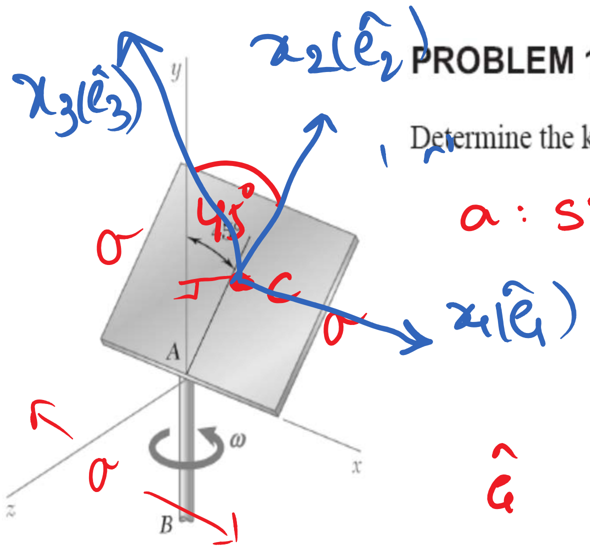
Set 10 A& B. Due on Oct 31




**PROBLEM 18.40**

Determine the kinetic energy of the plate of Problem 18.3.

a : side of the square plate

I : ground frame

$T_{|I}$ (K.E. w.r.t. frame I)

$= \frac{1}{2} m \underline{V}_{c|I} \cdot \underline{V}_{c|I} + \frac{1}{2} \underline{\omega}_{m|I} \cdot \underline{H}_{c|I}$

The shown csys has the plate in the plane $x_1(\hat{e}_1) - x_2(\hat{e}_2)$

$x_1(\hat{e}_1), x_2(\hat{e}_2), x_3(\hat{e}_3)$ are p-axes of body at C

$\underline{\omega}_{m|I} = \omega_1 \hat{e}_1 + \omega_2 \hat{e}_2 + \omega_3 \hat{e}_3$

$\underline{H}_{c|I} = I^C_{11} \omega_1 \hat{e}_1 + I^C_{22} \omega_2 \hat{e}_2 + I^C_{33} \omega_3 \hat{e}_3$

$\frac{1}{2} \underline{\omega}_{m|I} \cdot \underline{H}_{c|I} =$

$\frac{1}{2} I^C_{11} \omega_1^2 + \frac{1}{2} I^C_{22} \omega_2^2 + \frac{1}{2} I^C_{33} \omega_3^2$

$\underline{\omega}_{m|I} = \frac{\omega}{\sqrt{2}} \hat{e}_2 + \frac{\omega}{\sqrt{2}} \hat{e}_3$

$\omega_1 = 0, \quad \omega_2 = \frac{\omega}{\sqrt{2}}, \quad \omega_3 = \frac{\omega}{\sqrt{2}}$

$\underline{\omega}_{m|I} \cdot \underline{H}_{c|I}$

$= \frac{1}{2} I^C_{22} \frac{\omega^2}{2} + \frac{1}{2} I^C_{33} \frac{\omega^2}{2}$

$= \frac{\omega^2}{4} ma^2/12 + \omega^2/4 \, ma^2/6 = \frac{3 m a^2 \omega^2}{4 \times 12 \cdot 4}$

$$\underline{v}_{c|I} = -\omega\left(\frac{a}{2}\right)\left(\frac{1}{\sqrt{2}}\right)\hat{e}_1$$

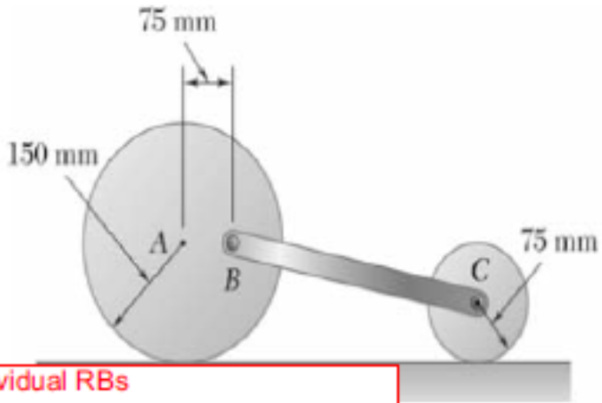
$\therefore\ T_{|I}$

$$= \frac{1}{2} m\, \underline{v}_{c|I} \cdot \underline{v}_{c|I} + \frac{1}{2}\, \underline{\omega}_{m|I} \cdot \underline{H}_{c|I}$$

