
— -01-24

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0.,

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$$\left(\quad \quad \quad \right) - \{B(t)\}_{t \geq 0},$$
$$\frac{dB(t)}{dt} = r(t) B(t), \quad r(t) - .$$

$$B(0) = 1.$$

$$B(t) = \exp\left(\int_0^t r(s) ds\right).$$

$$r(t) \equiv r : \quad$$

$$B(t) = e^{rt}.$$

() $B(t)$,
 r .

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■ **Numeraire —**

$$[0, T]$$

$$\Delta t:$$

$$0, \Delta t, 2\Delta t, \dots, n\Delta t = T, \quad \Delta t = \frac{T}{n}.$$

,

$$B(t + \Delta t) = B(t)(1 + r\Delta t).$$

$$n$$

$$B(T) = B(0)(1 + r\Delta t)^n.$$

$$\Delta t \rightarrow 0 \quad (n \rightarrow \infty),$$

$$\lim_{\Delta t \rightarrow 0} (1 + r\Delta t)^{T/\Delta t} = e^{rT}.$$

,

$$\frac{dB(t)}{dt} = \lim_{\Delta t \rightarrow 0} \frac{B(t + \Delta t) - B(t)}{\Delta t} = rB(t).$$

$r = 16,5\% = 0.165.$

$$B(0) = 100\,000$$

$$(T = 1)$$

1. 1 ():

$$B_1(1) = 100\,000 \left(1 + \frac{0.16}{1}\right)^1 = 100\,000 \cdot 1.16 = \mathbf{116\,000} \quad .$$

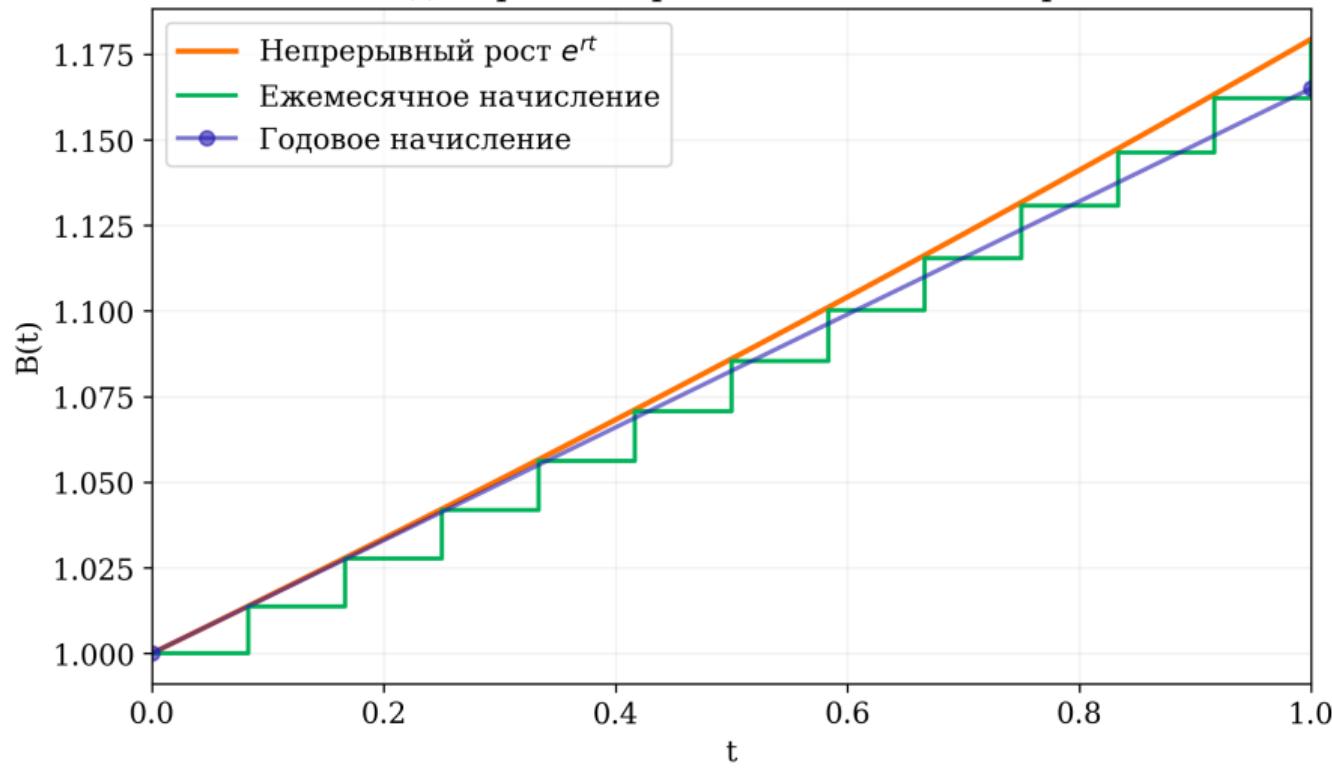
2. ($n = 12$):

