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Experiment Report

Computer Control of Mechanical Systems

(1) 1 DOF Plant Control

(2) 2 DOF Plant Control

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Introduction

To properly actuate a system (whether electrical or mechanical) and make it accurately meet the desired performance, controllers act crucial components. Controllers can be of different forms. However, as the development of computers achieves high, control theories can nowadays be simply realized by programming in computers.

This report will incorporate two main parts, 1-DOF and 2-DOF system computer/digital control. In the very beginning, system identification can be done by doing a clever approach, and then the hardware gain can be derived simply by the output of the open-loop/closed-loop system with unit step signal. Secondly by applying control theories to the 1-DOF system, we are expecting to see the advantages and the limits of different control methods. Then, with the knowledge of controlling 1-DOF system, we can further apply it to 2-DOF systems and be able to deal with them.

Finally, by mastering notch filters, we can have a more holistic scope for handling the unwanted situation, in which multi-degree-of-freedom systems get a drop to zero in velocity.

Description of Experiment

In this report, we are going to show how we implement computer/digital control theories onto a torsional plant (Model 205 Torsional Plant) by going through open-loop/closed-loop system identification, proportional-derivative (PD) control, proportional-integral-derivative (PID) control, and feed-forward control.

By the steps below:

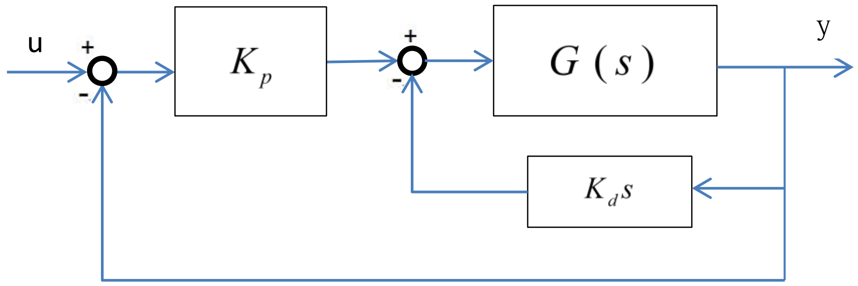
1) Make a proper assumption to the system and do the system identification by open-loop/closed-loop method.
2) Design PD and PID controllers to a 1-DOF system.
3) Apply feed-forward control theories.
4) Add notch filters into the control flow for a 2-DOF system.

We are able to do further analyses and step by step to see the influences casted to the plant being controlled so that we can compare the advantages and disadvantages of each control method.

Part A: Control of Rigid Body

I. System Model

For a computer/digital control system, the block diagram can be roughly represented as



Where

u is the input, counted in voltage (V)

y is the output, counted in position

For a permanent magnet DC motor, it follows

$$V(t) = Ri(t) + L\frac{di}{dt} + K_b\omega(t) \qquad (1)$$

$$T(t) = K_t i(t) \qquad (2)$$

Where K_b is the back electromotive force (EMF) voltage coefficient

R is the armature resistant

L is the armature inductance

ω is the angular speed

K_t is the torque coefficient

And without external loads, the motor also follows

$$J\frac{d\omega(t)}{dt} = T(t) - f\omega(t) \qquad (3)$$

Where

f is the bearing friction coefficient

By taking Laplace transform and making zero initial condition assumption for equation (1), (2), (3), combining them and making an arrangement, we get the transfer function to be

$$\frac{\omega(s)}{V(s)} = \frac{K_t}{(Ls+R)(Js+f)+K_tK_b} \qquad (5)$$

In general, $\frac{L}{R} \to 0$, so we can make a clever approach

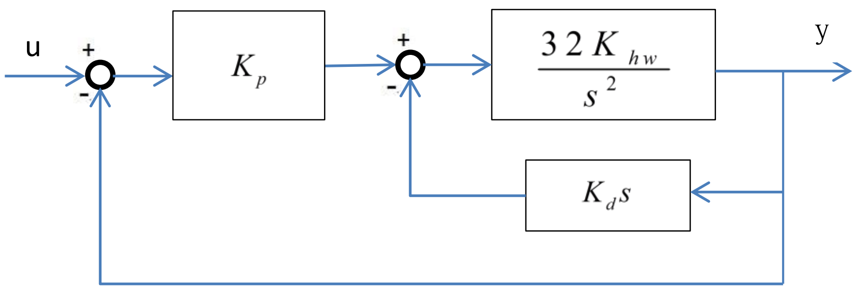
$$\frac{\omega(s)}{V(s)}=\frac{\frac{K_t}{Rf}}{[(\frac{L}{R})s+1][(\frac{J}{f})s+1]+\frac{K_tK_b}{Rf}}\cong\frac{\frac{K_t}{Rf}}{[(\frac{J}{f})s+1]+\frac{K_tK_b}{Rf}}=\frac{\frac{K_t}{(Rf+K_tK_b)}}{(\frac{RJ}{Rf+K_tK_b})s+1}=\frac{K}{\tau s+1} \tag{6}$$

