

Mean → centre of gravity / mes

- The most common type of Average out there.
- Most influenced by the outliers.
- It changes when symmetry of the distribution/graph is affected.

$$\bar{x} = \frac{\sum_{i=1}^N x_i}{N}$$

$$\bar{x} = \frac{\sum f_i}{\sum f} \rightarrow \text{frequency}$$

Mean  $\Rightarrow$  Sum of obs.  
No. of obs.

$$[10, 10, 10, 10] \Rightarrow 10 \Rightarrow \frac{4 \times 10}{4} = 10$$

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$$[10, 10, 10, 100] \Rightarrow 32.5$$

⇒ Mean is shifting towards higher value.

⇒ Mean is in love with outliers, so  
always shifts towards outliers.

$$\begin{bmatrix} 0 & 1 & 2 & 3 & 4 & 5 \\ 1, 2, 3, 4, 5, 6 \end{bmatrix}$$

X

$$\text{Mean} = \frac{1 + 2 + 3 + 4 + 5 + 6}{6}$$

$$\text{Mean} \Rightarrow \frac{\sum_{i=0}^5 X}{N}$$

summation

$x$	$f$
1	2
2	3
3	4

→ [ 1, 1, 2, 2, 2, 3, 3, 3, 3 ]

Mean ⇒  $\frac{1+1+2+2+2+3+3+3+3}{9}$

Distribution  
table

$\text{df}[\text{col}].\text{Count\_values}()$   
 $\text{df}[\text{col}].\text{mean}()$   
 ↓  
 Pandas

$$\Rightarrow \frac{1 \times 2 + 2 \times 3 + 3 \times 4}{9}$$

$$\Rightarrow \frac{\sum x f}{\sum f} \Rightarrow \text{Mean}$$

[ 1 2 3 4 5 ]

→ 3

[ 3 2 4 1 5 ]

Median

Sort ↗ [ 1 2 3 4 5 ] ⇒ Median ⇒ 3

- It is actually the middle value, dividing the data into 2 equal halves.
- Steps:

• Sort the data in ascending order

Median

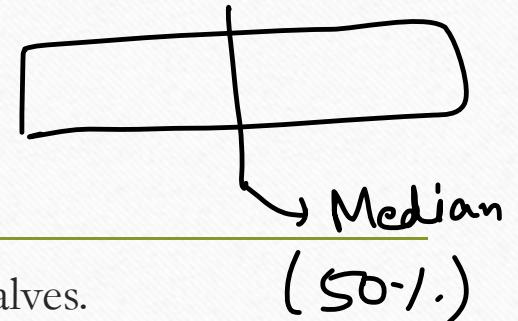
n is odd,  
 $\text{Median} = \left( \frac{n+1}{2} \right)^{\text{th}} \text{ observation}$

n is even,  
 $\left( \frac{n}{2} \right)^{\text{th}} + \left( \frac{n+1}{2} \right)^{\text{th}} \text{ observation}$

- It is ~~not~~ influenced by outliers.

less

↪ Special cases



X

$$\text{Median} = l + \left( \frac{\frac{n}{2} - F}{f} \right) \times h$$

$1^{st}$	$2^{nd}$	$3^{th}$	$4^{th}$	$5^{th}$
11	12	13	14	15

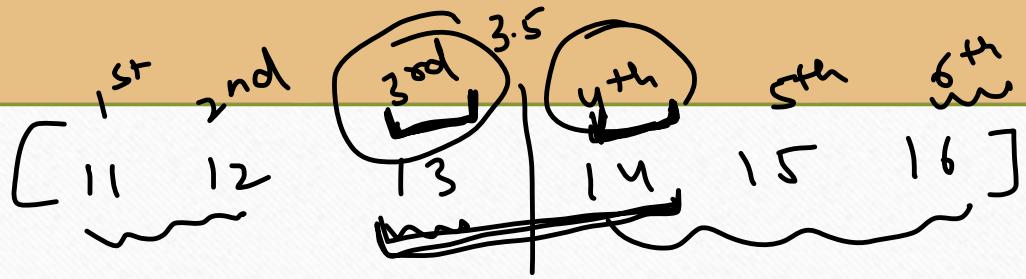
Median  $\Rightarrow$  13

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getting  
to right  
position

