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() — $\{B(t)\}_{t \geq 0}$,

$$\frac{dB(t)}{dt} = r(t) B(t), \quad r(t) \text{ — } .$$

$$B(0) = 1.$$

$$B(t) = \exp\left(\int_0^t r(s) ds\right).$$

$$r(t) \equiv r :$$

$$B(t) = e^{rt}.$$

() $B(t)$, ,
 r . :

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- **Numeraire** —

$[0, T]$

Δt :

$$0, \Delta t, 2\Delta t, \dots, n\Delta t = T, \quad \Delta t = \frac{T}{n}.$$

,

$$B(t + \Delta t) = B(t)(1 + r\Delta t).$$

n

$$B(T) = B(0) (1 + r\Delta t)^n.$$

$$\Delta t \rightarrow 0 \quad (n \rightarrow \infty),$$

$$\lim_{\Delta t \rightarrow 0} (1 + r\Delta t)^{T/\Delta t} = e^{rT}.$$

,

$$\frac{dB(t)}{dt} = \lim_{\Delta t \rightarrow 0} \frac{B(t + \Delta t) - B(t)}{\Delta t} = rB(t).$$

$r = 16,5\% = 0.165$. $B(0) = 100\,000$, :

$(T = 1)$.

1. $B_1(1)$ ($n = 1$):

$$B_1(1) = 100\,000 \left(1 + \frac{0.16}{1}\right)^1 = 100\,000 \cdot 1.16 = \mathbf{116\,000} \quad .$$

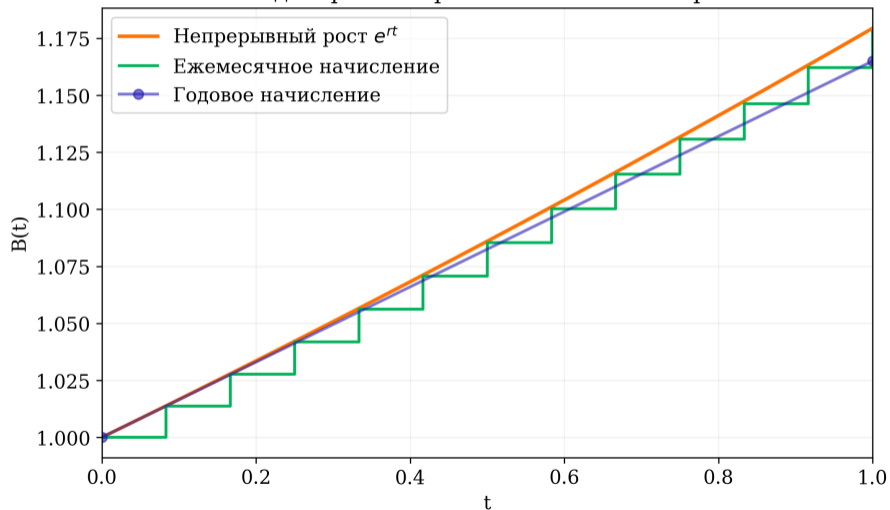
2. $B_{12}(1)$ ($n = 12$):

$$B_{12}(1) = 100\,000 \left(1 + \frac{0.16}{12}\right)^{12} \approx 100\,000 \cdot 1.17180 \approx \mathbf{117\,180} \quad .$$

3. $B_{\text{cont}}(1)$ ($n \rightarrow \infty$):

$$B_{\text{cont}}(1) = 100\,000 e^{0.16} \approx 100\,000 \cdot 1.17351 \approx \mathbf{117\,351} \quad .$$

Сближение дискретного роста к экспоненте при $r = 16.5\%$



$$\overbrace{116\,000}^{B_1(1)} < \overbrace{117\,180}^{B_{12}(1)} < \overbrace{117\,351}^{B_{\text{cont}}(1)}$$

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: e^r .

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1.

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: 99, 101, 98.5, 105 — , .

, , 100 .
?

: 99, 101, 98.5, 105 — , .
— , — , .



2. $(\Omega, \mathcal{F}, \mathbb{P})$

- $(\Omega, \mathcal{F}, \mathbb{P})$,
1. Ω — ();
 2. \mathcal{F} — σ - , Ω , ;
 3. \mathbb{P} — , $A \in \mathcal{F}$ $\mathbb{P}(A)$ $[0, 1]$.

