AME408: "Computer-Aid Design of Mechanical Systems"

Project #7 Due: November 21, 2013

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➢ Introduction

Finite element method (FEM) is a powerful numerical technique for finding approximate solutions to boundary value problems for differential equations. In this report, we are going to go through this method to examine the vibration modes, the corresponding natural frequencies, and the buckling condition occurring when parts are under different conditions and do a simple design.

➢ Method

Using Solidworks to realize finite element method (FEM) to do simulation for:

I. Task 1: Vibration Isolator

To find the first 4 non-rigid body natural frequencies and mode shapes of the symmetric alloy steel vibration isolator. To do this simulation, three boundary conditions are applied:

a) The two holes on the bottom are completely fixed.
b) The bottom surface is completely fixed.
c) The part is not fixed anywhere.

Frequency analysis is used, and solid mesh is applied. Mesh size spans from $1^{(in)}$ to $0.005^{(in)}$.

II. Task 2: Clamped-Clamped Beam

To find the followings of the clamped-clamped beam (whose length $L=4.7^{(mm)}$, height $H=0.132^{(mm)}$, width $W=1.02^{(mm)}$, $E=84.5^{(GPa)}$, $\rho = 2600^{(kg/m^3)}$, and $\upsilon = 0.3$):

a) Find the first 4 natural frequencies and illustrate the corresponding mode shapes.
b) Calculate the first 4 natural frequencies and the corresponding mode shapes under a tensile force $P=10^{(N)}$ actin on the beam's right support.
c) Calculate the lowest critical buckling force P_{cr}, and compare the results with the closed form solution.
d) Calculate the fundamental natural frequency under a compressive force $P=0.95\ P_{cr}$.

➢ Result & Discussion

I. Task 1

Under different boundary conditions (Figure 1), we can get different modes of vibration and corresponding natural frequencies. If the bottom holes are fixed, the first 4 natural frequencies are about $87^{(Hz)}$, $200^{(Hz)}$, $370^{(Hz)}$, and $434^{(Hz)}$ (Table 1 & 2). The corresponding vibration modes are 1) wobble back and forth, 2) wobble left and right, 3) the upper part moves up and down as a rigid body from the bottom, and 4) the upper part twists left and right (Figure 2).

