CHAPTER SEVEN

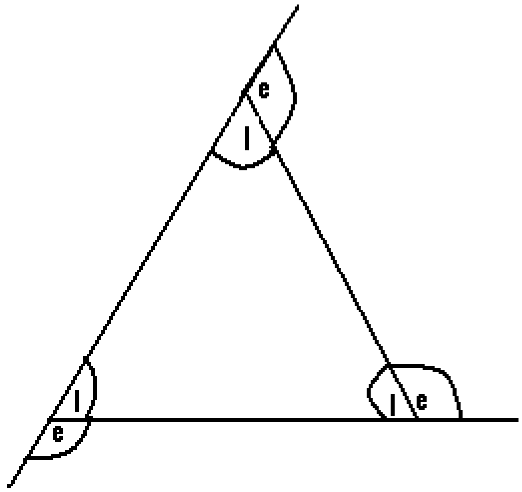
POLYGONS

Definition:

A polygon is a plane figure which is bounded by straight lines.

Polygons	
Number of sides	Name
3	triangle
4	quadrilateral
5	pentagon
6	hexagon
7	heptagon
8	octagon
9	nonagon
10	decagon

- A polygon has both interior as well as exterior angles.
- The interior angles of a polygon are those angles which lie within the polygon.
- The exterior angles of a polygon lie outside the polygon.



I = interior angle.

e = exterior angle.

N/B: For any polygon, the sum of the exterior angles = 360^0.

Q1. Calculate the value of each exterior angle of a regular decagon.

<u>Soln.</u>

Decagon has 10 sides and as such 10 exterior angles.

But the sum of the exterior angles of any polygon = 360^0.

$\Rightarrow$ 10 exterior angles = 360^0.

$\therefore$ 1 exterior angle = $\frac{1}{10} \times 360$

$= 36^0$.

$\Rightarrow$ each exterior angle of a decagon = 36^0.

Q2. Find the exterior angle of a regular pentagon.

<u>Soln.</u>

Pentagon has 5 sides, and as such 5 exterior angles. But the sum of the exterior angles of a polygon = 360^0

$\Rightarrow$ 5 exterior angles = 360

$\Rightarrow$ 1 exterior angle = $\frac{1}{5} \times 360$

$= 72^0$.

$\therefore$ Each exterior angle of the regular pentagon =72^0. For any polygon, the sum of the exterior angle and the exterior angle at any of its vertices = 180^0.

Determination of the interior angle of a regular polygon:

- We must first determine the value of the exterior angle.

- Using the fact that at any vertex, exterior angle + interior angle = 180^0.

$\Rightarrow$ interior angle = 180^0 - exterior angle.

Q1. Calculate the interior angles of a regular decagon.

Soln.

Decagon has 10 exterior angles

$\Rightarrow$ 10 exterior angles = 360^0.

$\therefore$ 1 exterior angle = $\frac{1}{10} \times 360$

= 36^0.

But at any vertex, exterior angle + interior angle = 180^0.

$\Rightarrow 36^0$ + interior angle = 180^0.

Interior angle =$180^0 - 36^0 = 144^0$.

The interior angle of the decagon = 144^0.

Q2. Find the value of each Interior angle of a triangle.

Soln.

A triangle has 3 sides and as such 3 exterior angles.

$\Rightarrow$ 3 exterior angles = 360^0

$\therefore$ 1 exterior angle = $\frac{1}{3} \times 360$

= 120^0.

But at any vertex, interior angle + exterior angle = 180^0

$\Rightarrow$ Interior angle + 120^0 = 180^0

$\therefore$ Interior angle = 60^0

Determination of the sum or the total interior angles of a polygon:

For any polygon, the sum of the interior angles = the number of sides of the polygon $\times$ the value of one interior angle.

Q1. Calculate the sum of the interior angles of a regular decagon.

Soln.

Decagon has 10 exterior angles

$\Rightarrow$ 10 exterior angles = 360^0

$\therefore$ 1 exterior angle = $\frac{1}{10} \times 360^0$

= 36^0.

