CHAPTER ONE

INDICES EXPONENTIAL EQUATIONS AND APPLICATION OF ALGEBRA

Indices:

1. $a^n \times a^m = a^{n+m}$
   This is the first law of indices.
2. $a^n \div a^m = a^{n-m}$
   This is the second law of indices.

Examples:

1. $2^3 \times 2^4 = 2^{3+4} = 2^7$
2. $2^6 \times 2^4 = 2^{6+4} = 2^{10}$
3. $3^4 \times 3 = 3^4 \times 3^1 = 3^{4+1} = 3^5$
4. $3^6 \times 3^2 = 3^{6+2} = 3^8$
5. $2^3 \times 2^4 \times 2^2 = 2^{3+4+2} = 2^9$
6. $4^3 \times 4 \times 4^2 = 4^3 \times 4^1 \times 4^2 = 4^{3+1+2} = 4^6$
7. $2^6 \div 2^4 = 2^{6-4} = 2^2$
8. $3^8 \div 3^2 = 3^{8-2} = 3^6$

Simplify the following:

1. $\dfrac{2^3 \times 2^6}{2^4}$

Soln.

$$\frac{2^3 \times 2^6}{2^4} = \frac{2^{3+6}}{2^4} = \frac{2^9}{2^4} = 2^9 \div 2^4 = 2^{9-4} = 2^5.$$

2. $\dfrac{3^4 \times 3^6}{3} = \dfrac{3^4 \times 3^6}{3^1} = \dfrac{3^{4+6}}{3^1} = \dfrac{3^{10}}{3^1} = 3^{10-1} = 3^9.$
3. $\dfrac{2^4 \times 2 \times 2^3}{2^6 \times 2^2} = \dfrac{2^{4+1+3}}{2^{6+2}} = \dfrac{2^8}{2^8} = 2^{8-8} = 2^0 = 1$

N/B: Any number raised to the power zero = 1.

4. $9^{a^2} \times 3^{a^{-6}} = 9 \times 3 \times a^2 \times a^{a-6} = 27 \times a^{2+-6} = 27x\, a^{2-6}$
   $= 27a^{-4}$

5. $a^{-4} \times a^{-2} = a^{-4+-2} = a^{-4-2} = a^{-6}$

6. $3a^{-4} \times 2a^{-3} = 3 \times 2\ a^{-4} \times a^{-3} = 6 \times a^{-4+-3}$
$= 6a^{-4-3} = 6a^{-7}$

N/B: $\frac{1}{a^2} = 1 \times a^{-2} = a^{-2}$

2. $\frac{1}{a^{-2}} = 1 \times a^2 = a^2$

3. $\frac{1}{3^2} = 1 \times 3^{-2} = 3^{-2}$

4. $\frac{1}{3^{-2}} = 1 \times 3^2 = 3^2$

5. $\frac{9}{3a^{-2}} = \frac{9}{3} \times a^2 = 3 \times a^2 = 3a^2$

6. $\frac{9}{3a^2} = \frac{9}{3} \times a^{-2} = 3a^{-2}$

7. $\frac{4}{2a^2} = \frac{4}{2} \times a^{-2} = 2 \times a^{-2} = 2a^{-2}$

8. $\frac{4}{2a^{-2}} = \frac{4}{2} \times a^2 = 2 \times a^2 = 2a^2$

9. $\frac{4a^{-6} \times 2a^2}{2a^{-3}} = \frac{4 \times 2 \times a^{-6} \times a^2}{2 \times a^{-3}}$
$= \frac{8 \times a^{-4}}{2 \times a^{-3}} = 4 \times a^{-4} \times a^3$
$= 4 \times a^{-4+3} = 4a^{-1}$

10. $\frac{2a^{-2} \times 6a^4}{3a} = \frac{2 \times 6 \times a^{-2} \times a^4}{3 \times a}$
$= \frac{12a^{-2+4}}{3 \times a^1} = \frac{4a^2}{a^1} = 4a^2 \times a^{-1}$
$= 4a^{2-1}$
$= 4a^1 = 4a$

11. $3a^2 b \times 4a^3 b^4 = 3 \times 4 \times a^2 \times a^3 \times b^1 \times b^4$
$= 12a^5 b^5$

12. $3a^{-3} b^2 \times 5a^{-2} b^{-4} = 3 \times 5 \times a^{-3} \times a^{-2} \times b^2 \times b^{-4}$
$= 15a^{-5} b^{-2}$

13. $\frac{3a^2 b \times 6a^3 b^4}{2ab^2} = \frac{3 \times 6 \times a^2 \times a^3 \times b \times b^4}{2 \times a \times b^2}$

$= \frac{18a^5b^5}{2 \times a \times b^2} = 9 \times a^5 \times a^{-1} \times b^5 \times b^{-2}$

$= 9a^4 b^3$

Exponential Equations:

N/B: 1. $4 = 2^2$

3. $8 = 2 \times 2 \times 2 = 2^3$
4. $32 = 2 \times 2 \times 2 \times 2 \times 2 = 2^5$
5. $64 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2^6$
6. $25 = 5 \times 5 = 5^2$
7. $125 = 5 \times 5 \times 5 = 5^3$
8. $16 = 4 \times 4 = 4^2$
9. $64 = 4 \times 4 \times 4 = 4^3$
10. $9 = 3 \times 3 = 3^2$
11. $27 = 3 \times 3 \times 3 = 3^3$
12. $81 = 9 \times 9 = 9^2$
13. $81 = 3 \times 3 \times 3 \times 3 = 3^4$

Q1. If $2^x = 8$, find x.

Soln.

Since $8 = 2 \times 2 \times 2 = 2^3$, then $2^x = 8 \Rightarrow 2^x = 2^3 \Rightarrow x = 3$.

Q2. Given that $2^{x+1} = 16$, find x.

Soln.

$16 = 2 \times 2 \times 2 \times 2 = 2^4$

$\therefore 2^{x+1} = 16 \Rightarrow 2^{x+1} = 2^4$

$\Rightarrow x + 1 = 4, \Rightarrow x = 4 - 1 = 3.$

Q3. If $4^{2x-1} = 64$, find x.

Soln.

$64 = 4 \times 4 \times 4 = 4^3$

$\therefore 4^{2x-1} = 64$

$\Rightarrow 4^{2x-1} = 4^3$

$\Rightarrow 2x - 1 = 3,$

$\Rightarrow \quad 2x = 3 + 1 = 4.$

$\therefore x = {}^4/_2 = 2.$

Q4. Given that $5^{2(n-1)} = 125$, find n.

Soln.

$125 = 5 \times 5 \times 5 = 5^3$

$\therefore 5^{2(n-1)} = 125$

$\Rightarrow 5^{2(n-1)} = 5^3$

$\Rightarrow 2(n - 1) = 3, \Rightarrow 2n - 2 = 3,$

$\therefore 2n = 3 + 2 \Rightarrow 2n = 5,$

$\therefore n = {}^5/_2 = 2.5.$

Q5. Given that $2^{x+1} = 8^x$, find x.

