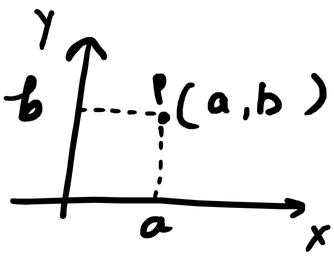
Linear Algebra



$\Rightarrow$ vector = [a   b]

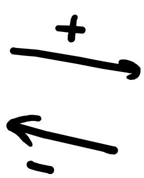
P → datapoint (row in your dataset)

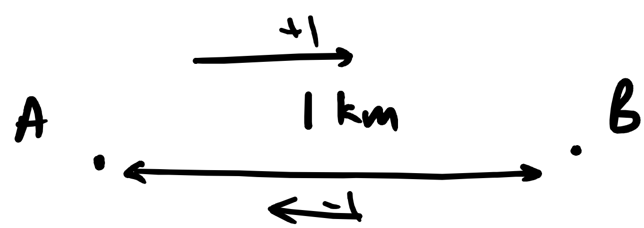
x, y → columns (axis/variables/features/dimensions)

Quantities

Scalar → magnitude

Vector → magnitude with direction

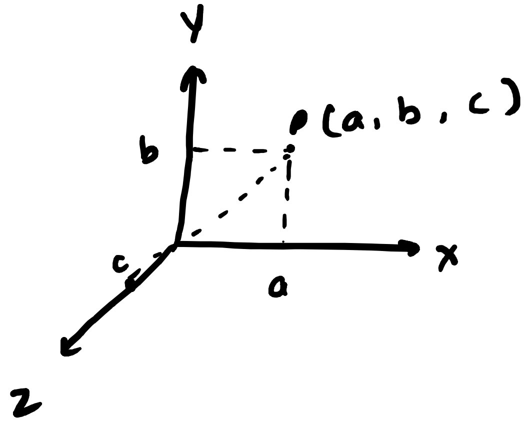




distance = 2 (1 + 1)

displacement = +1 - 1 = 0

vector in 3d



vector $(\vec{P})$ = $[a \quad b \quad c]$

vector in 6d ⇒ $[a \quad b \quad c \quad d \quad e \quad f]$

Vector in nd ⇒ $[a \quad b \quad c \quad - - - - - \quad n]$

Matrix ⇒ representation of data in rows & columns

$$\begin{array}{cc} & \text{columns} \\ \text{rows} & \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} \end{array}$$

shape of matrix ⇒ [Rows x columns]

⇒ [2 x 3]

Square matrix $\Rightarrow$ Rows = column

$$\downarrow$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \Rightarrow \text{Shape } [2 \times 2]$$

Linear Transformations.

$$I/p \longrightarrow \boxed{\text{Matrix}} \longrightarrow O/p$$

Matrix Addition:

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} + \begin{bmatrix} e & f \\ g & h \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} a+e & b+f \\ c+g & d+h \end{bmatrix}$$

Q $\begin{bmatrix} 2 & 4 \\ 1 & 5 \end{bmatrix} + \begin{bmatrix} 3 & 6 \\ 2 & 7 \end{bmatrix} \Rightarrow \begin{bmatrix} \overset{(2+3)}{5} & \overset{(4+6)}{10} \\ \overset{(1+2)}{3} & \overset{(5+7)}{12} \end{bmatrix}$

Matrix Multiflications:

$$\begin{array}{c} \\ \text{II} \to \text{I} \to \\ \text{IV} \to \text{III} \to \end{array} \begin{array}{c} \text{I} \quad \text{II} \\ \begin{bmatrix} a & b \\ c & d \end{bmatrix}_{2\times 2} \times \begin{bmatrix} e & f \\ g & h \end{bmatrix}_{2\times 2} \end{array}$$

$$\Rightarrow \begin{bmatrix} a\times e + b\times g & a\times f + b\times h \\ c\times e + d\times g & c\times f + d\times h \end{bmatrix}$$