$$= \frac{1}{2} m \left(\frac{\omega^2 a^2}{4} \frac{1}{2}\right) + \frac{m a^2 \omega^2}{16}$$

$$= \frac{m a^2 \omega^2}{16} + \frac{m a^2 \omega^2}{16}$$

$$T_{|I} = \frac{m a^2 \omega^2}{8}$$




## PROBLEM 17.35

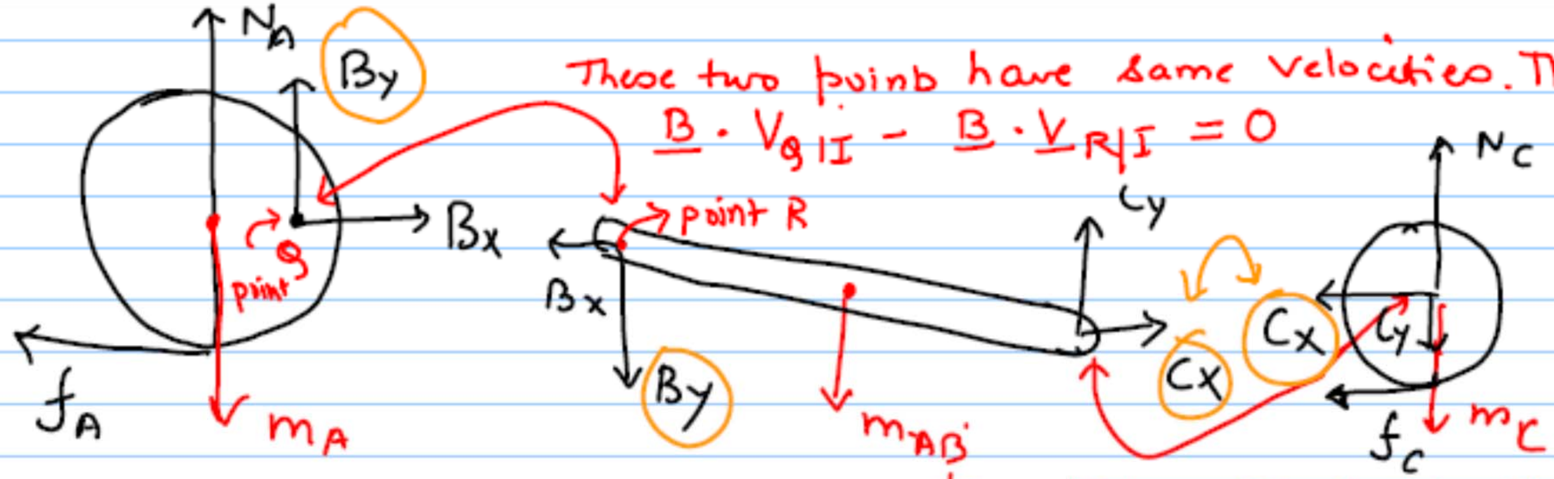
The 5-kg rod $BC$ is attached by pins to two uniform disks as shown. The mass of the 150-mm-radius disk is 6 kg and that of the 75-mm-radius disk is 1.5 kg. Knowing that the system is released from rest in the position shown, determine the velocity of the rod after disk $A$ has rotated through 90°.

FBD's of individual RBs

Assume that the disks roll without slip

These two points have same velocities. Therefore

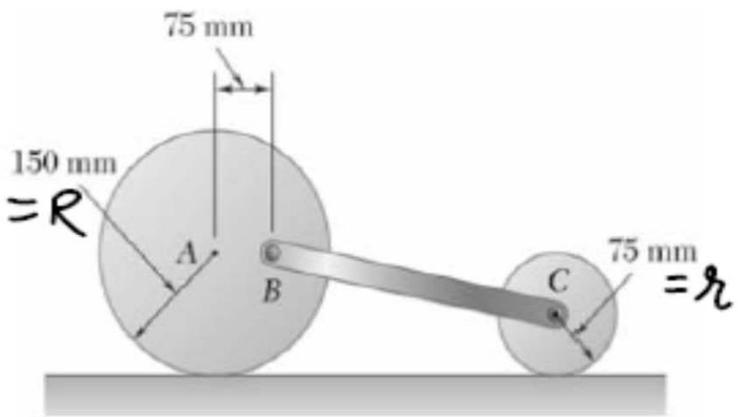
$$\underline{B} \cdot V_{Q|I} - \underline{B} \cdot \underline{V}_{R|I} = 0$$




