A Notation-Based Semantification Study

by

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MathSemantifier - a Notation-based Semantification Study

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Abstract

Mathematical formulae are a highly ambiguous content for which typesetting systems as LaTeX store only the rendering information. MathSemantifier is an open-source notation-based mathematical formula semantification system that attempts to tackle the problem of ambiguity in mathematical documents and produce knowledge-rich equivalents. The system extracts formulae (from formats such as LaTeX or MathML) and produces content-rich results (sTeX or Content MathML) that contain no semantic ambiguity. The disambiguation is achieved by matching the input formulae against a known database of notation definitions, which is aggregated into a Context Free Grammar. This paper outlines an implementation of MathSemantifier that focuses on helping researchers in semantifying their works, and the ultimate goal being a scalable implementation that would need next to no help from a human, and, therefore, could be used to semantify large collections of mathematical papers such as arXiV [ArX15].

Contents

1 Introduction

The scientific community produces a large number of mathematical papers (approximately 120 thousand new papers per year), which raises the importance of machine based processing of such documents. Unfortunately, the most popular formats in which these papers are found (for instance, LaTeX) do not contain much information that would allow the computers to infer the complex knowledge graph behind each paper. Since, at this point, changing these formats is not practically possible, the other solution is to add a semantic flavor to the existing documents by translating them into a more suitable format, for instance, Content MathML.

1.1 History and Motivation

As a system as complex as the current scientific community was created, it went through a series of evolutions in the attempt to introduce the best method of writing scientific documents. This process was highly influenced by the invention and spreading of the internet. Scientists understood the necessity of a standard that could help them write and exchange their findings in an efficient way. A lot of effort has gone into translating books into digital documents.

Now, scientists have found ways to represent their knowledge in a machine comprehensible manner, some of which are Content MathML and OMDoc [Koha]. These new methods do not directly store the rendering of the documents. Instead, what is actually stored is the knowledge graph hidden behind the ambiguity of the representation. Naturally, these documents can be used to also generate a human readable format, examples being Presentation MathML and LaTeX.

As previously mentioned, a lot of effort went into translating books into digital documents. Since the year of 1850, there have been produced approximately 3.5 million papers, and approximately 120 thousand new papers are written every year. Now, when all these digital documents need to be converted to these semantic representation, doing it by hand is not just unreasonable, but straight out impossible.

The next step in this evolution is translating digital documents into improved digital documents, that the computers can actually understand and not just store. This next step can only happen if a new, relatively painless, way of transition appears. As soon as the ease of transition and the benefits from doing it outweigh the difficulties associated with it, the scientific community will open the door into the world where computers can actively help researchers with more than just symbolic searches.

1.2 Content MathML and Presentation MathML

The two main formats this paper is focusing on are Content MathML (CMML) and Presentation MathML (PMML) [ABC+03].

PMML is used to describe the layout and structure of mathematical notations. PMML elements construct the basic kinds of symbols and expression-building structures present in traditional mathematical notations, containing also enough information for good renderings. The last part is exactly the motivation as to why MathML alone is not enough - because it only suggests specific ways of renderings, but does not require anything.

CMML, on the other hand, is used to provide an explicit encoding of the underlying mathematical meaning of mathematical expressions. This fact is important in this context because this implies that CMML contains no ambiguity, so, by choosing the final product to be CMML, it is indeed possible to achieve meaningful semantification. For example, considered "H multiplied by e". It can be often seen to be written as He in mathematics, however, this can be interpreted also as H applied to e in the context of lambda calculus, as well as a chemical.

An example of PMML and CMML code for the expression $x+y$ is provided in Figure 1. Note how the **mrow** tag is used to delimit nested expressions, while the **mo** tag indicate mathematical objects (such as arithmetic operators) and **mi** is used for variables and similar objects. This provides a clear structured format, that is parsed by MathSemantifier. In particular, PMML is such a small and convenient language that even building a parser for it as a part of MathSemantifier is certainly reasonable.

In the CMML part of the example, the **apply** tag is semantically the application of its first element (the arithmetic plus) on two identifiers x and y, which altogether means simply $x + y$.

PMML
```
<mrow>
    <mi>x</mi>
    <mo>+</mo>
    <mi>y</mi>
</mrow>
```

CMML
```
<apply>
    <csymbol cd="arith1">plus</csymbol>
    <ci>x</ci>
    <ci>y</ci>
</apply>
```

Figure 1: PMML and CMML

An important part of CMML is that all notations (like the arithmetic plus notations used in the example above), contain specifications of how these should be rendered into PMML, so the conversion is not a problem.

1.3 Ambiguity in Mathematical Documents

An important concept necessary in order to understand why semantification is a complex process is ambiguity. Mathematical documents are not a simple collection of symbols. The main use of these documents emerges only when the knowledge graph of a document is accessible. However, humans tend to be lazy in writing down the whole graph, but instead rely on implicit human knowledge to decipher these documents. This is where ambiguity comes into play, when the author relies on the ability of the human to use the context of document in order to pinpoint the actual meaning an expression. Ambiguities can be largely divided into two: structural and idiomatic ambiguities.

1.3.1 Structural Ambiguities

A simple example that demonstrates the concept of structural ambiguities can be $f(a(b))$. It can mean one of the following:

1. Application of function f to a times b
2. Application of function f to the application of a to b
3. Multiplication of f, a and b
4. Multiplication of f and the application of a and b

Note how such a simple example with just three identifiers has at three different possible interpretations. It is easy to notice that the number of readings has an exponential complexity in the length of the expression. For instance, the expression above has 2^{n-1} readings, where n is the number of identifiers, which is because we can considered each application as a multiplication too.

The concept above can be generalized as structural ambiguities being associated to different readings generated from different parse trees. This means that the document may provide some direct clue about what parse tree is the best, and the readings can be characterized by parse trees.

1.3.2 Idiomatic Ambiguities

Contrary to structural ambiguities, idiomatic ambiguities are not due to different parse trees. Given one single parse tree, some formulae allow for multiple readings. A standard example would be B_n. This could be:

1. The sequence of Bernoulli numbers
2. A user defined sequence
3. The vertex of one of a series of geometric objects

In other words, the same sequence of symbols, associated with the same parse tree can lead to multiple readings. The only feasible way at the moment of solving such ambiguities is having a large notation database and ultimately asking the user to choose from a list of possible readings. This is exactly how MathSemantifier works, which means that the final product is expect to excel at solving such tasks.

1.4 Extracting Semantics from Mathematical Documents

Ambiguity is the problem, so, now, a solution for it is required. Let us recall the example from the previous section $f(a(b))$. The first interpretation of the expression a human reader is likely to come up with is the application of f to the application of a to b. Moreover, the other readings may take the human reader by surprise, as he or she would assume that the first reading is the correct one.

Let us look into why this happens. First of all, f is a symbol humans usually use for functions, moreover, the brackets around $a(b)$ could be omitted if it were a multiplication rather than an application. Next, a and b are usually used for variables, but then again the brackets are useless if it is a multiplication. To sum up, the human throws away meanings that would imply useless work or usage of symbols in an unusual way. Imagine asking a mathematician which of the expressions is more natural, fa or ab, or, to be even more extreme, "Let $\epsilon > 0$" or "Let $\epsilon < 0$". All of these can be translated to heuristics that MathSemantifier can use to improve its results, or otherwise said, minimize the amount of work the human has to do by restricting the possible meanings as much as possible.

Up until now, different heuristics used were discussed. However, mathematics has better means of finding the one true meaning of the document. Imagine the formula $a = b$. As a human, we can deduce it may mean that some two objects are equal. We still have absolutely no understanding of what those objects are, and we just assume $=$ works as a relation on any 2 objects of the same type.

Now let's add a bit to the formula $(a = b) \wedge (b = 3)$. At this point, we deduce that $a = 3$, and $=$ is a relation of numbers by applying First Order Logic to the expression. Notice how more context reduced the ambiguity of the previous expression. Also, notice how we naturally assume First Order Logic is applicable to this situation. Notice also that we are not using just heuristics, but rather we are applying a deterministic approach to find out which meanings are impossible.

Let's get to a more interesting example $f(a + b)$. Normally, we would assume this is a function applied to $a+b$, but nothing in this context can help us decide against f multiplied by $a + b$. Adding an expression like $f : \mathrm{R} \to \mathrm{R}$ would certainly help to reason against the second interpretation, because it does not type-check.

To sum up, humans read mathematical documents bit by bit and throw away impossible interpretations until there is, hopefully, only one left. In doing so, they mainly apply two strategies - heuristics and proof by contradiction. This paper explores the possibility of MathSemantifier doing the same to some extent.

