{Log}_2\ 64 = x$

Ans: x = 6

c) $\text{Log}_2\ x = 1$

Ans: x = 2

d) $\text{Log}_{10}\ x = 0$

Ans: x = 1

(Q2) By means of logarithm, determine the value of the following:

a) 37.4×271

Ans: 10135

b) 0.411×12.5

Ans: 5.1

c) 0.032×0.571

Ans: 0.018272

d) $34.5^2 \times 71.5^3$

Ans: 435067173

e) $0.54^2 \times 6.41$

Ans: 1.869 or 1.87

$f) \sqrt{576} \times 2.77$

Ans: 66.5

g) $9261^{1/3} \times 0.049^{1/2}$

Ans: 4.5

(Q3)Evaluate the following, using or by means of logarithm (a) $\frac{4748}{272}$

Ans:17.5$b)$ (b)

$\frac{2.78}{0.34}$

Ans: 8.2

c) $\frac{271^3}{44.22^2}$

Ans: 10178

a) $\frac{\sqrt{625}}{\sqrt[3]{216}}$

Ans: 4.2

(Q4) Determine the value of each of the following by means of logarithm:

(a) $\frac{271 \times 4.45}{211}$

Ans: 5.72

(b) $\frac{0.88 \times 57.1^2}{3.11^3}$

$Ans: 95$

(c) $\frac{88.3 \times 273}{5.88 \times 20.6}$

Ans: 199

(d) $\frac{4.21^2 \times 22.6}{4.11 \times\ 0.68^3}$

$Ans: 310\ or\ 309.9$

(e) $(\frac{63.14^2 \times 1.22}{3.44}\ )^2$

Ans: 1999038

(Q5) Using logarithm, evaluate the following:

(a) 27^5

Ans: 14348907

(b) 37.1^3

Ans: 51065

(c) $475^{\frac{1}{2}}$

$Ans: 21.8$

(d) $374^{0.47}$

Ans: 16.2

(Q6) Determine the value of x, given that

(a) $2^x = 10.6$

Ans: x = 3.4

(b) $3^x = 19683$

Ans: x = 9

(c) $2^{x+2} = 64$

Ans: x = 4

(d) $3^{2x-1} = 19683$

Ans: x = 5

(e) $2^{x+3} = 4^3$

Ans: x = 3

(Q7) Evaluate the following:

a))Log 1 + log 10^2 – log1000

Ans: -1

b) $Log_2 4 + log_3 9$

Ans: 5

c)$Log_3 (3 \times 9) + 2 log_4 2$

Ans: 4

(Q8) Simplify the following:

(a) Log 37 – log 5 + log 8 – log 2.

Ans: $\log\left(\frac{37\times 8}{5\times 2}\right)$

(b) 2log5 + 3 log 2 – log7.

Ans: $\log\left(\frac{25\times 8}{7}\right)$