Similar cancellation of work done at the junction of rod BC and the smaller disk

$$\dot{T}_a + \dot{T}_b + \dot{T}_c = \sum \underline{F}_i^{ext, system}$$

(Work done by internal forces cancel out because of pin joints)



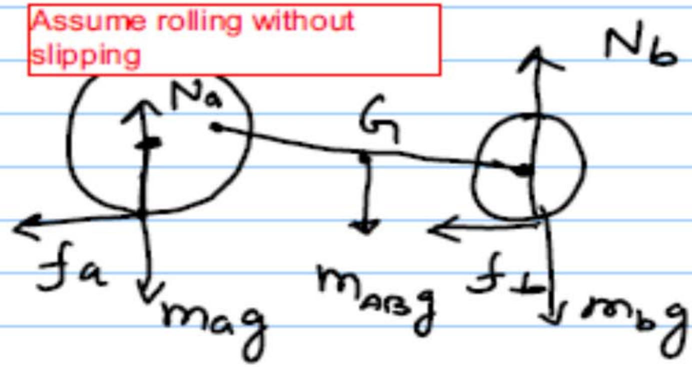

## PROBLEM 17.35

The 5-kg rod $BC$ is attached by pins to two uniform disks as shown. The mass of the 150-mm-radius disk is 6 kg and that of the 75-mm-radius disk is 1.5 kg. Knowing that the system is released from rest in the position shown, determine the velocity of the rod after disk $A$ has rotated through 90°.

$$\frac{dT^a}{dt} + \frac{dT^b}{dt} + \frac{dT^c}{dt} = \sum \underline{F}_i^{\text{external, system}} \cdot \underline{v}_{i/I} \quad \text{of (i)}$$

The only external force that does non zero work is: mg of the rod BC

Assume rolling without slipping




$\therefore$ RHS of (i) = m(of rod BC) $g(\Delta y)$

What is $\Delta y$?

$\Delta y$: vertical displacement of G going from State 1 to State 2

Since $m_{AB}g$ is a conservative force:

$$(T^a + T^b + T^c)_2 - (T^a + T^b + T^c)_1 = mg\Delta y$$

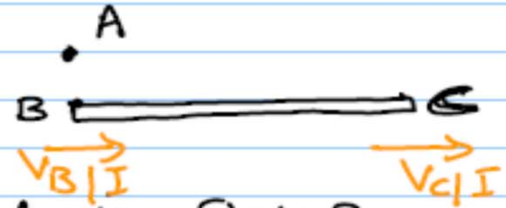
$$(T^a + T^b + T^c)_2 - 0 =$$

$$\Delta y = \frac{R-r}{2}$$

Configuration 1

$$T_2^b = \frac{1}{2} m^b V_G^2 + \frac{1}{2} \bar{I}_b \omega^{b\,2}$$

Configuration 2




Note that $\underline{V}_G = \underline{V}_B = \underline{V}_C$ }

Reason: $\underline{V}_B$, $\underline{V}_C$ are both horizontal in State 2.

and the position vector going from B to C is not perpendicular to these velocities

$\Rightarrow$ $\underline{\omega}^b = 0$ $\Rightarrow$ every pt. on BC has identical velocity

expressed in terms of V_G,

$$\therefore \boxed{T_2^b = \frac{m^b V_G^2}{2}}$$

Also $\omega_c = \frac{V_c}{r} = \frac{V_B}{r} = \frac{V_G}{r}$

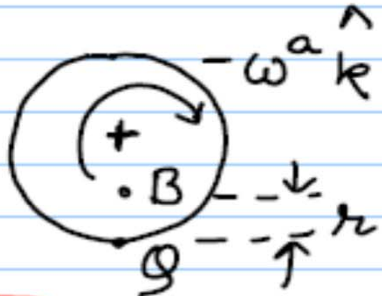
$$\therefore T_2^c = \frac{1}{2} m_c V_c^2 + \frac{I^c}{2}\omega_c^2 = \frac{1}{2} m_c V_G^2 + \frac{m_c r^2}{4}\frac{V_G^2}{r^2}.$$

