Combined Effect of Variable Viscosity and Variable Thermal Conductivity on Double-Diffusive Convection Flow of a Permeable Fluid in a Vertical Channel

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Combined Effect of Variable Viscosity and Variable Thermal Conductivity on Double-Diffusive Convection Flow of a Permeable Fluid in a Vertical Channel

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Abstract This paper reports a detailed analytical and numerical investigation on free convection flow of viscous fluid through a porous medium due to the combined effects of thermal and mass diffusion. Effect of temperature-dependent viscosity and thermal conductivity is investigated in the presence of first-order chemical reaction. The non-Darcy model is applied to define the porous matrix. The effects of viscous and Darcy dissipations are taken into account. The governing equations of continuity, momentum, energy, and concentration which are coupled nonlinear ordinary differential equations are solved analytically using regular perturbation method and numerically using Runge–Kutta shooting method. Brinkman number and variable thermal conductivity parameters are used as perturbation parameters. The velocity, temperature, and concentration distributions are discussed numerically and plotted in graphs. The effects of variable viscosity parameter, variable thermal conductivity parameter, thermal Grashof number, mass Grashof number, Brinkman number, wall temperature ratio, and first-order chemical reaction parameter on the flow fields are explored. The effects of physical parameters such as skin friction and Nusselt number at both the plates are derived and discussed, and the numerical values for various values of physical parameters are presented in tables. The solutions obtained using Runge–Kutta shooting method and using perturbation method are compared and the solutions agree very well in the absence of perturbation parameter.

Keywords Free convection · Variable viscosity · Variable thermal conductivity · Viscous dissipation

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List of symbols

Roman symbols

a, b_1	Empirical constant
b	Width of the channel
b_{v}	Viscosity variation parameter
Br	Brinkman number
C_w	Species concentration along the wall
C_0	Species concentration away from the wall
C	Species concentration of the fluid
ΔC	Différence in concentration
D	Effective diffusion coefficient
GR_T	Modified thermal Grashof number
Gr_T	Grashof number
GR_C	Modified local mass Grashof number
Gr_C	Local mass Grashof number
Re	Reynolds number
g	Acceleration due to gravity
K	Thermal conductivity of the fluid
m	Wall temperature ratio
T_0	Reference Temperature
T	Fluid Temperature
U	Velocity
u	Dimensionless velocity
$\bar{u}$	Mean velocity
Y	Coordinate axis
y	Dimensionless coordinate axis
ΔT	Difference in temperature

Greek symbols

α	Thermal diffusivity
β_{T}	Coefficient of thermal expansion
β_{C}	Concentration expansion coefficient
γ	Chemical reaction parameter
θ	Dimensionless temperature
κ	Permeability parameter
μ	Viscosity
μ_0	Viscosity at temperature T_0
υ	Kinematics viscosity
ρ_0	Density of the fluid
σ	Porous parameter
ϕ	Concentration of the fluid

1 Introduction

Natural convection in differently heated cavities filled with a fluid-saturated porous media plays an important role in many practical applications. It has attracted the attention of engineers in recent past owing to its relevance in high-performance insulations for cryogenic containers, petroleum reservoirs, sensible heat storage beds, coal combustors, risk assessment of radionuclide migration from depositories of nuclear waste, thermal performance of solar collectors, cooling of electronic systems, nuclear waste repositories, and ground water hydrology are just some examples of realistic applications where a porous cavity is basically differentially heated. Convective processes of fluid flow and associated heat transfer in porous cavities have been studied extensively for the past several decades. Free convective heat transfer in porous medium has been studied widely in the literature. Nield and Bejan (2013) gave an excellent summary of the subject.

The growing need for chemical reactions in chemical and hydrometallurgical industries requires the study of heat and mass transfer in the presence of chemical reaction. There are many transport processes that are governed by the simultaneous action of buoyancy forces due to both thermal and mass diffusion in the presence of chemical reaction effect. These processes are observed in nuclear reactor safety and combustion systems, solar collectors as well as chemical and metallurgical engineering. Their other applications include solidification of binary alloys and crystal growth dispersion of dissolved materials or particulate water in flows, drying and dehydration operations in chemical and food processing plants, evaporation at the surface of a water body, distribution of temperature and moisture over agricultural fields and groves of fruit trees, damage of crops due to freezing, energy transfer in a wet cooling tower and flow in a desert cooler, heat and mass transfer occur simultaneously. Chemical reaction can be modeled as either homogeneous or heterogeneous processes. This depends on whether they occur at an interface or a single-phase volume reaction. A homogeneous reaction is one that occurs uniformly throughout a given phase. The species generation in a homogeneous reaction is the same as the internal source of heat generation. On the other hand, heterogeneous reaction takes place in a restricted area or within the boundary of a phase. The order of the chemical reaction depends on several factors. One of the simplest chemical reactions is the first-order reaction in which the rate of reaction is directly proportional to the species concentration. In many chemical engineering processes, a chemical reaction between a foreign mass and the fluid does occur. These processes takes place in numerous industrial applications, such as the polymer production, the manufacturing of ceramics or glass ware, the food processing. The study of heat and mass transfer with chemical reaction is of great practical importance to engineers and scientists because of its almost universal occurrence in many branches of science and engineering. Muthucumaraswamy and Ganeshan (2000, 2001) have studied the impulsive motion of a vertical plate with heat flux/mass flux/suction and diffusion of chemically reactive species. Seddeek (2005) have studied the finite element method for the effect of chemical reaction, variable viscosity, thermophoresis and heat generation/absorption on a boundary layer hydromagnetic flow with heat and mass transfer over a heat surface. Kandasamy et al. (2005a, 2005b) have examined the effects of chemical reaction, heat and mass transfer with or without MHD flow with heat source/suction.

Most of the existing analytical studies for this problem are based on the constant physical properties of the ambient fluid. More accurate prediction for the flow and heat transfer can be achieved by taking into account variation of these properties with temperature (Herwig and Wicken 1986), especially for fluid viscosity and thermal conductivity. To accurately predict the flow and heat transfer rates, it is necessary to take into account the effects of these variations. For lubricating fluids, heat generated by internal friction and the corresponding rise in the temperature affects the physical properties of the fluid and are no longer assumed to be constant. The increase in temperature leads to increase in the transport phenomena by reducing the physical properties across the thermal boundary layer and so the heat transfer at the wall is also affected. Therefore to predict the flow and heat transfer rates, it is necessary to take into account the variable fluid properties. Klemp et al. (1990) studied the effect of temperature-dependent viscosity on the entrance flow in a channel in the hydrodynamic case. The effects of variable viscosity on hydromagnetic flow and heat transfer have been studied by Seddeek (2002). Natural convection flow of a viscous Newtonian fluid having temperature-dependent viscosity and thermal conductivity over an isothermal vertical wavy cone was studied by Hossain and Munir (2001a) and Hossain et al. (2001b) neglecting viscous dissipation effects. Attia (2008) considered unsteady hydromagnetic Couette flow of dusty fluid with temperature dependent viscosity and thermal conductivity under exponentially decaying pressure gradient and solved the governing equations numerically using finite difference method. Recently, Kalidas (2012) investigated the influence of thermophoresis and chemical reaction on MHD micropolar fluid flow with variable fluid properties. Barletta (1997) and Zanchini (1997) have pointed out that relevant effects of viscous dissipation in the fully developed laminar forced convection in tubes. Thus, an analysis of the effects of viscous and Darcy dissipation in the fully developed mixed convection in vertical duct appears as interesting.

