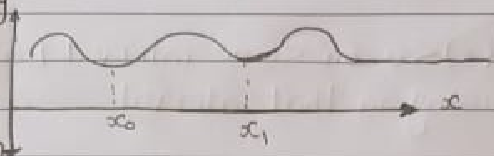
≪ Optimization ≫

≪ Lec "1" ≫

$y = F(x)$



$x_0 \rightarrow$ local minimum

$x_1 \rightarrow$ global minimum

$F(x_0) < F(x_1)$, $\forall x \in (x_0 - E, x_0 + E)$

⊛ Therom :-

→ If F is continuos on a local interval $[a, b]$, then F attains a global min and a global max

→ If C is a local min or max of a continous Function F, ~~there $F'(c) = 0$~~ then F'(c) = 0,if F'(c)exists

This is known as Fermat's theorem. It's important to note that this condition is necessary, but not sufficient. There are points where the derivative of a function is zero that are not local maxima or minima, these points are typically referred to as saddle points or inflection points.

ex_1 :- $F(x) = 3x - x^2$, Find all critical Point

$F'(x) = 3 - 2x$, $F'(x) = 0$

$3 - 2x = 0$, $x = 1.5$ (there is one critical Point)

Note → $F'(C) = 0$, then C is a critical Point

→ "x_0" is a "max" or "min" Point $\overset{\overset{X}{\longleftarrow}}{\underset{\checkmark}{\longrightarrow}}$ "x_0" is a "critical" Point

- if $f'(x) > 0$ on some interval $[a, b]$, then f is increasing in that interval.

- if $f'(x) < 0$ on some interval $[a, b]$, then f is decreasing in that interval

ex2:- Find interval of increase and the interval of decrease of: $F(x) = 3x^4 - 4x^3 - 12x^2 + 5$

$$F'(x) = 12x^3 - 12x^2 - 24x$$
$$= 12x(x^2 - x - 2)$$
$$= 12x(x-2)(x+1)$$

there is three critical Point $(0, 2, -1)$

$\mathbb{R}$

	$-\infty$ -2	-1 0	1 2	∞
$12x$	$-$	$-$	$+$	$+$
$(x-2)$	$-$	$-$	$-$	$+$
$(x+1)$	$-$	$+$	$+$	$+$
sighn (f)	$-$	$+$	$-$	$+$
shap (f)	↙	↗	↙	↗
	(Local) min	max	min	max

→ increase :- $(-1, 0) \cup (2, \infty)$

→ decrease:- $(-\infty, -1) \cup (0, 2)$

- Convex Function :- ∪ (min)
- Concave function :- ∩ (max)

→ If $f'(x) \geqslant 0$ in an interval "I", then it's a Convex

→ If $f'(x) \leqslant 0$ in an interval "I", then it's a Concave

ex$_3$:- $F(x) = x^3 - x^2$

$F'(x) = 3x^2 - 2x$

$= x(3x - 2)$, $x = 0$, $x = 2/3$

$F''(x) = 6x - 2$, $x = 1/3$

- $f'(x)$ Point's → critical Points
- $f''(x)$ Point's → inflection Points