CHAPTER FOUR

VARIATION

Introduction:

This shows the relationship which exists between two variables or quantities.
- For example we may choose to consider the relationship between the number of gallons of petrol used by a travelling car and the distance travelled.
- This relationship or variation will be that, the greater the distance travelled, the greater will be the number of gallons of petrol used.

- Variation truly speaking is the same as proportion.

Types:

- Basically there are three types and these are: :

1. Direct variation.

2. Inverse variation.

3. Partial variation.

NB: Associated with each of these is what certain individuals refer to as joint variation

DIRECT VARIATION :

-This may also be referred to as direct proportion:

-Two variables or quantities such as x and y are said to vary directly, or are directly proportional if

1. both x and y increases at the same time e.g

X	2	3	4	5	6	7
Y	4	8	12	16	20	24

2. both values of x and y decrease at the same time. i.e X decrease as y decreases. e.g.

X	100	80	60	40	20
Y	10	8	6	4	2

Now consider two variables x and y. If x varies directly as y, then we write

$x \propto y$, i.e x is directly proportional to y.

From $x \propto y$, in order to remove the proportional sign, we must introduce a constant,

i.e if $x \propto y$, $\Longrightarrow$ x = ky, where k = a constant, which may be referred to as the proportionality constant or the constant of proportionality.

Also if M varies directly as R, then $M \propto R \Longrightarrow M = K.R$, where K is a constant.

Q1. Two variables m and v are such that m varies directly as v. Given the constant as 10, calculate m when v = 20.

Solution

Since m varies directly as v

$\Longrightarrow m \propto v, \Longrightarrow m = kv$

Where k is a constant.

Since k = 10 $\Longrightarrow$ m = 10v.

When v = 20 $\Longrightarrow$ m = (10)(20) = 200.

Q2. The population, P of a nation is directly proportional to the birth rate, R. If the constant is 20, determine the population in million if R = 5.

Solution.

P varies directly as R

$\Longrightarrow p \propto R \Longrightarrow p = KR$,

Where k is a constant. Since k = 20 $\Longrightarrow$ p = 20 R.

When R = 5 $\Longrightarrow$ P = (20) (5) = 100

The population, P = 100 million.

Q3. The velocity v of a car is directly proportional to its mass m squared. Given the constant as 5 and the mass of the car as 60kg, calculate the velocity in m/s.

Solution.

V is directly proportional to m squared $\Longrightarrow v \propto m^2$,

$\Longrightarrow v = km^2$, where k is a constant.

Since $k = 5 \Rightarrow v = 5m.^2$

When $m = 60 \Longrightarrow v = 5(60)^2$,

$\Longrightarrow v = 5\ (3600) = 1800$,

$\Rightarrow V = 18000$ m\s.

Q4. The circumference of a circle varies directly as it radius cubed. Determine the circumference in cm, if the radius is 3cm. Take the proportionality constant to be 4.

Solution

Let c = the circumference and r = the radius.

Since the circumference varies directly as the radius cubed

$\Longrightarrow C \propto r^3 \Longrightarrow c = kr^3$,

where k = the constant. Since r = 3cm and k = 4

$\Longrightarrow c = (4)(3)^3 = (4)(27) = 108.$

The circumference = 108cm.

Q5. The density of a material is directly proportional to the square root of its mass. Given the variation constant as 5 and the mass as 25g, determine the density in g cm^{-3}.

Let d = density and m = mass.

Since the density is directly proportional to the square root of the mass

$\Longrightarrow d \propto \sqrt{m}$,

$\Longrightarrow d = k.\sqrt{m}$, where k = the constant. Since k = 5 and m =25 $\Longrightarrow d = 5.\sqrt{25}$, $\Longrightarrow d = (5)\ (5) = 25$,

$\Longrightarrow d = 25\ cm^{-3}$

Q6. The speed of a vehicle varies directly as it mass. The speed also varies directly

as its length. Find the speed in km/h, if the car is of mass 120kg and it is 50 m long. Take the variation constant to be 10.

Solution.

Let s = speed, m = mass and *l* = length.

1. The speed varies directly as the mass $\Longrightarrow s \propto m$………….(1)

11. Also the speed varies directly as the length $\Longrightarrow s \propto l$.........*(2)*

From 1 and 2 i.e $s \propto m$ and $s \propto l$

$\Longrightarrow s \propto m.l.$ Removing the proportion sign

$\Longrightarrow s = k.m.l.$

Bu since k = 10, m = 120 and l = 50

$\Longrightarrow s = (10)(120)(50) = 60{,}000$

$\Longrightarrow s = 60{,}000$ km/h.

