# Euclid's Elements
## Book 1

An Exercise by ISAAC Travers

Completed Entirely During the Summer
Camping trip of my 34th Year

Boulder Creek Campground
NEAR
COFFEE CREEK, CA

during the week of:
June 11th, 2019 – June 15th, 2019

Never forsake the Basics!

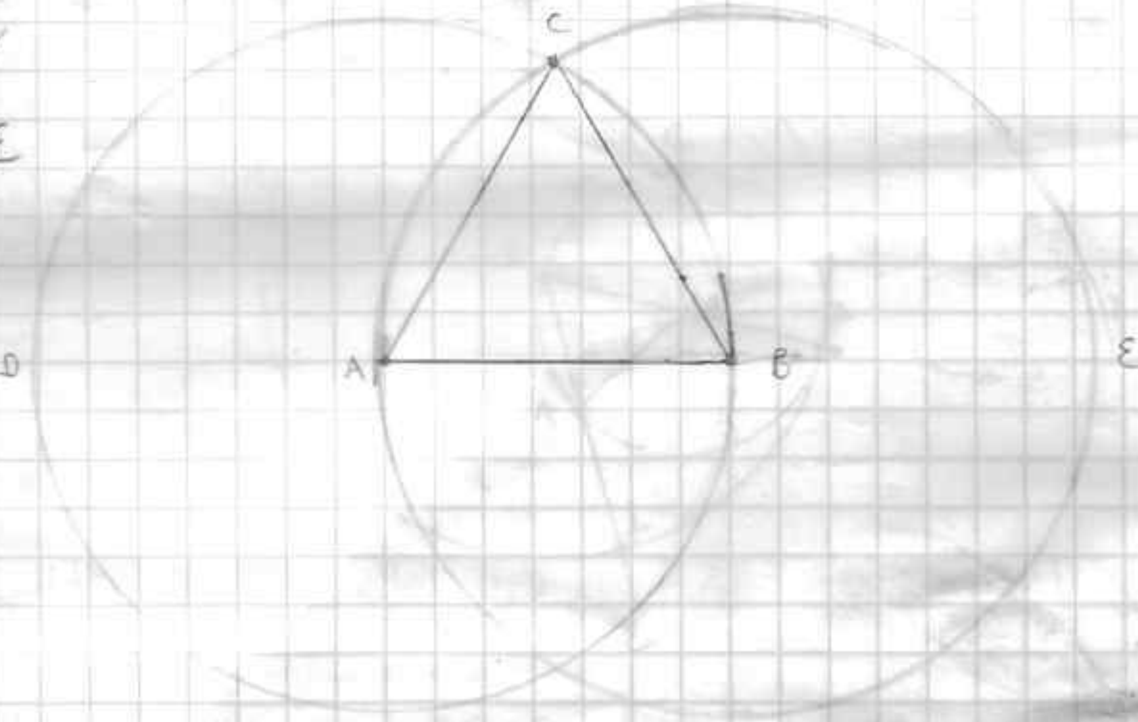
Proposition 1

Construct An Equalateral triangle on a given finite straight line

Let AB be the given ~~to~~ line
with center A and distance AB Inscribe circle BCD [Postulate 3]

Again, with center
B and Distance BA
Inscribe circle ACE
[Postulate 3]

And from point
C in which
the circles
intersect
make lines
CA and CB
[Postulate 1]



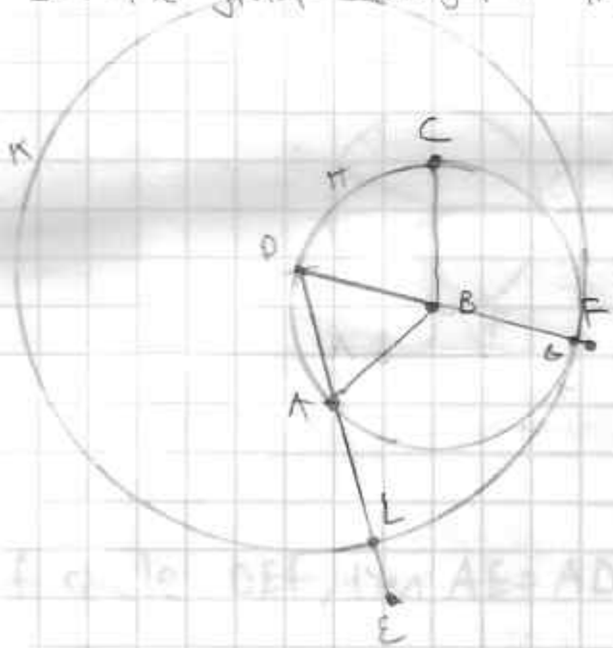

Point A is the center of circle CBD, AC is Equal to AB [illegible]
Point B is the center of circle CAE, BC is equal to BA [Def 15]
Since CA = AB and BC = AB, and since things which Are Equal [illegible]
to the Same things are Aslo Equal to each other [illegible]
then CA = BC
therefore CA = AB = BC, which means triangle
ABC is an Equalateral triangle. [QED]

Proposition 2

Place, At a given Point (At An Extremity), a straight line equal to a given straight line

Let A be the given Point, and BC the given straight line

From A to B construct straight line AB [Postulate 1]




On AB construct Equalateral triangle DAB [Proposition 1]

Let the straight lines AE and BF be Produced in a straight line with DA, DB [Postulate 2]

with center B and Distance BC inscribe circle CGH [Postulate 3]

And Again with center D and Distance DG let circle GKL be described [Posulate 3]

Since Point B is the center of circle CGH, BC = BG [Definition 15]

Since Point D is the center of circle GKL, DL = DG [Def 15]

DA = DB [Prop 1]

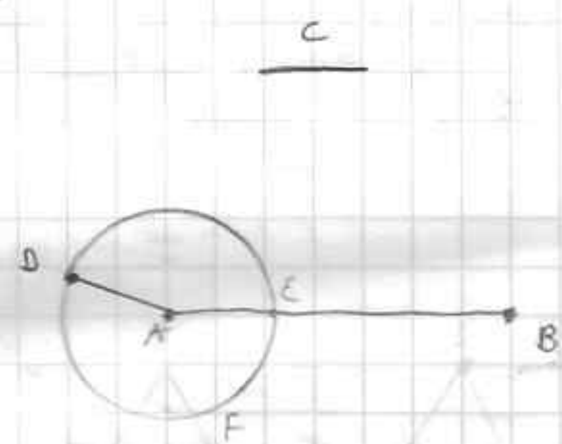
therefore, the remainders AL = BG [Common notion 3]

Since AL = BG, and BG = BC, and things which are Equal to the same thing are Equal to each other then AL = BC [Common Notion 1]

, which is the line we were trying to construct [QED]

Proposition 3

Given 2 unequal lines, cut off from the greater a straight line equal to the lessor




Let AB, and C be the two given unequal, straight lines, and let AB be the greater

At point A, let AD be placed equal to the straight line C [Proposition 2]

With center A, and distance AD, construct circle DEF [Postulate 3]

Since point A is the center of circle DEF, then AE = AD [Def. 15]
but AD = C [Prop 2]

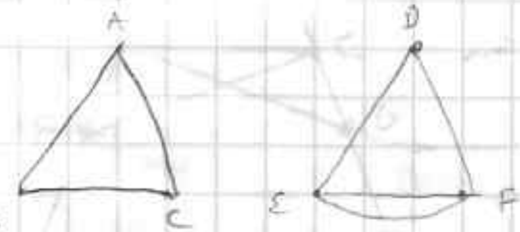
Since AE = AD and AD = C, then AE = C [Common Notion 1]

AE is what we set to construct [QED]

Propostion 4 (Side-Angle-Side)

If Two triangles have two sides equal to two sides respectively, And have the angles contained by the Equal sides Also being equal, then the triangles 3rd Sides will Also be Equal, The triangles and ALL their Sides And Angles will be Equal to Each other.

Let ABC, DEF be two triangles having two sides AB, AC Equal to the two sides DE and DF, AB = DE and AC = DF And the Angles BAC = EDF




I Say that base BC is Also Equal to base EF, the triangle ABC is Equal to triangle DEF, the remaining Angles will Also be Equal to their respective Angles

If triangle ABC is Applied to triangle DEF, and if Point A be Placed on point D, and Straight line AB on DE, then the Point B will coincide with Point E because AB = DE

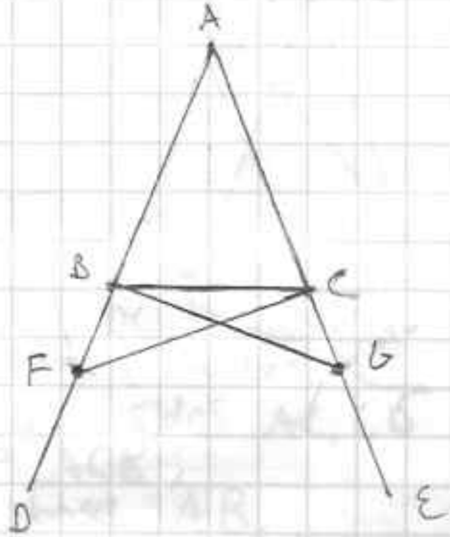
Again AB coincides with DE, the straight line AC will Also coincide with DF, because the Angle BAC = EDF, hence Point C will Also coincide with the point F, because AC = DF,

B is Also coincided with E, hence base BC coincides with base EF for if, when B coincides with E, and C with F, If the base BC does not Coincide with base EF, the two Straight lines would enclose a space, which is Impossible, therefore the base BC will Coincide with EF, and be Equal to it [Common Notion 4]

thus the whole triangle ABC will coincide with the whole triangle DEF, And All remaining Angles must be Equal As well $\boxed{\text{QED}}$

Proposition 5

In Isosceles triangles the Angles At the base Are Equal to one another, and if the equal straight lines be Produced further, the Angles under the base will be Equal to one another




Let ABC be an Isosceles triangle having the side AB Equal to the side AC;
Let the straight lines BD and CE be Produced further in a straight line with AB and AC $\boxed{\text{Post. 2}}$

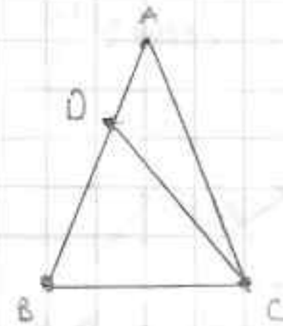
I Say that the Angle ABC = ACB and Angle CBD = BCE!