$$\frac{5+1}{2} \Rightarrow 3^{th} \rightarrow 13$$

↑  
Median



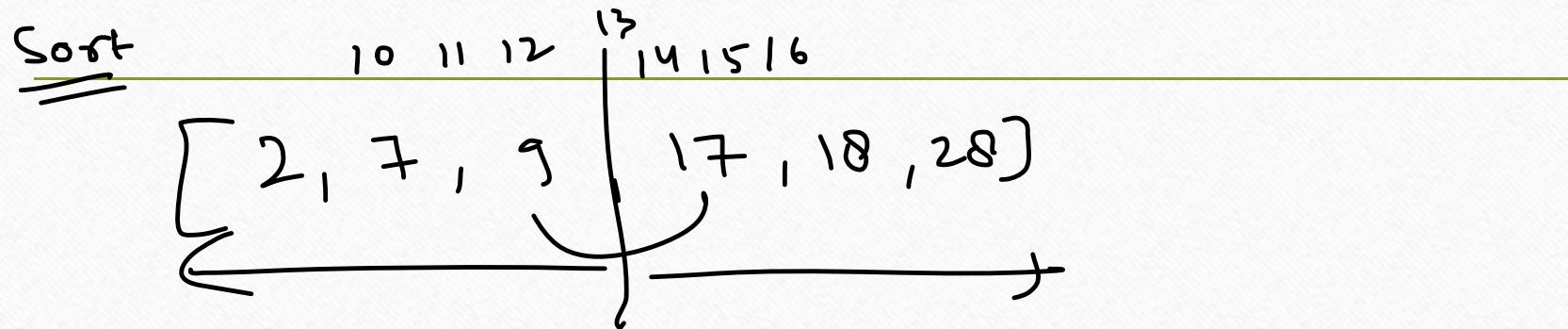
$\text{len}(\text{list}) = 6$

$$\frac{(3^{\text{rd}} + 4^{\text{th}})}{2}$$

*positions*

$$\left[ \frac{\frac{6}{2}}{2} + \left( \frac{\frac{6}{2} + 1}{(3^{\text{rd}}) + 1} \right) \right] \Rightarrow \frac{3^{\text{rd}} + 4^{\text{th}}}{2} = 13.5$$

$[7, 9, 2, 17, 18, 28]$



$$\text{Median} = \frac{9+17}{2} = 13$$

①

$$[1, 2, 3, 4, 5]$$



Median  $\Rightarrow$  ③

②

$$[1 \ 2 \ 3 \ 4 \ 500]$$



Median = 3

③

$$[1 \ 2 \ 3 \ 4 \ 500 \ 600]$$



Median = 3.5

④

$$\begin{matrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 1 & 2 & 3 & 4 & 500 & 600 & 700 & 800 \end{matrix}$$

$$\frac{4+5}{2} = \frac{4+500}{2} = 252$$

[B B B A C D E E]

Mode = B

Mode

[B B B A C D E E E]

Modes  $\Rightarrow$  (B) or E

Bi-modal

- It works for both numerical and categorical data.
- Observation with highest frequency is the mode.
- A dataset can be unimodal or multimodal.
- For ungrouped data, we can count or direct observe in the table.
- For grouped data, we use

$$\text{Mode} = l_1 + \left( \frac{f_0 - f_{-1}}{2f_0 - f_+ - f_-} \right).$$

df.value\_counts()

sns.countplot()

## Imputation of Missing Values

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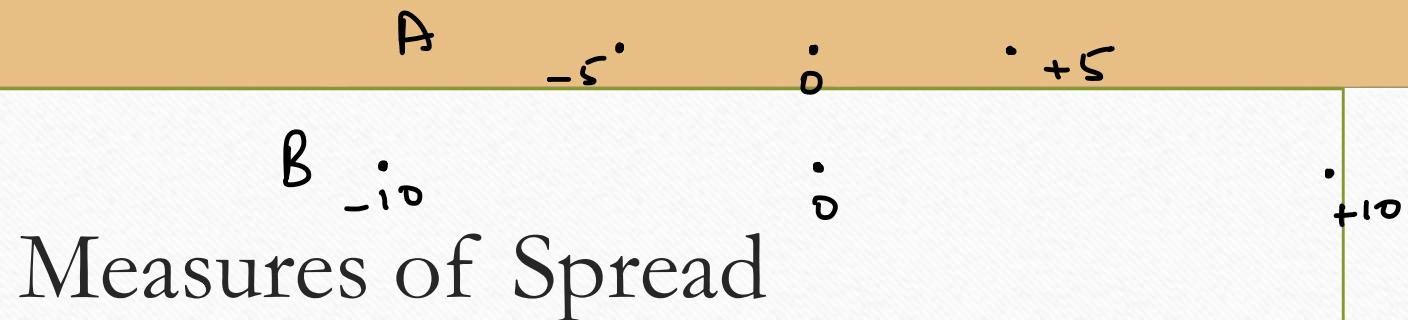
- If outliers aren't present & feature is numerical, mean value is used for imputation.
- If outliers are present, median is used
- If col is categorical the mode is used!

