CHAPTER NINE

TRANSFORMATION

Introduction:

There are various types of transformation and the types to be considered are:

1. Translation .
2. Reflection.
3. Rotation.
4. Enlargement.

Translation:

- This is the types of transformation in which every point moves the same distance, and in the same direction.
- Under translation, the lengths of lines and the sizes of angles do not change
- This implies that if a figure undergoes translation, its size as well as its angles remain unchanged.
- If the point (x, y) is translated by the vector $\binom{a}{b}$, then

  $(x, y) \longrightarrow (x + a, y + b)$,

  ie (x, y) $\xrightarrow{\text{tranlation by vector}\binom{a}{b}}$ $(x + a, y + b)$.

  Example (1)

  If $(x, y)$is translated by the vector $\binom{1}{4}$, then

$(x, y) \longrightarrow (x + 1, y + 4)$.

Example (2)

If $(2, 5)$ is translated by the vector $\binom{1}{3}$, then

$(2, 5) \longrightarrow (2 + 1, 5 + 3)$.

$(2,5) \longrightarrow (3,8)$.

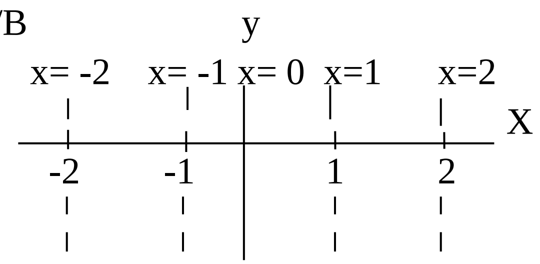
N/B: The point (3, 8) is called the image of the point (2, 5).

Reflection :

The reflection of a point or a figure can only be described, only when the position of the mirror line is well defined or known.

Under this type of transformation, the sizes of angles as well as the lengths of lines remain unchanged.

N/B



- The graph whose equation is x = 1, is a straight lines which is perpendicular to the x-axis, and passes through the point 1 on the x-axis.
- Also the line x = -2 passes through the point -2 on the x-axis.
- The y axis is also the same as the line x = 0

Types of reflections:

There are various types of reflections, and those to be considered are:

1. **Reflection in the y-axis or line x = 0:**

- For such a reflection,

  $(x, y) \longrightarrow (-x, y)$.

Example (1).

P (2, 5) reflection in the y- axis $\longrightarrow$ P_1(-2,5).

Example (2)

If the Q(3, 8) undergoes a reflection in the line x = 0, then for its image Q_1,
$Q(3,8) \longrightarrow Q_1(-3,8)$.

2. **Reflection in the line y = b:**

   For such a reflection, (x, y) $\longrightarrow$ (x, 2b – y).

Example (1)

If the point (2,4) undergoes a reflection in the line

y = 3, then

$(2, 4) \xrightarrow{\text{reflection in line } y = 3} \{2, 2(3) - 4\}$
$(2, 4) \longrightarrow (2, 6 - 4)$
$(2, 4) \longrightarrow (2, 2).$

Example (2)

If the point Q(2,3) undergoes a reflection in the line y = 5, then for its image Q_1,

$(x, y) \xrightarrow{\text{reflection in the line } y = b} (x, 2b - y),$

$\Longrightarrow Q(2,3) \xrightarrow{reflection\ in\ line\ y = 5} Q_1\{2, 2(5) - 3\}$

$Q(2, 3) \longrightarrow Q_1(2, 10 - 3)$

$Q(2, 3) \longrightarrow Q_1(2, 7).$

N/B: In this case, (x, y) = (2, 3) and y = b

becomes equal to y = 5.

$Therefore\ x = 2, y = 3\ and\ b = 5.$

There values are the substituted into the formula

$(x, y) \xrightarrow{\text{reflection in line } y = b} (x, 2b - y).$

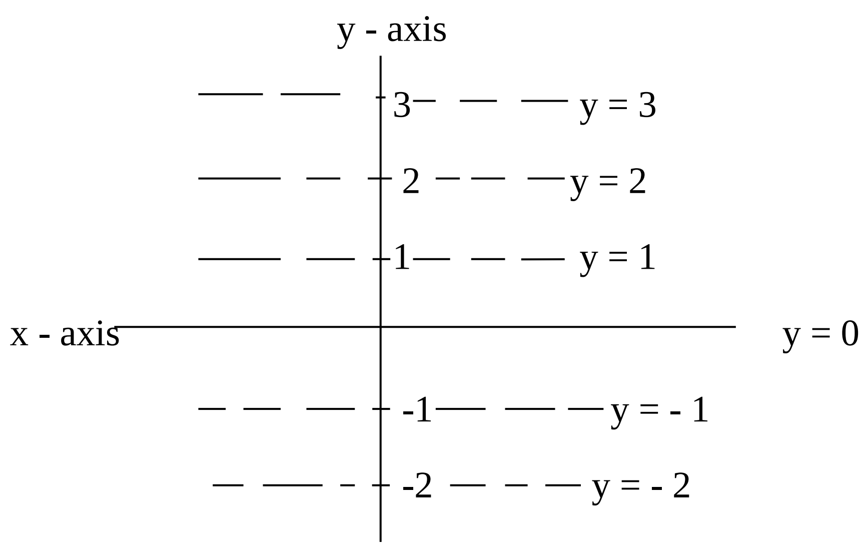
3. **<u>Reflection in the x-axis or the line y = 0:</u>**

- For such a reflection,
  $(x, y) \xrightarrow{\text{reflection in x-axis}} (x, -y)$
  Example (1)

If P(4, 3) undergoes a reflection in the x-axis, then its image P_1 is given by
$P(4,3) \xrightarrow{\text{reflection in x-axis}} P_1(4,-3).$

Example (2)

If the point A(-3,-4) undergoes a reflection in the x-axis, then its image A_1, is given by $A(-3, -4) \xrightarrow{\text{reflection in a-axis}} A_1(-3, 4)$.



- The line graph whose equation is y = 3, is a straight line which is perpendicular to the y-axis, and passes through the point 3 on they-axis.

- Also the line y = -2, passes through the point -2 on the y - axis.

- Lastly the x- axis is the same as the line y = 0.

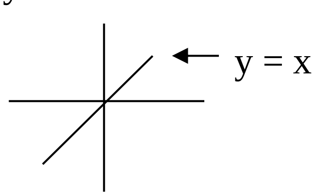
4. **<u>Reflection in line y = x, or the line y-x = 0, or line –y = -x:</u>**

- The line y = x is the same as the line y-x = 0, or the line –y = - x
- For such a reflection,
  $(x,y) \xrightarrow{\text{reflection in line } y = x} (y, x)$.

Example: if the point P(3, 5) undergoes a reflection in the line y = x, then for its image P_1,

$P(3,5) \xrightarrow{\text{reflection in line } y = x} P_1(5, 3)$.

The line y = x is shown next:



5. **Refle.ction in the line y = -x or the line y + x = 0 or the line – y = x:**

- The line $y = -x$ is the same as the line $y + x = 0$, or the line $-y = x$.
- For such a reflection,

  $(x, y) \xrightarrow{\text{reflection in line } y = -x} (-y,-x)$.

Example (1)

If the point B(2, 5) undergoes a reflection in the line $y = -x$, then for its image B_1,

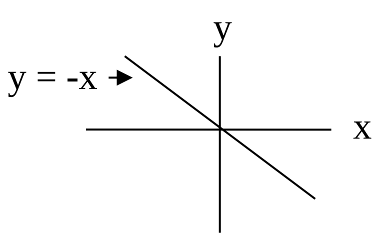
$B(2,5) \xrightarrow{\text{reflection in line } y = -x} B_1(-5, -2)$.

Example (2)

If the point C(-3, -2) undergoes a reflection in the line
$y + x = 0$, then for its image

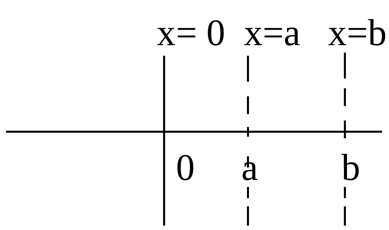
$C(-3, -2) \xrightarrow{\text{reflection in line } y + x = 0} C_1(2, 3)$.

- Next is a diagrammatic representation of the line
  $y = - x$



N/B: The line x = a is a straight which is perpendicular to the x-axis, and passes through the point a, on the x-axis.

Also the line x = b is perpendicular to the x-axis, and passes through the point b on the x-axis.



6. **Reflection in the line x = a:**
   If the point (x, y) undergoes a reflection in the line x = a, then
   $(x, y) \xrightarrow{\text{reflection in line } x = a} (2a - x, y)$.

Example (1)

If the point (4, 3) undergoes a reflection in the line x = 5, then (x, y) = (4, 3) and x = a becomes equal to x = 5. Therefore x = 4, y = 3 and a = 5

From $(x, y) \xrightarrow{\text{reflection in line } x = a} (2a-x, y)$.

$(4, 3) \xrightarrow{\text{reflection in line } x = 5} \{2(5)-4, 3\}$.

$(4, 3) \longrightarrow (10 - 4, 3)$

$(4, 3) \longrightarrow (6, 3)$.

Example (2)

If the point p(-3, 4) undergoes a reflection in the line

y = 8, then for its image P_1,

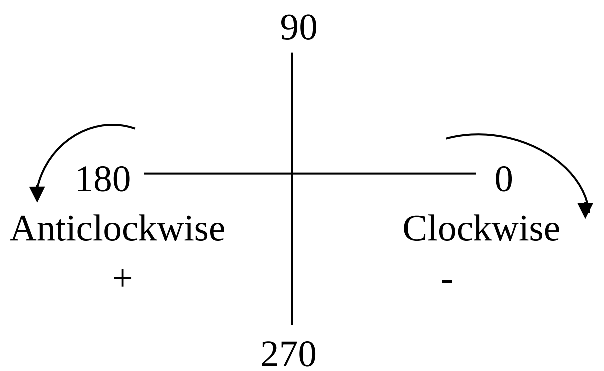
$P(-3, 4) \xrightarrow{\text{reflection in line } y = 8} P_1\{2(8) - (-3), 4\}$

$P(-3, 4) \longrightarrow P_1(16 + 3, 4)$.

$P(-3, 4) \longrightarrow P_1(19, 4)$.