Let a point F be taken at random on BD;
From AE the greater let AG be cut off Equal to AF the less $\boxed{\text{Proposition 3}}$
Let straight lines FC and GB be Joined $\boxed{\text{Post. 1}}$
Then since AF = AG and AB = AC, the two sides FA and AC are equal to the two sides GA and AB respectively.
~~namely those which the equal sides subtend, that is~~
and they contain a common Angle, the Angle FAG
therefore bases FC = GB, and the triangles AFC = AGB
and Angles ACF = ABG, AFC = AGB $\boxed{\text{Proposition 4}}$
And since the whole AF = AG, and in these AB = AC, the the remainder BF = CG;
But FC is equal to GB, therefore the two sides BF, FC are Equal to the two sides CG, GB respectively; and the Angle BFC is equal to the Angle CGB, while the base BC is common to them; therefore the triangles BFC = CGB, and the remaining ~~Eq~~ Angles will be Equal, therefore Angle FBC is equal to Angle GCB and Angle's BCF = CBG.

Accordingly, since the whole angle ABG was proved Equal to ACF, and in these the Angle CBG = BCF, the remaining Angle ABC = ACB and they are At the base of triangle ABC, but the Angles FBC = GCB, and they are under the base, which is what we wanted to prove $\boxed{\text{QED}}$

Proposition 6

If in a triangle two Angles be Equal to one another, the sides which subtend the Equal angles will Also be Equal to one another.



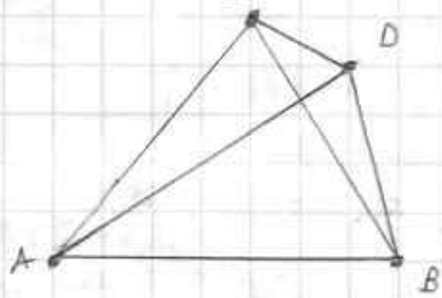

- Let ABC be a triangle having the angle ABC Equal to the Angle ACB;
- I Say side AB is equal to side AC;
- for If AB is unequal to AC, then one of them is greater.
- Let AB be greater; and from AB the greater let DB be cut off equal to AC the less; Let DC be Joined.
- then, since DB is equal to AC, And BC is Common, the two sides DB, BC are equal to the two sides AC, CB respectively, and the Angle DBC is equal to the Angle ACB;
- Therefor the base DC is equal to the base AB, and the triangle DBC will be equal to the triangle ACB, the less equal to the greater, which is AbsurD!
- therefore AB is not unequal to AC;

It is therefor equal to it.

QED

Proposition 7

Given two straight lines constructed on a straight line (from its extremities) and meeting in a point, there cannot be constructed on the same straight line (from its extremities), and on the same side of it two other straight lines meeting in another point and equal to the former two respectively, namely each to that which has the same extremity with it.




- For, if possible, given two straight lines AC, CB constructed on the straight line AB, and meeting at point C,
- Let two other straight lines AD, DB be constructed on the same straight line AB, on the same side of it, meeting in another point D, and equal to the former two respectively, namely each to that which has the same extremity with it, so that CA is equal to DA which has the same extremity A with it, and CB to DB which has the same extremity B with it;
- Let CD be Joined
- then since AC is equal to AD, the Angle ACD is Also equal to the Angle ADC; [Proposition 5]
- therefore the Angle ADC is greater than angle DCB;
- therefore the Angle CDB is much greater than Angle DCB
- Again Since CB is equal to DB, the Angle CDB is Also equal to the Angle DCB.

But it was Also Proved to be greater than It which is impossible

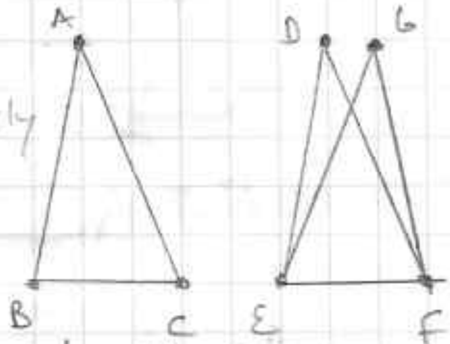
therefore

AC, CB, cannot be equal to AD, DB

QED

Proposition 8 Side-Side-Side Congruence

If two triangles have the two sides equal to two sides respectively, and have also the base equal to the base, they will also have angles equal which are contained by the equal straight lines




- Let ABC, DEF be two triangles having the two sides AB, AC equal to the two sides DE, DF respectively namely AB to DE and AC to DF; and let them have the base BC equal to the base EF;
- I say that the angle BAC is also equal to the angle EDF.
- for, if the triangle ABC be applied to the triangle DEF, and if the point B be placed on the point E and the straight line BC on EF, the point C will coincide with the point F, because BC is equal to EF.
- the BC coinciding with EF, → BA, AC will also coincide with ED, DF;
- for, if the base BC coincides with the base EF, and the sides BA, AC do not coincide with ED, DF, but fall beside them as EG, GF,
- then, given two straight lines constructed on a straight line (from its extremities and meeting) at a point, there will have been constructed on the same straight line (from its extremities), and on the same side of it, two other straight lines meeting in another point and equal to the former two respectively, namely each to that which has the same extremity with it, but they cannot be so constructed $\boxed{\text{Proposition 7}}$
- therefore it is not possible that, if the base BC be applied to the base EF, the sides BA, AC should not coincide with ED, DF;

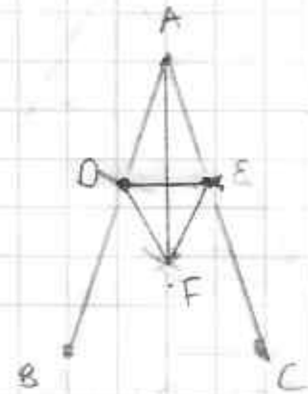
they will therefore coincide, so that the angle BAC will also coincide with the angle EDF, and will be equal to it

$\boxed{\text{QED}}$

Proposition 9

How to bisect A given rectilineal Angle

- Let the Angle BAC be the given rectilineal Angle
- We must bisect this Angle
- Let point D be taken randomly on AB;
- Let AE be Cut off from AC Equal to AD [Prop.3]
- Let DE be Joined;
- And on DE let the equalateral triangle DEF be constructed [Prop.1]
- Let AF be Joined

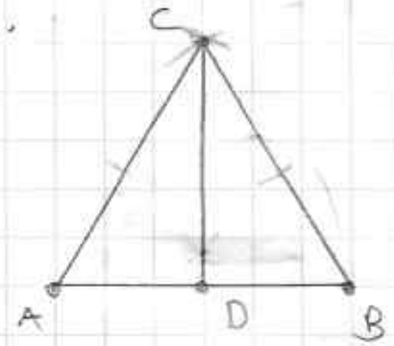



- I say that the Angle BAC has been bisected by AF.
- for since AD is equal to AE, and AF is common, the two sides DA, AF are equal to the two sides EA, AF respectively.
- And the base DF is equal to the base EF;
- therefore the Angle DAF is equal to the Angle EAF [Prop.8]
  - therefore the given rectilineal Angle BAC has been bisected by the straight line AF.

Q.E.D.

Proposition 10

How to bisect A given finite straight line.




- Let AB be the given straight line.
- thus ~~it~~ is required to bisect the straight line AB
- Let the Equalateral triangle ABC be constructed on it Proposition 1
- Let the Angle ACB be bisected by the straight line CD
- I say that the straight line AB has been bisected At Point D
- for since AC is equal to CB And CD is common, the two sides AC, CD are equal to the two sides -DC, CD respectively;
- And the Angle ACD is equal to the Angle BCD;
- therefore, the base AD is equal to the base BD Proposition 4

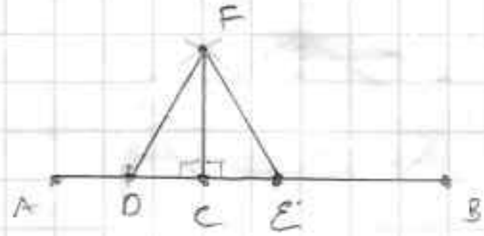
therefore the given finite straight line AB has been bisected At D

Q.E.D.

Proposition 11

How to draw a straight line at right Angles to a given straight line, from a given Point on it.

- Let AB be the given straight line, and C the given Point on it
- thus it is required to draw from the Point C a straight line At right Angles to the straight line AB
- Let Point D be taken at random on AC
- Let CE be made Equal to CD $\boxed{\text{Prop. 3}}$
- on DE Let the equalateral triangle FDE be constructed $\boxed{\text{Prop. 1}}$
- Let FC be Joined

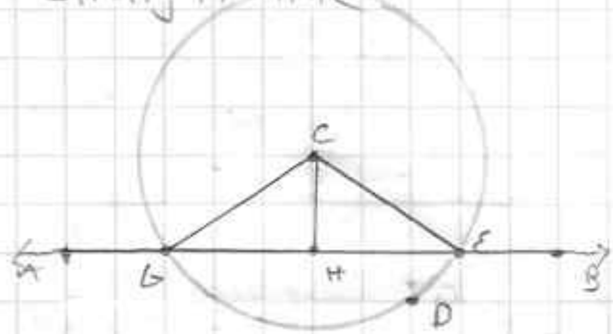



- I say that the straight line FC has been drawn at right Angles to the given straight line AB from C, the point given on It
- for since DC is equal to CE and CF is common the two sides DC, CF are equal to the two sides EC, CF respectively;
- And the base DF is equal to the base FE
- therefore the Angle DCF is equal to the Angle ECF $\boxed{\text{Proposition 8}}$ and they Are Adjacent Angles
- But, when a straight line set up on a straight line makes the Adjacent equal to one another, each of the Angles is right $\boxed{\text{Definition 10}}$
- therefor each of the Angles DCF, FCE is right.
- therefor the straight line CF has been drawn at right Angles to the given straight line AB from the given point C on it!