Mean A = 0

Mean B = 0

Median A = 0

Median B = 0



## Measures of Spread

### Measures

Range

Variance

Standard Deviation

Quartile Range(IQR)

[1 2 3 4 5]

$$\text{Range} = 5 - 1 = 4$$

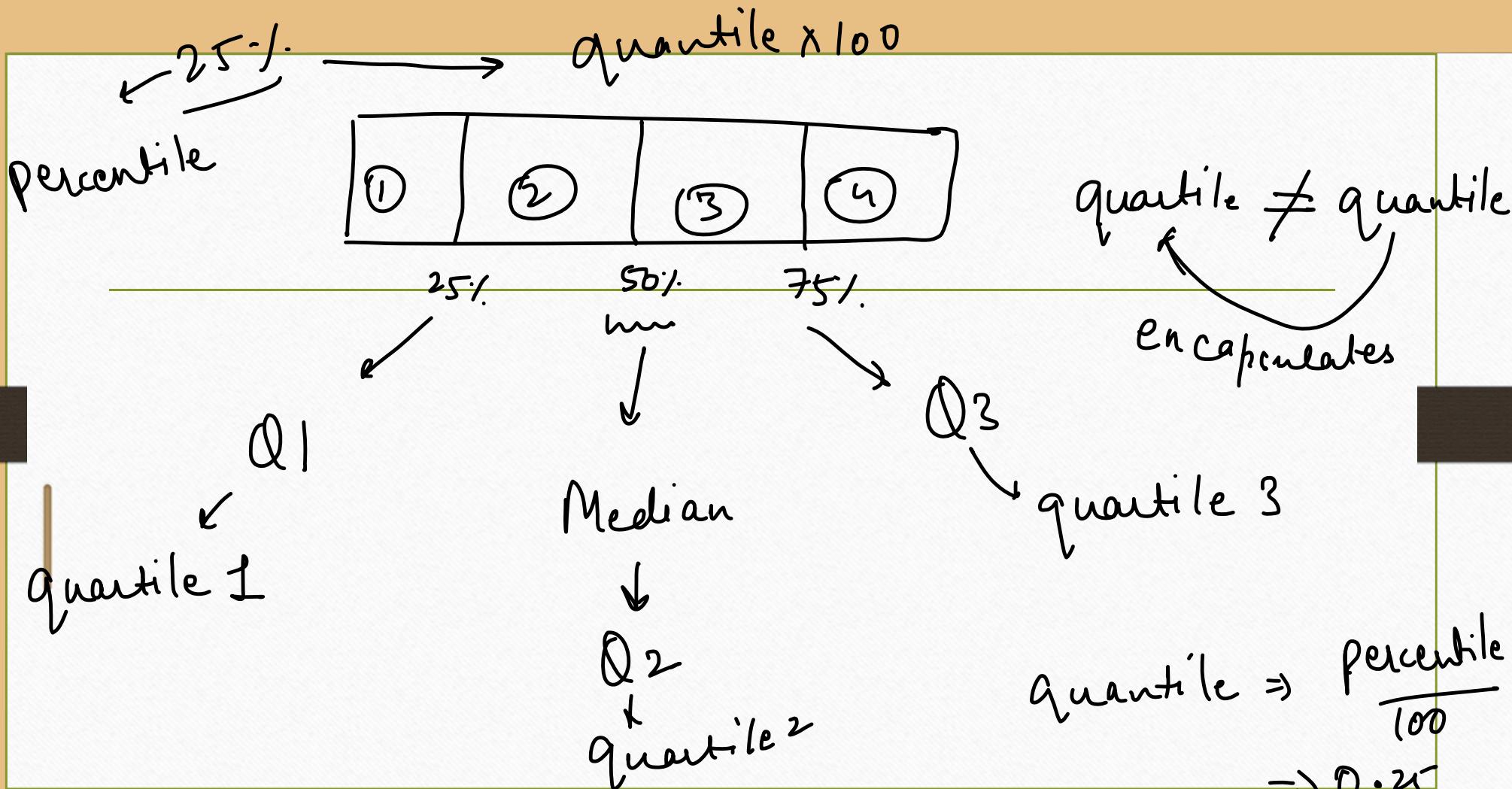
[1 3 5 10 200] Range  $\text{Range} = 200 - 1 = 199$