Where

$$\tau=\frac{RJ}{Rf+K_tK_b}$$

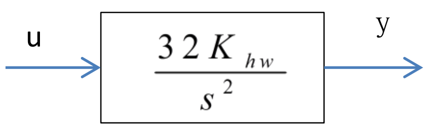
$$K=\frac{K_t}{(Rf+K_tK_b)}$$

Since the output y is counted in position and the input u in voltage, the transfer function $G(s)=\frac{\theta(s)}{V(s)}=\frac{K}{s(\tau s+1)}$. Fortunately, there is a powerful approach that $G(s)=\frac{K}{s^2}$, for $\tau=\frac{RJ}{Rf+K_tK_b}>>1$. On the other hand, in this experiment K is known as $32K_{hw}$ (K_{hw} is the hardware gain) so that the overall system with PD control can be built as:



II. System Identification

i. Open Loop



Since the open loop transfer function $G(s)=\frac{y(s)}{u(s)}=\frac{32K_{hw}}{s^2}$,

$\frac{d^2y(t)}{dt^2}=32K_{hw}u(t)$ by zero initial condition assumptions. Then, with the input $u(t) = 1^{(V)}$ the hardware gain K_{hw} can be easily got by $\frac{1}{32}$ times the acceleration $\frac{d^2y(t)}{dt^2}$, which is the slope of the velocity-time plot (Figure 1). By having $K_{hw} = \frac{1}{32}\frac{d^2y(t)}{dt^2}$, the hardware gain roughly comes to be 43.

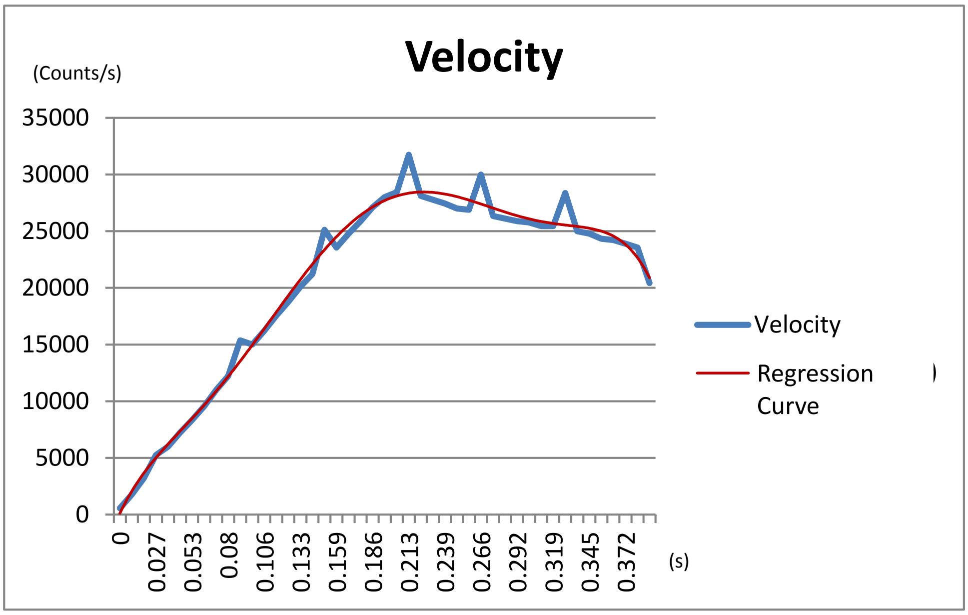
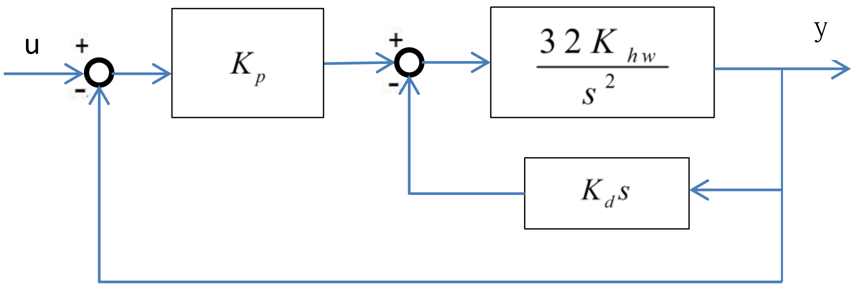


Figure 1

ii. Closed Loop



For a closed loop system above, the closed loop transfer function from u to y can be derived as $T(s) = \frac{y(s)}{u(s)} = \frac{32K_{hw}K_p}{s^2 + 32K_{hw}K_d s + 32K_{hw}K_p}$, which can be represented as $\frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$. By temporarily picking

$K_p = 0.1$, Figure 2 shows the response.