$\boxed{\text{Q.E.D.}}$

Proposition 12

To a given infinite straight line, from a given point which is not on it, to draw a perpendicular straight line




- Let AB be the given infinite straight line, and C be the given point not on it.
- Thus it is required to draw to the given infinite straight line AB, from the given point C (which is not on the line) a perpendicular straight line
- Let A point D, be taken at random on the other side of the straight line AB
- With center C and Distance CD, let circle EFG be described [Post. 3]
- Let the straight line EG be bisected At H [Proposition 10]
- Let CG, CH, CE be Joined; [Post. 1]

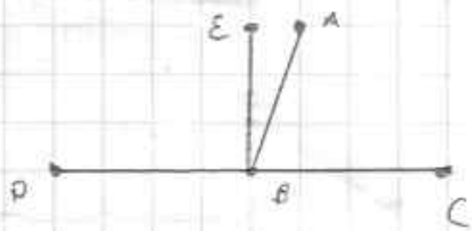
- I say that CH has been drawn perpendicular to the given Infinite straight line AB from the given point C, which is not on it.
- for since GH is equal to HE and HC is common, the two sides GH, HC are equal to the two sides EH, HC respectively; And the base CG is equal to the base CE;
- therefore the Angle CHG is equal to EHC [Proposition 8] And they Are Adjacent Angles.
- But when a straight line is set up on a straight line and makes the Adjacent Angles equal to another, each of the angles is right, and the straight line standing on the other is called perpendicular to that on which it stands [Definition 10]

therefore CH has been drawn perpendicular to the Given Infinite straight line AB from the Given Point C, which is not on it!

$\boxed{\text{Q.E.D,}}$

Proposition 13

If a straight line set up on a straight line make Angles, it will make either two right Angles, or angles equal to two right Angles

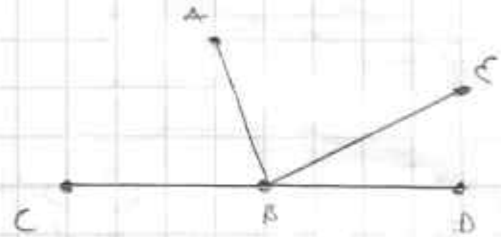



- for let Any straight line AB, set up on straight line CD make the Angles CBA and ABD;
- I say that Angles CBA, ABD are either two right Angles or equal to two right Angles
- Now if Angle CBA is equal to the Angle ABD they are two right Angles [Def. 10]
- But if Not, let BE be drawn from the point B at right Angles to CD [I, 11]
- therefore the Angles CDE, EBD are two right Angles
- then, since the angle CBE is equal to the angles CBA, ABE, let the Angle EBD be Added to each;
- therefore the Angles CBE, EBD are equal to the three Angles CBA, ABE, EBD [Common Notion 2]
- Again, Since the Angle DBA is equal to the Angles DBE, EBA let the Angle ABC be added to each;
- therefore the Angles DBA, ABC are equal to the three Angles DBE, EBA, ABC [Common Notion 2]
- But the Angles CBE, EBD are Also equal to the Same three Angles;
- and things which Are equal to the same thing are Also equal to one another [Common Notion 1]
- therefore the Angles CBE, EBD are also equal to the Angles DBA, ABC.
- But the angles CBE, EBD are two right Angles; therefore the Angles DBA, ABC are Also equal to two right Angles

[Q.E.D.]

Proposition 14

If with Any straight line, and at a point on it, two straight lines not lying on the same side make the adjacent Angles equal to two right Angles, the two straight lines will be in a straight line with one another



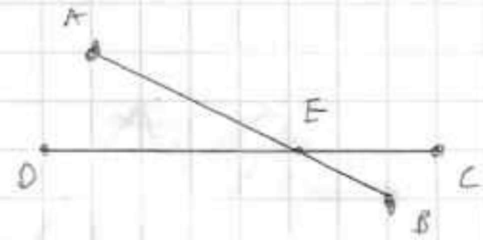

- for with Any straight line AB, and at the point B on it, let the two straight lines BC, BD not lying on the same side make the Adjacent Angles ABC, ABD equal too two right Angles [I. 13]
- I say BD is in a straight line with CB.
- for if BD is not in a straight line with BC, let BE be in a straight line with CB.
- then, since the straight line AB stands on the straight line CBE the Angles ABC, ABE Are equal to two right Angles [I. 13]
- But the Angles ABC, ABD are Also equal to two right Angles
- therefore the Angles CBA, ABE are equal to the Angles CBA, ABD [Post. 4] and [common notion 1]
- the Angle CBA can be subtracted from each;
- therefore, the remaining Angle ABE is equal to the remaining Angle ABD [common Notion 3]
- the less equal to the greater, which is impossible
- therefore BE is not in a straight line with CB,
- similarity we can prove that neither is Any other straight line except BD

therefore CB is in a straight line with BD.

Q.E.D.

Proposition 15

If two straight lines cut one Another, they make the vertical Angles equal to one another



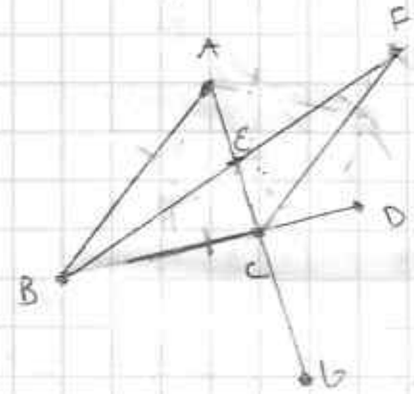

- Let the straight lines AB, CD cut one another at Point E;
- I Say, that the Angle AEC is equal to the Angle DEB, and the Angle CEB is equal to the Angle AED.
- for since the straight line AE stands on the straight line CD, making the Angles CEA, AED, which Are equal to two right Angles $\boxed{\text{I. 13}}$
- Again since the straight line DE stands on the straight line AB, making the Angles AED, DEB, which are equal to two right Angles $\boxed{\text{I. 13}}$
- But the Angles CEA, AED were Also proved equal to two right Angles; therefore the Angles CEA, AED are equal to the Angles AED, DEB $\boxed{\text{Post. 4}}$ and $\boxed{\text{Common Notion 1}}$
- Let the Angle AED be subtracted from each;
- therefore the remaining Angle CEA is equal to Angle BED. $\boxed{\text{Common Notion 3}}$
- Similarly it can be proved that the Angles CEB, DEA Are Also equal

$\boxed{\text{Q.E.D}}$

[Porism: from this it is manifest that, if two straight lines cut one another, they will make the Angles at the Point of Section equal to four right Angles.]

Proposition 16

In any triangle, if one of the sides be produced, the exterior angle is greater than either of the interior and opposite Angles

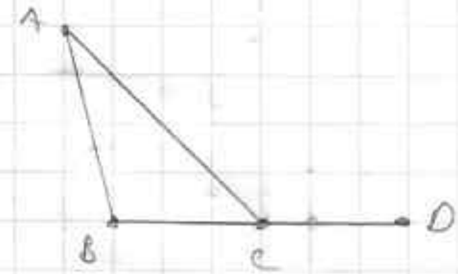



- Let ABC be a triangle, and let one side of it BC be produced to D;
- I say that the Exterior Angle ACD is greater than either of the interior And opposite Angles CBA, BAC!
- Let AC be bisected At E $\boxed{I.10}$
- Let BE be Joined And produced in a straight line with F;
- Let EF Be made equal to BE $\boxed{I.3}$
- Let FC Be Joined $\boxed{Post.1}$
- Let AC be drawn through to G $\boxed{Post\ 2}$
- then since AE is equal to EC and BE to EF, the two sides AE, EB are Equal to the two sides CE, EF respectively
- And the Angle AEB is equal to the Angle FEC for they are verticle Angles $\boxed{I.15}$
- therefore the base AB is equal to the base FC, and the triangle ABE is equal to the triangle CFE, and the remaining Angles are equal to the remaining angles respectively, namely those which the Equal Sides Subtend $\boxed{I.4}$
- therefore the Angle BAE is equal to the Angle ECF.
- But the Angle ECD is greater than the Angle ECF $\boxed{C.N.5}$
- therefore the Angle ACD is greater than the Angle BAE
- Similarly Also if BC be bisected, the Angle BCG, that is the Angle ACD $\boxed{I.15}$ can be proved Greater than the Angle ABC As well

$\boxed{Q.E.D.}$

Proposition 17

In Any triangle, two Angles taken together in Any manner are less than two right Angles.

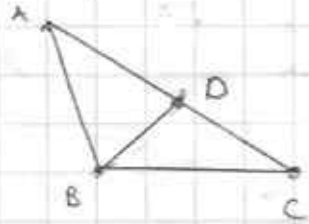



- Let ABC be a triangle;
- I say that two Angles of triangle ABC taken together in any manner are less than two right Angles
- Let BC be produced to D [Post. 2]
- then, since the Angle ACD is an exterior angle of the triangle ABC, it is greater than the interior and opposite Angle ABC [I, 16]
- Let the Angle ACB be added to each; therefore the Angles ACD, ACB are greater than the Angles ABC, BCA
- but the Angles ACD, ACB are equal to two right Angles [I, 13]
- Therefore the Angles ABC, BCA are less than 2 right Angles. Similarly we can prove that the Angles BAC, ACB are Also less than two right Angles, and so the Angles CAB, ABC As well,

Q.E.D.

Proposition 18

In Any triangle the greater side subtends the greater Angle

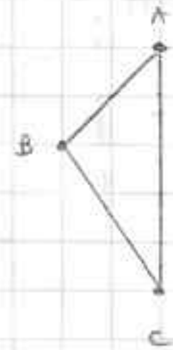



- Let ABC be a triangle having side AC greater than AB;
- I say that Angle ABC is also greater than the Angle BCA.
- For, since AC is greater than AB, let AD be made equal to AB $\boxed{I.3}$
- Let BD be Joined
- then, Since Angle ADB is an exterior Angle of the triangle BCD, it is greater than the Interior and opposite Angle DCB $\boxed{I.16}$
- But the Angle ADB is equal to the Angle ABD, since the side AB is equal to AD $\boxed{I.5}$
- therefore the Angle ABD is Also greater than ACB;
  - therefore the Angle ABC is much Greater than Angle ACB

$\boxed{Q.E.D.}$

Proposition 19

In any triangle, the greater Angle is subtended by the Greater Side.



