Go Go Green It

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1 Abstract

Distributed renewable energy resources are growing in popularity because they provide economic benefits to consumers and reduce energy losses, while also benefiting the power grid and reducing the climate impacts of energy use. Solar and wind energy generation are inherently intermittent, therefore adding energy storage is important to improving system reliability. The main disadvantage of these systems is high capital costs, so it is essential to design systems that will provide enough savings on energy costs for consumers to justify their initial investment. Optimization is used to evaluate economic viability of the system and make decisions about sizing and controls. A major challenge in modeling and optimizing energy systems is the uncertainty associated with renewable generation and energy demand. This project focuses on incorporating uncertainty while evaluating the economic viability of the system and determining optimal sizing and dispatch profiles. This is accomplished through a robust convex optimization program and the use of chance constraints through a Second Order Cone Program. Numerical simulations show that this program has the potential to provide significant economic benefits of up to 63% over a model that does not address uncertainty.

2 Introduction

2.1 Motivation & Background

Global atmospheric CO_2 concentrations are currently at 405 parts per million (ppm), 128 ppm above pre-industrial levels, and are unremittingly rising [1]. Global warming, extreme weather events, and sea level rise have already distressed and displaced communities around the world. Discussions of climate change mitigation often run the risk of deterring action by demanding an abrupt transformation of the entire energy system and disrupting the ingrained flow of electricity in the United States. While this strategy is attractively ambitious, it is also daunting and potentially dangerous to the stability of the grid. A more feasible and applicable approach is to utilize resources already available to support integration of renewable energy sources. Growing trends in U.S. industry lend way to supporting this integration.

Distributed renewable energy, particularly rooftop solar PV, has seen a dramatic increase in market penetration over the last six years. The capacity of residential PV installed in 2016 was approximately 13,000 MW, more than 10 times the amount installed in 2010 [2]. Furthermore, the energy storage market is projected to rise to 6 GW of annual installation in 2017 compared to a base of 0.34 GW annually in 2013 [3]. These technologies are 'exploding' only secondarily to growing concerns about climate change; the primary catalyst for the advancement of these technologies is falling prices. An opportunity exists to leverage these assets to store and discharge electricity at times less costly and less carbon intensive.

To optimally invest in the distributed energy resources (DER) described above, it is essential to evaluate in terms of economics by modeling the site-specific potential of these technologies. Current practice assumes perfect knowledge of future loads and solar generation in order to manage energy storage controls and sizing. Realistically, solar energy generation is intermittent and uncertain. Inaccuracies in forecasting can result in significant economic losses and issues in power system reliability [4]. More accurately modeling statistical variations around solar forecasting and evaluating the potential of future DERs could aid in ultimately enabling a 'smart grid' network fully powered by stored renewable energy.

Any incorporation of various dynamic DERs is inherently challenging. The most significant challenge to overcome, or at least manage successfully, is uncertainty. A multitude of constraints and variables dictate when, where, and how much energy is available at any given moment. This team is uniquely positioned to study this subject. Each group member possesses knowledge and experience in energy systems, particularly in the context of integrating renewable generation sources into the grid. Three of the group members are founding members of Go Green, Go Home, a residential scale demand response system (as part of a project in Berkeley's CE 186: Design of Cyber-Physical Systems course) that automates devices to actuate at renewable-favored times while incorporating users' constraints. Additionally, team members are studying in other complementary classes such as transportation sustainability, behavioral modeling, and electric power systems, all of which combine synergistically to address this energy system and control challenge.

2.2 Focus of this Study

Go Go Green It (GGGI) aims to develop a more realistic approach to optimizing energy system sizing and battery controls than is currently practiced by incorporating an uncertainty model around solar generation and building occupant loads.