But at any vertex, interior angle + exterior angle = 180^0

$\Rightarrow$ Interior angle + 36^0 = 180^0

$\Rightarrow$ Interior angle = 180 – 36

$\Rightarrow$ Interior angle = 144^0.

But the sum of the interior angles of a decagon = interior angle $\times$ the number of sides.

$\therefore$ Sum of interior angles of the decagon = $144^0 \times 10 = 1440^0$.

Q2. Find the sum of the interior angles of a regular octagon.

Soln.

Octagon has eight sides and as such eight exterior angles.

$\Rightarrow$ 8 exterior angles = 360^0

$\therefore$ 1 exterior angle = $\frac{1}{8} \times 360^0$

$= 45^0$.

But at any vertex, exterior angle + interior angle = 180^0

$\therefore 45^0$ + interior angle = 180^0

$\Rightarrow$ Interior angle = 180 – 45 = 135^0.

But the sum of interior angle = the number of sides of the polygon $\times$ interior angle = 8 $\times$ 135^0= 1080^0.

Q3.The interior angles of a regular triangle are marked $20^0 + 2x^0, 10^0 + 5x^0$and $40^0 + 4x^0$. Find the actual values of each of these angles.

N/B: First calculate the sum of the interior angles of the triangle.

<u>Soln.</u>

Triangle has 3 exterior angles

$\Rightarrow$ 3 exterior angles = 360^0

$\therefore$ 1 exterior angle = $\frac{1}{3} \times 360^0$

$= 120^0$.

But at any vertex, exterior angle + interior angle = 180^0

$\Rightarrow 120^0$ + interior angle = 180^0

$\Rightarrow$ Interior angle = 180^0 - 120^0= 60^0.

But the sum of the interior angles of the triangle = the number of sides $\times$ interior angle = 3 $\times$ 60 = 180^0.

But the interior angles of the triangle are given as $20^0 + 2x^0, 10^0 + 5x^0$and $40^0 + 4x^0$.The sum of these interior angles = $20^0 + 2x^0$+$10^0 + 5x^0$+ $40^0 + 4x^0$

= $20^0 + 10^0 + 40^0 + 2x^0 + 5x^0 + 4x^0 = 70^0 + 11x$.

But the sum of the interior angles of the polygon or triangle = 180^0

$\Rightarrow 70 + 11x = 180^0$

$\Rightarrow 11x = 180^0 - 70 = 110^0$

$\Rightarrow x = \frac{110}{11} = 10^0$.

$\therefore$ The angle marked $20^0 + 2x = 20 + 2(10) =$ $20^0 + 20^0 = 40^0$.

The angle marked $10^0 + 5x^0 = 10^0 + 50(10) = 10 + 50^0 = 60^0$.

Lastly, the angle marked $40^0 + 4x^0 = 40 + 4(10) = 40 + 40 = 80^0$.

Q4. The angles of a pentagon are marked $x^0, (x^0 + 20^0), (x^0 + 25^0), 2x^0$ *and* $(2x^0 + 5)$.

(a) Find the value of x.
(b) Determine the value of each of those angles.

<u>Soln.</u>

Pentagon has 5 exterior angles.

5 exterior angles = 360^0

$\therefore$ 1 exterior angle = $\frac{1}{5} \times 360$

= 72^0.

But at any vertex, exterior angle + interior angle = 180^0

$\Rightarrow 72^0 + interior\ angle = 180^0$

$\Rightarrow$ interior angle = $180 - 72 = 108^0$.

Sum of the interior angles of the pentagon = number of sides $\times$ interior angle

= $5 \times 108 = 540^0$.

The given angles which are $x^0,\ x + 20^0,\ x + 25^0, 2x$ *and* $2x + 5^0$ are the interior angles of the pentagon.

Sum of these interior angles = $x^0 + \ x + 20^0 + \ x + 25^0 + \ 2x + \ 2x + 5^0$

= 7x + 50.

Since the sum of the interior angles of the pentagon has been calculated to be equal to $540^0 \Rightarrow 7x + 50 = 540^0 \Rightarrow 7x = 540 - 50 \Rightarrow 7x = 490$

$\Rightarrow x = \frac{490}{7} = 70$, $\therefore$ x = 70^0.