$$B_{12}(1) = 100\,000 \left(1 + \frac{0.16}{12}\right)^{12} \approx 100\,000 \cdot 1.17180 \approx \mathbf{117\,180} \quad .$$

3. ($n \rightarrow \infty$):

$$B_{\text{cont}}(1) = 100\,000 e^{0.16} \approx 100\,000 \cdot 1.17351 \approx \mathbf{117\,351} \quad .$$

Сближение дискретного роста к экспоненте при $r = 16.5\%$



$$\overbrace{116\,000}^{B_1(1)} < \overbrace{117\,180}^{B_{12}(1)} < \overbrace{117\,351}^{B_{\text{cont}}(1)}$$

1

1.

, , 100 .
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?

100

: 99, 101, 98.5, 105 —

, , 100 .
?
—

: 99, 101, 98.5, 105 —

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■ ;
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■ ;
■ (. . . -) —

$(\Omega, \mathcal{F}, \mathbb{P})$

2.

- $(\Omega, \mathcal{F}, \mathbb{P})$,
1. Ω — ();
 2. \mathcal{F} — σ - , Ω , ;
 3. \mathbb{P} — , $A \in \mathcal{F}$ $\mathbb{P}(A)$ $[0, 1]$

()

:

1. ():

$$\Omega = \{\text{up}, \text{down}, \text{flat}\}.$$

2. — :
—

$$\mathcal{F} = \sigma(\{\text{up}\}, \{\text{down}\}, \{\text{flat}\}).$$

3. :
—

$$P(\text{up}) = 0.3, \quad P(\text{down}) = 0.4, \quad P(\text{flat}) = 0.3.$$

$$(\Omega, \mathcal{F}, P).$$

3.

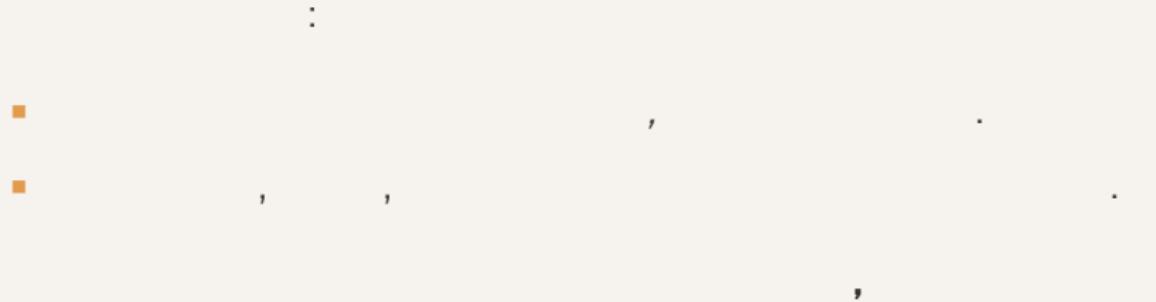
■ ,
■ ,
■ ,
■ .
■ P&L.
■ —
■ .

P&L

P&L

4.

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- (K) — ,

- (K) — ,
- (T) — , (). T

- (K) — ,
- (T) — , (). T

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— K T ,
 S_T .

Call- — $K.$

- , $S_T > K;$
- , $S_T \leq K.$

Put- — $K.$

- , $S_T < K;$
- , $S_T \geq K.$

- **Long Call:**
- **Short Call:**
- **Long Put:**
- **Short Put:**

(!!!)