Soln.

$2^{x+1} = 8^x$, but $8 = 2^3$

$\Rightarrow 2^{x+1} = 2^{3x}$

$\Rightarrow x + 1 = 3x, \Rightarrow 1 = 3x - x$

$\Rightarrow 1 = 2x,$

$\Rightarrow x = {}^1/_2$ or $x = 0.5.$

Q6. If $3^{2(x-1)} = 27^x$, determine the value of x.

Soln.

$27 = 3^3$

$\therefore 3^{2(x-1)} = 27^x$

$\Rightarrow 3^{2(x-1)} = 3^{3x}$

$\Rightarrow 2(x - 1) = 3x, \Rightarrow 2x - 2 = 3x,$

$\Rightarrow 2x - 3x = 2,$

$\Rightarrow -x = 2 \Rightarrow x = -2.$

Q7. Given that $2^{x-1} = 16^{x+1}$, find the value of x.

Soln.

$16 = 2^4$

$\therefore 2^{x-1} = 16^{x+1}$

$\Rightarrow 2^{x-1} = 2^{4(x+1)}$

$\Rightarrow x - 1 = 4(x + 1),$

$\Rightarrow x - 1 = 4x + 4,$

$\Rightarrow x - 4x = 4 + 1,$

$\Rightarrow -3x = 5$

$\Rightarrow x = {}^{-5}/_{3} = -1.6.$

N/B: If there is a plus or a minus sign between a number and a letter, they must be placed within a bracket before a number can be used to multiply them.

Q8. If $3^n = 81^{n-1}$, find the value of n.

Soln.

$81 = 3^4$

$\therefore 3^n = 81^{n-1} \Rightarrow 3^n = 3^{4(n-1)}$

$\Rightarrow n = 4(n - 1),$

$\Rightarrow n = 4n - 4$

$\Rightarrow n + 4 = 4n,$

$\Rightarrow 4 = 4n - n \Rightarrow 4 = 3n,$

$\Rightarrow n = {}^{4}/_{3} = 1.3.$

Q9. If $3^{2(x-1)} = 27^{x+2}$, find the value of x.

Soln.

$27 = 3^3$

$\therefore 3^{2(x-1)} = 27^{x+2}$

$\Rightarrow 3^{2(x-1)} = 3^{3(x+2)}$

$\Rightarrow 2(x - 1) = 3(x + 2),$

$\Rightarrow 2x - 2 = 3x + 6,$

$\Rightarrow -2 = 3x + 6 - 2x,$

$\Rightarrow -2 - 6 = 3x - 2x$

$\Rightarrow - 8 = x,$

$\therefore x = - 8.$

Q10. Given that $({}^{1}/_{2})^{n+1} = {}^{1}/_{4}$, determine the value of n.

Soln.

${}^{1}/_{4} = ({}^{1}/_{2})^2$

$\therefore (1/2)^{n+1} = 1/4$
$\Rightarrow (1/2)^{n+1} = (1/2)^2$
$\Rightarrow n + 1 = 2, \Rightarrow n = 2 - 1$
$\Rightarrow n = 1.$

Q11. If $(1/3)^n = 1/9$, find n.

Soln.

$Since\ 1/9 = (1/3)^2$
$\Rightarrow (1/3)^n = 1/9 =$
$(1/3)^n = (1/3)^2 \Rightarrow n = 2.$

Q12. Given that $(1/3)^{n-1} = 1/27$, evaluate n.

Soln.

$1/27 = (1/3)^3$
$(1/3)^{n-1} = 1/27$
$\Rightarrow (1/3)^{n-1} = (1/3)^3$
$\Rightarrow n - 1 = 3 \Rightarrow n = 3 + 1 = 4$
$\therefore n = 4.$

Q13. If $(1/3)^{n-1} = (1/27)^n$, find n.

Soln.

From $(1/3)^{n-1} = (1/27)^n$
$\Rightarrow (1/3)^{n-1} = (1/3)^{3n}$
$\Rightarrow n - 1 = 3n,$
$\Rightarrow n - 3n = 1,$
$\Rightarrow -2n = 1,$
$\Rightarrow n = 1/-2 = -0.5.$

Q14. If $1/2 = (1/64)^y$, find y.

Soln.

$1/64 = (1/2)^6$
$1/2 = (1/64)^y$
$\Rightarrow (1/2)^1 = (1/2)^{6y}$
$\Rightarrow 1 = 6y \Rightarrow y = 1/6.$

N/B: Every number, fraction or decimal is raised to the power one.
$\Rightarrow 2 = 2^1$, $3 = 3^1$, $^1/_2 = (^1/_2)^1$ and $^2/_3 = (^2/_3)^1$

Q15. If $(^1/_2)^{x-1} = {}^1/_{16}$, determine x.

Soln.

$(^1/_2)^{x-1} = {}^1/_{16}$
$\Rightarrow (^1/_2)^{x-1} = (^1/_2)^4$
$\Rightarrow x - 1 = 4, \Rightarrow x = 4 + 1 = 5.$

Q16. If $(^1/_2)^{x-1} = (^1/_{16})^x$, find x.

Soln.

$(^1/_2)^{x-1} = (^1/_2)^{4 x x}$
$\Rightarrow (^1/_2)^{x-1} = (^1/_2)^{4x}$
$\Rightarrow x - 1 = 4x \Rightarrow x - 4x = 1,$
$\Rightarrow -3x = 1 \Rightarrow x = {}^{-1}/_{3.}$

Q17. If $(^1/_2)^{x-1} = (^1/_{16})^{2x}$, find x.

Soln.

$(^1/_2)^{x-1} = (^1/_2)^{4 x 2x}$
$\Rightarrow (^1/_2)^{x-1} = (^1/_2)^{8x}$
$\Rightarrow x - 1 = 8x \Rightarrow x - 8x = 1,$
$\therefore -7x = 1 \Rightarrow x = {}^{-1}/_{7.}$

Q18. If $(^1/_2)^{x-1} = (^1/_{16})^{x-2}$, find x.

Soln.

$(^1/_2)^{x-1} = (^1/_{16})^{x-2}$
$\Rightarrow (^1/_2)^{x-1} = (^1/_2)^{4(x-2)}$
$\Rightarrow x - 1 = 4(x - 2),$
$\Rightarrow x - 1 = 4x - 8, \Rightarrow x - 4x = -8 + 1,$
$\Rightarrow -3x = -7, \Rightarrow x = {}^{-7}/_{-3} = {}^7/_3 = 2.3.$

Q19. Given that $^1/_9 = (^1/_{81})^{n-1}$, find n.

Soln.