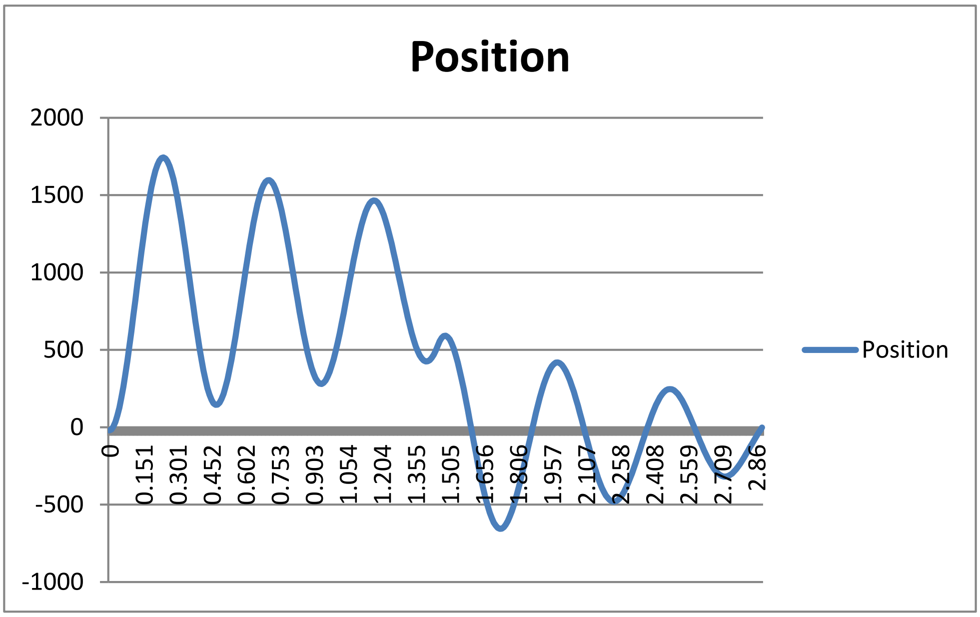


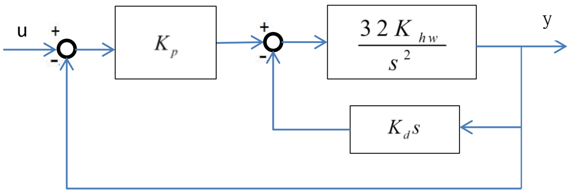
Figure 2

From Figure 2, the period T is read to be 0.465$^{(s)}$ so that $\omega_n = \frac{2\pi}{T} = \frac{2\pi}{0.465} = 13.5^{(rad/s)}$. Then the hardware gain K_{hw} can be got by the calculation of $\frac{\omega_n^2}{32K_p}$, by which $K_{hw} \cong 57$.

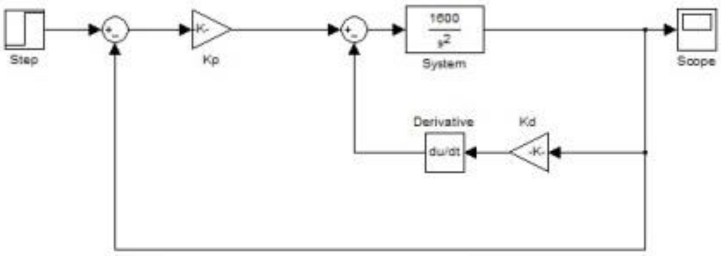
So, we can get the proper hardware gain K_{hw} by system verification to be approximately $K_{hw} \cong \frac{43+57}{2} = 50$.

III. Results

i. PD Control with Desired Natural Frequency of 4 Hz

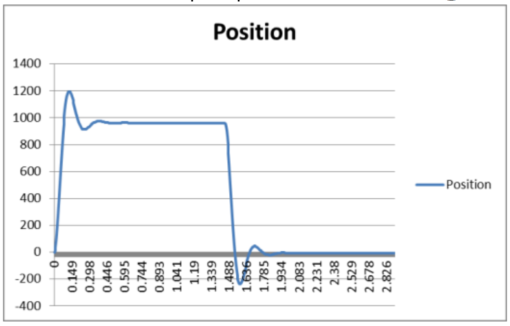


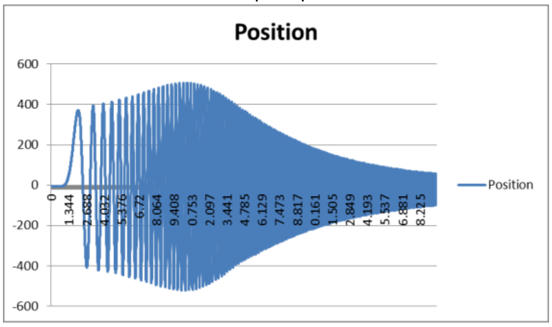
System Block Diagram



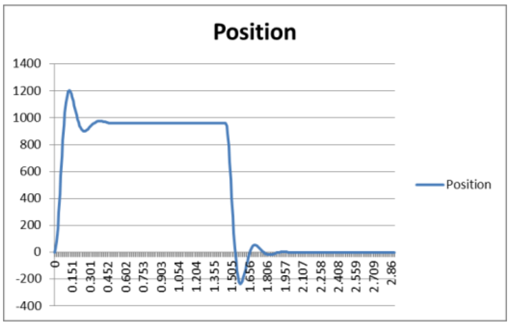
Simulation Block Diagram

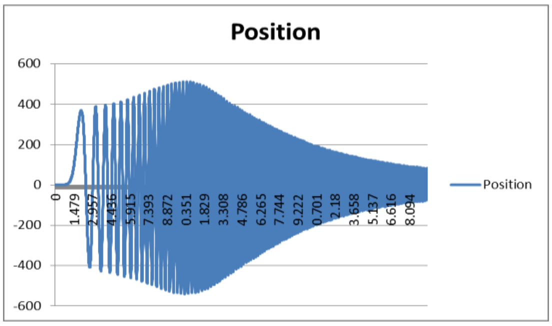
Step Response $\zeta = 0.4$ Sinesweep Responses