()

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1. ():

$$\Omega = \{\text{up}, \text{down}, \text{flat}\}.$$

2. — :

$$\mathcal{F} = \sigma(\{\text{up}\}, \{\text{down}\}, \{\text{flat}\}).$$

3. :

$$P(\text{up}) = 0.3, \quad P(\text{down}) = 0.4, \quad P(\text{flat}) = 0.3.$$

$$(\Omega, \mathcal{F}, P).$$

3.

■ $\frac{1}{2} \frac{d}{dt} \left(\frac{1}{2} \frac{d}{dt} \right)$

■ $\frac{1}{2} \frac{d}{dt} \left(\frac{1}{2} \frac{d}{dt} \right)$

■ $\frac{1}{2} \frac{d}{dt} \left(\frac{1}{2} \frac{d}{dt} \right)$

■ $\frac{1}{2} \frac{d}{dt} \left(\frac{1}{2} \frac{d}{dt} \right)$

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P&L

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■ (K) — ,

- (K) — ,
- (T) — , (). T

- (K) — ,
- (T) — , (). T

()

— K T ,
 S_T .

Call- — K .

- , $S_T > K$;

- , $S_T \leq K$.

Put- — K .

- , $S_T < K$;

- , $S_T \geq K$.

- Long Call: . .
- Short Call: . .
- Long Put: . .
- Short Put: . .

(!!!) : long short — , .

- Long Call: . .
- Short Call: . .
- Long Put: . .
- Short Put: . .

(!!!) : long short — ,

- (long) .
- (short) .

(...)

call- K . S_T :

$$\begin{cases} S_T \leq K, & \Rightarrow 0, \\ S_T > K, & K \quad S_T \Rightarrow S_T - K. \end{cases}$$

,

$$\text{Payoff}_{\text{call}} = \max(S_T - K, 0).$$

put

K :

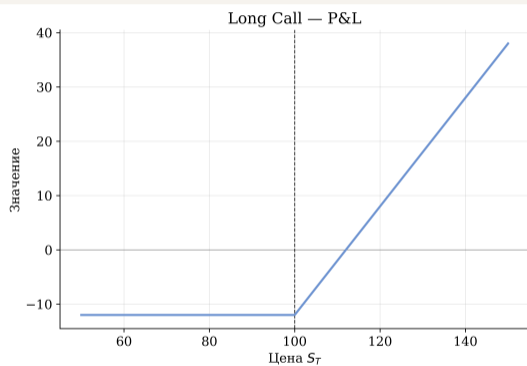
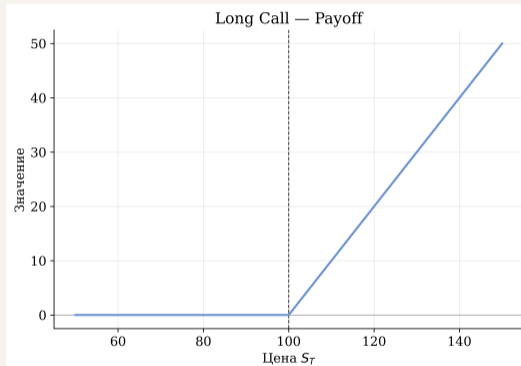
$$\left\{ \begin{array}{ll} S_T \geq K, & \Rightarrow 0, \\ S_T < K, & K - S_T \Rightarrow K - S_T. \end{array} \right.$$

:

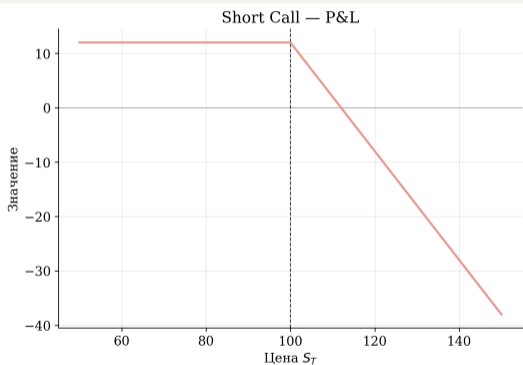
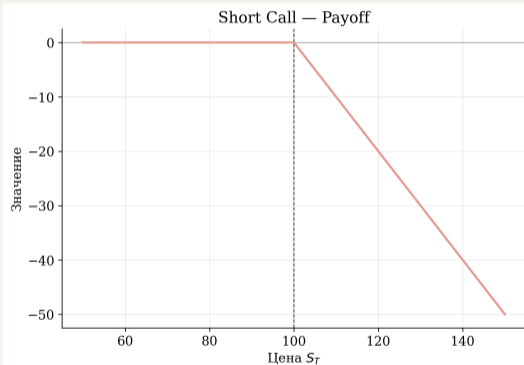
$$\text{Payoff}_{\text{put}} = \max(K - S_T, 0).$$

$$\begin{aligned}
 & : \\
 & \left\{ \begin{array}{ll} \text{Payoff}_{\text{Long Call}} & = \max(S_T - K, 0), \\ \text{Payoff}_{\text{Short Call}} & = -\max(S_T - K, 0), \\ \text{Payoff}_{\text{Long Put}} & = \max(K - S_T, 0), \\ \text{Payoff}_{\text{Short Put}} & = -\max(K - S_T, 0). \end{array} \right.
 \end{aligned}$$