$(^1/_{81}) = (^1/_9)^2$

$^1/_9 = (^1/_{81})^{n-1}$
$\Rightarrow (^1/_9) = (^1/_9)^{2(n-1)}$
$\Rightarrow 1 = 2(n-1) = 2n - 2,$
$\therefore 1 = 2n - 2 \Rightarrow 1 + 2 = 2n,$
$\therefore 3 = 2n \Rightarrow n = {}^3/_2 = 1.5.$

Q20. Given that $(^1/_5)^{2n} = (^1/_{125})^{n-1}$, find the value of n.

Soln.

$(^1/_5)^{2n} = (^1/_{125})^{n-1}$
$\Rightarrow (^1/_5)^{2n} = (^1/_5)^{3(n-1)}$
$\Rightarrow 2n = 3(n-1),$
$\Rightarrow 2n = 3n - 3,$
$\Rightarrow 2n + 3 = 3n,$
$\Rightarrow 3 = 3n - 2n,$
$\Rightarrow 3 = n$
$\Rightarrow n = 3.$

N/B: We are now going to apply the principle that
$^1/_{a^n} = a^{-n}$ and $^1/_{a^{-n}} = a^n$.

Q1. Given that $^1/_{2^n} = 8$, find the value of n.

Soln.

Since $8 = 2^3$
$^1/_{2^n} = 8 \Rightarrow {}^1/_{2^n} = 8,$
$\Rightarrow 1 \times 2^{-n} = 8 \Rightarrow 2^{-n} = 2^3$
$\Rightarrow -n = 3 \Rightarrow n = -3.$

Q2. If $^1/_{5^x} = 25$, find x.

Soln.

$25 = 5^2$
$\frac{1}{5^x} = 25 \Rightarrow \frac{1}{5^x} = 5^2$
$\Rightarrow 1 \times 5^{-x} = 5^2 \Rightarrow 5^{-x} = 5^2$
$\Rightarrow -x = 2 \Rightarrow x = -2.$

Q3. Given that $\frac{1}{\ }$ = 8, determine the value of x.

2^{x+1s}

Soln.

$8 = 2^3$

$\frac{1}{2^{x+1}} = 8 \Rightarrow 1 \times 2^{-(x+1)} = 8,$

$\Rightarrow 2^{-(x+1)} = 8 \Rightarrow 2^{-(x+1)} = 2^3,$

$\Rightarrow -(x+1) = 3 \Rightarrow -x - 1 = 3,$

$\Rightarrow -x = 3 + 1 \Rightarrow -x = 4, \Rightarrow x = -4.$

Q4. If $\frac{1}{3^{x-1}} = 81$, find the value of x.

Soln.

$81 = 3 \times 3 \times 3 \times 3 = 3^4$

$\frac{1}{3^{x-1}} = 81 \Rightarrow 1 \times 3^{-(x+1)} = 81,$

$\Rightarrow 3^{-(x+1)} = 81 \Rightarrow 3^{-(x+1)} = 3^4,$

$\Rightarrow -(x-1) = 4 \Rightarrow -x + 1 = 4,$

$\Rightarrow -x = 4 - 1 \Rightarrow -x = 3,$

$\Rightarrow x = -3.$

Q5. If $\frac{1}{3^{2n-4}} = 27$, determine the value of n.

Soln.

$27 = 3^3$, and $\frac{1}{3^{2n-4}} = 27$

$\Rightarrow \frac{1}{3^{2n-4}} = 3^3 \Rightarrow 1 \times 3^{-(2n-4)} = 3^3$

$\Rightarrow -(2n-4) = 3, \Rightarrow -2n + 4 = 3$

$\Rightarrow -2n = 3 - 4, \Rightarrow -2n = -1$

$\Rightarrow n = \frac{-1}{-2}, \Rightarrow n = \frac{1}{2} = 0.5.$

Q6. Given that $\frac{1}{64} = 2n$, find the value of n.

Soln.

$64 = 2^6$, but since $\frac{1}{64} = 2^n$

$\Rightarrow \frac{1}{2^6} = 2^n \Rightarrow 2^{-6} = 2^n$

$\Rightarrow n = -6.$

Q7. Given that $\frac{1}{8} = 2^{x-1}$, find x.

Soln.

$8 = 2^3$, and $\frac{1}{8} = 2^{x-1}$

$\therefore \frac{1}{2^3} = 2^{x-1} \Rightarrow 2^{-3} = 2^{x-1}$

$\Rightarrow x - 1 = -3, \Rightarrow x = -3 + 1$

$\Rightarrow x = -2.$

Q8. Given $\frac{1}{8^x} = 2^{x-1}$, find x.

Soln.

$\frac{1}{8^x} = 2^{x-1} \Rightarrow 8^{-x} = 2^{x-1}$

$\Rightarrow 2^{3(-x)} = 2^{x-1} \Rightarrow 2^{-3x} = 2^{x-1}$

$\Rightarrow -3x = x - 1, \Rightarrow -3x - x = -1,$

$\therefore -4x = -1 \Rightarrow x = {}^{-1}/_{-4} = {}^{1}/_{4}.$

Q9. If $\frac{1}{125^{2x}} = 5^{3x-2}$, find the value of x.

Soln.

$\frac{1}{125^{2x}} = 5^{3x-2}$

$\Rightarrow 1 \times 125^{-2x} = 5^{3x-2}$

$\Rightarrow 125^{-2x} = 5^{3x-2}$

$\Rightarrow 5^{3(-2x)} = 5^{3x-2}$

$\Rightarrow 5^{-6x} = 5^{3x-2}$

$\Rightarrow -6x = 3x - 2,$

=> - 6x – 3x = -2, ∴ - 9x = -2,

=> x = $\frac{-2}{-9}$ ⇒ x = $\frac{2}{9}$

=> x = 0.22.

Q10. Determine the value of x which satisfies the equation $\frac{1}{4^{x-1}}$ = 2x

Soln.

$\frac{1}{4^{x-1}}$ = 2^x => $4^{-(x-1)} = 2^x$

=>$2^{-2(x-1)} = 2^x$, => -2(x – 1) = x ,

=> -2x + 2 = x => - 2x – x = - 2,

=> - 3x = -2 => x = $\frac{-2}{-3}$,

=> x = $\frac{2}{3}$ => x = 0.66.

Q11. Find the value of x which satisfies the equation

$\frac{1}{81^{2x+1}}$ = 3^{x+1}

Soln.

$\frac{1}{81^{2x+1}}$ = 3^{x+1} => 1 x $81^{-(2x+1)} = 3^{x+1}$

=> $81^{-(2x+1)} = 3^{x+1}$

=> $3^{-4(2x+1)} = 3^{x+1}$

=> - 4(2x + 1) = x + 1,

∴ - 8x – 4 = x+ 1 => $-8x - x = 1 + 4$,

∴ - 9x = 5 => x = $\frac{-5}{9}$,

=> x = - 0.6.

Q12. If $\frac{1}{81^{-2x-1}}$ = 9^{x+3}, determine the value of x.

Soln.