: long short

—

,

- **Long Call:**
- **Short Call:**
- **Long Put:**
- **Short Put:**

(!!!) : long short

- (long)
- (short)

()

...

Call

call-

$K.$

$S_T:$

$$\begin{cases} S_T \leq K, & \Rightarrow 0, \\ S_T > K, & K \quad S_T \Rightarrow S_T - K. \end{cases}$$

,

$$\text{Payoff}_{\text{call}} = \max(S_T - K, 0).$$

Put

put

K :

$$\begin{cases} S_T \geq K, & \Rightarrow 0, \\ S_T < K, & K - S_T. \end{cases}$$

:

$$\text{Payoff}_{\text{put}} = \max(K - S_T, 0).$$

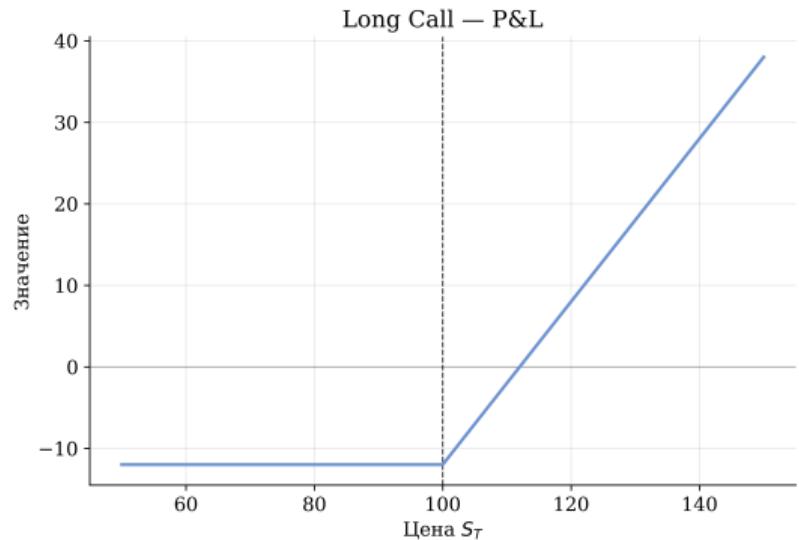
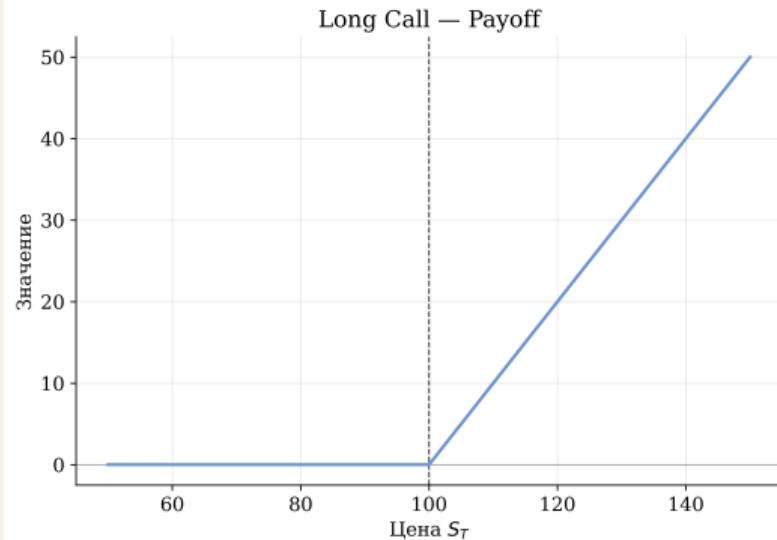
:

$$\begin{cases} \text{Payoff}_{\text{Long Call}} &= \max(S_T - K, 0), \\ \text{Payoff}_{\text{Short Call}} &= -\max(S_T - K, 0), \\ \text{Payoff}_{\text{Long Put}} &= \max(K - S_T, 0), \\ \text{Payoff}_{\text{Short Put}} &= -\max(K - S_T, 0). \end{cases}$$