1.5 An Introduction to MathSemantifier

In order to understand what **MathSemantifier** does, it is required that the Presentation Algorithm is explained first. The reason for that is that **MathSemantifier** is the exact opposite of the Presentation Algorithm, trying to convert PMML to CMML, as opposed to CMML to PMML.

1.5.1 The Presentation Algorithm

The presentation algorithm has its main goal to covert Content MathML to Presentation MathML, using a database of notation renderings. In Figure 2 a typical example of what the presentation algorithm produces is displayed. The first child of the **semantics** node

```
<math id="-1759683794" xmlns="http://www.w3.org/1998/Math/MathML">
  <semantics>
    <mrow>
      <mrow class="math-selected" data-mmt-position="">
        <mi data-mmt-position="1">a</mi>
        <mo data-mmt-symref="http://mathhub.info/smglom/numberfields
        /natarith.omdoc?natarith?addition" data-mmt-position="0">+</mo>
        <mi data-mmt-position="2">b</mi>
      </mrow>
    </mrow>
    <annotation-xml encoding="MathML-Content">
      <apply>
        <csymbol>http://mathhub.info/smglom/numberfields/natarith.o…</csymbol>
        <ci>a</ci>
        <ci>b</ci>
      </apply>
    </annotation-xml>
  </semantics>
</math>
```

```
<omdoc:notation cd="natarith" name="addition" stex:macro_name="natplus"
                stex:nargs="1" xml:id="natarith.notation8" about="#natarith.notation8"
                stex:srcref="smglom/numberfields/source/natarith.tex#textrange(from=7;0,to=7;76)">
   <omdoc:prototype>
     <om:OMA>
       <om:OMS cd="natarith" cr="fun" name="addition"/>
       <omdoc:exprlist name="args">
         <omdoc:expr name="arg"/>
       </omdoc:exprlist>
     </om:OMA>
   </omdoc:prototype>
   <omdoc:rendering precedence="500">
     <omdoc:iterate name="args">
       <omdoc:separator>
         <m:mo cr="fun">+</m:mo>
       </omdoc:separator>
       <omdoc:render name="arg"/>
     </omdoc:iterate>
   </omdoc:rendering>
  </omdoc:notation>
```

Figure 2: Presentation Algorithm Output and the corresponding Notation Definition

contains the PMML that corresponds to the CMML contained in the **annotation-xml** node. In order to produce this output, the algorithm used the notation **natarith addition** (shown in Figure 2 as well).

1.5.2 Definition of Valid Results

The Presentation Algorithm is conceptually simple because there is no ambiguity involved. A certain CMML expression is always presented the same way. Even more so, the implementation is free to include elements like **mrow** or **mspace** as long as those do not change the meaning.

However, different CMML expressions can be rendered the same way, which is were the ambiguity of the reverse process takes origin.

A valid result for **MathSemantifier** is, given a PMML expression, such a CMML expression that the Presentation Algorithm can render back to the original expression (up to **mrow** or **mspace** tags).

Note that, while this is certainly already a difficult task, a more proper definition would be, given a PMML expression, a CMML expression that can be rendered back to the original expression AND the resulting represents the actual meaning of the PMML expression (or at least, it is a valid

meaning for the expression according to the common sense of a human mathematician). **MathSemantifier** does not explore this task beyond using some heuristics mentioned in the further sections.

1.5.3 Approach to Semantification

MathSemantifier converts PMML into valid CMML as described above. In order to perform this task, it needs to match PMML against a list of notations. This is achieved by compiling the notations into a Context Free Grammar, and using a CFG Parsing Engine to parse the PMML. A parse returns a list of possible parse trees, out of which **MathSemantifier** extracts information regarding what notations matched at top level. This is then done recursively for the arguments of the found notation. The **Implementation** section describes in a lot more detail how the Notations are compiled into a CFG, and how the parse trees are converted to CMML. Parsing using CFGs is a known problem, so, instead of writing a Parsing Engine, a more reasonable approach is using an existing one. For that purpose, the **Marpa Grammar Engine** is used.

1.5.4 Possible Applications of Semantification

- **MathWebSearch [RPK14]** (discussed in more detail in subsection 2.3) is a search engine for mathematical formulas. Such search engines could greatly benefit from semantification. The idea is that a search engine is only as good as the database of information is. By improving the information it can search through by adding a semantic flavor to it, a new kind of queries could be possible - semantic queries. Rather than searching for strings, or formulas with free form subterms, the user could specify the meaning of the sought expression. This would improve a lot the relevance of the results, since there will be no result that matched just because it was presented in a similar manner.

- Another possibility for using semantification is theorem proving and correctness checking. One possible application would be realtime feedback to the user writing a paper about the correctness of the expressions used.

- The possibilities extend even beyond this. Using semantified content, rather than having a database of CMML expressions, it is possible to create a smart knowledge management system that could be used to create expert systems. The user could then ask questions, or create complex queries, to exploit the full power of semantic content.

Semantification will not make all of the above directly possible, however it is a necessary step towards achieving goals similar to the ones described above, that require more knowledge about the used content than just how it is rendered.

1.5.5 MathSemantifier in the Context of Disambiguation

In the endeavor of extracting semantics from mathematical documents MathSemantifier uses its notation database in order to attempt to explore the space of all possible readings, and the to throw away some using heuristics, ultimately letting the user decide which reading is the best.

This makes clear the parts the final product excels at, but also the main challenges related to that.

Regarding the good parts:

- Solving both structural and idiomatic ambiguities by exploring the space of all readings

- Notation addition can be easily added to the system, since the Context Free Grammar used is generated automatically

- Enhanced overall user experience when writing or semantifying existing mathematical documents by creating a dynamic, adaptive system by means of the above mentioned strengths

The main challenges are:

- Scalability in the context of such a system translates into finding the best ways to generate as few useless parse trees as possible

- Compiling a notation database into a Context Free Grammar when including multiple notation archives simultaneously

- The response time of a query to the system is a big issue, since, as mentioned before, everything related to ambiguities has an exponential time complexity. Finding a balance between exploring the space of all readings enough in order to find meaningful results while keeping the response time low is a very difficult challenge.

The rest of this paper demonstrates in detail how MathSemantifier harnesses its strengths and solves or avoids the challenges described.

2 State of the Art

2.1 Semantification

Nowadays extracting the semantics of mathematical documents is regarded as an interesting field, however the number of researchers in it is limited. This is caused by several domain specific problems.

First of all, mathematics is usually represented in a format where it mixed with natural language. Research on extracting mathematical expressions from such contexts has not been incredibly successful so far. Another important problem is the lack of a well prepared set of data. This kind of data could be used for Machine Learning applications as training data, or simply as test data for complex systems. As software that has not been tested cannot be called particularly useful, this poses a great problem.

2.1.1 Restricted Natural Language

There are multiple projects that try to use a subset of the natural language that is still not that ambiguous but allows for intuitive document creation. Such projects are FMathL [NS], MathLang [KMW], MathNat [HR] and Naproche [CKS].

Most such approaches try to analyze the context free parts of mathematical expressions. We will go into more detail about Context Free Grammar applications in semantification in the next sections. It is worthy to note though that most such projects are not satisfied by commonly found grammar engines, and some, like FMathL, are developing their own specialized parsers. This paper does not outline custom parsers, however, if further work is to be done on the subject, implementing heuristics as to reduce the disambiguation choice list could require at some point a similar approach.

2.1.2 Format Conversions

Mathematical documents in their majority are written in TEX/LATEX, but for storage and processing purposes converting them to an XML representation is a good idea. Large scale mathematical document archives such as *arXMLiv* [SKG+10] are using the LATEXML [Mil] converter to produce MathML and OpenMath.

LATEXML can actually already produce Content MathML, however the result produced in such a way from a knowledge poor setting contains only one of the possible readings, and, therefore, is not satisfactory from a semantic point of view as it not need satisfy the actual semantics of the document. Enhancing LATEXML's ability of converting LATEXinto knowledge rich XML format variations by exploring the space of possible readings is one of the main goal of this thesis.

2.2 Using Grammars for Semantification

2.2.1 Combinatory Categorial Grammars

The MSc Thesis of Deyan Ginev [GKW11] studies a similar approach that uses Combinatory Categorial Grammars as a part of a larger analysis pipeline that is applied to handpicked set of mathematical texts. The results show a close to perfect recall rate with a low degree of ambiguity.

This approach, however, uses a manually crafted grammar as opposed to the generated grammar used by MathSemantifier. While writing a grammar by hand can lead to good results as mentioned above, this approach does not scale, since it is hard to find someone who will write such grammars every time a new notation is needed. Notations, on the other hand, are much faster and easier to write. In other words, scalability and extensibility are the reasons behind MathSemantifier.