Rotation :

- This is measured in degrees and from the x-axis.
- It is either measured in a clockwise or an anticlockwise direction.
- Rotation in the clockwise direction is negative rotation, and that in the anticlockwise direction is positive rotation.

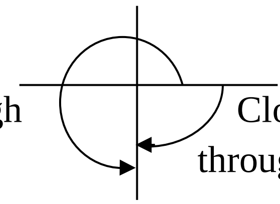


The different types of rotation to be considered are:

1. **<u>Clockwise rotation of 90 or rotation through -90°:</u>**

- This type of rotation is the same as an anticlockwise rotation through 270°or rotation through 270°.

Anticlockwise
Rotation through
270



Clockwise rotation
through 90

From the sketch made, it can be seen that from the same starting point, a clockwise rotation through 90°, and an anticlockwise rotation through 270°, all meet on the same line or at the same point.

For this reason the two are the same. For a clockwise rotation through 90° or an anticlockwise rotation through 270°, about the origin

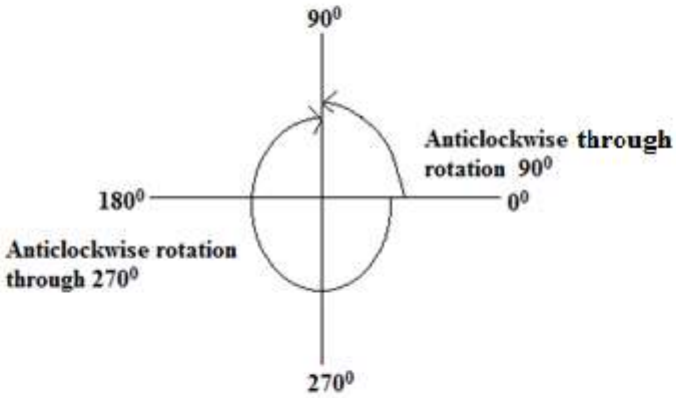
$(x, y) \longrightarrow (y, -x)$.

The following rotations are all the same, and as such the formula given must be used:

a) $(x, y) \xrightarrow{\text{Clockwise rotation through } 90° \text{ about the origin}} (y, -x)$.
b) $(x, y) \xrightarrow{\text{Rotation through } -90° \text{ about the origin}} (y, -x)$.
c) $(x, y) \xrightarrow{\text{Anticlockwise rotation through } 270°} (y, -x)$.
d) $(x, y) \xrightarrow{\text{Rotation through } 270°} (y, x)$.

2. **<u>Anticlockwise rotation through 90° or rotation 90°</u>**

- This types of rotation is the same as clockwise rotation through 270° or rotation through -270°.



From the diagram drawn, it can be seen that an anticlockwise rotation through 90, and a clockwise rotation through 270, originating from the same starting point or line, all end at the same starting point or line.

For this reason, they are the same. If the point (x, y) undergoes an anticlockwise rotation through 90°, or clockwise rotation through 270°, then

$(x, y) \longrightarrow (-y, x)$.

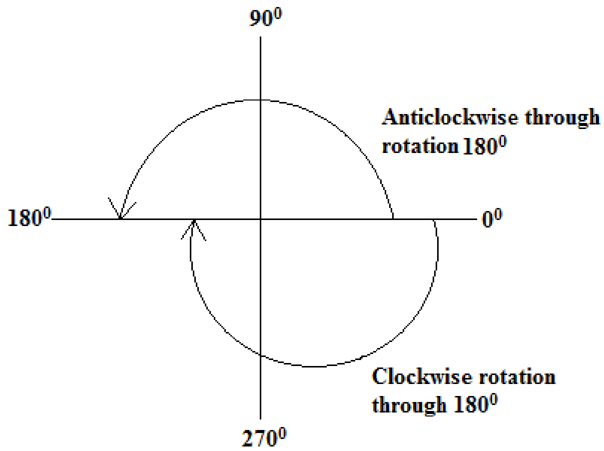
The following transformations are the same, and questions based on them must be solved using the given formula:

a. $(x, y) \xrightarrow{\text{Anticlockwise rotation through } 90^\circ \text{ about the origin}} (-y, x)$.
b. $(x, y) \xrightarrow{\text{rotation through } 90^\circ} (-y, x)$
c. $(x, y) \xrightarrow{\text{clockwise rotation through } 270^\circ} (-y, x)$
d. $(x, y) \xrightarrow{\text{rotation through } -270^\circ} (-y, x)$

Anticlockwise rotation through 90 or a clockwise rotation through 270 is known as or referred to as quarter turn.

Clockwise rotation through 180° or rotation through -180°

This type of rotation through which is referred to as half turn, is the same as an anticlockwise rotation through 180° or rotation through 180°



For this type of rotation $(x, y) \longrightarrow (-x, -y)$.

Example (1)

The image P_1 of the point P(2, 5), which undergoes a clockwise rotation of 180 about the origin, is given by $P(2,5) \longrightarrow P_1(-2, -5)$.

Example (2)

The image C_1 of the point C (-4, 7), after an anticlockwise rotation through 180°, is given by $C(-4, 7) \longrightarrow C_1(4, -7)$

Enlargement/ reduction:

- An important issue in connection with enlargement or reduction, is the scale factor (S.F).
- This scale factor tells us the number of times each length of the object has to be increased or decreased, so as to get the image.
- If the scale factor is greater than 1, then the size of the image is larger than that of the object.
- But if the scale factor is less than 1 or is a fraction, then the size of the image is smaller than that of the object.
- For example if the scale factor is 4, then the implication is that each length of the object had to be increased four times, in order to get the lengths of the image.
- On the other hand if the scale factor is $\frac{1}{4}$, then each length of the object had to be decreased to one quarter of its value, in order to get the lengths of the image.
- $\text{S.F} = \frac{\text{One length of the image}}{\text{Corresponding length of the object}}$

Example (1)

3cm

2cm $\xrightarrow{\text{enlargement}}$ 12cm, 8cm

Object Image

S.F = $\frac{12}{3} = 4 \; or \; S.F = \frac{8}{2} = 4.$

Example 2

9cm 3cm

Object 6cm $\xrightarrow{\text{enlargement}}$ image 2cm

S.F =$\frac{3}{9} = \frac{1}{3} \; or \; S.F = \frac{2}{6} = \frac{1}{3}$

S.F= $\frac{3}{9} = \frac{1}{3} \; or \; S.F = \frac{2}{6} = \frac{1}{3}$.

- The type of enlargement in which the scale factor is a fraction, is referred to as a reduction.

- If the point (x, y) is enlarged with a scale factor of K, then

(x, y) enlargement with $\xrightarrow{\text{S. F. K}}$ (kx, ky).

Example (1)

If (x, y) is enlarged with scale factor 2, then

(x, y) $\longrightarrow$ (2x, 2y).

Example (2)

If (2, 5) is enlarged with scale factor 4, then

(2, 5) $\longrightarrow$ {4(2), 4(5)}

(2,5) $\longrightarrow$ (8, 20).

Summary

A summary of formulae, which must be kept in memory are presented next.

1. (x, y) $\xrightarrow{\text{translation by vector } \binom{a}{b}}$ (x + a, y + b).
2. (x, y) $\xrightarrow{\text{reflection in the y – axis}}$ (-x , y).
3. (x, y) $\xrightarrow{\text{reflection in line y = b}}$ (x, 2b – y).
4. (x, y) $\xrightarrow{\text{reflection in the x-axis}}$ (x, -y)

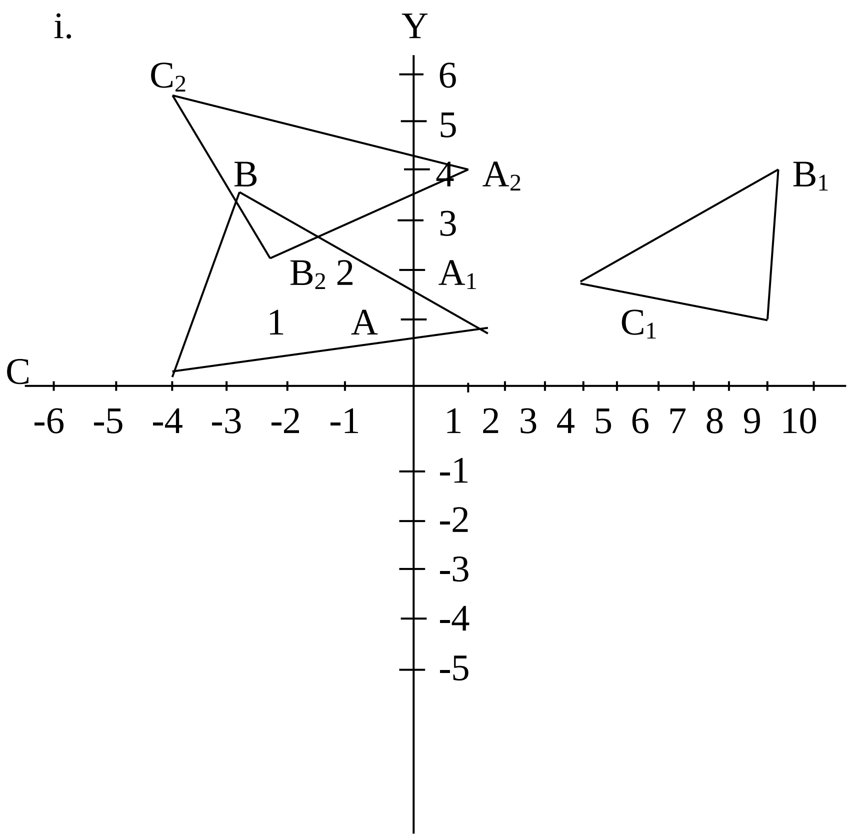
5. $(x, y) \xrightarrow{\text{reflection in line } y = x} (y, x)$.
6. $(x, y) \xrightarrow{\text{reflection in line } y = -x} (-y, -x)$
7. $(x, y) \xrightarrow{\text{reflection in line } x = a} (2a - x, y)$
8. Clockwise or anticlockwise rotation through 180 about the origin
   $(x, y) \longrightarrow (-x, y)$.
9. Clockwise rotation through 90 or anticlockwise rotation through 270, about the origin.
   $(x, y) \longrightarrow (y, -x)$.
10. Anticlockwise rotation through 90 or clockwise rotation through 270, about the origin
   $(x, y) \longrightarrow (-y, x)$.