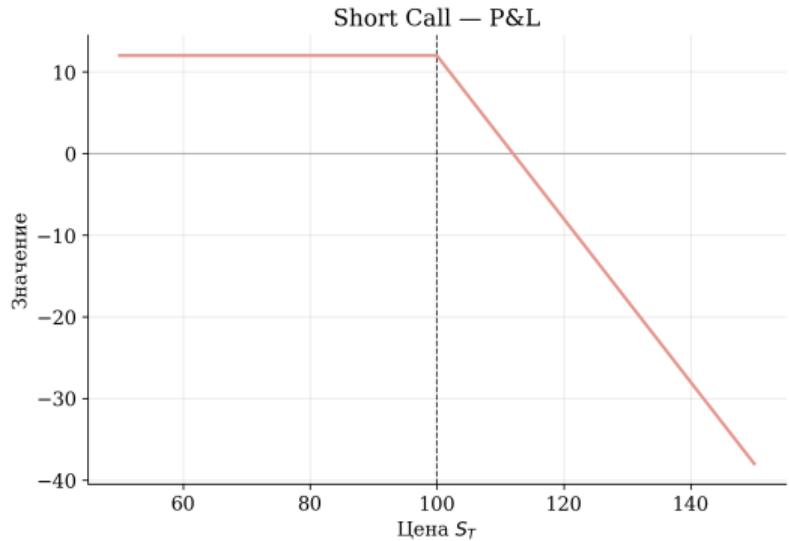
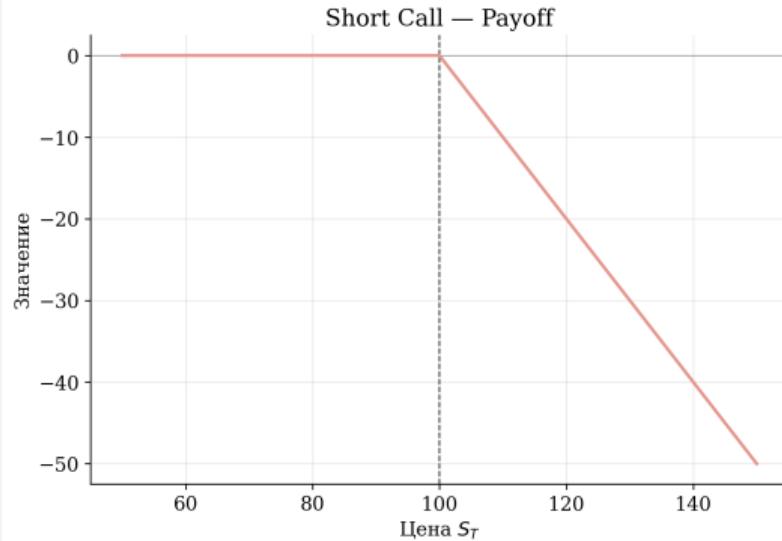
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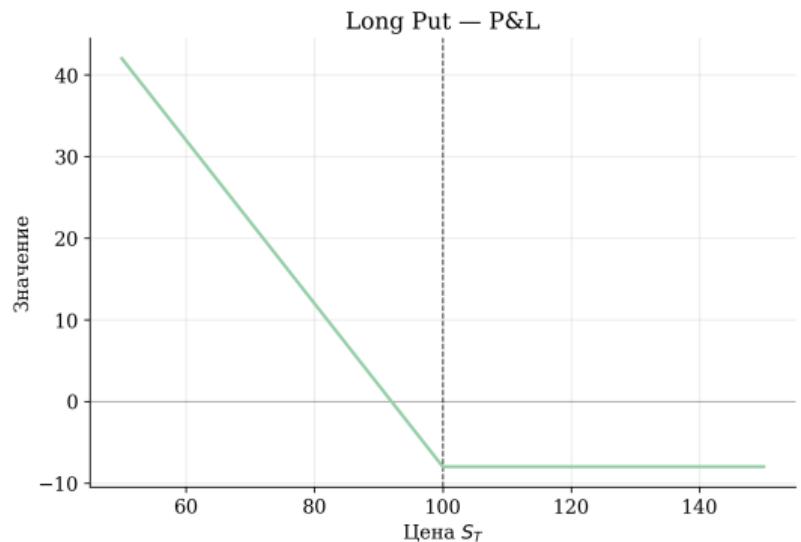
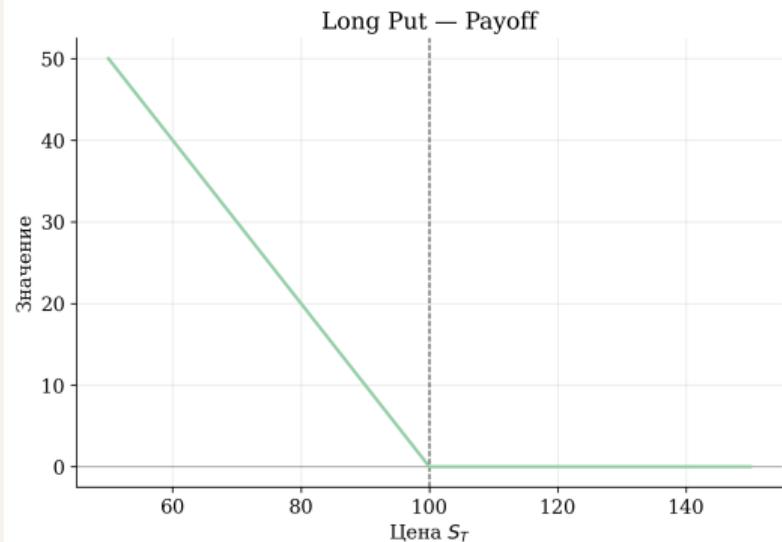
Payoff - Long Call $\max(S_T - K, 0)$