3 Relevant Literature

There are many papers available citing the economic risks associated with microgrid planning as motivation to incorporate uncertainty into energy system models. Uncertainty is addressed in various ways. Narayan et al. captured the stochastic nature of demand, renewable energy generation, and cost in the form of 200 scenarios of the random variables[5]. Khodaei et al. also considered uncertainty in load, generation and market prices[6]. They did so by assuming that uncertain data belong to bounded and convex uncertainty sets. Each of these uncertain parameters has a known nominal value and a range of uncertainty within which the parameter is expected to lie within a specified level of confidence. Wang et al. addressed the stochastic nature of renewable energy generation by allowing energy to fluctuate around a reference distribution according to past observations and empirical knowledge [7]. Abd-el-Motaleb et al. modeled the uncertainty associated with available wind energy as well as energy storage using the autoregressive moving average technique[8]. Like Wang et al., they did not include uncertainty associated with cost[7].

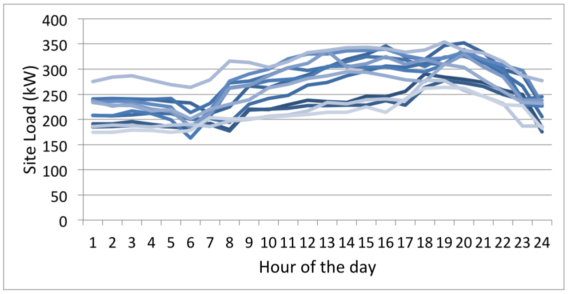
Narayan et al. and Khodaei et al. developed two stage models, an investment/sizing problem and an operation subproblem [5], [6]. Wang et al. does not optimally size the system and only provides optimal consumption and generation scheduling [7]. Abd-el-Motaleb et al. defines their scope as the optimal sizing of distributed generation, but they consider the operation in order to minimize annual operation costs [8]. Since Narayan et al. problem goes through 200 scenarios for each random parameter, it is computationally intensive and took 54 minutes to complete [5]. The other papers did not report long run times.

All of these papers sought to minimize costs and included capital costs and operational costs. The cost for compensating curtailed consumers when power demands were not met was included by Narayan et al. and Khodaei et al. [5], [6]. Narayan et al. also considered the possibility of a carbon tax[5]. Khodaei et al. included the possibility of selling power back to the grid through net metering [6]. All four papers reported that the incorporation of uncertainty produced economic benefits when compared to a deterministic approach. Demand response and user elasticity are considered in Wang et al., and it was reported that more interruptible loads and higher user elasticity leads to significant energy cost savings [7]. In Abd-el-Motaleb et al. the effect of the cycle efficiency and charging/discharging rate of different energy storage units is investigated under various reliability and load shifting levels [8]. It was found that with increasing reliability level with no load shifting, the battery cycle efficiency and discharge rate are the dominant factor for determining the system cost, while the battery unit cost is the dominant factor with increasing load shifting at constant reliability.

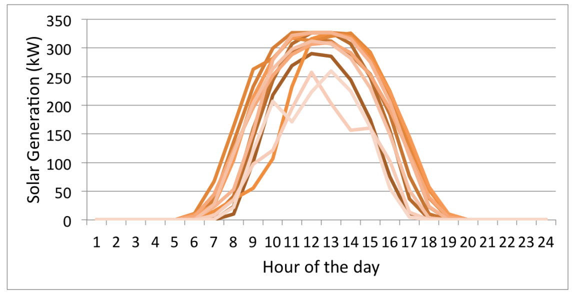
In this paper, sizing and operation are optimized for energy systems. Like the papers discussed, both capital and operational costs are taken into account. Many of the constraints are similar to those seen in the literature, including battery dynamics, supply-demand balance and capacity limits. Uncertainty is incorporated, in order to lessen investment risks, through chance constraints that use Gaussian distributions from site data. The optimization was formulated as a Second Order Cone Program (SOCP) because there are methods to efficiently solve SOCPs, they are scalable in that they require a low number of iterations, and they allow the use of site specific data. [9].