Q1. i. Using a scale of 2cm to 2units on each axis, draw the line ox and oy for $-6 \leq x \leq 10 \; and \; -5 \leq y \leq 6$

i. Draw triangle ABC with vertices A(1,2), B(-3,4) and C(-4,1).
ii. On the same axes, draw the image $A_1B_1C_1$of ABC, after a reflection in the line x = 3.
iii. Draw the image $A_2B_2C_2$ of ABC under a reflection in the line y = 3..

Soln

i.



$(x, y) \xrightarrow{\text{reflection in line } x = a} (2a - x, y)$

$\Rightarrow (x, y) \xrightarrow{\text{reflection in line } x = 3} \{2(3) - x, y\}$

$\Rightarrow (x, y) \longrightarrow (6 - x, y)$.

$A \longrightarrow A_1$, $\Rightarrow A(1, 2) \xrightarrow{\text{reflection in line } x = 3} A_1(6 - 1, 2)$,

$\Rightarrow A(1, 2) \longrightarrow A_1(5,2)$

$B \longrightarrow B1$ and $(x, y) \longrightarrow (6 - x, y)$.

$\therefore B(-3, 4) \xrightarrow{\text{reflection in line } x = 3} B_1(6 - \bar{3}, 4)$,

$\Rightarrow B(-3, 4) \longrightarrow B_1(6 + 3, 4)$,

$\Rightarrow B(-3, 4) \longrightarrow B_1(9, 4)$.

$C \longrightarrow C_1 \Rightarrow C(-4, 1) \longrightarrow C_1(6 - x, y)$,

$\Rightarrow C(-4, 1) \longrightarrow C_1(6 - \bar{4}, 1)$,

$\Rightarrow C(-4, 1) \longrightarrow C_1(6+4, 1) \Rightarrow C(-4, 1) \longrightarrow C_1(10,1)$.

To get $\Delta A_1B_1C_1$, we plot the point A_1,B_1 and $C_1$and join one to the other.

ii. $(x, y) \xrightarrow{\text{reflection in line } y = b} (x, 2b - y)$,

$\Rightarrow (x, y) \xrightarrow{\text{reflection in line } y = 3} (x, 2(3) - y)$,

$\Rightarrow (x, y) \longrightarrow (x, 6 - y)$.

$A \longrightarrow A_2 \Rightarrow A(1,2) \longrightarrow A_2(1, 6 - 2)$,

$\Rightarrow A(1,2) \longrightarrow A_2(1,4)$.Since $B \longrightarrow B_2$

and $(x, y) \longrightarrow (x, 6 - y)$,

$B(-3, 4) \longrightarrow B_2(-3, 6 - 4)$,

$\Rightarrow B(-3, 4) \longrightarrow B_2(-3, 2)$.

$C(-4, 1) \longrightarrow C_2(-4, 6 - 1)$,

$\Rightarrow C(-4, 1) \longrightarrow C_2(-s4, 5)$.

We finally draw $\Delta A_2B_2C_2$ using the points A_2, B_2 and C_2.

Q2. i. Using a scale of 2cm to 2units on each axis, draw on a sheet of graph paper two perpendicular axes ox and oy, for the interval $-10 \leq x \leq 10$ and $-10 \leq y \leq 10$.

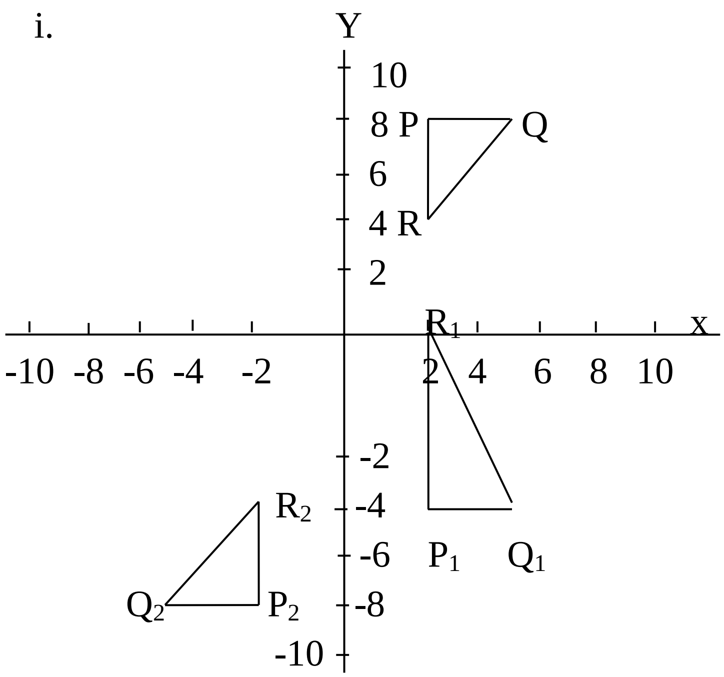
ii. Draw ΔPOR with vertices P(2, 8), Q(5, 8) and R(2, 4).

iii Draw the image $\Delta P_1Q_1R_1$of ΔPQR under a reflection in the line y = 2, where $P \longrightarrow P_1$, $Q \longrightarrow Q_1$ and $R \rightarrow R_1$.

iii. Draw also the image $\Delta P_2Q_2R_2$ of ΔPQR after a half turn about the origin o, where $P \longrightarrow P_2$,$Q \longrightarrow Q_2$ and $R \longrightarrow R_2$

Soln.

i.



ii. $(x, y) \xrightarrow{\text{reflection in line } y = b} (x, 2b - y)$

$(x, y) \xrightarrow{\text{reflection in line } y = 2} \{x, 2(2) - y\}$

$(x, y) \longrightarrow (x, 4 - y)$.

Since $P \longrightarrow P_1 \Longrightarrow P(2,8) \longrightarrow P_1(2,4-8)$,

$\Longrightarrow P(2,8) \longrightarrow P_1(2,-4)$.

Since $Q \longrightarrow Q_1$ and $(x, y) \longrightarrow (x, 4 - y)$,

$\Longrightarrow Q\ (5, 8) \longrightarrow Q_1\ (5, 4 - 8)$,

$\Longrightarrow Q\ (5, 8) \rightarrow Q_1\ (5, -4)$.

Lastly $R \longrightarrow R_1$ and $(x, y) \longrightarrow (x, 4 - y)$, =>

$R(2, 4) \longrightarrow R_1(2, 4 - 4), \Longrightarrow R(2,4) \longrightarrow R_1(2,0)$.

Using P_1,Q_1 and R_1, we draw $\Delta P_1Q_1R_1$.

iii. For a half turn about the origin $(x, y) \longrightarrow (-x, -y)$

$P \longrightarrow P_2 \Longrightarrow P(2, 8) \longrightarrow P_2(-2, -8)$..

$Q \longrightarrow Q_2 \Longrightarrow Q(5,8) \longrightarrow Q_2(-5, -8)$.

$R \longrightarrow R_2 \Longrightarrow R(2, 4) \longrightarrow R_2(-2, -4)$.

Q3. Using a scale of 2cm to 1unit on both axes, draw the x and the y axes for $0 \leq x \leq 8$ and $-6 \leq y \leq 6$.

i. Plot A(3, 1), B(1, 1) and C(1, 5).

ii. Find the equation of line AC.

iii. Draw $\Delta A_1B_1C_1$ where A $\longrightarrow$ A_1, B $\longrightarrow$ B_1 and C $\longrightarrow$ C_1.

iv. Draw $\Delta A_2B_2C_2$ which is the image of ΔABC under the mapping $\begin{pmatrix} x \\ y \end{pmatrix} \longrightarrow \begin{pmatrix} 2x \\ 1-y \end{pmatrix}$, where A $\longrightarrow$ A_2, B $\longrightarrow$ B_2 and C $\longrightarrow$ C_2.

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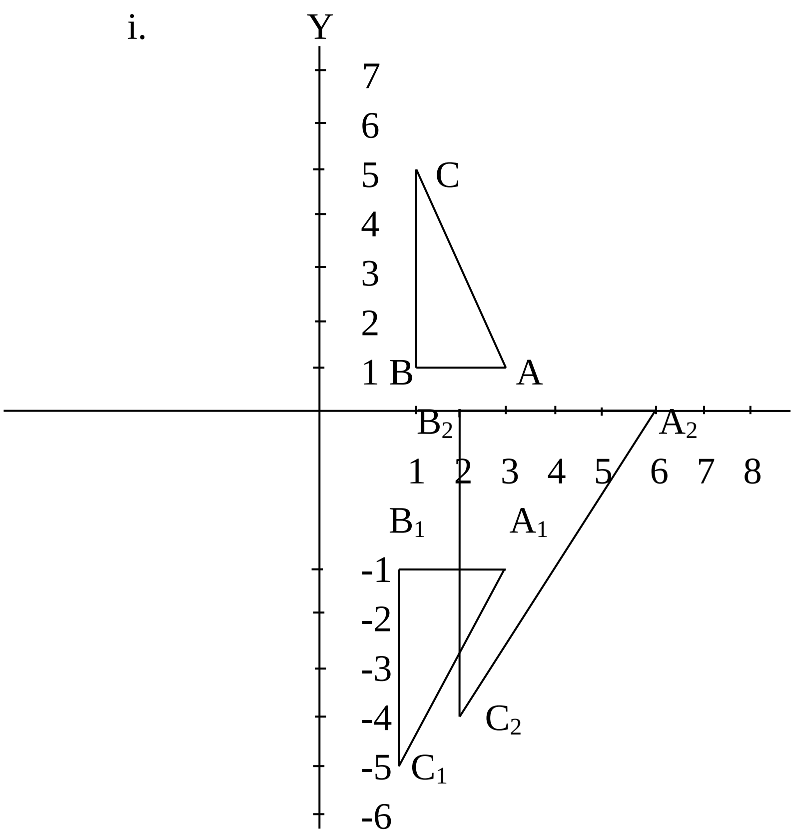
. Soln.

i.



ii. Line AC passes through the points (3 1) and (1, 5). Let $(x_1, y_1) = (3, 1)$ and $(x_2, y_2) = (1, 5) \Longrightarrow x_1 = 3$, $y_1 = 1$, $x_2 = 1$ and $y_2 = 5$.