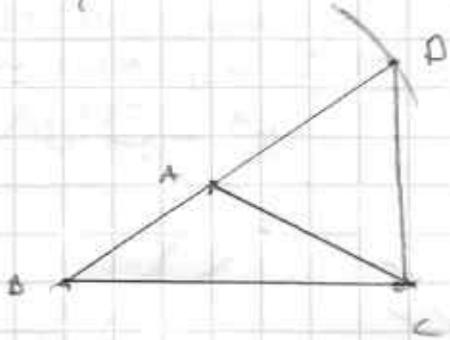

- Let triangle ABC be a triangle having the Angle ABC Greater than BCA
- I say that the side AC is Also Greater than the side AB.
- for if Not, AC is either equal to AB, or less.
- Now AC is Not Equal to AB; for then the Angle ABC would Also be equal to ACB $\boxed{I.5}$ but it is NOT!
- therefore AC is not equal to AB.
- Neither is AC less than AB, for then the Angle ABC would Also have been less than the Angle ACB $\boxed{I.18}$ but it is Not!
- therefore AC is not less than AB. And it was proved that it is Not Equal either

- therefore AC is greater than AB

$\boxed{Q.E.D.}$

Proposition 20

In Any triangle, two sides taken together in Any manner are greater than the remaining one.




- Let ABC be A triangle
- I say then in the triangle ABC two sides taken together in any manner are greater than the remaining one, namely
  BA, AC greater than BC
  AB, BC greater than AC
  BC, CA greater than AB.

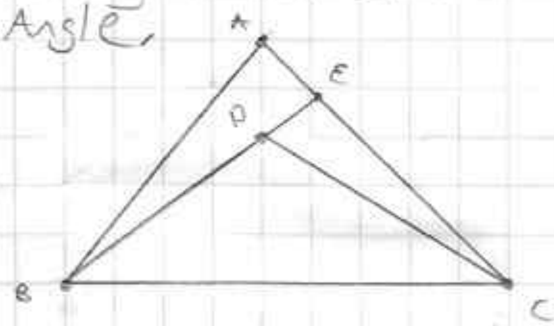
For let BA be drawn through to point D
- Let DA be mad equal to CA
- let DC be Joined
- then, since DA is equal to AC, the Angle ADC is equal to ACD $\boxed{I.5}$
- therefore the Angle BCD is greater than ADC $\boxed{C.N.5}$
- And since DCB is a triangle having the Angle BCD greater than the Angle BDC and the greater Angle is Subtended by the greater Side $\boxed{I.19}$
- therefore DB is greater than BC.
- But DA is equal to AC; therefore BA, AC are greater than BC.

Similarly we can prove that AB, BC are Also greater than CA, and BC, CA greater than AB

$\boxed{\text{Q.E.D.}}$

Proposition 21

If on one of the sides of a triangle, from its extemities, there be constructed two straight lines meeting within the triangle, the straight lines so constructed will be less than the remaining two sides of the triangle, but will contain a greater Angle.




- On BC, one of the sides of triangle ABC, from its extemities B, C let the two straight lines BD, DC be constructed meeting within the triangle;
- I say that BD, DC are less than the remaining two sides BA, AC, but contain an Angle BDC greater than the Angle BAC.
- for, let BD be drawn through to E
- then, since in any triangle two sides are greater than the remaining one [I.20]
- therefore, in the triangle ABE, the two sides AB, AE are greater than BE.
- Let EC be added to each;
- therefore BA, AC are greater than BE, EC.
- Again since, in the triangle CED the two sides CE, ED are greater than CD, let DB be added to each,
- therefore CE, EB are greater than CD, DB
- but BA, AC were proved greater than BE, EC
- therefore BA, AC are much greater than BD, DC.
- Again, since in any triangle the exterior Angle is greater than the interior and opposite Angle [I.16]
  therefore, in the triangle CDE, the exterior Angle BDC is greater than the Angle CED.
- for the same reason, moreover, in the triangle ABE also, the exterior Angle CEB is greater than the Angle BAC.
- But the Angle BDC was proved greater than CEB;
  - therefore the Angle BDC is much greater than Angle BAC

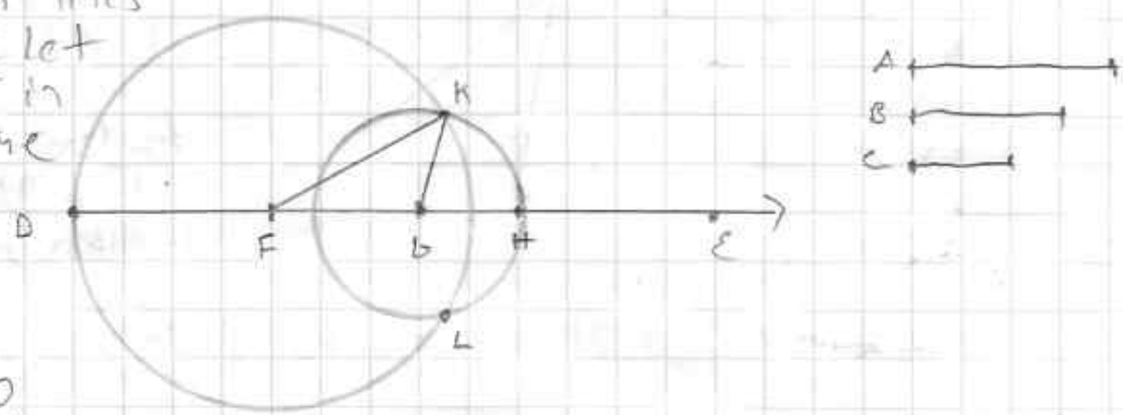
Q.E.D.

Proposition 22

out of three straight lines, which are equal to three given straight lines to construct A triangle; thus it is necessary that two of the straight lines taken together in any manner should be greater than the remaining one.

I.20

Let the three given straight lines be A, B, C, and of these let two be taken together in any manner be greater than the remaining one, namely
A, B greater than C
A, C greater than B
B, C greater than A;
thus it is required to construct a triangle out of straight lines A, B, C.



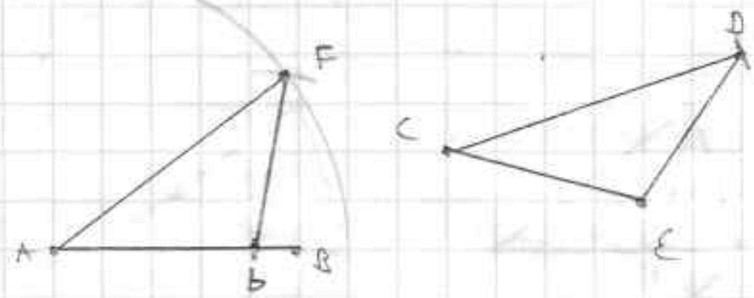

- Let there be set out A straight line DE, terminated at D, but of infinite length in the direction of E;
- Let DF, be made equal to A; FG equal to B, and GH equal to C I.3
- with center F, and distance FD let circle DKL be described;
- Again, with center G, and distance GH, let circle KLH be described/
- Let KF, KG be joined
- I say that triangle KFG has been constructed out of three straight lines which Are Equal to A, B, C
- for since Point F is the center of DKL, FD is equal to FK,
- but FD is equal to A, therefore KF is equal to A
- Again, since the point G is the center of circle LKH, GH is equal to GK
- but GH is equal to C; therefore KG is Also equal to C.
- And FG is equal to B, the three straight lines KF, FG, GK are equal to the three straight lines A, B, C

- therefore out of the three straight lines KF, FG, GK, which are equal to A, B, C, the triangle KFG has been constructed!

Q.E.D.

Proposition 23

On a given straight line and at a point on it to construct A rectilineal Angle equal to a given rectilineal Angle



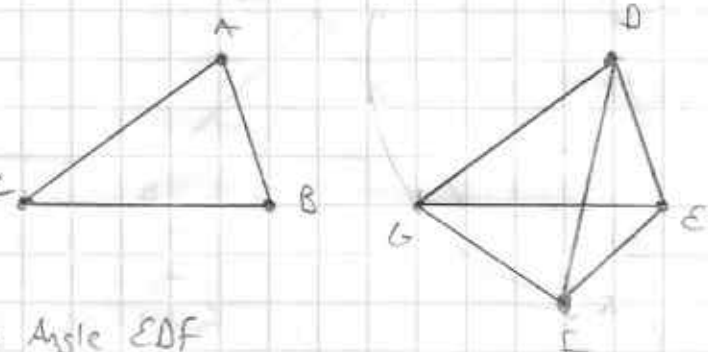

- Let AB Be the given straight line, A the point on it and the Angle DCE the given rectilineal Angle;
- Thus it is required to construct on given straight line AB and At the point A on it, a rectilineal Angle equal to DCE
- On straight lines CD, CE respectively let the points D, E be taken at random,
- Let DE be Joined
- and out of three ~~st~~ straight lines which are equal to the three straight lines CD, DE, CE let triangle AFG be constructed in such a way that CD is equal to AF, CE to Ag and DE to FG $\boxed{I.22}$
- then, since the two sides DC, CE are equal to the the two sides FA, AG respectively, And the base DE is equal to FG the Angle DCE is equal to Angle FAG $\boxed{I.8}$

- therefore, on the given straight line AB, at the point A on it, the rectilineal Angle FAG has been constructed equal to the given rectilineal Angle DCE

$\boxed{Q.E.D.}$

Proposition 24

If two triangles have the two sides equal to two sides respectively, but have the one of the Angles contained by the equal straight lines greater than the other, they will Also have the base greater than the base.

