

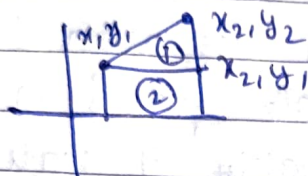
SACHIN

2019CS10722

COL 216

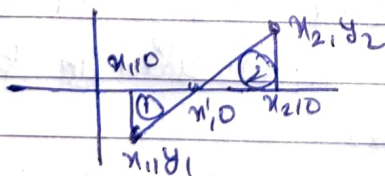
COL216: Assignment 1Approach:

Area under the curve can be found by breaking area in area formed by successive points & adding them.

a) if $y_1, y_2 > 0$ 

$$\begin{aligned} \text{Area} &= A_1 + A_2 = \frac{1}{2} (x_1(y_1 - y_2) + x_2(y_2 - y_1) + 0) + (x_2 - x_1)y_1 \\ &= \frac{(x_2 - x_1)(y_1 + y_2)}{2} \end{aligned}$$

if y_1, y_2 both +ve consider this as area
& if y_1, y_2 both -ve take absolute value.

b) if $y_1, y_2 < 0$ 

$$\text{Area} = A_2 - A_1$$

using slope of line $\frac{y_2 - y_1}{x_2 - x_1} = \frac{y_2}{x_2 - x_1}$

$$\Rightarrow x' = x_1 - y_1 \left(\frac{x_2 - x_1}{y_2 - y_1} \right)$$

$$\& \text{Area} = \frac{(x_2 - x_1)(y_1^2 + y_2^2)}{2(y_2 - y_1)}$$

if $y_2 > 0, y_1 < 0$ take this as area
& if $y_2 < 0, y_1 > 0$ take abs value.

$$A_{\text{total}} = \sum_{i=1}^{n-1} A_{i+1,i}, \text{ where } A_{i+1,i} = \begin{cases} \frac{1}{2} |(x_2 - x_1)y_2 + y_1| & \text{if } y_1, y_2 \geq 0 \\ \frac{1}{2} (x_2 - x_1)(y_1^2 + y_2^2) & \text{if } y_1, y_2 < 0 \end{cases}$$

Constraints:

- 1) While deriving formula we assumed $x_2 \geq x_1$, so input should be sorted w.r.t x .
- 2) No. of points given by user should be > 0 .
- 3) Area under single point is initially 0.
- 4) Area computed accurately if values calculated at any step is \geq in b/w 2^{-32} to $2^{32}-1$.
ie $2^{-32} < \text{area} < 2^{32}$
 $2^{-32} < y_1^2, y_2^2 < 2^{32}-1$
 $2^{-32} < x_1, x_2 < 2^{32}-1$.

5)

Input taking:

Firstly no. of points is taken as input then one by one in single line x & y coordinates are taken (NOTE: first x_1 is taken in 1 line then y_1 in new line).

- At any point if constraints are violated appropriate error is thrown.

Code in words:

- registers used:

$$t_0, t_1, t_2, t_3 = x_i, x_{i+1}, y_i, y_{i+1}$$

Registers used:

- $t_0, t_1, t_2, t_3 = x_i, x_{i+1}, y_i, y_{i+1}$
- $f_0, f_2 =$ temp registers for appropriate use.
- $f_{1c} =$ Storing partial sum
- $f_4 = 2.0$
- $S_0 =$ no. of points
- $S_1 =$ counter to check how many points read
- $S_2 =$ checker (0 if first point is stored else 1).

Code in Words:

no. of inputs stored in S_0 then loop is started till S_1 reach S_0 , at first point S_2 is make 1 & then at each reading of y_i "area" subprogram is called.

in area, condition is check for ~~for~~ on y_1, y_2 for finding correct area.