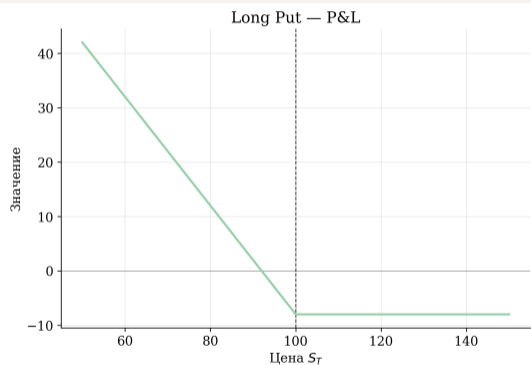
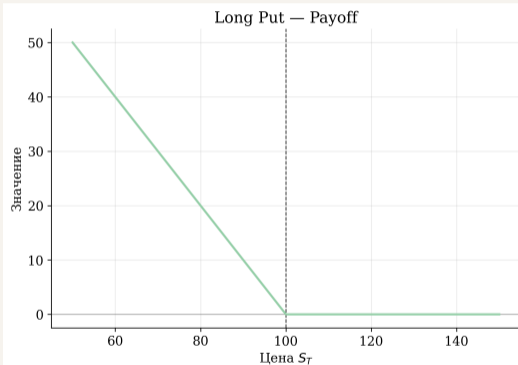
Payoff - Long Call $\max(S_T - K, 0)$



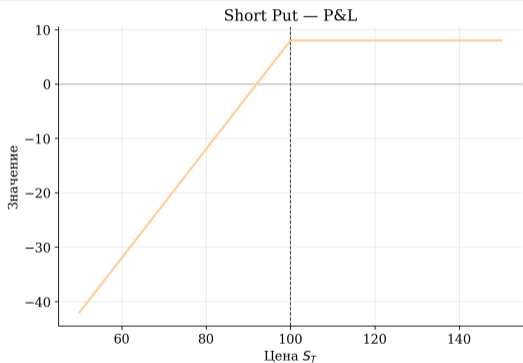
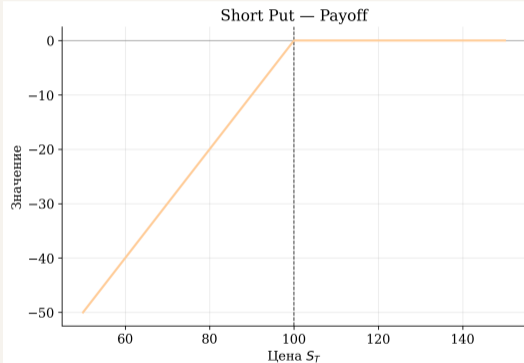
Payoff - Short Call — $\max(S_T - K, 0)$



Payoff - Long Put $\max(K - S_T, 0)$



Payoff - Short Put — $\max(K - S_T, 0)$



:

$$S_T \in \{80, 100, 130\}.$$

$$K = 100.$$

:

$$S_T$$

:

Long Call (LC), Short Call (SC), Long Put (LP), Short Put (SP).

CALL-OPTION

1. Long Call :

$$\text{Payoff}_{LC}(S_T) = \max(S_T - K, 0).$$

$$S_T = 80 : \quad \max(80 - 100, 0) = 0,$$

$$S_T = 100 : \quad \max(100 - 100, 0) = 0,$$

$$S_T = 130 : \quad \max(130 - 100, 0) = 30.$$

2. Short Call - call :

$$\text{Payoff}_{SC}(S_T) = -\max(S_T - K, 0).$$

$$S_T = 80 : \quad 0,$$

$$S_T = 100 : \quad 0,$$

$$S_T = 130 : \quad -30.$$

3. Long Put - :

$$\text{Payoff}_{LP}(S_T) = \max(K - S_T, 0).$$

$$S_T = 80 : \quad \max(100 - 80, 0) = 20,$$

$$S_T = 100 : \quad \max(100 - 100, 0) = 0,$$

$$S_T = 130 : \quad \max(100 - 130, 0) = 0.$$

4. Short Put - put « »:

$$\text{Payoff}_{SP}(S_T) = -\max(K - S_T, 0).$$

$$S_T = 80 : \quad -20,$$

$$S_T = 100 : \quad 0,$$

$$S_T = 130 : \quad 0.$$

S_T	LC	SC	LP	SP
80	0	0	20	-20
100	0	0	0	0
130	30	-30	0	0

- $(S_T = 80)$ put: 100 20.
- $(S_T = 100)$ — payoff .
- $(S_T = 130)$ call: 100 130 30.

