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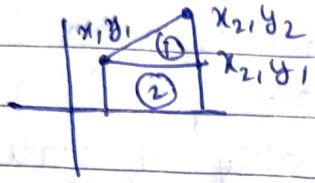
SACHIN 2019CS10722 COL 216

# COL216: Assignment 1

## Approach:

Area under the curve can be found by breaking area in area formed by successive points & adding them.

a) if $y_1 * y_2 > 0$

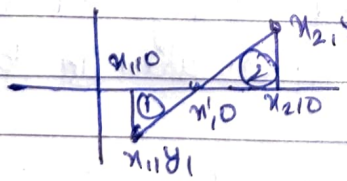



$$\text{Area} = A_1 + A_2 = \frac{1}{2}\left(x_1(y_1 - y_2) + x_2(y_2 - y_1) + 0\right) + (x_2 - x_1)\,y_1$$

$$= \frac{(x_2 - x_1)(y_1 + y_2)}{2}$$

if $y_1$ & $y_2$ both +ve consider this as area & if $y_1$ & $y_2$ both -ve take absolute value.

b) if $y_1 * y_2 < 0$




$\text{Area} = A_2 - A_1$

using slope of line $\frac{y_2 - y_1}{x_2 - x_1} = \frac{y_2}{x_2 - x'}$

$$\Rightarrow x' = x_1 - y_1\left(\frac{x_2 - x_1}{y_2 - y_1}\right)$$

$$\& \text{ Area} = \frac{(x_2 - x_1)(y_1^2 + y_2^2)}{2(y_2 - y_1)}$$

if $y_2 > 0, y_1 < 0$ take this as area & if $y_2 < 0, y_1 > 0$ take abs value.

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$$A_{total} = \sum_{i=1}^{n-1} A_{i+1,i}, \text{ where } A_{i+1,i} = \begin{cases} \frac{1}{2}|(x_2-x_1)(y_2+y_1)| & \text{if } y_1y_2 \geq 0 \\ \frac{1}{2}\frac{(x_2-x_1)(y_1^2+y_2^2)}{y_2-y_1} & \text{if } y_1y_2 < 0 \end{cases}$$

## Constraints:

1.) While deriving formula we assumed $x_2 \geq x_1$ so input should be sorted wrt x.
2.) No. of points given by user should be $> 0$.
3.) Area under single point is trivially 0.
4.) Area computed accurately if values calculated at any step is to in b/w $2^{-32}$ to $2^{32}-1$.
ie $2^{-32} < area < 2^{32}-1$
$2^{-32} < y_1y_2 < 2^{32}-1$
$2^{-32} < x_1x_2 < 2^{32}-1$.
5.)

## Input taking:

Firstly no. of points is taken as input then one by one in single line x & y coordinates are taken (NOTE: first $x_1$ is taken in taken in 1 line then $y_1$ in new line).
- At any point if constraints are violated appropriate error is thrown.

## Code in words

- ~~registers used:~~
~~$t_0, t_1, t_2, t_3 = x_i, x_{i+1}, y_i, y_{i+1}$~~

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## Registers used:

- $t_0, t_1, t_2, t_3$ = $x_i, x_{i+1}, y_i, y_{i+1}$
- $f_0, f_2$ = temp registers for appropriate use.
- $f_{12}$ = Storing partial sum
- $f_4$ = 2.0
- $S_0$ = no. of points
- $S_1$ = counter to check how many points readd
- $S_2$ = checker ( 0 if first point is stored else 1).

## Code in Words:

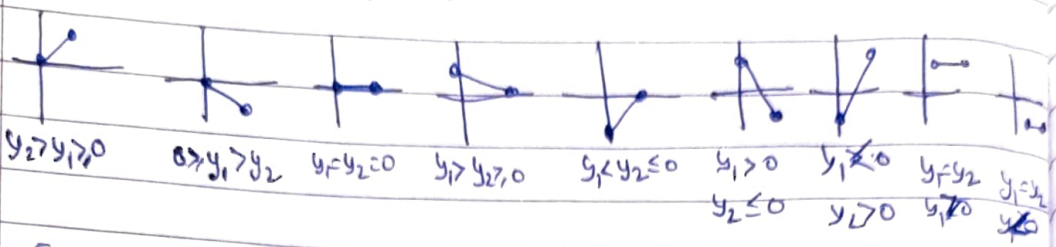
no. of inputs stored in $S_0$ then loop is started till $S_1$ reach $S_0$, at first point $S_2$ is make 1 & then at each reading of $y_i$ "area" subprogram is called.
in area, condition is check for ~~for~~ on $y_1, y_2$ for fiding correct area.