Using $y - y_1 = \frac{y_2-y_1}{x_2-x_1}(x - x_1) \Rightarrow y - 1 = \frac{5-1}{1-3}(x - x_1), \Rightarrow y - 1 = \frac{4}{-2}(x - x_1), \Rightarrow y - 1 = -2(x - 3) \Rightarrow y - 1 = -2x + 6 \Rightarrow y = -2x + 6 + 1 \Rightarrow y = -2x + 7$

iii. For a reflection in the x – axis, $(x, y) \rightarrow (x, -y)$

Since $A \longrightarrow A_1 \Rightarrow A(3,1) \longrightarrow A_1(3, -1)$

Since $B \longrightarrow B_1 \Rightarrow B(1, 1) \longrightarrow B_1(1, -1)$

Since $C \longrightarrow C_1 \Rightarrow C(1,5) \longrightarrow C_1(1, -5)$

Using $A_1(3, -1)$, $B_1(1, -1)$ and $C_1(1, -5)$, we draw $\Delta A_1B_1C_1$

iv. $\begin{pmatrix} x \\ y \end{pmatrix} \longrightarrow \begin{pmatrix} 2x \\ 1-y \end{pmatrix} \Rightarrow (x, y) \longrightarrow (2x, 1 - y)$.

Since $A \longrightarrow A_2$, then $A(3, 1) \longrightarrow A_2\ \{2(3), 1 - 1\}$,

$\Rightarrow A(3, 1) \longrightarrow A_2 6, 0)$.

Since $B \rightarrow B_2$, then $B(1, 1) \longrightarrow B_2\{2(1), 1 - 1\}$,

$\Rightarrow B(1,1) \longrightarrow B_2(2, 0)$.

Since $C \longrightarrow C_2$, then $C(1, 5) \longrightarrow C_2\{2(1), 1 - 5)\}, \Rightarrow$

$C_2(1,5) \longrightarrow C_2(2, -4)$.

Using $A_2(6, 0)$, $B_2\ (2, 0)$ and $C_2(2, -4)$, we draw $\Delta A_2B_2C_2$.

Q4. Draw on a sheet of graph paper two perpendicular axes, ox and oy for $-10 \leq x \leq 10$ and $-10 \leq y \leq 10$, using a scale of 2cm to 2units on both axes. Given the point A(1, 2) and the vector $\overrightarrow{AB} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}, \overrightarrow{BC} = \begin{pmatrix} -1 \\ 3 \end{pmatrix}$ and $\overrightarrow{CD} = \begin{pmatrix} -2 \\ -2 \end{pmatrix}$, draw on the same graph

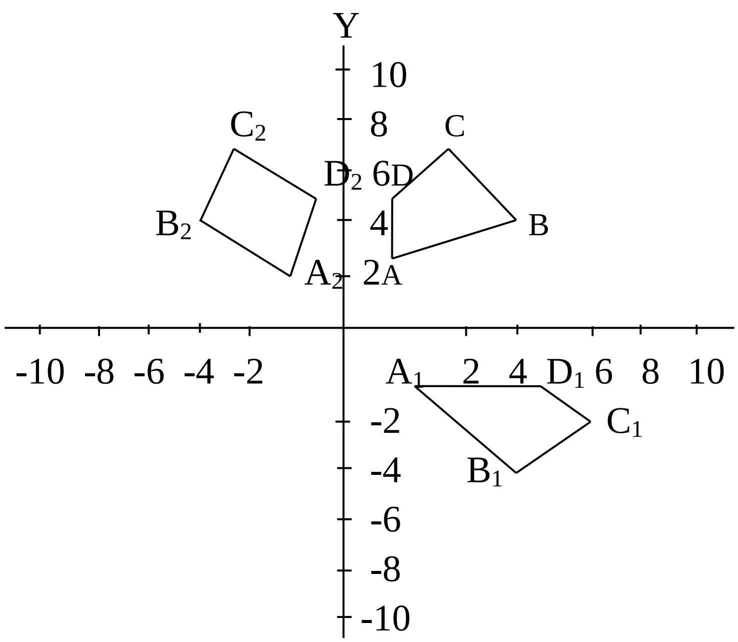
a. the quadrilateral ABCD.
b. the image $A_1B_1C_1D_1$ of ABCD under a clockwise rotation of 90 about the origin (0, 0), Where $A \longrightarrow A_1$, $B \longrightarrow B_1$, $C \quad C_1$ and $D \quad D_1$.
c. The image $A_2B_2C_2D_2$ of $A_1B_1C_1D_1$ under a reflection in the line y = x, where $A_1 \longrightarrow A_2$, $B_1 \longrightarrow B_2$, $C_1 \longrightarrow C_2$ and $D_1 \longrightarrow D_2$
d. Determine the length A_2C_2.

Soln.

a. Given A(1, 2), and $\overline{AB} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$

$\Rightarrow B = (1 + 3, 2 + 2) \Rightarrow B(4,4)$.

Now B(4, 4) and $\overline{BC} = \begin{pmatrix}-1\\ 3\end{pmatrix} \Rightarrow C(4+\bar{1}, 4+3) \Rightarrow C(3,7). C(3,7) and\ \overline{CD} = \begin{pmatrix}-2\\ -2\end{pmatrix} \Rightarrow D(3+\bar{2}, 7+\bar{2}) \Rightarrow D(1,5)$. At this stage, since we now know the coordinates of A,B,C and D, we can now draw the quadrilateral ABCD



b. Under a clockwise rotation of 90 about the origin (0,0),

c. (x, y) $\rightarrow$ (y, -x). Since A $\rightarrow$ A_1, then
   A(1, 2) $\rightarrow$ A_1(2, -1)

Since B $\rightarrow$ B_1, then B(4, 4) $\rightarrow$ B_1(4, -4). Also since
C $\rightarrow$ C_1 then C(3, 7) $\rightarrow$ C_1(7, -3).
Lastly since D $\rightarrow$ D_1, then D(1, 5) $\rightarrow$ D_1(5,-1). With the coordinate of $A_1B_1C_1$ and D_1 known, we can now draw quadrilateral $A_1B_1C_1D_1$.

d. Under a reflection in the line y = x, (x, y) $\rightarrow$ (y, ,x)

Since $A_1 \rightarrow A_2$, then A_1(2,-1) $\rightarrow$ A_2(-1,2). Since
$B_1 \rightarrow B_2$, then B_1(4,- 4) $\rightarrow$ B_2(- 4, 4).
Also $C_1 \rightarrow C_2$, => C_1(7,-3) $\rightarrow$ C_2(-3,7).
Lastly $D_1 \rightarrow D_2$, =>D_1(5,-1) $\rightarrow$ D_2(-1,5)..
We then draw the quadrilateral $A_2B_2C_2D_2$.
N/B: If $A_1 \rightarrow A_2$, then the coordinates of A_1 must be used but not those of A.

e. A_2 = (-1, 2) and C_2 = (-3, 7). Let (x_1, y_1) = (-1, 2) and (x_2, y_2) = (-3, 7), $\Rightarrow x_1$ = -1, y_1 = 2, x2 = -3 and y_2 = 7. If l = the length of $A_2C_2 \Rightarrow l =$

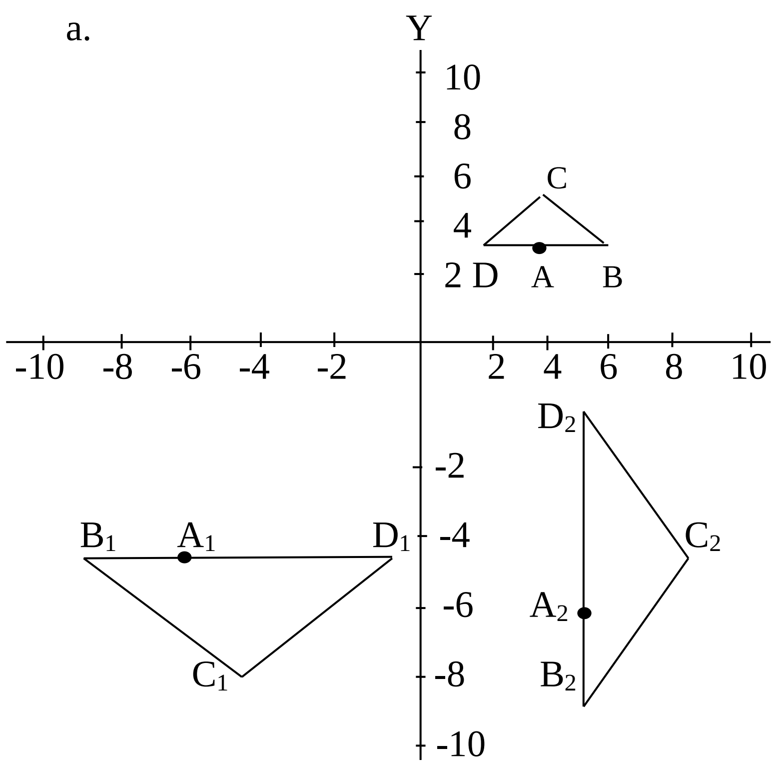
$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \Longrightarrow l = \sqrt{(-3+1)^2 + 5^2} = \sqrt{(-2)^2 + 5^2} =$
$\sqrt{4 + 25} = \sqrt{29} = 5.4$

Q5. Draw on the graph sheet, using a scale of 2cm to 2units, two perpendicular axes ox and oy for $-10 \le x \le 10\ and - 10 \le y \le 10$

a. Plot the points A(4,3), B(6, 3), C(3,5) and D(1,3).
b. Draw the image $A_1B_1C_1D_1$ of ABCD under an enlargement from the origin (0, 0) with a scale factor of $\frac{-3}{2}$, where $A \to A_1$, $B \to B_1$, $C \to C_1$ and $D \to D_1$.
c. Draw the image $A_2B_2C_2D_2$ of $A_1B_1C_1D_1$ under an anticlockwise rotation of 90 about origin 0(0,0), where $A_1 \to A_2$, $B_1 \to B_2$, $C_1 \to C_2$ and $D_1 \to D_2$..

Soln.

a.



b. If (x,y) undergoes an enlargement with scale factor k, then
(x, y) $\longrightarrow$ (kx, ky),
$\Longrightarrow$ if (x, y) undergoes an enlargement with scale factor $\frac{-3}{2}$, then
. (x, y) $\longrightarrow$ $(\frac{-3}{2}x,\ \frac{-3}{2}y)$.

Since A $\longrightarrow$ A_1, then A(4, 3) $\longrightarrow A_1\{\frac{-3}{2}(4), \frac{-3}{2}(3)\}, \Rightarrow A(4,3) \Longrightarrow$ $A_1\left(\frac{-12}{2}, \frac{-9}{2}\right)$, $=> A(4,3) \longrightarrow A_1(-6, -4.5)$.