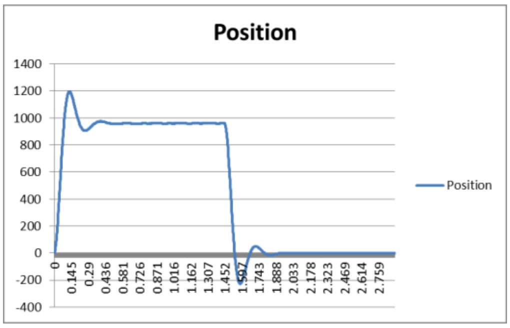


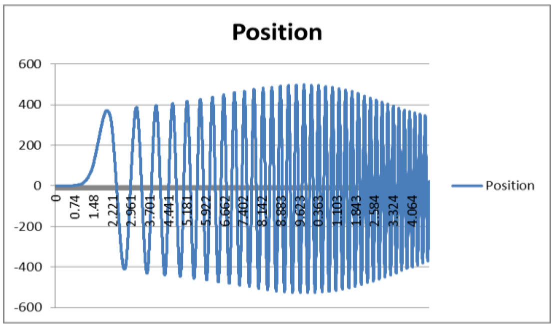
Ts=0.002652 Kp= 0.396127 Kd=0.012615508





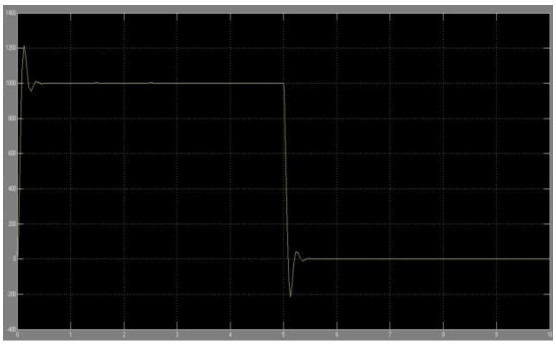
Ts=0.00442





Ts=0.000884

Figure 3



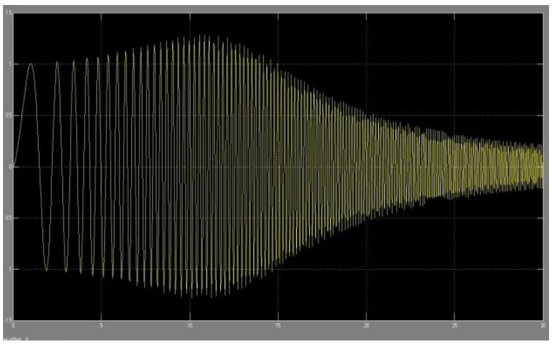
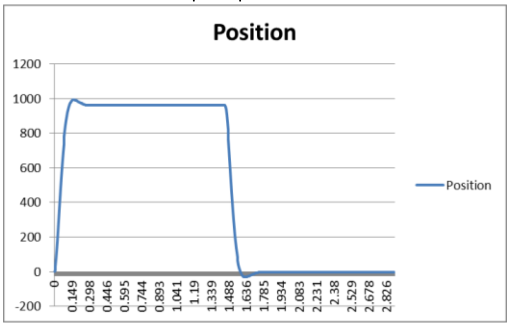
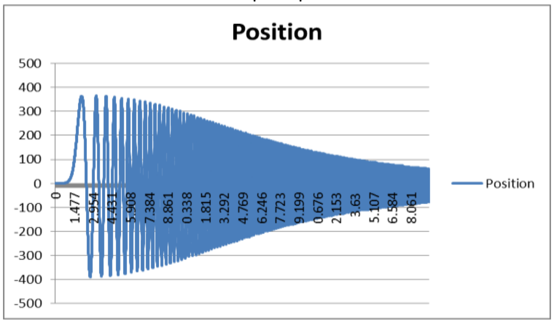


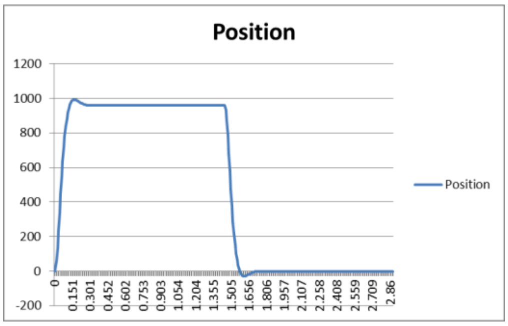
Figure 4 Simulation for ζ=0.4

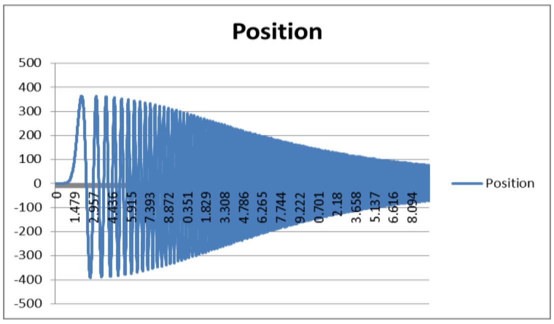
Step Response $\zeta = 0.7$ Sinesweep Responses