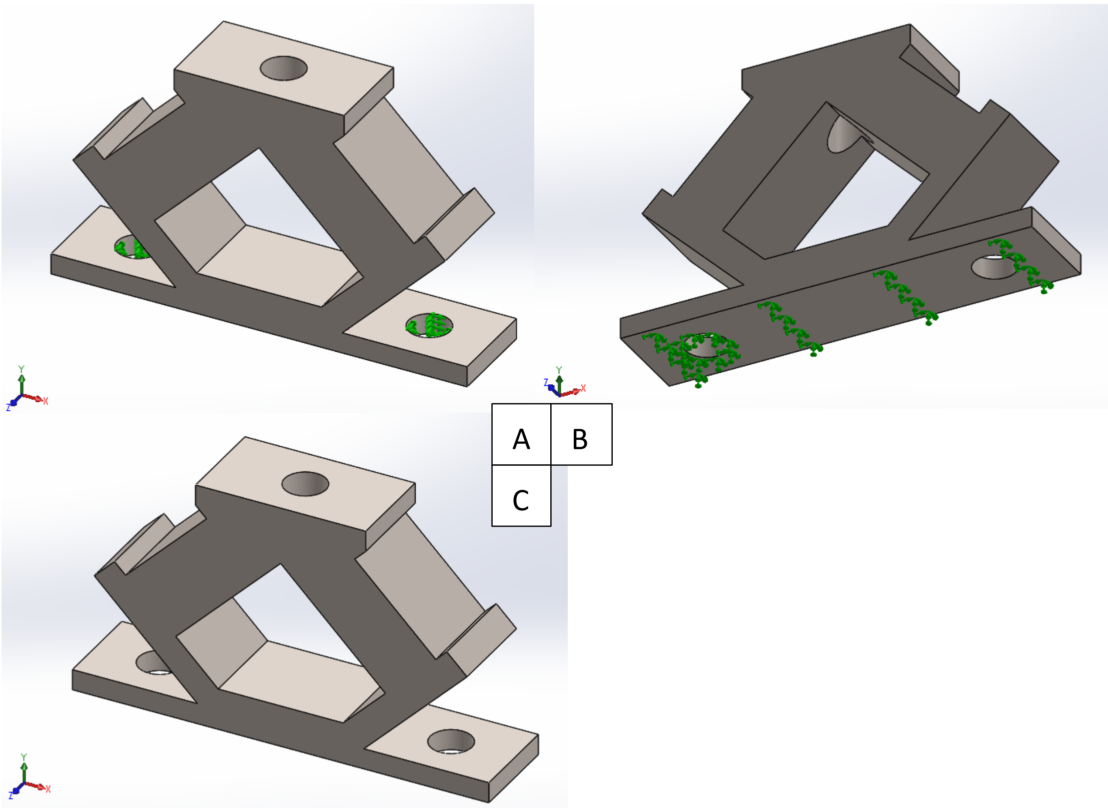

Figure 1 Loads & Constraints (A: Bottom Holes Fixed; B: Bottom Surface Fixed; C: Free Fixture)

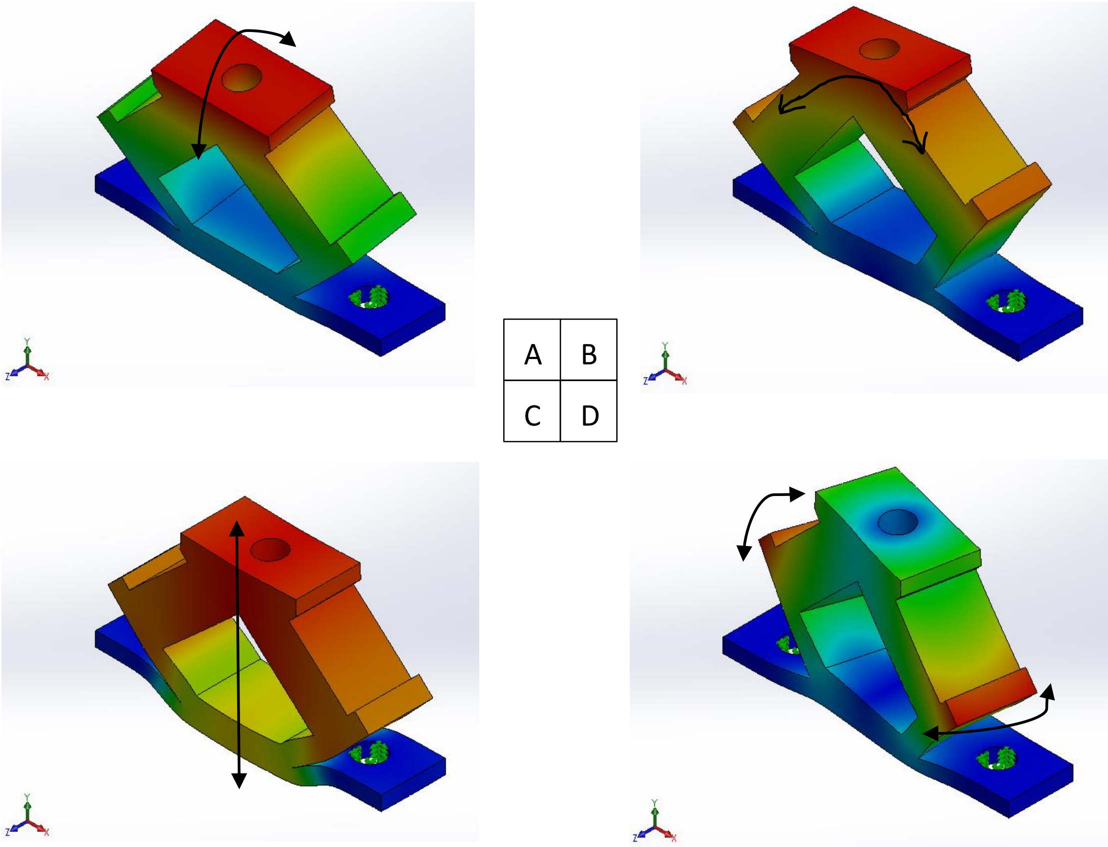

Figure 2 Modes (Bottom Holes Fixed): 1(A), 2(B), 3(C), and 4(D)