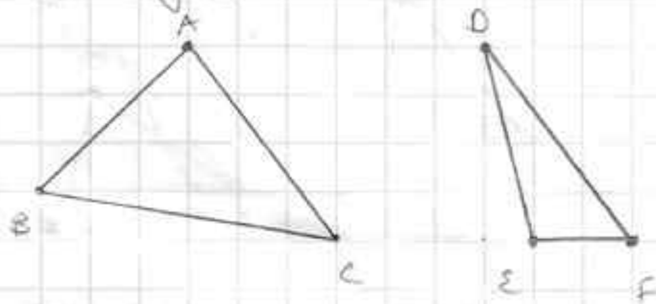



- Let ABC, DEF be two triangles having sides AB = DE and AC = DF, and Let the Angle At A Be greater than the Angle At D
- I Say that the base BC is also greater than the base EF
- For, since the Angle BAC is greater than the Angle EDF Let there be constructed, on the straight line DE at point D on it, the angle EDG equal to the Angle BAC $\boxed{I.23}$
- Let DG be made equal to either AC or DF, and let EG, FG be Joined
- Then since AB is equal to DE and AC to DG, the two sides BA, AC are equal to the two sides ED, DG respectively; And the Angle BAC is equal to EDG; therefore, the base BC is equal to EG $\boxed{I.4}$
- Again since DF is equal to DG, the Angle DGF is Also equal to the Angle DFG $\boxed{I.5}$
- therefore the Angle DFG is greater than the Angle EGF $\boxed{Why?}$
- therefore the Angle EFG is much greater than EGF
- And since, EFG is a triangle having the Angle EFG greater than the Angle EGF, and the greater Angle is subtended by the greater side $\boxed{I.19}$
  - the side EG is greater than EF
  - but EG is equal to BC
  - therefore BC is greater than EF

$\boxed{Q.E.D.}$

Proposition 25

If two triangles have the two sides equal to two sides respectively, but have the base greater than the base, they will Also have the one of the Angles contained by the equal Straight lines greater than the other.



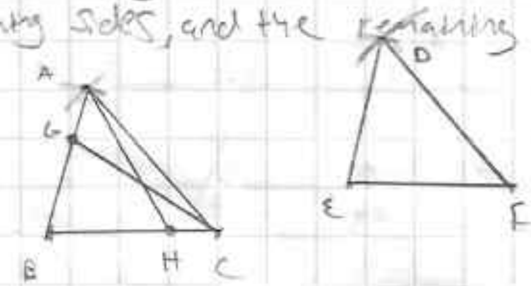

Let ABC, DEF be two triangles having the two sides AB = DE and AC = DF;
and Let the base BC be greater than the base EF;
- I say that the Angle BAC is Also greater than the Angle EDF
- If Not greater, than it is equal to it, or less
- Now, the Angle BAC is not equal to Angle EDF, for then the base BC would also have been equal to the base EF $\boxed{I.4}$ but it is Not!
- Therefore the Angle BAC IS NOT Equal to Angle EDF.
- Neither Again is the Angle BAC less than the Angle EDF, for then BC would have been less than EF $\boxed{I.24}$ But it is Not!
- therefore the Angle BAC is not less than the Angle EDF, But it was Proved Not Equal either;

therefore the Angle BAC is greater than the Angle EDF

$\boxed{Q.E.D}$

Proposition 26

If two triangles have the two Angles equal to the two Angles respectively, and one side equal to one side, namely, either the side Adjoining the equal Angles, or that Subtending one of the equal Angles, they will Also have the remaining sides equal to the remaining sides, and the remaining Angle to the remaining Angle

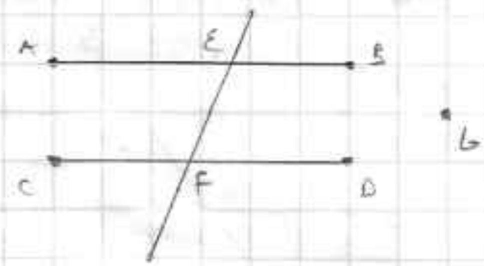



- Let ABC, DEF be two triangles having the two Angles ABC, BCA equal to the two Angles DEF, EFD respectively, namely ABC = DEF and BCA = EFD; and let them Also have one Side equal to one side, namely BC to EF;
- I say that they will Also have remaining sides equal to remaining sides, namely AB = DE, AC = DF and remaining Angle to remaining Angle namely, BAC = EDF.
- for if AB is unequal to DE, one of them is greater.
- Let AB be greater, and let BG be made Equal to DE, and Let GC be Joined.
- then since BG is equal to DE, and BC = EF, the two sides GB, BC are equal to DE, EF respectively
- And the Angle GBC is equal to DEF; therefore base GC is equal to base DF, and the triangle And All remaining angles of GBC equal DEF [I.4]
- therefore the Angle GCB is equal to the Angle DFE.
- But the Angle DFE is by hypothesis equal to the Angle BCA; therefore the Angle BCG is equal to the Angle BCA, the less to the greater, which is impossible.
- therefore AB is Not unequal to DE, and is therefore Equal to it IE AB = DE
- But BC is Also equal to EF; therefore the two sides AB, BC are equal to DE, EF respectively, and the Angle ABC is equal to DEF; therefore the base is equal the base IE AC = DF, and the remaining Angle BAC is equal to EDF [I.4]
- Again Let Sides, subtending equal Angles be equal as AB = DE
- I say Again, that the remaining sides will be equal to the remaining sides, namely AC = DF and BC = EF, and further the remaining Angles BAC = EDF.
- for if BC is unequal to EF, then one of them is Greater.
- Let BC be greater, if possible, and let BH be made equal to EF, Join AH.
- then, since BH is equal to EF, and AB to DE, the two Sides AB, BH are equal to the two sides DE, EF respectively, and they contain equal Angles; therefore the base AH equals DF, and the triangle ABH is equal to ΔDEF and the remaining angles are equal to the remaining Angles [I.4]; therefore the Angle BHA = EFD
- But the Angle EFD is equal to BCA; therefore in triangle AHC, the exterior Angle BHA is equal to the interior and opposite Angle BCA: which isn't possible [I.16]
- therefore BC is not unequal to EF, and is therefore equal to it.
- But AB is Also Equal to DE; therefore the two sides AB, BC are equal to DE, EF respectively and they Contain equal Angles; therefore base AC = DF, the triangle ABC is equal to triangle DEF, and the remaining Angle BAC is equal to the remaining Angle EDF [I.4]

Q.E.D.

Proposition 27

If a straight line falling on two straight lines make the Alternate angles equal to one another, the Straight lines Are parallel to eachother.



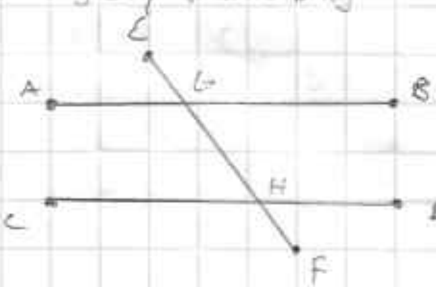

- for let the straight line EF, falling on the two straight lines AB, CD make the Alternate Angles AEF, EFD equal to one another,
- I say that AB is parallel to CD.
- for if Not, AB, CD when produced will meet either in direction of B, D, or towards A, C.
- Let them be Produced to meet in the direction B, D at point G
- then in triangle GEF, the exterior angle AEF is equal to the interior and opposite Angle EFG, which is Impossible [I.16]
- therefore AB, CD when produced will not meet in the Direction of B, D.
- Similarly it can be proved that neither will they meet towards A, C.
- But Straight lines that do not meet in either Direction Are parallel [Definition. 23]

therefore AB is Parallel to CD

Q.E.D.

Proposition 28

If a straight line falling on two straight lines makes the exterior Angle equal to the Interior and opposite Angle on the same side, or the the interior Angles on the same side equal to two right Angles, the straight lines will be parallel to one another

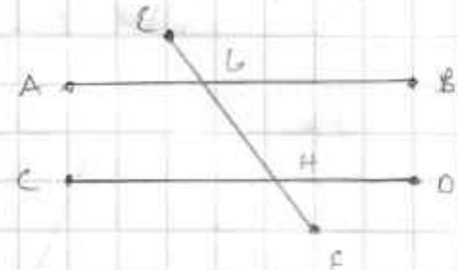



- Let A straight line EF fall on two straight lines AB, CD, making the exterior Angle EGB equal to the interior and opposite Angle GHD, or the interior Angles on the same side, namely BGH, GHD equal to two right Angles.
- I say that AB is Parallel to CD.
- for since the Angle EGB is equal to the Angle GHD, while the Angle EGB is equal to Angle AGH [I.15]
- and Angle AGH is Also equal to GHD; and they are Alternate; therefore AB is Parallel to CD [I.27]
- Again, since the Angles BGH, GHD are equal to two right Angles and the Angles AGH, BGH are Also Equal to two right Angles [I.13]
- the Angles AGH, BGH are equal to the Angles BGH, GHD.
- Let the Angle BGH be subtracted from each.
- therefore the remaining Angles AGH equals GHD, and they Are Alternate
- therefore AB is Parallel to CD [I.27]

Q.E.D.

Proposition 29

A straight line falling on parallel straight lines makes the Alternate Angles equal to one another, the exterior Angle equal to the interior and opposite Angle, and the Interior Angles on the same side equal to two right Angles



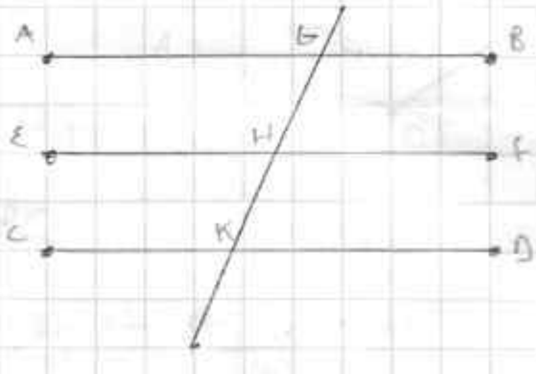

- let the line EF fall on the parallel straight lines AB, CD;
- I say that it makes the Alternate Angles AGH, GHD equal; the exterior Angle EGB equal to the interior and opposite Angle GHD, and the Interior Angles on the same side, namely BGH, GHD equal to two right Angles
- for, if the Angle AGH is unequal to the Angle GHD, the one of them is bigger/greater
- let AGH be greater,
- Let the Angle BGH be added to each;
- therefore the Angles AGH, BGH are greater than BGH, GHD.
- But the Angles AGH, BGH are equal to two right Angles $\boxed{I.13}$
- therefore the Angles BGH, GHD are less than two right Angles.
- But straight lines produced indefinitely from Angles less than 2 right Angles meet; $\boxed{Post. 5}$
- therefore AB, CD if produced indefinitely will meet;
- but they do not meet, because by Hypothesis they Are Parallel
- therefore the Angle AGH is not unequal to GHD, but is equal to it.
- Again Angles AGH equals EGB $\boxed{I.15}$
- therefore the Angle EGB is Also equal to GHD $\boxed{C.N. 1}$
- Let the Angle BGH be Added to each; therefore the Angles EGB, BGH are equal to the Angles BGH, GHD $\boxed{C.N. 2}$
- But the Angles EGB, BGH are equal to two right Angles $\boxed{I.13}$
  therefore the Angles BGH, GHD are Also equal to two right Angles

$\boxed{\text{Q.E.D.}}$

Proposition 30

Straight lines parallel to the same straight line are Also parallel to eachother.

