Solution

Roll No. _____ Name_____ Group_____

Computer Science & Engineering Department
Thapar Institute of Engineering & Technology, Patiala
Discrete Mathematical Structures (UCS405)
Quiz-2

Time: 10 min **Max Marks: 10**

Instructions for students:

- *Any cutting or overwriting will be considered as a wrong answer.*
- *Missing roll no. or name will be considered as an absent.*
- *Write the answer only in the space provided otherwise no marks will be given.*

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1. Write the correct translation of the following statement into mathematical logic using quantifier. "Some real numbers are rational".
Ans: $\exists x (real(x) \wedge rational(x))$

2. Suppose the predicate F(x, y, t) is used to represent the statement that person x can fool person y at time t. No one can fool everyone all the time. statement express best the meaning of the formula $\forall x \exists y \exists t (\neg F(x, y, t))$?

3. Write the correct translation of the following statement into mathematical logic using quantifier. "Tigers and lions attack if they are hungry or threatened."
Ans: $\forall x [(tiger(x) \vee lion(x)) \rightarrow \{(hungry(x) \vee threatened(x)) \rightarrow attacks(x)\}]$

4. A bag contains 10 red marbles, 10 white marbles, and 10 blue marbles. What is the minimum no. of marbles you have to choose randomly from the bag to ensure that we get 4 marbles of same color?
(a) 9 (b) 10 (c) 11 (d) None of the above

5. Suppose a laundry bag contains many red, white and blue socks. Find the minimum number of socks that one needs to choose in order to get two pairs of the same color.
Ans: 10

6. Which of the following represent the sequence 1, 2, 5, 11, 26, ...
(a) $t_n = t_{n-1} + t_{n-2}$ $t_0 = 1, t_1 = 2$
(b) $t_n = t_{n-1} + 3t_{n-2}$ $t_0 = 1, t_1 = 2$
(c) $t_n = 2t_{n-1} + 1$ $t_0 = 1, t_1 = 2$
(d) $t_n = 2t_{n-1} + 2$ $t_0 = 1, t_1 = 2$

7. The maximum degree of any vertex in a simple graph with n vertices are ___n-1___.

8. Consider the following graph: V = {V_0, V_1, V_2, V_3, V_4, V_5, V_6}. There are the following twelve edges, with edge costs listed as the third item in the triplet:
E = {(V_0,V_2, 4), (V_1, V_0, 2), (V_1, V_3, 3), (V_3, V_0, 1), (V_3, V_2, 2), (V_3, V_5, 8), (V_3, V_6, 4), (V_4,V_1, 10), (V_4, V_3, 2), (V_4, V_6, 7), (V_5, V_2, 2), (V_6, V_5, 1)}

What is the cost of its minimum spanning tree?
(a) 1 (b) 10 (c) 11 (d) 12

9. In Euler Circuit all the vertices are of ___even___ degree.

10. A Complement of a cyclic graph on 5 vertices has a Hamiltonian Circuit ___True___. (True/ False)

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Tutorial Evaluation **Time: 20 min** **Max Marks: 10**

Q 1: Use mathematical induction to prove De Moivre's theorem $[R(\cos t + i \sin t)]^n = R^n(\cos nt + i \sin nt)$ for every positive integer n.

Step 1: For $n=1$

$[R(\cos t + i \sin t)]^1 = R^1(\cos 1*t + i \sin 1*t)$

It can easily be seen that the two sides are equal.