Q7. Three quantities, R, Q and M are such that R varies directly as Q. R also varies directly as M. If the proportionality constant is 20, find R when Q = 1 and m = 7.

Solution

1. R varies directly as Q

$\Longrightarrow R \propto Q$.

11. Also R varies directly as M.

$\Longrightarrow R \propto M$.

From $R \propto Q$ and $R \propto M$

$\Longrightarrow R \propto Q.\ M \Longrightarrow R = KQ.M.$

But since Q = 1, M = 7 and k = 20

$\Longrightarrow R = (20)(1)(7) = 140$,

$\Longrightarrow R = 140$.

Q8. The velocity of a car varies directly as its mass squared. It also varies directly as the length of the car. Given that the car has a length of 10m and a mass of 30kg, calculate its velocity in km/h. Take the proportionality constant as 200.

Solution

Let v = velocity, m = mass and l = length.

1. Since the velocity varies directly as the mass squared

$\Longrightarrow v \propto m^2$.

2. Also since the velocity varies directly as the length

$\Longrightarrow$ V $\propto$ l.

From 1. V $\propto$ m^2 and from 2. V $\propto$ l,=> $v \propto m^2 l \Rightarrow v = km^2 l$.

But m = 30, k = 200 and l = 10 .

From $v = km^2l \Longrightarrow v = (200)(30)^2(10)$,

$\Longrightarrow v = (200)(900)(10)$,

$\Longrightarrow$ v = 18 000 00 km/h.

Q9. The quantities Q, L and Z are such that L varies directly as Q cubed as well as Z. Given that L = 160 when Q = 2 and Z = 5, calculate the variation constant.

Solution

1. L varies directly as Q cubed $\Longrightarrow L \propto Q^3$.

11. L varies directly as Z $\Longrightarrow L \propto Z$.

Now from $L \propto Q^3$ and $L \propto Z$

$\Longrightarrow L \propto Q^3. Z, \Longrightarrow L = K.Q^3. Z.$

But since L= 160, when Q = 2 and Z = 5,$\Longrightarrow$ L= k $(2)^3(5)$,

=> 160 = K(8)(5) $\Longrightarrow$160 = 40K,

$\Longrightarrow$ K = 160/40 = 4.

The constant = 4.

10. The distance d covered by a vehicle varies directly as its length, *l* cubed as well as the square root of its mass m. Given that when m = 9kg,*l* = 2m and d = 120km,

1. determine the value of the constant.

11. deduce an expression for d in terms of m, l and k, where k is the constant

111. determine the distance travelled by the car in kilometers, when it is 40m long and has a mass of 60kg.

Solution

1.Since d varies directly as l cubed $\Longrightarrow d \propto l^3$

11. Since d also varies directly as the square root of m

$\Longrightarrow$ d $\propto \sqrt{m}$.

Now since d $\propto l^3$ and d $\propto \sqrt{m}$,

$\Longrightarrow$ d $\propto l^3.\sqrt{m}$ => d = k.$l^3.\sqrt{m}$..

But d = 120 when l = 2 and m = 9, and since d = k $l^3\sqrt{m}$,

$\Longrightarrow$120 = k$(2)^3(\sqrt{9})$ = k(8)(3) = 24k,

$\Longrightarrow$120 = 24k $\Longrightarrow$ k = $^{120}/_{24}$ = 5.

$\therefore$ K = 5.

11. We have already noted that d = $kl^3\sqrt{m}$.

To deduce an expression for d in terms of k, l and , m, we only substitute the value of the constant (i.e. 5) into the just written expression.

From d = $kl^3\sqrt{m}$ $\Longrightarrow$d = $5l^3\sqrt{m}$.

111. When l = 40m and m = 60kg,

$\Longrightarrow$ d = 5 $l^3\sqrt{m}$ $\Longrightarrow$d = $5(40)^3\sqrt{60}$,

$\Longrightarrow$ d = 5 (64000) (7.7),

$\Longrightarrow$ d = 2,464 000.

Q12. The period T of a pendulum is directly proportional to the cube root of the acceleration due to gravity ′g′. T also varies directly as the length l of the pendulum. Give that T = 120, when g = 27 and l = 10,

1.deduce an expression for T in terms of g and l, as well as the constant.

11. find T when g = 64m s^{-2} and l = 30m.

111. deduce an expression for l in terms of T, g and the constant.

Solution

Since T varies directly as the cube root of g $\Longrightarrow$ T $\propto \sqrt[3]{g}$.

11. Since also T varies directly as the length $\Longrightarrow$ T $\propto$ l.

Now since T $\propto \sqrt[3]{g}$ and T $\propto$ l,

$\Longrightarrow T \propto \sqrt[3]{g}\ l, \Longrightarrow T = k.\sqrt[3]{g}.l,$

where k = the constant.

