COL352: Assignment 1

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1 Question 1

2 Question 2

An all-NFA M is a 5-tuple $(Q, \Sigma, \delta, q_0, F)$ that accepts $x \in \Sigma^*$ if every possible state that M could be in after reading input x is a state from F . Note, in contrast, that an ordinary NFA accepts a string if some state among these possible states is an accept state. Prove that all-NFAs recognize the class of regular languages.

To prove that all-NFA's recognise the class of regular languages we need to show two things, firstly that the language accepted by all-NFA's is regular, and secondly given any regular language there exists an all-NFA which accepts it. Following are the proofs of these parts,

To Prove: Language accepted by all-NFA is regular.
Proof: Now by the definition, all-NFA M is a 5-tuple $(Q, \Sigma, \delta, q_0, F)$ that accepts $x \in \Sigma^*$ if every possible state that M could be in after reading input x is a state from F . This would mean the all-NFA's are NFA because NFA a ccepts the string even if some of the states reached after reading an input x is in accept state F. NOw we know that the language accepted by NFA is regular. Therefore the language accepted by all-NFA is also regular. Hence proved.

To Prove: For every regular language there exists an all-NFA that accepts it.
Proof: We know that for every regular language there exists a DFA which accepts it. Now the definition of a DFA M is that it is a 5-tuple $(Q, \Sigma, \delta, q_0, F)$ that accepts $x \in \Sigma^*$ if the state that M could be in after reading input x is a state from F. Now we also know that the set of states DFA M would be in after reading the input x is a singleton set (Deterministic nature) and the state belongs to F if x is accepted by DFA. So every DFA is an all-NFA. Therefore for every regular language, there exists an all-NFA that accepts it. Hence proved.

Now above two facts would imply that the all-NFA's recognize the class of regular languages.

3 Question 3

4 Question 4

5 Question 5

For any string $w = w_1 w_2 ... w_n$ the reverse of w written w^R is the string $w_n ... w_2 w_1$. For any language A, let $A^R = w^R | w \in A$. Show that if A is regular, then so is A^R. In other words, regular languages are closed under the reverse operation.

As every regular language has a DFA which accepts it, let D be $(Q, \Sigma, \delta, q_0, F)$ that accepts the language A. Now we will construct an NFA N from this DFA. The steps of the construction are given below. We will then show that the language accepted by NFA is indeed A^R.

Construction: To construct this NFA N we will have to reverse all the edges of the DFA D. Also make the start state of the D as the accepting state of the N. Add ϵ transitions from the accepting states of D to a new state s in NFA. Make s the start state of the NFA.
So N is $(Q_1, \Sigma, \delta', q_0', F')$ such that

$$Q_1 = Q \cup \{s\}$$
$$F' = q_0$$
$$q_0' = s$$
$$\delta'(q_1, a) = (q_2 \mid \delta(q_2, a) = q_1 \ and \ q_1, q_2 \in Q) \cup (s \mid q_1 \in F \ and \ a = \epsilon)$$

To Prove: The language accepted by N is A^R.
Proof: Now take any string $w_1 w_2 .... w_n$ from the language A. The path(path is state then alphabet taken then next state reached) taken by this string to accept state in D would be $q_0, w_1, q_1, w_2, q_2 ..... q_n, w_n, f$ where $q_i \in Q$ and $f \in F$. Now take the reverse of the string $w_n w_{n-1} .... w_1$. There exist a path from start to accept state in N as which is as follows $s, \epsilon, f, w_n, q_n, w_{n-1} ..... q_1, w_1, q_0$. We know that q_0 is an accept state of N. Thus we have got an NFA that has an accepting path for any string w in A^R.
Now we have proved that for every string in A we have a accepting path in N. Now we can also similarly prove the reverse direction too i.e. for every string w in A^R we have accepting path in D for reverse of w. The proof of this goes as follows:
Take any string $w_1 w_2 .... w_n$ from the language A^R. The pathtaken by this string to accept state in N would be $s, \epsilon, f, w_1, q_n, w_2 ..... q_1, w_n, q)$ where $q_i \in Q$ and $f \in F$. Now take the reverse of the string $w_n w_{n-1} .... w_1$. There exist a path from

start to accept state in D as which is as follows $q_0, w_n, q_1, w_{n-1}, q_2.....q_n, w_1, f$. We know that f is an accept state of D.Thus we have got an DFA that has an accepting path for any string w in A.

Hence we have proved that regular languages are closed under the reverse operation.

6 Question 6