$\underline{2\times 2} \Rightarrow$ 4 elements

* if columns of first Matrix = rows of second matrix then only MM is possible

Q $A \Rightarrow p\times q$, $B = q\times c$

$C = A\times B \Rightarrow p\times c$

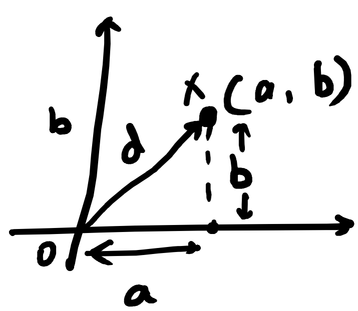
Q $\begin{bmatrix} 2 & 4 \\ 3 & 6 \end{bmatrix} \times \begin{bmatrix} 1 & 4 \\ 2 & 5 \end{bmatrix}$

$$\Rightarrow \begin{bmatrix} 2\times1+4\times2 & 2\times4+4\times5 \\ 3\times1+6\times2 & 3\times4+6\times5 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 10 & 28 \\ 15 & 42 \end{bmatrix}$$

Distance

Distance of a point from origin:



By Pythagoras Theorem,

$$H^2 = P^2 + B^2$$

$$d^2 = b^2 + a^2$$

or

$$d^2 = a^2 + b^2$$

$$\boxed{d = \sqrt{a^2+b^2}} \rightarrow 2d$$

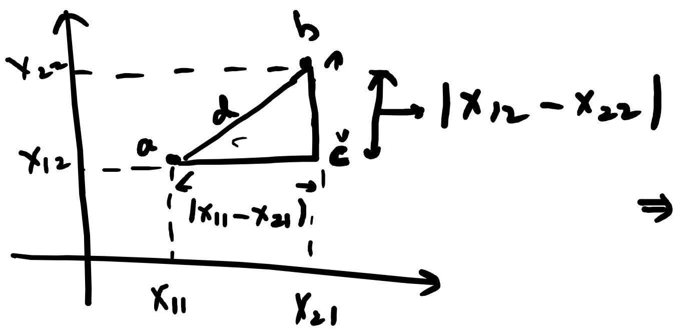
3d

$$d = \sqrt{a^2+b^2+c^2}$$

nd

$$d = \sqrt{a^2+b^2+c^2+\dots+n^2}$$

Distance b/w 2 points



By Pythagoras Theorem,

$$\Rightarrow \quad ab^2 = ac^2 + bc^2$$

$$d^2 = |x_{11} - x_{21}|^2 + |x_{12} - x_{22}|^2$$

$a \Rightarrow [x_{11} \quad x_{12}]$

$b = [x_{21} \quad x_{22}]$

$$\boxed{d = \sqrt{|x_{11} - x_{21}|^2 + |x_{12} - x_{22}|^2}}$$

Euclidean distance

$$d = \left[\sum_{i=1}^{n} |x_{1i} - x_{2i}|^2\right]^{1/2}$$

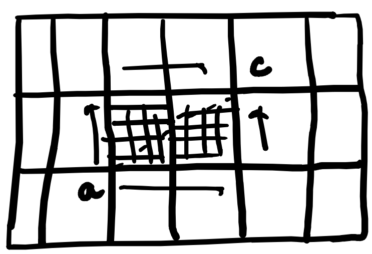
3d

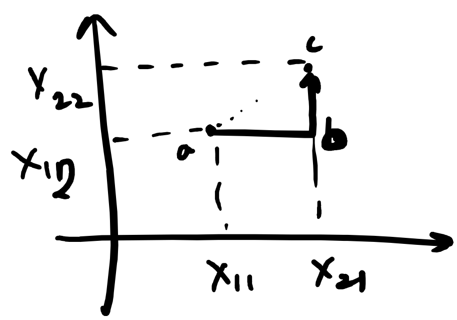
$$d = \sqrt{|x_{11} - x_{21}|^2 + |x_{12} - x_{22}|^2 + |x_{13} - x_{23}|^2}$$

$+ \cdots + |x_{1n} - x_{2n}|^2$

In a space with a lot of dimentions, euclidean distance loses its meaning.

