# FIT3139 Assignment 2 Q 1

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## Part A:

Using an RK2 integrator (set to Heun's for simplicity's sake, though other inspections were done with minimal change) we can inspect the system when $r + s = 0$.

For a typical starting arrangement, we may observe:

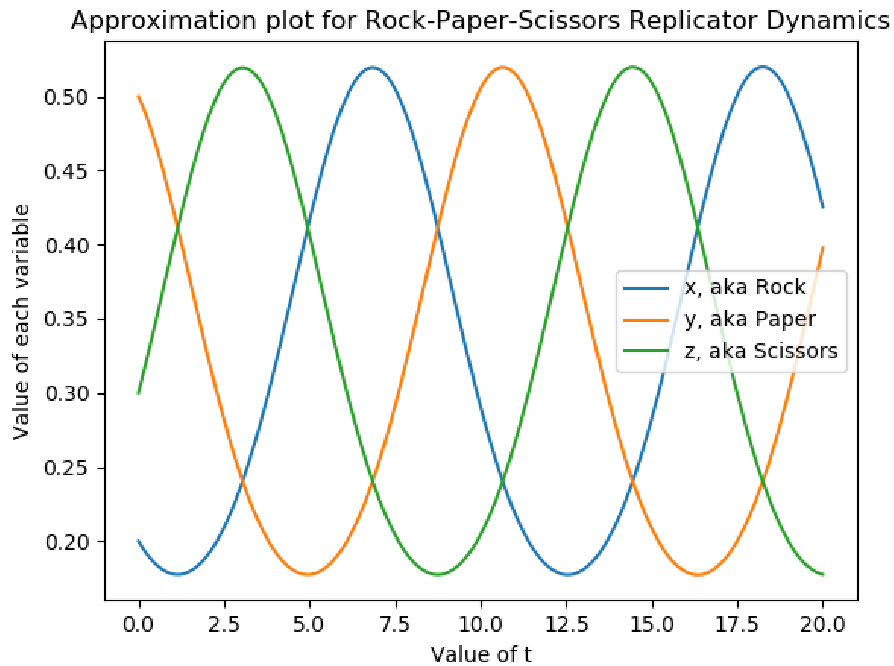
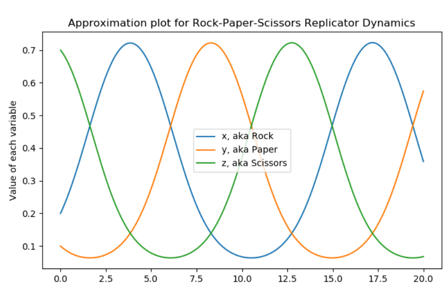
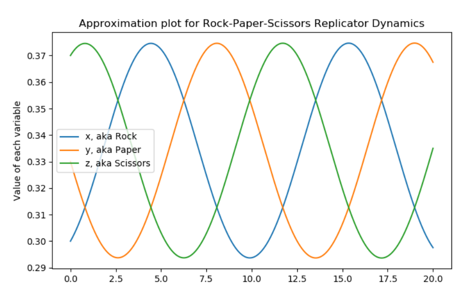



*Fig 1.a.1: RK2 inspection with initial conditions* $(x = 0.2, y = 0.5, z = 0.3)$

It is worth noting that the range between the largest and smallest input values significantly effects the height of the sinusoidal function we end up observing. Here are two differing initial values for comparison, note the difference in the axes markings for the value of each variable:




*Left: Fig 1.a.2: RK2 inspection with initial conditions* $(x = 0.2, y = 0.1, z = 0.7)$
*Right: Fig 1.a.3: RK2 inspection with initial conditions* $(x = 0.3, y = 0.33, z = 0.37)$

The period of these two charts may also be observed to be different, this likewise effect effected by the initial values.