- Let each of the straight lines AB, CD be parallel to EF; —
- I say that AB is Also parallel to CD.
- for let the straight line GK fall upon them,



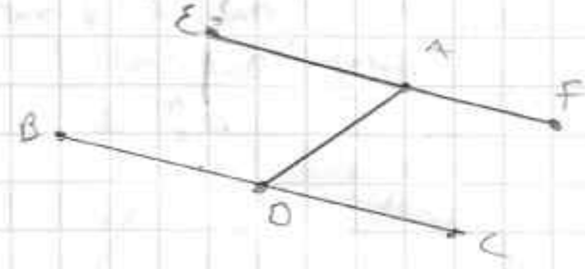

- then, since the straight line GK has fallen on the parallel straight lines AB, EF, the Angle AGK is equal to GHF $\boxed{\text{I.29}}$
- Again, since the straight line GK has fallen on the parallel straight lines EF, CD, the Angle GHF is equal to GKD $\boxed{\text{I.29}}$
- But, the Angle AGK was Also proved equal to GHF; therefore the Angle AGK is equal to GKD $\boxed{\text{C.N.1}}$ and they Are Alternate

therefore AB is parallel to CD $\boxed{\text{I.27}}$

Q.E.D.

Proposition 31

through a given point to draw a straight line parallel to a given straight line

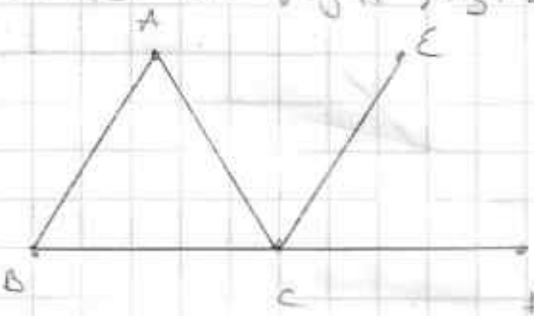



- Let A be the given point, and BC be the given straight line
- It is required to draw through point A, A straight line parallel to BC
- Let point D be taken at random on BC
- Let AD be Joined
- on the straight line DA, and at the point A, let the Angle DAE be constructed equal to Angle ADC [I.23]
and let the straight line AF be produced in A straight line with EA
- then, since the straight line AD falling on two straight lines BC, EF has made the Alternate Angles EAD, ADC equal to one another, therefore EAF is parallel to BC [I.27]
  - therefore, through A given point A, the straight line EAF Has been drawn parallel to BC

Q.E.D.

Proposition 32

In any triangle, if one of the sides be produced, the exterior Angle is equal two the two interior and opposite Angles, and the three interior Angles of the triangle are equal to two right Angles.

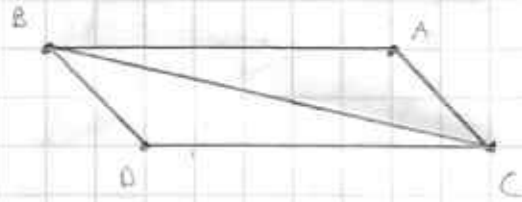



- Let ABC be a triangle, and let one side of it BC, be produced to D;
- I say that the exterior Angle ACD is equal to the two interior and opposite Angles CAB, ABC, and that the three interior angles of the triangle IE. ABC, BCA and CAB are equal to two right Angles
- for let CE be drawn through the point C parallel to AB. $\boxed{I, 31}$
- then since AB is Parallel to CE, and AC has fallen upon them, the Alternate angles BAC equals ACE $\boxed{I, 29}$
- Again, since AB is parallel to CE, and BD has fallen upon them, the exterior Angle ECD is equal to ABC $\boxed{I, 29}$
- But the Angle ACE was also proved equal to the Angle BAC, therefore the whole Angle ACD is equal to the two interior & opposite Angles BAC, ABC.
- Let the Angle ACB be Added to each;
- therefore the Angles ACD, ACB are equal to the three Angles ABC, BCA, CAB
- But, the Angles ACD, ACB are equal to two right Angles $\boxed{I, 13}$
- therefore Angles ABC, BCA, CAB are equal to two right Angles

$\boxed{Q.E.D.}$

Proposition 33

the straight lines Joining equal and parallel straight lines (At the extremities) in the same directions respectively, are themselves also equal And parallel




- Let AB, CD be equal And parallel and let the straight lines AC, BD Join them, at the exteremities and in the same direction respectively.
- I say that AC, BD are Also equal And Parallel!
- Let BC be Joined
- then since AB is parallel to CD, and BC has fallen upon them, the Alternate Angles ABC, BCD are equal to one another. [I,29]
- And Since AB is equal to CD, and BC is common, then; AB = DC and BC = CB, and Angles ABC = BCD, and AC = BD and triangle ABC = DCB [I,4]
- therefore Angle ACB = Angle CBD
- and Since the straight line BC falls on AC, BD and has made Alternate Angles equal to each other, therefore AC is parallel to BD [I.27]

And it was Also proved equal to it

Q.E.D.

Proposition 34

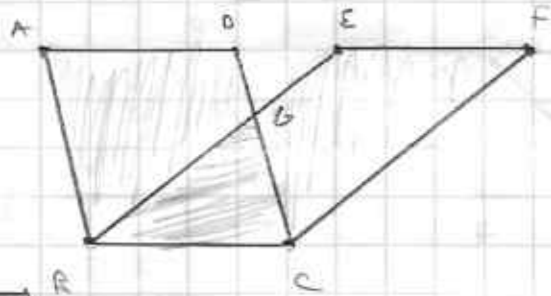
In parallelogrammatic areas the opposite sides and Angles are equal to one another, and the diameter bisects the Areas.


- Let ACDB be a parallelogrammatic Area and BC be its diameter;
- I say that the opposite sides, and Angles of parallelogram ACDB are equal to one another, and the diameter BC bisects it.
- for since AB is parallel to CD, and the straight line BC has fallen upon them, the Alternate angles ABC, BCD are equal [I.29]
- Again since AC is parallel to BD, and BC has fallen upon them, the Alternate Angles ACB, CBD are Equal [I.29]
- therefore ABC, DCB are two triangles having two Angles ABC, BCA equal to the two Angles DCB, CBD, respectively, and one side equal to one side, namely BC, which is common. the remaining sides and remaining Angles will be equal respectively [I.26]
- therefore AB = CD and AC = BD
- further the Angle BAC is equal to CDB, and since Angle ABC is equal to BCD, and Angle CBD to ACB the whole Angle ABD is equal to ACD [C.N.2]
- ~~therefore in parallelogrammatic areas~~
- And BAC was proved Equal to CDB
- therefore in parallelogrammatic Areas the opposite sides and Angles are equal to one another!
- I say, next, that the Diameter also bisects the Areas.
- for since AB is equal to CD, and BC is common, the two sides AB, BC are equal to DC, CB respectively, and ABC = BCD, and base AC = DB and triangle ABC = DCB [I.4]
- therefore BC Bisects the parallelogram ACDB

Q.E.D.

Proposition 35

Parallelograms which are on the same base, and in the same parallels are equal to one another!



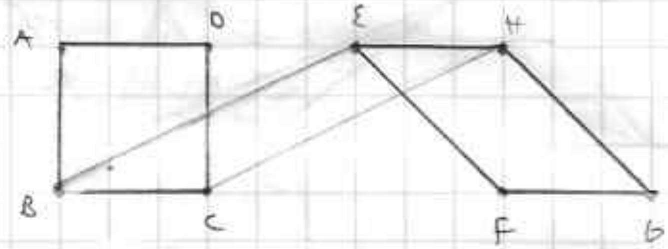

- Let ABCD, EBCF be parallelograms on the same base BC and in the same parallels AF, BC;
- I say that ABCD is equal to EBCF
- For since ABCD is a parallelogram, AD is equal to BC $\boxed{I.34}$
- for the the same reason EF equals BC $\boxed{I.34}$
- So AD is Also Equal to EF; $\boxed{C.N.1}$ and DE is common;
- therefore the whole AE is equal to the whole DF $\boxed{C.N.2}$
- But AB is Also equal to DC $\boxed{I.34}$
- therefore the two sides EA, AB are equal to the two sides FD, DC respectively and Angle FDC is equal to EAB, the exterior to the interior $\boxed{I.29}$
- therefore the base EB is equal to FC
- and the triangle EAB will be equal to triangle FDC $\boxed{I4}$
- Let DGE be subtracted from each; therefore the trapezium ABGD which remains is equal to the trapezium EGCF, which Also remains $\boxed{C.N.3}$
- Let triangle GBC be Added to each therefore the whole Parallelogram ABCD is equal to the whole Parallelogram EBCF

QED.

[Equality of Parallelograms and trapezium are seemingly talking About the Equality of the Area Described by the figures]

Proposition 36

Parallelograms which are on equal bases and in the same parallels are equal to one another.



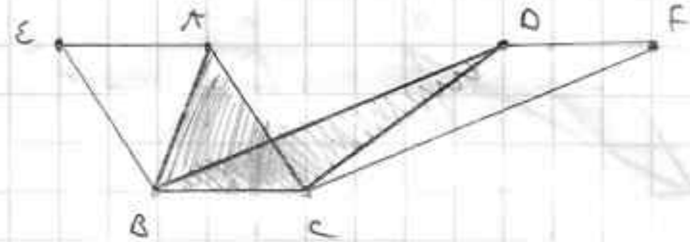

- Let ABCD, EFGH be parallelograms which are on equal bases BC, FG and in the same parallels AH, BG
- I say that parallelogram ABCD is equal to parallelogram EFGH.
- for, let BE, CH be joined
- Then since BC is equal to FG, while FG is equal to EH, then BC is also equal to EH $\boxed{C.N1}$ Hypothetical Syllogism.
- But they are also parallel, and EB, HC join them,
- but straight lines joining ~~are~~ equal and parallel straight lines (At the extremities) in the same direction respectively are also equal and parallel $\boxed{I.33}$
- therefore EBCH is a parallelogram and it is equal to ABCD, for it has the same base BC, and is in the same parallels BC, AH $\boxed{I.35}$
- for the same reason EFGH is also equal to EBCH $\boxed{I.35}$

therefore the parallelogram ABCD is also equal to parallelogram EFGH $\boxed{C.N.1}$

$\boxed{Q.E.D.}$

Proposition 37

Triangles which are on the same base, and in the same parallels are equal to each other.




