COL352: Assignment 1

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1 Question 1

2 Question 2

An all-NFA M is a 5-tuple $(Q, \Sigma, \delta, q_0, F)$ that accepts $x \in \Sigma^*$ if every possible state that M could be in after reading input x is a state from F . Note, in contrast, that an ordinary NFA accepts a string if some state among these possible states is an accept state. Prove that all-NFAs recognize the class of regular languages.

To prove that all-NFA's recognise the class of regular languages we need to show two things, firstly that the language accepted by all-NFA's is regular, and secondly given any regular language there exists an all-NFA which accepts it. Following are the proofs of these parts,

To Prove: Language accepted by all-NFA is regular.
Proof: Now by the definition, all-NFA M is a 5-tuple $(Q, \Sigma, \delta, q_0, F)$ that accepts $x \in \Sigma^*$ if every possible state that M could be in after reading input x is a state from F . This would mean the all-NFA's are NFA because NFA a ccepts the string even if some of the states reached after reading an input x is in accept state F. NOw we know that the language accepted by NFA is regular. Therefore the language accepted by all-NFA is also regular. Hence proved.

To Prove: For every regular language there exists an all-NFA that accepts it.
Proof: We know that for every regular language there exists a DFA which accepts it. Now the definition of a DFA M is that it is a 5-tuple $(Q, \Sigma, \delta, q_0, F)$ that accepts $x \in \Sigma^*$ if the state that M could be in after reading input x is a state from F. Now we also know that the set of states DFA M would be in after reading the input x is a singleton set (Deterministic nature) and the state belongs to F if x is accepted by DFA. So every DFA is an all-NFA. Therefore for every regular language, there exists an all-NFA that accepts it. Hence proved.

Now above two facts would imply that the all-NFA's recognize the class of regular languages.

3 Question 3

4 Question 4

5 Question 5

For any string $w = w_1w_2...w_n$ the reverse of w written w^R is the string $w_n...w_2w_1$. For any language A, let $A^R = w^R|w \in A$. Show that if A is regular, then so is A^R. In other words, regular languages are closed under the reverse operation.

As every regular language has a DFA which accepts it, let D be $(Q, \Sigma, \delta, q_0, F)$ that accepts the language A. Now we will construct an NFA N from this DFA. The steps of the construction are given below. We will then show that the language accepted by NFA is indeed A^R.

Construction: To construct this NFA N we will have to reverse all the edges of the DFA D. Also make the start state of the D as the accepting state of the N. Add ϵ transitions from the accepting states of D to a new state s in NFA. Make s the start state of the NFA.
So N is $(Q_1, \Sigma, \delta^{'}, q_0^{'}, F^{'})$ such that

$$Q_1 = Q \cup \{s\}$$
$$F^{'} = q_0$$
$$q_0^{'} = s$$
$$\delta^{'}(q_1, a) = (q_2 \mid \delta(q_2, a) = q_1 \; and \; q_1, q_2 \in Q) \cup (s \mid q_1 \in F \; and \; a = \epsilon)$$

To Prove: The language accepted by N is A^R.
Proof: Now take any string $w_1w_2....w_n$ from the language A. The path(path is state then alphabet taken then next state reached) taken by this string to accept state in D would be $q_0, w_1, q_1, w_2, q_2.....q_n, w_n, f$ where $q_i \in Q$ and $f \in F$. Now take the reverse of the string $w_nw_{n-1}....w_1$. There exist a path from start to accept state in N as which is as follows $s, \epsilon, f, w_n, q_n, w_{n-1}.....q_1, w_1, q_0)$. We know that q_0 is an accept state of N. Thus we have got an NFA that has an accepting path for any string w in A^R, hence A^R is regular. Hence we have proved that regular languages are closed under the reverse operation.

6 Question 6