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0. ,

$$(\qquad) - \qquad \{B(t)\}_{t \geq 0},$$

$$\frac{dB(t)}{dt} = r(t)\,B(t), \quad r(t) - \qquad .$$

$$B(0) = 1.$$

$$B(t) = \exp\!\left(\int_0^t r(s)\,ds \right).$$

$r(t) \equiv r \qquad :$

$$B(t) = e^{rt}.$$

(   ) $B(t),$,

$r.$:

- -
- -
- **Numeraire** —

$$[0, T] \qquad \Delta t:$$

$$0, \ \Delta t, \ 2\Delta t, \ \ldots, \ n\Delta t = T, \qquad \Delta t = \frac{T}{n}.$$

,

$$B(t + \Delta t) = B(t)\big(1 + r\Delta t\big).$$

n

$$B(T) = B(0) \, (1 + r\Delta t)^n.$$

$$\Delta t \to 0 \ (\qquad n \to \infty),$$

$$\lim_{\Delta t \to 0} (1 + r\Delta t)^{T/\Delta t} = e^{rT}.$$

,

$$\frac{dB(t)}{dt} = \lim_{\Delta t \to 0} \frac{B(t + \Delta t) - B(t)}{\Delta t} = rB(t).$$

$$B(0) = 100\,000 \qquad , \qquad :$$

$$r = 16,5\% = 0.165.$$

$$(T = 1) \qquad .$$

1. 1 ( ):

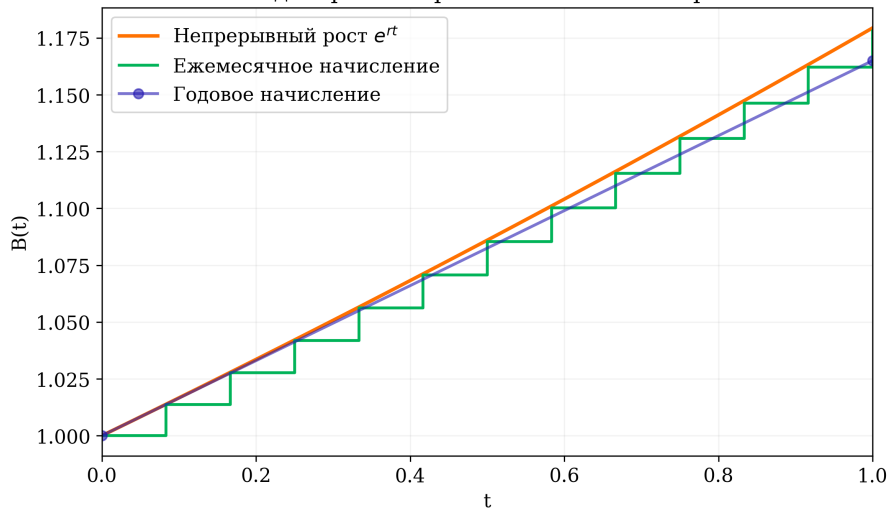$$B_1(1) = 100\,000 \left(1 + \frac{0.16}{1}\right)^1 = 100\,000 \cdot 1.16 = \mathbf{116\,000} \quad .$$

2. $(n = 12)$:

$$B_{12}(1) = 100\,000 \left(1 + \frac{0.16}{12}\right)^{12} \approx 100\,000 \cdot 1.17180 \approx \mathbf{117\,180} \quad .$$

3. $(n \to \infty)$:

$$B_{\mathsf{cont}}(1) = 100\,000\, e^{0.16} \approx 100\,000 \cdot 1.17351 \approx \mathbf{117\,351} \quad .$$

Сближение дискретного роста к экспоненте при r = 16.5%

Непрерывный рост e^{rt}
Ежемесячное начисление
Годовое начисление

B(t)

t

$$\overbrace{116\,000}^{B_1(1)} < \overbrace{117\,180}^{B_{12}(1)} < \overbrace{117\,351}^{B_{\text{cont}}(1)}$$

-      —   .
-      ,   .
-      : e^r.
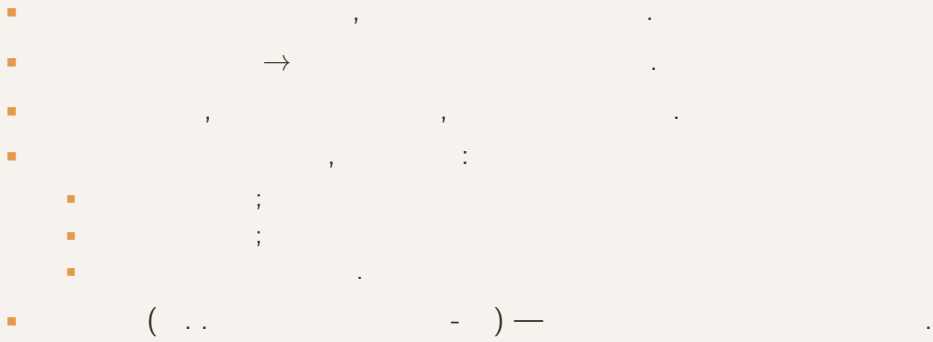
     —   .

1.

,   ,   100   .
                    ?

, , 100 .
?

: 99, 101, 98.5, 105 — , .

, , 100 .
?

: 99, 101, 98.5, 105 — , .
— , — , .

-           ,   .
- $\longrightarrow$.
-    ,   ,   .
-     ,   :
  -   ;
  -   ;
  -   .
-   (. .   -) —   .

$$(\Omega, \mathcal{F}, \mathbb{P})$$

$(\Omega, \mathcal{F}, \mathbb{P})$,

1. Ω — ( );
2. $\mathcal{F}$ — σ- , Ω, ;
3. $\mathbb{P}$ — , $A \in \mathcal{F}$ $\mathbb{P}(A)$ $[0,1]$
.

:

1.   (   ):

$$\Omega = \{\text{up, down, flat}\}.$$

2.   —   :

$$\mathcal{F} = \sigma(\{\text{up}\}, \{\text{down}\}, \{\text{flat}\}).$$

3.   :

$$P(\text{up}) = 0.3, \quad P(\text{down}) = 0.4, \quad P(\text{flat}) = 0.3.$$

$$(\Omega, \mathcal{F}, P).$$

- :
  - ,
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- P&L.
- .
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- — P&L .

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-    ,   —   **P&L**.
-
-    :