But T =120, when g = 27 and l =10.

Since $T= k\sqrt[3]{g}.l$,

$\Longrightarrow 120 = (k)(\sqrt[3]{27})(10),$

$\Longrightarrow 120 = k(3)(10) \Longrightarrow 120 = 30k,$

$\Longrightarrow k = 120/30 \Longrightarrow k = 4.$

1. As already seen or noted, $T= k\sqrt[3]{g}.l.$ and substituting k = 4,

=> $T= 4\sqrt[3]{g}.l.$ which is an expression for T, in terms of g and l.

11. $T = 4\sqrt[3]{g}\ l$. When g = 64 and l = 30, => $T = (4)(\sqrt[3]{64})(30)$,

$\Longrightarrow T = (4)(4)(30) \Longrightarrow T = 480.$

111. To deduce an expression for l in terms of T, g and k, we first consider the equation $T= 4\sqrt[3]{g}.l$ and make l the subject.

From $T = 4\sqrt[3]{g}l$, divide through using $4\sqrt[3]{g}$

ie. $\dfrac{T}{4\sqrt[3]{g}} = \dfrac{4\sqrt[3]{g}\ l,}{4\sqrt[3]{g}}$

$\Longrightarrow l = \dfrac{T}{4\sqrt[3]{g}},$

which is an expression for l in terms of T and g.

13. The quantities d, w, l and T are such that d varies directly as w squared. Also d varies directly as l as well as the square root of T. Given that d = 32 when w = 2, l = 1 and T= 4,

a. deduce an expression for d in terms of k, w, l and T, where k is the constant.

b. calculate d when w = 1, l = 10 and T = 25.

c. deduce an expression for l in terms of the constant, T and w.

d. calculate l when d =20, T=25 and w =1 *solution*

i. Since d varies directly as w squared $\Longrightarrow$ d $\propto w^2$.

ii. Since d varies directly as l $\Longrightarrow$ d $\propto$ l.

iii. Since d also varies directly as the square root of T $\Longrightarrow$ d $\propto \sqrt{T}$.

Now from d $\propto w^2$, d $\propto$ l and d $\propto \sqrt{T}$ $\Longrightarrow$d $\propto w^2.l.\sqrt{T}$,

$\Longrightarrow$ d = k w^2 l $\sqrt{T}$. We then determine the value of the constant .

Now d = 32, when w = 2, l = 1 and T = 4.

Since d = kw^2 l $\sqrt{T}$

$\Longrightarrow$32= $K(2)^2(1)(\sqrt{4})$,

$\Longrightarrow$32 = k(4)(1)(2),

$\Longrightarrow$32 = 8k $\Longrightarrow$ k= 32/8,

$\Longrightarrow$ k = 4.

a. For an expression for d, d = $kw^2l\sqrt{T}$ $\Longrightarrow$ d = $4w^2$ l $\sqrt{T}$.

b. To calculate d when w = 1, l = 10 and T = 25,

$\Longrightarrow$ d = $4(1)^2(10)(\sqrt{25})$,

$\Longrightarrow$ d = 4(1)(10)(5) =200.

c. To deduce an expression for l, we must first consider the equation d = kw^2 l $\sqrt{T}$ and make l the subject.

From d = kw^2 l. $\sqrt{T}$, divide through using $kw^2\sqrt{T}$

$$=> \frac{d}{kw^2\sqrt{T}} = \frac{kw^2 l\sqrt{T}}{kw^2\sqrt{T}}$$

$$=> \frac{d}{kw^2\sqrt{T}} = I, \Rightarrow I = \frac{d}{kw^2\sqrt{T}}$$

Substituting k = 4

$\Rightarrow l = \dfrac{d}{4w^2\sqrt{T}}$ which is an expression for l in terms of k, w and T.

d. When d =20, T = 25 and w = 1,

$\Rightarrow l = \dfrac{d}{4w^2\sqrt{T}} \Rightarrow l = \dfrac{20}{4(1)^2(\sqrt{25})}$

$= \dfrac{20}{4(1)(5)} = \dfrac{20}{20} = 1,$

$\Rightarrow l = 1$..

Inverse variation:

This may also be referred to as inverse proportion. Two variables such as x and y exhibit or show this type of variation, when as one variable increases, the other decreases.

Examples.

(1)

X	100	90	80	70	60	50
Y	1	2	3	4	5	6

(2)

X	200	300	400	500	600
Y	20	19	18	17	16

If x varies inversely as y, we write $x \propto 1/y$.

In or In order to remove the proportional sign, we introduce a constant.

From i.e $x \propto 1/y \Rightarrow x = k.1/y$, =>

=> $x = k/y$, where k is a constant.