2.2.2 S-graph grammars

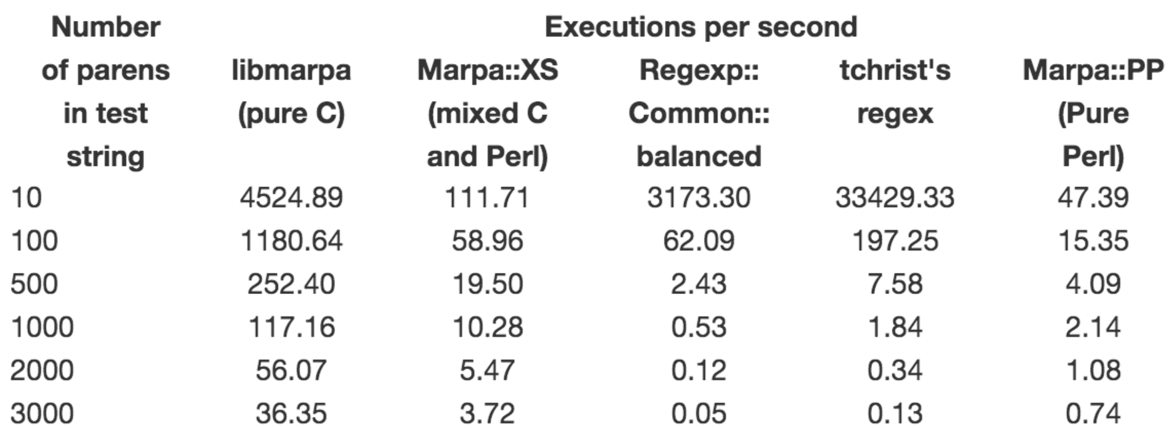
The research of Alexander Koller of University of Potsdam exhibits an interesting way of utilizing grammars for semantic construction. S-graph grammars [Kol] constitute a new grammar formalism for computing graph based semantic representations. What distinguishes this line of research from the common data-driven systems trained for such purposes is that S-graph grammars use graphs as semantic representations in a way that is consistent with more classical views on semantic construction.

S-graph grammars are introduced as a synchronous grammar formalism that describes relations between strings and graphs, which can be used for a graph based compositional semantic construction, which is essentially what this paper outlines in simpler terms - using Context Free Grammars.

2.2.3 Marpa Grammar Engine

As the idea of the thesis is to use a Grammar Engine, we need to introduce a concrete solution. Text parsing using Context Free Grammars is quite popular, and, therefore, there is quite a number of solutions, which can, however, make the choice of the best grammar engine for a particular purpose difficult. After comparing several such tools, the conclusion reached was that the Marpa Grammar Engine [Kegb] stands out as a flexible, powerful and efficient tool suited for the purpose of semantification.

To understand better the claim about efficiency, Figure 3 provides a comparison between multiple regex parsers and Marpa made by the creator of Marpa is provided.[Keg11]



Number of parens in test string	Executions per second				
	libmarpa (pure C)	Marpa::XS (mixed C and Perl)	Regexp::Common::balanced	tchrist's regex	Marpa::PP (Pure Perl)
10	4524.89	111.71	3173.30	33429.33	47.39
100	1180.64	58.96	62.09	197.25	15.35
500	252.40	19.50	2.43	7.58	4.09
1000	117.16	10.28	0.53	1.84	2.14
2000	56.07	5.47	0.12	0.34	1.08
3000	36.35	3.72	0.05	0.13	0.74

Figure 3: Marpa Parser - Comparison with other Parsers

The results are presented in executions per second. We can see that standard solutions may be faster for a small nesting level, but they go quadratic as it rises.

Another important point is that Marpa Grammar Engine has a quite simple interface, allowing for extensive manipulations during the parsing process. Each rules can be given an “Action” - a Perl routine - that can define what the engine is supposed to do when the rules is used.

As it was mentioned in the previous sections, the grammar generated is highly ambiguous, and the Marpa Engine allows for ambiguous grammar of any kind that are required for this application.

2.3 Extracting Formulae from Mathematical Documents

As MathSemantifier as described in this paper need not be restricted to processing just MathML formulae, but also mathematical documents written in LaTeX, a method of extracting MathML formulae from mathematical documents needs to be provided. A simple way of achieving that would be by using the same method as MathWebSearch [RPK14] is using in order to extract MathML formulae from documents containing MathML formulae and other information. The structure of MWS is shown in Figure 4. MathWebSearch accepts MwsHarvest [MWS15] as from the crawler as seen in Figure 4. These harvests are in Presentation MathML, therefore perfectly suited for this application.

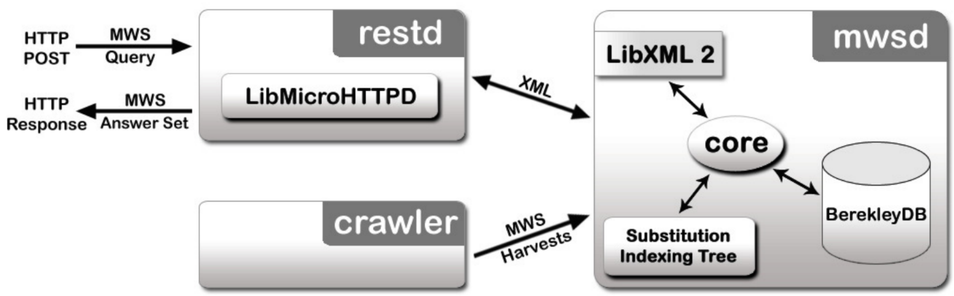


Figure 4: MathWebSearch Architecture

3 Preliminaries

3.1 The Notation Database of MMT

MathSemantifier uses a Notation Database to create the grammar it uses. Therefore, it needs to be explained where those notations come from. MMT [Rab] (Module system for Mathematical Theories) is a language developed as a scalable representation and interchange language for mathematical knowledge. It permits natural representations of the syntax and semantics of virtually all declarative languages. The decisive factor about MMT is that there is already a large database of notations written in sTEX, which is transformed and stored in MMT in an original format that MathSemantifier is processing in order to generate a Context Free Grammar.

SMGloM [GIJ+] is a part of the notation database of MMT that is especially relevant to the MathSemantifier. It contains notations from vastly different topics of mathematics. The ones MathSemantifier uses are number fields, algebra, calculus, geometry, graphs, magic, sets, topology, mv, smglom, primes and numbers. SMGloM is also available online [Kohb].

3.2 MMT Notation Storage Format

The original format that MMT uses to store the notations is of high relevance, since it the raw material from which the Context Free Grammar is generated. The format consists of a Scala **case class** tree. Several examples are provided in Figure 5.

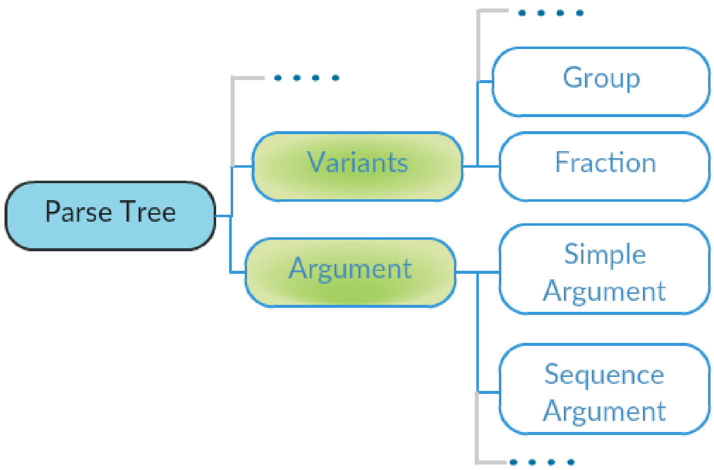


Figure 5: MMT Notation Markers

Since there are about 40 types of different Markers, the tree above is meant to simply give an idea of the structure, rather than explain it fully. The **Marker** class is the tree of all possible notation components. The most basic types of Markers include **Presentation Markers** and **Argument Markers.** They have a great diversity of subtypes. **Simple Argument** is used for single arguments, while **Sequence Argument** is a placeholder for a sequence with a specified delimiter. The most basic example of a sequence argument is, if considering the example $2 + 3 + 5$, the sequence $2, 3, 5$ and the delimiter $+$.

Presentation Markers have yet another purpose of simulating the PMML structure. **Group Marker** stands for the **mrow** MathML tag, **Fraction Marker** - for **mfrac** and so on. MMT has Markers for every possible MathML tag, which goes to explain the total number of Markers to an extent.

