Theoretische Physik IX: Superstringtheorie, Exercise 87

Prof. Dr. Farnsworth, SS 2015, 22. April 2042

The Yukawa potential is given by the expression

$$\phi_Y(\mathbf{r}) = \frac{\mathrm{e}^{-mr}}{r} \tag{0.1}$$

where $r = |\mathbf{r}|$ and $m > 0$ is the mass of the particle that mediates the potential. If photons had a rest mass, the Coulomb potential would have to be replaced by the Yukawa potential. We can see that the Coulomb potential is the limiting case of $\phi_Y(\mathbf{r})$ in the zero-mass limit (infinite-range limit).

1. Proof of important statement (Written) [2pt]

In this exercise, we are going to prove the important statement

$$\int\limits_0^1 \mathrm{d}s\, s^2 + \frac{2}{3} = 1. \tag{1.1}$$

a) Solve the integral by on the left hand side (you may use Mathematica).

b) Perform the addition to show that equation (1.1) holds.

Solution

a) Using Mathematica, we find that $\int\limits_0^1 \mathrm{d}s\, s^2 = \frac{1}{3}$.

b) Then, we have $\frac{1}{3} + \frac{2}{3} = \frac{1+2}{3} = 1$. This is the result on the right hand side. $\square$

2. Proof of other important statement (Oral)

Use the trick with the commutative law to prove the “other important statement”:

$$\frac{2}{3} + \int\limits_0^1 \mathrm{d}s\, s^2 = 1 \tag{2.1}$$

Solution

In 1a, we have seen that $\int\limits_0^1 \mathrm{d}s\, s^2 = \frac{1}{3}$. Then, we have $\frac{2}{3} + \frac{1}{3}$ on the left hand side. Using the commutative law, this is equal to $\frac{1}{3} + \frac{2}{3}$. According to ex. 1b, this equals 1. $\square$