- $y_1$ & $y_2$ could be used to check condition but that demands of storing a large quantity, rather I used case by case analysis of $y_1$ & $y_2$ (ie comparing them wrt 0)
- for $A = (x_2 - x_1)(y_1 + y_2)/2$ case I first computed $y_1 + y_2/2$ then multiplied it with $(x_2 - x_1)$ Because direct multiplication might create $> 2^{32}$ value for large input & cause inaccuracy
- Similarly for $\frac{(x_2 - x_1)(y_1^2 + y_2^2)}{2}$ I first computed $\frac{y_1^2 + y_2^2}{2}$ then multiplied.

## Testing:

Inducing on no of inputs.

- Base case:
  If $n = 2$
  For 2 points, 9 possible cases.




For all cases my code gives correct area.
[ALL TEST CASES ATTACHED ON PAGE -5].

- Now I tested code for $n=3$ points (one such case) It gave correct ans hence, addition of two successive area is also computed correctly

- Inductive Hypothesis:
  let for $n = k$ code gives correct area ($k > 2$)

- Induction step:
  for $n = k+1$, $A_{k+1} = A_k + A_{(k+1,k)}$

  $A_k$: computed correctly by I.H
  $A_{(k+1,k)}$: computed correctly by base case
  addition done correctly by ⁂.

Hence by induction code is correct.

- For further rigorous testing I computed some small problems (4-5 points) with hand & some cases when all points are in straight line, code shows correct results.
- I checked ans for some big test cases with my friend.
- For overflow I tested code where ans is big & code shows wrong ans around b/w $6 \times 10^8$ to $8 \times 10^9$ ie $\approx 2^{32}$ (hence code overflow constraint proved)

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--------------------------------Test cases for BASE CASE----------------------
Number of Points: 2
Point 1 X Co-ordinate: 0
Point 1 Y co-ordinate: 0
Point 2 X Co-ordinate: 1
Point 2 Y co-ordinate: 1
Area under the curve formed by joining given points with straight line is: 0.5
Number of Points: 2
Point 1 X Co-ordinate: 0
Point 1 Y co-ordinate: -1
Point 2 X Co-ordinate: 1
Point 2 Y co-ordinate: 0
Area under the curve formed by joining given points with straight line is: 0.5
Number of Points: 2
Point 1 X Co-ordinate: 0
Point 1 Y co-ordinate: 1
Point 2 X Co-ordinate: 1
Point 2 Y co-ordinate: 0
Area under the curve formed by joining given points with straight line is: 0.5
Number of Points: 2
Point 1 X Co-ordinate: 0
Point 1 Y co-ordinate: 0
Point 2 X Co-ordinate: 1
Point 2 Y co-ordinate: -1
Area under the curve formed by joining given points with straight line is: 0.5
Number of Points: 2
Point 1 X Co-ordinate: 0
Point 1 Y co-ordinate: 1
Point 2 X Co-ordinate: 1
Point 2 Y co-ordinate: -1
Area under the curve formed by joining given points with straight line is: 0.5
Number of Points: 2
Point 1 X Co-ordinate: 0
Point 1 Y co-ordinate: -1
Point 2 X Co-ordinate: 1
Point 2 Y co-ordinate: 1
Area under the curve formed by joining given points with straight line is: 0.5
-------------------------Test case for Addition check for BASE CASE-----------------
Number of Points: 3
Point 1 X Co-ordinate: 1
Point 1 Y co-ordinate: 1
Point 2 X Co-ordinate: 2
Point 2 Y co-ordinate: 2
Point 3 X Co-ordinate: 3
Point 3 Y co-ordinate: 3
Area under the curve formed by joining given points with straight line is: 4
-----------------------Test case for WRONG INPUT---------------------------------
Number of Points: -9
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----------Test case for WRONG INPUT----------
Number of Points: -9
INVALID-N : Number of points should be greater than 0 to compute area.
Number of Points: 2
Point 1 X Co-ordinate: 12
Point 1 Y co-ordinate: 1
Point 2 X Co-ordinate: -9
Point 2 Y co-ordinate: 1
UNSORTED-X : X Co-ordinates should be provided sorted.
----------Test case provided in the question----------
Number of Points: 5
Point 1 X Co-ordinate: 1
Point 1 Y co-ordinate: 1
Point 2 X Co-ordinate: 3
Point 2 Y co-ordinate: 4
Point 3 X Co-ordinate: 5
Point 3 Y co-ordinate: 3
Point 4 X Co-ordinate: 6
Point 4 Y co-ordinate: 7
Point 5 X Co-ordinate: 9
Point 5 Y co-ordinate: 5
Area under the curve formed by joining given points with straight line is: 35
----------Small test case checked by hand computation----------
Number of Points: 2
Point 1 X Co-ordinate: -9
Point 1 Y co-ordinate: 1
Point 2 X Co-ordinate: 1
Point 2 Y co-ordinate: 1
Area under the curve formed by joining given points with straight line is: 10
Number of Points: 5
Point 1 X Co-ordinate: -8
Point 1 Y co-ordinate: 33
Point 2 X Co-ordinate: -3
Point 2 Y co-ordinate: -4
Point 3 X Co-ordinate: 0
Point 3 Y co-ordinate: 5
Point 4 X Co-ordinate: 23
Point 4 Y co-ordinate: 12
Point 5 X Co-ordinate: 44
Point 5 Y co-ordinate: 21
Area under the curve formed by joining given points with straight line is: 623.495495495495561
Number of Points: 4
Point 1 X Co-ordinate: 11
Point 1 Y co-ordinate: -88
Point 2 X Co-ordinate: 42
Point 2 Y co-ordinate: 12
Point 3 X Co-ordinate: 44
Point 3 Y co-ordinate: -9
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Point 3 Y co-ordinate: -9
Point 4 X Co-ordinate: 50
Point 4 Y co-ordinate: -55
Area under the curve formed by joining given points with straight line is: 1425.35428571428565
Number of Points: 5
Point 1 X Co-ordinate: -20
Point 1 Y co-ordinate: -20
Point 2 X Co-ordinate: -10
Point 2 Y co-ordinate: 20
Point 3 X Co-ordinate: 0
Point 3 Y co-ordinate: -20
Point 4 X Co-ordinate: 10
Point 4 Y co-ordinate: 20
Point 5 X Co-ordinate: 20
Point 5 Y co-ordinate: -20
Area under the curve formed by joining given points with straight line is: 400
----------Small test case checked with friend. ----------
Number of Points: 5
Point 1 X Co-ordinate: 477
Point 1 Y co-ordinate: -61
Point 2 X Co-ordinate: 1081
Point 2 Y co-ordinate: -342
Point 3 X Co-ordinate: 1901
Point 3 Y co-ordinate: -67
Point 4 X Co-ordinate: 2116
Point 4 Y co-ordinate: -455
Point 5 X Co-ordinate: 2651
Point 5 Y co-ordinate: -218
Area under the curve formed by joining given points with straight line is: 525538.5
Number of Points: 4
Point 1 X Co-ordinate: 1
Point 1 Y co-ordinate: 10
Point 2 X Co-ordinate: 3
Point 2 Y co-ordinate: -23
Point 3 X Co-ordinate: 21
Point 3 Y co-ordinate: 7
Point 4 X Co-ordinate: 22
Point 4 Y co-ordinate: -1
Area under the curve formed by joining given points with straight line is: 195.58560606060604
----------Test cases with large values----------
Number of Points: 2
Point 1 X Co-ordinate: 1234567890
Point 1 Y co-ordinate: 1
Point 2 X Co-ordinate: 1234567891
Point 2 Y co-ordinate: 1
Area under the curve formed by joining given points with straight line is: 1
Number of Points: 2
Point 1 X Co-ordinate: 45
Point 1 Y co-ordinate: 1234567890