3.3 LaTeXML Pre-processing

Since sTeX, as an extension of LaTeX, is just as complex to parse, MMT uses LaTeXML in order to convert sTeX to OMDoc [Koha]. OMDoc is then processed and stored in MMT.

Figure 6 contains an example of the arithmetic plus notation written in sTeX and the equivalent in MathML.

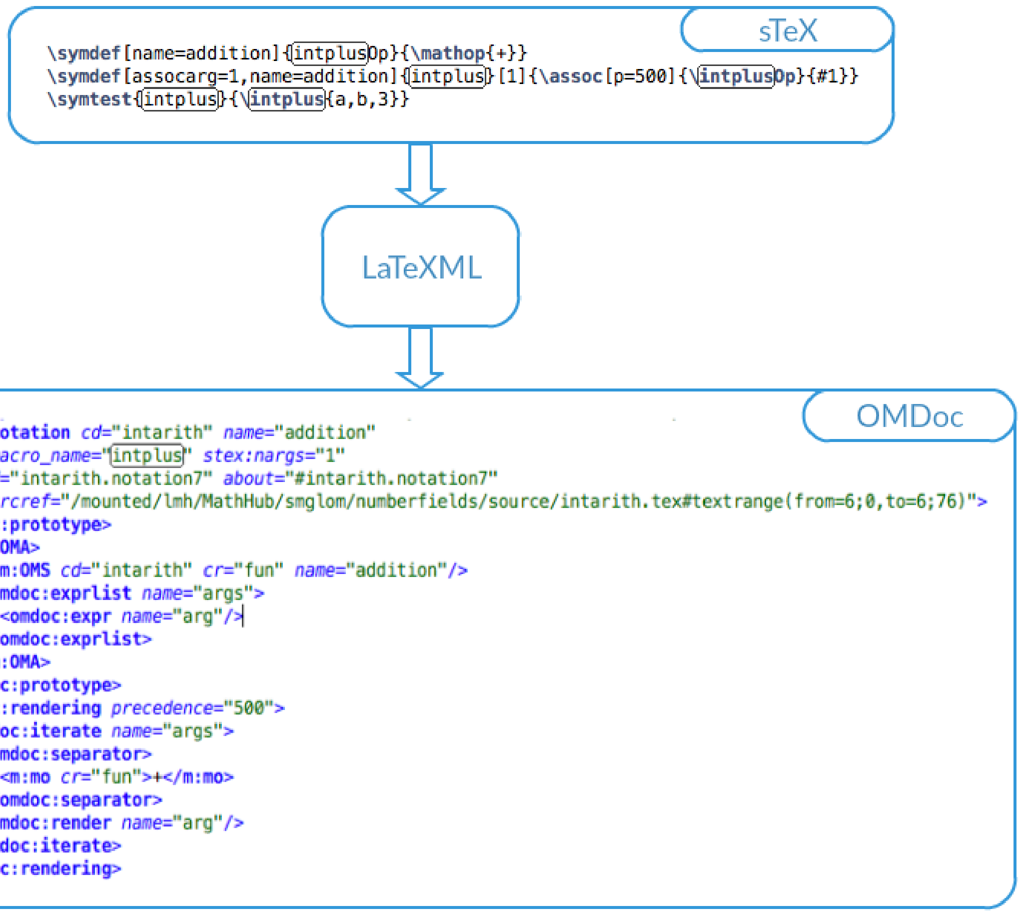


Figure 6: sTEX to OMDoc conversion

3.4 Marpa Context Free Grammar Parser

MathSemantifier takes the an auto-generated CFG and input from the **Web UI** (discussed in more details in further sections) in order to produce possible parse trees of the input. Marpa Grammar Engine is the proposed. It was already discussed above, so let us summarize the important points that matter for this particular application.

Marpa is [Kega]

- Fast - it parses grammars in linear time. This is critical since the size of the grammar is likely to be at least linear in the number of notations
- Powerful - it can parse left, right and middle recursions. Since it is unclear what kind of recursions can occur in generated grammars, and it would be a pity if the parse tree generator would need to be changed midway because of it, it is better to choose something that can handle all kinds of recursion

- Convenient - Parser generation is unrestricted and exact, all that needs to be provided is a CFG in BNF form. Also, if several alternatives may yield a parse, all of them are considered. This is critical, since this directly implies that it can deal with ambiguous grammars
- Flexibility - it provides control over the parsing process, giving out information about which rules have been recognized so far and their locations. This is again extremely important because otherwise, once again, semantification would be simply impossible

Since the CFG is ambiguous this is the most computationally intensive part. The performance of the final product mostly depends on the efficiency of this step.

As a side note, Marpa does not have a Scala API, so, instead, the Perl API is used. This implies that the result parse trees need to be transported somehow to the core application in MMT. For that purpose LWP is used. [LWP15]. LWP is a set of Perl modules which provide a simple and consistent API to the World Wide Web. After experimenting with the API, the conclusion was that the API is simple and powerful enough for the current purpose.

4 The MathSemantifier System

The major idea of **MathSemantifier** is, as already described in the introduction, finding possible Content MathML readings for Presentation MathML input expressions.

The general flow of a single semantification can be described as follows:

1. Context Free Grammar generation from the MMT Notations
2. Parsing using the Marpa Grammar Engine and the generated CFG to detect the top level notation
3. Parsing the arguments of the top level notation recursively
4. Using the parse trees from step 2 and 3 to generate an internal representation of the meaning trees
5. Converting the meaning trees to Content MathML
6. Displaying the Content MathML trees in the frontend

The reason it was decided to use a CFG based solution is that there exist already parsing frameworks like the Marpa Grammar Engine. It solves all the parsing related technical problems, like parsing ambiguous expressions, different kinds of recursion, while also providing a high degree of freedom.

This sections goes into the details of how exactly semantification is accomplished. First, a general overview of the goals of the project and its high level architecture is given. Then, each component is described in further detail.

4.1 Project Goals and Challenges

The main goals of the project can be expressed in a concise manner as follows:

1. Generating the correct set of parses efficiently and effectively
2. Providing opportunities for improvement for further research in the area

The challenges on the way of achieving a way of generating dynamically the full set of semantic parse trees are:

1. Aggregating the mathematical notation database that the MMT system into a CFG
2. Dealing with scalability issues which are entailed by ambiguity
3. The set of correct parses should be a subset of the set of generated parses
4. The set of generated parses should ideally be a subset of the set of correct parses too
5. Handle character encodings correctly

This following subsections how **MathSemantifier** achieves the above goals.

Section 6 shows to what extent the challenges described are overcome and how.

4.2 **MathSemantifier** Architecture

The architecture can be roughly divided into four parts as shown in Figure 7. The components will be discussed in more details in the further subsections.

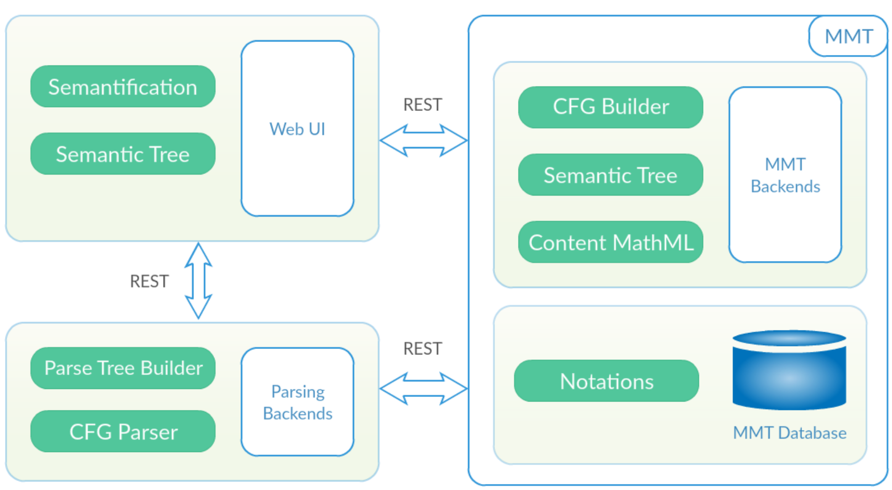


Figure 7: **MathSemantifier** Architecture

4.3 Component communication

Regarding the communication between parts:

- MMT Backends and MMT Notation DB are both part of the MMT Server Application
- All the other connections are implemented using the **REST** protocol, using simple **POST** requests

This also implies that the all the different components do not need to run on one machine, especially the **Web UI** and the reset of the components.

The frontend / backend separation reveals opportunities for distributed solutions of such a system, in case scalability becomes an issue.