Manhattan Distance





a to c $\Rightarrow$ a to b + b to c

Manhattan distance $\leftarrow$ $d = |x_{11} - x_{21}| + |x_{12} - x_{22}|$

$$\boxed{d \Rightarrow \sum_{i=1}^{n} |x_{1i} - x_{2i}|}$$

Minkowski Distance

$$d = \left[\sum_{i=1}^{n} |x_{1i} - x_{2i}|^{p} \right]^{1/p}$$

where $P = 1, 2, 3, \ldots$

lets
$P = 1$

$$d = \left[\sum_{i=1}^{n} |x_{1i} - x_{2i}|^{1} \right]^{1} \rightarrow \text{Manhattan Distance}$$

lets $p = 2$

$$d = \left[\sum_{i=1}^{n} |x_{1i} - x_{2i}|^{2} \right]^{1/2} \rightarrow \text{euclidean distance}$$

Vector Multiplication

dot product
↓
scalar (number)

$\times$ cross product
↓
direction

Linear Algebra

Row format $\Rightarrow \begin{bmatrix} a & b \end{bmatrix}$

column format is the default one

$$\begin{bmatrix} a \\ b \end{bmatrix}$$

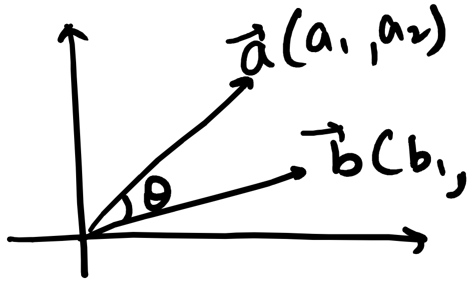
$$\vec{a} = \begin{bmatrix} a_1 \\ \vdots \\ a_n \end{bmatrix} \qquad \vec{b} = \begin{bmatrix} b_1 \\ \vdots \\ b_n \end{bmatrix}$$

$$\vec{a} \cdot \vec{b} = \begin{bmatrix} a_1 \\ \vdots \\ a_n \end{bmatrix}_{n \times 1} \begin{bmatrix} b_1 \\ \vdots \\ b_n \end{bmatrix}_{n \times 1}$$

$$a^T \cdot b = \begin{bmatrix} a_1 & \cdots & a_n \end{bmatrix}_{1 \times n} \begin{bmatrix} b_1 \\ \vdots \\ b_n \end{bmatrix}_{n \times 1}$$

$$\Rightarrow \begin{bmatrix} a_1 b_1 + a_2 b_2 + \cdots + a_n b_n \end{bmatrix}_{1 \times 1}$$

Angle b/w vectors



θ = angle b/w a & b

(I) → $\vec{a} \cdot \vec{b} = \|\vec{a}\| \, \|\vec{b}\| \cos\theta$ $\qquad \|\vec{a}\| = \sqrt{a_1^2 + a_2^2}$

(II) → $a^T \cdot b = [a_1 b_1 + a_2 b_2]$ $\qquad \|\vec{b}\| = \sqrt{b_1^2 + b_2^2}$

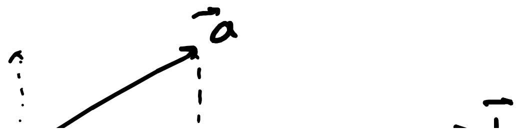
(Geo) $\vec{a} \cdot \vec{b} = a^T \cdot b$ (LA)

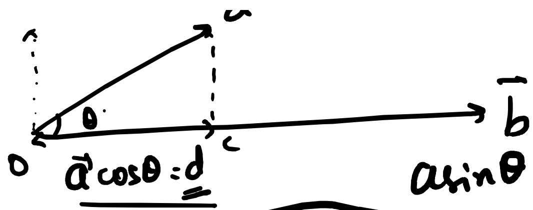
$$\|\vec{a}\| \, \|\vec{b}\| \cos\theta = [a_1 b_1 + a_2 b_2]$$

$$\cos\theta = \frac{[a_1 b_1 + a_2 b_2]}{\|\vec{a}\| \, \|\vec{b}\|}$$

angle b/w vectors → $\theta = \cos^{-1}\left[\frac{a_1 b_1 + a_2 b_2}{\|\vec{a}\| \, \|\vec{b}\|}\right]$

Projection of vectors





$$\vec{a} \cdot \vec{b} = \|\vec{a}\| \|\vec{b}\| \cos\theta$$

$$\vec{a} \cdot \vec{b} = d \cdot \|\vec{b}\|$$

$$\boxed{d = \frac{\vec{a} \cdot \vec{b}}{\|\vec{b}\|}}$$ — projection of vector a on b.

Unit Vector ⇒ vector with magnitude 1.