4 Technical Description

GGGI implemented a robust convex optimization program that minimizes the sum of solar PV and battery capital costs as well as operating costs of purchasing electricity from the grid. Uncertainty (error) in the building electricity load and solar irradiance is inherent in the projected values and was built into this model. Figure 1 illustrates the underlying uncertainty in these values by plotting a small sample of the load and generation data over a year for a large retail building in Southern California, provided by NRG's Station A. The hourly standard deviations are shown in Figure 2.

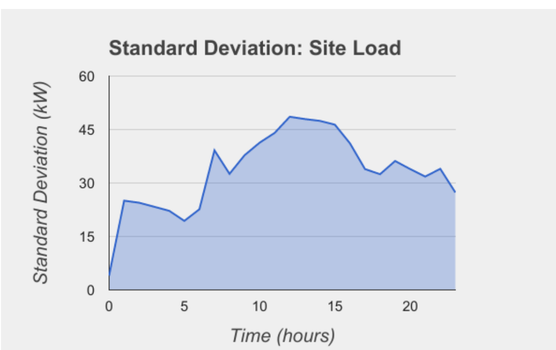


(a) Load



(b) Solar Generation

Figure 1: Sample of site load and solar generation uncertainty



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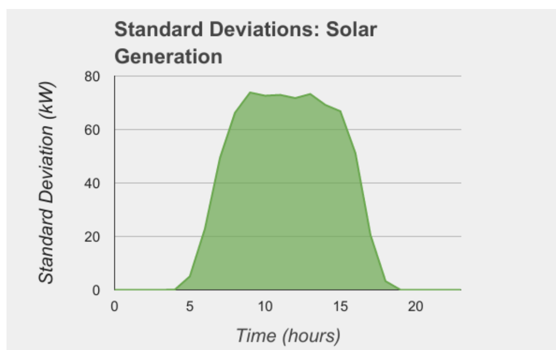

(b) Solar Generation

Figure 2: Standard deviations for site load and solar generation

4.1 Approach

The solution of this program specifically provides optimal energy storage capacity, photovoltaic system capacity, and an optimized battery charging/discharging rate for each time step for the average day. To arrive at the solution, the following steps were taken.

1. A statistical analysis was performed on the site data for a retail building located in Irvine, California. Mean and standard deviation values of solar generation and site demand load were computed based on hourly timesteps within a day extracted from data for an entire year capturing various meteorological phenomena and seasonal fluctuations. This ultimately aided in construction of the daily distribution utilized in the chance constraints explained in detail later.

2. The optimization incorporated bi-directional flow between the system and the grid utilizing Net Energy Metering (NEM) as well as for consumption. The following system parameters were researched and recorded: maximum bidirectional electricity flow between the site and the grid, energy rates for purchase and sale, and capital costs of battery and solar PV installation amortized to a daily USD per kW.

3. A convex, linear optimization to determine battery and PV size and operational controls was formulated. The optimization was designed to minimize capital and use-phase costs, incorporating net metering streams. The constraints were identified as a combination of battery dynamics, battery energy limits, battery discharge/charge limits, system power conservation, and electric grid power restrictions.

4. Chance constraints were substituted to relax certain constraints of choice to handle infrequent stochastic irregularities in the load and/or solar irradiation through a SOCP. These were based on the variances calculated from Step 1.

Figure 3 shows a schematic of the GGGI system and the modeled flow of power within a battery-backed rooftop solar PV building.