4.4 Web User Interface

The **Web UI** is a core component of **MathSemantifier.** It is intended to be a lightweight solution that queries a server for the results of more computationally intensive tasks.

The interface consists of an input area, where **MathML** needs to be inputted, and three options:

1. Semantify (The user can guide the semantification of the top symbol directly, by choosing the correct matching range, notation and argument positions)
2. Show Semantic Tree (The other option is to ask for all the possibilities and get all the semantic trees)
3. Evaluation (In this case, by repeatedly pressing this, the user is walked through a series of examples to demo the functionality)

The **Evaluation** option will be discussed in more detail in section 6.

4.4.1 User Guided Semantification

The user provides Presentation MathML as input to the system, then uses the **Semantify** button to reveal a list of top level notations as shown in Figure 8.

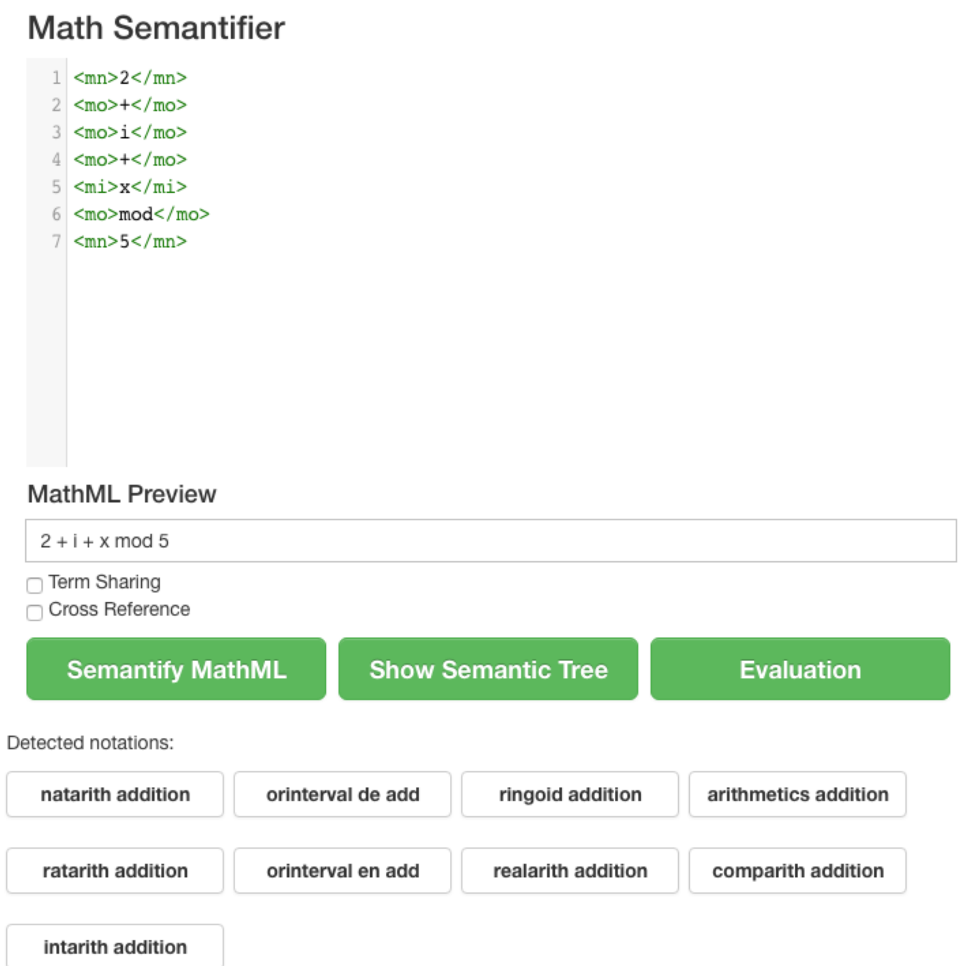
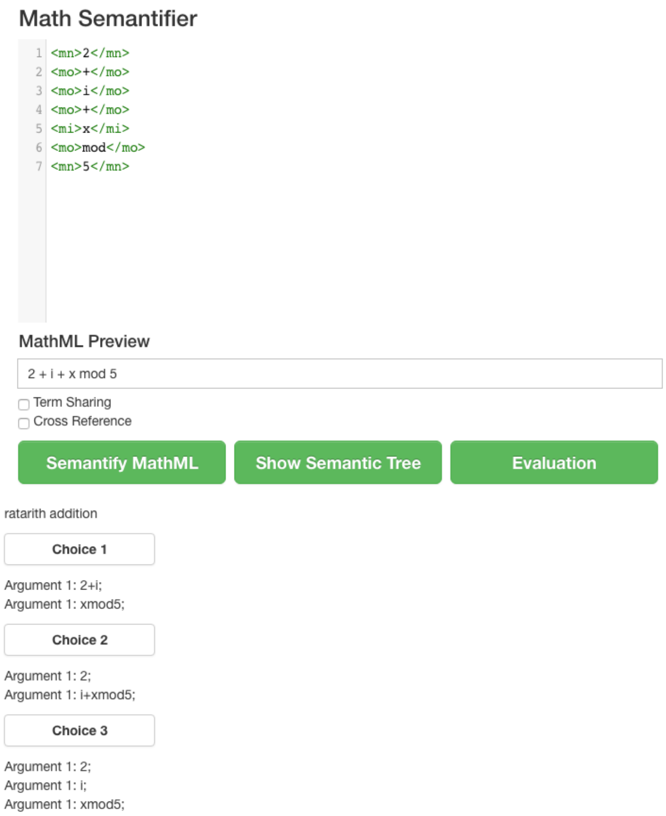
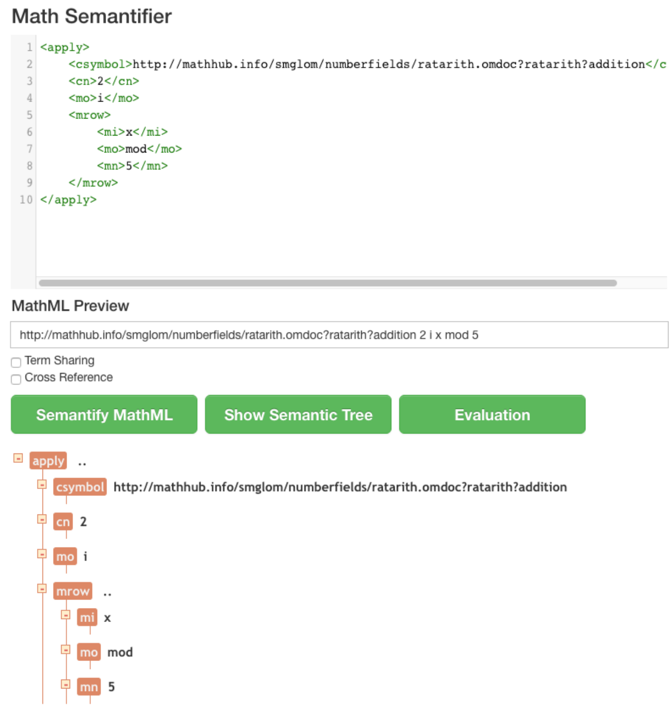


Figure 8: Top level notations

The names of the notations are derived from the notation paths as follows: **archive name** + **symbol name.** The result is humanly readable in most cases as seen in Figure 8. For example, **natarith addition** refers to the addition of natural numbers, **comparith addition** - to the addition of complex numbers and so on.



(a) Argument choice



(b) Content MathML

Figure 9: Semantification

After determining which notation is the correct one, the user needs to make sure that the arguments were detected properly (see 9a).

Finally, the resulting Content MathML is displayed, as shown in 9b.

4.4.2 Semantic Tree Generation

The easier but computationally more expensive alternative is to simply generate all the possible parse trees at once and display them. The user simply needs to click the **Show Semantic Tree** option.