The value of the angle marked $x^0 = 70^0$.

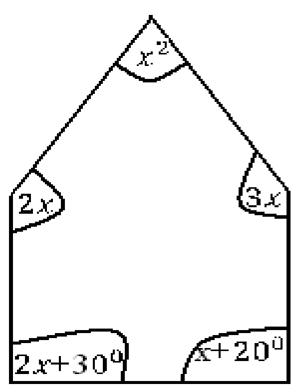
The value of the one marked x + $20^0 = 70 + 20 = 90^0$.

The angle marked x + 25 = 70 + 25 = 95^0.

The angle marked 2x = $2 \times 70 = 140^0$.

Lastly, the angle marked 2x + 5 = 2(70) + 5 = 140 + 5 = 145^0

Q5.



Determine the value of *x*.

<u>Soln.</u>

The given figure has five sides (a pentagon) and as such has five exterior angles.

5 exterior angles = 360^0

$\Rightarrow$ 1 exterior angle = $\frac{1}{5} \times 360$

= 72^0.

But at any vertex, interior angle + exterior angle $= 180^0$

$\Rightarrow 72^0$ + interior angle = 180^0

$\Rightarrow$ interior angle = 180 – 72 = 108^0.

The sum of the interior angles of the pentagon = number of sides $\times$ $interior\ angle = 5 \times 108 = 540^0$.

The sum of the interior angles of the given figure = x + 2x + 3x + 2x + $30^0 + x + 20^0 = 9x + 50$

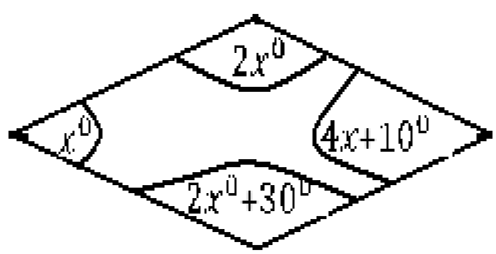
$\Rightarrow$ 9x + $50^0 = 540^0$

$\Rightarrow$ 9x = 540 – 50 = 490

$\Rightarrow$ 9x = 490 $\Rightarrow$ x = $\frac{490}{9} = 54$

$\therefore$ x = 54^0

Q6.



Calculate the value of *x.*

<u>Soln.</u>

The given figure is a quadrilateral and as much has four exteriior angles.

4 exterior angles = 360^0

$\therefore$ 1 exterior angle = $\frac{1}{4} \times 360^0$

= 90^0.

But at a vertex, exterior angle + interior angle = 180^0

$\Rightarrow 90^0 + \ interior\ angle = \ 180^0$

$\Rightarrow$ interior angle = 180 – 90 = 90.

Sum of the interior angles of a polygon = number of sides $\times$ interior angle

= $4 \times 90 = 360^0$.

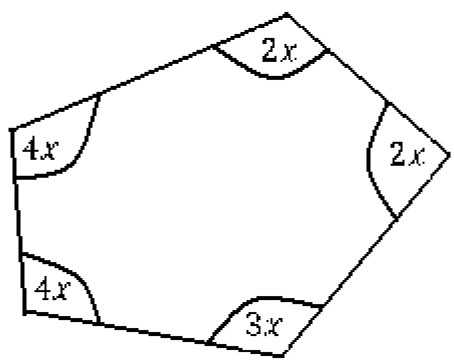
The sum of the interior angles of the given polygon = $x^0 + 2x^0 + 2x^0 + 30^0 + 4x^0 + 10^0 = 9x + 40^0$

$\Rightarrow 9x + 40^0 = \ 360^0$

$\Rightarrow$ 9x = $360^0 - 40 = 320^0$

$\therefore$ x = 35.5^0.

Q7.



Find the value of x.

<u>Soln.</u>

The given figure has five sides (pentagon), and as such has five exterior angles.

5 exterior angles = 360^0

$\therefore$ 1 exterior angle = $\frac{1}{5} \times 360$

= 72^0.