Table 1 Bottom Holes Fixed_1

Point #	Mesh Size (in)	Natural Freq_1 (Hz)	Convergence (%)	Natural Freq_2 (Hz)	Convergence (%)
1	1	87.934	N/A	204	N/A
2	0.5	87.887	-0.0534%	204.1	0.0490%
3	0.25	87.271	-0.7009%	201.57	-1.2396%
4	0.1	87.035	-0.2704%	200.63	-0.4663%
5	0.05	86.938	-0.1114%	200.3	-0.1645%
6	0.025	86.901	-0.0426%	200.11	-0.0949%
7	0.01	86.893	-0.0092%	200.02	-0.0450%
8	0.005	86.859	-0.0391%	200.01	-0.0050%

Table 2 Bottom Holes Fixed_2

Point #	Mesh Size (in)	Natural Freq_3 (Hz)	Convergence (%)	Natural Freq_4 (Hz)	Convergence (%)
1	1	376.62	N/A	435.32	N/A
2	0.5	376.65	0.0080%	435.28	-0.0092%
3	0.25	372.41	-1.1257%	434.39	-0.2045%

4	0.1	370.8	-0.4323%	434.15	-0.0552%
5	0.05	370.29	-0.1375%	434.09	-0.0138%
6	0.025	370	-0.0783%	434.04	-0.0115%
7	0.01	369.91	-0.0243%	434.01	-0.0069%
8	0.005	369.9	-0.0027%	434	-0.0023%

If bottom surface fixed, the first 4 natural frequencies are about 229$^{(Hz)}$, 272$^{(Hz)}$, 610$^{(Hz)}$, and 619$^{(Hz)}$ (Table 3 & 4). The corresponding vibration modes are 1) wobble back and forth, 2) wobble left and right, 3) the upper part twists left and right, and 4) the upper part moves up and down as a rigid body from the bottom (Figure 3).