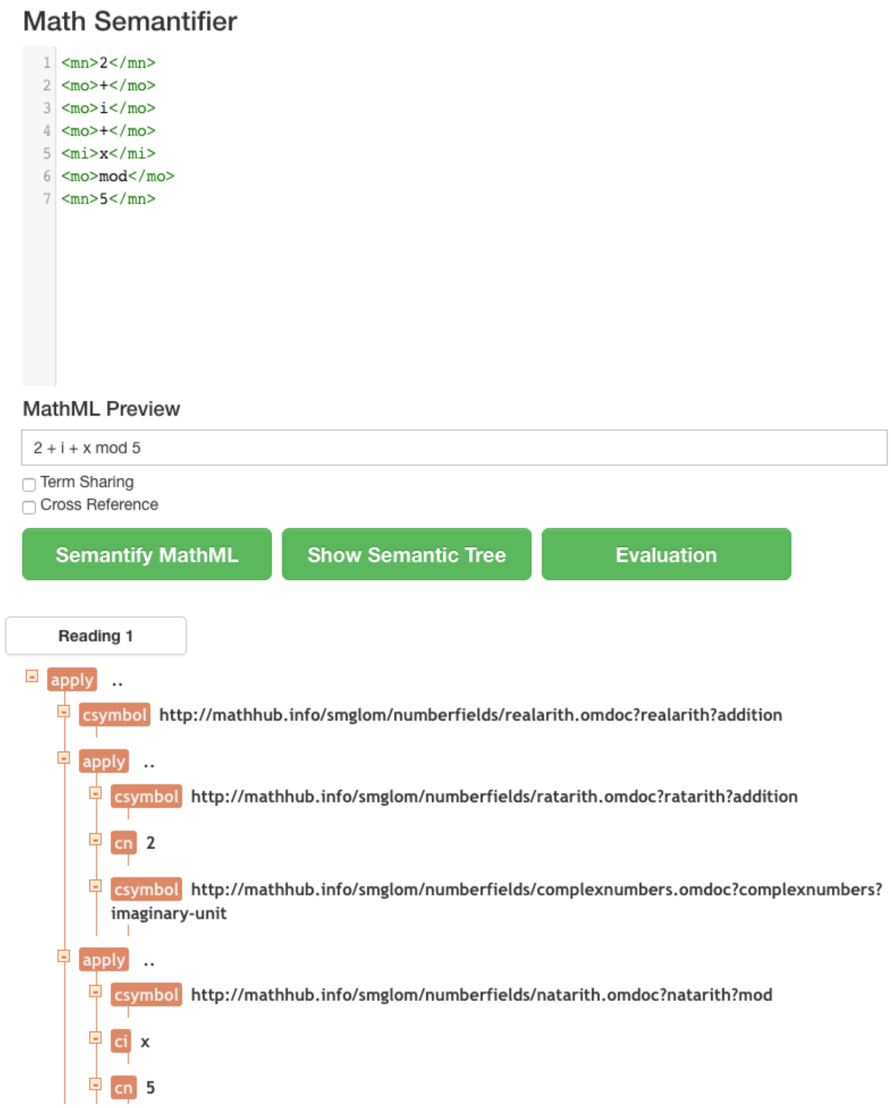


Figure 10: Semantic Tree Results

For the example shown in Figure 10, there is a total of 72 different readings. This can be easily explained, since **Invisible Times** and **Arithmetic Plus** have each 6 different notations, and **Mod** has 2. $6 \cdot 6 \cdot 2 = 72$ explains exactly where the 72 readings come from, since each notation can be combined with any other notation without any restrictions.

4.4.3 Term Sharing

In order to minimize the Content MathML, the standard allows subtree sharing. To enable this option, the **Use term sharing** checkbox should be checked. In that case, the terms is shared along different readings. This feature could be useful for an application that requires all the possible readings, to minimize user interaction, for instance.

The current implementation shares the CML generated at any recursion level. This implies that only **apply** or **csymbol** which correspond to some part of the input can be shared. This can be improved upon, by also sharing ground terms and other parts of the notations. More importantly, this type of sharing is incompatible with cross-referencing, since it may share subtrees that have common ancestors. For instance, $1 + 1$ has two instances of one, however, it may not be shared, because each instance corresponds to a different presentation element.

4.5 MMT Backend

The MMT Backend is a **Server Extension** that is part of MMT. As shown in Figure 7, it is linked via **REST** with the **Web UI** and the **Parsing Backends.**

Its role can be summarized to the following core functions:

1. Compile the **MMT Notations** into a CFG Grammar
2. Receive the input from the **Web UI** and build its Semantic Tree
3. Delegate the parsing to the **Parsing Backends**

I decided to put the core logic of the application in MMT in order to make it easier to interoperate with the MMT Notation Database, as well as with any other MMT components that may want to need **MathSemantifier.** By the current design, using **MathSemantifier** within MMT is as simple as a function call that takes the input as a string.

Let us look at the components of the MMT Backends in more detail below.

4.5.1 Context Free Grammar Generator

The Grammar Generator aggregates all the knowledge contained in the MMT Notations into one Context Free Grammar. The grammar is shaped into the normal form accepted by the Marpa Grammar Engine. To achieve this, the format used to store the notations in MMT is decomposed into CFG rules. Otherwise said, the tree-like structure of each formula, that is stored as nested applications of **MMT Markers** (discussed previously) needs to be serialized into CFG rules.

This is done in several steps:

1. Break apart the **MMT Marker** trees into level by level representations
2. Transform the intermediate representation into valid CFG rules
3. Optimize if possible (will be discussed in detail in the subsequent sections)

The fundamental structure of the CFG is established using a preamble as shown in Figure 11. Note the default action is the **Grammar Entry Point.** It is precisely what tells the Grammar Engine to build parse trees, and also determines their structure.

The entry point into the grammar is the **Expression** rule. It can be an MMT Notation, or Presentation MathML. The **prec0** rule should be read as precedence zero, that is - the lowest precedence there is.

```
#GRAMMAR ENTRY POINT
:default ::= action => [name, start, length, values]
lexeme default = latm => 1
:start ::= Expression
ExpressionList ::= Expression+
Expression ::= Presentation
             | Notation

#Notation Precedences
Notation ::= prec0
prec0 ::= prec1 | #Rules with precedence zero
prec1 ::= prec2 | #Rules with precedence one
...

#Argument Precedences (depends on the number of precedences)
argRuleP0 ::= prec0 | Presentation
argRuleP1 ::= prec1 | Presentation
argRuleP2 ::= prec2 | Presentation
...

#Presentation MathML
Presentation ::= mrowB Notation mrowE
 | mrowB ExpressionList mrowE
 | moB '(' moE ExpressionList moB ')' moE
 | moB text moE
 | miB text miE
 | mnB text mnE
 ...

#Presentation MathML Parts
mrowB ::=  '<mrow' attribs '>'
mrowE ::=  '</mrow>'
mathB ::=  '<math' attribs '>'
mathE ::=  '</math>'
miB ::=  '<mi' attribs '>'
miE ::=  '</mi>'
...

#Lexemes
ws ::= spaces
spaces ~ space+
space ~ [\s]
text ::= textRule
textRule::= char | char textRule
char ~ [^<>]
...
```

Figure 11: CFG Preamble

Precedence handling is done using a commonly used method for including it in CFGs. The **Notation Precedences** section in Figure 11 gives a quick glance at how exactly it is done. The number of precedences needs to be known in advance, then, for each precedence value a corresponding **precN** rule is created.

The rule contains all the notations with that precedence, and, one of the alternatives is going to a higher precedence value. In the example below there are 15 used precedence values (**prec0** corresponds to precedence $-\infty$). However, the **Notation Precedences** section shows only half of the concept.

To make this approach work, what is also needed is that the arguments in a rule of a certain precedence N can only contain notations with precedence K if $K > N$. This is done by the **Argument Precedences** part in the preamble (as shown in Figure 11).

Finally, Figure 12 presents an example of what the **natharith addition** MMT notation from the **smglom/numberfields** archive translated to CFG rules looks like.

```
_natarith_additionP7N213::= rule443
rule443::= argRuleN213A1ArgSeq rule32 rule443_
rule443_::= argRuleN213A1ArgSeq | argRuleN213A1ArgSeq rule32 rule443_
rule32::= moB '+' moE
```

Figure 12: MMT Notation in CFG

4.5.2 Semantic Tree Generator

The **Semantic Tree Generator** works by recursively querying the **Parsing Backend** and using the result to construct a the tree of possible meanings.

The parse trees stored in MMT have the following structure:

```
ParseTree
    toCML: List[String]
Variants
    treeList: List[ParseTree]
    toCML: List[String]
Notation
    name: String
    arguments: List[Argument]
    toCML: List[String]
Argument
    name: String
    value: List[Variants]
    toCML: List[String]
    argToCML: List[List[String]]
RawString
    value: String
    toCML: List[String]
```

Figure 13: Semantic Parse Tree Structure

This simple structure shown in Figure 13 allows for a flexible way of parse tree storage.

- **Variants -** represents a list of possible readings. It is always be the top node in any parse tree.
- **Notation -** a notation detected in the input. It contains its name and arguments.
- **Argument -** an argument of a notation. The plugin is recursively called on it to construct its meaning subtree as well.
- **RawString -** the ground term representation

This structure is important because, in case another application wants to access the parse trees and process them, this structure will need to be dealt with.