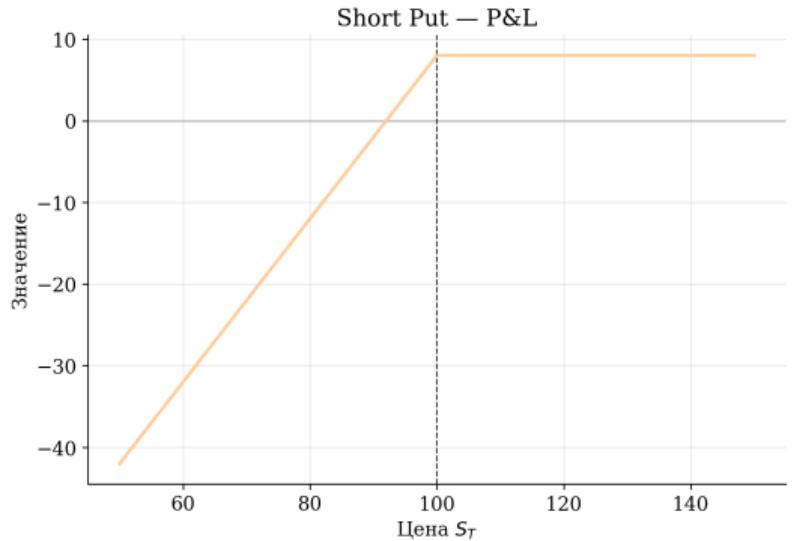
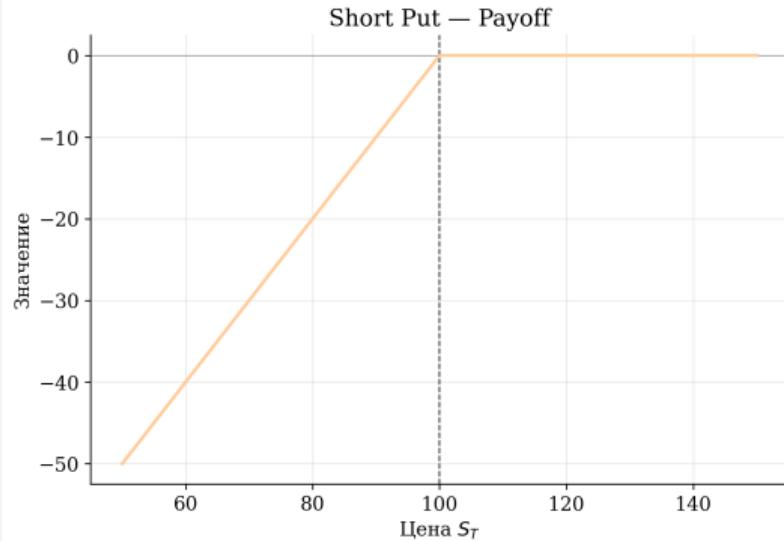
Payoff - Short Call – $\max(S_T - K, 0)$