It is also worth noting that starting the system in equilibrium, with all strategies equally represented, maintains equilibrium:

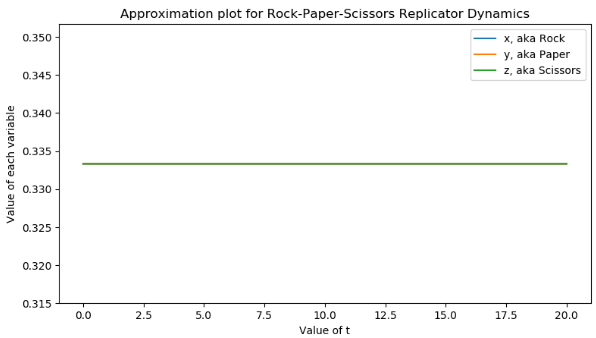
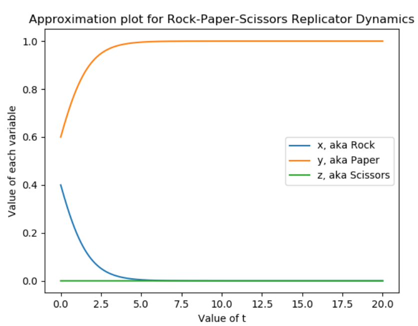
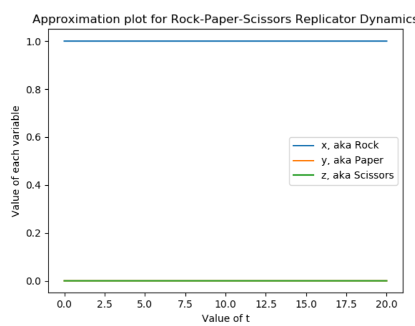



*Fig 1.a.4: RK2 inspection with initial conditions* $(x = 1/3, y = 1/3, z = 1/3)$

Finally, it may be observed that initial conditions with only one or two variable represented result in the system converging to only the 'winning' variable being represented (as a fight between paper and rock will always be won by paper) in a game with two initially represented choices, and only the initial choice in a single initial value game.




*Left: Fig 1.a.5: RK2 inspection with initial conditions* $(x = 0.4, y = 0.6, z = 0)$
*Right: Fig 1.a.6: RK2 inspection with initial conditions* $(x = 1, y = 0, z = 0)$

## Part B:

Let us examine each of the two situations specified:

**Case $r + s > 0$:**

When in this situation, we may observe that our system converges to towards an equilibrium state of $(1/3, 1/3, 1/3)$:

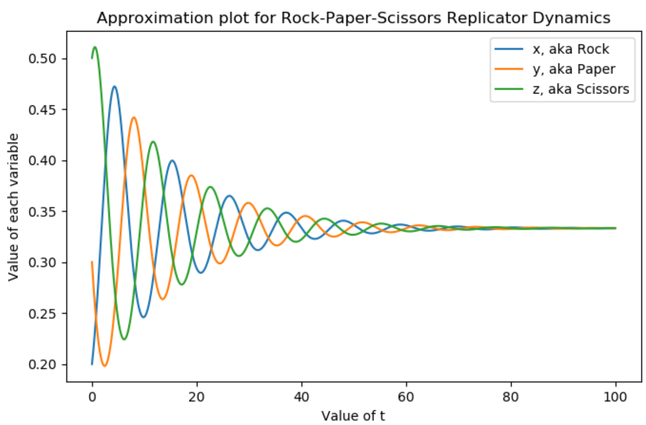



*Fig 1.b.1: RK2 inspection with initial conditions* $(x = 0.2, y = 0.3, z = 0.5)$ *and* $r = -0.8,\ s = 1.2$

The total size of $r + s$ dictates how quickly the system converges, as may be observed by the below result:

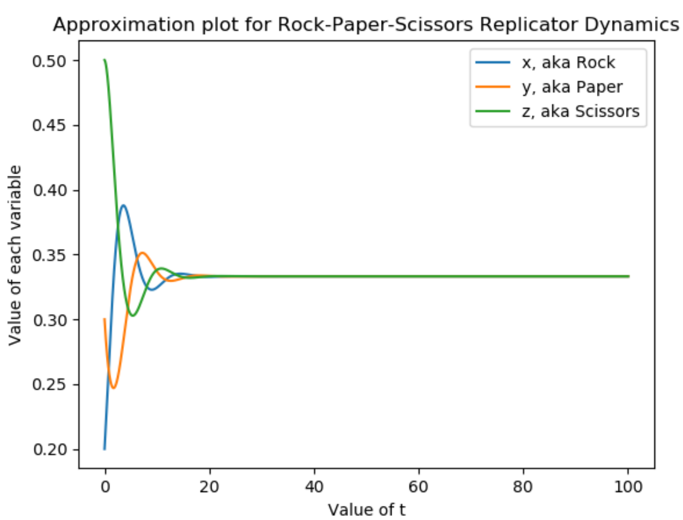



*Fig 1.b.2: RK2 inspection with initial conditions* $(x = 0.2, y = 0.3, z = 0.5)$ *and* $r = -0.1,\ s = 1.9$

From this we may conclude that having $r + s > 0$ causes our system to converge. We may note that this arises from our system's penalty term becoming positive in this instance. From a basic interpretation, $r$ and $s$ are analogous to how we weight responding to being a losing strategy and being a winning strategy. This may be observed in how $r$ is negative, reducing the strategy's proportion when it is losing, and $s$ positive, increasing the strategy when the strategy it beats is on the ascendant. From this, one would think making $r+s > 0$ would result in some sort of behaviour reflecting an exploding winning strategy, perhaps. However, because of $phi$, this is not the case.

The penalty term, being $\phi = (r + s)(xy + xz + yz)$, reduces the change in each variable when $r + s > 0$. Much like a mute on a guitar string, this pulls the values towards the stationary distribution. !!! Fix this paragraph from final lecture !!! From the game theory point of view, what we are observing is the emergence of a Nash Equilibrium in Mixed Strategies.

**Case $r + s < 0$:**

By contrast with the first case, when $r+s < 0$, our system essentially diverges to pure strategies. Instead of penalising movement, $\phi$ is instead encouraging it, as our situation makes $-\phi$ a positive value in this instance. From a Game Theory point of view, we observe divergence from the mixed strategies Nash Equilibrium towards pure strategies, oscillating due to an absence of Pure Strategies Nash Equilibria in this problem.

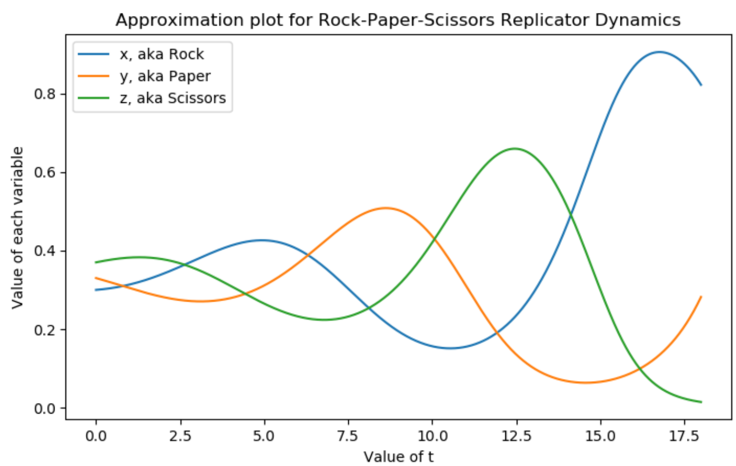



*Fig 1.b.2: RK2 inspection with initial values* $(x = 0.3, y = 0.33, z = 0.37)$ *and* $r = -1.5,\ s = 0.5$