Since B $\longrightarrow$ B_1, then B(6, 3) $\longrightarrow$ $B_1\left(\frac{-3}{2}(6), \frac{-3}{2}(3)\right), \Rightarrow B(6,3) -\rightarrow$ $B_1\left(\frac{-18}{2}, \frac{-9}{2}\right), \Rightarrow B(6,3) - B_1(-9, -4.5)$. C $\rightarrow$ C1$\Rightarrow C(3,5) \longrightarrow$ $C_1\left(\frac{-3}{2}(3), \frac{-3}{2}(5)\right), \Rightarrow C(3,5) \longrightarrow C_1\left(\frac{-9}{2}, -\frac{15}{2}\right), \Rightarrow C(3,5) \longrightarrow$ $C_1(-4.5, -7.5)$

Lastly D$\longrightarrow D_1 \Rightarrow D(1,3) \longrightarrow D_1\left(\frac{-3}{2}(1), \frac{-3}{2}(3)\right), \Rightarrow D(1,3) \longrightarrow$ $D_1\left(\frac{-3}{2}, \frac{-9}{2}\right), \Rightarrow D(1,3) \longrightarrow D_1(-1.5, -4.5)$

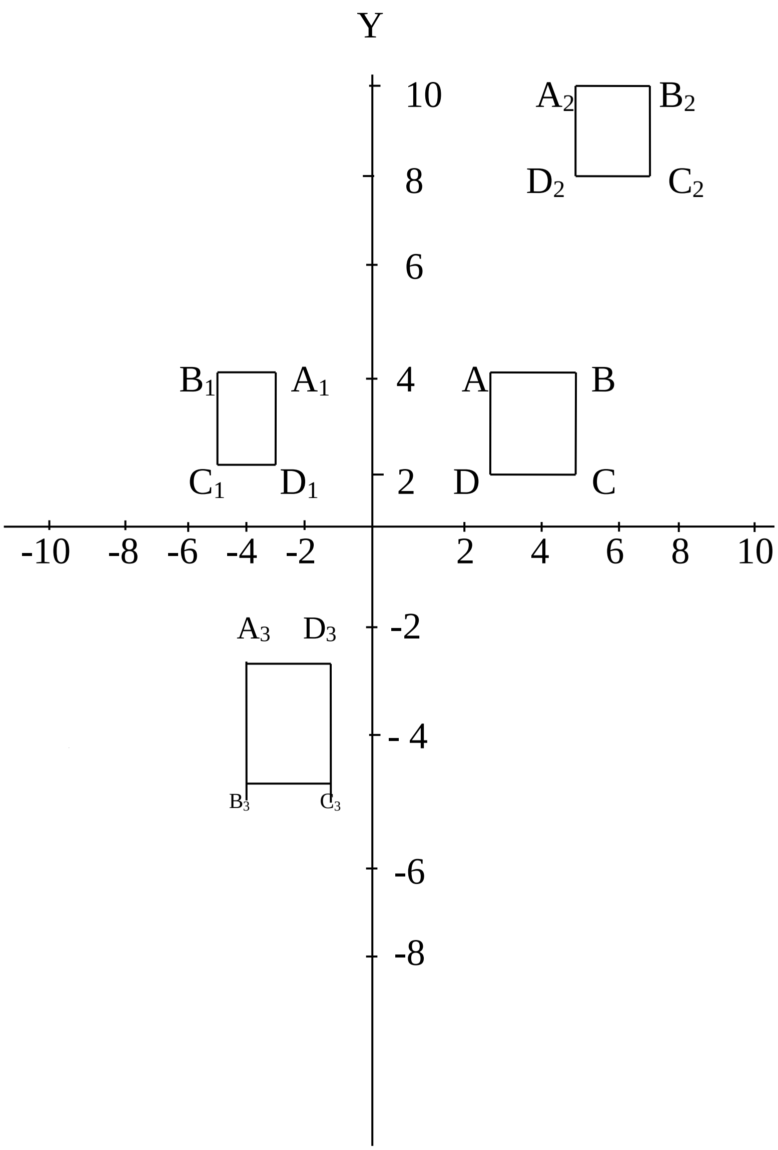
c. For an anticlockwise rotation of 90 about the origin 0(0, 0), (x, y) (-y, x)'
Since $A_1 \longrightarrow A_2, then\ A_1(-6, -4.5) \longrightarrow A_2(4.5, -6)$.
$B_1 \longrightarrow B_2 \Rightarrow B_1(-9, -4.5) \longrightarrow B_2(4.5, -9). Since$
$C_1 \longrightarrow C_2 \Rightarrow C_1(-4.5, -7.5) \longrightarrow C_2(7.5, -4.5)$.
$lastly\ D_1 \longrightarrow D_2 \Rightarrow D_1(-1.5, -4.5) \longrightarrow D_2(4.5, -1.5)$.

We then draw figure $A_2B_2C_2D_2$.

Q6. Using a scale of 1cm to 1 unit on each axis, draw the x and y axes.

i. Draw quadrilateral ABCD whose vertices are A(3, 4), B(5,4), C(5, 2) and D(3, 2).
ii. Draw the quadrilateral $A_1B_1C_1D_1$ which is the image of ABCD, after a reflection in the y-axis, where A$\longrightarrow A_1, B \longrightarrow B_1, C \longrightarrow C_1\ and\ D \longrightarrow D_1$.
iii. Draw the image $A_2B_2C_2D_2$ of ABCD after a translation by the vector $\binom{2}{6}$.
iv. Draw quadrilateral $A_3B_3C_3D_3$ which is the image of $A_1B_1C_1D_1$, after a rotation through 270 about the origin in the clockwise direction.

Soln



(ii) For a reflection in the y- axis, (x,y) ⟶ (-x,y). Since

A ⟶ $A_1, then\ A(3,4)$ ⟶ A(-3,4).

B ⟶ B_1 ⇒ B$(5,\ 4)$ ⟶ $B_1(-5,\ 4)$. Since C ⟶ C_1, then

C(5 ,2) ⟶ $C_1(-5,\ 2)$. Lastly D ⟶ D_1

⇒ D (3,2) ⟶ $D_1(-3{,}2)$.

We then draw quadrilateral A_1, B_1, C_1, D_1.

(i) If (x, y) undergoes a translational by the vector $\binom{a}{b}$, then

(x, y) $\longrightarrow$ (x + a, y+ b).

$\Rightarrow$ If (x, y) undergoes a translation by the vector $\binom{2}{6}$, then

(x, y) $\longrightarrow$ (x + 2, y + 6).

Since $A_2\ B_2\ C_2 D_2\ is\ the\ image\ of\ ABCD, then$

$A \longrightarrow A_2, B \longrightarrow B_2$, C $\longrightarrow C_2\ \ and\ D \longrightarrow D_2.$

Since A $\longrightarrow A_2, then$ A(3, 4) $\longrightarrow A_2(3+2, 4+6),$

$\Rightarrow$A(3, 4) $\longrightarrow A_2(5, 10)$'

B $\longrightarrow B_2 \Rightarrow$ B(5, 4) $\longrightarrow B_2(5+2, 4+6),$

$\Rightarrow$B(5, 4) $\longrightarrow B_2\ (7, 10)$..

C $\longrightarrow C_2 \Rightarrow$C(5, 2) $\longrightarrow C_2(5+2, 2+6),$

$\Rightarrow$C(5,2) $\longrightarrow C_2(7,\ 8)$.

Lastly D $\longrightarrow D_2 \Rightarrow$ D(3, 2) $\longrightarrow D_2(3+2, 2+6)$

$\Rightarrow$D(3, 2) $\longrightarrow D_2$(5, 8).

With these coordinates, draw $A_2\ B_2\ C_2 D_2$.

(iv) Under a clockwise rotation through 270^0,

(x, y) $\longrightarrow$ (-y, x).

Since $A_3\ B_3 C_3 D_3$ is the image of $A_1 B_1 C_1 D_1$, then

$A_1 \longrightarrow A_2,\ B_1 \longrightarrow B_3,\ C_1 \longrightarrow C_3\ and\ D_1 \longrightarrow D_3.$

Since $A_1 \longrightarrow A_3,$ then$A_1(-3, 4) \longrightarrow A_1(-4, -3).$

Since $B_1 \longrightarrow B_3$, then $B_1(-5, 4) \longrightarrow B_3(-4, -5).$

Since $C_1 \longrightarrow C_3,$ then $C_1(-5, 2) \longrightarrow C_3(-2, -5).$

Lastly Since $D_1 \longrightarrow D_3$, then D_1(-3, 2) $\longrightarrow D_3$(-2,-3)

Q7)(i). Using a scale of 1cm to 1 unit on each axis, draw the quadrilateral whose vertices are A(-5, 3), B(0, 3), C(-1,2) and D(-4,2).

(ii) Construct $A_1B_1C_1D_1$ which is a reflection of $A_1B_1C_1D_1$ the line y – x= 0, where A → A_1, B → B_1, C→ C_1 and D → D_1.