Payoff - Long Put $\max(K - S_T, 0)$



Payoff - Short Put – $\max(K - S_T, 0)$



10

$$S_T \in \{80, 100, 130\}.$$

$$K = 100.$$

•

S_T

2

Long Call (LC), Short Call (SC), Long Put (LP), Short Put (SP).

CALL-OPTION

1. Long Call :

$$\text{Payoff}_{LC}(S_T) = \max(S_T - K, 0).$$

$$S_T = 80 : \quad \max(80 - 100, 0) = 0,$$

$$S_T = 100 : \quad \max(100 - 100, 0) = 0,$$

$$S_T = 130 : \quad \max(130 - 100, 0) = 30.$$

2. Short Call - call :

$$\text{Payoff}_{SC}(S_T) = -\max(S_T - K, 0).$$

$$S_T = 80 : \quad 0,$$

$$S_T = 100 : \quad 0,$$

$$S_T = 130 : \quad -30.$$

PUT-OPTION

3. Long Put - :

$$\text{Payoff}_{LP}(S_T) = \max(K - S_T, 0).$$

$$S_T = 80 : \quad \max(100 - 80, 0) = 20,$$

$$S_T = 100 : \quad \max(100 - 100, 0) = 0,$$

$$S_T = 130 : \quad \max(100 - 130, 0) = 0.$$

4. Short Put - put « »:

$$\text{Payoff}_{SP}(S_T) = -\max(K - S_T, 0).$$

$$S_T = 80 : \quad -20,$$

$$S_T = 100 : \quad 0,$$

$$S_T = 130 : \quad 0.$$

S_T	LC	SC	LP	SP
80	0	0	20	-20
100	0	0	0	0
130	30	-30	0	0

- $(S_T = 80)$ put: 100 20.
- $(S_T = 100)$ — payoff .
- $(S_T = 130)$ call: 100 130 30.

- .

?

: ()