4.5.3 Content MathML Generator

The final part of any of the semantification processes is converting the internal representation to a standard one, which is CMML in this case. MMT provides a simple API which requires:

1. The notation path
2. The argument maps

Both of which are available in my representation structure. The argument path is obtained by extracting the argument number from the argument name, and looking up in a map of paths created at the time of grammar creation. This implies the grammar rule names are overloaded with meaning, however, the possibilities are very limited in this aspect since the parsing framework used does not give complete control over the parsing process.

4.6 Parsing Backend

The parsing backend consists of two parts.

4.6.1 Context Free Grammar Parser

First of all, the the CFG needs to be queried and parsed. This is implemented using lazy evaluation, which means that it is only done when a request actually comes.

The serialized CFG is unpacked and feed to the Marpa Parser Generator.

4.6.2 Parse Tree Generator

The more complex of the two parts is actually going through the parse trees and extracting useful information. Note that going through all the parse trees is not practical, so only the first N (currently 1000) parse trees are processed. This still gives the correct results in most cases since the grammar rules are optimized for giving preference to parse trees that are more likely to be correct (discussed in more detail in section 6).

4.7 Encoding issues

Since **MathSemantifier** deals with unicode, and has parts written in Javascript, Scala and Perl, which treat unicode symbols differently, it needed a solution for this problem. Javascript and Scala have **UTF-16** strings, while Perl has **UTF-8** strings. The implemented solution simply passes around the string encoded using **encodeURIComponent** in Javascript or its equivalents in other languages, and only the parsing backend in Perl actually deals meaningfully with substrings. The result of the parsing backend contains the complete substrings for the matched rules and arguments. This also represents an opportunity for improvement, since passing around positions in a string is more efficient that substrings.

5 Optimizations and Heuristics

This section describes the attempts made in order to deal with the overwhelming ambiguity of mathematical notations and produce actual results.

1. Only notations on the whole input are matched. Subterms are matched directly by the **Semantic Tree** plugin.

2. The number of parse trees is limited to $N = 1000$. The number of parse trees was empirically determined to include the set of correct parses for the examples used (see the **Evaluation** section**).**

3. The non-empty alternatives in the CFG are put first. The Grammar Parser has a predictable behavior of going through the alternatives left to right, and since empty rules match **always**, it is imperative that other alternative are attempted first. Note that only after this optimization any kind of results were possible to achieve on non-trivial input.

4. The CFG includes precedences correctly. This weeds out a large number of incorrect readings.

5. Sub-term sharing is part of the **MathML** standard, and within one request, if sub-term sharing is enabled, the response tries to share its terms as much as possible in order to compress the output.

6. **The Semantic Tree** plugin makes use of memoization to reduce one dimension of the exponential blowup as explained on the following example. $2 + 3$ has 6 possible readings, because $+$ is defined 6 times, between natural, integer, real numbers and so on. However, 2 and 3 - these subterms do not contain any notations, yet, the plugin needs to know that for every possibility of $+$. Memoization allows to omit a large number of **POST** requests even in this simple example, which results into a significant speedup of a factor of approximately 4 (13 **POST** requests vs 3 **POST** requests). Also, slight modification of the input benefit of a significant speed-up, as well as using a previous input as a subterm in a new input.

6 Evaluation

This section presents an evaluation of **MathSemantifier** from the point of view of efficiency and effectiveness of semantic tree generation. Next, the interoperability of the system with other possible applications is examined, which mostly depends on the back-end APIs.

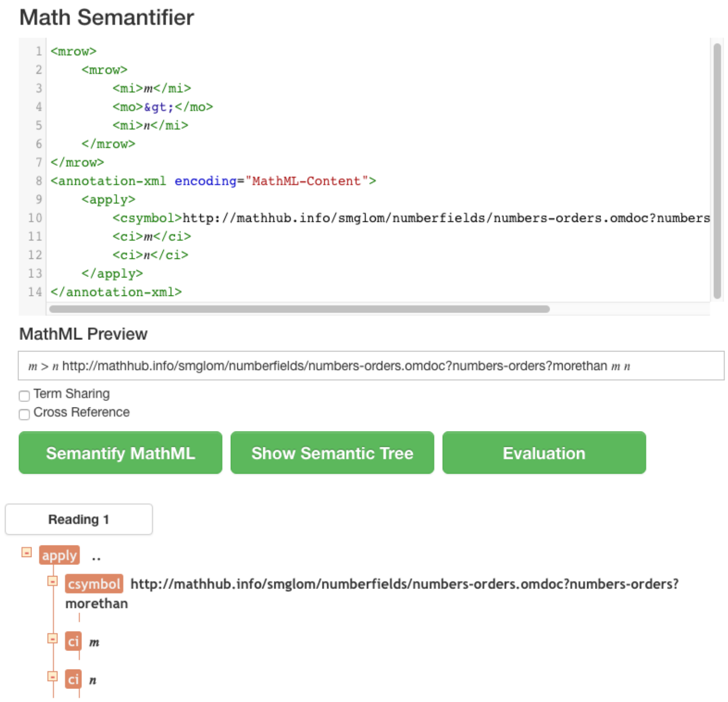


Figure 14: Glossary Example

6.1 Semantic Tree Generation

Testing **MathSemantifier** through normal operation is not a trivial task, because the results need a human expert to check whether the results are indeed correct. Fortunately, the **MathHub Glossary [KWA]** contains a sufficient number (about 3000) of examples of Presentation MathML with Content MathML annotations that result from applying the **Presentation Algorithm** discussed in the introduction. Since **MathSemantifier** is the partial inverse of the **Presentation Algorithm,** checking whether the results are indeed correct boils down to comparing two Content MathML trees.

For the purpose of testing **MathSemantifier** on the **MathHub Glossary,** the **Evaluation** option mentioned in the **Web UI** section is used. It walks the user through the examples extracted from the **MathHub Glossary.** A typical example looks as shown in **??** (the example actually produces seven readings, which are not all shown for brevity).

The editor displays the example itself, which contains the PMML and the corresponding CMML it was generated from using the Presentation algorithm. The readings are displayed below in a manner that makes reading more efficient.

6.1.1 Results

A visual control revealed the following approximate results for the testing on the **MathHub Glossary.** About 40% of the examples are semantified correctly. However, this is largely because of reasons that do not depend on **MathSemantifier** itself.

Contrary to what the numerical result suggests, for expressions with less than 1000 parse trees that are correctly rendered, **MathSemantifier** produces correct results with a very high probability.

Let us look into the reasons that make **MathSemantifier** fail. First of all, the reasons that do not depend on the system itself.

1. The Presentation Algorithm fails to render CMML into PMML properly. This may very well be the main reason for failure. I estimate that about 30-40% of the **Glossary** examples are not rendered properly.

2. About 10% of the examples include symbols from archives that were not included when generating the CFG that **MathSemantifier** is using. This is a limitation that could be easily overcome by adding those symbols, but, as a proof of concept, having a complete coverage of the symbols was not the primary focus.

Up to this point, if we exclude the above mentioned examples, the effective success rate of **MathSemantifier** is about 80-90%.

1. Some notations are omitted (mainly because their rendering consists only of one argument rendering, which creates cycles in the CFG). This limitation is considerably hard to work around, since it increase drastically the ambiguity of the input if we allow free form subterms anywhere.

2. Too long inputs / **MathSemantifier** only considers the first 1000 parse trees as mentioned in the **Heuristics** section above. An obvious fix would be increasing the number of considered parse trees, however this would result into a significantly worse performance. Even for further attempts I would recommend failing early (allowing for 80-90% of requests to complete on average) and retrying again with other parameters rather than waiting for too long. In any case, using a huge auto-generated CFG is certainly a big limitation for the performance.

3. **MathSemantifier** assumes that the input has only one top level **mrow** or a single bracketed list of expressions. This technical limitation is still present because the number of notations that actually do that is quite low as seen in practice, however it is not impossible to fix, even in the current setup.

4. One last interesting effect is over-semantification. This happens for symbols like zero or one, that have custom notation definitions in some of the archives. Since the process of semantification is greedy, if given **<mn>1</mn>** **MathSemantifier** will try to semantify it as much as it can, meaning the result will be **<csymbol>... one</csymbol>** rather than just **<cn>1</cn>.** While this can be solved easily for particular cases of zero or one, solving this generally may pose an interesting problem. Sometimes parts of the input or the whole input may need to be semantified without using any notations at all. When exactly this should be done could probably be determined by some smart heuristics at best.