But at a vertex, interior angle + exterior angle = 180^0

$\Rightarrow$ interior angle + $72^0 = 180^0$

$\therefore$ interior angle = $180^0 - 72^0$

$\Rightarrow$ Interior angle = 108^0.

Sum of the interior angles of the given figure = number of sides $\times$ interior angle

= $5\times 108^0 = 540^0$.

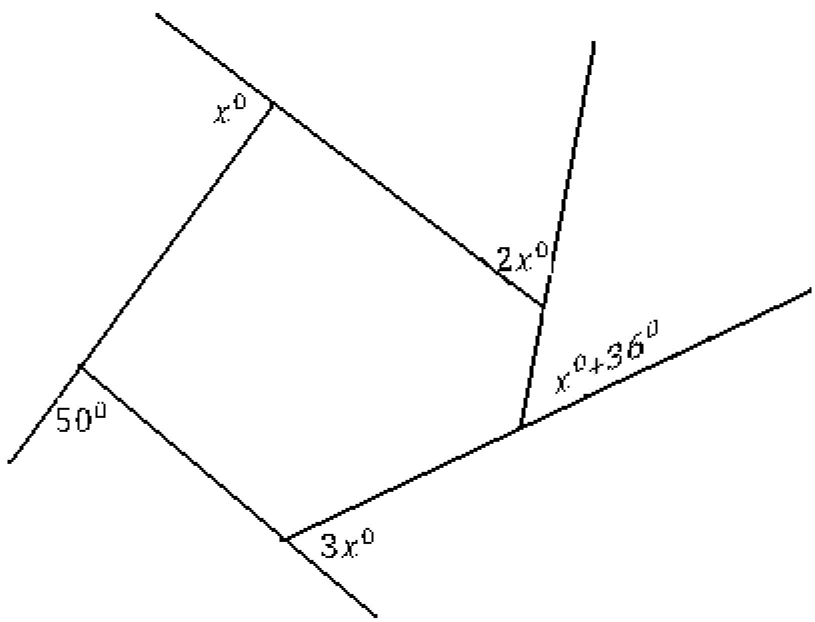
The sum of the interior angles of the given figure = 4x + 4x + 2x + 2x + 3x = 15x

$\therefore$ 15x = $540^0 \Rightarrow$ x $= \frac{540}{15} = 36$

$\Rightarrow$ x = 36^0.

N/B: The sum of the exterior angles of any polygon is equal to 360^0.

Q8.



Find the value of x.

N/B: All the given angles are exterior angles.

Soln.

The given figure is a polygon (pentagon) and has five exterior angles.

But the sum of the exterior angles of a polygon = 360^0

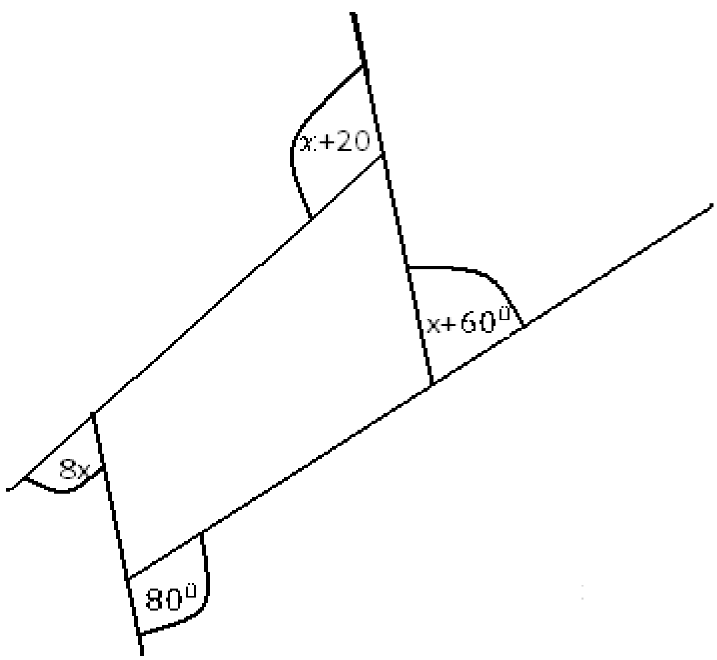
⇒ $x^0 + 50^0 + 3x^0 + x^0 + 36^0 + 2x^0 = 360^0$

⇒ 7x + 86 = 360 => 7x = 360 - 86 = 274^0.