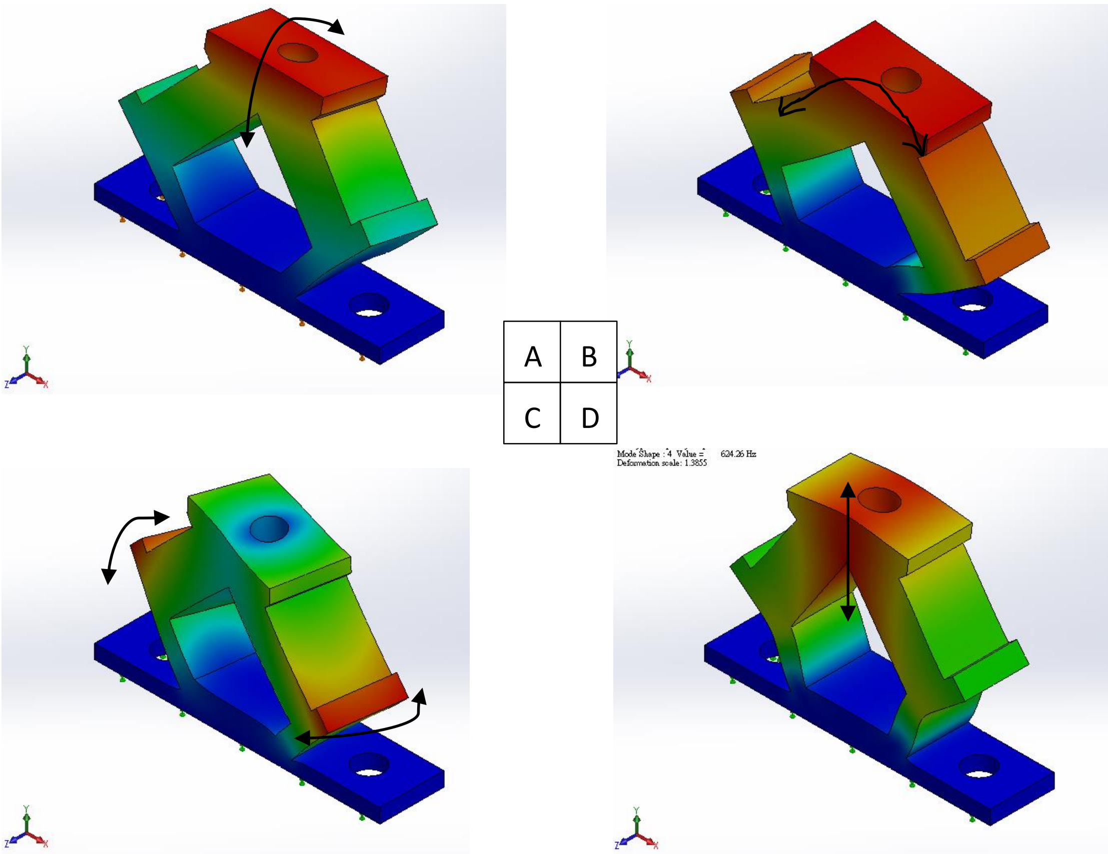

Figure 3 Modes (Bottom Surface Fixed): 1(A), 2(B), 3(C), and 4(D)

Table 3 Bottom Surface Fixed_1

Point #	Mesh Size (in)	Natural Freq_1 (Hz)	Convergence (%)	Natural Freq_2 (Hz)	Convergence (%)
1	1	230.62	N/A	275.12	N/A
2	0.5	230.24	-0.1648%	274.16	-0.3489%
3	0.25	229.65	-0.2563%	273.1	-0.3866%

4	0.1	229.33	-0.1393%	272.46	-0.2343%
5	0.05	229.25	-0.0349%	272.21	-0.0918%
6	0.025	229.18	-0.0305%	272.03	-0.0661%
7	0.01	229.13	-0.0218%	271.93	-0.0368%
8	0.005	229.15	0.0087%	271.92	-0.0037%

Table 4 Bottom Surface Fixed_2

Point #	Mesh Size (in)	Natural Freq_3 (Hz)	Convergence (%)	Natural Freq_4 (Hz)	Convergence (%)
1	1	613.08	N/A	624.26	N/A
2	0.5	612.4	-0.1109%	622.53	-0.2771%
3	0.25	611.17	-0.2008%	620.05	-0.3984%
4	0.1	610.68	-0.0802%	619.78	-0.0435%
5	0.05	610.56	-0.0197%	619.35	-0.0694%
6	0.025	610.48	-0.0131%	618.93	-0.0678%
7	0.01	610.41	-0.0115%	618.76	-0.0275%
8	0.005	610.39	-0.0033%	618.79	0.0048%

If the part is free, the first 4 non rigid-body natural frequencies will occur at the 7^{th} to 10^{th} ones since there are 6 rigid body frequencies (3 translation and 3 rotation) before them. The non rigid-body frequencies are about $372^{(Hz)}$, $387^{(Hz)}$, $732^{(Hz)}$, and $830^{(Hz)}$ (Table 7 & 8). The corresponding vibration modes are 1) the upper part moves up and down and the bottom strip moves correspondently, 2) the upper part wobbles left and right and the bottom strip moves left and right correspondently, 3) the upper part twists left and right and the bottom strip twists left and right correspondently, and 4) the whole part stretches vertically and squeezes in horizontally and then acts inversely (Figure 4).