The aim of the present paper is to study the velocity, temperature and concentration distribution within a vertical channel with variable properties. The combined effect of viscosity and thermal conductivity on temperature is analyzed. The effects of various governing parameters on the flow and concentration fields are explored.

2 Mathematical Formulation

Consider a steady laminar, fully developed flow of an incompressible, viscous, permeable fluid between two parallel plates. The distance between the plates is $2b$, and the origin of coordinate axis is located in the mid-plane of the channel. The two plates are kept at two constant temperatures T_1 for the left plate and T_2 for the right plate. The channel is assumed to occupy the region of space$-b \leq Y \leq b$. A fluid rises in the channel driven by buoyancy forces. The no-slip boundary condition is imposed on the parallel plates for the velocity, and since the plates are infinite in the X-direction, the physical variables are invariant in these directions and the problem is essentially one dimensional with velocity component $U(Y)$ along the X axis.

The physical properties characterizing the fluid except density, viscosity and thermal conductivity are assumed to be constant. As customary, the Boussinesq approximation and the equation of state

$$\rho = \rho_0 \left[1 - \beta_T (T - T_0) - \beta_C (C - C_0)\right]. \tag{1}$$

will be adopted. The flow and heat transfer of viscous, permeable fluid is examined considering viscosity and thermal conductivity dependent temperature.

The momentum equations governing the motion of an incompressible permeable fluid in the presence of viscous and Darcy dissipations with the variable viscosity and variable thermal conductivity are given by

$$\frac{\mathrm{d}}{\mathrm{d}Y}\left(\mu \frac{\mathrm{d}U}{\mathrm{d}Y}\right) - \frac{\mu U}{\kappa} + \rho_0 g \beta_T (T - T_0) + \rho_0 g \beta_C (C - C_0) = 0 \tag{2}$$

$$\frac{\mathrm{d}}{\mathrm{d}Y}\left(K\frac{\mathrm{d}T}{\mathrm{d}Y}\right)+\mu\left(\frac{\mathrm{d}U}{\mathrm{d}Y}\right)^2+\frac{\mu}{\kappa}U^2=0 \tag{3}$$

$$D\frac{\mathrm{d}^2C}{\mathrm{d}Y^2}-\gamma C=0 \tag{4}$$

where U is the velocity of the fluid, T the temperature of the fluid, D is the diffusion coefficient, C is the concentration of the fluid, ρ_0 the static density, β_T the coefficient of thermal expansion, β_C the concentration expansion coefficient, κ the permeability parameter, g the acceleration due to gravity, μ the viscosity and K the thermal conductivity of the fluid.

The boundary conditions on the velocity and temperature fields are given as

$$U=0 \text{ at } Y=\pm b \tag{5}$$

$$T=T_1 \text{ at } Y=-b,\ T=T_2 \text{ at } Y=b \tag{6}$$

$$C=C_w \text{ at } Y=-b,\ C=C_0 \text{ at } Y=b \tag{7}$$

The fluid viscosity μ is assumed to vary with temperature as (Saravanan and Kandaswamy 2004; Attia 2006)

$$\mu=\mu_0 e^{-a(T-T_0)} \tag{8}$$

where the subscript 0 denotes the reference state and a an empirical constant.

In Eq. (8), the viscosity μ is assumed to depend on temperature exponentially. The parameter a may take positive values for liquids such as water, benzene or crude oil. In some gases like air, helium or methane a may be negative, i.e., the coefficient viscosity increases with temperature (Sutton and Sherman 1965; Schlichting 1968). This type of model can find applications in many processes where preheating of the fuel is used as a means to enhance heat transfer effects. In addition, for many fluids such as lubricants, polymers, and coal slurries where viscous dissipation is substantial, an appropriate constitutive relation where viscosity is a function of temperature should be used.

The thermal conductivity of the fluid is assumed as (Attia 2006)

$$K=K_0\left(1+b_1\left(T-T_0\right)\right) \tag{9}$$

The thermal conductivity of the fluid is assumed to vary with temperature as can be seen in Eq. (9) where the parameter b may be positive for some fluids such as air or water vapor or negative for others like liquid water or benzene (Schlichting 1968; White 1991). The thermal conductivity changes approximately linearly with temperature in the range from 0 to 400° F.

Equations (2–4) determine the velocity, temperature, and concentration distribution, and they can be written in a dimensionless form by means of the following dimensionless parameters

$$u^*=\frac{U}{\bar{u}};\quad y^*=\frac{Y}{b};\quad m=\frac{T_1-T_2}{\Delta T};\quad \phi=\frac{c-c_0}{\Delta c};\quad \theta=\frac{T-T_0}{\Delta T};\quad Gr_T=\frac{g\beta_\mathrm{T}b^3\Delta T}{\upsilon^2};$$
$$GR_T=\frac{Gr_T}{\mathrm{Re}};\quad Gr_C=\frac{g\beta_\mathrm{C}b^3\Delta C}{\upsilon^2};\quad GR_C=\frac{Gr_C}{\mathrm{Re}};\quad \mathrm{Re}=\frac{\bar{u}b}{\upsilon};\quad \sigma=\frac{b^2}{\kappa};$$
$$Br=\frac{\mu_0\bar{u}^2}{K_0\Delta T} \tag{10}$$

In terms of the non-dimensional variables as in Eq. (10), Eqs. (2–4) take the form

$$\frac{\mathrm{d}^2u}{\mathrm{d}y^2}-b_\mathrm{v}\frac{\mathrm{d}\theta}{\mathrm{d}y}\frac{\mathrm{d}u}{\mathrm{d}y}-\sigma^2u+(1+b_\mathrm{v}\theta)\left(GR_T\theta+GR_C\phi\right)=0 \tag{11}$$

$$\frac{d^2\theta}{dy^2} - b_k\left(\frac{d\theta}{dy}\right)^2 + Br\left(\frac{du}{dy}\right)^2 + (b_k - b_v)\,Br\theta\left[\left(\frac{du}{dy}\right)^2 + \sigma^2u^2\right]$$
$$-b_kb_vBr\theta^2\left[\left(\frac{du}{dy}\right)^2 + \sigma^2u^2\right] + Br\sigma^2u^2 = 0 \quad (12)$$

$$\frac{d^2\phi}{dy^2} - \alpha\phi = 0 \quad (13)$$

where $b_v = a\Delta T$ is the variable viscosity parameter, $b_k = b_1\Delta T$ is the variable conductivity parameter, $m = \frac{T_1 - T_2}{\Delta T}$ the wall temperature ratio, σ is the porous parameter and Br is the Brinkman number. The non-dimensional form of boundary conditions is

$$u = 0 \text{ at } y = \pm 1 \quad (14)$$

$$\theta = 1 + m \text{ at } y = -1,\ \theta = 1 \text{ at } y = 1 \quad (15)$$

$$\phi = 1 \text{ at } y = -1, \quad \phi = 0 \text{ at } y = 1 \quad (16)$$

Equations (11–13) show that the dimensionless velocity, temperature, and concentration fields depend on the parameters: the viscosity parameter b_v, the conductivity parameter b_k, the porous parameter σ, the Brinkman number Br, chemical reaction parameter α and the wall temperature ratio m.

3 Solutions

The solution of Eq. (13) can be obtained directly by integrating Eq. (13), and the solution is

$$\phi = c_1e^{-\sqrt{\alpha}y} + c_2e^{\sqrt{\alpha}y} \quad (17)$$

where c_1 and c_2 are integrating constants and can be evaluated using boundary conditions as defined in Eq. (16).