(iii)Draw $A_2\ B_2\ C_2D_2$ which is the image of$A_1B_1C_1D_1$ after a reflection in the line x= 5, where$A \longrightarrow A_1, B_1 \longrightarrow B_2, C_1 \longrightarrow C_2$ *and*

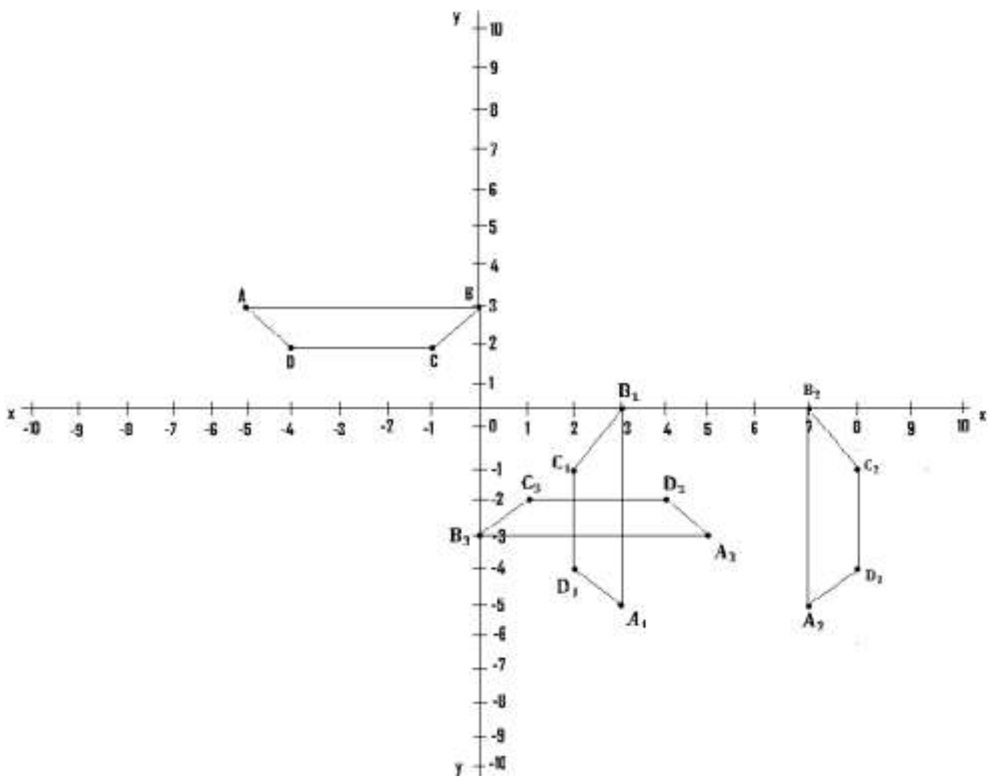
$D_1 \longrightarrow D_2$..

(iv) Draw $A_3B_3C_3D_3$ which is the image of ABCD after a half turn, where A$\longrightarrow$ A_3, B $\longrightarrow$ B_3, C $\longrightarrow$ C_3 and D $\longrightarrow$ D_3..

(v) Find the gradient of the line B_1C_1..

(vi) Find the equation of the lineB_1C_1.

(vii) Determine also the equation of line AD.



(ii) Line y – x =0 is the same as the line y = x. If (x, y) is reflected in the line y = x, (x , y)$\longrightarrow$ (y, x).

Since A$\longrightarrow A_1$, then A(-5, 3) $\longrightarrow A_1(3, -5)$.

Since B$\longrightarrow B_1$, then B(0, 3) $\longrightarrow B_1(3,0)$.

Also C $\longrightarrow$ $C_1 \Rightarrow$ C(-1, 2) $\longrightarrow$ $C_1(2,-1)$.

Lastly D $\rightarrow D_1 \Rightarrow$ D(-4, 2) $\longrightarrow D_1(2,-4.)$

We then draw $A_1B_1C_1D_1$.

(iii)

$$(x, y) \xrightarrow{\text{Reflection in line } x = a} (2a - x, y)$$

$$\Rightarrow (x, y) \xrightarrow{\text{Reflection in line } x = 5} (2(5) - x, y)$$

$\Rightarrow$**(x, y)** $\longrightarrow$**(10 – x, y).**

$A_1 \longrightarrow A_2 \Rightarrow A_1(3,-5) \longrightarrow A_2(10-3,-5)$,

$\Rightarrow A_1(3,5) \longrightarrow A_2(7,-5)$,

$B_1 \longrightarrow B_2 \Rightarrow$ $B_1(3,0) \longrightarrow B_2(10-3,0)$,

$\Rightarrow B_1(3,0) \longrightarrow B_2(7,0)$.

$C_1 \longrightarrow C_2 \Rightarrow C_1(2,-1) \longrightarrow C_2(10-2,-1), \Rightarrow C_1(2,-1) \longrightarrow C_2(8,-1)$.

$D_1 \longrightarrow D_2 \Rightarrow D_1(2,-4) \longrightarrow D_2$(10 - 2, -4),

$\Rightarrow D_1(2,-4) \longrightarrow D_2(8,-4)$. .

We then draw $A_2B_2C_2D_2$.

(iv) If (x, y) undergoes a half turn, then

(x,y)$\longrightarrow$ (-x,-y).

Since A$\longrightarrow A_3$, then A(-5, 3) $\longrightarrow A_3$ {-(-5), -3},

A(-5, 3) $\longrightarrow A_3(5,-3)$.

Also B $\longrightarrow$ $B_3 \Rightarrow$ B(0, 3) $\longrightarrow B_3(0,-3)$.

C $\longrightarrow C_3 \Rightarrow$ C(-1, 2) $\longrightarrow$ $C_3(1,-2)$.

Lastly since D$\longrightarrow D_3$, then D(-4, -2) $\longrightarrow$ $D_3(4,-2)$

We now draw$A_3B_3C_3D_{3.}$.

(v) The line B_1C_1 has coordinatedes B_1(3,0) and C_1(2, -1).

Let (x_1, y_1) = (3, 0) and $(x_{2,}y_2)$ = (2, -1) ⇒$x_1 = 3$

$y_1 = 0,\ x_2 = 2\ and\ y_2 = 1$.The gradient of $B_1C_1 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-1-0}{2-3} = \frac{-1}{-1} = 1.$

(vi) The line B_1C_1 has a gradient of 1 and passes through the point $C_1(2, -1). Let\ x_1$= 2 and $y_1 = -1.$

Using y - y_1= m(x - x_1) ⇒ y –(1) = 1(x – 2),

⇒y - 1 = -x - 2 ⇒ y = x - 2 + 1

⇒ y = x + 1.

(vii) The line AD passes through the points A(-5, 3) and D(-4, 2). Let $(x_1, y_1) = (-5, 3)\ and\ let\ (x_2, y_2)$

= (- 4, 2) ⇒ x_1=-5, $y_1 = 3$, $x_2 = -4\ and\ y_2 = 2.$

Now y-y_1 = $\frac{y_2 - y_1}{x_2 - x_1}(x - x_1),$

⇒ y – 3 = $\frac{2 - 3}{-4 - -5}\{x - (-5)\},$

⇒ y -3 = $\frac{-1}{-4+5}(x + 5),$

⇒ y – 3 = $\frac{-1}{1}(x + 5)$⇒$y - 3 = -1(x + 5),$

⇒ $\mathrm{y} - 3 = -\mathrm{x} - 5$,⇒y=-x – 5+3, ⇒ y = -x – 2.

Q8) Using a scale of 2cm to 2 units on both axes, draw on a sheet of graph paper two perpendicular axes ox and oy for -10 ≤ x ≤ 10 and -12 ≤ y ≤ 12. Draw

(i) on the same graph sheet the △PQR with P(4, 8), $\overrightarrow{QR}$= $\binom{-2}{2}$ and $\overrightarrow{RP} = \binom{2}{4}$.

(ii) $\triangle P_1Q_1R_1$ which is the image of PQR under a reflection in the line y = -2, whereP⟶P_I, Q⟶Q_I and R⟶R_1.

(iii) the image △ $P_2Q_2R_2$ of △PQR under a translation by the vector $\binom{-8}{2}$, where P ⟶ P_2, Q ⟶ Q_2 and R ⟶ R_2.

(iv) the image $\triangle P_3Q_3R_3$ of $\triangle$PQR under a rotation through 180^0 about the origin.

Soln.

(i) We are given P(4, 8) and $\overline{RP} = \binom{2}{4}$.

From $\overrightarrow{RP} = \binom{2}{4} \Rightarrow \overline{PR} = -\binom{2}{4}, \Rightarrow \overrightarrow{PR} = \binom{-2}{-4}$.

Now P(4, 8) and $\overrightarrow{PR} = \binom{-2}{-4}, \Rightarrow$ R(4 + - 2, 8+ - 4),

$\Rightarrow$ R(2, 4) $\Rightarrow$ the coordinates of R = (2, 4).

Now R = (2, 4) and we are also given that $\overrightarrow{QR} = \binom{-2}{2}$.

From $\overrightarrow{QR} = \binom{-2}{2} \Rightarrow \overrightarrow{RQ} = -\binom{-2}{2}$, => $\overrightarrow{RQ} = -\binom{-2}{2} = \binom{2}{-2}$.

Since R= (2, 4) and $\overrightarrow{RQ} = \binom{2}{-2}$

$\Rightarrow$ Q(2+2, 4+-2) => Q(4, 2) => Q has coordinates (4,2)

With the help of or using P(4, 8), R(2, 4) and Q(4, 2) we draw $\triangle$PQR .

(ii) $(x, y) \xrightarrow{\text{Reflection in the line } y = b} (x, 2b - y)$

$\Rightarrow$ (x, y) $\xrightarrow{\text{reflection in the line } y = -2,}$ {(x, 2(-2) – }

$\Rightarrow (x, y) \longrightarrow (x, -4 - y)$.

$P \longrightarrow P_1, \Rightarrow P(4,8) \longrightarrow P_1(4, -4 - 8)$,

$\Rightarrow p(4, 8) \longrightarrow P_1(4,-12)$.

$R \rightarrow R_1, \Rightarrow R(2, 4) \longrightarrow R_1(2, -4 - 4)$,

$\Rightarrow R(2, 4) \longrightarrow R_1(2, -8)$.

$Q \rightarrow Q_1 \Rightarrow Q(4, 2) \longrightarrow Q_1(4, -4-2), \Rightarrow Q(4, 2) \rightarrow (4,-6)$.

Now using $P_1(4,-12)$, $R_1(2,-8)$ and $Q_1(4,-6)$, we draw $\triangle P_1R_1Q_1$.

iii.) (x, y) $\xrightarrow{\text{translation by the vector } \binom{a}{b}}$ (x+a, y+b),

$\Rightarrow$(x, y) $\xrightarrow{\text{translation by the vector } \binom{-8}{2}}$ (x+- 8 , y+2),

$\Rightarrow$(x, y) $\longrightarrow$ (x - 8, y+2)

P $\longrightarrow P_2 \Rightarrow$ P(4, 8) $\longrightarrow P_2$(4-8, 8+2),

$\Rightarrow$ P(4, 8) $\longrightarrow$ (- 4, 10).

R $\rightarrow R_2, \Rightarrow$R(2, 4) $\longrightarrow R_2$ (2-8, 4+2),

$\Rightarrow$R (2, 4) $\longrightarrow R_2$(- 6, 6).

Q $\longrightarrow Q_2 \Rightarrow$ Q(4, 10) $\longrightarrow Q_2$ (4 – 8, 10 + 2),

$\Rightarrow$ Q(4, 10) $\longrightarrow Q_2$ (- 4, 12).

