Matrix Project

Jayasimha Reddy Rahul Devaraj

IITH

ee15btech11024@iith.ac.in

ce15btech11002@iith.ac.in

February 14, 2019

Geometry Question

JEE Problems in Linear Algebra - Q33

A circle passes through $\begin{bmatrix} -2 \\ 4 \end{bmatrix}$ and touches the y-axis at $\begin{bmatrix} 0 \\ 2 \end{bmatrix}$

Which one of the following equations can represent a diameter of this circle?

a) $\begin{bmatrix} 4 & 5 \end{bmatrix} x = 6$

b) $\begin{bmatrix} 2 & -3 \end{bmatrix} x + 10 = 0$

c) $\begin{bmatrix} 3 & 4 \end{bmatrix} x = 3$

d) $\begin{bmatrix} 5 & 2 \end{bmatrix} x + 4 = 0$

Matrix Transformation of Question

JEE Problems in Linear Algebra - Q3

Let's say point $\begin{bmatrix} -2 \\ 4 \end{bmatrix}$ is P_1, $\begin{bmatrix} 0 \\ 2 \end{bmatrix}$ is P_2 and centre C as $\begin{bmatrix} x \\ 2 \end{bmatrix}$

We know that in a circle $\mid \overline{CP_1} \mid = \mid \overline{CP_2} \mid$,

$$(P_1 - C)^T (P_1 - C) = (P_2 - C)^T (P_2 - C)$$

Solution

$$C^T(P_2 - P_1) = (C^T(P_2 - P_1))^T \qquad [integer]$$

$$C^T(P_2 - P_1) = \frac{P2^T P_2 - P_1^T P_1}{2} = \frac{\mid P_2 \mid^2 - \mid P_1 \mid^2}{2}$$

$$[\text{x} \;\; 2]\begin{bmatrix} 2 \\ -2 \end{bmatrix} = \frac{\mid P_2 \mid^2 - \mid P_1 \mid^2}{2} = -8$$

$$2x - 4 = -8$$

$$\implies x = -2$$

Solution

$$\implies \mathit{CentreC} = \begin{bmatrix} -2 \\ 2 \end{bmatrix}$$

$$\implies \text{Option a)} \begin{bmatrix} 4 & 5 \end{bmatrix} \begin{bmatrix} -2 \\ 2 \end{bmatrix} = 6$$

$$2 = 6 \qquad (\mathit{False})$$

$$\implies \text{Option b)} \begin{bmatrix} 2 & -3 \end{bmatrix} \begin{bmatrix} -2 \\ 2 \end{bmatrix} + 10 = 0$$

$$-10 + 10 = 0 \qquad (\mathit{True})$$

Solution

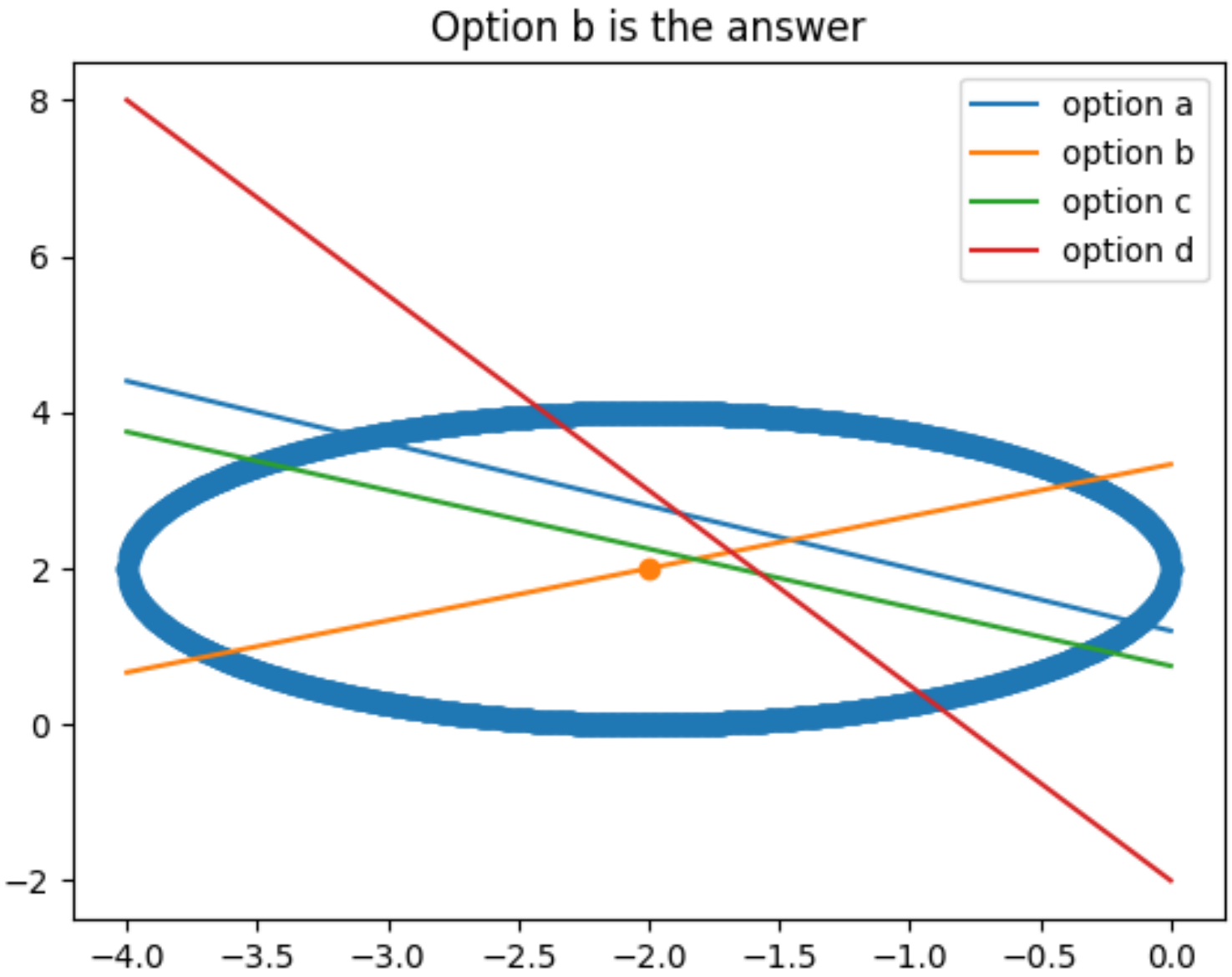
$$\implies \text{Option c) } \begin{bmatrix} 3 & 4 \end{bmatrix} \begin{bmatrix} -2 \\ 2 \end{bmatrix} + 10 = 0$$

$$2 = 3 \qquad (False)$$

$$\implies \text{Option d) } \begin{bmatrix} 5 & 2 \end{bmatrix} \begin{bmatrix} -2 \\ 2 \end{bmatrix} + 4 = 0$$

$$-6 + 4 = 0 \qquad (False)$$

Diagram


Option b is the answer
option a
option b
option c
option d
8
6
4
2
0
−2
−4.0
−3.5
−3.0
−2.5
−2.0
−1.5
−1.0
−0.5
0.0