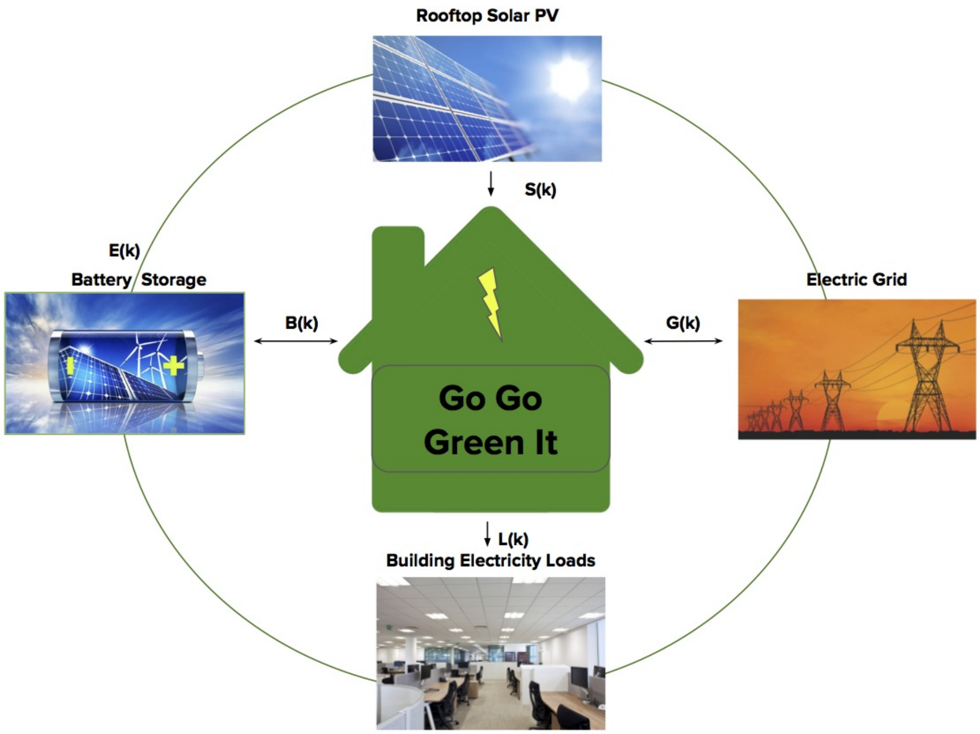

Figure 3: GGGI Schematic

The modeling objective for this scenario is to evaluate the economic viability of DERs (namely solar PV and batteries) and determine optimal sizing and dispatch profiles for the system. As introduced previously, current practice accomplishes this by using convex optimization to solve the scenario as a single problem, assuming the future is certain. Before incorporating uncertainty through chance constraints, this optimization problem was developed with all relevant cost functions and constraints defined.

4.2 Linear Program Formulation

Table 1 provides a reference for the variables, notation, and units applied within the equations and discussions throughout the report.

Table 1: List of Variables and Notation

Variable	**Units**	**Description**
b, s	-	Scale factor for: battery, PV
$G(k)$	kW	Amount of electricity used from the grid
$S(k)$	kW	PV power generated
$L(k)$	kW	Load from the building
$B_c(k)$,$B_d(k)$	kW	Power for the Battery: charging, discharging
$E(k)$	kWh	Energy stored in the battery
c_b, c_S	\$/day	Amortized capital cost of the: battery, PV
$c_G(k)$	\$/kWh	Price of grid electricity
$E_{max}(k), B_{max}(k)$	kWh, kW	Maximum battery capacity in terms of: energy, power
η_c, η_d	-	Efficiency of Battery for: charging, discharging
α	-	Reliability constant (Uncertainty Factor)
σ_S, σ_L	kW	Standard Deviation for: PV generation, load
Δk	hr	Timestep
s_{min}, s_{max}	-	Solar scaling limits
b_{min}, b_{max}	-	Battery scaling limits

The optimization program formulated before incorporating uncertainty is shown in the equations below, with the optimization variables highlighted in blue.