* y_1, y_2 could be used to check condition but that demands of storing a large quantity, rather I used case by case analysis of y_1 & y_2 i.e. comparing them w.r.t 0)

* for $A = (x_2 - x_1)(y_1 + y_2)/2$ case I first computed $y_1 + y_2 / 2$ then multiplied it with $(x_2 - x_1)$ because direct multiplication might create $\rightarrow 2^{32}$ value for large input & cause inaccuracy.

* similarly for $\frac{(x_2 - x_1)(y_1^2 + y_2^2)}{2}$ I first computed

$\frac{y_1^2 + y_2^2}{2}$ then multiplied.

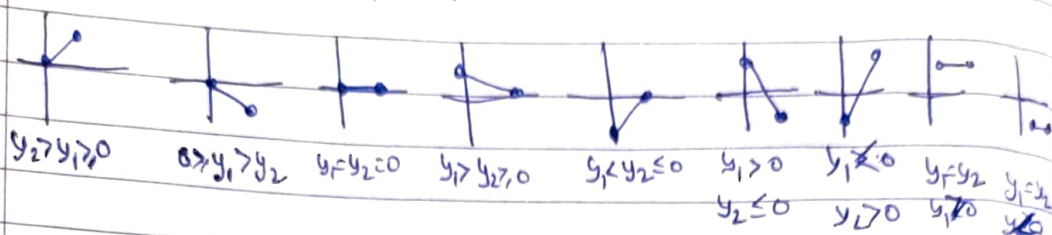
Testing:

Inducing on no. of inputs.

- Base Case:

if $n = 2$

For 2 points, 9 possible cases.



For all cases my code gives correct area.
[ALL TEST CASES ATTACHED ON PAGE - 5].

Now I tested code for $n=3$ points (one such case)
It gave correct ans hence, addition of two successive area is also computed correctly.

- Inductive Hypothesis:

let for $n=k$ code gives correct area ($k \geq 2$)

- Induction Step:

$$\text{for } n=k+1, A_{k+1} = A_k + A_{(k+1, k)}$$

Diagram illustrating the induction step:

A_k is computed correctly by I.H. (Inductive Hypothesis).

$A_{(k+1, k)}$ is computed correctly by base case.

The addition is done correctly by ~~the~~ code.

Hence by induction code is correct.

- For further rigorous testing I computed some smaller problems (4-5 points) with hand & some cases where all points are in straight line, code shows correct results.

- I checked ans for some big test cases with my friend.

- For overflow I tested code where ans is big & code shows wrong ans around 6×10^8 to 10^9 i.e. $\approx 2^{32}$ (hence code overflow constraint broken)

2 -----Test cases for BASE CASE-----

3 Number of Points: 2
4 Point 1 X Co-ordinate: 0
5 Point 1 Y co-ordinate: 0
6 Point 2 X Co-ordinate: 1
7 Point 2 Y co-ordinate: 1
8 Area under the curve formed by joining given points with straight line is: 0.5
9 Number of Points: 2
10 Point 1 X Co-ordinate: 0
11 Point 1 Y co-ordinate: -1
12 Point 2 X Co-ordinate: 1
13 Point 2 Y co-ordinate: 0
14 Area under the curve formed by joining given points with straight line is: 0.5
15 Number of Points: 2
16 Point 1 X Co-ordinate: 0
17 Point 1 Y co-ordinate: 1
18 Point 2 X Co-ordinate: 1
19 Point 2 Y co-ordinate: 0
20 Area under the curve formed by joining given points with straight line is: 0.5
21 Number of Points: 2
22 Point 1 X Co-ordinate: 0
23 Point 1 Y co-ordinate: 0
24 Point 2 X Co-ordinate: 1
25 Point 2 Y co-ordinate: -1
26 Area under the curve formed by joining given points with straight line is: 0.5
27 Number of Points: 2
28 Point 1 X Co-ordinate: 0
29 Point 1 Y co-ordinate: 1
30 Point 2 X Co-ordinate: 1
31 Point 2 Y co-ordinate: -1
32 Area under the curve formed by joining given points with straight line is: 0.5
33 Number of Points: 2
34 Point 1 X Co-ordinate: 0
35 Point 1 Y co-ordinate: -1
36 Point 2 X Co-ordinate: 1
37 Point 2 Y co-ordinate: 1
38 Area under the curve formed by joining given points with straight line is: 0.5