$$= , (a+b)^n.$$

- $n = a - b;$
- $a^k b^{n-k};$
- , $k \leq a \leq n;$
- $\binom{n}{k}.$

$$: (a+b)^n = a - b - n .$$

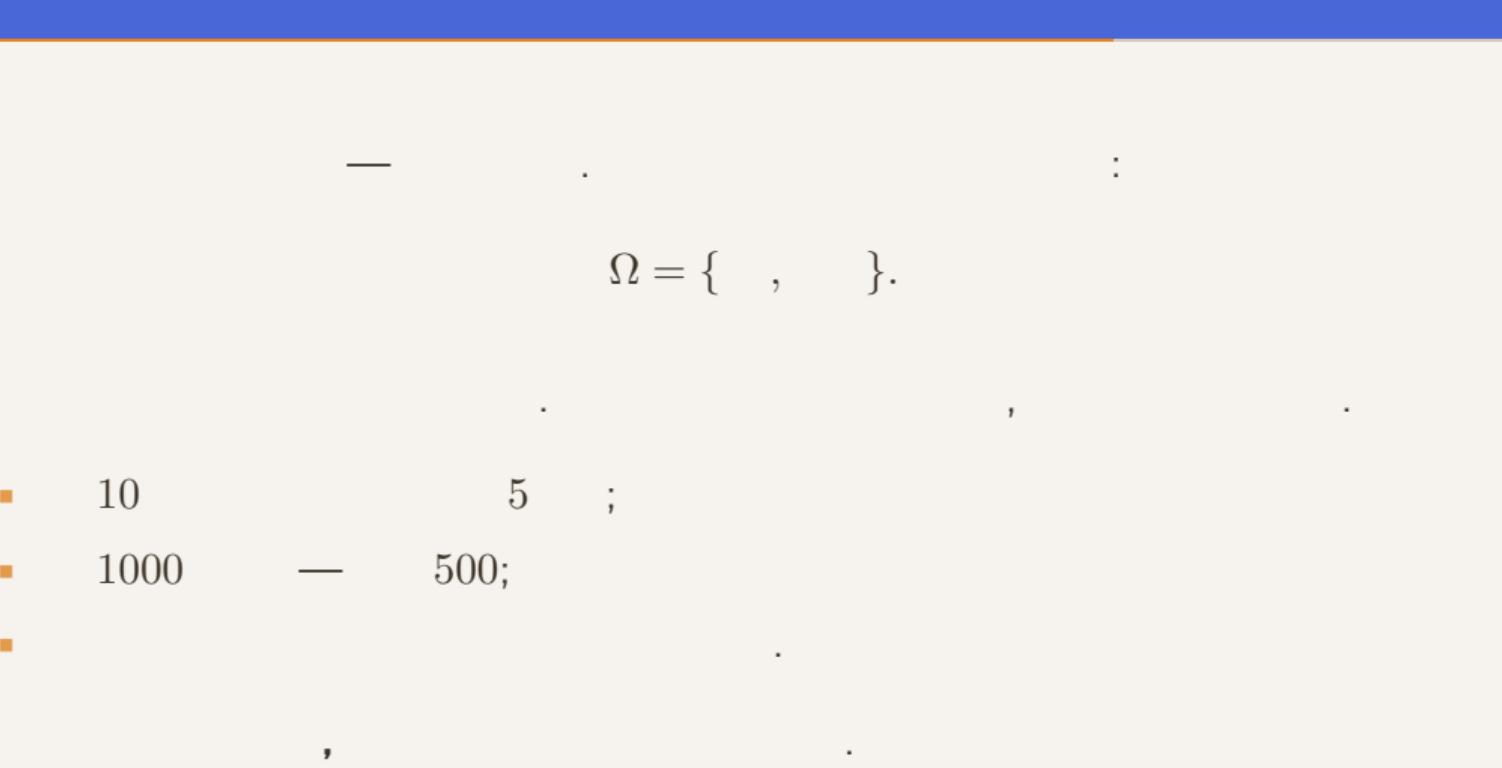
:

:

$$(a + b)^n = \sum_{k=0}^n \binom{n}{k} a^{n-k} b^k.$$

:

$$\binom{n}{k} = \frac{n!}{k! (n - k)!}.$$



$$\Omega = \{ \quad , \quad \}.$$

()

— « » . : ;

$$\xi : \Omega \rightarrow \mathbb{R}.$$

$\omega \in \Omega$ $\xi(\omega)$. : ;

■ 5 : — \mathbb{H}/\mathbb{T} . : ;

$$X(\omega) = \dots \omega.$$

■ S_T — : ;

$$S_T(\omega) = \dots T.$$

— , .

\vdots $\overline{\quad}$,
 \vdots $\overline{\quad}$ $\mathbb{R},$ $\xi:$

$$P_\xi(A) = P(\xi^{-1}(A)).$$

■ , $\xi \in A,$ $\omega,$

■

· · · , 5 3 ? · · ·

- : $n = 5$;
- « »: $k = 3$;
- : $p = \frac{1}{2}$;
- : $q = 1 - p = \frac{1}{2}$.

· · ·

$$P(X = k) = \binom{n}{k} p^k q^{n-k}.$$

· · ·

$$P(X = 3) = \binom{5}{3} \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^2.$$

:

:

$$\binom{5}{3} = \frac{5!}{3! 2!} = \frac{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{(3 \cdot 2 \cdot 1)(2 \cdot 1)} = 10.$$

:

$$P(X = 3) = 10 \cdot \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^2 = 10 \cdot \frac{1}{8} \cdot \frac{1}{4} = \frac{10}{32} = \frac{5}{16}.$$

∴ $\frac{5}{16}$.

S_0 .

S_T , S_T

- \iff ;
- \iff .

100

— , 102;

— , 98.

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— —

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($\alpha_1, \alpha_2, \alpha_3, \dots$);

-

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.