$$\underset{b,s,B_c(k),B_d(k),G(k)}{\text{minimize}} \quad c_b * b + c_s * s + \sum_{k=0}^{N} c_G(k) * G(k) \tag{1}$$

$$\text{subject to} \quad s * S(k) + G(k) - B_c(k) + B_d(k) = L(k) \tag{2}$$

$$E(k+1) = E(k) + [\eta_c B_c(k) - \frac{1}{\eta_d} B_d(k)]\Delta k \tag{3}$$

$$0 \leq E(k) \leq b * E_{max} \tag{4}$$

$$0 \leq B_c(k) \leq b * B_{max}, 0 \leq B_d(k) \leq b * B_{max} \tag{5}$$

$$- G_{max} \leq G(k) \leq G_{max} \tag{6}$$

$$s_{min} \leq s \leq s_{max}, b_{min} \leq b \leq b_{max} \tag{7}$$

This optimization minimizes the capital cost of battery and PV plus the money paid to the utility for importing grid electricity (1), while incorporating the battery dynamics (3), system energy and power limitations (4-5) and electric grid restrictions (6). This is executed all while satisfying the supply and demand model of the site from 2. Equations 1-7 formulate a linear program (LP) with respect to the optimization variables encoded in blue. S(k) and L(k) are non-decision variables due to their stochastic therefore, uncontrollable inputs to the program.

4.3 Second Order Cone Program Formulation

As discussed, the objective of this project was to develop a more realistic model that incorporates uncertainty about site load and PV output in the optimization of battery controls and system sizing. To incorporate uncertainty in solar output and consumer load, a SOCP was developed using the chance constraint method.

First, a simple reduction is performed solving for $G(k) = L(k) - s * S(k) - B_d(k) + B_c(k)$ which is substituted into equations 1 and 6. Variables $L(k)$ and $S(k)$ are denoted as random variables in red.

$$L(k) \sim \mathcal{N}(\bar{L}(k), \sigma_L^2(k)) \tag{8}$$

$$S(k) \sim \mathcal{N}(\bar{L}(k), \sigma_S^2(k)) \tag{9}$$

Second, the chance constraints are applied to constraint equation 2 above since this equation involves uncertainty. We involve a reliability factor α and denote a mean $u = L(k) - s * S(k)$ and similarly the mean $\bar{u} = \bar{L}(k) - s * \bar{S}(k)$ The variance is of the Gaussian random variable $\sigma_u^2(k) = \sigma_L^2(k) + s^2 * \sigma_S(k)^2$ A probability constraint is formulated. Note that $\bar{L}(k)$ and $\bar{S}(k)$ are the expected values of the random variables. It is assumed $\alpha \geq 0$.

$$Pr\left(\frac{u-\bar{u}}{\sigma_u} \leq \frac{B_d(k) - B_c(k) + G_{max} - \bar{u}}{\sigma_u}\right) \geq \alpha \tag{10}$$

Note that the mean of $\frac{u-\bar{u}}{\sigma_u}$ is zero. Rearranging the probability constraint above and using the CDF function, the SOCP constraint is defined.

$$\bar{u} + \Phi^{-1}(\alpha) * \sigma_u \leq B_d(k) - B_c(k) + G_{max} \qquad \text{where } \Phi(x) = \frac{1}{\sqrt{2\pi)}} \int_{-\infty}^{x} e^{(\frac{-y^2}{2})}\, dy \tag{11}$$

Substituting $\bar{u} = \bar{L}(k) - s * \bar{S}(k)$ and variance $\sigma_u^2(k) = \sigma_L^2(k) + s^2 * \sigma_S^2(k)$ the SOCP constraints are finalized:

$$\sqrt{\sigma_S^2(k) * s^2 + \sigma_L^2(k)} \leq \frac{1}{\Phi^{-1}(\alpha)}[\pm(s * \bar{S}(k) + B_d(k) - B_c(k) - \bar{L}(k)) + G_{max}] \tag{12}$$