Equations (11) and (12) are coupled nonlinear equations, and it is difficult in general to find analytical solutions. However approximate analytical solutions can be found using regular perturbation method. We shall perform a perturbation analysis for the nonlinear Eqs. (11) and (12) considering

$$u = u_0 + Bru_1 \text{ and } \theta = \theta_0 + Br\theta_1 \quad (18)$$

Substituting (18) into Eqs. (11) and (12) and equating terms with the same powers of *Br,* we obtain the following sequence of boundary value problems for u_0, θ_0, u_1 and θ_1
Zeroth-Order equations

$$\frac{d^2u_0}{dy^2} - b_v\frac{d\theta_0}{dy}\frac{du_0}{dy} - \sigma^2u_0 + (1 + b_v\theta_0)\,GR_T\theta_0 + GR_C\,(1 + b_v\theta_0)\,\phi = 0 \quad (19)$$

$$\frac{d^2\theta_0}{dy^2} - b_k\left(\frac{d\theta_0}{dy}\right)^2 = 0 \quad (20)$$

First-order equations

$$\frac{d^2u_1}{dy^2} - b_v\frac{du_0}{dy}\frac{d\theta_1}{dy} - b_v\frac{du_1}{dy}\frac{d\theta_0}{dy} - \sigma^2u_1 + GR_Tb_v\theta_0\theta_1$$
$$+ (1 + b_v\theta_0)\,GR_T\theta_1 + b_v\theta_1GR_C\phi = 0 \quad (21)$$

$$
\begin{aligned}
&\frac{d^2\theta_1}{dy^2} - 2b_k \frac{d\theta_0}{dy}\frac{d\theta_1}{dy} + \left(\frac{du_0}{dy}\right)^2 + (b_k - b_\mathrm{v})\,\theta_0 \left(\frac{du_0}{dy}\right)^2 - b_\mathrm{v} b_k \theta_0^2 \left(\frac{du_0}{dy}\right)^2 \\
&+\sigma^2 u_0 + \sigma^2 (b_k - b_\mathrm{v})\,\theta_0 u_0^2 - \sigma^2 b_\mathrm{v} b_k \theta_0^2 u_0^2 = 0
\end{aligned} \tag{22}
$$

The Eqs. (19–22) are still nonlinear equations and hence closed form solutions cannot be found. Hence we apply once again the perturbation method using variable conductivity parameter as the perturbation parameter only for the zeroth-order Eqs. (19) and (20) as follows

$$
u_0 = u_{00} + b_k u_{01}; \quad \theta_0 = \theta_{00} + b_k \theta_{01} \tag{23}
$$

Substituting (23) into Eqs. (19) and (20) and equating terms with the same powers of b_k, we obtain the following sequence of boundary values problems for u_{00}, θ_{00}, u_{01} and θ_{01}
Zeroth-order equations

$$
\frac{\mathrm{d}^2\theta_{00}}{\mathrm{d}y^2} = 0 \tag{24}
$$

$$
\frac{\mathrm{d}^2 u_{00}}{\mathrm{d}y^2} - b_\mathrm{v} \frac{\mathrm{d}\theta_{00}}{\mathrm{d}y}\frac{\mathrm{d}u_{00}}{\mathrm{d}y} - \sigma^2 u_{00} + (1 + b_\mathrm{v}\theta_{00})\, GR_T \theta_{00} + GR_C\,(1 + b_\mathrm{v}\theta_{00})\,\phi = 0 \tag{25}
$$

First-order equations

$$
\frac{\mathrm{d}^2\theta_{01}}{\mathrm{d}y^2} - \left(\frac{\mathrm{d}\theta_{00}}{\mathrm{d}y}\right)^2 = 0 \tag{26}
$$

$$
\begin{aligned}
&\frac{\mathrm{d}^2 u_{01}}{\mathrm{d}y^2} - b_\mathrm{v} \frac{\mathrm{d}\theta_{00}}{\mathrm{d}y}\frac{\mathrm{d}u_{01}}{\mathrm{d}y} - b_\mathrm{v} \frac{\mathrm{d}\theta_{01}}{\mathrm{d}y}\frac{\mathrm{d}u_{00}}{\mathrm{d}y} - \sigma^2 u_{01} + b_\mathrm{v}\theta_{00}\theta_{01} GR_T \\
&+ (1 + b_\mathrm{v}\theta_{00})\, GR_T \theta_{01} + GR_C \theta_{01}\phi = 0
\end{aligned} \tag{27}
$$

Integrating Eqs. (24), (25) and (26) yield

$$
\begin{gathered}
\theta_{00} = c_3 y + c_4 \\
u_{00} = c_7 e^{r_1 y} + c_8 e^{r_2 y} + p_1 y e^{-\sqrt{\alpha} y} + p_2 y e^{\sqrt{\alpha} y} + p_3 e^{-\sqrt{\alpha} y} + p_4 e^{\sqrt{\alpha} y} + p_5 y^2 + p_6 y + p_7 \\
\theta_{01} = l_1 y^2 + c_5 y + c_6
\end{gathered}
$$

where c_3, c_4, c_5, c_6, c_7 and c_8 are integrating constants which can be evaluated using boundary conditions (14) and (15) after substituting (18) and (23). The solution of Eq. (27) is not found as it is very tedious.

3.1 Numerical Solutions

The analytical solutions obtained in the above section are valid for small values of perturbation parameters. Further it is seen in the above section that it is not possible to find solutions of even the first order. Hence we resort to solve the governing equations by numerical methods using Runge–Kutta shooting method (RKSM). The validity of RKSM is justified by comparing the solutions with the results obtained by the perturbation method, and the values are displayed in tables. The perturbation method and RKSM solutions agree very well in the absence of perturbation parameter.

3.2 Skin Friction and Nusselt Number

In addition to the velocity and temperature fields, the following physical quantities can be defined: The dimensionless skin friction at each boundary can be defined as

$$\tau_1 = \left.\frac{\mathrm{d}u}{\mathrm{d}y}\right|_{y=-1} \quad \text{and} \quad \tau_2 = \left.\frac{\mathrm{d}u}{\mathrm{d}y}\right|_{y=1} \tag{28}$$

The dimensionless Nusselt number at each boundary can be defined as follows:

$$Nu_1 = \left.\frac{\mathrm{d}\theta}{\mathrm{d}y}\right|_{y=-1} \quad \text{and} \quad Nu_2 = \left.\frac{\mathrm{d}\theta}{\mathrm{d}y}\right|_{y=1} \tag{29}$$

The above equations are solved and the results are tabulated in Table 1 for different governing parameters.

4 Results and Discussion

In this section, the results obtained for the heat and mass transfer in a vertical channel containing fluid-saturated porous medium are discussed. The Darcy–Brinkman model is used to define the governing equations. The exponential dependent of the viscosity and linear dependence of thermal conductivity on temperature is studied. The variations of these resulting terms with the variable viscosity parameter b_{v} and thermal conductivity parameter b_k and their relative magnitude have an importance on the velocity, temperature and concentration fields. The major parameter such as thermal Grashof number GR_T, mass Grashof number GR_C, Brinkman number Br, the wall temperature ratio m, porous parameter σ and first-order chemical reaction parameter α on the flow for positive and negative values of viscosity parameter b_{v} and the conductivity parameter b_k is numerically evaluated and depicted graphically. The governing equations which are coupled and highly nonlinear are solved analytically using regular perturbation method (PM) and numerically using Runge–Kutta shooting method (RKSM). The validity of RKSM is justified by comparing the solutions of RKSM with PM.