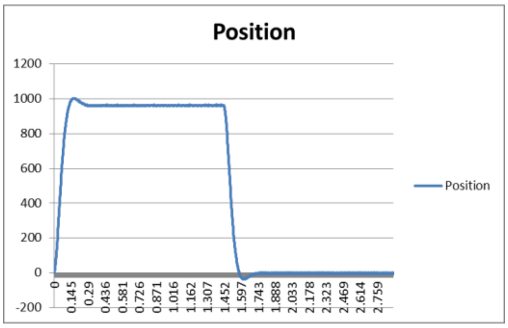


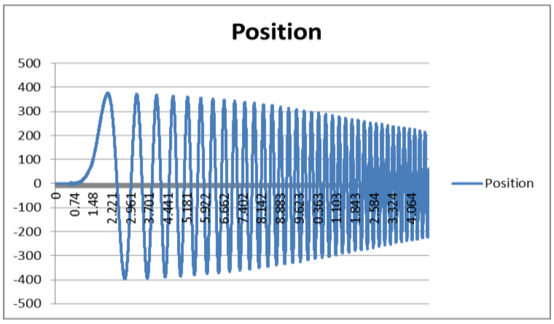
Ts=0.002652 Kp=0.396126959 Kd=0.022077139



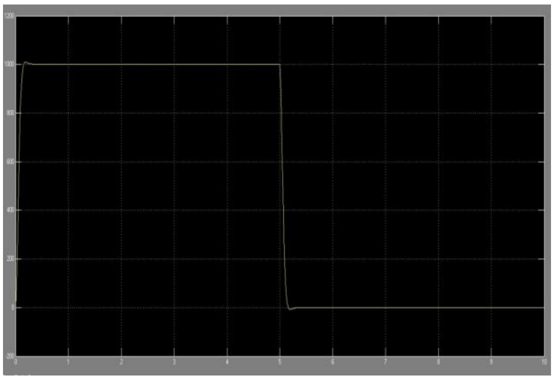


Ts=0.00442





Ts=0.000884



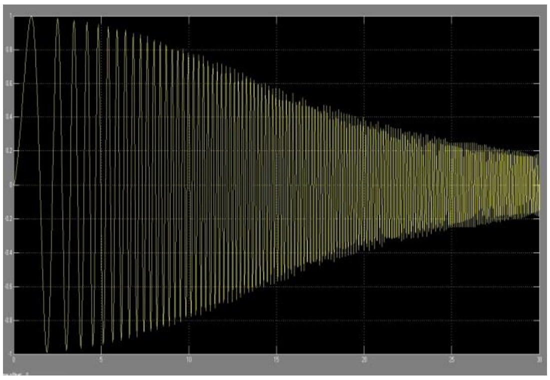
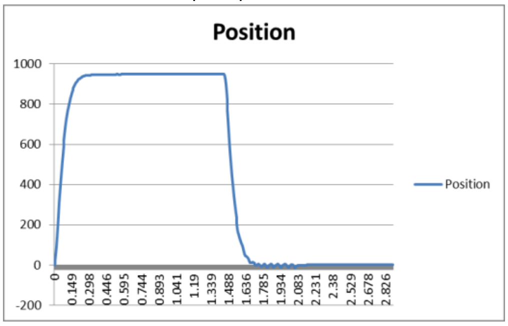
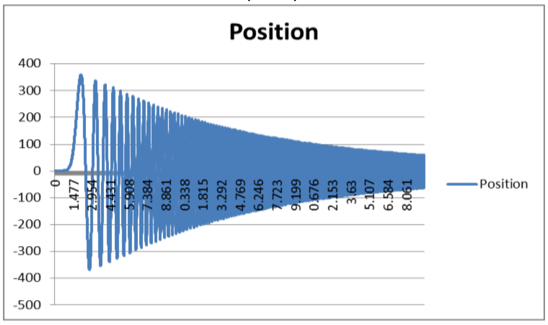


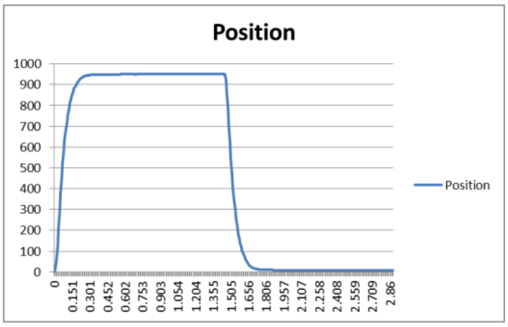
Figure 5 Simulation for ζ=0.7

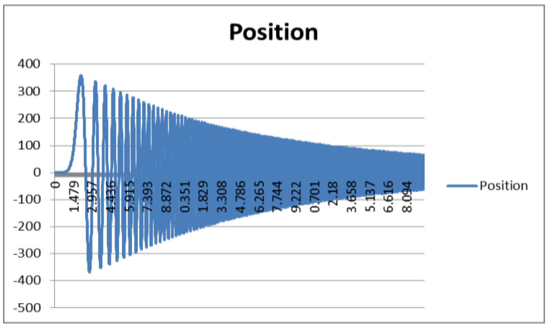
Step Response $\zeta = 1.0$ Sinesweep Responses