Step 2: We now assume that the theorem is true for $n=k$ hence

$[R(\cos t + i \sin t)]^k = R^k(\cos kt + i \sin kt)$

Multiply both sides of the above equation by $R(\cos t + i \sin t)$

$[R(\cos t + i \sin t)]^k R(\cos t + i \sin t) = R^k(\cos kt + i \sin kt) R(\cos t + i \sin t)$

Rewrite the above as follows

$[R(\cos t + i \sin t)]^{k+1} = R^{k+1}[(\cos kt \cos t - \sin kt \sin t) + i(\sin kt \cos t + \cos kt \sin t)]$

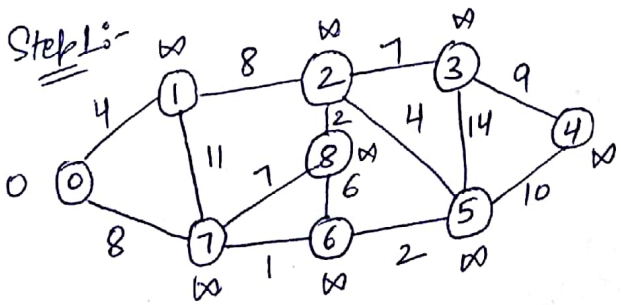
$(\cos kt \cos t - \sin kt \sin t) = \cos(kt+t) = \cos(k+1)t$

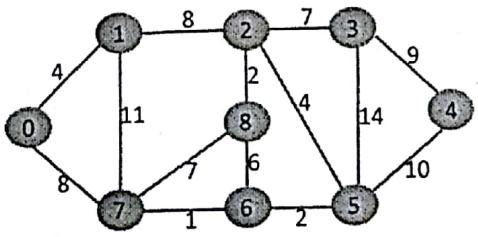
$(\sin kt \cos t + \cos kt \sin t) = \sin(kt+t) = \sin(k+1)t$

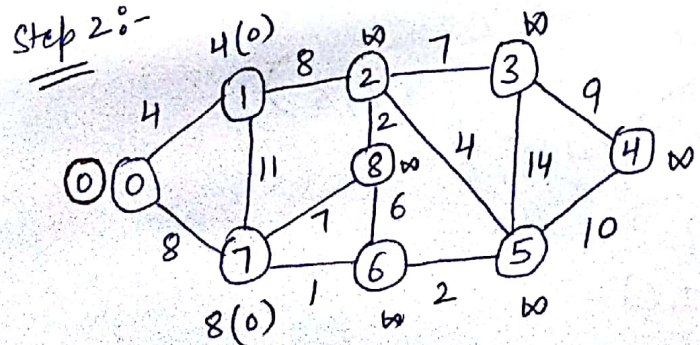
Substitute the above

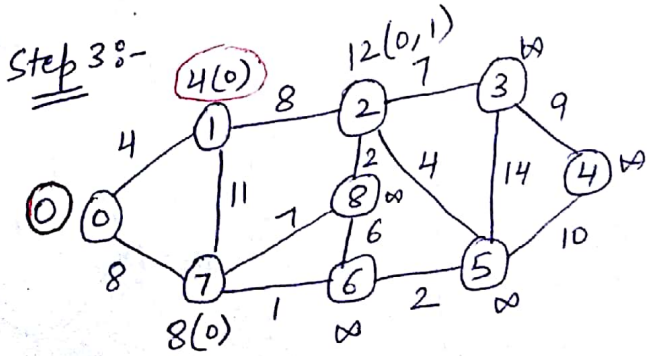
$[R(\cos t + i \sin t)]^{k+1} = R^{k+1}[\cos(k+1)t + \sin(k+1)t]$ Hence proved

Q 2: Implement Dijkstra's algorithm step by step on the following graph to find the minimum distance and path from vertex 0 to vertex 4.