$\frac{1}{\quad}$ = 9^{x+3},

81^{-2x-1}

$\Rightarrow 1 \times \frac{1}{81^{-2x-1}} = 9^{x+3}$

$\Rightarrow 81^{-(-2x-1)} = 9^{x+3}$

$\Rightarrow 9^{-2(-2x-1)} = 9^{x+3}$

$\Rightarrow -2(-2x-1) = x + 3,$

$\Rightarrow 4x + 2 = x + 3,$

$\Rightarrow 4x - x = 3 - 2 = 1,$

$\Rightarrow 3x = 1 \Rightarrow x = 1/3 = 0.33.$

Q13. Given that $\frac{1}{2^{-x}} = 16$, find x.

Soln.

$\frac{1}{2^{-x}} = 16 \Rightarrow 1 \times 2^{x} = 16,$

$\Rightarrow 2^{x} = 16 \Rightarrow 2^{x} = 2^{4}$

$\Rightarrow x = 4.$

Q14. Find the value of x which satisfies the equation $\frac{1}{5^{-2x}} = 625.$

Soln.

$\frac{1}{5^{-2x}} = 625$

$\Rightarrow 1 \times 5^{2x} = 625 \Rightarrow 5^{2x} = 5^{4}$

$\Rightarrow 2x = 4, \Rightarrow x = \frac{4}{2}$

Q15. Given that $\frac{1}{4^{-(x-2)}} = 64$, find x.

Soln.

$\frac{1}{4^{-(x-2)}} = 64 \Rightarrow 1 \times 4^{(x-2)} = 64,$

$\Rightarrow 4^{(x-2)} = 64 \Rightarrow 4^{(x-2)} = 4^{3}$

=> x – 2 = 3, => x = 3 + 2 = 5.

=> x = 5.

Q16. Find n, if $\frac{1}{4^{-2(n+4)}} = 16$

Soln.

$\frac{1}{4^{-2(n+4)}} = 16 \Rightarrow 1 \times 4^{2(n+4)} = 16,$

$\Rightarrow 4^{2(n+4)} = 4^2, \Rightarrow 2(n + 4) = 2,$

$\Rightarrow 2n+8 = 2 \Rightarrow 2n = 2 - 8,$

$\Rightarrow 2n = -6 \Rightarrow n = \frac{-6}{2} = -3, \therefore n = -3.$

Q17. Given that $\frac{1}{3^{-4(x-1)}} = 81^{2x}$, find the value of x.

Soln.

$\frac{1}{3^{-4(x-1)}} = 81^{2x} \Rightarrow 1 \times 3^{4(x-1)} = 81^{2x},$

$\Rightarrow 3^{4(x-1)} = 3^{4(2x)} \Rightarrow 3^{4(x-1)} = 3^{8x},$

$\Rightarrow 4(x-1) = 8x \Rightarrow 4x - 4 = 8x,$

$\Rightarrow -4 = 8x - 4x \Rightarrow -4 = 4x,$

$\Rightarrow x = -4/4 = -1 \Rightarrow x = -1.$

Q18. Given that $\frac{1}{3^{-2(x+1)}} = 81^{x-2}$, find x.

Soln.

$\frac{1}{3^{-2(x+1)}} = 81^{x-2} \Rightarrow 1 \times 3^{2(x+1)} = 81^{x-2},$

$\Rightarrow 3^{2(x+1)} = 81^{x-2}, \Rightarrow 3^{2(x+1)}$

$= 3^{4(x-2)}, \Rightarrow 2(x+1) = 4(x-2), \Rightarrow 2x + 2 = 4x - 8,$

$\Rightarrow 2x - 4x = -8 - 2, \Rightarrow -2x = -10 \Rightarrow x = \frac{-10}{-2} \Rightarrow x = 5.$

Q19. Given that $\frac{1}{3^{-x+1}} = \frac{1}{9}$, find the value of x.

Soln.

$\frac{1}{3^{-x+1}} = \frac{1}{9} \Rightarrow \frac{1}{3^{-x+1}} = \frac{1}{3^2} \Rightarrow -x + 1 = 2, \Rightarrow -x = 2 - 1$

=> - x = 1, => x = - 1.

Q20. If $\frac{1}{9^{2(n-1)}} = \frac{1}{81}$, find n.

Soln.

$\frac{1}{9^{2(n-1)}} = \frac{1}{81} \Rightarrow \frac{1}{9^{2(n-1)}} = \frac{1}{9^2} \Rightarrow 2(n-1) = 2,$

$\Rightarrow 2n - 2 = 2 \Rightarrow 2n = 2 + 2,$

$\Rightarrow 2n = 4 \Rightarrow n = 2$

Any number raised to the power zero = 1.

N/B: At certain times, it will be necessary to convert two numbers into a particular factor, before solving the question.

Q1. If $4^x = 8$, find x.

Soln.

$4 = 2^2$ and $8 = 2^3$

$\therefore 4^x = 8 \Rightarrow 2^{2x} = 2^3 \Rightarrow 2x = 3, \Rightarrow x = {}^3/_2 = 1.5.$

$\therefore x = 1.5.$

Q2. If $27^x = 81$, find x.

Soln.

$27 = 3^3$ and $81 = 3$. Since $27^x = 81 \Rightarrow 3^{3x} = 3^4,$

$\Rightarrow 3x = 4 \Rightarrow x = 4/3 = 1.33, \Rightarrow x = 1.33.$

Q3. Given that $25^{x+1} = 125$, determine the value of x.

Soln.

$25 = 5^2$ and $125 = 5^3$

$25^{x+1} = 125 \Rightarrow 5^{2(x+1)} = 5^3,$

$\Rightarrow 2(x+1) = 3 \Rightarrow 2x + 2 = 3, \Rightarrow 2x = 3 - 2 = 1,$

$\therefore 2x = 1 \Rightarrow x = ½ = 0.5.$

Q4. If $16^{n+1} = 8$, find n.

Soln.

$16 = 2^4$ and $8 = 2^3$

$\therefore 16^{n+1} = 8 \Rightarrow 2^{4(n+1)} = 2^3, \Rightarrow 4(n+1) = 3 \Rightarrow 4n + 4 = 3,$

$\Rightarrow 4n = 3 - 4 = -1, \ \therefore \ 4n = -1 \Rightarrow n = \frac{-1}{4} = -0.25.$

Q5. Given that $81^{n+1} = 27^n$, find n.

Soln.

$81 = 3^4$ and $27 = 3^3$

$\therefore 81^{n+1} = 27^n \Rightarrow 3^{4(n+1)} = 3^{3n}$

$\Rightarrow 4(n+1) = 3n, \Rightarrow 4n+4 = 3n \Rightarrow 4n - 3n = -4,$

$\Rightarrow n = -4.$

Q6. Given that $n^2 = 25$, find the value of n.

Soln.

$25 = 5^2$, but $n^2 = 25$,

$\Rightarrow n^2 = 5^2 \Rightarrow n = 5.$

Q7. If $x^2 = 4$, find x.