Also if M varies inversely as Q, then $M \propto \frac{1}{Q}, \Rightarrow M = K.\frac{1}{Q}$

$\Rightarrow M = K/Q$, where k is a constant.

Q1 The velocity V of a car varies inversely as its mass m. Calculate the velocity in m/s, given that the mass of the car is 60kg and the variation constant is 240.

Solution.

Since v varies inversely as m,

$\Rightarrow$ v $\propto$ 1/m , $\Rightarrow$ V = k. 1/m $\Rightarrow$ V = k/m.

If If k = 240 and m = 60

$\Rightarrow$ v = $\frac{240}{60}$ = 4 $\Rightarrow$ V = 4m/s .

Q2. The rate of diffusion r of oxygen is inversely proportional to the square root of the mass of oxygen present. If the mass of oxygen present is 36g and the constant of variation is 10, find the rate.

Solution.

Since the rate of diffusion varies inversely as the square root of the mass

$\Rightarrow r \propto \frac{1}{\sqrt{m}} \Rightarrow r = \frac{k.1}{\sqrt{m}}$,

$\Rightarrow r = \frac{k}{\sqrt{m}}$. Since m = 36 and k = 10, then $r = \frac{10}{\sqrt{36}} = \frac{10}{6} = 1.66$,

$\Rightarrow$ r = 1.66.

Q3 The radius of a circle varies inversely as its diameter squared.

Given that the radius was 4 cm when the diameter was 2 cm,

i. determine the value of the constant.

ii deduce an expression for the radius in terms of the constant and diameter.

Solution

Let r = radius and d = diameter.

Since the radius varies inversely as the diameter squared $\Rightarrow r \propto \frac{1}{d^2}$,

==> $r = \frac{k.1}{d^2}, \Rightarrow r = \frac{k}{d^2}$..

If r = 4cm and d =2cm, =>

$\Rightarrow 4 = \frac{k}{2^2} \Rightarrow 4 = \frac{k}{4}, \Rightarrow k = 4 \times 4 = 16.$

11. T To get the required expression, substitute k = 16 into $r = \frac{k}{d^2}$

$\Longrightarrow$ => r = $^{16}/_{d^2}$.

Q4. The mass of a body m varies inversely as its density d. The mass also varies inversely as the radius, r. Given that the constant is 60, calculate the mass in grams when the radius is 2cm and the density is 3g cm/3.

Solution

Since m varies inversely as d

$\Longrightarrow$ m => m $\propto \frac{1}{d}$.

2. Also since m varies inversely as the radius $\Longrightarrow$ m $\propto$ 1/r

Since m $\propto \frac{1}{d}$ and m $\propto \frac{1}{r}$, then

m $\propto \frac{1}{d} \cdot \frac{1}{r} \Longrightarrow$ m $\propto \frac{1}{dr} \Longrightarrow$ m = $\frac{k.1}{dr}$

$\Longrightarrow$ m = $\frac{k}{dr}$. But since k = 60, r = 2 and d = 3,

then m = $\frac{60}{(3)(2)} = \frac{60}{6} = 10$

$\Longrightarrow$ m = 10g.

Q5. The quantities P, Q and R are such that P varies inversely as Q squared.

P also varies inversely as R. Calculate P when Q = 2, R = 10 and the variation constant = 400.

Solution

1. Since P varies inversely as Q squared $\Longrightarrow$ P $\propto \frac{1}{Q^2}$.

11. P also varies inversely as R

$\Longrightarrow$ p $\propto \frac{1}{R}$.

Siince p $\propto \frac{1}{Q^2}$ and p $\propto \frac{1}{R}$, then $P \propto \frac{1}{Q^2} \cdot \frac{1}{R} \Rightarrow P \propto \frac{1}{Q^2 R}$,

$\Longrightarrow$ $P = \frac{K \cdot 1}{Q^2 R}$ $\Longrightarrow$ $P = \frac{K}{Q^2 R}$

When Q = 2, R = 10 and k = 400

$\Rightarrow p = \frac{400}{(2)^2(10)} = \frac{400}{4x10} = \frac{400}{40}$

$\Rightarrow p = 10$..

Q6. The time T of a pendulum to complete one oscillation varies inversely as m, the mass of the bob. T was also found to vary inversely as the square root of the length l. Given the mass of the bob as 2g and the length of the pendulum to be equal to 25 cm when T was 80 seconds,

a) .i. determine the constant.

b. ii. calculate T when m = 4g and l = 9cm.

Solution.

(a) .1. Since T varies inversely as m

=> $T \propto \frac{1}{m}$.