The effects of viscosity variation parameter b_{v} on the velocity and temperature fields are seen in Figs. 1 and 2, respectively, fixing the variable thermal conductivity parameter. As the viscosity variation parameter b_{v} increases, flow increases and the profiles for constant viscosity ($b_{\mathrm{v}} = 0$) lie above $b_{\mathrm{v}} < 0$ and below $b_{\mathrm{v}} > 0$ on velocity and temperature fields. The variable viscosity parameter b_{v} on the flow was the similar result observed by Attia (2001) on the MHD channel flow of dusty fluids. Keeping the value of variable viscosity parameter b_{v} fixed, the effect of variable thermal conductivity parameter b_k on the flow is shown in Figs. 3 and 4, respectively. It is seen that as the thermal conductivity parameter increases, both the velocity and temperature fields are suppressed. The profiles for constant thermal conductivity ($b_k = 0$) lie above $b_k > 0$ and below $b_k < 0$. The effect of thermal conductivity parameter b_k is in contrast to the effect of variable viscosity parameter b_{v} (Figs. 1, 2). The effect of b_k on the flow was the similar result observed by Attia (1999) and Palani and Kim (2010).

The effect of thermal Grashof number GR_T and mass Grashof number GR_C on the velocity and temperature fields is displayed in Figs. 5, 6, 7, and 8. As the thermal Grashof number GR_T and mass Grashof number GR_C increase, the velocity and temperature fields increase. Physically an increase in the thermal Grashof number GR_T and mass Grashof number GR_C

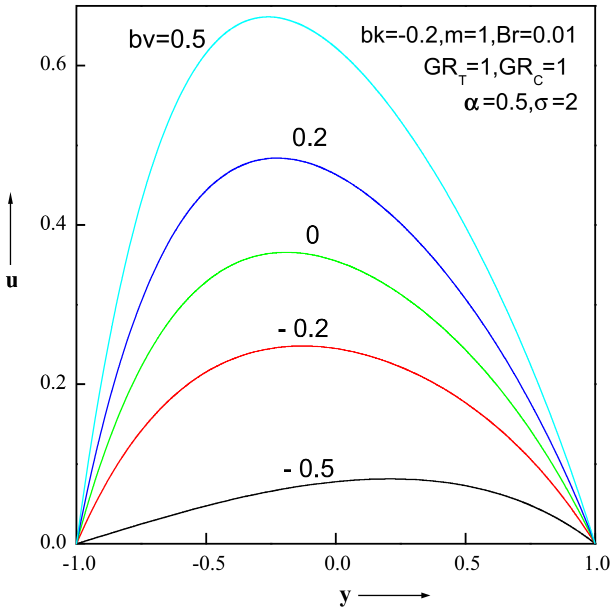


Fig. 1 Velocity profiles for different values of b_V

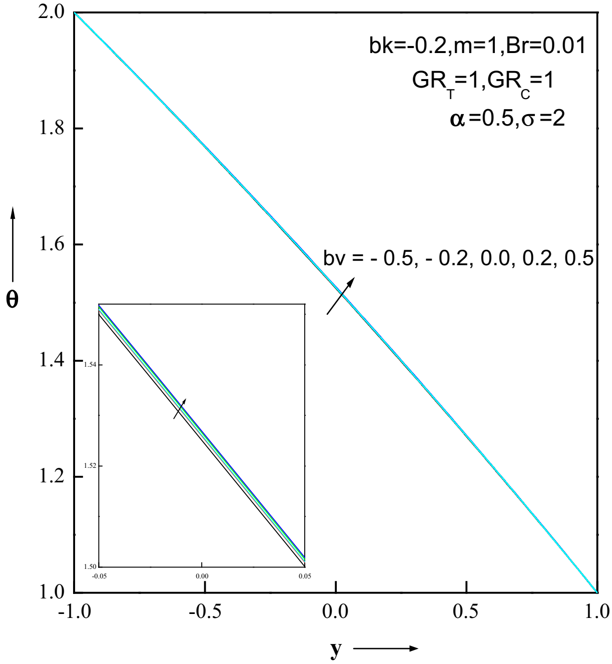


Fig. 2 Temperature profiles for different values of b_V

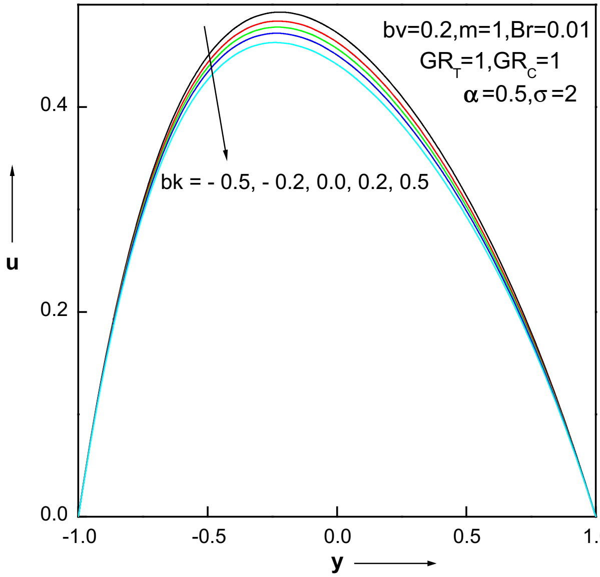
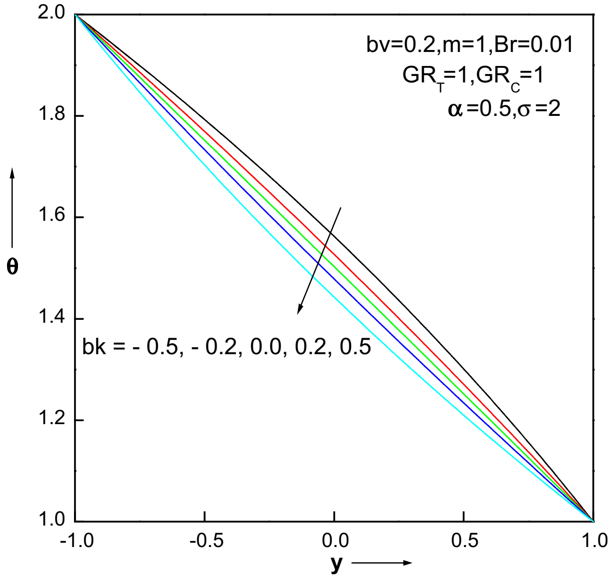


Fig. 3 Velocity profiles for different values of b_k

Fig. 4 Temperature profiles for different values of b_k



implies an increase in buoyancy force which supports the motion. Since the thermal Grashof number GR_T acts as the driving mechanism of the driving force in the momentum equation, the velocity and/or velocity gradient increases and therefore the effect of dissipation increases, which results in the enhancement of temperature field. The effects of GR_T and GR_C on the flow were the similar results observed by Shivaiah and Anandrao (2012) for the flow past a porous vertical plate for constant properties. The effect of Brinkman number Br on the velocity and temperature field is shown in Figs. 9 and 10, respectively. It is evident from