Soln.

$x^2 = 4 \Rightarrow x^2 = 2^2 \Rightarrow x = 2.$

Q8. Given that $4n^2 = 100$, find the value of n.

Soln.

$4n^2 = 100 \Rightarrow n^2 = \frac{100}{4} = 25,$

$\Rightarrow n^2 = 25 \Rightarrow n^2 = 5^2 \Rightarrow n = 5.$

Q9. If $n^3 = 8$, find n.

Soln.

$n^3 = 8 \Rightarrow n^3 = 2^3 \Rightarrow n = 2.$

Q10. If $5n^3 = 320$, find n.

Soln.

$5n^3 = 320 \Rightarrow n^3 = \frac{320}{5}$,
$\Rightarrow n^3 = 64 \Rightarrow n^3 = 4^3, \Rightarrow n = 4$

Q11. If $\frac{1}{2}(81^n) = \frac{1}{54}$, find the value of n.

Soln.

$\frac{1}{2}(81^n) = \frac{1}{54}$.
Multiply through using 2 in order to remove the $^1/_2$.
$\Rightarrow 2 \times \frac{1}{2}(81^n) = 2 \times \frac{1}{54}$
$\Rightarrow 81^n = \frac{2}{54}, \Rightarrow 81^n = \frac{1}{27},$
$\Rightarrow (3^4)^n = \frac{1}{3^3}$, $\Rightarrow 3^{4n} = 3^{-3},$
$\Rightarrow 4n = -3 \Rightarrow n = \frac{-3}{4} \Rightarrow n = -0.75.$

Q12. Given that $\frac{1}{5}(2^x) = \frac{1}{40}$, determine the value of x.

Soln.

$\frac{1}{5}(2^x) = \frac{1}{40}$
Multiply through using 5 in order to remove the 5.
$\Rightarrow 5 \times \frac{1}{5}(2^x) = 5 \times \frac{1}{40}$
$\Rightarrow 2^x = \frac{5}{40} \Rightarrow 2^x = \frac{1}{8},$
$\Rightarrow 2^x = \frac{1}{2^3} \Rightarrow 2^x = 2^{-3} \Rightarrow x = -3.$

Q13. If $\frac{1}{2}(8^{n-1)} = 32$, find the value of n.

Soln.

$\frac{1}{2}(8^{n-1}) = 32.$
Multiply through using 2,
$\Rightarrow 2 \times \frac{1}{2}(8^{n-1}) = 2 \times 3^2, \Rightarrow 8^{n-1} = 64, \Rightarrow 8^{n-1} = 8^2,$

$\Rightarrow$ n – 1 = 2 $\Rightarrow$ n = 2 + 1, $\Rightarrow$ n = 3.

Application of Algebra:

Q1. Simplify $\sqrt{\frac{x^3y^2}{xy^7}}$

Where $x > 0$ and $y > 0$.

N/B: $\sqrt{a} = a^{1/2}$, $\sqrt{3} = 3^{1/2}$ and $\sqrt{20} = 20^{1/2}$.

Soln.

$$\sqrt{\frac{x^3y^6}{xy^7}} = \sqrt{\frac{x^3y^6}{x^1y^7}}$$

$$= \sqrt{x^3 \times x^{-1} \times y^6 \times y^{-7}}$$

$$= \sqrt{x^2y^{-1}} = (x^2y^{-1})^{1/2}$$

$$= (x^2y^{-1})^{0.5} = x^{2 \times 0.5}\, y^{-1 \times 0.5}$$

$$= x^1 \times y^{-0.5} = xy^{-0.5}$$

Q2. Simplify

$\frac{m+1}{m-1} - \frac{m-1}{m+1} + \frac{4}{m^2-1}$.

N/B: $m^2 - 1 = m^2 - 1^2 = (m+1)(m-1)$.

Find the L.C.M, which is equal to (m + 1)(m – 1).

Soln.

$$\frac{m+1}{m-1} - \frac{m-1}{m+1} + \frac{4}{(m+1)(m-1)}$$

$$\frac{(m+1)(m+1) - (m-1)(m-1) + 4}{(m+1)(m-1)}$$

$$=\frac{m^2+m+m+1-(m^2-m-m+1)+4}{(m+1)(m-1)}$$

$$=\frac{m^2+2m+1-m^2+m+m-1+4}{(m+1)(m-1)}$$

$$=\frac{m^2-m^2+2m+m+m+1-1+4}{(m+1)(m-1)}$$

$$=\frac{4m+4}{(m+1)(m-1)}$$

N/B: $m^2-m^2=0$ and $2m+m+m=4m$.

Q3. Simplify

$\sqrt{5\frac{44}{49}} \times (11\frac{1}{3})^{-1}$, and leave your answer in the form $\frac{p}{q}$, where p and q are positive integers.

Soln.

$$\sqrt{5\frac{44}{49}} = \sqrt{\frac{289}{49}} = \frac{\sqrt{289}}{\sqrt{49}}$$

$$=\frac{17}{7}.$$

$$(11\frac{1}{3})^{-1} = \frac{1}{(11\frac{1}{3})^1} = \frac{1}{\left(11\frac{1}{3}\right)}$$

$$=\frac{1}{\left(\frac{34}{3}\right)}.$$

$$\Rightarrow\sqrt{5\frac{44}{49}} \times \left(11\frac{1}{3}\right)^{-1}$$

$= \frac{17}{7} \times \frac{1}{(\frac{34}{3})} = \frac{17}{7} \times \frac{3}{34}$

$= \frac{3}{14}$, which is of the form $\frac{p}{q}$, where p = 3 and q = 14.

N/B: (5 x 49) + 44 = 245 + 44 == 289.

Also (11 x 3) + 1 = 33 + 1 = 34.

Q4. In the sequence a_1 , a_2 , a_3 , a_4 , a_5 , each number is twice the preceding number. If $a_5 - a_1 = 20$, determine the value of a_1.

Soln.

Since each number is twice the preceding one and the first number is a_1 , then the second one will be $2a_1$, the third one will be $2(2a_1) = 4a_1$, the fourth one will be $2(4a_1) = 8a_1$, and finally the fifth number will be $2(8a_1) = 16a_1$.

$\therefore a_1 = a_1$ and $a_5 = 16a_1$.

But $a_5 - a_1 = 20, \Rightarrow 16a_1 - a_1 = 20 \Rightarrow 15a_1 = 20$

$\Rightarrow a_1 = \frac{20}{15} = 1.33$.

Q5. Two tanks X and Y are filled to a capacity with petrol. Tank X holds 600 litres more than tank Y. If 100 litres of petrol were pumped out of each tank, tank x would contain three times as much petrol as in tank Y. Find the capacity of each tank.

Soln.

Let the initial amount of petrol in tank Y = K litres. Since tank X contains 600 litres more than tank Y, then the initial amount of petrol in the tank X = (K+600) litres. When 100 litres of petrol is pumped from each tank, then

(I) the amount of petrol left in tank X = (K + 600 – 100) litres. = (K + 500) litres.