11. Also T varies inversely as the square root of l

=> $T \propto \frac{1}{\sqrt{l}}$

From $T \propto \frac{1}{m}$ and $T \propto \frac{1}{\sqrt{l}}$ =>$T \propto \frac{1}{m\sqrt{l}}$,

$\Rightarrow T = \frac{K}{m.\sqrt{L}}$

When m = 2, T = 80 and l = 25 => $T = \frac{k}{(2)\sqrt{25)}}$

$\Rightarrow 80 = \frac{k}{(2)(5)}$ $\Rightarrow 80 = \frac{K}{10}$,

$\Rightarrow k = 80 \times 10 = 800$.

b. If Since m = 4g and l = 9,

=> $T = \frac{800}{(4)(\sqrt{9})}$

=>$T = \frac{800}{(4)(3)}$ $\Rightarrow T = \frac{800}{12} = 6.7$.

T = 6.7 sec.

Q7 The variables P, Q, R and Z are such that P varies inversely as R as well as the cube root of Z. Calculate p when Q = 2, R = 1 and Z = 27.Take the constant to be 60.

Solution

(i) P varies inversely as Q

$\Rightarrow p \propto \frac{1}{Q}$.

(ii) P varies inversely as R

$\Rightarrow P \propto \frac{1}{R}$.

(iii) . Lastly P varies inversely as the cube root of Z =>

$P \propto \frac{1}{\sqrt[3]{z}}$.

Si $P \propto \frac{1}{Q}$, $p \propto \frac{1}{R}$ and $p \propto \frac{1}{\sqrt[3]{Z}}$ $\Rightarrow P \propto \frac{1}{Q} \cdot \frac{1}{R} \cdot \frac{1}{\sqrt[3]{Z}} \Rightarrow P \propto \frac{1}{Q.R.\sqrt[3]{Z}}$

$\Rightarrow p = \frac{k.1}{Q\,R.\sqrt[3]{Z}}$. => $p = \frac{K}{Q\,R.\sqrt[3]{Z}}$

But since when Q = 2, R=1, Z =27 and k = 60,

$\Rightarrow p = \frac{60}{(2)(1)(\sqrt[3]{27})} =$

$\frac{60}{(2)(1)(3)} = \frac{60}{6} = 10,$

$\Rightarrow P = 10$..

Q8. The variable p varies inversely as the square of (Q + 1), and p = 2 when Q = 3,

a. Write the expression connecting p and Q.

b. Find the possible values of Q when p = 8.

Solution.

(a) Since P varies inversely as the square of (Q + 1) $\Longrightarrow$ p $\propto \frac{1}{(Q+1)^2}$

=> P = $\frac{K.1}{(Q+1)^2}$ $\Longrightarrow$ p $= \frac{k}{(Q+1)^2}$

But since p = 2 when Q =3,

=> 2 = $\frac{k}{(3+1)^2}$ => 2 = $\frac{k}{4^2}$

=> 2 = k/16 => k = 2 x 16 = 32

Where k = the constant .

The expression connecting

P and Q is p = $\frac{k}{(Q+1)^2}$

=> P = $\frac{32}{(Q+1)^2}$

b. If p = 8 => 8 = $\frac{32}{(Q+1)^2}$

=> 8 x $(Q+1)^2$ = 32,

=> 8 $(Q+1)^2$ = 32, => $\frac{8\,(Q+1)^2}{8} = \frac{32}{8}$

i.e divide through using 8

=> $(Q+1)^2 = 4$

Find the square root of both sides => $\sqrt{(Q+1)^2} = \sqrt{4}$,

=> Q + 1 = $\sqrt{4}$, but since $\sqrt{4}$ = 2 or - 2

=> Q + 1 = 2 => Q = 2-1 =1,

or Q + 1= -2 => Q = -2 - 1 = - 3

=> The two possible values of Q are 1 and -3

Q9. If y is inversely proportional to x+ 2 and y = 48 when x = 10, , find x when y = 30.

Solution

Since y is inversely proportional to x+2 , then y $\propto\frac{1}{x+2}$ => y = k. $\frac{1}{x+2}$, $\Rightarrow$ $y = \frac{k}{x+2}$

But since when y = 48, x =10

=> 48 = $\frac{k}{10+2}$ => 48 (10+2) = k

(i.e cross multiply.)

=> 48(12) = k, => 48 x12 = k,

=> k = 576.

The formula y = $\frac{k}{x+2}$ therefore

becomes y = $\frac{576}{x+2}$

When y =30 => 30 = $\frac{576}{x+2}$

=> 30(x+2) = 576, =>

30 x + 60 = 576,

$\Longrightarrow$30 x = 576 – 60 = 516, ==>

30 x = 516 $\Longrightarrow$ x = $\frac{516}{30}$,

$\Longrightarrow$ x = 17.2

Q10. R varies inversely as the cube of S. When R = 9, S = 2. Find S when R = $\frac{243}{64}$.

