

## **Memo:** On Small-Batch Irrigation of Regulated Substances

This memorandum outlines a preliminary framework for the academic consideration of small-batch irrigation techniques involving substances classified under regulatory oversight. The focus is on low-volume, high-precision delivery systems employed in controlled environments, particularly where substance tracking, environmental load, and legal compliance intersect.

Given the dual pressures of climate volatility and heightened regulatory scrutiny, small-batch irrigation offers a promising route to reconcile resource efficiency with traceability. However, current regulatory language—designed for bulk or industrial-scale movement—often lacks clarity when applied to micro-dosed or site-specific applications.

This memo advocates for the recognition of “regulated movement” as a dynamic continuum rather than a fixed threshold, accounting for both mass and intention. It proposes that academic and technical inquiry center on three dimensions: temporal dosing patterns, spatial delivery fidelity, and the interface between sensing technologies and compliance frameworks.

Further research is needed to define permissible thresholds, develop auditing protocols that scale with batch size, and simulate unintended diffusion under variable conditions.

# Small-Batch Irrigation (SBI)

## 1. Overview

This document formalizes a mathematical framework for analyzing the deployment of regulated substances through small-batch irrigation. The core idea involves treating regulated movements as a three-dimensional analytical space:

1. Temporal Dosing Patterns ( $T$ )
2. Spatial Delivery Fidelity ( $S$ )
3. Regulatory Interface Compliance ( $R$ )

Each axis is treated as a continuous dimension with bounded uncertainty and specific thresholds defining compliant vs. non-compliant regimes.

## 2. Dimensional Definitions

Let the overall system state be a vector in  $\mathbb{R}^3$ :

$$\vec{\Psi} = \begin{bmatrix} T(t) \\ S(x, y, z) \\ R(\lambda) \end{bmatrix} \quad (1)$$

### 2.1 Temporal Dosing Pattern

Temporal behavior of dosing is described by:

$$T(t) = \sum_{i=1}^n d_i \cdot \delta(t - t_i) \quad (2)$$

Where:

- $d_i$  is the dose delivered at time  $t_i$
- $\delta$  is the Dirac delta function representing instantaneous delivery

## 2.2 Spatial Delivery Fidelity

Modeled as a Gaussian distribution around the intended delivery point:

$$S(x, y, z) = \frac{1}{(2\pi)^{3/2} |\Sigma|^{1/2}} \exp \left( -\frac{1}{2} \mathbf{r}^T \Sigma^{-1} \mathbf{r} \right) \quad (3)$$

Where:

- $\mathbf{r} = [x - \mu_x, y - \mu_y, z - \mu_z]^T$  is the position vector from the intended target
- $\Sigma$  is the spatial covariance matrix describing delivery uncertainty

## 2.3 Regulatory Interface Function

Compliance level is modeled as a sigmoid function:

$$R(\lambda) = \frac{1}{1 + e^{-\alpha(\lambda - \lambda_0)}} \quad (4)$$

Where:

- $\lambda$  is a derived metric (e.g., cumulative dose or exposure score)
- $\lambda_0$  is the regulatory threshold
- $\alpha$  controls the sensitivity of the response

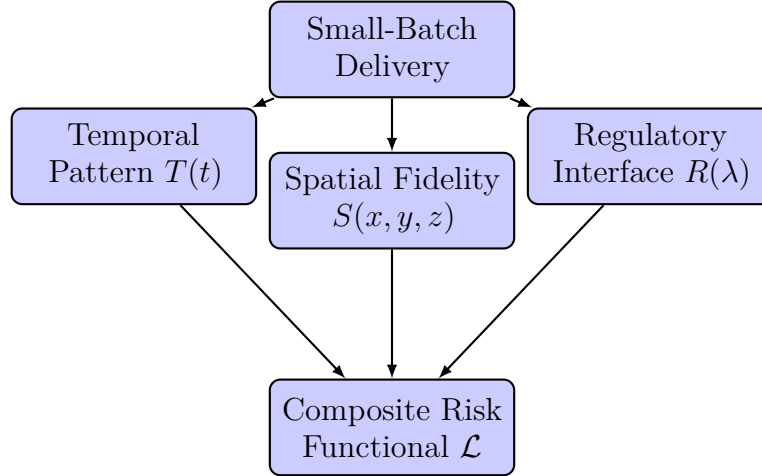
## 3. Interaction Term and Objective Functional

Define an overall system risk functional:

$$\mathcal{L} = \int_{t_0}^{t_f} \int_V [T(t) \cdot S(x, y, z) \cdot (1 - R(\lambda))] dV dt \quad (5)$$

This functional penalizes dosing that is both spatially dispersed and temporally misaligned, particularly when regulatory compliance drops below threshold.

## 4. Conceptual Flowchart



## 5. Conclusion

We’re modeling small-batch irrigation of regulated substances as a system moving through three key dimensions:

First, Temporal Dosing Patterns track when doses are delivered. Think of each dose as a sharp spike on a timeline—high precision, not continuous spraying.

Second, Spatial Delivery Fidelity measures how tightly the substance hits the intended area. Ideally, you want pinpoint delivery, not spray-and-pray. That’s modeled with a 3D Gaussian to reflect diffusion or drift.

Third, Regulatory Interface checks how close the operation is to legal or safety thresholds. That’s the sigmoid curve—once you pass a critical point, compliance quickly drops.

We combine all three into a risk functional, which penalizes dosing that’s too dispersed, poorly timed, or non-compliant. The goal? Maximize efficiency and compliance in tiny, deliberate irrigation events.

The flowchart just shows how those three inputs funnel into that risk calculation.

## References

- [1] Tiffen, M., “Variability in Water Supply, Incomes & Fees: Illustrations of Vicious Circles from Sudan & Zimbabwe” odi-IIMI, Available at: <https://media.odi.org/documents/8164.pdf>