(II) the amount of petrol left on tank Y = (K – 100) litres.

But after the removal of these 100 litres, the amount of petrol left in tank X is three times the amount left in Y.
Now, amount of petrol left in tank X = (K + 500) litres, and that left in tank Y = (K – 100) litres.
Since the amount of petrol left in tank X after the removal = three times that left in tank Y, then K + 500 = 3(K – 100) $\Rightarrow$ K + 500 = 3K – 300, $\Rightarrow$ 500 = 3K – 300 – K, $\Rightarrow$ 500 + 300 = 3K – K, $\Rightarrow$ 800 = 2K $\Rightarrow$ K = 400
$\therefore$ The capacity of tank Y = K = 400 litres.
Also, the capacity of tank X = k + 600 = 400+600
=1000 litres.

Q6. The sum of the ages of two brothers is 38 years. Four years ago, the ages of the older brother was the square of the younger brother`s age. Find their ages.

Soln.
Let y = the present age of the senior brother, and let x = the present age of the junior brother. Since the sum of their ages is 38 years $\Rightarrow$ x + y = 38…….eqn (1).
Four years ago, the age of the senior brother = y – 4.
Also four years ago, the age of the junior brother = x - 4.
Since 4 years ago, the age of the senior brother was the square of that of the junior brother, then y – 4 = $(x-4)^2$,
$\Rightarrow$ y – 4 = $x^2 - 8x + 16$
$\Rightarrow$ y = $x^2 - 8x + 16 + 4$,
$\Rightarrow$ y = $x^2 - 8x + 20$ ... ... eqn (2).
But from eqn (1), x + y = 38 $\Rightarrow$ y = 38 – x.
Substitute this into eqn (2)
i.e y = $x^2 - 8x + 20$
$\Rightarrow$ 38 – $x = x^2 - 8x + 20$,
$\Rightarrow$ 38 = $x^2 - 8x + 20 + x$
$\Rightarrow$ 38 = $x^2 - 8x + x + 20$,
$\Rightarrow$ 38 = $x^2 - 7x + 20$
$\Rightarrow$ 0 = $x^2 - 7x + 20 - 38$,

$\Rightarrow 0 = x^2 - 7x - 18$, which is a quadratic in x, in which a = 1, b = -7 and c = -18.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\Rightarrow x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(1)(-18)}}{2(1)}$$

$$\Rightarrow x = \frac{7 \pm \sqrt{49 + 72}}{2}$$

$$\Rightarrow x = \frac{7 \pm \sqrt{121}}{2}$$

$$\Rightarrow x = \frac{7 \pm 11}{2}$$

Considering the positive sign $\Rightarrow x = \frac{7 + 11}{2} = \frac{18}{2} = 9$.

Considering the negative sign $\Rightarrow x = \frac{7 - 11}{2} = \frac{-4}{2} = -2$,

Since we do not have negative age, then x = -2 is inadmissible $\Rightarrow x$ = 9. From eqn (1) x + y = 38

$\Rightarrow$ 9 + y = 38 $\Rightarrow$ y = 29.

The age or the present age of the junior brother = x = 9yrs, and that of his senior brother = y = 29yrs.

Q7. The sum of Kofi`s age and that of John`s is 22 years. But two years ago, when 2 years was added to Kofi`s age, his age became the same as that of John.

(a) Determine their present ages.

(b) What will be Kofi`s age in ten years time?

Soln.

Let x = Kofi`s present age and y = John`s present age. Since the sum of their age is 22,

$\Rightarrow$ x + y = 22…..…….eqn (1).

Two years ago, Kofi`s age = x – 2 and adding 2 to it, this becomes x – 2 + 2 = x. In short, after adding 2 years to Kofi`s age two years ago, his age = x.

John`s age two years ago = y – 2.

Since the ages of these two boys was the same two years ago, after the addition of 2 years to that of Kofi`s, then x = y – 2……..eqn (2).

Substitute this into eqn (1)

i.e x + y = 22 $\Rightarrow$ y – 2 + y = 22,

$\Rightarrow$ 2y – 2 = 22

$\Rightarrow$ 2y = 22 + 2 = 24, $\Rightarrow$ y = 12.

From eqn (1), x + y = 22

$\Rightarrow$ x + 12 = 22 $\Rightarrow$ x = 10.

(a) Kofi`s present age = x = 10 yrs and John`s present age = y = 12yrs.

(b) Kofi`s age in 10 yrs time = 10 + 10 = 20 yrs.

Q8. Kofi is twice as old as John. If four years ago, Kofi was four times as old as John, determine

(a) the sum as well as the product of their present ages.

(b) their ages two year ago.

(c) their ages in five years time.

Soln.

Let John`s age = x.

Then since Kofi is twice as old as John,$\Rightarrow$ Kofi`s age = 2x. John`s age four years ago = x – 4 and Kofi`s age four years ago = 2x – 4.

Since four years ago Kofi was four times as old as John $\Rightarrow$ Kofi`s age = 4 $\times$ John`s age

$\Rightarrow 2x - 4 = 4 \times (x - 4)$,

$\Rightarrow 2x - 4 = 4x - 16 \Rightarrow 2x - 4x = -16 + 4$,

$\Rightarrow -2x = -12 \Rightarrow x = \frac{-12}{-2} = 6$.

(a) John`s age = x = 6 yrs and Kofi`s age = 2x = 2(6) = 12 yrs. The sum of their ages = 12 + 6 = 18 yrs and the product of their ages = $12 \times 6 = 72$ yrs.

(b) The age of Kofi two years ago = 12 -2 = 10 yrs, and that of John, two years ago = 6 – 2 = 4 yrs.

(c) In 5 years time, Kofi`s age will be 12 + 5 = 17 yrs and that of John will be 6 + 5 = 11 yrs.

Q9. The product of the ages of two brothers is 50yrs, and the difference between their ages is 5 yrs. Determine their ages two years ago.

Soln.

Let x = the age of the senior brother and y = that of the junior brother. Since the product of their ages = 50 yrs

$\Rightarrow x \times y = 50 \Rightarrow xy = 50$ ... ... eqn (1).

Since the difference between their ages = 5 yrs,

$\Rightarrow x - y = 5$……eqn (2).