(iv) If (x, y) undergoes a rotation through 180^0 about the origin, then
(x, y). $\longrightarrow$ (-x, -y).

Since $\triangle P_3Q_3R_3$ is the image of $\triangle$PQR, then P $\rightarrow P_3$,

Q $\rightarrow Q_3$ and R $\rightarrow R_3$.

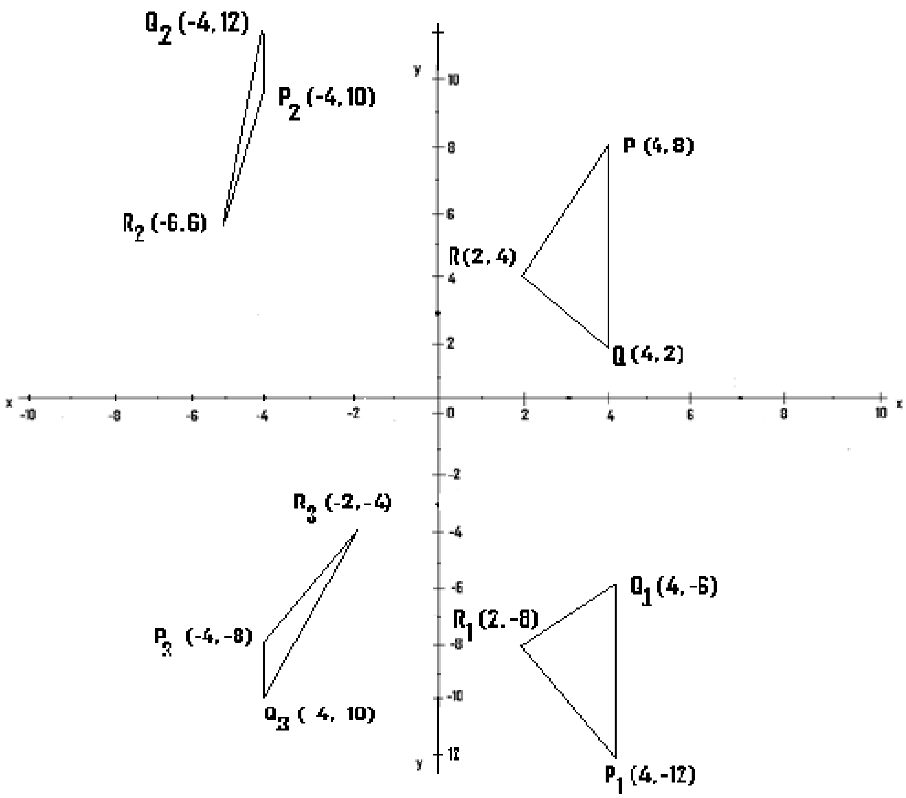
P $\rightarrow P_3 \Rightarrow$ P(4,8) $\longrightarrow P_3$(- 4, - 8).

Q $\rightarrow Q_3 \Rightarrow$ Q(4,10) $\longrightarrow Q_3$(-4, -10).

R $\rightarrow R_3 \Rightarrow$ R(2, 4) $\longrightarrow R_3$(-2, - 4).

Using P_3(- 4, - 8), Q_3(- 4, -10) and R_3 (-2, - 4), we draw

$\triangle P_3Q_3R_3$.



Using A_1(4, -2), B_1(8, 2) and C_1(10, 0), we draw$\triangle$ $A_{1.}B_1C_{1.}$.

(d) Consider the line $\frac{1}{2}x = \frac{1}{2}y$.Multiply through using 2

$\Rightarrow 2 \times \frac{1}{2}x = 2\times \frac{1}{2}y \Rightarrow$ x = y. Therefore the reflection in the line $\frac{1}{2}x = \frac{1}{2}y$, is the same as the reflection in the line y = x, and for such a reflection, (x, y) $\rightarrow$ (y, x).

Since$A_1 \longrightarrow A_2$, then A_1(4, -2) $\rightarrow A_2$(-2, 4).

Also since $B_1 \longrightarrow B_2$, then B_1(8, 2) $\rightarrow$ B_2(2, 8).

Lastly since $C_1 \longrightarrow C_2$, then C_1(10, 0)$\rightarrow$ C_2(0, 10).

Using A_2(-2, 4), B_2(2, 8) and C_2(0, 10), we draw

$\triangle A_2B_2C_2$..

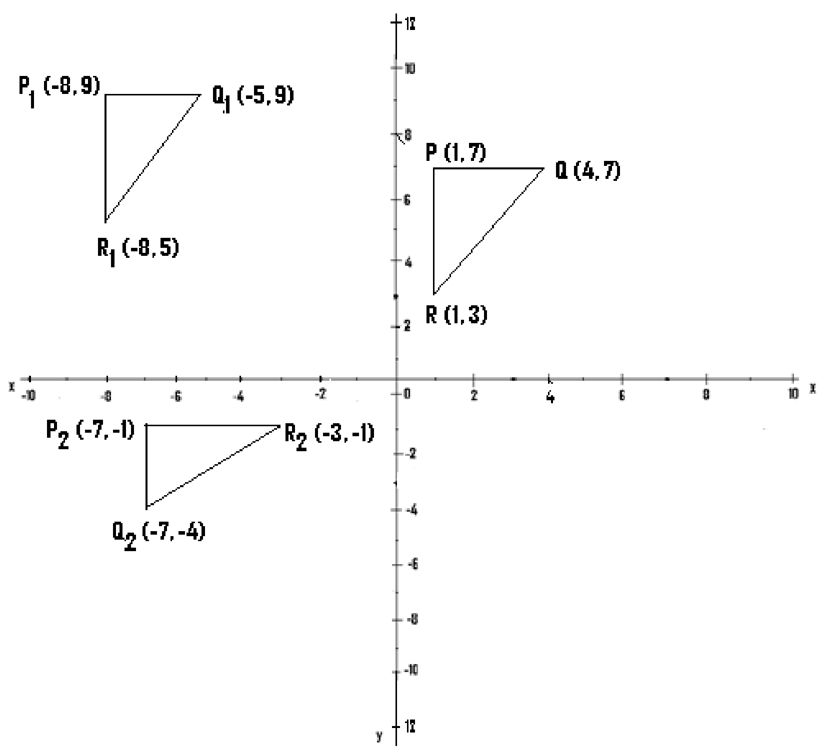
Questions:

Q1)(a) Using an appropriate scale, draw on a graph paper △PQR with vertices P(1, 7), Q(4, 7) and R(1, 3).

(b) Draw △ $P_1Q_1R_1$ which is the image of △PQR after a translation by the vector $\binom{-9}{2}$, where P $\longrightarrow P_1$, Q $\longrightarrow Q_1$ and R $\longrightarrow R_1$.

(C) Draw also the image △$P_2Q_2R_2$ of △PQR, after a reflection in the line y = -x, where P $\longrightarrow P_2$, Q $\longrightarrow Q_2$ and R $\longrightarrow R_2$.

Ans: △



(Q2)(a) Using a scale of 2cm to 2units on both axes, draw △ ABC with the vertices A(4, 4), B(0, 7) and C(3, -6).

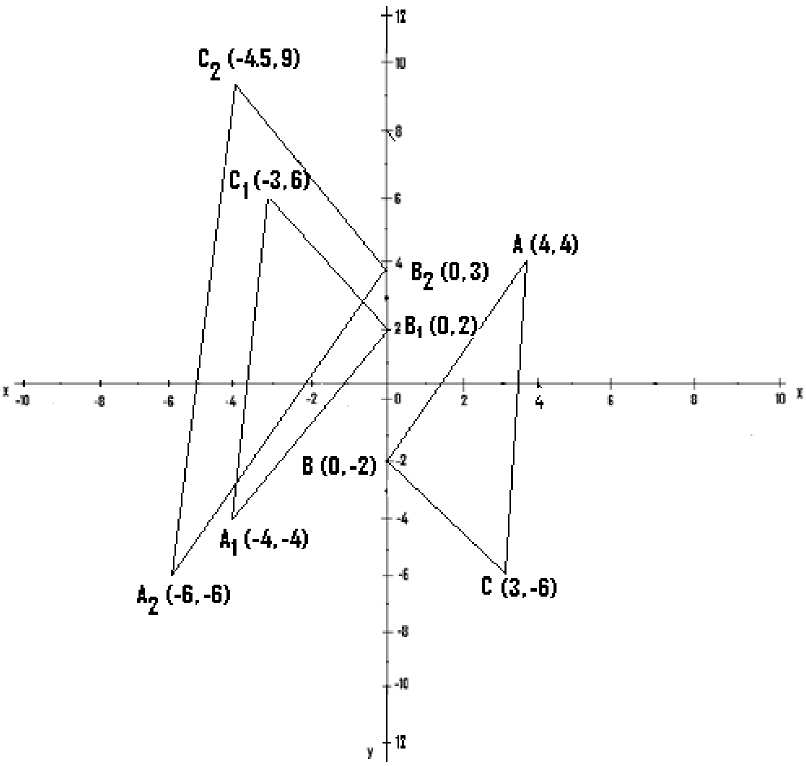
(b) Draw the image △$A_1B_1C_1$ of △ABC, after an anticlockwise rotation through 180^0 about the origin, where A $\longrightarrow A_1$, B $\longrightarrow B_1$ and C $\longrightarrow C_1$.

(c) Draw the image $\triangle A_2B_2C_2$ of $\triangle A_1B_1C_1$ after an enlargement with a scale factor of of 1.5, where $B_1 \rightarrow B_2$, $C_1 \rightarrow C_2$. and so on

(d) Find the slope of the line A_1B_1.

Ans:

N/B: (-4, -4), B_1(0, 2) and C_1(-3, 6)
A_2(-6, -6), B_2(0,3) and C_2(-4.5, 9).



(Q3)(i) Using a scale of 2cm to 2 units on each axis, draw on a sheet of graph paper two perpendicular axes ox and oy for the interval

-10 $\leq x \leq 10$ and -10 $\leq y \leq 10$.

(ii) Draw quadrilateral ABCD with the vertices

A(3, 6), B(7, 6), C(9, 2) and D(2, 2).

(iii) Draw quadrilateral$A_1B_1C_1D_1$ which is the image of quadrilateral ABCD after a clockwise rotation through 90^0, about the origin where A $\longrightarrow A_1$, B $\longrightarrow B_1$,

C $\longrightarrow C_1$ and D $\longrightarrow D_1$.

