Theoretische Physik IX: Superstringtheorie, Exercise 87

Prof. Dr. Farnsworth, SS 2015, 22. April 2042

The Yukawa potential is given by the expression

$$\phi_Y(\mathbf{r}) = \frac{e^{-mr}}{r} \tag{0.1}$$

where $r = |\mathbf{r}|$ and m > 0 is the mass of the particle that mediates the potential. If photons had a rest mass, the Coulomb potential would have to be replaced by the Yukawa potential. We can see that the Coulomb potential is the limiting case of $\phi_Y(\mathbf{r})$ in the zero-mass limit (infinite-range limit).

1. Proof of important statement (Written) [2pt]

In this exercise, we are going to prove the important statement

$$\int_{0}^{1} \mathrm{d}s \, s^2 + \frac{2}{3} = 1. \tag{1.1}$$

- a) Solve the integral by on the left hand side (you may use Mathematica).
- b) Perform the addition to show that equation (1.1) holds.

2. Proof of other important statement (Oral)

Use the trick with the commutative law to prove the "other important statement":

$$\frac{2}{3} + \int_{0}^{1} \mathrm{d}s \, s^2 = 1 \tag{2.1}$$