From eqn (1), $xy = 50 \Rightarrow x = \frac{50}{y}$. Substitute this into eqn (2)

i.e $x - y = 5 \Rightarrow \frac{50}{y} - y = 5$. Multiply through this using y

$\Rightarrow y \times \frac{50}{y} - y \times y = y \times 5,$

$\Rightarrow 50 - y^2 = 5y, \Rightarrow 50 - y^2 - 5y = 0,$

$\Rightarrow -y^2 - 5y + 50 = 0$, which is a quadratic in y in which a = -1, b = -5, and c = 50.

$$y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\Rightarrow y = \frac{-(-5) \pm \sqrt{25 - 4(-1)(50)}}{2(-1)}$$

$$\Rightarrow y = \frac{5 \pm \sqrt{225}}{-2}$$

$$\Rightarrow y = \frac{5 \pm 15}{-2}$$

Considering the positive sign,

$$\Rightarrow y = \frac{5 + 15}{-2} = -10$$

Considering the negative sign,

$$\Rightarrow y = \frac{5 - 15}{-2} = \frac{-10}{-2} = 5.$$

Since age cannot be negative, then x = -10 is inadmissible

$\Rightarrow x = 5.$

Put x = 5 into eqn (2) i.e $x - y = 5 \Rightarrow x - 5 = 5$

$\Rightarrow x = 10.$

The present age of the senior brother = x = 10.

His age two years ago = 10 – 2 = 8 yrs.

The present age of the junior brother = y = 5, and his age two years ago = 5 – 2 = 3 yrs.

Q10. The product of the ages of Ama and Esi is 75 yrs, but Esi is thrice as old as Ama. Determine the sum of their ages in 4 years time.

Soln.

If Ama`s age = x, then Esi`s age = 3x.
Since the product of their ages = 75 yrs,
$\Rightarrow x \times 3x = 75, \Rightarrow 3x^2 = 75 \Rightarrow x^2 = \frac{75}{3},$
$\Rightarrow x^2 = 25 \Rightarrow x = \sqrt{25},$
$\Rightarrow x = 5.$
Ama`s present age = x = 5 yrs. In 4 yrs time, her age will be 5 +4 = 9 yrs. Esi`s present age = 3x = 3x5 = 15yrs and in4 yrs time, her age will be 15+ 4 = 19yrs. Sum of their age in 4 yrs time = 9 + 19 = 22 yrs.

(Q11) Three times the age of Filicia is four more than the age of Asare. In three years, the sum of their ages will be 30 years. Find their present ages.

Soln;

Let x = the present age of Filicia.

Then three times her age = 3x.

Since 3 times Filicia`s age is 4 more than Asare`s age, then Asare`s age is 4 less than three times Filicia`s age.

∴ Asare`s present age = 3x – 4.

Filicia`s age in 3 year's time = 3x + 3, and Asare`s age in 3 years time = 3x – 4 + 3 = 3x – 1.

Since the sum of their ages in three years time will be 30 years, then 3x + 3 + 3x – 1 = 30,

=> 6x + 2 = 30 => 6x = 30 – 2 = 28,

=> x = $\frac{28}{6}$ = 4.7 yrs.

Filicia`s present age = x = 4.7 yrs and Asare`s present age = 3x – 4 = 3(4.7) – 4 = 14 – 4 = 10 yrs.

Q12. Kofi and Jojo are carrying loads but Kofi`s load is 2kg heavier than that of Jojo. When the load being carried by each boy is reduced by 5kg, Kofi`s load became twice as heavy as Jojo`s own. Determine the weight of the load being carried by each boy.

Soln.

Let the weight of Jojo`s load = x. Then since Kofi`s load is 2kg heavier than that of Jojo $\Rightarrow$ weight of Kofi`s load =
x + 2. When the load of each boy is reduced by 5kg, then the weight of Jojo`s load = x – 5, and that of Kofi
= (x +2) – 5 = x + 2 – 5 = x – 3. Since after this, Kofi`s load became twice as heavy as that of Jojo, then x – 3 = 2(x – 5),
$\Rightarrow$ x – 3 = 2x – 10, $\Rightarrow$ x – 2x = -10 + 3, $\Rightarrow$ -x = -7 $\Rightarrow$ x = 7.
Weight of Jojo`s load = x = 7kg.
Weight of Kojo`s load = x + 2 = 7 + 2 = 9kg.

Q13. The difference between the ages of Ama and John is 10 years. If 5 years ago, Ama`s age was twice that of John, determine their present ages.

Soln.

Let Ama`s age = x and that of John = y. Since the difference between their ages is 10, then
x – y = 10……..eqn (1).
Ama`s age 5 yrs ago = x – 5, and John`s age 5 yrs ago = y – 5.
Since 5 yrs ago, Ama`s age was twice as that of John`s, then x – 5 = 2(y – 5), $\Rightarrow$ x – 5 = 2y – 10 $\Rightarrow$ x – 2y = -10 + 5 $\Rightarrow$
x – 2y = -5……eqn (2).
Solving eqn (1) and eqn (2) simultaneously $\Rightarrow$ x = 25

and y = 15.
Ama`s age = x = 25 yrs.
John`s age = y = 15 yrs.

Q14. Abu and Ibrahim are each carrying mangoes in a basket, but the number of mangoes being carried by Ibrahim is 5 less than what Abu is carrying. If 5 mangoes are removed from each basket, the number of mangoes being carried by Abu will be twice as that being carried by Ibrahim. Find the number of mangoes being carried by each of the two boys.

Soln.

Let x = the number of mangoes being carried by Abu.
Then the number of mangoes being carried by Ibrahim
= x – 5.
If 5 mangoes are removed from each basket, then the number of mangoes being carried by Abu = x – 5, and that being carried by Ibrahim = (x – 5) -5 = x – 5 – 5 = x – 10.
Since after this, the number of mangoes being carried by Abu is twice of that being carried by Ibrahim, then x – 5 = 2(x – 10) $\Rightarrow$ x – 5 = 2x – 20 $\Rightarrow$ x – 2x = -20 + 5
$\Rightarrow$ -x = -15 $\Rightarrow$ x = 15.

$\Rightarrow$ The number of mangoes being carried by Abu = x = 15,
And that being carried by Ibrahim = x – 5 = 15 – 5 = 10 mangoes.

Q15. If $\frac{3}{4}$ of a number added to $\frac{5}{6}$ gives the same result as subtracting $\frac{7}{8}$ of the number from $20^{1}/_{3}$, find the number.

Soln.

Let x = the number.
Then $\frac{3}{4}$ of the number = $\frac{3}{4}x$,
and $\frac{7}{8}$ of the number = $\frac{7}{8}x$.
Adding $\frac{3}{4}$ of the number to $\frac{5}{6} \Rightarrow \frac{5}{6} + \frac{3x}{4} = 0.83 + 0.75x$.
Also subtracting $\frac{7}{8}$ of the number from $20^{1}/_{3} \Rightarrow 20^{1}/_{3} - \frac{7}{8}x$

$= \frac{61}{3} - \frac{7}{8}x = 20.3 - 0.875x.$

Since the addition of $\frac{3}{4}$ of the number to $\frac{5}{6}$ gives the same result as subtracting $\frac{7}{8}$ of the number from $20^{1}/_{2}$, then

$0.83 + 0.75x = 20.3 - 0.875x,$

$\Rightarrow 0.75x + 0.875x = 20.3 - 0.83,$

$\Rightarrow 1.625x = 19.47 \Rightarrow x = \frac{19.47}{1.625} = 11.98 = 12\ approx.$

The number = 12.