- . ?

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— , $(a + b)^n$.

■ n — a b ;

■ $a^k b^{n-k}$;

■ , k a n ;

■ $\binom{n}{k}$.

: $(a + b)^n$ — a b n .

:

:

$$(a + b)^n = \sum_{k=0}^n \binom{n}{k} a^{n-k} b^k.$$

:

$$\binom{n}{k} = \frac{n!}{k! (n - k)!}.$$

:

— . :

$$\Omega = \{ \quad , \quad \}.$$

. , .

- 10 5 ;
 - 1000 — 500;
 - .
- ,

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— « ». , :

$$\xi : \Omega \rightarrow \mathbb{R}.$$

$\omega \in \Omega$ $\xi(\omega)$. :

■ 5 : — H/T. :

$$X(\omega) = \omega.$$

■ S_T — :

$$S_T(\omega) = T.$$

— , .

ξ is a random variable with values in \mathbb{R} ,
 ξ is a random variable with values in \mathbb{R} .

$$P_\xi(A) = P(\xi^{-1}(A)).$$

ξ is a random variable with values in \mathbb{R} , $\xi \in A$, ω .

ξ is a random variable with values in \mathbb{R} .

:

. , 5 3 ? :

- : $n = 5$;
- « »: $k = 3$;
- : $p = \frac{1}{2}$;
- : $q = 1 - p = \frac{1}{2}$.

:

$$P(X = k) = \binom{n}{k} p^k q^{n-k}.$$

:

$$P(X = 3) = \binom{5}{3} \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^2.$$

:

:

$$\binom{5}{3} = \frac{5!}{3!2!} = \frac{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{(3 \cdot 2 \cdot 1)(2 \cdot 1)} = 10.$$

:

$$P(X = 3) = 10 \cdot \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^2 = 10 \cdot \frac{1}{8} \cdot \frac{1}{4} = \frac{10}{32} = \frac{5}{16}.$$

$$\therefore \frac{5}{16}.$$

S_0 , . :

S_T , S_T

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■ \Leftrightarrow .

100 .

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■ — , 98.

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S_0 . :

$$S_1 = \begin{cases} uS_0, \\ dS_0, \end{cases} \quad u > d > 0.$$

■

$$u = \frac{S_1(up)}{S_0};$$

■

$$d = \frac{S_1(down)}{S_0}.$$

u d — , : = « ».

$$S_0 = 100, \quad :$$

$$S_1(up) = 105 \quad S_1(down) = 97 \quad \rightarrow \begin{cases} u = \frac{105}{100} = 1.05, \\ d = \frac{97}{100} = 0.97. \end{cases}$$

$$u - 1 = \quad (\quad +5\%), \quad 1 - d = \quad (\quad -3\%)$$

:

$$S_1 = uS_0 \qquad S_1 = dS_0.$$

:

$$S_0 \longrightarrow \begin{cases} uS_0 & (\quad) \\ dS_0 & (\quad) \end{cases}$$

$$u > d > 0. \quad \text{—}$$

.

.

$$P(\quad) = p, \quad P(\quad) = 1 - p,$$

$$0 < p < 1. \quad p?$$

- $p > \frac{1}{2}$ — $\frac{1}{2}$ (more than half).
- $p < \frac{1}{2}$ — $\frac{1}{2}$ (less than half).
- $p = \frac{1}{2}$ — $\frac{1}{2}$ (exactly half).

$$r \Delta t.$$

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$$B_1 = B_0(1 + r\Delta t).$$

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B_t

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B_t .

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: u , : d , : $1 + r\Delta t$.

:

$$d < 1 + r\Delta t < u$$

:

- $1 + r\Delta t \leq d$ — \rightarrow ;
- $1 + r\Delta t \geq u$ — r \rightarrow .

:

 $C_u, C_d.$

() :

$$C_0 = \frac{1}{1 + r\Delta t} \left(qC_u + (1 - q)C_d \right)$$

:

- - q ;

- ;

- .

.