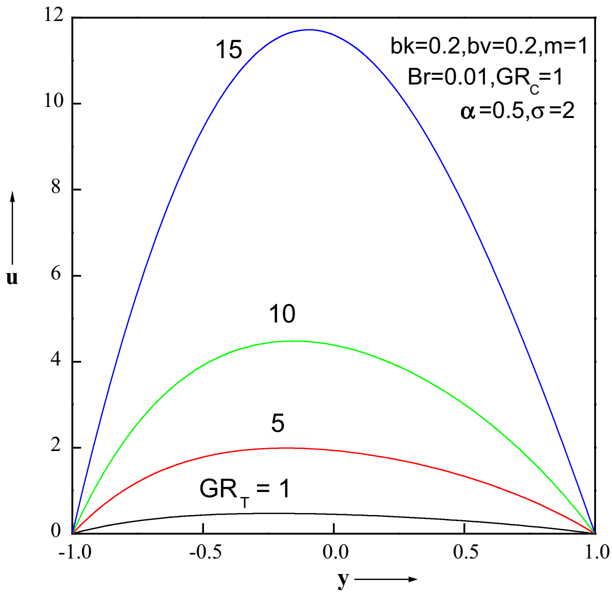


Fig. 5 Velocity profiles for different values of GR_r

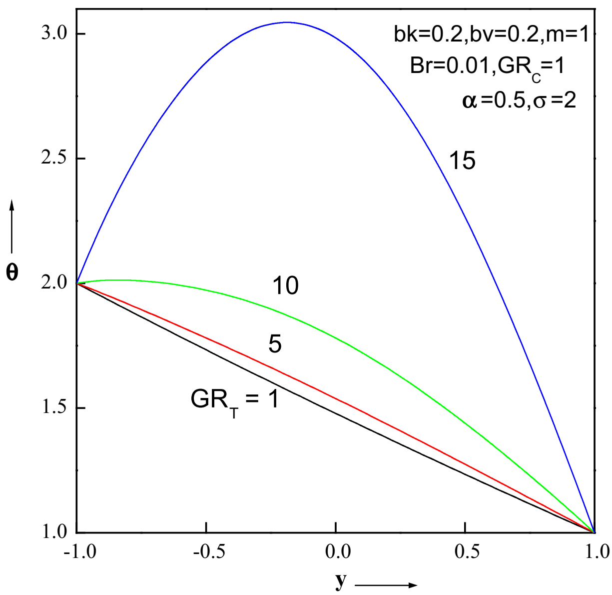


Fig. 6 Temperature profiles for different values of GR_r

these figures that as the Brinkman number Br increases, the flow is promoted. This is due to the fact that an increase in the Brinkman number Br results in the increase in dissipation effects which results in the increase in temperature and as a consequence velocity increases for the increase in buoyancy force in the momentum equation. The wall temperature ratio m

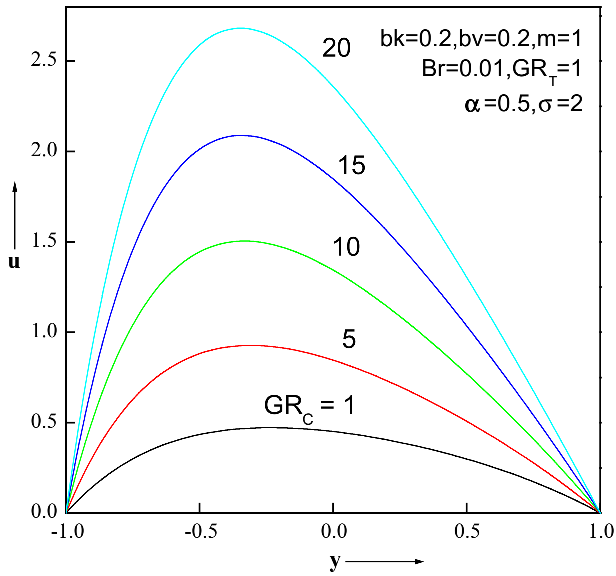


Fig. 7 Velocity profiles for different values of GR_c

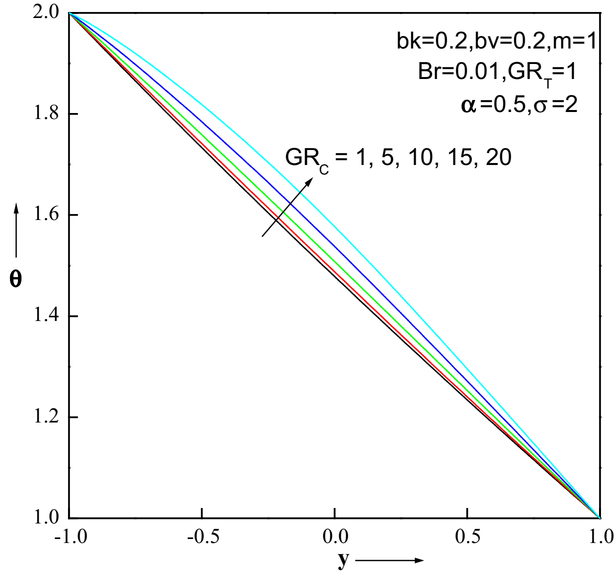


Fig. 8 Temperature profiles for different values of GR_c

fixing the values of b_v, b_k, GR_T, GR_C, Br, σ and α can be viewed in Figs. 11 and 12. It is seen that as the wall temperature ratio m increases, the flow is enhanced. It is interesting to note that for negative values of m, there is a flow reversal at the left wall. Since the wall temperature boundary conditions are taken as $1 + m$ at left wall and fixed as 1 at the right wall, the temperature profiles show the variation at the left wall and remain constant at the right wall. The effect of porous parameter σ (inverse of Darcy number) is to suppress the flow

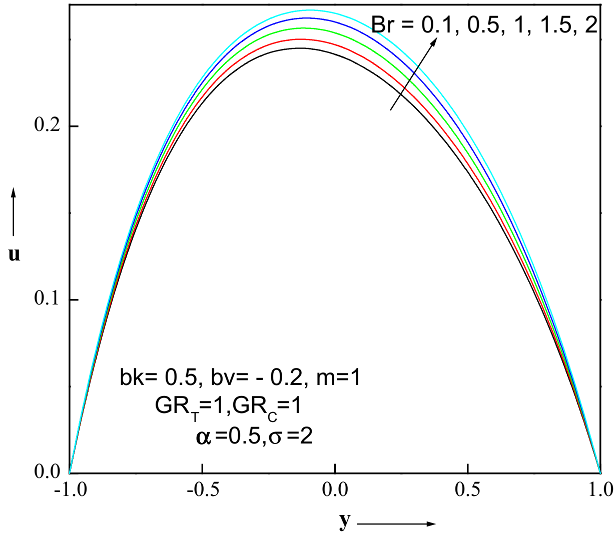


Fig. 9 Velocity profiles for different values of Br

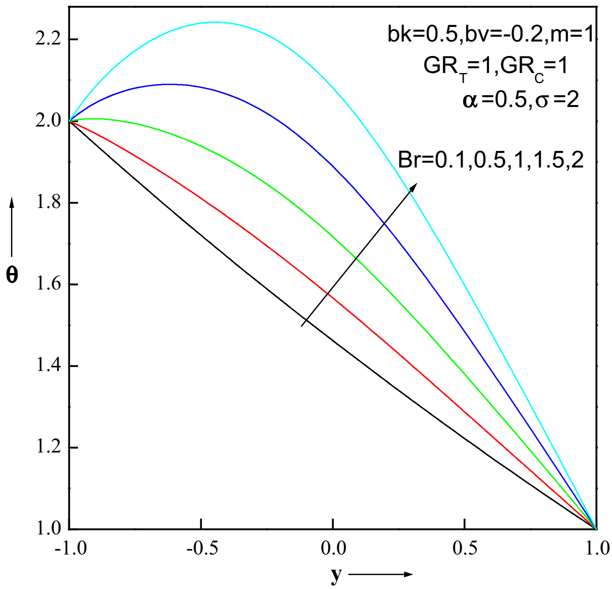


Fig. 10 Temperature profiles for different values of Br

as seen in Figs. 13 and 14. This is also an expected result because physically, large values of porous parameter σ correspond to densely packed porous medium and hence flow rate will be reduced. The effect of porous parameter σ on the flow was the similar result observed by Umavathi and Veershetty (2012) for constant properties. The effects of first-order chemical reaction parameter α on the velocity, temperature, and concentration fields are depicted in Figs. 15, 16, and 17. It is evident from these figures that the velocity, temperature, and