Solution

Since R varies inversely as the cube of S $\Longrightarrow$ R $\propto \frac{1}{S^3}$, $\Longrightarrow$ R = k.$\frac{1}{S^3}$,

$\Longrightarrow$ R = $\underline{K}$. Since when R = 9, S = 2

5^3

$\Rightarrow 9 = \frac{K}{2^3}$ => $9 = \frac{K}{8}$

$\Rightarrow 9 \times 8 = k \Rightarrow k = 72.$

Since $R = \frac{k}{S^3}$ $\Rightarrow R = \frac{72}{S^3}$

But it is given that

$R = \frac{243}{64} = 3.7$. From $R = \frac{72}{S^3}$ => $3.7 = \frac{72}{S^3}$, => $3.7S^3 = 72$ =>

$S^3 = \frac{72}{3.7,}$ => $S^3 = 19.9$ => $S = \sqrt[3]{19.5}$

=> S = 2.7.

Combination of direct and inverse variation:

It is possible to get or come across questions associated with a combination of both direct and inverse variation, and the mode of solving such questions is almost the same, as what we have already learnt.

Q1. The speed of a car in km/h varies directly as its length . It also varies inversely as its mass. Find the speed if the car is 20m long and has a mass of 5kg. Take the constant = 10.

Solution.

Let s = the speed of the car,

L = length of the car and m= the mass of the car.

1). Since the speed varies directly as the length

$\Rightarrow s \propto l$

2). Since the speed varies inversely as the mass

$\Rightarrow s \propto \frac{1}{m}$. From $s \propto l$ and $s \propto \frac{1}{m}$, $\Rightarrow s \propto l .. \frac{1}{m}$, $\Rightarrow s \propto \frac{l}{m}$

$\Rightarrow s = \frac{k.\,l}{m}$ => $S = \frac{kl}{m}$,

where k is the constant. But since when

$k = 10$, $l = 20m$ and $m = 5kg$

$\Rightarrow s = \frac{(10)(20)}{5}$ $\Rightarrow s = 40km/h$

Q2. The temperature of a moving body varies directly as its velocity squared.

The temperature also varies inversely as its length. Given the constant of variation

to be 5 and the velocity to be 20m/s, determine the temperature in degrees celsius if the length of the body is 50m.

Solution

Let V = the velocity. L = the length and k = the constant.

1). Since the temperature varies directly as the velocity squared

$\Rightarrow T \propto V^2$.

2).Also since the temperature also varies inversely as the length

$\Rightarrow T \propto \frac{1}{L}$

Now since $T \propto V^2$ and $T \propto \frac{1}{L}$

$\Rightarrow T \propto V^2 . \frac{1}{L}$ $\Rightarrow T \propto \frac{V^2}{L}$

$\Rightarrow T = \frac{kv^2}{L}$. Since when $k = 5$, $v = 20m/s$ and $l = 50m$, then $T = \frac{5\,(20)^2}{50} = \frac{5\,(400)}{50}$

$= 40^{\circ}c$

Q3. The resistance to the flow of current r varies directly as the square root of the current c. It also varies inversely as the diameter d, of the wire cubed. Given that when $r = 18\Omega$, $c = 9A$ and $d = 1cm$, calculate r when $c = 25A$ and $d = 20cm$.

Solution

1).Since r varies directly as the square root of c $\Rightarrow r \propto \sqrt{c}$.

11) Since. r also varies inversely as the diameter cubed $\Rightarrow r \propto \frac{1}{}$.

d^3

Since $r \propto \sqrt{c}$ and $r \propto \frac{1}{d^3}$ $\Rightarrow r \propto \sqrt{c} \cdot \frac{1}{d^3}$, $\Rightarrow r \propto \frac{\sqrt{c}}{d^3}$, $\Rightarrow r = \frac{k.\sqrt{c}}{d^3}$, .

where k = the constant.

NB. Since the value of the constant is not given, we first have to find it.

Now $r = \frac{k.\sqrt{c}}{d^3}$, and since when r = 18, c = 9 and d = 1 $\Rightarrow 18 = \frac{k\sqrt{9}}{1^3}$, =>

$18 = \frac{k(3)}{1^3}$ $\Rightarrow 18 = \frac{3k}{1}$, (since $1^3 = 1 \times 1 \times 1 = 1$).

By cross multiplication $18 \times 1 = 3k \Rightarrow 3k = 18, \Rightarrow k = 18/3$

=> k = 6..

