**CHAPTER THIRTEEN**

**SURDS**

When a square roots sign is squared or raised to the second power, the square root sign disappears.

(1) ( √2)2 = 2 (2)(√3)2 = 3

(3) (√a)2 = a (4)(√b)2 = b

(5) 2(√3)2 = 2 (3) = 6

(6) 5(√2)2 = 5 (2) = 10

(7) 2(√a)2 = 2 (a) = 2a

(8) (2√3)2 = 22 (√3)2 = 4 (3) = 12

(9) (5√2)2 = 52 (√2)2 = 25 (2) = 50

(10) (a√b)2 = a2 (√b)2 = a2 (b) = a2b

**The Perfect Squares:**

The Perfect squares are:

(1) 2 x 2 = 4 (2)3 x 3 = 9

(3) 4 x 4 = 16 (4) 5 x 5 = 25

(5) 6 x 6 = 36 (6)7 x 7 = 49

(7) 8 x 8 = 64 (8) 9 x 9 = 81

(9) 10 x 10 = 100

-- In surd manipulation, a number which is a multiple of a perfect square must be converted into the multiple of that perfect square

Examples:

1. √8 = = √4 x √2 = 2 x √2 = 2√2.

2. √12 = = √4 x √3 = 2 x √3 = 2√3.

3. √32 = = √16 x √2

= 4 x √2 = 4√2

4. √27 = = √ 9 x √3 = 3√3

5. √69 = = √9 x √7 = 3√7

6. √80 = = √16 x √5 = 4√5

(7) √50 = = √25 x √2 = 5√2

(8) √147 = = √49 x √3 = 7√3

(9) √125 = = √25 x √3 = 5√3

**How to determine whether a number is a multiple of a perfect square:**

-- The perfect squares 4, 9, 16, 25, 36, 49, 64, 81 and 100 are what we shall make use of.

-- When a number is given and we want to know whether it is a multiple of a perfect square, we start dividing the numbers with the perfect squares in turn, starting with the highest which is 100.

-- If we get an answer which is a whole number but not a decimal, then that particular number is a multiple of a perfect square.

-- Far example if we are given √448 and we want to know whether or not we can break it down, we first divide 448 by 100 which gives us 4.48.

-- Since this is a decimal, then we try the next perfect square which is 81.

-- Dividing 448 by 81 gives us 5.5 which is also a decimal. We then try the next perfect square which is 64.

-- Dividing 448 by 64 gives us 7, which is a whole numbers.

- This implies that √448

= = √64 x √7 = 8√7.

Example (2) Let us now determine whether √294 can be simplified or broken down.

- Dividing 294 by 100 gives us 2.94 which is a decimal.

- Dividing 294 by 81 gives us 3.6 which is a decimal.

- Diving 294 by 64 gives us 4.59 which is also a decimal.

- Dividing 294 by 49 gives us 6 which is a whole number

- This implies that √294

= = √49 x √6 = 7√6.

Examples (3) Now let us determine whether √150 can be simplified or broken down.

- Dividing 150 by 100 gives us 1.50 which is a decimal

- Dividing 150 by 81 gives us 1.85 which is a decimal.

- Dividing 150 by 49 gives us 3.06 which is a decimal.

- Dividing 150 by 36 gives us 4.2 which is a decimal.

- Dividing 150 by 25 gives us 6 which is a whole number.

- This implies that √150

= = √25 x √6 = 5√6.

NB: If we divide a given number by all the perfect

Squares we are supposed to use, and in each case get

a decimal, then that number cannot be simplified or

broken down.

For example, assume that we want to know whether

or not √271 can be simplified or broken down.

-- Dividing 271 by 100 gives us 2.71 which is a decimal.

-- Dividing 271 by 81 gives us 3.34 which is a decimal

- Dividing 271 by 64 gives us 4.2 which is decimal.

- Dividing 271 by 49 gives us 5.5 which is decimal.

-- Dividing 271 by 36 gives us 7.5 which is a decimal.

-- Dividing 271 by 25 gives us 10.8 which is decimal.

-- Dividing 271 by 16 gives us 16.9 which is decimal.

-- Dividing 271 by 9 gives us 30.1 which is a decimal.

-- Dividing 271 by 4 gives us 67.7 which is also a decimal.

Since when all the perfect squares concerned, when used to divide the given number gave us decimals as our answer, then √271 cannot be simplified and should be left as √271.