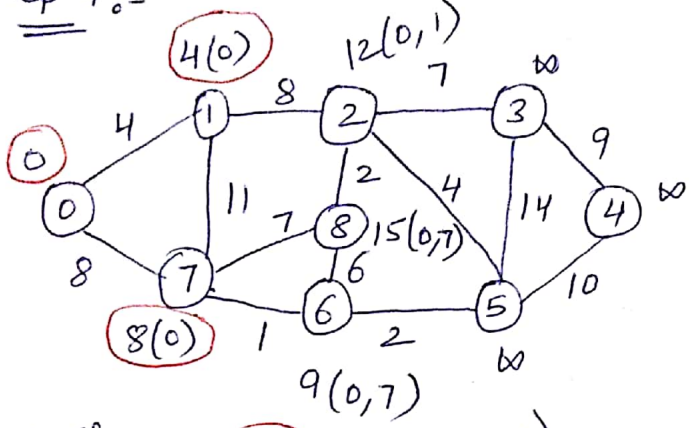








Step 4:-



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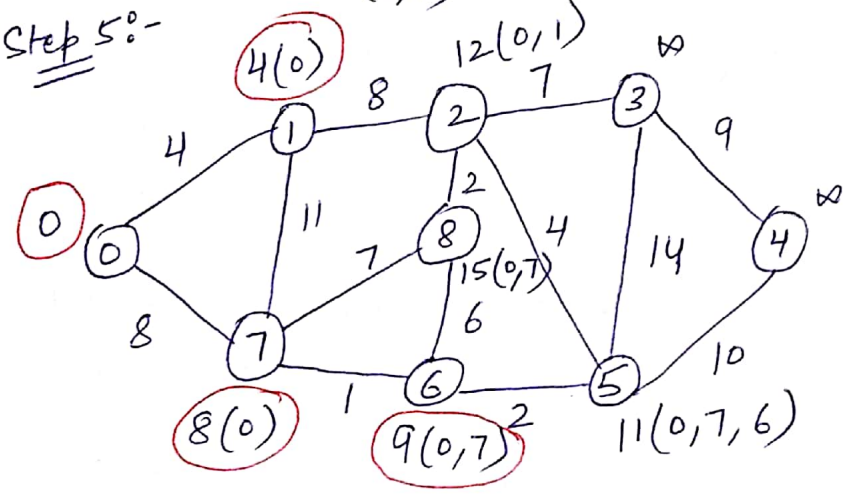