- Let ABC, DBC be triangles on the same base BC, and in the same parallels AD, BC
- I say that the triangle ABC is equal to the triangle DBC,
- Let AD be produced in both directions to E and F
- through B let BE be drawn parallel to CA [I.31]
- through C let CF be drawn parallel to BD [I.31]
- then each of the figures EBCA, DBCF is a parallelogram; and they are Equal for they are on the same base BC and same parallels BC, EF [I.35]
- Moreover, the triangle ABC is half of the parallelogram DBCF; for the diameter AB bisects it [I.34]
- And triangle DBC is half of the parallelogram DBCF for Diameter DC bisects it [I.34]

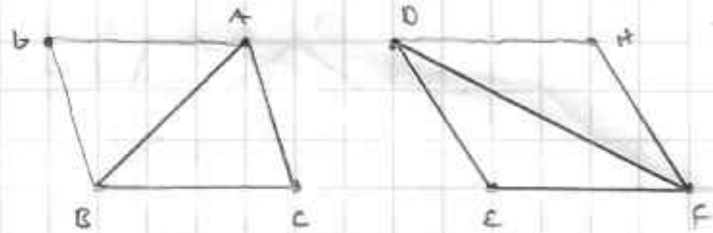
  but the halves of equal things are also equal to each other [C.N1?]

  therefore triangle ABC is equal to triangle DBC

QED.

Proposition 38

Triangles which are on equal bases and in the same parallels are equal to one another




- Let ABC, DEF be triangles on equal bases BC, EF and in the same parallels BC, AD;
- I say that the triangle ABC is equal to triangle DEF
- for let AD be produced in both directions to G, H;
- through B let be drawn BG parallel to CA (I.31)
- And through F let FH be drawn parallel to DE (I.31)
- then each of the figures GBCA, DEFH is a parallelogram; and GBCA is equal to DEFH; for they are on equal bases BC, EF, and they are in the same parallels BF, GH (I.36)
- Moreover triangle ABC is half parallelogram GBCA, for the diameter AB bisects it (I.34)
- And the triangle FED is half of the parallelogram DEFH, for the diameter DF bisects it

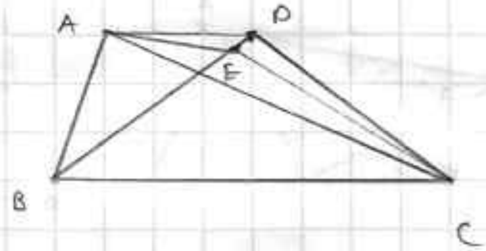
But the halves of equal things are equal (C.N.1.)

therefore triangle ABC is equal to triangle DEF

$\boxed{\text{Q.E.D.}}$

Proposition 39

Equal triangles which are on the same base and on the same side are Also in the same parallels.

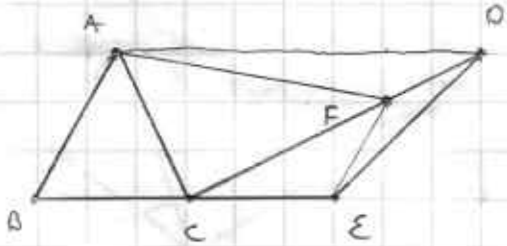



- Let ABC, DBC be equal triangles which are on the same base BC and on the same side of it.
- I say that these triangles are Also on the same parallels
- Let AD Be Joined
- I Say that AD is Parallel to BC
- for if it is not, let AE be drawn through the point A parallel to the straight line BC $\boxed{I.31}$ (Impossible) and let EC be Joined
- therefore the triangle ABC is equal to EBC, for it is on the same base BC with it, and in the same parallels $\boxed{I.37}$
- But ABC is equal to DBC; therefore DBC is Also equal to EBC $\boxed{CN1}$ the greater to the less, which is Impossible!
  - therefore AE is Not Parallel to BC
  - Similarly we can prove that neither is any other Straight line except AD; therefore AD is Parallel to BC

$\boxed{Q.E.D}$

Proposition 40

Equal triangles which are on equal bases and on the same side are also in the same parallels,

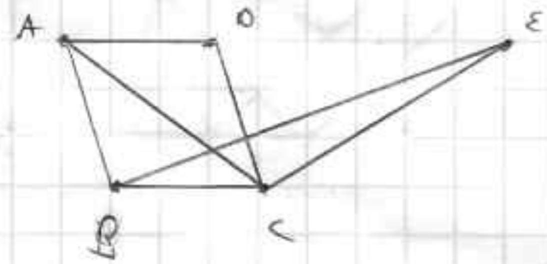



- Let ABC, CDE be equal triangles on equal bases BC, CE and on the same side;
- Let AD be joined
- I say that they are on the same parallels, and that AD is parallel to BE;
- for if AD is not parallel to BE, let AF be drawn through A parallel to BE $\boxed{I.31}$
  - And let FE be joined
- therefore the triangle ABC is equal to triangle FCE for they would be on equal bases BC, CE and in the same parallels BE, AF. $\boxed{I.38}$
- But the triangle ABC is equal to the triangle DCE; therefore the triangle DCE would be equal to triangle FCE $\boxed{C.N.1}$
  - the greater to the less, which is impossible
  therefore AF is not parallel to BE
  similarly we can prove that neither is any other straight line, except AD;
  therefore AD is parallel to BE

$\boxed{Q.E.D.}$

Proposition 41

If a parallelogram have the same base with a triangle, and be in the same parallels, the parallelogram is double the triangle!



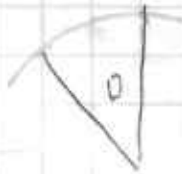

- Let the parallelogram ABCD have the same base BC with the triangle EBC, let it be in the same parallels BC, AE;
- I say that parallelogram ABCD is double of the triangle BEC;
- Let AC be joined
- then triangle ABC is equal to the triangle EBC; for it is on the same base with BC and in the same parallels BC, AE $\boxed{I.37}$
- But the parallelogram ABCD is double of the triangle ABC, for the diameter AC bisects it $\boxed{I.34}$

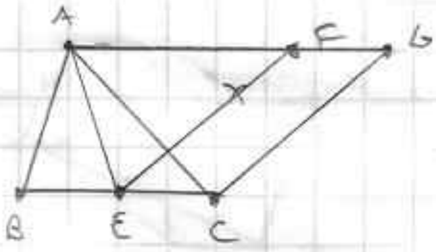
So that the parallelogram ABCD is also double to EBC $\boxed{CN1}$

$\boxed{Q.E.D.}$

Proposition 42

To construct, in a given rectilineal Angle, a parallelogram equal to a given triangle



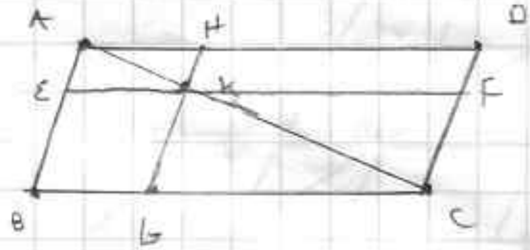




- Let ABC be the given triangle and D be the given Angle, thus
- it is required to construct in the rectilineal Angle D a parallelogram equal to triangle ABC
- Let BC be bisected At E
- AND let AE be joined;
- On the straight line EC, and at point E on it, let Angle CEF be constructed Equal to Angle D [I.23]
- through A let AG be drawn parallel to EC [I.31]
  through C let CG be drawn parallel to EF [I.31]
- then FECG is A parallelogram
- and since BE is equal to EC, ~~and~~ the triangle ABE is Also equal to the triangle AEC, for they are on equal bases BE, EC and in the same parallels BC, AG [I.38]
- therefore the triangle ABC is double triangle AEC
  But the parallelogram FECG is Also double the triangle AEC for it has the same base and is in the same parallels with it [I.41]
  - therefore the parallelogram FECG is equal to the triangle ABC
  - And it has the Angle CEF equal to Angle D
    - therefore the parallelogram FECG Has been Constructed equal to the given triangle ABC, with the Angle CEF which is equal to D

$\boxed{\text{QED.}}$

Proposition 43

In Any Parallelogram the compliments of the parallelograms About the diameter Are equal to one another.

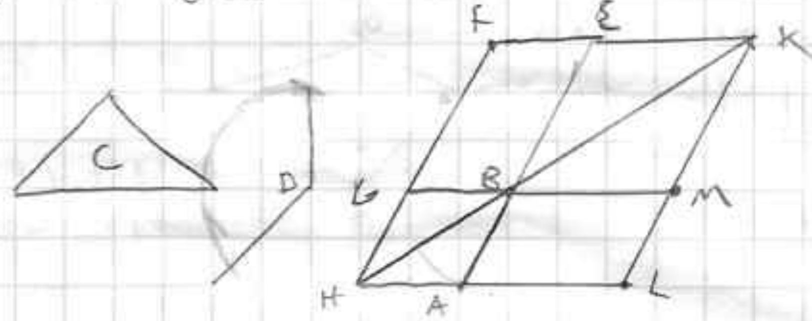



- Let ABCD be a parallelogram, and AC its diameter,
- And About AC let EH, FG be Parallelograms
- And BK, KD the so called compliments
- I say that the compliment BK is equal to compliment KD
- for since ABCD is a parallelogram, and AC its diameter then the triangle ABC is Equal to triangle ACD [I.34]
- Again, since EH is a parallelogram and AK its diameter the triangle AEK is equal to AHK [I.34]
- for the same reason KFC is equal to KGC [I.34]
- Now since the triangle AEK is equal to the triangle AHK and KFC to KGC the triangle AEK together with triangle KGC is equal to the triangle AHK together with KFC. [C.N.2]
- And the whole triangle ABC is equal to the whole ADC;
- therefore the compliments BK which remains is equal to the compliment KD, which Also remains [C.N.3]

Q.E.D.

