

$$\begin{pmatrix} 2 \\ 1 \\ -7 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ -7 \end{pmatrix}$$

$$\text{Cond}(A) = \|A\| \|A^{-1}\|$$

if $\text{Cond}(A) \approx 1$ Well Conditioned
if $\text{Cond}(A)$ is large, ill-Conditioned

$$A = \begin{pmatrix} 1 & -7 \\ -2 & -8 \end{pmatrix}$$

$$\|A\|_1 = \max\{8, 10\} = 10$$

$$\|A\|_\infty = \max\{8, 5\} = 8$$

$$\|A\|_2 = \sqrt{2 + 6 + 2 + 6 + 4 + 4} = \sqrt{24} \approx 4.9$$

$$a) \begin{cases} x+y=2 \\ 2+1 \cdot 0.01y=2 \end{cases} \quad b) \begin{cases} x+y=2 \\ 2+1 \cdot 0.01y=2.001 \end{cases}$$

$$0.001y=0 \quad (y=0) \Rightarrow x=2$$

$$0.001y=0.001 \quad (y=1) \Rightarrow x=1$$

$$A = \begin{pmatrix} 1 & 1 \\ 1 & 1.001 \end{pmatrix} \quad \|A\|_1 = 2.001$$

$$\|A\|_1 = 2.001 \quad \text{Cond}(A) = \|A\| \|A^{-1}\|$$

$$A^{-1} = \frac{1}{0.001} \begin{pmatrix} 1.001 & -1 \\ -1 & 1 \end{pmatrix} = \begin{pmatrix} 1001 & -1000 \\ -1000 & 1000 \end{pmatrix}$$

$$0.001x+y=1 \quad x+y=2$$

$$A = \begin{pmatrix} 0.001 & 1 \\ 1 & 1 \end{pmatrix}$$

$$\text{Cond}(A) \approx 4$$

$$0.001x+y=1 \quad 0.001x+y=1$$

$$-999y = -998 \quad y = \frac{998}{999}$$

$$\tilde{A} = \begin{pmatrix} 0.001 & 1 \\ 0 & 1 \end{pmatrix}, \text{Cond}(\tilde{A}) = 200$$

$$0.001x+y=1 \quad x+y=2$$

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = b_1$$

$$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 = b_2$$

$$a_{31}x_1 + a_{32}x_2 + a_{33}x_3 = b_3$$

$$A \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \cdot b = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$$

$$A = LU$$

$$AX = b$$

$$L(UX) = b$$

$$Lz = b \quad \leftarrow \text{forward substn}$$

$$UX = z \quad \leftarrow \text{back substn}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 1/2 & 1 & 0 & 0 \\ -2 & -2 & 1 & 0 \\ -1 & 1 & -2 & 1 \end{pmatrix} \begin{pmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{pmatrix} = \begin{pmatrix} 2 \\ -4 \\ 8 \\ -43 \end{pmatrix}$$

$$\text{forward substn: } z_1 = 2$$

$$z_1 + z_2 = -4 \quad z_2 = -6$$

$$z_2 + z_3 = -4 \quad z_3 = 2$$

$$z_3 + z_4 = -43 \quad z_4 = -45$$

$$UX = z$$

$$\begin{pmatrix} 6 & -2 & -4 & 4 \\ 0 & -2 & -4 & -1 \\ 0 & 0 & 5 & -2 \\ 0 & 0 & 0 & 8 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 2 \\ -5 \\ 2 \\ -32 \end{pmatrix}$$

$$\text{back substn: } 8x_4 = -32 \Rightarrow x_4 = -4$$

$$5x_3 - 2x_4 = 2 \Rightarrow x_3 = -1.2$$

$$-2x_2 - 4x_3 - x_4 = -5 \Rightarrow x_2 = 6.9$$

$$x_1 = 4.5 \quad X = \begin{pmatrix} 4.5 \\ 6.9 \\ -1.2 \\ -4 \end{pmatrix}$$

$$x_1 + x_2 + 0x_3$$

$$A = \begin{pmatrix} 1 & 2 & -1 \\ 2 & 1 & -2 \\ -2 & 3 & 1 \end{pmatrix}$$

$$-2R_1 + R_2$$

$$3R_1 + R_3$$

$$7/3R_2 + R_3$$

$$\begin{pmatrix} 1 & 2 & -1 \\ 0 & -3 & 0 \\ 0 & 0 & 2 \end{pmatrix} = U$$

$$L = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -3 & -2 & 1 \end{pmatrix}$$