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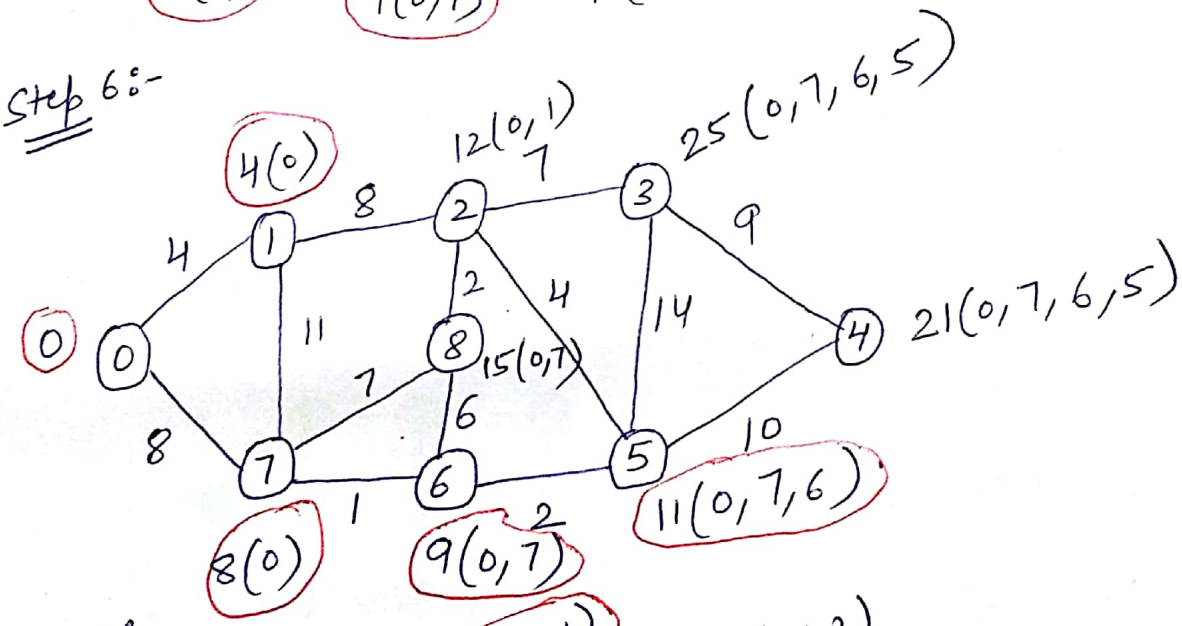
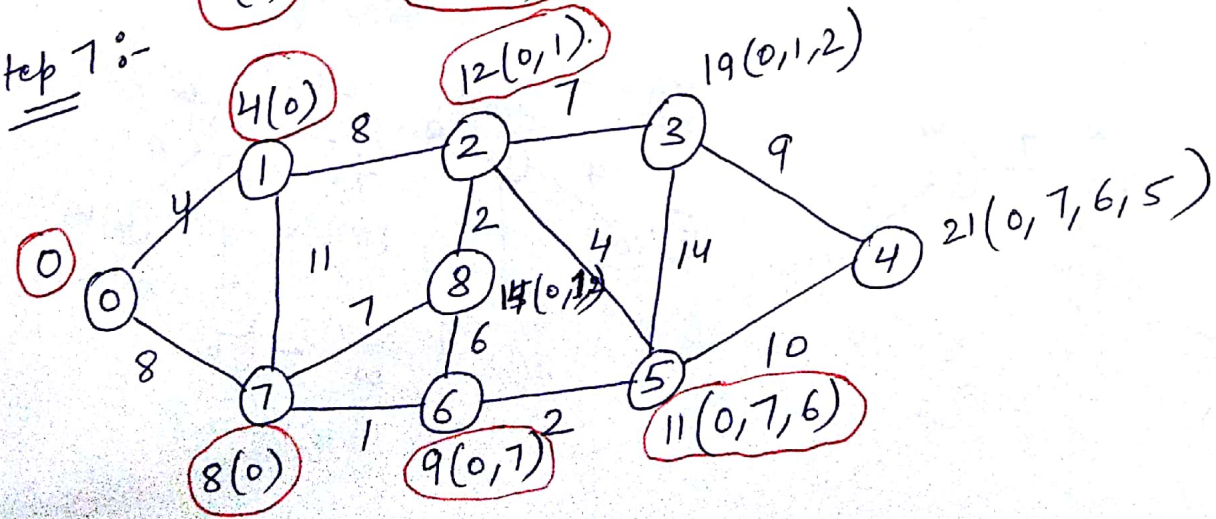


Step 7:-



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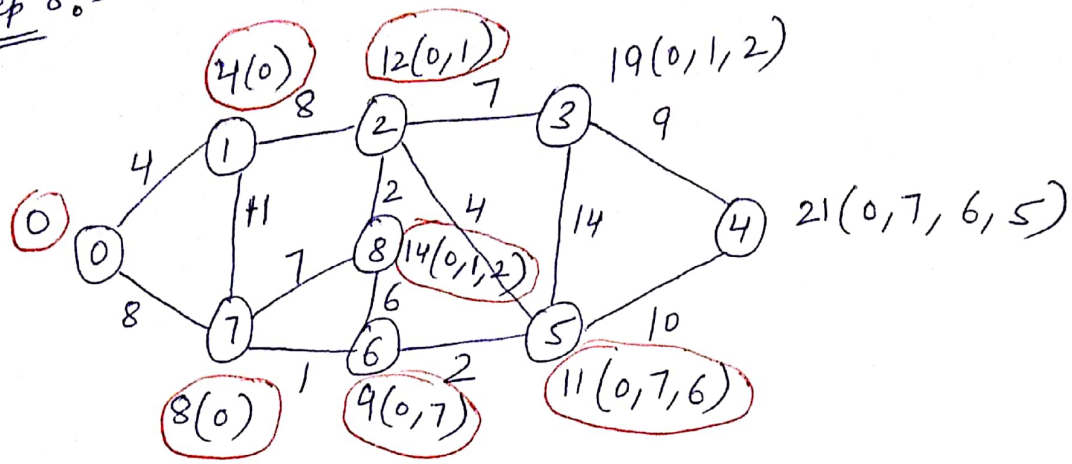