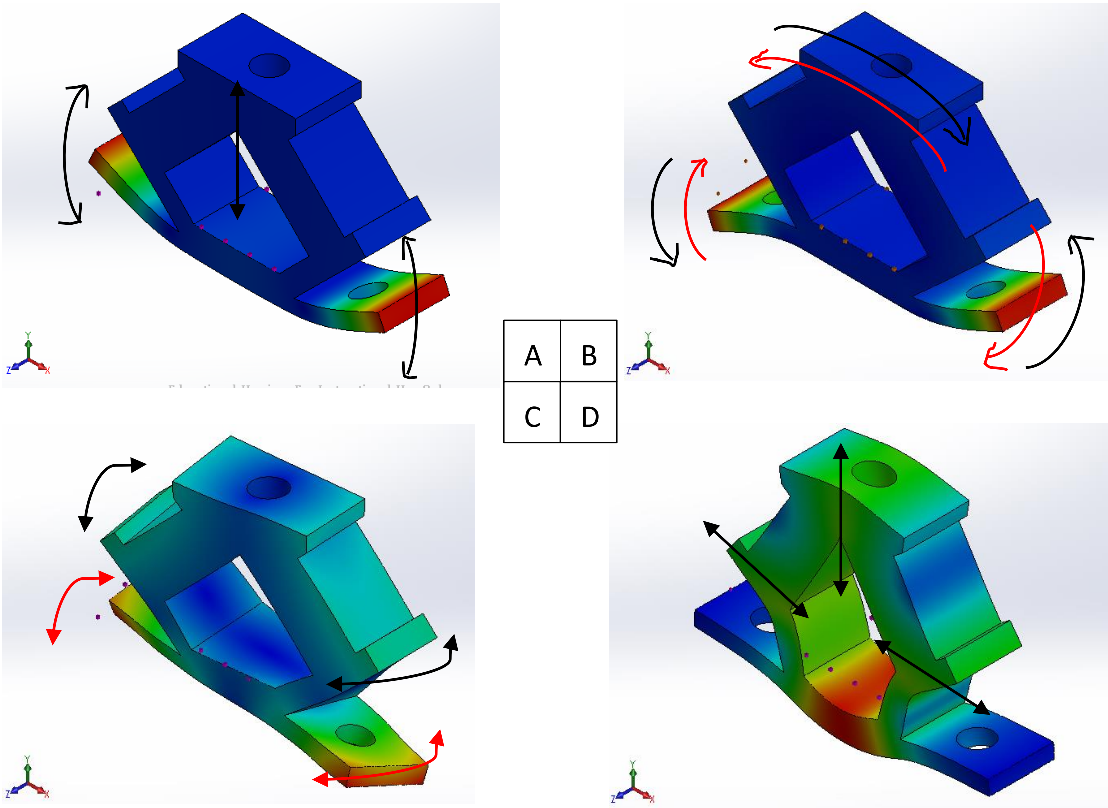

Figure 4 Non-Rigid Modes (Free Fixture): 1(A), 2(B), 3(C), and 4(D)

Table 5 Free Fixture_1

Point #	Mesh Size (in)	Natural Freq_1 (Hz)	Convergence (%)	Natural Freq_2 (Hz)	Convergence (%)
1	1	373.29	N/A	388.41	N/A
2	0.5	372.84	-0.1205%	387.9	-0.1313%
3	0.25	372.83	-0.0027%	387.9	0.0000%
4	0.1	372.47	-0.0966%	387.5	-0.1031%
5	0.05	372.13	-0.0913%	387.08	-0.1084%
6	0.025	372.65	0.1397%	387.68	0.1550%
7	0.01	371.92	-0.1959%	386.83	-0.2193%
8	0.005	372.48	0.1506%	387.49	0.1706%

Table 6 Free Fixture_2

Point #	Mesh Size (in)	Natural Freq_3 (Hz)	Convergence (%)	Natural Freq_4 (Hz)	Convergence (%)
1	1	733.41	N/A	831.32	N/A
2	0.5	733.15	-0.0355%	831.09	-0.0277%
3	0.25	732.91	-0.0327%	831.04	-0.0060%
4	0.1	732.72	-0.0259%	831.08	0.0048%

5	0.05	732.51	-0.0287%	830.4	-0.0818%
6	0.025	732.65	0.0191%	830.76	0.0434%
7	0.01	732.44	-0.0287%	830.44	-0.0385%
8	0.005	732.63	0.0259%	830.91	0.0566%

Generally speaking in this simulation, the vibration isolator gets higher natural frequencies with the bottom surface being fixed than with the bottom holes being fixed. On the other hand, the isolator with no fixture needs higher frequencies than both of the previous cases, and the non rigid-body frequencies happens at the 7th to 10th frequencies (after 6 rigid-body frequencies).