(iv) Draw quadrilateral $A_2B_2C_2D_2$ which is the image of quadrilateral ABCD after a reflection in the line Y = -x, where A $\longrightarrow A_2$, B $\longrightarrow B_2$, C $\longrightarrow C_2$ and

D $\longrightarrow D_2$.

(v) Lastly draw quadrilateral $A_3B_3C_3D_3$ which is the image of quadrilateral $A_2B_2C_2D_2$, after a reflection in the line y = 0, where $A_2 \rightarrow A_3$, $B_2 \rightarrow B_3$, $C_2 \rightarrow C_3$ and $D_2 \rightarrow D_3$.

$A_2 \longrightarrow A_3$, $B_2 \longrightarrow B_3 C_2 \longrightarrow C_3$ and D_2 --->D_3.

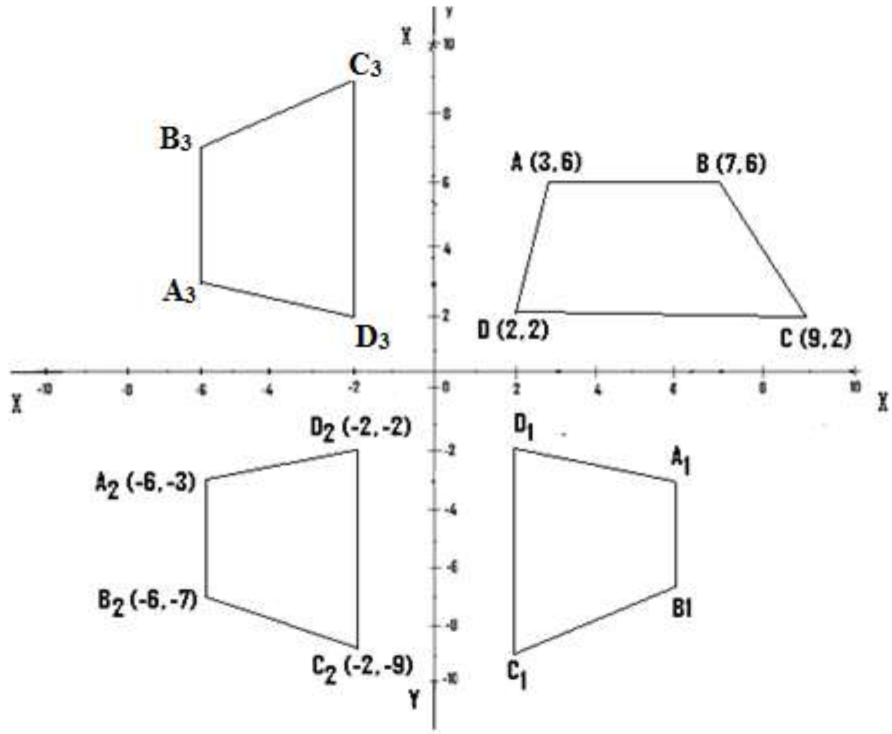
(v) Determine
  (a) the slope of the line AD. Ans: 4
  (b) the equation of line BC.
    Ans: y = -2x + 20.

Ans:

N/B: $A_1(6_1, -3)$, $B_1(6, -7)$, $C_1(2, -9)$ and $D_1(2, -2)$

$A_2(-6, -3)$, $B_2(-6, -7)$, $C_2(-2, -9)$ and $D_2(-2, -2)$

$A_3(-6, 3)$, $B_3(-6, 7)$, $C_3(-2, 9)$ and $D_3(-2, 2)$



(4) (a) Using a scale of 1cm to 1 unit on each axis, draw the ox and oy axes for the interval -5≤ x ≤8and -10≤ y ≤10. Draw

(b) the quadrilateral ABCD whose vertices are A(3, 4) B(5, 4), C(5, 2) and D(3, 2).

(c) draw the image $A_1B_1C_1D_1$ of ABCD after a reflection in the line x = 1, where A $\rightarrow A_1$,B$\rightarrow B_1$, C$\rightarrow C_1$ and D $\rightarrow D_1$.

(d) the image $A_2B_2C_2D_2$ of ABCD after a clockwise rotation through 90^0 about the the origin, where

A $\rightarrow A_2$, B $\rightarrow$ $B_2, C \rightarrow C_2$ and D$\rightarrow D_2$.

(e) the image$A_3B_3C_3D_3$ of $A_1B_1C_1D_1$ after an enlargement with a scale factor of -2, where $A_1 \rightarrow A_3$, $B_1 \rightarrow B_3$, $C_1 \rightarrow C_3$ and $D_1 \rightarrow D_3$.

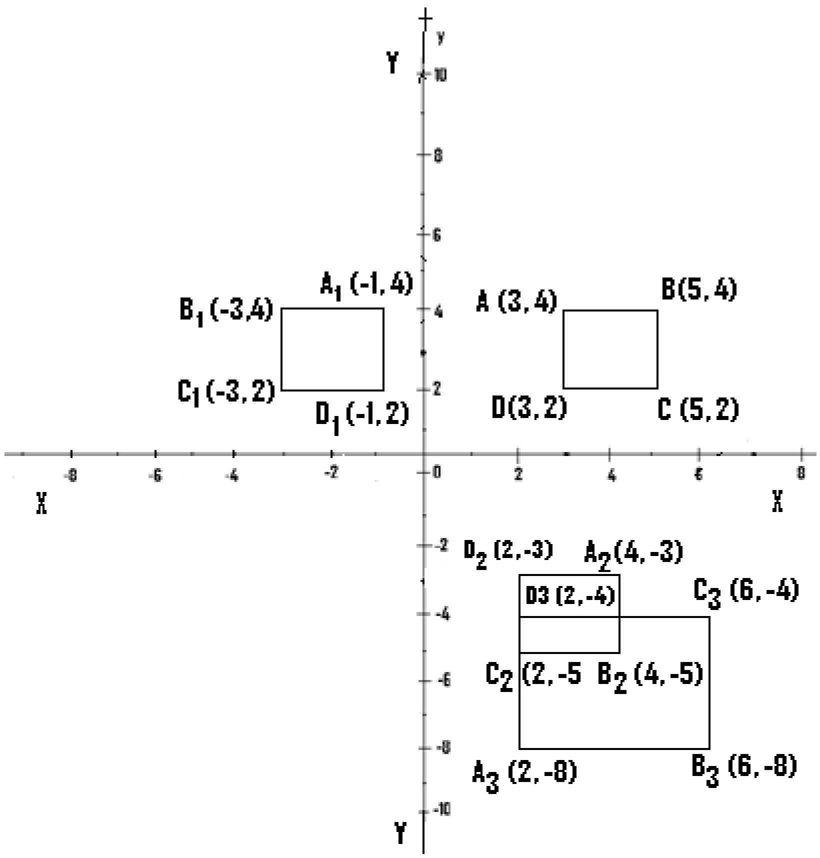
(f) Determine the length of line AD.

Ans:

N/B: A_1(-1, 4) B_1(-3, 4), C_1(-3, 2) and D_1(-1, 2)

$A_2(4, -3)$, $B_2(4, -5)$, $C_2(2, -5)$ and $D_2(2, -3)$

$A_3(2, -8)$, $B_3(6, -8)$, $C_3(6, -4)$ and $D_3(2, -4)$.

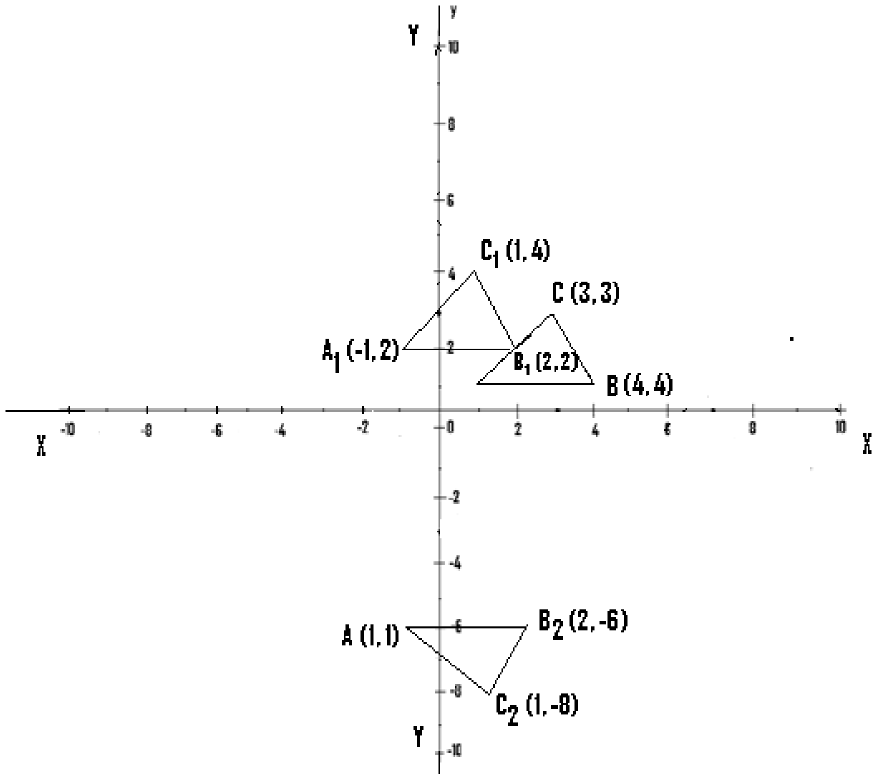


(Q5) By using an appropriate scale, draw

(i) $\triangle$ABC with vertices A(1, 1), B(4, 1) and C(3, 3).

(ii) the image $\triangle A_1B_1C_1$ of $\triangle$ABC after a translation in which B(4, 1) $\longrightarrow$ B_1(2, -2), A $\longrightarrow$ A_1 and C $\longrightarrow$ C_1.

(iii) the image $\Delta\, A_2B_2C_2$ of $\Delta\, A_1B_1C_1$ after a reflection in the line y + 2 = 0, where $A_1 \longrightarrow A_2$, $B_1 \longrightarrow B_2$ and $C_1 \longrightarrow C_2$.



Ans:

N/B: $A_1(-1, 2)$, $B_1(2, 2)$ and $C_1(1, 4)$.

$A_2(-1, -6)$, $B_2(2, -6)$ and $C_2(1, -8)$.