# IA353A - Neural Networks
# EC1

Rafael Claro Ito
(R.A.: 118430)

May 2020

## Question 1

## Question 2

$$\|Ax - b\|_P^2 + \|x - x_0\|_Q^2$$

**Since we are searching for $x$ that minimizes the previous expression, we will calculate the derivative with relation to $x$ and set it equal to zero:**

$$\frac{d}{dx}(\|Ax - b\|_P^2 + \|x - x_0\|_Q^2) = 0$$

**Property used:** $\|x\|_Q^2 = x^T Q x$

$$\frac{d}{dx}[\overbrace{(Ax - b)^T P(Ax - b)}^{\|Ax-b\|_P^2} + \overbrace{(x - x_0)^T Q(x - x_0)}^{\|x-x_0\|_Q^2}] = 0$$

**Property used:** $(M + N)^T = M^T + N^T$

$$\frac{d}{dx}\{[(Ax)^T - b^T]P(Ax - b) + (x^T - x_0^T)Q(x - x_0)\} = 0$$

**Property used:** $(MN)^T = N^T M^T$

$$\frac{d}{dx}\{[x^T A^T - b^T]P(Ax - b) + (x^T - x_0^T)Q(x - x_0)\} = 0$$

$$\frac{d}{dx}[(x^T A^T PAx - x^T A^T Pb - b^T PAx + b^T Pb) + (x^T Qx - x^T Qx_0 - x_0^T Qx + x_0^T Qx_0)] = 0$$

**Properties used:**

- $\frac{d}{dy}(y^T My) = M^T y + My$.
- $\frac{d}{dy}(y^T My) = 2My$, if $M = M^T$ (i.e. $M$ is simetric)
- $\frac{d(Ax)}{dx} = A$
- $\frac{d(x^T A)}{dx} = A^T$
- obs.: $A^T PA$ is simetric, since $(A^T PA)^T = (PA)^T (A^T)^T = A^T P^T A$, but $P^T = P$, since P is simetric. Then $A^T PA = (A^T PA)^T$

**Using the previous properties, we have:**

$$\frac{d}{dx}[(x^T A^T PAx - x^T A^T Pb - b^T PAx + b^T Pb) + (x^T Qx - x^T Qx_0 - x_0^T Qx + x_0^T Qx_0)] = 0$$

$$[2A^T PAx - (A^T Pb)^T - b^T PA + 0] + [2Qx - (Qx_0)^T - x_0^T Q + 0] = 0$$

$$2A^T PAx - (Pb)^T (A^T)^T - b^T PA + 2Qx - x_0^T Q^T - x_0^T Q = 0$$

$$2A^T PAx - b^T P^T A - b^T PA + 2Qx - x_0^T Q - x_0^T Q = 0$$

$$2A^T PAx - b^T PA - b^T PA + 2Qx - x_0^T Q - x_0^T Q = 0$$

$$2A^T PAx - 2b^T PA + 2Qx - 2x_0^T Q = 0$$

$$2A^T PAx + 2Qx = 2b^T PA + 2x_0^T Q$$

$$A^T PAx + Qx = b^T PA + x_0^T Q$$

$$(A^T PA + Q)^{-1}(A^T PA + Q)x = (A^T PA + Q)^{-1}(b^T PA + x_0^T Q)$$

$$x = (A^T PA + Q)^{-1}(b^T PA + x_0^T Q)$$

$$x = (A^T PA + Q)^{-1}(b^T P^T A + x_0^T Q^T)$$

$$x = (A^T PA + Q)^{-1}[(Pb)^T A + (Qx_0)^T]$$

$$x = (A^T PA + Q)^{-1}[(A^T Pb)^T + (Qx_0)^T]$$

$$\boxed{x = (A^T PA + Q)^{-1}(A^T Pb + Qx_0)^T}$$

**Question 3**

**Question 4**

**Question 5**

**Question 6**