EFC3 - Exercise 3

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November 2019

1 Source files

All code cited and all figures showed here can be found at the following GitHub repository:
`https://github.com/ito-rafael/IA006C-MachineLearning/tree/master/efc2`
In this repository, one can found the following files:

- Jupyter Notebook
  - efc2_pre-ex1.ipynb
  - efc2_ex1_binary_classification.ipynb
  - efc2_ex2_multiclass_classification.ipynb
  - efc2_ex2_knn.ipynb
- LaTeX
  - efc2.tex

The notebook "efc2_pre-ex1" plots the histograms for the exercise 1 and it is used for data visualization. It shows the input features histograms for the raw data and after a data standardization. Also, it shows the correlation between these data.

The notebook "efc2_ex1_binary_classification" effectively implements the logistic regression used to perform a binary classification proposed in exercise 1.

The notebooks "efc2_ex2_multiclass_classification" and "efc2_ex2_knn" implements the algorithms to perform a multiclass classification proposed in exercise 2. The former one uses the softmax approach while the latter one implements the K-Nearest Neighbors (kNN) algorithm.

2 Part 1 - Error backpropagation

$$u_1 = 1 \cdot v_{00} + x_1 \cdot v_{10} + x_2 \cdot v_{20}$$
$$u_2 = 1 \cdot v_{01} + x_1 \cdot v_{11} + x_2 \cdot v_{21}$$
$$u_3 = 1 \cdot v_{02} + x_1 \cdot v_{12} + x_2 \cdot v_{22}$$

$$s_1 = f(u_1)$$
$$s_2 = f(u_2)$$
$$s_3 = f(u_3)$$

$$y_1 = 1 \cdot w_{00} + s_1 \cdot w_{10} + s_2 \cdot w_{20} + s_3 \cdot w_{30}$$
$$y_2 = 1 \cdot w_{01} + s_1 \cdot w_{11} + s_2 \cdot w_{21} + s_3 \cdot w_{31}$$

$$J = e_1^2 + e_2^2$$

$$\delta_3 = \frac{\partial J}{\partial u_3} = \frac{\partial[(d_1 - y_1)^2 + (d_2 - y_2)^2]}{\partial u_3}$$
$$\delta_3 = \frac{\partial (d_1 - y_1)^2}{\partial u_3} + \frac{\partial (d_2 - y_2)^2}{\partial u_3}$$
$$\delta_3 = \frac{\partial (d_1 - y_1)^2}{\partial (d_1 - y_1)} \cdot \frac{\partial (d_1 - y_1)}{\partial u_3} + \frac{\partial (d_2 - y_2)^2}{\partial (d_2 - y_2)} \cdot \frac{\partial (d_2 - y_2)}{\partial u_3}$$
$$\delta_3 = 2(d_1 - y_1) \cdot \left(-\frac{\partial y_1}{\partial u_3}\right) + 2(d_2 - y_2) \cdot \left(-\frac{\partial y_2}{\partial u_3}\right)$$
$$\delta_3 = -2(d_1 - y_1) \cdot \frac{\partial y_1}{\partial s_3} \cdot \frac{\partial s_3}{\partial u_3} - 2(d_2 - y_2) \cdot \frac{\partial y_2}{\partial s_3} \cdot \frac{\partial s_3}{\partial u_3}$$
$$\delta_3 = -2(d_1 - y_1) \cdot \frac{\partial (1 \cdot w_{00} + s_1 \cdot w_{10} + s_2 \cdot w_{20} + s_3 \cdot w_{30})}{\partial s_3} \cdot \frac{\partial f(u_3)}{\partial u_3} - 2(d_2 - y_2) \cdot \frac{\partial (1 \cdot w_{01} + s_1 \cdot w_{11} + s_2 \cdot w_{21} + s_3 \cdot w_{31})}{\partial s_3} \cdot$$

$$\frac{\partial f(u_3)}{\partial u_3}$$
$$\delta_3 = -2(d_1 - y_1)\dot{f}(u_3)w_{30} - 2(d_2 - y_2)\dot{f}(u_3)w_{31}$$

$$\frac{\partial J}{\partial v_{12}} = \frac{\partial J}{\partial u_3} \cdot \frac{\partial u_3}{\partial v_{12}}$$
$$\frac{\partial J}{\partial v_{12}} = \frac{\delta_3 \cdot \partial(1 \cdot v_{02} + x_1 \cdot v_{12} + x_2 \cdot v_{22})}{\partial v_{12}}$$
$$\frac{\partial J}{\partial v_{12}} = \delta_3 \cdot x_1$$
$$\frac{\partial J}{\partial v_{12}} = -2x_1\dot{f}(u_3)[w_{30}(d_1 - y_1) + w_{31}(d_2 - y_2)]$$

3 Part 2 - Multiclass Classification