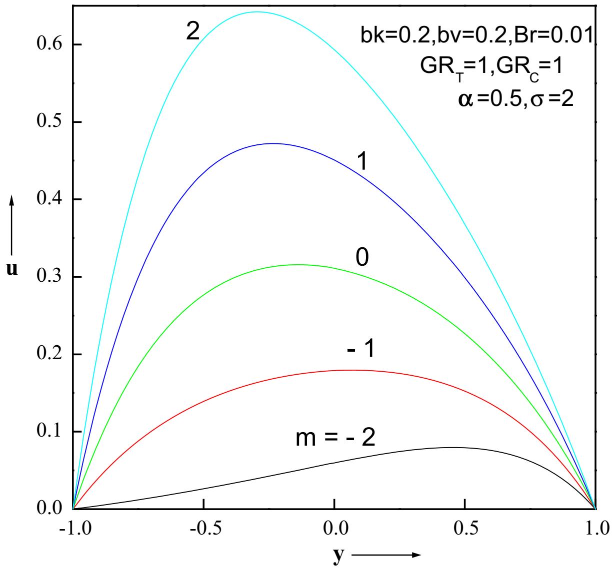


Fig. 11 Velocity profiles for different values of *m*

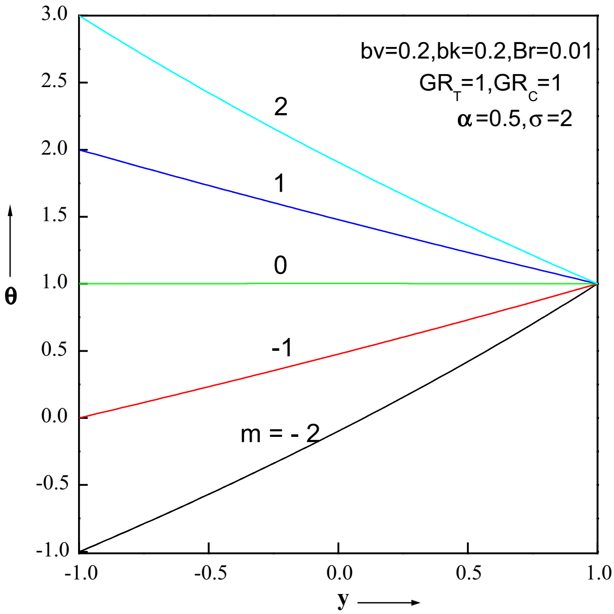


Fig. 12 Temperature profiles for different values of *m*

concentration fields are reduced with an increase in α. Physically an increase in α leads to the increase in the number of solute molecules undergoing chemical reaction which decreases the fluid field. This is the similar result observed by Damesh and Shannak (2010) for viscoelastic fluid, and Krishnendu (2012) for viscous fluid with constant properties. The effect of variable

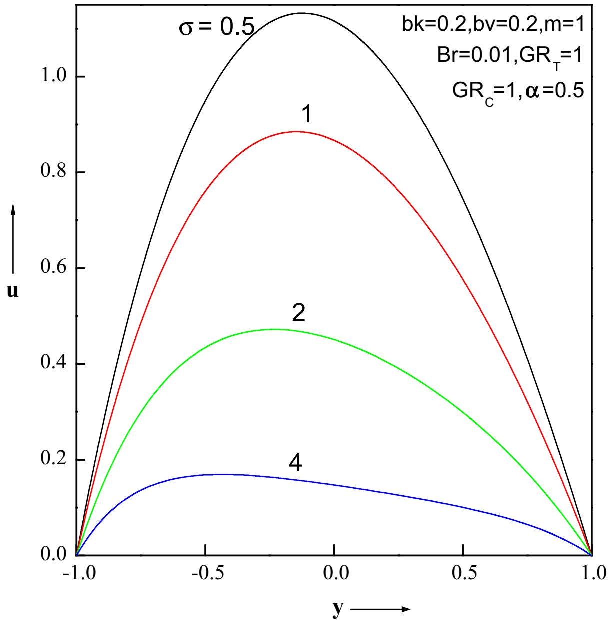


Fig. 13 Velocity profiles for different values of σ

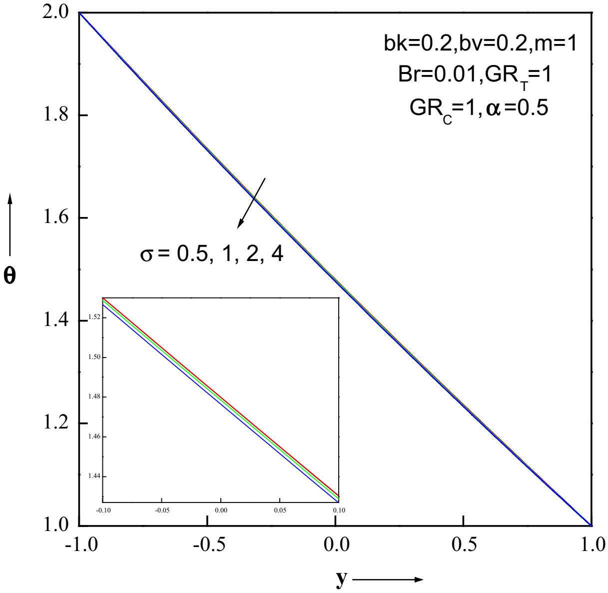


Fig. 14 Temperature profiles for different values of σ

viscosity parameter b_v, variable thermal conductivity parameter b_k, thermal Grashof number GR_T, mass Grashof number GR_C, Brinkman number Br, the wall temperature ratio m, and porous parameter σ on the concentration fields is invariant, because the concentration equation does not include these parameters.

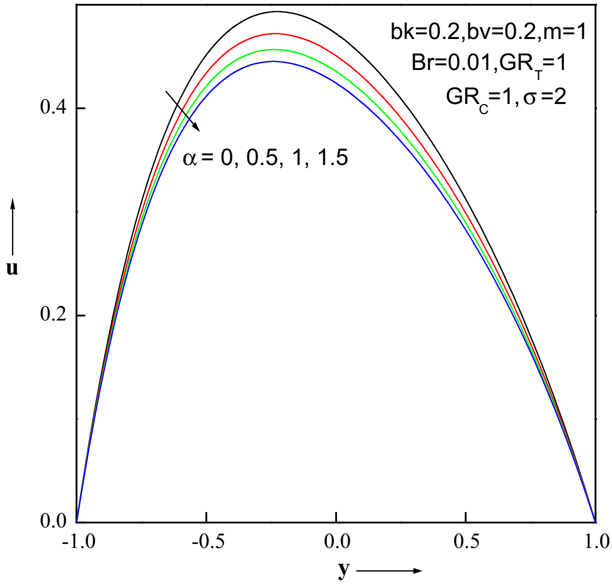


Fig. 15 Velocity profiles for different values of α

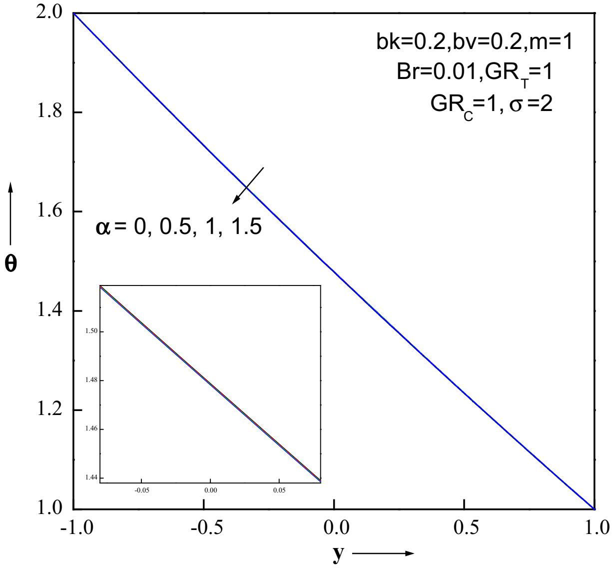


Fig. 16 Temperature profiles for different values of α

The effect of variable viscosity, variable thermal conductivity, thermal Grashof number GR_T, mass Grashof number GR_C, Brinkman number Br, the wall temperature ratio m, porous parameter σ, and first-order chemical reaction parameter α on skin friction and Nusselt number is shown in Table 1. Fixing $b_v = 0.2$, increasing the values of b_k decreases the skin friction and Nusselt number at the left wall and increases at the right wall. Fixing b_k and