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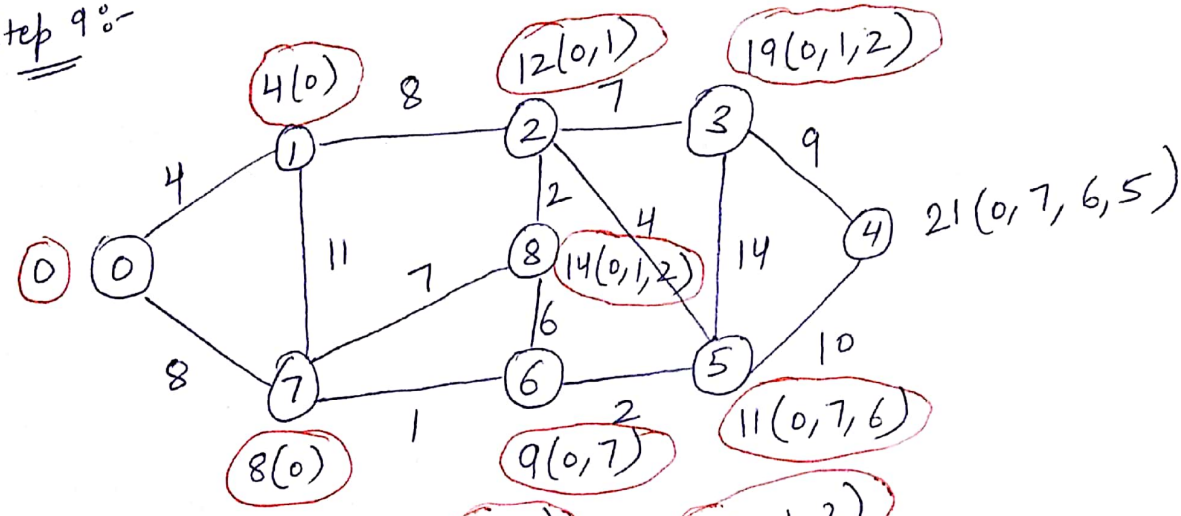

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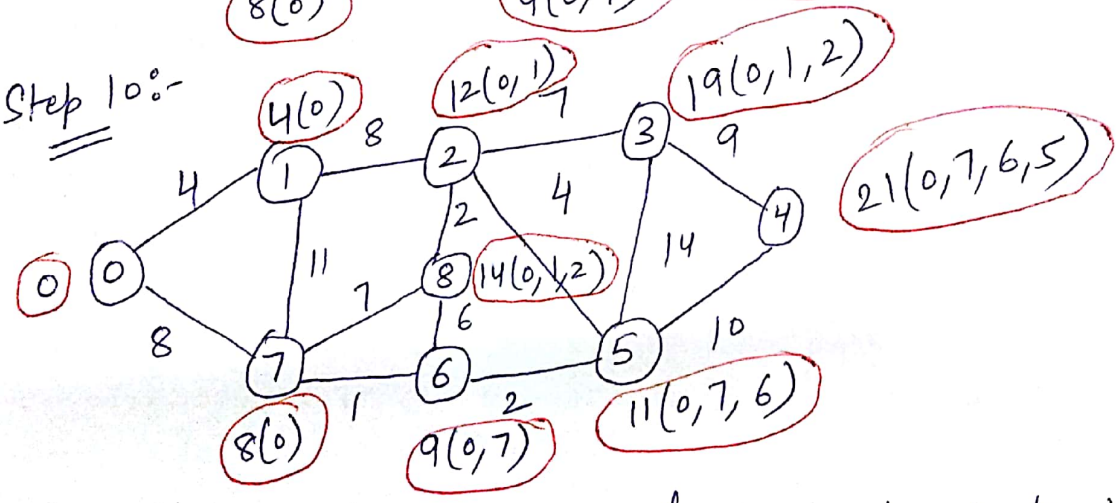

Minimum distance is 21 from Vertex 0 to Vertex 4.