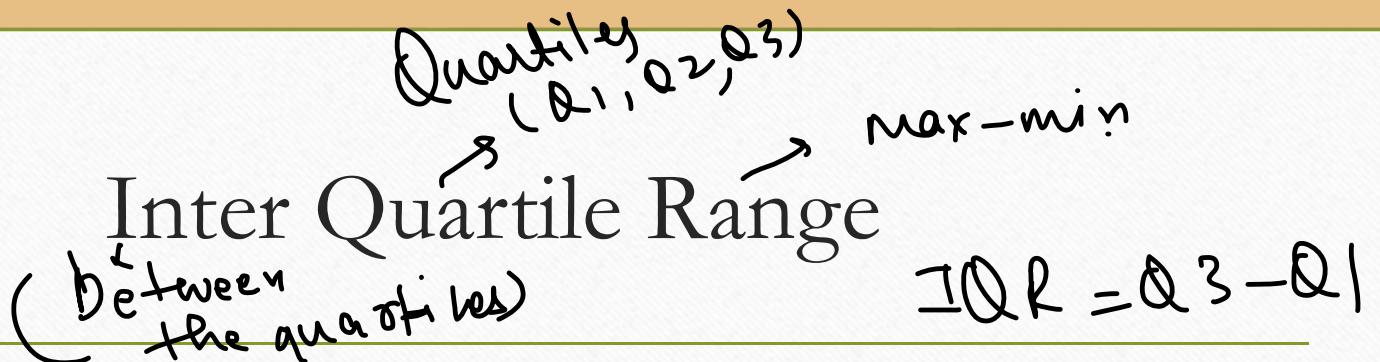
X

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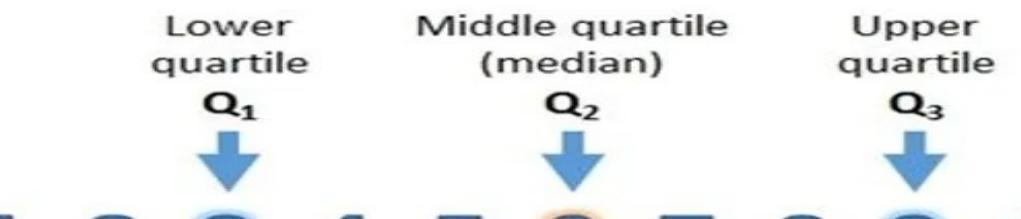
- The range is a way of measuring how spread out a set of values are.
- The range only describes the width of the data, not how it's dispersed between the bounds.
- Range is very sensitive to outliers.