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Point 1 X Co-ordinate: 45
Point 1 Y co-ordinate: 1234567890
Point 2 X Co-ordinate: 46
Point 2 Y co-ordinate: 1234567890
 Exception 12 [Arithmetic overflow] occurred and ignored
Area under the curve formed by joining given points with straight line is: 617283945
Number of Points: INVALID-N : Number of points should be greater than 0 to compute area.
Number of Points: 2
Point 1 X Co-ordinate: 45
Point 1 Y co-ordinate: 61111111
Point 2 X Co-ordinate: 46
Point 2 Y co-ordinate: 611111111
Area under the curve formed by joining given points with straight line is: 336111111
Number of Points: 2
Point 1 X Co-ordinate: 2
Point 1 Y co-ordinate: 1
Point 2 X Co-ordinate: 60000000
Point 2 Y co-ordinate: 1
Area under the curve formed by joining given points with straight line is: 59999998
Number of Points: 2
Point 1 X Co-ordinate: 0
Point 1 Y co-ordinate: 1
Point 2 X Co-ordinate: 600000000
Point 2 Y co-ordinate: 1
Area under the curve formed by joining given points with straight line is: 600000000
Number of Points: 2
Point 1 X Co-ordinate: 1
Point 1 Y co-ordinate: 00
Point 2 X Co-ordinate: 600000000
Point 2 Y co-ordinate: 2
Area under the curve formed by joining given points with straight line is: 599999999
Number of Points: 2
Point 1 X Co-ordinate: 0
Point 1 Y co-ordinate: 600000000
Point 2 X Co-ordinate: 1
Point 2 Y co-ordinate: 600000000
Area under the curve formed by joining given points with straight line is: 600000000

----------------------LIMIT REACHED from (6*10^8 to 6*10^9 ie around 2^32)----------------------
----------------------hence justifying the overflow contraint in design--------------------------

Number of Points: 2
Point 1 X Co-ordinate: 0
Point 1 Y co-ordinate: 6000000000
Point 2 X Co-ordinate: 1
Point 2 Y co-ordinate: 6000000000
 Exception 12 [Arithmetic overflow] occurred and ignored
Area under the curve formed by joining given points with straight line is: 852516352

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