II. Task 2

For the clamped-clamped beam without any preload (Figure 5A), the first 4 natural frequencies are about 36695$^{(Hz)}$, 1.01E5$^{(Hz)}$, 1.98E5$^{(Hz)}$, and 3.18E5$^{(Hz)}$ (table 7 & 8), and the corresponding vibration modes are with one node, two nodes, three nodes and four nodes (Figure 6).

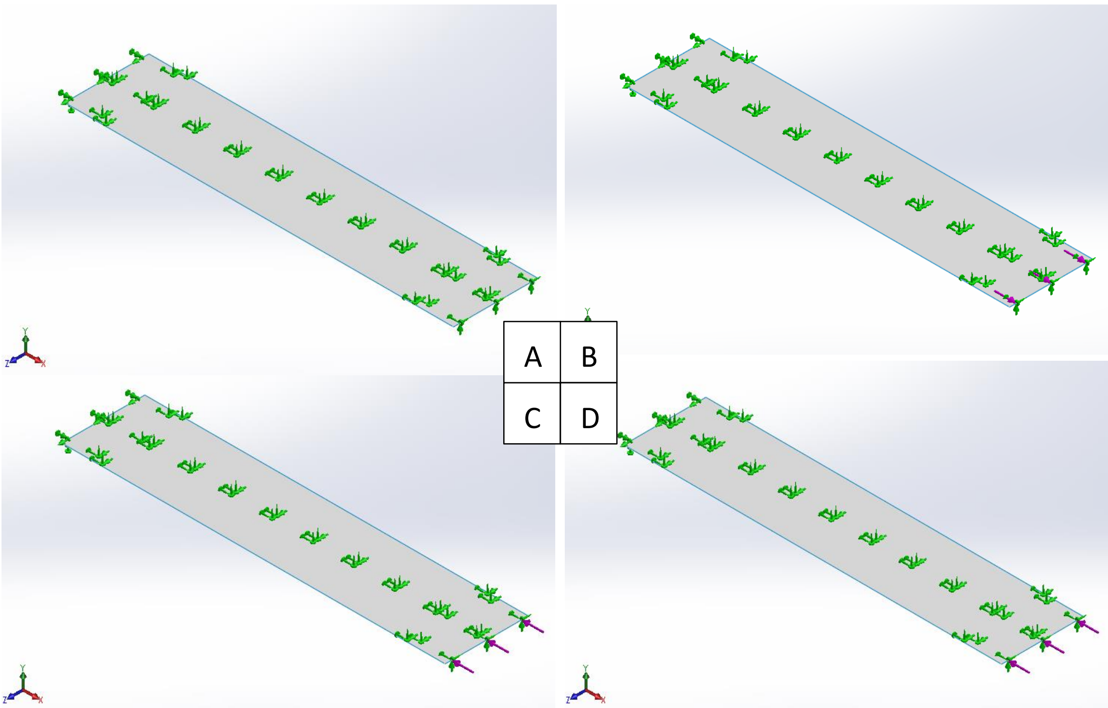

Figure 5 Loads & Constraints

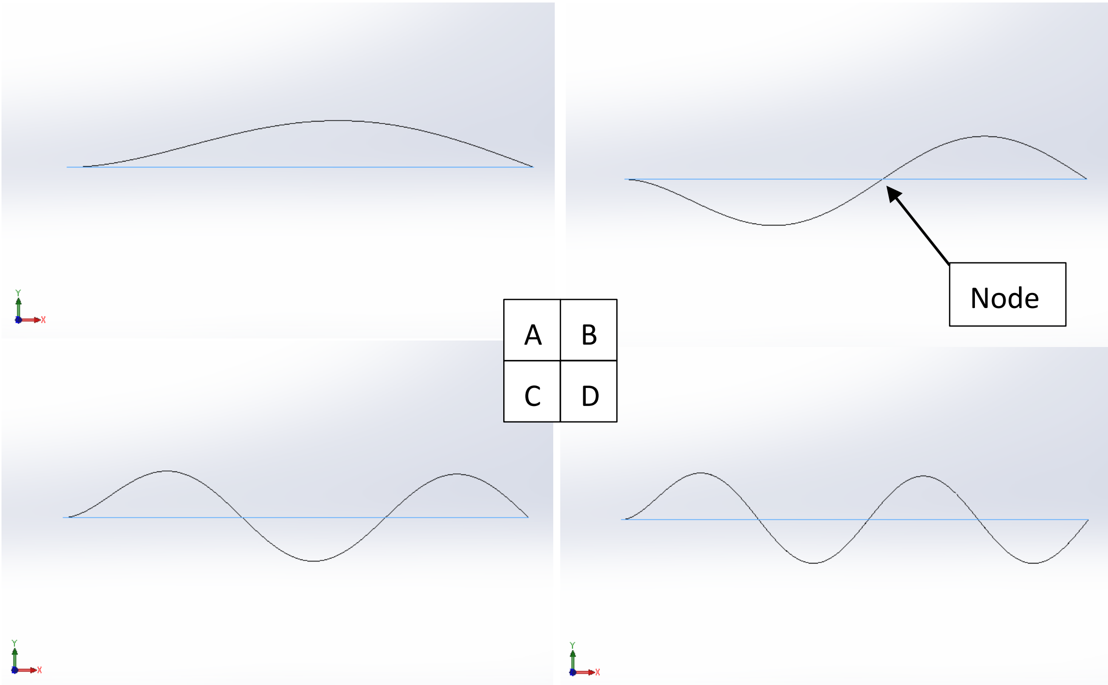

Figure 6 Modes 1(A), 2(B), 3(C), and 4(D)

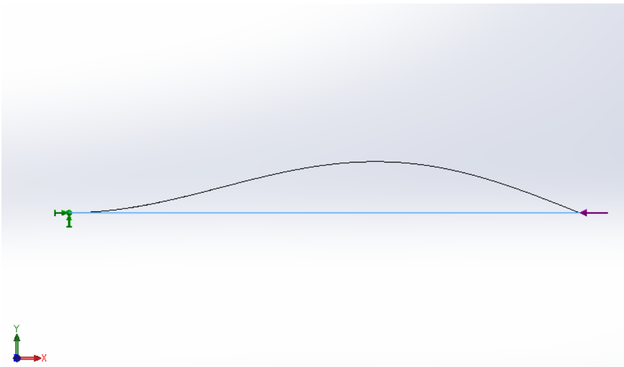

Figure 7 Fundamental Mode

Table 7 Free Load_1

Point #	Mesh Size (mm)	Natural Freq_1 (Hz)	Convergence (%)	Natural Freq_2 (Hz)	Convergence (%)
1	0.066	36693	N/A	1.01E+05	N/A
2	0.033	36692	-0.0027%	1.01E+05	0.0000%
3	0.016	36692	0.0000%	1.01E+05	0.0000%
4	0.008	36693	0.0027%	1.01E+05	0.0000%
5	0.004	36697	0.0109%	1.01E+05	0.0000%

Table 8 Free Load_2

Point #	Mesh Size (mm)	Natural Freq_3 (Hz)	Convergence (%)	Natural Freq_4 (Hz)	Convergence (%)
1	0.066	1.98E+05	N/A	3.18E+05	N/A
2	0.033	1.98E+05	-0.0051%	3.18E+05	0.0000%
3	0.016	1.98E+05	0.0000%	3.18E+05	0.0000%
4	0.008	1.98E+05	0.0000%	3.18E+05	0.0000%
5	0.004	1.98E+05	0.0000%	3.18E+05	0.0000%

Under a preload of tensile force P=10$^{(N)}$ (Figure 5B), the beam gets the first 4 natural frequencies to be 41787$^{(Hz)}$, 1.08E5$^{(Hz)}$, 2.06E5$^{(Hz)}$, and 3.18E5$^{(Hz)}$ (table 9 & 10). The corresponding vibration modes remain the same as the previous case (Figure 6).

