Lecturer: Dr. Torabi
Spring 1402 - 1403

Game Theory
Homework 2

Submitted: 1403/01/22 00:00
Deadline: 1403/02/15 23:59

Upload your answers as a PDF file on Course-Ware with the name: StudentNumber.pdf

1. Consider two companies, I and II, manufacturing products A and B, respectively. Here the players choose product prices as their strategies. Assume that company I declares the unit prices of c_1, while company II declares the unit prices of c_2. As the result of prices quotation, one observes the demands for each product on the market, i.e., $Q_1(c_1, c_2) = q - c_1 + kc_2$ and $Q_2(c_1, c_2) = q - c_2 + kc_1$. The symbol q means an initial demand, and the coefficient k reflects the interchangeability of products A and B. The unit cost will be specified by c for both companies. Consequently, the players' payoffs acquire the form $H_1(c_1, c_2) = (q - c_1 + kc_2)(c_1 - c)$ and $H_2(c_1, c_2) = (q - c_2 + kc_1)(c_2 - c)$. Find the (pure) Nash equilibrium of the corresponded game.

2. With the assumptions of Cournot's game with two players discussed in class, find the maximum sum of the payoff of the players when they collaborate. Compare that with sum in the Nash equilibrium case.

3. Consider Cournot's game in the case of an arbitrary number n of firms; retain the assumptions that the inverse demand function takes the form

$$P(Q) = \begin{cases} \alpha - Q & Q \leq \alpha \\ 0 & Q > \alpha \end{cases}$$

where $\alpha > 0$ and the cost function of each firm i is $C_i(q_i) = cq_i$ for all q_i, with $0 \leq c < \alpha$. Find the best response function of each firm and set up the conditions for $(q_1^*, ..., q_n^*)$ to be a (pure) Nash equilibrium, assuming that there is a Nash equilibrium in which all firms' outputs are positive. Solve these equations to find the Nash equilibrium. First show that in an equilibrium all firms produce the same output, then solve for that output. Find the price at which output is sold in a Nash equilibrium and show that this price decreases as n increases, approaching c as the number of firms increases.

4. Two firms are developing competing products for a market of fixed size. The longer a firm spends on development, the better its product. But the first firm to release its product has an advantage: the customers it obtains will not subsequently switch to its rival. (Once a person starts using a product, the cost of switching to an alternative, even one significantly better, is too high to make a switch worthwhile.) A firm that releases its product first, at time t, captures the share $h(t)$ of the market, where h is a function that increases from time 0 to time T, with $h(0) = 0$ and $h(T) = 1$. The remaining market share is left for the other firm. If the firms release their products at the same time, each obtains half of the market. Each firm wishes to obtain the highest possible market share. Model this situation as a strategic game and find its (pure) Nash equilibria (When finding firm i's best response to firm j's release time t_j, there are three cases: that in which $h(t_j) < \frac{1}{2}$ (firm j gets less than half of the market if it is the first to release its product), that in which $h(t_j) = \frac{1}{2}$, and that in which $h(t_j) > \frac{1}{2}$).

5. Player I chooses a positive integer $x > 0$ and player II chooses a positive integer $y > 0$. The player with the lower number pays a dollar to the player with the higher number unless the higher number is more than twice larger in which case the payments are reversed; in other words, the payoff function is:

$$u_1(x,y) = \begin{cases} 1 & y < x \leq 2y \quad \text{or} \quad x < \dfrac{y}{2} \\ 0 & x = y \\ -1 & x < y \leq 2x \quad \text{or} \quad y < \dfrac{x}{2} \end{cases}$$

and similarly for player II. Find a (mixed) Nash equilibrium strategy in this game (you do not have to find every single mixed equilibrium).

6. Consider the game

1,1	a,0
0,0	2,1

where $a \in \mathbb{R}$. For every value of a, determine the (mixed) Nash equilibria of the game.

7. Find the (mixed) Nash equilibria of the following game by using two different methods:

2,2	0,3	1,2
3,1	1,0	0,2

I) By eliminating strictly dominated actions and then using best response functions.

II) By using the algorithm described in class which results from the characterization of mixed Nash equilibria of finite games.