Note that this SOCP notation above represents two constraints: the upper and lower bounds of G_{max}. These equations help address uncertainty by relaxing the supply and demand constraint (2) based on the variance obtained in the forecasting statistical analysis to obtain (8) and (9). This is calculated by incorporating a desired confidence level or probability and applying it to the cumulative density function equations as shown in (10) and (11). The updated optimization program is then able to provide a more realistic and flexible solution for sizing DER assets and managing battery controls by replacing (2) and (6) with chance constraints (19) and (20). The final SOCP was implemented in CVX, a Matlab-based modeling system for convex optimization, and is shown below:

$$\underset{b,s,B_c(k),B_d(k),E(k)}{\text{minimize}} \; c_b * b + c_s * s + \sum_{k=0}^{N} c_G(k) * [red\bar{L}(k) - s * \bar{S}(k) - B_d(k) + B_c(k)] \tag{13}$$

$$\text{subject to} \quad E(k+1) = E(k) + [\eta_c B_c(k) - \frac{1}{\eta_d} B_d(k)]\Delta t \tag{14}$$

$$0 \leq E(k) \leq b * E_{max} \tag{15}$$

$$0 \leq B_c(k) \leq b * B_{max}, 0 \leq B_d(k) \leq b * B_{max} \tag{16}$$

$$-G_{max} \leq G(k) \leq G_{max} \tag{17}$$

$$s_{min} \leq s \leq s_{max}, b_{min} \leq b \leq b_{max} \tag{18}$$

$$\sqrt{\sigma_S^2(k) * s + \sigma_L^2(k)} \leq \frac{1}{\Phi^{-1}(\alpha)}[s * \bar{S}(k) + G_{max} - B_c(k) + B_d(k) - \bar{L}(k)] \tag{19}$$

$$\sqrt{\sigma_S^2(k) * s + \sigma_L^2(k)} \leq \frac{1}{\Phi^{-1}(\alpha)}[-s * \bar{S}(k) + G_{max} + B_c(k) - B_d(k) + \bar{L}(k)] \tag{20}$$

Table 2 below shows the parameters and assumptions implemented before solving the program in the CVX system package within a constructed Python Jupyter notebook. These parameters represent given information from Station A and assumed engineering design specifications.

Table 2: Parameter Assumptions

B_{max}	0.8* Peak Load
E_{max}	B_{max} *4
G_{max}	600 Kw
η Discharge	0.9
η Charge	0.85

4.4 Variable Grid Capacity

An initial limitation was identified with this reduced formulation. The value of G_{max}, 1000kW according to the California Public Utilities Commission's net metering specifications, was found to render the results infeasible because of its magnitude ([10]). To address this limitation, G_{max} was lowered to 600 kW to ensure that the constraint was active and feasible. Then, an exploratory iteration of the program was developed to validate this assumption. Instead of substituting the supply demand balance equality constraint into the G_{max} equations when adding the chance constraints, this equality constraint was altered to create an inequality constraint so that the uncertainty factor could be incorporated directly. In other words, (19-20) were substituted with (21) and (22) below:

$$\sqrt{\sigma_S^2(k) * s + \sigma_L^2(k)} \leq \frac{1}{\Phi^{-1}(\alpha)}[s * \bar{S}(k) - B_c(k) + B_d(k) + G(k) - \bar{L}(k)] \tag{21}$$

$$-G_{max} \leq G(k) \leq G_{max} \tag{22}$$

where G(k) is now an optimization variable. The results of these methods are compared in the following section.

5 Results and Sensitivity Analysis

The optimization results for a 24 hour period are shown in Figure 4. The reliability for this simulation was 95%. The battery charges between midnight and 10 AM when power coming from the grid is relatively inexpensive. At 10 AM, pricing increases by almost 70% according to the time of use pricing implemented. The maximum cost of electricity occurs between 1 PM and 7 PM. The battery discharges during this time to profit from selling back to the grid. The yellow line indicates the net load as seen by the grid. When the yellow line is below 0, power is being provided to the grid.