While the effective success rate of 80-90% is inspiring for a proof of concept system, it does not reflect the main problem that prevents **MathSemantifier** from becoming a practically useful tool rather than just a proof of concept study. **MathSemantifier** has critical efficiency issues, which could have been expected since it parses the input against a CFG [Tol] of more than 2800 rules that contains over 700 MMT Notations. It is the greedy free form subterm matching that, when applied in a context of such ambiguity results into unsatisfactory performance for an industrial setting.

Overall, the results can be said to meet the purpose of proving that MMT Notations can be used for semantification purposes. Therefore, **MathSemantifier** is a successful proof of concept, but not yet the practical tool mathematicians need right now in order to harness the power of semantic content.

6.2 Backend APIs

It is important that the MMT backend provides a simple **HTTP** endpoint, which can be accessed with a **POST** request that contains just the input in a suitable encoding. The backend then returns a **JSON** object that contains all the possible readings. The backend could be modified to accept parameters to fine tune the system, like the maximum number of processed parse trees, but even then accessing it would be just as simple.

7 Conclusions

The conclusion of this study is the proof of concept architecture and implementation of a system capable of converting **Presentation MathML** to all the possible meanings, which is a list of **Content MathML**. The testing revealed that the system is able to recognize correctly single top level symbols, as well as the whole set of readings of expressions with less than 1000 parse trees (this is not the limit, but no testing in section 8 is done beyond that). This shows that it is certainly possible to aggregate the knowledge from the MMT Notations and to use it for parsing purposes. However, both the Notations and the Parsing Framework need significant improvements in order for the system to be scalable beyond what is presented in the previous section. The most important part of this study is, therefore, the optimizations and heuristics used, and other techniques presented below that further research could benefit from.

7.1 Further work

The study presented in this paper reached certain results, however there is a long way yet to a fully automatic and scalable system that could handle large collections of papers without any supervision. Below there are some suggestions that further research could make use of in order to get closer to this goal.

7.1.1 Suggestions for further optimizations

1. The CFG could be checked for unused rules, and, more importantly, converted to BNF, for instance

2. Theory based optimization - either let the user specify which theories to create the grammar from, or take into consideration that notations from the same theory are likely to be close in the input (for instance, $-$ and $+$ are more likely to both be used as arithmetic symbols in an expression, than only one of them)

3. Sequence arguments (for instance, $2 + 3 + 4 + 5 + 6$) currently generate a very high number of possible parses. If more control over parsing were possible, parsing of sequence arguments should be greedy - attempting to take in as many delimiter argument pairs as there are.

7.1.2 Requirements for a more suitable Notation Database

1. The most important optimization would be adding types to the notation input arguments and output. This would allow for type based parsing, which would certainly be more efficient and generate more relevant results.

2. Grouping similar notations within a theory. For example, arithmetic plus on natural, integer or real number should be possible to connect somehow. If the notations have types, the it will naturally occur by grouping the notations with the same types into single rules.

3. For the **csymbol** used in an **apply** tag, the notation should cross-reference it with the part of the representation it corresponds to, since it is not otherwise clearly specified.

7.1.3 Requirements for a more suitable Parsing Framework

1. One of the biggest problems with the current implementation is that it greedily matches the argument renderings, which are free form subterms. If a custom parsing framework would be used, lazy matching of such subterms would greatly increase the performance.

2. The ability to handle types, precedences and associativity.

3. The ability to handle greedy parsing - most CFG parsing frameworks have the counted rule $Expression ::= Expression+$, which has a similar meaning to the $+$ found in a **regex.** However, **regexes** actually will do a greedy match, the more appropriate match being then $Expression+?$, the non-greedy version. This is critical to reduce the number of useless parse trees when dealing with Sequence Arguments

7.1.4 Term Indexing

Term Indexing is a standard technique for finding substitutions that allow term unification. This exactly applies to the subject of semantification since we can think about the Presentation Algorithm as if it generates substitutions for the CMML into the notation renderings. In other words, finding the substitution that unifies an input PMML and a notation rendering allows for semantification. There are multiple standard techniques, like substitution tree indexing, discrimination tree indexing or path indexing that make this possible. The main advantage of this approach over the current approach is that it matches free form subterms (argument renderings) in a lazy way. This may result into a significant speed improvement. For instance, determining the top level notation would be linear in the number of notations rather than exponential. Another advantage would be the possibility to parallelize by top-level notations (or of any level, though I expect splitting just by the number of the top level notations should be enough). This scenario is a typical application of MapReduce [Apa]. Moreover, a cutoff heuristic can be applied to ensure the performance is acceptable. If the result is not acceptable, the job could be rerun without the cutoff heuristic. I expect that prioritizing failure and retrial over waiting could improve significantly the average case performance.

References

[ABC+03] Ron Ausbrooks, Stephen Buswell, David Carlisle, Stephane Dalmas, Stan Devitt, Angel Diaz, Max Froumentin, Roger Hunter, Patrick Ion, Michael Kohlhase, Robert Miner, Nico Poppelier, Bruce Smith, Neil Soiffer, Robert Sutor et Stephen Watt : Mathematical Markup Language (MathML) version 2.0 (second edition). *W3C recommendation, World Wide Web Consortium (W3C)*, 2003.

[Apa] Apache : MapReduce. https://hadoop.apache.org. [Online; accessed 4-May-2016].

[ArX15] ArXiV Online. *URL, as seen November 2015, http://arxiv.org*, 2015.

[CKS] Marcos Cramer, Peter Koepke et Bernhard Schroder : Parsing and disambiguation of symbolic mathematics in the naproche system.

[GIJ+] Deyan Ginev, Mihnea Iancu, Constantin Jucovschi, Andrea Kohlhase, Michael Kohlhase, Heinz Kroger, Jurgen Schefter et Wolfram Sperber : The SMGloM Project and System.

[GKW11] Deyan Ginev, Michael Kohlhase et Magdalena Wolska : The Structure of Mathematical Expressions. 2011.

[HR] Muhammad HUMAYOUN et Christophe RAFFALLI : Mathnat - mathematical text in a controlled natural language. *Special issue: Natural Language Processing and its Applications*.

[Kega] Jeffrey KEGLER : The Marpa Parser. https://jeffreykegler.github.io/Marpa-web-site/. [Online; accessed 19-April-2016].

[Kegb] Jeffrey KEGLER : The Marpa parser. Web Manual at https://jeffreykegler.github.io/Marpa-web-site/, seen November 2015.

[Keg11] Jeffrey KEGLER : Marpa v. Perl regexes: some numbers. http://blogs.perl.org/users/jeffrey_kegler/2011/11/marpa-v-perl-regexes-some-numbers.html, 2011. [Online; accessed 19-April-2016].

[KMW] Fairouz KAMAREDDINE, Manuel MAAREK et Joe B. WELLS : MathLang: An experience driven language of mathematics. *Electronic Notes in Theoretical Computer Science 93C*.

[Koha] Michael KOHLHASE : OMDoc - An open markup format for mathematical documents (Version 1.2).

[Kohb] Michael KOHLHASE : The SMGLOM Archive. https://mathhub.info/smglom. [Online; accessed 4-May-2016].

[Kol] Alexander KOLLER : Semantic construction with S-graph grammars.

[KWA] KWARC : MathHub Glossary. https://mathhub.info/mh/glossary. [Online; accessed 1-May-2016].

[LWP15] LWP - The World-Wide Web library for Perl, URL: http://search.cpan.org/dist/libwww-perl/lib/LWP.pm, seen November 2015. 2015.

[Mil] Bruce MILLER : LaTeXML: A LaTeX to XML converter. Web Manual at http://dlmf.nist.gov/LaTeXML, seen November 2015.

[MWS15] MwsHarvest, URL: https://trac.mathweb.org/ MWS/wiki/MwsHarvest (seen November 2015). 2015.

[NS] Arnold NEUMAIER et Peter SCHODL : A framework for representing and processing arbitrary mathematics. *In Proceedings of the International Conference on Knowledge Engineering and Ontology Development*.

[Rab] Florian RABE : The MMT API: A Generic MKM System.

[RPK14] Hambasan RADU, Corneliu C. PRODESCU et Michael KOHLHASE : MathWebSearch at NTCIR-11. 2014.

[SKG+10] Heinrich Stamerjohanns, Michael Kohlhase, Deyan Ginev, Catalin David et Bruce Miller : Transforming large collections of scientific publications to XML. *Mathematics in Computer Science*, 2010.

[Tol] Ion Toloaca : MathSemantifier Auto-Generated Context Free Grammar Example. https://github.com/itoloaca/MathSemantifier/blob/master/CFGexample.json. [Online; accessed 4-May-2016].