39 -----Test case for Addition check for BASE CASE-----

40 Number of Points: 3
41 Point 1 X Co-ordinate: 1
42 Point 1 Y co-ordinate: 1
43 Point 2 X Co-ordinate: 2
44 Point 2 Y co-ordinate: 2
45 Point 3 X Co-ordinate: 3
46 Point 3 Y co-ordinate: 3
47 Area under the curve formed by joining given points with straight line is: 4

48 -----Test case for WRONG INPUT-----

49 Number of Points: -9

48 -----Test case for WRONG INPUT-----
49 Number of Points: -9
50 INVALID-N : Number of points should be greater than 0 to compute area.
51 Number of Points: 2
52 Point 1 X Co-ordinate: 12
53 Point 1 Y co-ordinate: 1
54 Point 2 X Co-ordinate: -9
55 Point 2 Y co-ordinate: 1
56 UNSORTED-X : X Co-ordinates should be provided sorted.
57 -----Test case provided in the question-----
58 Number of Points: 5
59 Point 1 X Co-ordinate: 1
60 Point 1 Y co-ordinate: 1
61 Point 2 X Co-ordinate: 3
62 Point 2 Y co-ordinate: 4
63 Point 3 X Co-ordinate: 5
64 Point 3 Y co-ordinate: 3
65 Point 4 X Co-ordinate: 6
66 Point 4 Y co-ordinate: 7
67 Point 5 X Co-ordinate: 9
68 Point 5 Y co-ordinate: 5
69 Area under the curve formed by joining given points with straight line is: 35
70 -----Small test case checked by hand computation-----
71 Number of Points: 2
72 Point 1 X Co-ordinate: -9
73 Point 1 Y co-ordinate: 1
74 Point 2 X Co-ordinate: 1
75 Point 2 Y co-ordinate: 1
76 Area under the curve formed by joining given points with straight line is: 10
77 Number of Points: 5
78 Point 1 X Co-ordinate: -8
79 Point 1 Y co-ordinate: 33
80 Point 2 X Co-ordinate: -3
81 Point 2 Y co-ordinate: -4
82 Point 3 X Co-ordinate: 0
83 Point 3 Y co-ordinate: 5
84 Point 4 X Co-ordinate: 23
85 Point 4 Y co-ordinate: 12
86 Point 5 X Co-ordinate: 44
87 Point 5 Y co-ordinate: 21
88 Area under the curve formed by joining given points with straight line is: 623.495495495495561
89 Number of Points: 4
90 Point 1 X Co-ordinate: 11
91 Point 1 Y co-ordinate: -88
92 Point 2 X Co-ordinate: 42
93 Point 2 Y co-ordinate: 12
94 Point 3 X Co-ordinate: 44
95 Point 3 Y co-ordinate: -9