SUBSTITUTION IN ALGEBRA:

Q1. Evaluate, correct to two decimal places $q = \frac{y(x-m)}{m-p}$,

When y = 2.5, m = 15, x = 10 and p = 18.

Soln.

When y = 2.5, m = 15, x = 10 and p = 18, then $q = \frac{y(x-m)}{m-p}$

$= \frac{2.5(10-15)}{(15-18)} = \frac{2.5(-5)}{-3} = \frac{-12.5}{-3}$

$= 4.17$

Q2. Given $Q = \frac{(2a-b)^2}{b^2-a}$, determine the value of Q when a = 3 and b = 2.

Soln.

If a = 3 and b = 2, then $Q = \frac{(2a-b)^2}{b^2-a} = \frac{\{2(3)-2\}^2}{2^2-3}$

$= \frac{\{6-2\}^2}{4-3} = \frac{4^2}{1} = \frac{16}{1} = 16.$

Q3. Given that $T = \frac{3x-2y^2}{(x^2+1)}$, evaluate T when x = -2 and y = 4.

Soln.

If x = -2 and y = 4, => $T = \frac{3x-2y^2}{(x^2+1)}$

$= \frac{3(-2) - 2(4)^2}{\{(-2)^2 + 1\}} = \frac{-6 - 2(16)}{\{4 + 1\}}$

$= \frac{-6 - 32}{5} = \frac{-38}{5} = -7.6$

Q4. Simplify $\frac{2(4x + 1) + y}{(2x^2 + 1)(x - y)^2}$, when x =1 and y =3

Soln.

When x = 1 and y = 3,

$=> \frac{2\{4(1) + 1\} + 3}{\{2(1)^2 + 1\}\{1 - 3\}^2} = \frac{2\{4+1\} + 3}{\{2(1) + 1\}\{-2\}^2} = \frac{2\{5\}+3}{\{2+1\}\{4\}} = \frac{10+3}{\{3\}\{4\}} = \frac{13}{12} = 1.08.$

Q5. Given x = -2 and y = -1, simplify $\frac{(x + 2)(2y)}{1 + x^2y^2}$.

Soln.

If x = -2 and y = -1, then $\frac{(x + 2)(2y)}{1 + x^2y^2}$

$= \frac{\{(-2) + (2)\}\{2(-1)\}}{1 + (-2)^2(-1)^2} = \frac{\{-2+2\}\{-2\}}{1 + (4)(1)}$

$= \frac{\{0\}\{-2\}}{1 + 4} = \frac{0}{5} = 0.$

.

Q6. If a = 2, b = -3 and c = -1, simplify $\frac{a(2b - c)}{4ab(3c - a)}$

Soln.

If a = 2, b = -3 and c = -1, then $\frac{a(2b - c)}{4ab(3c - a)}$

$= \frac{2\{2(-3) - (-1)\}}{4(2)(-3)\ \{3(-1) - (2)\}} = \frac{2\{-6 - (-1)\}}{-24\{-3 - 2\}}$

$= \frac{2\{-6 + 1\}}{-24\{-5\}} = \frac{2\{-5\}}{120} = \frac{-10}{120} = -0.083$

Q7. Determine the value of $2a^2b^3 - (3a + b)$, when a = -1 and b = -2.

N/B: $(-2)^3 = -2 \times -2 \times -2 = 4 \times -2 = -8.$

Soln.

If a = -1 and b = -2, then $2a^2b^3 - (3a + b)$

$= 2(-1)^2(-2)^3 - \{3(-1) + (-2)\}$

$= 2(1)(-8) - \{-3 + (-2)\}$

$= -16 - \{-3\ -2\}$

$= -16 - \{-5\}$

$= -16 + 5 = -11.$

Questions.

Simplify the following:

1. $4a^2b \times 2a^3b$, Ans: $8a^5b^2$
2. $5x^{-4}y^2 \times 3x^{-2}y^{-6}$, Ans: $15x^{-6}y^{-4}$
3. $\frac{8a^2b^{-1}}{2a^3}$ Ans: $4a^{-1}b^{-1}$
4. $\frac{3a^2 \times 5a^6}{a^{-2}}$ Ans: $15a^{10}$
5. $\frac{3a^2b^{-1} \times 4a^6b}{6a^2b^{-3}}$, Ans: $2a^6b^3$
6. $\frac{2a^{-2}b^3 \times 8a^{-4}b^6}{4a^{-2}b^4}$, Ans: $4a^{-4}b^5$
7. $\frac{3a^{-2}b^{-4} \times 4a^{-3}b^6}{6a^3 \times 2ab^2}$, Ans: a^{-9}
8. If $2^{x-1} = 32$, find x.
   Ans: x = 6
9. If $2^{2(x-1)} = 64$, find the value of x.
   Ans: x = 4
10. Given that $3^{3(2x-1)} = 81$, determine the value of x.
    Ans: x = 1.2
11. Given that $2^{2x-1} = 32^{x-1}$, find x.
    Ans: x = 1.3
12. Given that $(\frac{1}{3})^n = (\frac{1}{81})$, find n.
    Ans: n = 4
13. If $\left(\frac{1}{2}\right)^{x-1} = \frac{1}{64}$, find the value of x.
    Ans: x = 7

Q14. If $\left(\frac{1}{4}\right)^{2n-1} = \frac{1}{64}$, find the value of n.

Ans : n = 2

Q15. Given that $\left(\frac{1}{2}\right)^{9x-1} = \left(\frac{1}{16}\right)^{2x}$, determine the value of x.

Ans : x = 1.

Q16. Given that $\frac{1}{3^{2x-1}} = 81$, determine the value of x.

Ans : x = -1.5.

Q17. Find y, given that $\frac{1}{2^{3y}} = 16$.

Ans: y = -1.3.

Q18. Find n, given that $\frac{1}{3^{-3n}} = 27$.

Ans: n =1 .

Q19. If $\frac{1}{5^{-3x+1}} = 25$,. find x.

Ans. X = 1.

Q20. If $\frac{1}{5^{-(2x-1)}} = 125,$ find x.

Ans: x = 2.

Q21. If $\frac{1}{3^{2x-1}} = \frac{1}{27}$ find the value of x.

Ans: x = 2.

Q22. Given that $\frac{1}{6^{2(n+1)}} = \frac{1}{36}$, find the value of n.

Ans: n = 0.

Q23. Twice the product of Esi`s age and that of Ama is 40 years, and the difference between their ages is 8 years. If Esi is older than Ama, calculate their present ages.

Ans: Esi`s age = 10 yrs.

Ama`s age = 2 yrs.

Q24. The sum of one third Musa`s age and half of that of Atta is 12 years. But 4 years ago, the difference between their ages was 6 years. If Musa is older than Atta, determine their ages.

Ans: Musa`s age = 18 yrs.

Atta`s age = 12 yrs.