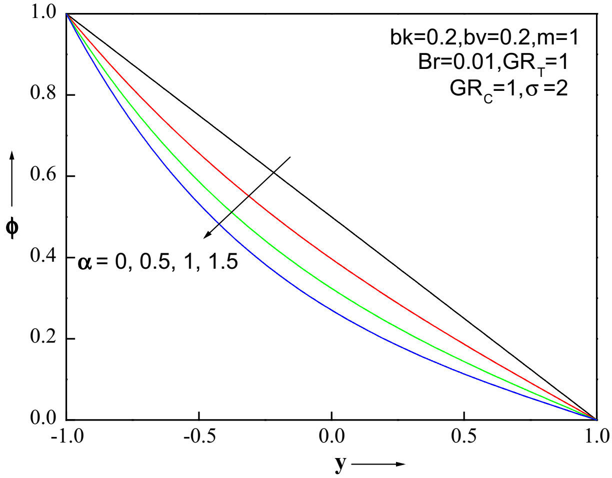


Fig. 17 Connection profiles for different α

increasing the values of b_v show that skin friction and Nusselt number increase at the left wall and decrease at the right wall. The thermal Grashof number GR_T, mass Grashof number GR_C, and Brinkman number Br increase the skin friction and Nusselt number at the left wall and decrease at the right wall. For increase in the wall temperature ratio m, skin friction increases at the left wall and decreases at the right wall whereas the Nusselt number decreases at both the walls. The porous parameter σ and first-order chemical reaction parameter α decrease the skin friction and Nusselt number at the left wall and increase at the right wall.

The analytical solutions obtained by regular perturbation methods are valid only for small values of perturbation parameter. To overcome this restriction, the governing equations are solved using Runge–Kutta shooting method. The validity of Runge–Kutta shooting method is justified by comparing the results obtained by perturbation method and Runge–Kutta shooting method in the absence of Brinkman number Br and is displayed in Table 2. It is viewed that the analytical and numerical solutions are exact for $b_k = 0$. The error increases as the second perturbation parameter b_k increases.

5 Conclusion

The problem of free convective flow, heat and mass transfer in a vertical channel filled with porous medium was analyzed for the variation of viscosity and thermal conductivity on the temperature. The analytical solutions were found by perturbation parameter method valid for small values of perturbation parameter, and numerical solutions were found by Runge–Kutta shooting method valid for any values of governing parameters. The Runge–Kutta shooting method and perturbation method show good agreement in the absence of Brinkman number and conductivity variation parameter. The following results were drawn.

1. Increase in the variable viscosity parameter enhances the flow and heat transfer, whereas increase in the variable thermal conductivity parameter suppresses the flow and heat transfer.
2. The thermal Grashof number, mass Grashof number, wall temperature ratio, and Brinkman number enhance the flow.

Table 1 Computations showing the effect of parameter variations on skin friction and Nusselt number

	$\frac{du}{dy}\Big\vert_{y=-1}$	$\frac{du}{dy}\Big\vert_{y=1}$	$\frac{d\theta}{dy}\Big\vert_{y=-1}$	$\frac{d\theta}{dy}\Big\vert_{y=1}$
b_k	$b_v = 0.2, m = 1, Br = 0.01, GR_T = 1, GR_C = 1, \sigma = 2, \alpha = 0.5$			
−0.5	1.68933436	−0.84123343	−0.39278627	−0.65049652
−0.2	1.67710432	−0.82868984	−0.44896098	−0.55742372
0	1.66842914	−0.82067749	−0.49329682	−0.50492406
0.2	1.65936388	−0.81298052	−0.54414787	−0.45887457
0.5	1.64510430	−0.80207567	−0.63493222	−0.40004395
b_v	$b_k = -0.2, m = 1, Br = 0.01, GR_T = 1, GR_C = 1, \sigma = 2, \alpha = 0.5$			
−0.5	0.09882489	−0.25552172	−0.45296576	−0.55387021
−0.2	0.75397621	−0.51847213	−0.45102697	−0.55556520
0	1.20848763	−0.67921321	−0.44955719	−0.55676898
0.2	1.67710432	−0.82868984	−0.44896098	−0.55742372
0.5	2.40606293	−1.03245921	−0.45168486	−0.55631542
GR_T	$b_k = 0.2, b_v = 0.2, m = 1, Br = 0.01, GR_C = 1, \sigma = 2, \alpha = 0.5$			
1	1.65936388	−0.81298052	−0.54414787	−0.45887457
5	6.49732931	−3.74482867	−0.39454005	−0.55960546
10	13.42845003	−8.11877853	0.17969868	−0.96303850
15	26.26123072	−17.46181959	2.62231362	−2.86606223
GR_C	$b_k = 0.2, b_v = 0.2, m = 1, Br = 0.01, GR_T = 1, \sigma = 2, \alpha = 0.5$			
1	1.65936388	−0.81298052	−0.54414787	−0.45887457
5	3.57688102	−1.22604264	−0.51614086	−0.47214423
10	5.97740107	−1.74499541	−0.45447735	−0.50001365
15	8.38202929	−2.26703069	−0.36311997	−0.54047831
20	10.79077891	−2.79229920	−0.24208431	−0.59355329
Br	$b_k = 0.5, b_v = -0.2, m = 1, GR_T = 1, GR_C = 1, \sigma = 2, \alpha = 0.5$			
0.1	0.74731223	−0.50850163	−0.57624723	−0.42922657
0.5	0.75951284	−0.52223702	−0.27437911	−0.58034492
1	0.77534917	−0.54071559	0.12855478	−0.78766072
1.5	0.79111059	−0.56013806	0.55345452	−1.01351819
2	0.80556956	−0.57944683	0.98575804	−1.25147476
m	$b_k = 0.2, b_v = 0.2, Br = 0.01, GR_T = 1, GR_C = 1, \sigma = 2, \alpha = 0.5$			
−2	0.04580486	−0.38034388	0.82429545	1.22920888
−1	0.45354382	−0.53412063	0.45429147	0.55211963
0	0.99227404	−0.67911247	0.00407286	−0.00324447
1	1.65936388	−0.81298052	−0.54414787	−0.45887457
2	2.45304731	−0.93456176	−1.21316995	−0.83262044
σ	$b_k = 0.2, b_v = 0.2, m = 1, Br = 0.01, GR_T = 1, GR_C = 1, \alpha = 0.5$			
0.5	2.89619323	−1.78148048	−0.53299537	−0.46719182
1	2.45131944	−1.41671329	−0.53704813	−0.46403654
2	1.65936388	−0.81298052	−0.54414787	−0.45887457
4	0.93718307	−0.36578303	−0.54988646	−0.45513947
α	$b_k = 0.2, b_v = 0.2, m = 1, Br = 0.01, GR_T = 1, GR_C = 1, \sigma = 2$			
0	1.70219637	−0.84150278	−0.54337226	−0.45940143
0.5	1.65936388	−0.81298052	−0.54414787	−0.45887457
1	1.62743587	−0.79332917	−0.54468928	−0.45851694
1.5	1.60241888	−0.77910997	−0.54509017	−0.45825924