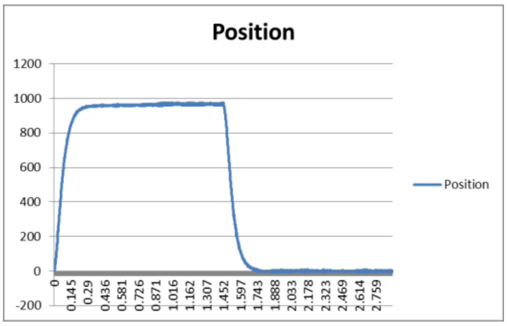


Ts=0.002652 Kp=0.396126959 Kd=0.031538771

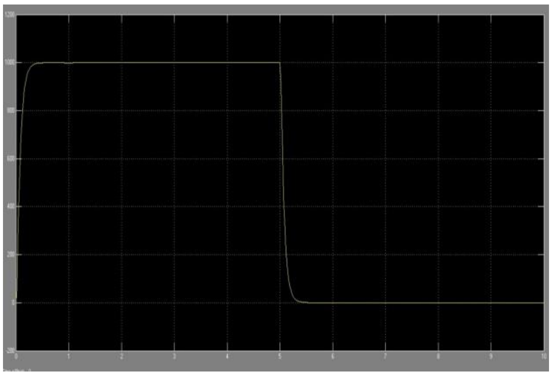




Ts=0.00442



Ts=0.000884



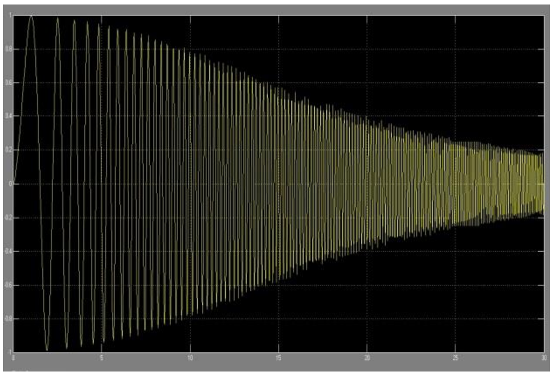
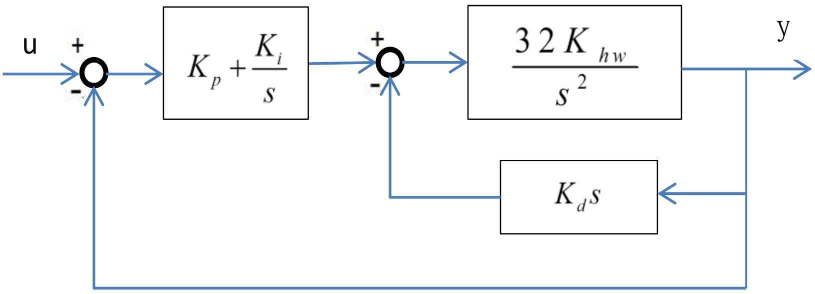


