COL352: Assignment 4

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1   Question 1

Show that every infinite Turing-recognizable language has an infinite decidable subset.

2 Question 2

Show that single-tape TMs that cannot write on the portion of the tape containing the input string recognize only regular languages.

3 Question 3

Let C be a language. Prove that C is Turing-recognizable iff a decidable language D exists such that

$$C = \{x | \exists y (< x, y > \in D)\}$$

4   Question 4

Say that a variable A in CFL G is usable if it appears in some derivation of some string w ∈ G. Given a CFG G and a variable A, consider the problem of testing whether A is usable. Formulate this problem as a language and show that it is decidable

5   Question 5

Consider the problem of determining whether a Turing machine M on an input w ever at- tempts to move its head left when its head is on the left-most tape cell. Formulate this problem as a language and show that it is undecidable

6   Question 6

Consider the problem of determining whether a Turing machine M on an input w ever attempts to move its head left at any point during its computation on w. Formulate this problem as a language and show that it is decidable.

7 Question 7

Let $AM_{BCFG} = \{ <G> \,|\, G$ is an ambiguous CFG $\}$
Show that AM_{BCFG} is undecidable via a reduction from PCP

8   Question 8

In the Silly Post Correspondence Problem (SPCP), the top string in each pair has the same length as the bottom string. Show that the SPCP is decidable.