Table 9 With a Tensile Force P=10$^{(N)}$_1

Point #	Mesh Size (mm)	Natural Freq_1 (Hz)	Convergence (%)	Natural Freq_2 (Hz)	Convergence (%)
1	0.066	41787	N/A	1.08E+05	N/A
2	0.033	41787	0.0000%	1.08E+05	0.0000%
3	0.016	41787	0.0000%	1.08E+05	0.0000%
4	0.008	41787	0.0000%	1.08E+05	0.0000%
5	0.004	41787	0.0000%	1.08E+05	0.0000%

Table 10 With a Tensile Force P=10$^{(N)}$_2

Point #	Mesh Size (mm)	Natural Freq_3 (Hz)	Convergence (%)	Natural Freq_4 (Hz)	Convergence (%)
1	0.066	2.06E+05	N/A	3.18E+05	N/A
2	0.033	2.06E+05	-0.0049%	3.18E+05	0.0031%
3	0.016	2.06E+05	0.0000%	3.18E+05	0.0000%
4	0.008	2.06E+05	0.0000%	3.18E+05	0.0000%
5	0.004	2.06E+05	0.0049%	3.18E+05	0.0000%

By applying a compressive force P=1$^{(N)}$ to the beam, the critical buckling force P_{cr}=32.5$^{(N)}$ by doing FEM (Table 12), and from Euler's equation, $P_{cr} = \pi^2 \frac{EI}{L_e^2}$, where $L_e = \frac{L}{2}$, the critical buckling force P_{cr}=29.49$^{(N)}$, which is close to what we get from FEM and thus makes the result acceptable.

Table 11

Length (m)	Height (mm)	Width (mm)	Section Area (m^2)	Moment of Inertia (m^4)	E (Pa)	P_cr (N)
0.0047	0.000132	0.00102	1.3464E-07	1.95497E-16	8.4500E+10	29.49

Table 12 Lowest Critical Buckling Force P_{cr}

Point #	Mesh Size (mm)	Buckling Factor of Safety (λ)	Convergence (%)	P_applied (N)	P_cr (N)	Convergence (%)
1	0.066	32.447	N/A	1	32.447	N/A
2	0.033	32.446	-0.0031%		32.446	-0.0031%
3	0.016	32.445	-0.0031%		32.445	-0.0031%
4	0.008	32.445	0.0000%		32.445	0.0000%
5	0.004	32.445	0.0000%		32.445	0.0000%

By applying a compressive force 0.95 times the lowest critical buckling load back to the beam, we can see the fundamental natural frequency is about $8149^{(Hz)}$, which is far lower than the fundamental frequencies we get so far.

Table 13 Fundamental Natural Frequency with a Compressive Force P=0.95 P_{cr}

Point #	Mesh Size (mm)	P_critical (N)	P (N)	Fundamental Natural Freq. (Hz)	Convergence (%)
1	0.066	32.5	30.875	8154.1	N/A
2	0.033			8151.1	-0.0368%
3	0.016			8149.1	-0.0245%
4	0.008			8148.8	-0.0037%
5	0.004			8148.4	-0.0049%

➢ Conclusion

I. Task 1

In this task, the first 4 natural frequencies of the vibration isolator are $87^{(Hz)}$, $200^{(Hz)}$, $370^{(Hz)}$, and $434^{(Hz)}$ with its bottom holes fixed and are $229^{(Hz)}$, $272^{(Hz)}$, $610^{(Hz)}$, and $619^{(Hz)}$ with its bottom surface fixed. The corresponding vibration modes in each case are similar to each other. The first 4 non rigid-body vibrations of the isolator with free fixture occurs at the 7^{th} to the 10^{th} natural frequencies, which come after the 6 rigid-body vibration. The frequencies are $372^{(Hz)}$, $387^{(Hz)}$, $732^{(Hz)}$, and $830^{(Hz)}$.

II. Task 2

For the clamped-clamped beam without any preload (Figure 5A), the first 4 natural frequencies are $36695^{(Hz)}$, $1.01E5^{(Hz)}$, $1.98E5^{(Hz)}$, and $3.18E5^{(Hz)}$, and if a tensile load ($10^{(N)}$) is applied, the first 4 natural frequencies become $41787^{(Hz)}$, $1.08E5^{(Hz)}$, $2.06E5^{(Hz)}$, and $3.18E5^{(Hz)}$. It is clearly seen here that a tensile load can increase natural frequencies. The lowest critical buckling force of this beam is around $30^{(N)}$, and the fundamental natural frequency under a compressive force 0.95 times the lowest critical buckling load is $8149^{(Hz)}$.