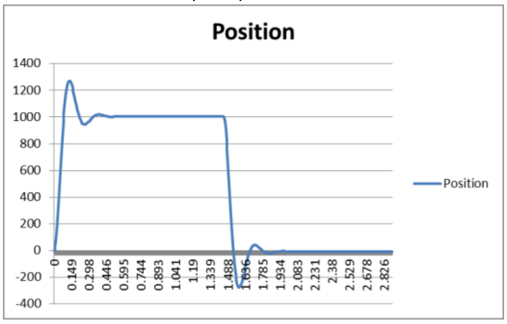
Figure 6 Simulation for ζ=1.0

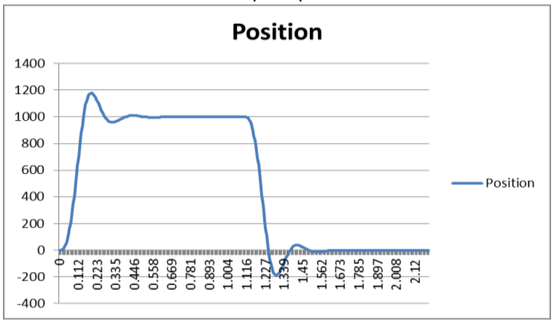
ii. PID Control



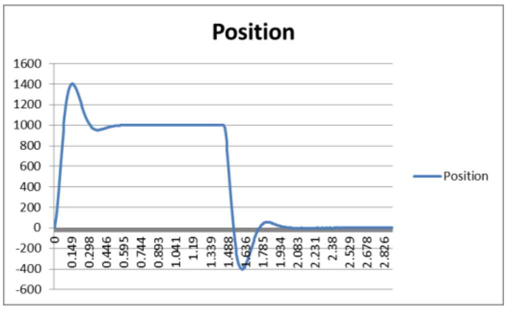
Step Response

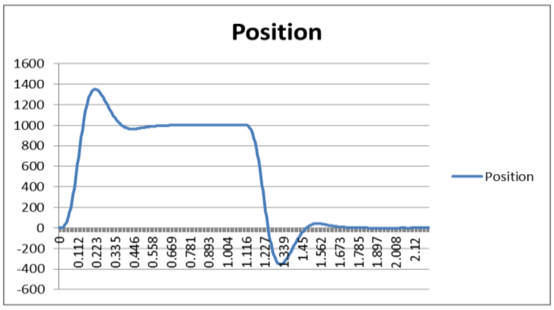
Sinesweep Responses





Ki=0.001

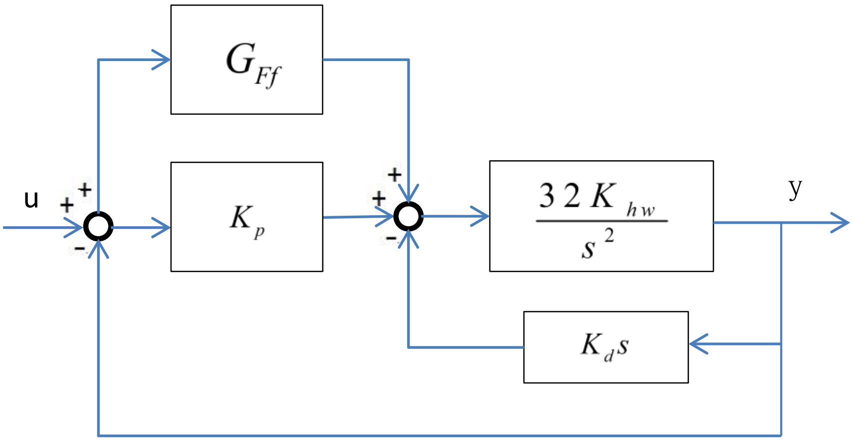




Ki=0.01

Figure 7

iii. Feed-forward control



To do perfect feed-forward control, the transfer function $G_{Ff}(s)$ can be chosen as $\frac{1}{G'(s)}$ (Figure 6), and the overall transfer function becomes $G(s)=\frac{y(s)}{u(s)}=1$.

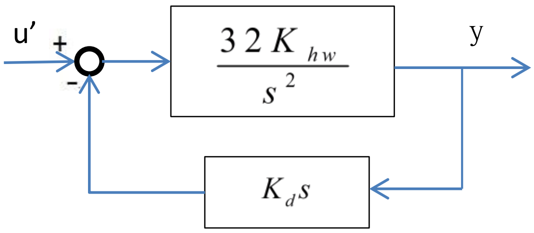


Figure 8 Block Diagram of G’(s)

Where $$G'(s)=\frac{y(s)}{u'(s)}=\frac{32K_{hw}}{s^2+32K_{hw}K_d s}$$

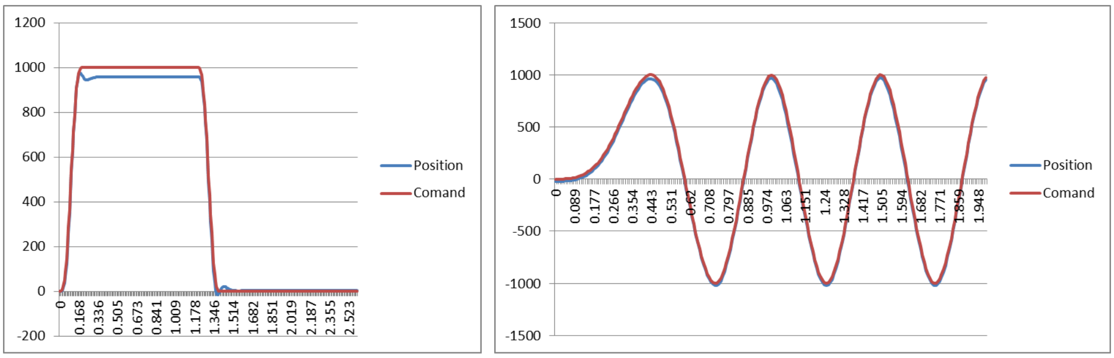
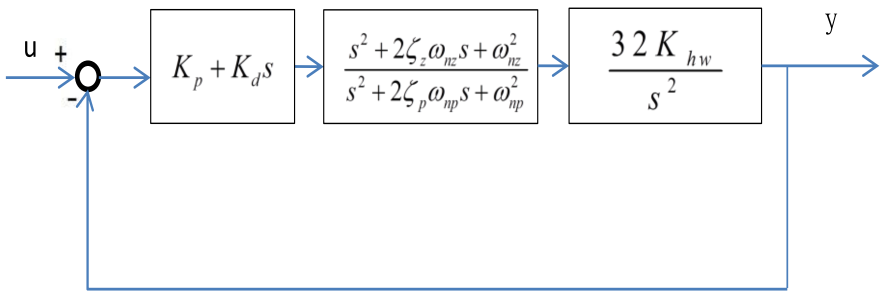


Figure 9

Part B: 2-DOF System (Control of Flexible System)



