**CHAPTRE EIGHT**

**VECTORS**

* A vector is a physical quantity which has both magnitude and direction.
* Example are

1. A force of 20N acting North.
2. A velocity of 5km/h East.

**Types of vectors:**

* In general the are two types and these are

1. Free vector.
2. Position vector.

**Free vector:**

Y

A B

O x

* A free vector is a vector which does not pass through any specific position.
* They are usually represented by small letters e.g

**Position vector :**

y

A

O

This is a vector which passes through the origin or a specified point.

**Vector notation:**

* A vector may be represented by a line segment as shown next:

A B

* This given vector can be represented by

**The Triangle law:**

C

A B

According to the triangle law,

**The unit vector:**

* This is a vector whose magnitude is one in the direction under consideration.
* The unit vector along a vector is written as
* Also the unit vector along a vector is written as
* The unit vector along the vector is written as
* Consider the vector A B = 1
* The vector is written as and its unit vector is written as

**Equal vectors:**

* Two vectors are said to be equal if their magnitudes and directions are equal
* Example are

**The negative vector:**

* The negative of the vector written as –a
* If the negative vector of the vector
* The vector of the same magnitude as but it is opposite in direction.
* It must be noted that
* Also if
* If we consider a vector then its negative vector is .

**The zero vector (null vector):**

* This is a vector where magnitude is zero and its direction is undefined.
* It is represented by 0 =

**Notation of the magnitude of a vectors:**

* If is a vector, then its magnitude is written as
* Similarly the magnitude of the vector is written as
* If magnitude =

Y

P (a,b)

b

O a

Q1. i. If OP = magnitude of .

ii. Find the angle between and the x – axis

Soln.

Y

P

5

O 6 x

1. .

**Scalar multiplication of vector:**

* If ^ is the scalar and is the vector, then the scalar x the vector
* When a scalar multiplies a vector, the product is also a vector, and for this reason is also a vector.
* The vector is parallel to and is in the same direction as but has ^ times the magnitude of
* For example the vectors have the same direction.

i.e

* But the vectors and and are opposite in direction.
* ^( e.g 6(
* Also (2 + 4)
* Finally , e.g 3(2

N/B: y

P(x1,y1)

y1

O x1

* If P(x1,y1) is a point in the x – y plane, then the position vector of P relative to the origin, O is defined by
* Also if A = (0,6), then

Q2. Find the numbers m and n such that

M

Soln.

Solve eqns (1) and (2) simultaneously

Q3. If mp + nq = where m and n are scalar, given that p =

Soln.

p =

Solve eqns (1) and (2) simultaneously to get the values of m and n.

Q4. If r = evaluate 6(r + 25)

Soln.

Consider 6(r + 2s), solve what is inside the bracket first

Q5. If p = , find 2p – q + r

Soln.

2p – q + r = 2.

Q6. If the vector p =

Find the vector r.

Soln.

r =

N/B: Given the points A and B, then .

Examples: If A =

Also if C =

Q7. If A = (4, 5) and B = (6, 2), find

Soln.

A = Also B = (6, 2) .

N/B: If

Also if

Q8. If A and B are the points (2, 1) and (1, 2) respectively, find

Soln.

A =

.

Q9. Given A =

evaluate i. ii. 2 iii. 2(A – B)

Soln.

1. A =
2. A = , then

Since

1. 2(A – B) = ? but A = and B =

A – B =

Since A – B =

Q10. If A = (2, 4) and B = (4, 9), find ie the magnitude of AB.

Soln.

A =

Q11. If A = (-5, 2) and B(-8 – -9),

1. Find the vector
2. Calculate the length of

Soln.

1. A =
2. The length of = the magnitude of =11.4

Q12. If C = (4, 1) and D = (2, 6),

1. find the vector
2. calculate the length of

Soln.

1. .
2. The length of

Q13. If C = (1, 3) and D = (2, 4) find

Soln.

Q14. If

1. ii.

Soln.

i.

Q15. If Q is the point (2,4) and find the coordinates of R.

Soln.

Q = (2, 4) and then the coordinates of R

= (2+1, 4+3) = (3, 7)

The coordinates of R = (3, 7)

Q16. If , find the coordinates of y.

Soln.

Since z = (1,2) and then the coordinates of

y = (.

Q17. If A = (1, 5) and find the coordinates of B.

N/B: Since the point given is A and the vector given is then must first be changed into

Soln.

Since A is given as (1, 5), we must findthen .

Now since A = (1, 5) and, then

B = (1+2, 5+3).

Q18. If Q = (4, 1) and coordinates of R.

Soln.

Since Q = (4, 1) and we must first find

Now Q = (4, 1) and

Q19. If C =

Soln.

Now C =

Q20. If A = (1,2),

Soln.

Since A = (1, 2) and

Also Since A = (1, 2) and

Q21. Given B(4,2),

determine the coordinates of C and D.

Soln.