**Addition of surds:**

-- In surd addition, we can only add if the numbers under the square root signs are the same.

-- If they are not the same, then nothing can be done.

**Examples**

(1) a √b + c √b = (a + c ) √b

(2) 5 √2 + = (5 + 4) √2 = 9 √2

(3) 6 √7 + 2 √7 = (6 + 2) √7 = 8√7

(4) 3√5 + 2√ 5 = (3+ 2) √5 = 5√5

(5) 2√7 +3√7 = (2 + 3) √7 = 5√7

(6) 2√3 + √3 = 2√3 + 1√3 = (2 + 1) √3 = 3√3

(7) 5√2 + √2 = 5√2 + 1√2 = (5 + 1) √2 = 6√2

(8) 2√5 + 4√5 + 3√5 = (2 + 4 + 3)√5 = 9√5

(9) 5√2 + 2√2 + 4√2 = (5 + 2 + 4) √2 = 11√2

(10) √3 + 4√3 + 6√3 = 1√3 + 4√3 + 6√3 = (1 + 4 + 6)√3 = 11√3

(11) 2√3 + 4√2 = 2√3 + 4√2

(12) 5√7 + 2√6 = 5√7 + 2√6

(13) 4√2 + 3√5 = 4√2 + 3√5

(14) 5√2 + 3√3 + 2√7 = 5√2 + 3√3 + 2√7

Simplify each of the following:

(Q1) 2 +3√5 + 6 + 4√5

Solution

2 + 3√5 + 6 + 4√5 = 2 + 6 + 3√5 + 4√5

= 8 + (3 + 4)√5 = 8 + 7√5

(Q2) 6√7 + 1 + 4√7 + 3 + 2√7

Solution

6√7 + 1 + 4√7 + 3 + 2√7 = 6√7 + 4√7 + 2√7 +1 +3

= (6 + 4 + 2) √7 + 4 = 12√7 + 4

(Q3) 2√3 + 5√7 + 5√3 + 2

Solution

2√3 + 5√7 + 5√3 + 2 = 2√3 + 5√3 + 5√7 + 2

= (2 + 5) √3 + 5√7 + 2 = 7√3 + 5√7 + 2

(Q4) 2 + 5√2 + √3 + 6√2 + 7√3 + 6

Solution

2 + 5√2 + √3 + 6√2 + 7√3 + 6 = 2 + 5√2 + 1√3 +6√2 + 7√3 + 6

= 2 + 6 + 5√2 + 6√2 + 1√3 +7√3

= 8 + (5 + 6) √2 + (1 + 7) √3

= 8 + 11√2 + 8√3

(Q5) 51/2 + 3√2 + 6√2 + 7√3 + 1/2

Solution

51/2 + 3√2 + 6√2 + 7√3 + 1/2

= 51/2 + 1/2 + 3√2 + 6√2 + 7√3

= 6 + (3 + 6) √2 + 7√3 = 6 + 9√2 + 7√3

(Q6) 2√8 + 3√3 + 1

Solution

2√8 + 3√3 + 1 = 2+ 3√3 + 1

= 2x√4 x √2 + 3√3 + 1

= 2 x 2 x √2 + 3√3 + 1 = 4√2 + 3√3 + 1

(Q7) 3√2 + 2 + 2√8 + 4√2 + 6

Solution

3√2 + 2 + 2√8 + 4√2 + 6

= 3√2 + 2 + 2 + 4√2 + 6

= 3√2 + 2 + 2x√4x√2 + 4√2 + 6

= 3√2 + 2 + 2 x 2 x√2 + 4√2 + 6

= 3√2 + 4√2 + 4√2 + 6 + 2

= (3 + 4 + 4) √2 + 8

=11√2 + 8

(Q8) 5+3√27 + 2 + 6√3 + 2√2 + √12 + 1

Solution

5+3√27 + 2 + 6√3 + 2√2 + √12 + 1

= 5 + 3√27 + 2 + 6√3 + 2√2 + √12 + 1

= 5 + 3x√9x√3 + 2 + 6√3 + 2√2 + √4x√3 +1

= 5 + 3x3x√3 + 2 + 6√3 + 2√2 + 2x√3 + 1

= 5 + 9√3 + 2 + 6√3 + 2√2 + 2√3 + 1

= 5 + 2 + 1 + 9√3 + 6√3 + 2√3 + 2√2

= 8 + (9 + 6 + 2) √3 + 2√2

= 8 + 17√3 + 2√2

(Q9) 4 + 2√32 + 3√2 + 1 + 2√50

Solution

4 + 2√32 + 3√2 + 1 + 2√50

= 4 + 2+ 3√2 + 1 + 2

= 4 + 2x√16 x √2 + 3√2 + 1 + 2x√25 x √2

= 4 + 2 x 4 x √2 + 3√2 + 1 + 2 x 5 x √2

= 4 + 8√2 + 3√2 + 1 + 10√2

= 4 + 1 + 8√2 + 3√2 + 10√2

= 5+ (8 + 3 + 10) √2

= 5+21√2

(Q10) 3√7 + 5 + 2√7 + 3√16 + 2√25 + 4√128

Solution

3√7 + 5 + 2√7 + 3√16 + 2√25 + 4√128

= 3√7 + 5 + 2√7 + 3 (4) + 2 (5) + 4

= 3√7 + 5 + 2√7 + 12 + 10 + 4x√64 x √2

= 3√7 + 2√7 + 5 + 12 + 10 + 4 x 8 x √2

= (3 + 2) √7 + 27 + 32√2

= 5√7 + 27 + 32√2 = 5√7 +32√2 + 27

NB: You must first check whether 128 is a multiple of a perfect square or not.

(Q11) 5 + √243 + 2√2 + √18 +4√162 + 2√4

NB: You must first check whether or not 243, 162 and 18 are multiples of perfect squares.

Solution

5 + √243 + 2√2 + √18 +4√162 + 2√4

= 5 + + 2√2 + + 4+ 2 (2)

= 5 + √81 x √3 + 2√2 + x √ 2 + 4

= 5 + 9 x √3 + 2√2 + 3 x√2 + 4 x 9 x√2 + 4

= 5 + 9√3 + 2√2 + 3√2 + 36√2 +4

= 5 +4 + 9√3 + 2√2 +3√2 + 36√2

= 9 + 9√3 + (2 + 3 + 36 ) √2

= 9 + 9√3 + 41√2

NB: √4 = 2, √25 = 5, √36 = 6, √49 = 7, √64 = 8,

√81 = 9 and √100 = 10

**Subtraction of Surds:**

In surds subtraction, we can only subtract when the numbers under the square root signs are the same.

Examples:

1. 5√2 – 3√2 = (5 -3)√2 = 2√2

2. 8√3 – 2√3 = (8 – 2)√3 = 6√3

3. 5√6 – 9√6 = (5 – 9)√6 = -4√6

4. 8√2 – 2√2 – 3√2 = (8 – 2 – 3)√2 = 3√2

5. 4√3 – 2√5 = 4√3 – 2√5

6. 6√2 – 7√7 = 6√2 – 7√7

7. 4√5 – 2√3 = 4√5 – 2√3

Simplify each of the following:

(Q1) 2√75 – 4√3 – 2

Solution

2√75 – 4√3 – 2

= 2– 4√3 – 2

= 2 x√25 x√3 – 4√3 – 2

= 2 x 5 x √3 – 4√3 – 2

= 10√3 – 4√3 – 2

= (10 – 4)√3 – 2 = 6√3 – 2

(Q2) 5√8 – 2 – 2√2 – 3√3

Solution

5√8 – 2 – 2√2 – 3√3

= 5– 2 – 2√2 – 3√3

= 5 x √4 x √2 – 2 – 2√2 – 3√3 = 5 x 2 x - 2 - 2 - 3

= 10√2 – 2 – 2√2 – 3√3

= 10√2 – 2√2 – 3√3 – 2

= (10 – 2)√2 – 3√3 - 2

= 8√2 – 3√3 – 2

(Q3) - 4√7 – 3√36 – 2 – 2√7 – 3√72

Solution

- 4√7 – 3√36 – 2 – 2√7 – 3√72

= - 4√7 – 3– 2 – 2√7 –

= - 4√7 – 3x √9 x √4 – 2 – 2√7 – 3x√36x√2

= - 4√7 – 3x3x2 – 2 – 2√7 – 3 x 6 x √2

= - 4√7 – 18 – 2 – 2√7 – 18√2

= - 4√7 – 2√7 – 18 – 2 – 18√2

= (- 4 – 2)√7 – 20 – 18√2

= - 6√7 – 18√2 – 20

**Combination of addition and subtractions of surds:**

Simplify the following:

1). 3√2 + 5√3 – 2√18 +

Solution

3√2 + 5√3 – 2√18 +

= 3√2 +5√3 – 2+ √3

= 3√2 +5√3 – 2x√9x√ 2 +

= 3√2 + 5√3 – 2 x 3 x √2 + √3

= 3√2 + 5√3 – 6√2 + √3

= 3√2 + 5√3 – 6√2 + 1√3 = 3 - 6 + 5 + 1

= (3 – 6)√2 + 5√3 + 1√3

= - 3√2 + (5 +1) √3

= - 3√2 + 6√3

(Q2). 3√8 – 8 + 2√3 – 4√2 – 2√27 + 5

Solution

3√8 – 8 + 2√3 – 4√2 – 2√27 + 5

= 3– 8 + 2√3 – 4√2 – 2 +5

= 3x√4 x √2 – 8 + 2√3 – 4√2 – 2x√9x√3 + 5

=3 x 2 x √2 – 8 + 2√3 – 4√2 – 2 x 3 x + 5

= 6√2 – 8 + 2√3 – 4√2 – 6√3 + 5

= 6√2 – 4√2 + 2√3 – 6√3 – 8 +5

= (6 – 4) √2 + 2√3 – 6√3 – 3

= 2√2 + (2 – 6) √3 -3

= 2√2 + (-4)√3 – 3

= 2√2 – 4√3 – 3

(Q3). (2 + 3√5) –(3+ 2√5)

Solution

(2 + 3√5) – (3+ 2√5)

= 2 + 3√5 – 3 – 2√5

= 2 – 3 + 3√5 – 2√5

= -1 + (3 – 2) √5

= -1 + 1√5 = -1 + √5

(Q4) . 1 + 2√7 – 4√63 + 3√12 – 5√3 – 4√9

Solution

1 + 2√7 – 4√63 + 3√12 – 5√3 – 4√9

= 1+ 2√7 – 4 + 3 – 5√3 – 4 (3)

= 1 + 2√7 – 4x√9x√7 + 3x√4x√3 – 5√3 - 12

= 1 + 2√7 – 4 x 3 x√7 + 3 x 2 x√3 – 5√3 – 12

= 1 + 2√7 – 12√7 + 6√3 – 5√3 – 12

= 1 + (2 – 12) + 6√3 -5√3 – 12 = 1 + (-10) + (6-5) - 12

= 1 -

= 1 – 12 - 10

= -11 - 10

(Q5). 6√5 – 2√20 - √45 + 3 + 1 + 5√3 + √125 + 3√5

Solution

6√5 – 2√20 - √45 + 3 + 1 + 5√3 + √125 + 3√5

= 6√5 – 2 - + 4+ 5√3 + + 3√5

= 6√5 – 2x√4 x√5 -√9x√5 + 4 + 5√3 +√25 x √5 + 3√5.

= 6√5 – 2x2x√5 – 3x√5+4 +5√3 +5 x√5 + 3√5

= 6√5 – 4√5 – 3√5 + 4+5√3 + 5√5 + 3√5

= 6√5 – 4√5 – 3√5 + 5√5 + 3√5 + 5√3 + 4

= (6 – 4 – 3 + 5 + 3) √5 + 5√3 + 4

= 7√5 + 5√3 + 4

(Q6). √24 – 3√6 – 216 + √294

Solution

√24 – 3√6 – 216 + √294

=– 3 +

= √4 x √6 – 3√6 - √36 x √6 + √49 x √6

= 2√6 – 3√6 – 6√6 + 7√6

= (2 – 3 – 6 + 7) √6 = 0√(6) = 0

(Q7). √128 - √50 + 2/3 √162

Solution

√128 - √50 + 2/3 √162

= - √50 + 2/3

= √64 x√2 - √25 x √2 + 2/3 x √81 x √2

= 8√2 – 5√2 + 2/3 x 9 x √2

= (8 -5) √2 + 18/3 x √2

= 3√2 + 6√2

**Multiplication of Surds:**

1, a√z x b√y = a x b x √z x √y

= ab x √zy = ab√zy

2. 2√3 x 5√7 = 2 x 5 x √3 x √7

= 10 x = 10√21

3. 5√2 x 6√7 = 5 x 6 x √2 x √ 7

= 30 x = 30√14

4. 2√3 x 4√15 = 2 x 4 x√3 x √15

= 8 x = 8 x√45 = 8 x

= 8 x √9 x √5 = 8 x 3 x√5 = 24√5

5. 2√9 x 4√3 = 2 x 4 x √9 x √3

= 8 x √27 = 8 x = 8 x√9 x√3

= 8 x 3 x √3 = 24√3

**Simplify the following**:

(Q1) √10 (2√2 + √5)