Table 2 Comparison of velocity and temperature for various values of $b_V = 0.2, m = 1, Br = 0, GR_T = 1, GR_C = 1, \alpha = 0.5, \sigma = 2$ with Runge–Kutta shooting method

y	Velocity		Temperature					
	$b_k = 0$		$b_k = 0$		$b_k = 0.3$		$b_k = 0.6$	
	Analytical	Numerical	Analytical	Numerical	Analytical	Numerical	Analytical	Numerical
1.0	0.000000	0.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000
0.6	0.255938	0.255938	1.200000	1.200000	1.176000	1.177427	1.152000	1.157619
0.2	0.410465	0.410466	1.400000	1.400000	1.364000	1.364832	1.328000	1.331717
−0.2	0.477004	0.477004	1.600000	1.600000	1.564000	1.563404	1.528000	1.526146
−0.6	0.398282	0.398282	1.800000	1.800000	1.776000	1.774559	1.752000	1.746289
−1	0.000000	0.000000	2.000000	2.000000	2.000000	2.000000	2.000000	2.000000

3. The increase in porous parameter and chemical reaction parameter suppresses the flow. The variable viscosity parameter, variable thermal conductivity parameter, thermal Grashof number, mass Grashof number, wall temperature ratio, Brinkman number, and porous parameter were invariant on concentration field.
4. The solutions obtained by Runge–Kutta shooting method and perturbation method are exact in the absence of Brinkman number and variable conductivity parameter, and the error increases as the variable conductivity parameter increases.
5. The obtained results agreed very well with Attia (2006) for variable properties and with Umavathi (1996) for constant properties.

References

Attia, H.A.: Transient MHD flow and heat transfer between two parallel plates with temperature dependent viscosity. Mech. Res. Commun. **26**, 115–121 (1999)

Attia, H.A.: Influence of temperature dependent viscosity on the MHD-channel flow of dusty fluid with heat transfer. Acta Mech. **151**, 89–101 (2001)

Attia, H.A.: Unsteady hydromagnetic channel flow of dusty fluid with temperature dependent viscosity and thermal conductivity. Heat Mass Transf. **42**, 779–787 (2006)

Attia, H.A.: Unsteady hydromagnetic Couette flow of dusty fluid with temperature dependent viscosity and thermal conductivity under exponentially decaying pressure gradient. Commun. Nonlinear Sci. Numer. Simul. **13**, 1077–1088 (2008)

Barletta, A.: Fully developed laminar forced convection in circular ducts for power-law fluids with viscous dissipation. Int. J. Heat Mass Transf. **40**, 15–26 (1997)

Damesh, R.A., Shannak, B.A.: Viscoelastic fluid flow past an infinite vertical porous plate in the presence of first order chemical reaction. Appl. Math. Mech **31**(8), 955–962 (2010)

Herwig, H., Wicken, G.: The effect of variable properties on laminar boundary layer flow. Warme-und Stoffubertrag. **20**, 47–57 (1986)

Hossain, M.A., Munir, M.S.: Natural convection flow of a viscous fluid about a truncated cone with temperature-dependent viscosity and thermal conductivity. Int. J. Numer. Methods Heat Fluid Flow **11**(6), 494–510 (2001a)

Hossain, M.A., Munir, M.S., Pop, I.: Natural convection with variable viscosity and thermal conductivity from a vertical wavy cone. Int. J. Therm. Sci. **40**, 437–443 (2001b)

Kalidas, D.: Influence of thermophoresis and chemical reaction on MHD micropolar fluid flow with variable fluid properties. Int. J. Heat Mass Transf. **55**, 7166–7174 (2012)

Kandasamy, R., Perisamy, K., Sivagnana Prabhu, K.K.: Effects of chemical reaction, heat and mass transfer along a wedge with heat source and concentration in the presence of suction or injection. Int. J. Heat Mass Transf. **48**, 1388–1394 (2005a)

Kandasamy, R., Perisamy, K., Sivagnana Prabhu, K.K.: Chemical reaction, heat and mass transfer on MHD flow over a vertical stretching surface with heat source and thermal stratification effects. Int. J. Heat Mass Transf. **48**, 4557–4561 (2005b)

Klemp, K., Herwig, H., Selmann, M.: Entrance flow in channel with temperature dependent viscosity including viscous dissipation effects. In: Proceedings of the Third International Congress of Fluid Mechanics, Cairo, Egypt **3**, 1257–1266 (1990)

Krishnendu, B.: Slip effects on boundary layer flow and mass transfer with chemical reaction over a permeable flat plate in porous medium. Front. Heat Mass Transf. **3**, 1–6 (2012)

Muthucumaraswamy, R., Ganesan, P.: On impulsive motion of a vertical plate with heat flux and diffusion of chemically reactive species. Forch Ing. **66**, 17–23 (2000)

Muthucumaraswamy, R., Ganesan, P.: First order chemical reaction on flow past an impulsively started vertical plate with uniform heat and mass flux. Acta Mech. **147**, 45–57 (2001)

Nield, D.A., Bejan, A.: Convection in Porous Media, 4th edn. Springer, New York (2013)

Palani, G., Kim, K.Y.: Numerical study on vertical plate with variable viscosity and thermal conductivity. Arch. Appl. Mech. **80**, 711–725 (2010)

Saravanan, S., Kandaswamy, P.: Hydromagnetic stability of convective flow of variable viscosity fluids generated by internal heat sources. Z. Angew. Math. Phys. **55**, 451–467 (2004)

Schlichting, H.: Boundary Layer Theory. McGraw-Hill, New York (1968)

Seddeek, M.A.: Effects of radiation and variable viscosity on a MHD free convection flow past a semi-infinite flat plate with an aligned magnetic field in the case of unsteady flow. Int. J. Heat Mass Transf. **45**, 931–935 (2002)

Seddeek, M.A.: Finite element method for the effects of chemical reaction, variable viscosity, thermophoresis and heat generation/absorption on a boundary layer hydromagnetic flow with heat and mass transfer over a heat surface. Acta Mech. **177**, 1–18 (2005)

Shivaiah, S., Anandrao, J.: Chemical reaction effect on unsteady MHD free convection flow past a vertical porous plate in the presence of suction or injection. Appl. Mech. **39**(2), 185–208 (2012)

Sutton, G.W., Sherman, A.: Engineering Magnetohydrodynamics. McGraw-Hill, New York (1965)

Umavathi, J.C.: A note on magneto convection in a vertical enclosure. Int. J. Nonlinear Mech. **31**, 371–376 (1996)

Umavathi, J.C., Veershetty, S.: Mixed convection of a permeable fluid in a vertical channel with boundary conditions of third kind. Heat Transf. Asian Res. **41**(6), 516–535 (2012)

White, M.F.: Viscous Fluid Flow. McGraw-Hill, New York (1991)

Zanchini, E.: Effect of viscous dissipation on the asymptotic behavior of laminar forced convection in circular tubes. Int. J. Heat Mass Transf. **40**, 169–178 (1997)