# COL352: Assignment 2

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## 1 Question 1

**We say that a context-free grammar G is self-referential if for some non-terminal symbol $X$ we have $X \to^* \alpha X \beta$, where $\alpha, \beta \neq \varepsilon$. Show that a CFG that is not self-referential is regular.**

# 2 Question 2

**Prove that the class of context-free languages is closed under intersection with regular languages. That is, prove that if $L_1$ is a context-free language and $L_2$ is a regular language, then $L_1 \cap L_2$ is a context-free language. Do this by starting with a DF**

## 3 Question 3

**Given two languages $L, L'$, denote by**

$$L||L' := \{x_1y_1x_2y_2\dots x_ny_n \mid x_1x_2\dots x_n \in L, y_1y_2\dots y_n \in L'\}$$

**Show that if $L$ is a CFL and $L'$ is regular, then $L||L'$ is a CFL by constructing a PDA for $L||L'$. Is $L||L'$ a CFL if both $L$ and $L'$ are CFLs? Justify your answer.**

# 4 Question 4

**For $A \subseteq \Sigma^*$, define**

$$cycle(A) = \{yx \mid xy \in A\}$$

**For example if $A = \{aaabc\}$, then**

$$cycle(A) = \{aaabc, aabca, abcaa, bcaaa, caaab\}$$

**Show that if $A$ is a CFL then so is $cycle(A)$**

## 5 Question 5

**Let**

$$A = \{wtw^R \mid w, t, \in \{0,1\}^* \ \ \textbf{and} \ \ |w| = |t|\}$$

**Show that $A$ is not a CFL.**

# 6 Question 6

**Prove the following stronger version of pumping lemma for CFLs: If $A$ is a CFL, then there is a number $k$ where if $s$ is any string in $A$ of length at least $k$ then $s$ may be divided into five pieces $s = uvxyz$, satisfying the conditions:**

1. **for each $i \geq 0$, $uv^ixy^iz \in A$**
2. **$v \neq \varepsilon$, and $y \neq \varepsilon$, and**
3. **$|vxy| \leq k$.**

# 7 Question 7

**Give an example of a language that is not a CFL but nevertheless acts like a CFL in the pumping lemma for CFL (Recall we saw such an example in class while studying pumping lemma for regular languages).**