Since B = (4, 2) and

Q22. If A is the point (2, 3),

N/B: Since the point given is point A, then must be changed into Also must be changed into .

Soln.

.

Also

Now since A = (2, 3) and , then the coordinates of

Also since A = (2,3) and coordinates of C = (2+1, 3+5)= (3, 8)

Q23. The point C is given as (4, 1),

Soln.

Since C = (4, 1) and

Now D = (5, 3) and

Q24. If the point A is given as

and C.

Soln.

the coordinates of B = (3+

Now B = (1, 5) and

= (1+5, 5+1) = (6, 6)

Q25. Given A(4, 1), the coordinates of B and C.

N/B: The point given is point A and the vector given is

First find

Soln.

Now A = (4, 1) and

Now B = (3, -5) and

Q26. B is given as the point (4, 8),

Soln.

Since B = (4, 8) and

Now since C = (5, 15) and CD =

Q27. If A = (1, 3), find the coordinates of B and C.

Soln.

s

Now A = (1, 3) and

Since CB = .

Now B(0, 1) and

.

Q28. P is the point (4, 1) and Q is (-3, 2). If

1. find the coordinates of S and T.
2. find also .

Soln.

1. Since P = (4, 1) and

Q29. If A = (4, 3) and B = (1, 1), .

N/B: Before we can find we must first determine the coordinates of C and D.

Soln.

Also

N/B:

1. M

O P

In the figure drawn, moving from M to O, and then from O to P is the same as moving from M directly to P, since in both cases we end at the same point, which is P.

O X

Y

In the given figure

Q1. If find xy .

Soln.

O Y

X

Q2. If

Soln.

O Y

X  
From the diagram,

Since .

Q3. Given that .

Soln.

O Y

X

**The inverse of a vector or the negative vector.**

* If
* *the inverse or the negative vector of*
* A vector and its inverse have the same magnitude, but have opposite direction
* For example if
* Also if

**The direction of a vector:**

* This is the angle ,which the vector makes with the x-axis
* If P(x, y) and Q(), then the direction of is given by

Q1. Given A(5, 4) and B(3, 1), find the direction of

Soln.

Let () = (5, 4)and () = (3,1) . =

.

Q2.

1. Find the magnitude and the direction of the displacement vector A and B are the points (2, 1) and (8, 9) respectively.
2. Determine the magnitude of the vector .

Soln.

Let (2, 1) = () and (8, 9) =

**Parallel vectors:**

* Two vectors are said to be parallel vectors, if one is the scalar multiplication of the other.
* Consider the vectors These are parallel vectors, since one is the scalar multiple of the other, i.e 2 x .
* If the scalar is positive or a positive number, as in the example just given, then the two given vectors are in the same direction.
* But if the scalar is negative, then the two vectors are in the opposite direction
* Also the vectors are parallel vectors, since one is the scalar multiple of the other i.e 3 x C = .
* In this case, the scalar is 3 and since it is positive, then the two vectors are in the same direction.
* Now consider These are parallel vectors, since one is the scalar multiple of the other i.e .
* In this case, since the scalar is negative i.e – 4, then the two given vectors are in the opposite direction, eventhough they are parallel.

Q1. Determine whether the vector are parallel to each other, and determine whether they are in the same or opposite in direction.

Soln.

One is a scalar multiple of the other they are parallel vectors. Since the scalar is negative or a negative number i.e -3, then the two vectors are opposite in direction.

Q2. Determine whether the vectors are parallel to each other, and determine also whether they are in the same direction

Soln.

Since one of the vectors is a scalar multiple of the other, the two given vectors are parallel. Since the scalar = 2 which is positive two given vectors are in the same direction.

**Determination whether two vectors are parallel – method two:**

Let if ad – bc = o, then the two vectors are parallel.

Q1. Show that the vectors are parallel vectors.

Soln.

If is parallel to , then (4 x 4) – (2 x 8) = 0 i.e if the left hand side is equal to zero, then they are parallel.

Now L.H.S = (4 x 4) – (2 x 8) = 16 – 16 = 0the two vectors are parallel

Q2. Determine whether or not the vectors vectors. If and are parallel vectors, then

(-2 x 3) – (-3 x 2) = 0

L.H.S = (-6) – (-6) = -6 + 6 = 0

Since L.H.S = 0 the two vectors are parallel vectors.

Q3. Determine whether or not are parallel vectors.

Soln.

are parallel vectors, then

(4 x 6) – (1 x 8) = 0

L.H.S = 24 – 8 = 16.

Since the L.H.S i.e not equal to zero, then the two vectors are not parallel.

Q4. Find the value of x, such that the vector will be parallel to the vector .

Soln.

. For them to be parallel, then .

Q5. Given the value of x such that the two vectors become parallel.

Soln.

are given. If they are parallel, then

Q6. Given find the value of y such that the two vectors are parallel.

Soln.

