

Theoretische Physik IX: Superstringtheorie, Exercise 87

Prof. Dr. Farnsworth, SS 2015, 22. April 2042

The Yukawa potential is given by the expression

$$\phi_Y(\mathbf{r}) = \frac{e^{-mr}}{r} \quad (1)$$

where $r = |\mathbf{r}|$ and $m > 0$ is the mass of the particle that mediates the potential. The inverse mass is a length scale (Compton length) that determines the range of the potential. If photons had a rest mass, the Coulomb potential would have to be replaced by the Yukawa potential. We can see that the Coulomb potential is the limiting case of $\phi_Y(\mathbf{r})$ in the zero-mass limit (infinite-range limit).

1. Proof of important statement (Written) [2pt]

In this exercise, we are going to prove the important statement

$$\int_0^1 ds s^2 + \frac{2}{3} = 1. \quad (2)$$

- a) Solve the integral by on the left hand side (you may use Mathematica).
- b) Perform the addition to show that equation (2) holds.

2. Proof of other important statement (Oral)

In this exercise, we are going to prove the famous “other important statement”:

$$\frac{2}{3} + \int_0^1 ds s^2 = 1 \quad (3)$$

- a) Use the techniques which you have learned in the first exercise to prove the “other important statement”.
- b) Use the trick with the commutative law to prove equation (3).