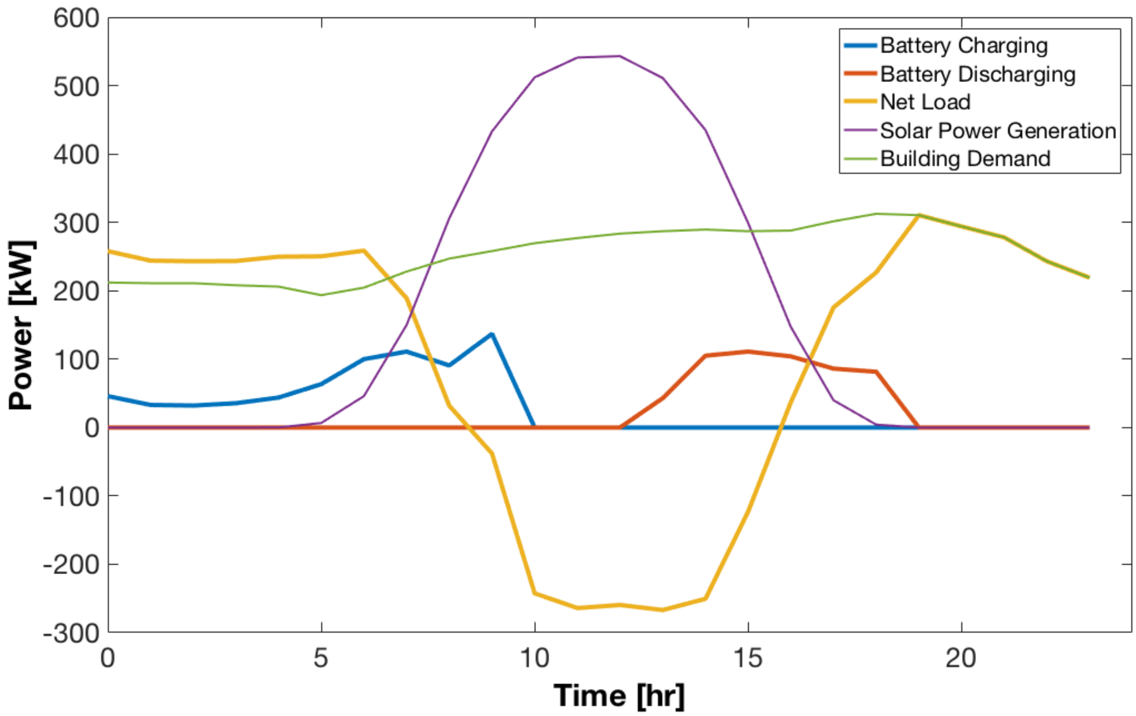

Figure 4: Load, Generation, and Battery Dynamics

Table 3 below shows the sensitivity of the solar size, battery size, and daily cost of power to varying the reliability factor, α. The operating expenses (OPEX) found in the furthest right column demonstrate the magnitude of power usage in the building analyzed. At daily consumption of over 6,000 kWh, the potential for cost savings is immense.

Table 3: Sensitivity Analysis - Varying α

	Solar Life	12 yrs	
	Battery Life	12.3 yrs	
α	**Battery Size (kWh)**	**Solar Size (kW)**	**Cost of Power ($/day)**
90%	802	720	759.26
95%	584	720	759.26
99%	571	720	796.16
99.9%	652	648	893.58
100% (no SOCP)	340	360	1020.20
No DERs	0	0	1471.39

Table 4 and Table 5 illustrate the sensitivity of the optimization variables to varying of the solar life and battery life, respectively. The underlying assumption here is the longer lifetime of the assets spreads out the amortized daily capital cost over a longer period.

Table 4: Sensitivity Analysis - Varying Solar Life

	α	99.9%	
	Battery Life	12.3 yrs	
Solar Life (yrs)	**Battery Size (kWh)**	**Solar Size (kW)**	**Cost of Power ($/day)**
10	652	655	957.15
12	652	655	893.58
14	1019	720	845.36
20	1019	720	755.52

Table 5: Sensitivity Analysis - Varying Battery Life

	α	99.9%	
	Solar Life	12 yrs	
Battery Life (yrs)	**Battery Size (kWh)**	**Solar Size(kW)**	**Cost of Power ($/day)**
10	652	655	912.71
12	652	655	895.85
14	1019	720	879.84
20	1359	655	839.77

The alternative method used when specifying G(k) as an optimization variable and limiting the reliance on the accuracy of G_{max} yielded results on a similar order of magnitude with slight variation. For instance, the cost for the 99.9% reliability was still $759 with battery size of 0.42 and solar size of 2.0. In other words, without restricting the solar net metering to 600kW, it is optimal to size the solar panels at full capacity even at high reliability because the savings of selling power during peak solar hours pays off.