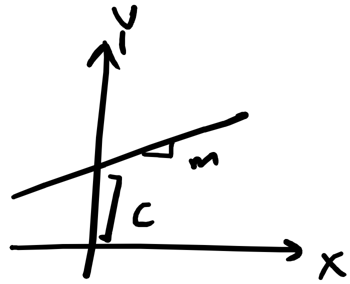
↳ used to get direction.

$$\hat{a} = \frac{\vec{a}}{\|\vec{a}\|}$$

Eigen Value & Eigen vector

$$A\vec{x} = \lambda \vec{x}$$

Lines & Planes



equation of line

$y = mx + c$

m ↙ slope ↳ rate of change of y wrt x

c ↳ intercept

$$m \Rightarrow \frac{\text{change in } y}{\text{change in } x}$$

$$Y = MX + C$$

$$\begin{bmatrix} Y_1 \\ Y_2 \\ \vdots \\ Y_3 \end{bmatrix} = \begin{bmatrix} M_1, M_2 \cdots M_n \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ \vdots \\ X_n \end{bmatrix} + \begin{bmatrix} c \\ c \\ c \\ \vdots \\ c \end{bmatrix}$$

$$Y = [M_1X_1 + M_2X_2 + M_3X_3 \cdots + M_nX_n] + c$$

eq^n of a line $\boxed{Y = \underbrace{M^T x}_{\text{dot product}} + C}$

Newspaper	TV	Sales
—	—	—

Newspaper	TV	Sales
—	—	—

I/p ⇒ [X] ⇒ Newspaper, TV

O/p ⇒ Y ⇒ sales

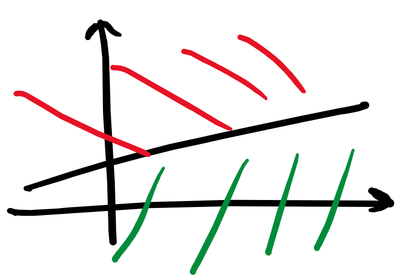
$$\boxed{Y = M_1 \text{Newspaper} + M_2 \text{TV} + C}$$

weights

$\boxed{Y = M^T X}$ → eq^n of line passing through origin

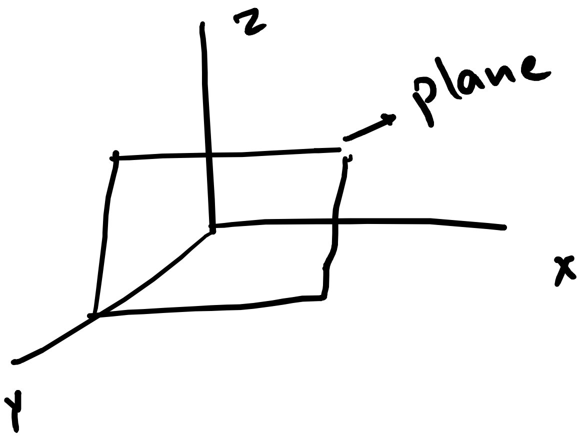
||

$\boxed{Y = W^T X}$ → eq^n of line ⇒ $Y = w_1 x_1 + w_2 x_2$



→ line divides 2d space in two parts

Plane



eq^n: $Ax + By + Cz + D = 0$

coefficients (slopes/weights)

↓ axis name

$Ax_1 + Bx_2 + Cx_3 + D = 0$

↓ coeff. name

$\underbrace{w_1 x_1 + w_2 x_2 + w_3 x_3} + w_0 = 0$

↓

eq^n of plane ← $\boxed{W^T x + w_0 = 0}$

intercept

↓

plane passes through origin

plane passes through origin

$\downarrow$

$\boxed{W^T X = 0} \rightarrow Y$

$\swarrow$

$\boxed{w_1 x_1 + w_2 x_2 + w_3 x_3 = Y}$

Above 3d $\rightarrow$ Hyperplane

$\underline{w_0 + w_1 x_1 + w_2 x_2 + w_3 x_3} + \cdots + w_n x_n = 0$

$\downarrow$

eqⁿ of hyperplane

$\boxed{w_0 + w^T x = 0} \rightarrow$ eqⁿ of hyperplane

$$\boxed{\begin{aligned} Y &= \theta_0 + \theta^T x \\ 0 &= \theta_0 + \theta^T x \end{aligned}}$$

$\rightarrow$ Blogs / Research paper