$$\boxed{T_2^c = \frac{3}{4} m_c V_G^2}$$

expressed in terms of V_G

$$T_2^a = \frac{1}{2} m_a V_A^2 + \frac{1}{2} I^A \omega_a^2 = \frac{1}{2} m_a \omega_a^2 R^2 + \frac{1}{2}\frac{m_a R^2}{2}\omega_a^2.$$

$$T_2^a = \frac{3}{4} m_a \omega_a^2 R^2.$$




But $\underline{V_B} = \cancel{V_{g|I}}^{\,0} + (-\omega_a \hat{k} \times r\hat{j}) = \omega_a r \hat{i}$

$\Rightarrow \omega_a = V_B/r = V_G/r$

$$\boxed{T_2^a = \frac{3}{4} m_a R^2 \frac{V_G^2}{r^2} = \frac{3}{4} m_a \left(\frac{R^2}{r^2}\right) V_G^2}$$

expressed in terms of V_G

$$V_G = V_B$$

$$\therefore \quad T_2^a + T_2^b + T_2^c = \frac{m^b V_B^2}{2} + \frac{3}{4} m^c V_B^2 + \frac{3}{4} m^a V_B^2 \left(\frac{R^2}{r^2}\right)$$

$$= \frac{5V_B^2}{2} + \frac{3}{4}\, 1.5\, V_B^2 + \frac{3}{\cancel{4}} \times 6 \times V_B^2 \times \cancel{4}$$

$$= \left(\frac{5}{2} + \frac{4.5}{4} + 18\right) V_B^2 = \left(2.5 + 18 + 1.1\right) V_B^2 = 21.6\, V_B^2$$

Equating this to work done by gravitational force

$$21.6\, V_B^2 = m^b g \frac{(R-r)}{2}$$

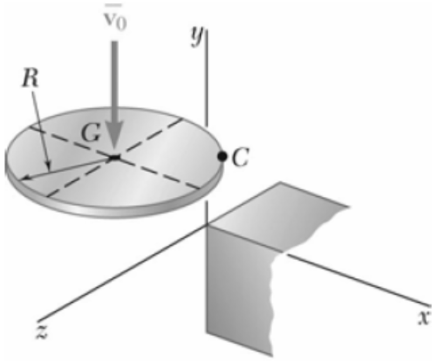
$$\therefore \quad V_B^2 = \frac{m^b g (R-r)}{2(21.6)} = \frac{5 \times 9.8 \times .075}{2 \times 21.6}$$

$$\boxed{V_B = .29\ ms^{-1}}$$

Note that work-energy relationship is a single scalar eq$^n$.

[redacted]

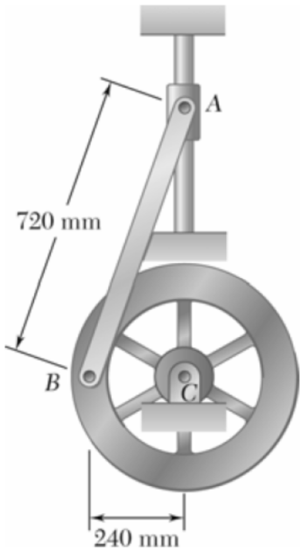
## Set 10 B




### PROBLEM 18.51

Determine the kinetic energy lost when edge $C$ of the plate of Problem 18.29 hits the obstruction.

$$\frac{1}{10} m\bar{v}_0^2 \blacktriangleleft$$




### PROBLEM 17.43

The 4-kg rod $AB$ is attached to a collar of negligible mass at $A$ and to a flywheel at $B$. The flywheel has a mass of 16 kg and a radius of gyration of 180 mm. Knowing that in the position shown the angular velocity of the flywheel is 60 rpm clockwise, determine the velocity of the flywheel when Point $B$ is directly below $C$.

$$\omega_2 = 84.7 \text{ rpm} \blacktriangleleft$$

For more practice problems: Chapter 17 & 18 of B&J