6 Discussion

From the results of this study, it is apparent that the addition of chance constraints allows assets to be sized to incorporate varying levels of reliability that result in correspondingly varying levels of savings. Table 3 in particular, highlights the advantages of implementing a reliability factor by measuring daily operational costs over time. Without chance constraints under current industry practice (100% reliability), operators are able to save $1.9 million over the assumed lifetime of 12 years. This is noteworthy as the base case of no DER assets yields a lifetime utility bill of $6.4 million. However, with the implementation of the SOCP with a 95% reliability, the program developed by GGGI has the potential to save an additional 63%, or $3.1 million total, over the same 12 year time scale.

Because information about solar PV generation and demand load is stochastic, sizing and optimization of assets clearly should incorporate an uncertainty factor to provide more realistic model inputs. The advantage of the SOCP, aside from the resulting high cost savings, lies in its scalability and efficiency as previously discussed. A noteable limitation is that the program is sizing based on the expected cost of the variables. While this is necessary as the future is not known, it also provides favorable results based on the data available. Keeping in mind that at 95% reliability the demand will not be met approximately 5% of the time, increased costs will be accrued when using grid electricity to cover the load. Therefore the $1.2M projected savings will likely be reduced, but not by a factor significant enough to limit the impact of the results. A method to resolve this would be to 'train' the optimization forecasts based on realtime data across NRG's asset locations. More accurate forecasting yields less uncertainty and thus more optimal results.

Although GGGI's optimization algorithm calls for a larger and more expensive battery and solar installation, the relaxed constraint means there is potentially more opportunity to rely on DER assets than forecasted. This assessment, if further investigated, provides confidence that the larger assets can satisfy higher loads and store more incoming solar energy than before the SOCP. This is important as industry suggests that building owners want to be assured they are getting the most value for their money hence the need for optimization, but they also want to have a level of confidence in the product to meet their demands most of the time. The sensitivity analysis in Tables 4 and 5 were included to show how the change in asset lifetime affect the daily savings. The cost of power varies highly with asset lifetime, yet the projected savings still rival the 100% reliability scenario even with the conservative lifetime estimations.

It is interesting to note the effect net metering and various peak and off-peak pricing structures have on the result. In Figure 4, the net load drops below 0 kW indicating a discharge to the grid from the battery and solar PV during the peak price time periods throughout the day. If net metering were not allowed, or capped less than our current G_{max}, then the savings would not be as substantial. By manipulating the chance constraints one can discern which of the restrictions are the most limiting. This model can be used to demonstrate how changes in policy such as the amount a customer can sell to the grid impacts the economic viability of the project. The easy manipulation of the program allows the user to quickly size batteries and solar PVs in different locations under varying policy and pricing structures.

7 Summary

The goal of this project was to find a more robust method to size PVs and batteries given a customer's location and demand. Often, building owners are risk averse, so the conclusion of this study is that the GGGI model is more valuable than current practice because it incorporates the expected natural daily variability in generation and demand. The economic advantages of a model with perfect generation and load predictions compared to one with uncertainty are made clear in the technical description section of this report. Future work in this area could include adding uncertainty to the utility pricing structure which would undoubtedly change the investment to return ratio of a battery and PV installation project. Uncertainty can be found in many real world energy systems, and its effects should be quantified in order to give decisions makers the tools they need. With the tools provided in this report, building owners and decision makers can make the decisions that lead to reduced utility energy demand and thereby lower carbon emissions.

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