Solution

√10 (2√2 + √5) = √10 x 2√2 + √10 x √5

= 2x √2 x √ 10 + √10 x √5

= 2 x +

= 2√20 + √50

= 2 +

= 2 x √4 x √5 + √25 x √2

= 2 x 2 x √5 + 5√2 = 4√5 + 5√2

(Q2).2 (2√6 – 3√24 + 4√10)

Solution

2 (2√6 – 3√24 + 4√10)

= 2 x 2√6 – 2 x 3√24 + 2 x 4√10

= 4√6 – 6√24 + 8√10

= 4√6 - 6 + 8√10

= 4√6 – 6 x √4 x√6 + 8 √10

=4√6 – 6 x 2 x √6 + 8√10

= 4√6 – 12√6 +8√10

= (4 – 12) √6 + 8√10

= -8 + 8

(Q3). 2√2 (2√6 – 3√25 + 4√16)

Solution

2√2 (2√6 – 3√25 + 4√16)

= 2√2 x 2√6 – 2√2 x 3√25 + 2√2 x 4√16

= 2 x2 x – 2x3x + 2x4x

= 4√12 – 6√50 + 8√32

= 4– 6 + 8

= 4 x √4 x √3 – 6x √25 x √2 + 8 x√16 x √2

= 4 x 2 x √3 – 6 x 5 x √2 + 8 x 4 x √2

= 8√3 – 30√2 + 32√2

= 8√3 + 32√2 – 30 √2

= 8√3 + (32 – 30) √2

= 8√3 + 2√2

N/B:

1). (a + b) (c + d) = a x c + a x d + b x c + b x d

= ac + ad + bc + bd.

(2) (a – b) (c + d) = a x c + a x d – b x c – b x d

= ac + ad – bc – bd

(3) (a + b) (c – d) = a x c – a x d + b x c – b x d

= ac – ad + bc – bd

(4) (a – b) (c – d) = a x c – a x d – b x c + b

= ac – ad – bc +bd

Examples:

1. (2 + 3) (4 + 5) = 2 x 4 + 2 x 5 + 3 x 4 + 3 x 5

= 8 + 10 + 12 + 15 = 45

2. (2 + 3) ( 4 – 5) = 2 x 4 – 2 x 5 + 3 x 4 – 3 x 5

= 8 – 10 + 12 – 15 = -5

3. (2 – 3) (4 + 5) = 2 x 4 + 2 x 5 – 3 x 4 – 3 x5

= 8 + 10 – 12 – 15 = -9

4. (2 – 3) ( 4 – 5) = 2 x 4 – 2 x 5 – 3 x 4 + 3 x5

= 8 – 10 – 12 + 15 = 1

Evaluate the following:

(1) (√2 + √3) (√4 + √5)

Solution

(√2 + √3) (√4 + √5)

= √2 x√4 + √2 x √5 + √3 x√4 + √3 x √5

= √8 + √10 + √12 + √15

= + √10 + + √15

= √4 x √2 + √10 + √4 x √3 + √15

= 2√2 + √10 + 2√3 + √15

(2) (√3 + √6) (√4 - √2)

Solution

(√3 + √6) (√4 - √2)

= √3x√4 - √3x√2 + √6x√4 - √6x√2

= √12 - √6 + √24 - √12

= - √6 + -

= √4x√3 - √6 + √4x√6 - √4x√3

= 2√3 - √6 + 2√6 – 2√3

= 2√3 – 2√3 - √6 + 2√6

= (2 – 2) √3 – 1√6 +2√6

= 0√3 – 1√6 + 2√6

= 0 - 1√6 + 2√6 = -1√6 + 2√6

= (-1 + 2) √6 = 1√6 = √6

(3). (√5 - √3) (√6 + √2)

Solution

(√5 - √3) (√6 + √2)

= √5x√6 + √5x√2 - √3x√6 - √3x√2 =

= √30 + √10 - - √6

= √30 + √10 - √9 x √2 - √6

= √30 + √10 – 3√2 - √6

(4). (√2 - √6) (√3 - √2)

Solution

(√2 - √6) (√3 - √2)

= √2 x √3 - √2 x√2 - √6 x √3 + √6 x √2

= √6 - √4 - √18 +

= √6 – 2 - +

= √6 – 2 - √ 9 x √2 + √4 x √3

= √6 – 2 – 3√2 + 2√3

Solution

(5). (2√3 + 4√2) (4√4 + √2) .