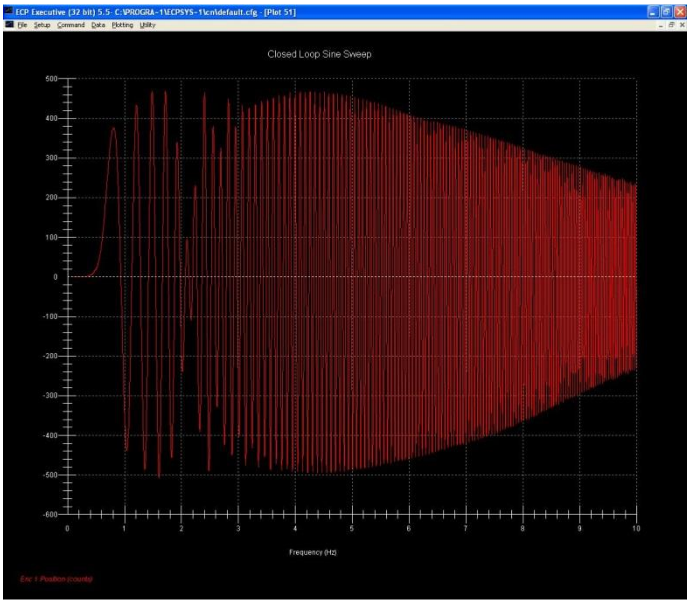


Figure 10 System without Notch Filter Control

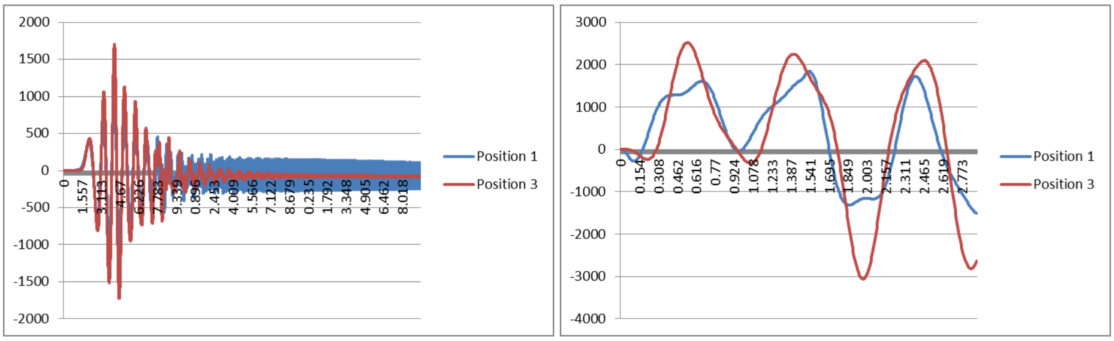


Figure 11 System with Notch Filter Control

Discussion & Conclusions

- PD/PID Control

For a second-order system with transfer function $G(s)$ ($G(s)=\frac{32K_{hw}}{s^2}$ here), PD control method can be thought of as adding damping factors to the system. It can be easily seen (Figure 3 & 4) that the derivative gain Kd influences the damping ratio ζ ($\zeta = \frac{32K_{hw}K_d}{2\omega_n} \propto K_d$) and further the unforced and forced responses. In the unforced mode, the overshoot can be decreasing with the damping ratio going high, and on the other hand, the amplitude decay under sinesweep input can be going quicker with larger damping ratio.

This is reasonable. For dampers can be thought of as energy dissipaters and damping ratio is proportional to velocity, the high frequency vibration goes harder to perform with the same ζ, and the vibration with the same frequency goes down in amplitude with higher ζ.

Otherwise, If adding an integrator to a feedback system, G(0) would be infinite. That implies a rather charming result that the steady state error to a step input can be zero, and apparently from the experiment result, the time needed to get to the steady state can be shorter. However, using integrators in control flow may cause higher overshoot (Figure 7).

- Effect of Sampling Time

While implementing control theories by computers, discretization is introduced. By sampling the input signal at discretized time point, discretized signal can only be an approach to continuous signal. Thus, the sampling time (Ts) can be crucial to make the digital and analog signal meet. When Ts goes bigger, the discretized signal may not be fitting the analog signal anymore. This can cast huge error to the output and make it totally deviate from the input. However, it’s not obviously shown in the experiment results since for the inputs we use in the experiment are rather traceable in the Ts period.

On the other hand, by shortening the sampling time to a limit in order to greatly fit the discretized input to the continuous one, this way may otherwise cause high frequency vibration to the mechanical system (i.e. the system will be buzzing), and this effect will cast damages.

- Feed-forward Control

Amongst lots of control theories, Feed-forward control acts the most ideal one, theoretically. By adding a feed-forward controller (with transfer function $G_{Ff}(s)$ equal to the reciprocal of the whole system to be controlled) to the system, we can get a perfect overall transfer function from input to output $G(s) = \frac{y(s)}{u(s)} = 1$. Nevertheless, this ideal situation can be hardly seen in real life. Because of the inertia of mechanical systems, error more or less exists. As Figure 9 shows, the error would be bigger especially when undergoing cubic inputs (such as step, square pulses, etc.), and the more the inertia is, the bigger the error is.

- Notch Filter

While controlling 2-DOF or higher degree systems, vibration brings problems. As what Figure 10 shows, there happens a total stop (or say pseudo-static state) to the system when the exerting force goes from low frequencies to high frequencies. This phenomenon really matters because it means the distal part falls totally out of control and further make the whole system goes unstable. By the method of notch filter, we can improve this uncertainty by replacing and adjusting the poles and zeros of the original system. After adding a notch filter to the system, the response (Figure 11) looks more like a rigid-body response than that in Figure 10. To this stage, the notch filter is still not well designed. However, the effects and the advantages of this method can be foreseen.