The formula $r = \frac{k\sqrt{c}}{d^3}$ becomes $r = \frac{6\sqrt{c}}{d^3}$

11). When c = 25A and d = 2cm, then, $r = \frac{(6)(\sqrt{25})}{2^3} = \frac{(6)(5)}{8} = \frac{30}{8}$

$\Rightarrow r = 3.7$.

Q4. Four quantities P,Q, R and Z are such that P varies directly as Q squared, as well as R. P also varies inversely as the square root of z . Given that P = 20 when Q = 2, R= 1 and z = 4, calculate P when Q = 5, R = 2 and Z = 25.

Solution

(i)Since. p varies directly as Q squared

$\Rightarrow$. $P \propto Q^2$.

(ii).Since P varies directly as R

$\Rightarrow P \propto R$.

(iii) Since P also varies inversely as the square root of z $\Longrightarrow$ p $\propto \frac{1}{\sqrt{z}}$.

Now since P $\propto Q^2$, $p \propto$ R and P $\propto \frac{1}{\sqrt{Z}}$, ,

$$\Longrightarrow p \propto Q^2 . R. \frac{1,}{\sqrt{z}} \quad \Longrightarrow p \propto \frac{Q^2 R}{\sqrt{z}}$$

$$\therefore P = \frac{K Q^2 R}{\sqrt{z}}$$, where k = a constant.

Determine the value of the constant first. But P= 20, when Q = 2, R = 1 and z = 4. From p = $\frac{k\,Q^2 R}{\sqrt{Z}}$

$$\Longrightarrow 20 = \frac{k(2)^2 (1)}{\sqrt{4}} \quad \Longrightarrow 20 = \frac{4k}{2}$$

$$\Longrightarrow 4k = 2(20) \Rightarrow k = \frac{40}{4} \quad \Longrightarrow k=10.$$

But since $P = \frac{KQ^2R}{\sqrt{z}} \Rightarrow P = \frac{10\,Q^2 R}{\sqrt{z}} \Longrightarrow P = \frac{10\,(5)^2(2)}{\sqrt{25}}$

$$\Rightarrow P = \frac{10(25)(2)}{5} = 100.$$

Q5 Three quantities P,Q and R are connected such that P varies directly as R and inversely as the square root of Q . If p = 6 when R = 12 and Q = 25, find

i). an expression for P in terms of Q,R and the constant.

ii). the value of P when Q = 16 and R= 40.

Solution

i)1. P varies directly as R

$\Longrightarrow$ P $\propto$ R

11. P also varies inversely as the square root of Q =>

$$P \propto \frac{1}{\sqrt{Q}}$$

Now since $p \propto R$ and $p \propto \frac{1}{\sqrt{Q}}$ $\Rightarrow$ $P \propto R.\frac{1}{\sqrt{Q}}$ $\Rightarrow P \propto \frac{R}{\sqrt{Q}}$

$\Rightarrow P = \frac{KR}{\sqrt{Q}}$, but since P = 6 when R = 12 and Q = 25,

$\Rightarrow 6 = \frac{k\,(12)}{\sqrt{25}}$ $\Rightarrow 6 = \frac{12k}{5}$,

$\Rightarrow$ 12k = 5 x 6 = 30.

12k =30 $\Rightarrow k = \frac{30}{12}$

$\Rightarrow$ k = 2.5.

. To get the expression for P in terms of Q, R and the constant, substitute k = 2.5 into the formula $P = \frac{KR}{\sqrt{Q}} \Rightarrow$

$P = \frac{2.5R}{\sqrt{Q}}$

ii).When R = 40 and Q = 16 $\Rightarrow P = \frac{2.5\,(40)}{\sqrt{16}}$, $\Rightarrow P = \frac{2.5\,(40)}{4} = 25.$

Q6. The quantities R,P, Z,W and Q are such that R varies directly as P as well as Z

Squared. R also varies inversely as Q cubed as well as W. Given that when

R = 4, P = 2, Z = 8, Q = 4 and W = 5,

a). determine the value of the variation constant.

b). deduce an expression for R in terms of the constant, P, Z, Q and W.

c). find R when p = 4, z = 2, Q = 2 and W = 1.