(2√3 + 4√2) (4√4 + √2)

= 2√3 x 4√4 + 2√3 x √2 + 4√2 x 4√4 + 4√2 x √2

= 8√12 + 2√6 + 16√8 + 4√4

= 8 + 2√6 + 16 + 4(2)

= 8 x √4 x √3 + 2√6 + 16 x √4 x √2 + 8

= 8 x 2 x √3 + 2√6 + 16 x 2 x√2 + 8

= 16√3 + 2√6 + 32√2 + 8

(6). (2√5 + 4√3) (3√3 – 2√2)

Solution

(2√5 + 4√3) (3√3 – 2√2)

= 2√5 x 3√3 – 2√5 x 2√2 + 4√3 x 3√3 – 4√3 x 2√2

= 2x3x – 2x2x√5x + 4x3x√3x3 – 4x2x√3x

= 6√15 – 4√10 + 12√9 – 8√6

= 6√15 – 4√10 + 12(3) – 8√6

= 6√15 – 4√10 + 36 – 8√6

(7). (5√3 – 4√2) (√9 + 2√2)

Solution

(5√3 – 4√2) (√9 + 2√2)

= 5√3 x√9 +5√3 x 2√2 – 4√2 x √9 – 4√2 x 2√2

= 5√27 + 10√6 – 4√18 – 8√4

= 5+10√6 – 4 – 8(2)

= 5x√9x√3 + 10√6 – 4x√9 x√2 – 16

= 5 x 3 x √3 + 10√6 – 4 x 3 x √2 – 16

= 15√3 + 10√6 – 12√2 – 16

(8). (2√2 – 3√7) (4√6 – 3√3)

Solution

(2√2 – 3√7) (4√6 – 3√3)

= 2√2x4√6 – 2√2x3√3 – 3√7x4√6 + 3√7x3√3

= 8√12 – 6√6 – 12√42 + 9√21

= 8 – 6√6 – 12√42 + 9√21

= 8x√4x√3 – 6√6 – 12√42 + 9√21

= 8x2x√3 – 6√6 – 12√42 + 9√21

= 16√3 – 6√6 – 12√42 + 9√21

(9).i. Simplify 2√3 (2 - √3) + 3√2(√2 – 1).

ii. Hence evaluate your answer to 2.d.p, given that √2 = 1.414 and √3 = 1.732

Solution

2√3 (2 - √3) + 3√2(√2 – 1)

= 2√3 x2 – 2√3x√3 + 3√2x√2 – 3√2 x1

= 4√3 – 2√9 +3√4 – 3√2

= 4√3 – 2(3) + 3(2) – 3√2

= 4√3 – 6 + 6 – 3√2

= 4√3 – 3√2, since – 6 +6 = 0

(ii). If √2 = 1.414 and √3 = 1.732,

then 4√3 – 3√2 = 4(1.732) – 3(1.414)

= 6.93 – 4.242 = 2.69

(10). Simplify (1 - √3) (1/3 + √3), and leave your answer in the form a + b√3

Solution

(1 - √3) (1/3 + √3)

= 1 x 1/3 + 1 x √3 - √3 x 1/3 - √3 x√3

= 1/3 + √3 – 1/3 √3 -√9

= 0.33 + 1√3 – 0.33√3 – 3

= 0.33 – 3 + 1√3 – 0.33√3

= - 2.67+(1- 0.33)√3

= - 2.67 + (0.67) √3

Which is of the form a + b√3, where a = - 2.67 and b = 0.67

(11). Simplify (2 + 3√5) (4 – 3√5)

Solution

(2 + 3√5) (4 – 3√5)

= 2 x 4 – 2 x 3√5 + 3√5 x 4 – 3√5 x 3√5

= 8 – 6√5 + 12√5 – 9√25

= 8 - 6 + 12 - 9(5)

= 8 + 12√5 – 6√5 – 9(5)