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95 Point 3 Y co-ordinate: -9
96 Point 4 X Co-ordinate: 50
97 Point 4 Y co-ordinate: -55
98 Area under the curve formed by joining given points with straight line is: 1425.35428571428565
99 Number of Points: 5
100 Point 1 X Co-ordinate: -20
101 Point 1 Y co-ordinate: -20
102 Point 2 X Co-ordinate: -10
103 Point 2 Y co-ordinate: 20
104 Point 3 X Co-ordinate: 0
105 Point 3 Y co-ordinate: -20
106 Point 4 X Co-ordinate: 10
107 Point 4 Y co-ordinate: 20
108 Point 5 X Co-ordinate: 20
109 Point 5 Y co-ordinate: -20
110 Area under the curve formed by joining given points with straight line is: 400
111 -----Small test case checked with friend. -----
112 Number of Points: 5
113 Point 1 X Co-ordinate: 477
114 Point 1 Y co-ordinate: -61
115 Point 2 X Co-ordinate: 1081
116 Point 2 Y co-ordinate: -342
117 Point 3 X Co-ordinate: 1901
118 Point 3 Y co-ordinate: -67
119 Point 4 X Co-ordinate: 2116
120 Point 4 Y co-ordinate: -455
121 Point 5 X Co-ordinate: 2651
122 Point 5 Y co-ordinate: -218
123 Area under the curve formed by joining given points with straight line is: 525538.5
124 Number of Points: 4
125 Point 1 X Co-ordinate: 1
126 Point 1 Y co-ordinate: 10
127 Point 2 X Co-ordinate: 3
128 Point 2 Y co-ordinate: -23
129 Point 3 X Co-ordinate: 21
130 Point 3 Y co-ordinate: 7
131 Point 4 X Co-ordinate: 22
132 Point 4 Y co-ordinate: -1
133 Area under the curve formed by joining given points with straight line is: 195.58560606060604
134 -----Test cases with large values-----
135 Number of Points: 2
136 Point 1 X Co-ordinate: 1234567890
137 Point 1 Y co-ordinate: 1
138 Point 2 X Co-ordinate: 1234567891
139 Point 2 Y co-ordinate: 1
140 Area under the curve formed by joining given points with straight line is: 1
141 Number of Points: 2
142 Point 1 X Co-ordinate: 45
143 Point 1 Y co-ordinate: 1234567890
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142 Point 1 X Co-ordinate: 45
143 Point 1 Y co-ordinate: 1234567890
144 Point 2 X Co-ordinate: 46
145 Point 2 Y co-ordinate: 1234567890
146 Exception 12 [Arithmetic overflow] occurred and ignored
147 Area under the curve formed by joining given points with straight line is: 617283945
148 Number of Points: INVALID-N: Number of points should be greater than 0 to compute area.
149 Number of Points: 2
150 Point 1 X Co-ordinate: 45
151 Point 1 Y co-ordinate: 61111111
152 Point 2 X Co-ordinate: 46
153 Point 2 Y co-ordinate: 61111111
154 Area under the curve formed by joining given points with straight line is: 336111111
155 Number of Points: 2
156 Point 1 X Co-ordinate: 2
157 Point 1 Y co-ordinate: 1
158 Point 2 X Co-ordinate: 600000000
159 Point 2 Y co-ordinate: 1
160 Area under the curve formed by joining given points with straight line is: 599999998
161 Number of Points: 2
162 Point 1 X Co-ordinate: 0
163 Point 1 Y co-ordinate: 1
164 Point 2 X Co-ordinate: 6000000000
165 Point 2 Y co-ordinate: 1
166 Area under the curve formed by joining given points with straight line is: 6000000000
167 Number of Points: 2
168 Point 1 X Co-ordinate: 1
169 Point 1 Y co-ordinate: 00
170 Point 2 X Co-ordinate: 6000000000
171 Point 2 Y co-ordinate: 2
172 Area under the curve formed by joining given points with straight line is: 5999999999
173 Number of Points: 2
174 Point 1 X Co-ordinate: 0
175 Point 1 Y co-ordinate: 6000000000
176 Point 2 X Co-ordinate: 1
177 Point 2 Y co-ordinate: 6000000000
178 Area under the curve formed by joining given points with straight line is: 6000000000
179
180 -----LIMIT REACHED from (6*10^8 to 6*10^9 ie around 2^32)-----
181 -----hence justifying the overflow constraint in design-----
182
183 Number of Points: 2
184 Point 1 X Co-ordinate: 0
185 Point 1 Y co-ordinate: 6000000000
186 Point 2 X Co-ordinate: 1
187 Point 2 Y co-ordinate: 6000000000
188 Exception 12 [Arithmetic overflow] occurred and ignored
189 Area under the curve formed by joining given points with straight line is: 852516352
190
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