. For them to be parallel, then

(y x 6) – (2 x 9) = 0

**Perpendicular vector**s**:**

Consider the vectors if these two vectors are perpendicular, then ac + db = 0

Q1. Show that the vectors are perpendicular

Soln.

For to be perpendicular, then

(4 x 2) + (-2 x 4) = 0 i.e the L.H.S must be equal to zero.

L.H.S =

Since L.H.S = 0 the two vectors are perpendicular.

Q2. Determine whether or not are perpendicular vectors.

Soln.

Given . If these two vectors are perpendicular, then (8 x 4) + (2 x 5) = 0 i.e

The L.H.S must be equal to zero. L.H.S 32 + 10 = 42.

Since L.H.S 0, then the two vectors are not perpendicular.

Q3. Given whether these two given vectors are perpendicular vectors.

Soln.

. For these two vectors to be perpendicular, then

L.H.S = (-12) + (12) = 0. Since L.H.S = 0, then the two vectors are perpendicular.

Q4. Are the vectors

Soln.

. If these two vectors are perpendicular, then

L.H.S = (-10) + (6) = - 4

Since L.H.S then the two given vectors are not perpendicular

Q5. Find the value of x such that the vectors

perpendicular to each other.

Soln.

. If these two vectors are to be perpendicular, then

Q6. If are two vectors, determine the value of y, if these two vectors are perpendicular.

Soln.

If are perpendicular, then

Q7. Find the values of x and y such that .

Soln.

Equating corresponding component

Also

Solve eqn (1) and eqn (2) simultaneously

Q8. If p =

Soln.

Q9. Triangle ABC has vertices A(-2, -4), B(10, 1) and C(3, 8).

1. Find the length of the side AB
2. Show that the triangle is isosceles

Soln.

1. A(-2, -4)

B(10, 1) C(3, 8)

The length of is the same as the magnitude of .

A =

Since

Also C = (3, 8) =

Now for the given triangle,. Since two lengths of the given triangle are equal, (ie , then it is an isosceles triangle

**Questions:**

Q1. Find the values of K and M such that

Ans: K = 2 and M = 3

Q2. Determine the values of Q and R such that

Ans: Q = -1 and R = 3

Q3. Given that and y. Ans: x = 4 and y = 2

Q4. Given A (6, 4) and B(3, 2), evaluate i. ii.

Ans: i. ii. =

Q5. If

1. Ans:
2. Ans: 9.2
3. Ans:

Q6. Given that x = (2, 4) and y = (4, 9), determine the length of Ans: 5.4

Q7. Given that evaluate

1. Ans:
2. Ans:
3. Ans: 5.8
4. Ans: 5.1
5. Ans: 5.8

Q8. Given the coordinates of Q. Ans: (5, 10)

Q9. Given determine the coordinates of Q. Ans: (4, 8).

Q10. Given A(3, 2), determine the coordinates of

1. the point B Ans: (4,7)
2. The point C Ans: (7,8)

Q11. Given A(2, 1), determine the coordinates of

1. the point B Ans:
2. the point C Ans:

Q12. Given x (-2, 1), , determine the coordinates of

1. the point Y. Ans:
2. the point Z Ans:

Q13. Given C(2, 3), coordinates of

1. point D. Ans: (4, 4)
2. point E. Ans: (9, 8)

Q14. Given A(3,2), determine the coordiantes of

1. point B. Ans:
2. point C Ans:

Q15. Given A(4,2), , determine the coordinates of

1. the point B. Ans: (-1, -4)
2. the point C Ans: (-2, -7)

Q16. Given x(2, 4), y =(3,6). Determine the coordinate of

1. Point P Ans: (4,5)
2. Point Z Ans:(4, 7)

Q17. Given x(2, 3), y(4, 1)

find the coordinates of

1. the point B. Ans: (4, 4)
2. the point M. Ans (-2, 6)

Q18. If

1. . Ans: ii. Ans: 7.6

Q19. If find

1. ii.

Q20. Given that

Q21. Given that .

Ans:

Q22. If M = evaluate

1. 4(3M + 3k) Ans:
2. 2(2M – k) Ans:

Q23. If P =

Q24. Given

Q25. Find the direction of the displacement vector where A and B are the points (8, 4) and (6, 2) respectively. Ans:

Q26. Find the direction of the displacement vector where A and B are the points (2, -4) and (-6, -10) respectively. Ans:

Q27. Determine whether or not these pairs of vectors and parallel.

Ans: They are parallel

Ans: They are not parallel

Ans : They are parallel

Q28. Given that the vector are parallel vectors, determine the value of x. Ans: 4

Q29. If find the value of y, so that P and Q become parallel vectors. Ans: 15

Q30. Determine whether or not the following pairs of vectors are perpendiculars

Ans: They are not perpendicular

Ans: They are perpendicular

Ans: They are perpendicualr

Ans: They are not perpendicular

Q31. Given that are two perpendicular vector, find x. Ans: -6

Q32. Find the values of x and y such that Ans: x = 2 and y = -1