Solution

a)i. R varies directly as P

$\Rightarrow R \propto P$.

ii. R also varies directly as Z squared $\Rightarrow R \propto Z^2$

iii. R varies inversely as w

$\Longrightarrow R \propto \frac{1}{W}$ iv. R varies inversely as Q cubed $\Longrightarrow R \propto \frac{1}{Q^3}$

Now $R \propto P$, $R \propto Z^2$. $R \propto \frac{1}{Q^3}$ and $R \propto \frac{1}{W}$ $\Longrightarrow R \propto P.Z^2.\ \frac{1}{Q^3}.\ \frac{1}{W}$

$\Longrightarrow R \propto \frac{P.\ Z^2}{Q^3.\ W}$

$\Longrightarrow R = \frac{K\ P\ Z^2}{Q^3.\ W}$.

b).Since when R = 4, P = 2, Z = 8, Q = 4 and W = 5,

$\Longrightarrow 4 = \frac{K(2)(8)^2}{(4)^3(5)} \Longrightarrow 4 = \frac{128K}{(64)(5)}$

$\Longrightarrow 4 = \frac{128k}{320}$

$\therefore 4 \times 320 = 128k \Longrightarrow k = \frac{4 \times 320}{128}$

$\Longrightarrow k = 10.$

c). $R = \frac{KPZ^2}{Q^3W}$ => $R = \frac{10PZ^2}{Q^3W}$

When P = 4, Z = 2, Q = 2, and W = 1 $\Longrightarrow R = \frac{10p\ z^2}{Q^3W}$

$= \frac{10(4)(2)^2}{(2)^3\ (1)} = \frac{10(4)(4)}{8} = 20$

$\Longrightarrow R = 20.$

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Q8.

W	R	Q
3	4	4
W2	1	2
8	r_3	6

In the table $W \propto \frac{Q}{R^2}$ where W, R and Q are positive integers. Solve for W_2 and r_3.

Solution

$W \propto \frac{Q}{R^2} \Rightarrow W = K\frac{Q}{R^2}$,

where k = the constant. From the table, when W = 3, R = 4 and Q = 4 .

Substitute these values into

$W = \frac{KQ}{R^2} \Rightarrow 3 = \frac{K(4)}{4^2} \Rightarrow 3 = \frac{4K}{16}$

$\Rightarrow 4K = 3 \times 16 =>$

$4K = 48 => K = \frac{48}{4} = 12.$

Since $W = \frac{KQ}{R^2} \Rightarrow W = \frac{12Q}{R^2}$

From the table also, when W = W_2, R = 1 and Q = 2.

Substituting these values into the formula $\Rightarrow W2 = \frac{12(2)}{(1)^2}$

$\Rightarrow W2 = \frac{24}{1} \Rightarrow W2 = 24.$

From the table when w = 8 R = r_3 and Q = 6.

Substitute these values into

$W = \frac{12\,Q}{R^2} \Rightarrow 8 = \frac{12\,(6)}{r_{3^2}} \Longrightarrow 8 = \frac{72}{r_{3^2}} \Rightarrow 8 \times r_{3^2} = 72, \Longrightarrow r_{3^2} = \frac{72}{8}, \Longrightarrow r_{3^2} = 9.$

$\Rightarrow r_3 = \sqrt{9} \Longrightarrow r_3 = 3.$

Questions:

Q1. The weight of a body varies directly as it's mass. It also varies directly as its length. If the constant of variation is 20, find the weight when the body has a mass of 10g, and a length of 5m.

Ans 1000kg

Q2. The speed of a space rocket is directly proportional to the square root of it's length as well as it's weight. Given the constant as 4, find the speed in m/s if the length of the rocket is 16m and has a weight of 20kg.

Ans 320 m/s

Q3 The quantities x,y and z are such that x varies directly as y squared, as well as the square root of z . When y = 2, z = 25 and x = 100.

1. Express x in terms of y and z and the constant

Ans. $X = 5y^2 \sqrt{z}$

11. Find x when y = 3 and z =9 Ans. 135

Q4. The variables R,Q,M and Z are such that R varies directly as Q as well as M raised to the third power. R also varies inversely as Z squared. Taking the constant to be 10, find R when Q = 2, M= 3 and Z = 6. Ans. 15.

Q5. The perimeter P of a certain figure varies directly as its length L squared as well it's breadth B. The perimeter also varies inversely as the weight W as well as the density D. When L = 2m, B = 6m, W = 1kg,D = $4kgm^{-3}$ and P = 120.

1. Determine the variation constant Ans. 20

11.Deduce an expression for P in terms of L,B,W, D, and the variation constant.

Ans $P = \frac{20 L^2 B}{WD}$

111. Find P when L= 6, B = 18, W = 9 and D = 3

Ans 480

Q6. The rate of diffusion r of a gas varies directly as π (pie) as well as its length squared. The rate was also detected to vary inversely as the square root of the temperature T as well as the quantity of gas present Q. Determine the rate of diffusion in cm^3/s if the constant is 2, π = 3.14, L = 5, T= 81 and Q = 2.

Ans $8.7cm^3/s$

Q7. The variable m varies inversely as V - 3. When V = 13, m = 2. Find m when V= 8.

Ans ; 4