:

$$S_0 \rightarrow \begin{cases} uS_0 & (\quad) \\ dS_0 & (\quad) \end{cases}$$

:

$$B_1 = B_0(1 + r\Delta t)$$

,

:

■ $S - B,$

■ ,

■ .

$$u \leq 1 + r\Delta t$$

\rightarrow

$$d \geq 1 + r\Delta t$$

\rightarrow

r

$$d < 1 + r\Delta t < u.$$

$$\mathbb{E}^{\mathbb{Q}}[S_1] = S_0(1 + r\Delta t).$$

$$q = \frac{(1 + r\Delta t) - d}{u - d}.$$

$$q = \frac{(1 + r\Delta t) - d}{u - d}$$

- ;
- ;
- ;
- ,

call-

$$C_u = \max(uS_0 - K, 0), \quad C_d = \max(dS_0 - K, 0).$$

call

$$C_0 = \frac{1}{1 + r\Delta t} (qC_u + (1 - q)C_d).$$

$$S_0 \rightarrow \begin{cases} uS_0 & (C_u) \\ dS_0 & (C_d) \end{cases}$$
$$C_0 = \frac{qC_u + (1-q)C_d}{1 + r\Delta t}$$

$$\Pi = \Delta S_0 - B_0,$$

$$\Pi_u = \Delta u S_0 - B_0(1 + r\Delta t), \quad \Pi_d = \Delta d S_0 - B_0(1 + r\Delta t).$$

$$\Pi_u = C_u, \quad \Pi_d = C_d.$$

$$(\Delta, B_0).$$

1:

:

$$\Delta u S_0 - B_0(1 + r\Delta t) = C_u,$$

$$\Delta d S_0 - B_0(1 + r\Delta t) = C_d.$$

—

$$\Delta \quad B_0.$$

2:

⋮

$$(\Delta u S_0 - B_0(1 + r\Delta t)) - (\Delta d S_0 - B_0(1 + r\Delta t)) = C_u - C_d.$$

$$B_0 \quad : \quad$$

$$\Delta S_0(u - d) = C_u - C_d.$$

⋮

$$\Delta = \frac{C_u - C_d}{(u - d)S_0}.$$

3:

$$B_0$$

$$\Delta = \frac{C_u - C_d}{(u - d)S_0}$$

, :

$$\Delta u S_0 - B_0(1 + r\Delta t) = C_u.$$

:

$$B_0(1 + r\Delta t) = \Delta u S_0 - C_u.$$

:

$$B_0 = \frac{\Delta u S_0 - C_u}{1 + r\Delta t}.$$

($t = 0$):

$$\Pi_0 = \Delta S_0 - B_0.$$

, :

$$C_0 = \Pi_0.$$

:

$$C_0 = \Delta S_0 - B_0.$$

:

$$\Delta S_0 = \frac{C_u - C_d}{u - d},$$

$$B_0 = \frac{\Delta u S_0 - C_u}{1 + r\Delta t}.$$

:

$$C_0 = \frac{C_u - C_d}{u - d} - \frac{\Delta u S_0 - C_u}{1 + r\Delta t}.$$

6:

$$\Delta S_0 = \frac{C_u - C_d}{u - d} \quad : \quad$$

$$\Delta u S_0 = u \cdot \frac{C_u - C_d}{u - d}.$$

:

$$C_0 = \frac{C_u - C_d}{u - d} - \frac{u(C_u - C_d)/(u - d) - C_u}{1 + r\Delta t}.$$

$$(u - d)(1 + r\Delta t).$$

:

$$C_0 = \frac{1}{1 + r\Delta t} \left[\frac{(1 + r\Delta t) - d}{u - d} C_u + \frac{u - (1 + r\Delta t)}{u - d} C_d \right].$$

7:

-

:

$$q = \frac{(1 + r\Delta t) - d}{u - d}, \quad 1 - q = \frac{u - (1 + r\Delta t)}{u - d}.$$

:

$$C_0 = \frac{qC_u + (1 - q)C_d}{1 + r\Delta t}$$

call

.

$$C_0 = \frac{qC_u + (1-q)C_d}{1 + r\Delta t}$$

:

- q — ;
- , ;
- q — ;
- = payoff.