Proposition 44

To a given straight line to apply in a given rectilineal Angle, a Parallelogram equal to a given triangle.



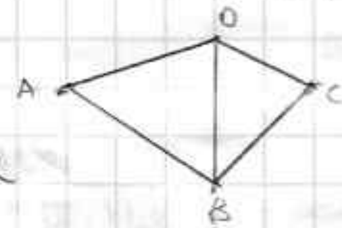

- Let AB be the given straight line, C the given triangle, and D the given rectilineal Angle;
- Thus it is required to Apply to the given straight line AB, in an Angle equal to D, A Parallelogram equal to the given triangle C.
- Let the Parallelogram BEFG, be constructed equal to triangle C, in the Angle EBG, which is equal to D. $\boxed{\text{I.42}}$
- Let it be placed so that BE is in a straight line with AB
- Let FG be drawn through to H, and let AH be drawn through A parallel to either BG or EF $\boxed{\text{I.31}}$
- Let HB be Joined,
- then since the straight line HF falls upon the Parallels AH, EF, the Angles AHF, HFE Are equal to two right Angles I.29
- therefore the Angles BHG, GFE are less than two right Angles; and straight lines produced indefinitely from Angles Less than 2 right Angles meet $\boxed{\text{Post 5}}$
- therefore HB, FE, when produced, will meet.
- Let them be produced and meet At K; through point K let KL be drawn parallel to either EA or FH, and let HA, GB be produced to the Points L, M
- then HLKF is a ~~pa~~ parallelogram, HK is its diameter, and AG, ME are Parallelograms, and LB, BF, the compliments Around HK.
- therefore LB is equal to BF $\boxed{\text{I.43}}$
- But BF is equal to the triangle C; therefore LB is Also equal to C $\boxed{\text{C.N.1}}$
- And since the Angle GBE is equal to Angle ABM $\boxed{\text{I.15}}$ while Angle GBE is equal to Angle D, and the Angle ABM is Also equal to D

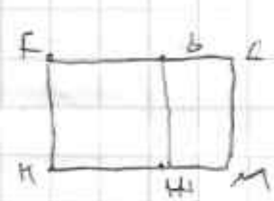
therefore the parallelogram LB equal to the given triangle C, has been Applied to the given straight line AB, in the Angle ABM, which is equal to D.

$\boxed{\text{Q.E.D.}}$

Proposition 45

To construct, in a given rectilineal Angle, A parallelogram equal to a given rectilineal figure.




- Let ABCD be the given rectilineal figure and E be the given Angle
- thus it is required to construct, in the Given Angle E, A Parallelogram equal to the rectilineal figure ABCD


- Let DB be joined, and let the parallelogram FH be constructed equal to the triangle ABD in the Angle HKF, which is equal to E [I.42]
- Let the parallelogram GM equal to the triangle DBC be Applied to the straight line GH, in the Angle GHM, which is equal to E [I.44]
- then since Angle E is equal to Angles HKF and GHM, ~~the~~ HKF = GHM [C.N.1]
- Let the Angle KHG be Added to each; therefore the Angles FKH, KHG are equal to the Angles KHG, GHM, but the Angles FKH, KHG are equal to two right Angles [I.29]
- therefore the Angles KHG, GHM are Also Equal to two right Angles [C.N.1]
- Thus with a straight line GH, At the point H on it, two straight lines KH, HM, not lying on the same side make adjacent Angles equal two right Angles;
- therefore KH is in a straight line with HM [I.14]

And, Since the straight line HG falls upon the parallels KM, FG, the Alternate Angles MHG, HGF are equal to one another [I.29]

- Let the Angle HGL be Added to each; therefore the Angles MHG, HGL are equal to the Angles HGF, HGL [C.N.2]
- But the Angles MHG, HGL are equal to two right Angles [I.29]
- therefore Angles HGF, HGL are Also equal to two right Angles [C.N.1]
- therefore FG is in a straight line with GL [I.14]
- And since FK is equal to HG [I.34] and HG to ML Also [I.34]
- KF is Also equal to and Parallel to ML [C.N.1] and [I.30]
- And the straight lines KM, FL Join them at their extremities, therefore KM, FL are also equal And parallel [I.33]
- therefore KFLM is a parallelogram

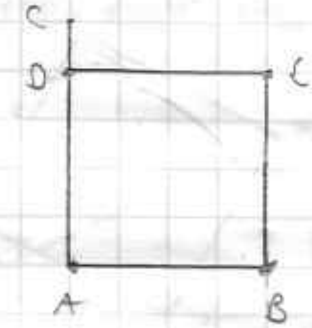
And, since the triangle ABD is equal to the parallelogram FH, and DBC to GM, the whole rectilineal figure ABCD is equal to the whole parallelogram KFLM [C.N1 CN2]

- therefore the Parallelogram KFLM Has been constructed equal to the given rectilineal figure ABCD, in the Angle FKM which is equal to the given Angle E

QED.

Proposition 46

On a given straight line to describe a square

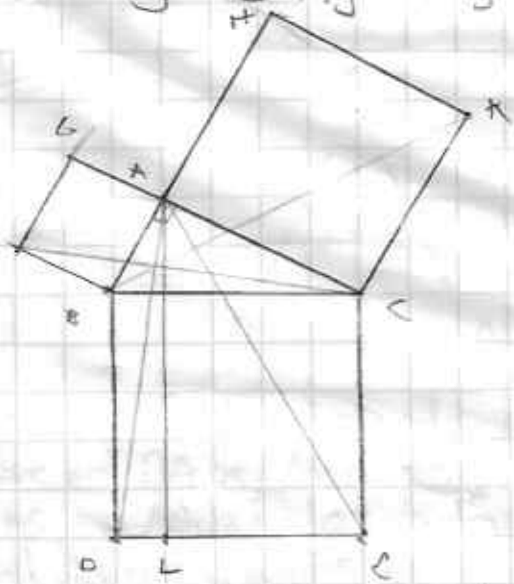



- Let AB be the given straight line;
- thus it is required to describe a square on straight line AB
- Let AC be drawn at right angles to the straight line AB from the point A on it [I.11]
- And let AD be made equal to AB;
- through the point D let DE be drawn parallel to AB [I.31]
- through point B let BE be drawn parallel to AD [I.31]
- therefore ADEB is a parallelogram; therefore AB is equal to DE, and AD to BE [I.34]
- But AB is equal to AD;
- therefore the four straight lines BA, AD, DE, EB are equal to each other; therefore the parallelogram ADEB is equalateral.
- I say next, that it is also right-angled.
- For, since the straight line AD falls upon the parallels AB, DE, the angles BAD, ADE are equal to two right angles [I.29]
- But the angle BAD is right; therefore the angle ADE is also right. And in parallelogrammic areas the opposite sides and angles are equal to one another [I.34]
- therefore each of the opposite angles ABE, BED are also right.
- therefore ADEB is right-angled, It was also proved equalateral.
- therefore, it is a square; and it is described on the straight line AB. [Def. 22]

Q.E.D.

Proposition 47 Pythagorean theorem

In right-angled triangles the square on the side subtending the right angle is equal to the squares on the sides containing the right angle

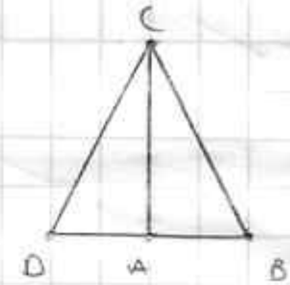



- Let ABC be a right-angled triangle having the angle BAC right;
- I say that the square on BC is equal to the squares on BA, AC,
- for let there be described on BC the square BDEC, and on BA, AC the squares GB, HC [I.46]
- through A, let AL be drawn parallel to either BD or CE, and let AD, FC be joined.
- then since each of the angles BAC, BAG is right, it follows that with a straight line BA, and at the point A on it, the two straight lines AC, AG, not lying on the same side make the adjacent angles equal to two right angles; therefore CA, is in a straight line with AG. [I.14]
- for the same reason BA is in a straight line with AH [I.14]
- And since the angle DBC is equal to the angle FBA; for each is right: let the angle ABC be added to each; therefore the whole angle DBA is equal to the whole angle FBC [C.N.2]
- And since DB is equal to BC, and FB to BA, the two sides AB, BD are equal to the two sides FB, BC respectively; and the angle ABD is equal to FBC; therefore the base AD is equal to the base FC, and the triangle ABD = FBC [I.4]
- Now the parallelogram BL is double the triangle ABD, for they have the same base BD and are in the same parallels BD, AL [I.41]
- And the square GB is double of the triangle FBC, for they again have the same base FB, and are in the same parallels FB, GC [I.41]
- But doubles of equals are also equal to one another!
- therefore the parallelogram BL is also equal to the square GB.
- Similarly, if AE, BK be joined, the parallelogram CL can also be proved equal to the square HC;
- therefore the whole square BDEC is equal to the two squares GB, HC [C.N.2]
- and the square BDEC is described on BC, and the squares GB, HC on BA, AC
  - therefore the square on side BC is equal to the squares on the sides BA, AC!

QED

Proposition 48

If in a triangle the square on one of the sides be equal to the squares on the remaining two sides of the triangle, the Angle contained by the remaining two sides of the triangle, is right




- for in triangle ABC, let the square on one side BC be equal to the squares on the sides BA, AC;
- I say the Angle BAC is right.
- for let AD be drawn from point A at right Angles to the straight line AC, let AD be made equal to BA, and Let DC be joined.
- Since DA is equal to AB, and the square on DA is Also equal to the square on AB. Let the square on AC be Added to each;
therefore the squares on DA, AC are equal to the Squares on BA, AC;
But the square on DC is equal to the squares on DA, AC for the Angle DAC is right; $\boxed{\text{I.47}}$
and the square on BC is equal to the squares on BA, AC for this is the Hypothesis;
therefore the square on DC is equal to the square on BC.
- And, since DA is equal to AB, and AC is common, the two sides DA, AC are equal to the two sides BA, AC;
and the base DC is equal to the base BC;
- therefore the DAC is equal to the Angle BAC $\boxed{\text{I.8}}$
- But the Angle DAC is right;
- therefore the Angle BAC is Also right $\boxed{\text{C.N.1}}$

$\boxed{\text{Q.E.D.}}$

Book 1 of Euclid's elements

complete!