Theoretische Physik IX: Superstringtheorie, Solution 87

Prof. Dr. Farnsworth, SS 2015, 22. April 2042

The Yukawa potential is given by the expression

$$\phi_Y(\mathbf{r}) = \frac{\mathrm{e}^{-mr}}{r} \tag{1}$$

where $r = |\mathbf{r}|$ and $m > 0$ is the mass of the particle that mediates the potential. The inverse mass is a length scale (Compton length) that determines the range of the potential. If photons had a rest mass, the Coulomb potential would have to be replaced by the Yukawa potential. We can see that the Coulomb potential is the limiting case of $\phi_Y(\mathbf{r})$ in the zero-mass limit (infinite-range limit).

1. Proof of important statement (Written) [2pt]

In this exercise, we are going to prove the important statement

$$\int_0^1 \mathrm{d}s\, s^2 + \frac{2}{3} = 1. \tag{2}$$

a) Solve the integral by on the left hand side (you may use Mathematica).

b) Perform the addition to show that equation (2) holds.

Solution

a) Using Mathematica, we find that $\int_0^1 \mathrm{d}s\, s^2 = \frac{1}{3}$.

b) Then, we have $\frac{1}{3} + \frac{2}{3} = \frac{1+2}{3} = 1$. This is the result on the right hand side. □

2. Proof of other important statement (Oral)

In this exercise, we are going to prove the famous “other important statement”:

$$\frac{2}{3} + \int\limits_0^1 \mathrm{d}s\, s^2 = 1 \tag{3}$$

a) Use the techniques which you have learned in the first exercise to prove the “other important statement”.

b) Use the trick with the commutative law to prove equation (3).

Solution

a) In exercise 1a, we have seen that $\int\limits_0^1 \mathrm{d}s\, s^2 = \frac{1}{3}$.

b) Then, we have $\frac{2}{3} + \frac{1}{3}$ on the left hand side. Using the commutative law, this is equal to $\frac{1}{3} + \frac{2}{3}$. This is exactly what we have calculated in 1b! $\square$