$$\text{Range} = X_{\max} - X_{\min}$$



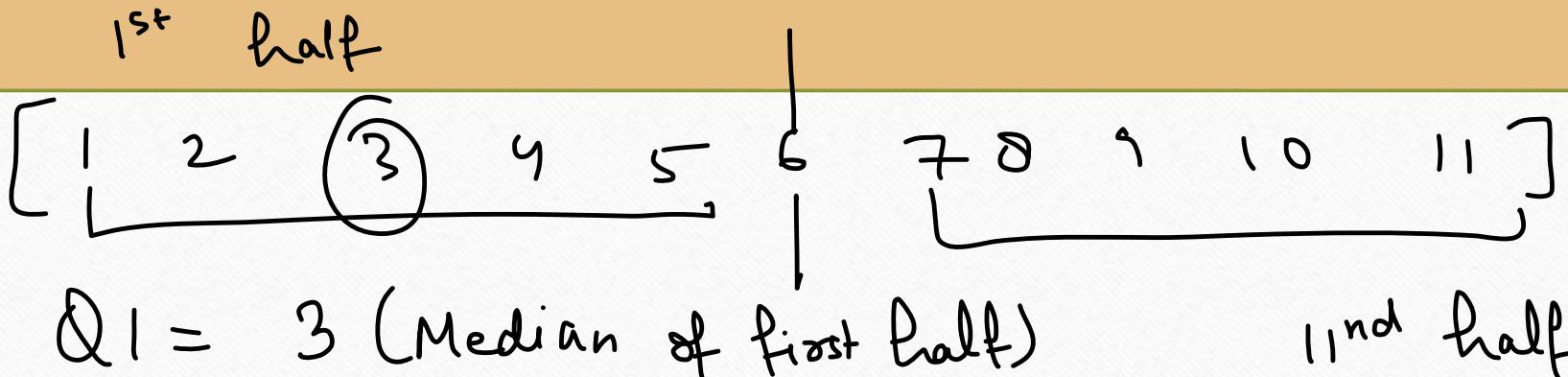


- Quartiles: The numbers that separate data into 4 equal parts.



- IQR: difference between the 3<sup>rd</sup> quartile and 1<sup>st</sup> quartile.
- Less sensitive to outliers

$$IQR = Q_3 - Q_1$$



$$Q_2 = \text{Median} \Rightarrow 6$$

$$Q_3 = 9 \text{ (Median of second half)}$$

$$IQR = 9 - 3 = 6$$

$$IQR = 6$$

$$IQR = 4$$

B

A

"Outlier  
Detection"

$$UL = 9 + 1.5 \times 6$$

$$= 18$$

$$LL = 3 - 1.5 \times 6 = -6$$

[ 1    2    3    4    5    6    7    8    9    10    11 ]

$$Q1 = 3$$

$$IQR = 6$$

$$Q2 = 6$$

$$\text{UL} \Rightarrow Q3 + 1.5 \xrightarrow{\text{?}} \text{IQR} \rightarrow \begin{array}{l} \text{beyond it} \\ \text{everything is} \\ \text{an outlier} \end{array}$$

$$Q3 = 9$$

$$LL \Rightarrow Q1 - 1.5 IQR \rightarrow \begin{array}{l} \text{below it everything} \\ \text{is an outlier} \end{array}$$

$$Q = [17, 17, 18, 19, 20, 22, 23, 25, 33, \underline{64}] \rightarrow \text{outlier}$$

$$Q_1 = 18$$

$$UL = Q_3 + 1.5 IQR = 25 + 1.5 \times 7 = 35.5$$

$$Q_2 = 21$$

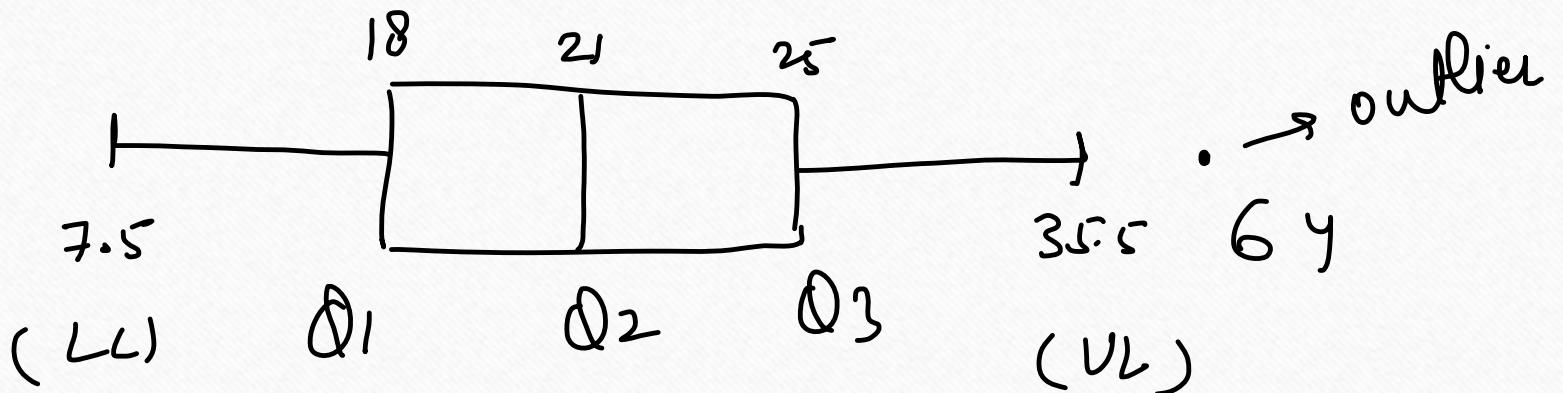
$$LL = Q_1 - 1.5 IQR = 18 - 1.5 \times 7$$