= 8+ (12 – 6) √5 – 45

= 8 + 6√5 – 45 = 8 – 45 + 6√5

= -37 + 6√5

(12). Evaluate (2√3 – 4√2)2

Solution

(2√3 – 4√2)2

= (2√3 – 4√2) (2√3 – 4√2)

= 2√3 x 2√3 – 2√3 x 4√2 – 4√2 x 2√3 + 4√2 x 4√2

= 4√9 – 8√6 – 8√6 + 16√4

= 4(3) – 8√6 – 8√6 +16(2)

= 12 – 8√6 – 8√6 + 32

= 12 + 32 – 8√6 – 8√6

= 44 – 8√6 – 8√6

= 44 – 16√6

(13) Show that or prove that 2√45 + 3√8 – 4√20 – 5√2 = √2 – 2√5

NB: In such a question, either the left hand side of the equation (L.H.S) is simplified to get the right hand side (R.H.S) or vice versa.

Solution

Consider the L. H. S

2√45 + 3√8 – 4√20 – 5√2

= 2+ 3– 4 – 5√2

= 2x√9x√5 + 3x√4x√2 – 4x√4x√5 – 5√2

= 2x3x√5 + 3x2x√2 – 4x2x√5 – 5√2

= 6√5 + 6√2 – 8√5 – 5√2

= 6√5 – 8√5 + 6√2 – 5√2

= (6 – 8) √5 + 6√2 – 5√2

= - 2√5 + (6 – 5)√2

= -2√5 + 1√2 = -2√5 + √2

= √2 - 2√5, which is equal to the R.H.S

Questions

(Q1) Evaluate the following:

(a) (√9)2 Ans: 9

(b) (3√2)2 Ans: 18

(c) 3(√2)2 Ans: 6

(d) (3√2)2 + (4√2)2 Ans: 50

(e) 4(√3)2 – (2√2)2 Ans: 4

(Q2) Simplify the following:

1. √16 + √320 + √75

Ans: 4 + 8√5 + 5√3

1. 2√2 + 3√3 + √27 + √8

Ans: 4√2 + 6√3

1. 3 + 2√8 + 5√72 + √27

Ans: 3 + 34√2 +3√3

1. √147 + 3√3 - √243 – 2√2

Ans: √3 – 2√2

1. 6 + 4√2 + 2√64 – 3√45 - √2

Ans: 22 + 3√2 – 9√5

1. √200 + 20 – 2√20 + √175 + 3 – 2√7

Ans: 10√2 + 23 – 4√5 + 3√7

1. 1 – 2√5 + √125 - √567 + 2√7 + 2

Ans: 3 + 3√5 – 7√7

1. (2 + 4√2) – (1 – 3√2)

Ans: 1 + 7√2

1. 5 +3√3 + 4√2 - √3 + 4√3 – 2√2 – 1

Ans: 4 + 6√3 + 2√2

1. 3√180 + 2 – 2√5 – 5 + – 6√2

Ans: 16√5 + 8√2 - 3

(Q3) Simplify each of the following:

(a) √4 (√3 + √10)

Ans: 2√3 + 2√10

(b) 2√3 (2√5 + 4√12)

Ans: 4√15 + 48

(c) 3√4 (2√10 – 2√5)

Ans: 12√10 – 12√5

(d) (2 + √5) (√3 + √4)

Ans: √6 + 2√2 + √15 + 2√5

(e) (2√3 + √4) (√2 – 3√2)

Ans: - 4√6 – 4√2

(f) (2√3 – 4√2) (3√5 + 2√6)

Ans: 6√15 + 12√2 – 12√10 – 16√3

(g) (2√5 – 3√2) (4√5 – 3√6)

Ans: 40 – 6√30 – 12√10 +18√3

(h) 2√2 (3√2 – 4√3) + 3√3( – 2√4)

Ans: 12 – 8√6 – 6√3

(i) 2√5(2√2 + 4√3) -3√4(2√2 – 2√5)

Ans: 4√10 + 8√15 – 12√2 + 12√5

(Q4)a. Simplify 2√5 (4 –√2) + 3√5(√2 +1).

b. Hence evaluate your answer to 2 d.p, given that √5 = 2.34 and √10 = 3.16

Ans: a. 11√5 + √10

b. 28.9

(Q5) Evaluate (1 – 2√2) (1/2 + √2)

Ans: -3.5

(Q6) Evaluate the following:

1. (2√2 + 3√5)2

Ans: 12√10 +53

1. (3√4 – 2√2)2

Ans: 44 – 24√2