∴ x = $\frac{274}{7} = 39$

⇒ x = 39^0 .

Q9



Determine the value of x.

N/B: The exterior angles of the given figure are $8x^0$, $80^0, x$ + 60^0 and $x + 20^0$.

Soln.

Sum of the exterior angles of the given figure = $8x^0 + 80^0 + x$ + 60^0 + $x + 20^0$

= 10x + 160.

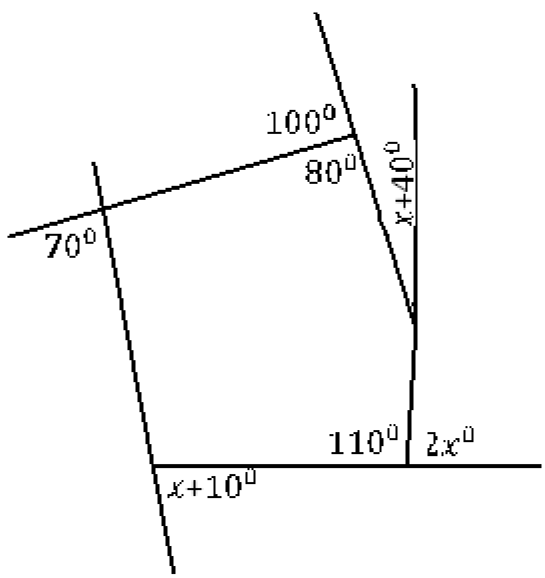
Since the given figure is a polygon (i.e a quadrilateral), then the sum of its exterior angles is 360^0.

⇒ 10x + 160 = 360^0

⇒ 10x = 360 – 160 = 200

⇒ x = $\frac{200}{10} = 20^0$.

Q10.



Calculate the values of the angles marked $2x^0$and x + 4.

Soln.

The sum of the exterior angles of the given figure which is a polygon

= 70 + 100 + x + 40 + 2x + x + 10 = 220 + 4x.

Since the sum of the exterior angles of a polygon = $360^0, then\ 220 + 4x = 360$

$\Rightarrow$ 4x = 360 – 220 = 140

$\Rightarrow$ x = $\frac{140}{4} = 35.$

The value of the angle marked 2x = 2(35^0) $= \ 70^0$.

Also the value of the angle marked x + 40 = 35 + 40 = 75..

Determination of the number of sides of a polygon:

The number of sides of any polygon = $\frac{360^0}{exterior\ angle}$

Q1. The exterior angle of a polygon is 72^0. How many sides has this polygon?

Soln.

Number of sides = $\frac{360^0}{exterior\ angle} = \frac{360^0}{72^0} = 5\ sides.$

Q2.Given that the exterior angle of a polygon is 45^0, determine its number of sides.

Soln.

Number of sides = $\frac{360^0}{exterior\ angle}$

= $\frac{360}{45} = 8\ sides.$

Q3. Determine the number of sides of a polygon, whose interior angle is 140^0.

N/B: First find the exterior angle and use it to divide 360^0.

Soln.

At any vertex, exterior angle + interior angle = 180^0

$\Rightarrow$ exterior angle + $140^0 = 180^0$

$\Rightarrow$ exterior angle = $180^0 - 140^0$

$\Rightarrow$ exterior angle $= 40^0$.

Number of sides = $\frac{360^0}{exterior\ angle}$

$= \frac{360}{40} = 9\ sides.$

Q4. Determine the name of a polygon, whose interior angle is 135^0.

N/B: By determining the number of sides, we can know the name of such a polygon.

<u>Soln.</u>

At any vertex, exterior angle + interior angle = 180^0.

$\therefore$ exterior angle + $135^0 = 180^0$

$\Rightarrow$ exterior angle = 180 – 135 = 45.

Number of sides = $\frac{360^0}{exterior\ angle}$ $= \frac{360}{45} = 8\ sides.$

$\therefore$ The polygon has 8 sides and as such it is an octagon.

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