$$Q_3 = 25$$

$$= 7.5$$

$$IQR = 7$$

Box plot → Documentation of  
Seaborn / Matplotlib  
What is UL & LL?



# Checking for Outliers

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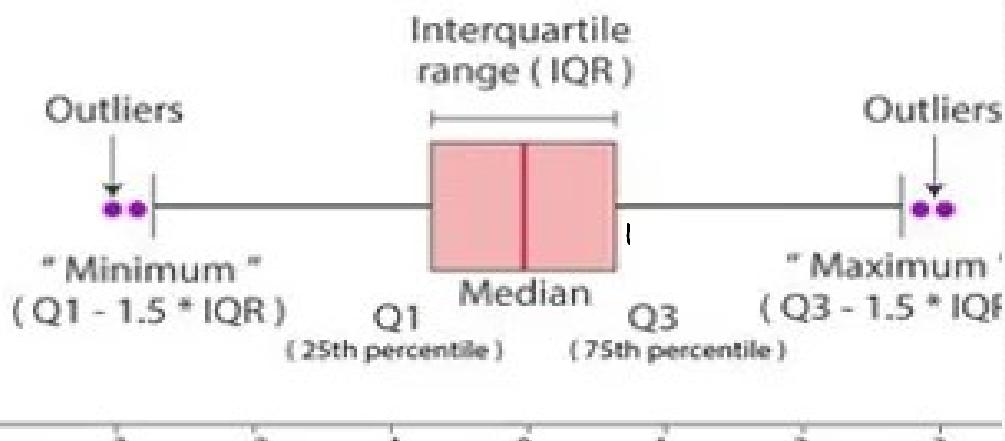
- With Range, Outlier detection isn't possible but with IQR it is possible!!

$$L = Q1 - (1.5 * \text{IQR})$$

$$H = Q3 + (1.5 * \text{IQR})$$

- Where L is the lower outlier
- H is the higher outlier
- Q1 and Q3 are the average values of those quartiles

# Box Plot & 5 Number Summary



$VK: [80, 0, 1, 40, 25] \cup$   
 $[10, 10, 10, 10]$

$MSD: [40, 41, 40, 39, 40] \cup$

$[10, 0, 5, -10, 0]$

Variance  $\sigma^2 :=$



- The variance is a way of measuring spread, and it's the average of the distance of values from the mean squared.

(1)  $VAR = \frac{1}{n} \sum_{i=0}^n (x - \mu)^2$  population mean

(2)  $VAR = \frac{1}{n-1} \sum_{i=0}^n (x - \bar{x})^2$  sample mean

$$\mu = \frac{\sum_{i=1}^n x_i}{n}$$
$$v = \frac{\sum_{i=1}^n x_i^2}{n} - \mu^2$$

- The problem with the variance is that it can be quite difficult to think about spread in terms of distances squared.

(2) → sample variance → estimating pop. variance  
from sample

[1 2 3 4 5] → Mean ≥ 3

variance  $\Rightarrow \frac{1}{n} \sum (x - \mu)^2$

avg

$$= \frac{1}{5} [(1-3)^2 + (2-3)^2 + (3-3)^2 + (4-3)^2 + (5-3)^2]$$

$$= \frac{1}{5} [4 + 1 + 0 + 1 + 4] \Rightarrow \frac{10}{5} = 2$$

$\begin{bmatrix} -5 & 0 & 5 \end{bmatrix} \Rightarrow \text{python} \rightarrow \text{variance}$

$$\frac{(-5-0)^2 + (0-0)^2 + (5-0)^2}{3} = \frac{50}{3}$$

$$\Downarrow \frac{50}{3-1} = \frac{50}{2} = 25$$

Python  $\Rightarrow 25$