DAA

Tutorial-1

Q1 What do you understand by asymptotic notation. Define different asymptotic notation with examples.

Solⁿ i) Big $O(n)$

$f(n) = O(g(n))$

iff

$f(n) \leq c\,g(n)$

$\forall\ n \geq n_0$

for some constant, $c > 0$

$g(n)$ is "tight" upper bound of $f(n)$

Ex:- $f(n) = n^2 + n$

$g(n) = n^3$

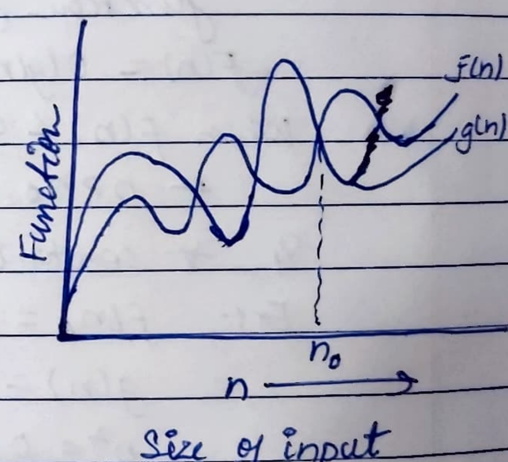
$n^2 + n \leq c.n^3$

$n^2 + n = O(n^3)$

ii) Big Omega (Ω)

$f(n) = \Omega(g(n))$

$g(n)$ is "tight" lower bound of function $f(n)$



$f(n) = \Omega(g(n))$

iff

$f(n) \geq c\,g(n)$

$\forall\ n \geq n_0$

For some constant $c > 0$

Ex:- $f(n) = n^3 + 4n^2$

$g(n) = n^2$

$n^3 + 4n^2 = \Omega(n^2)$

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(iii) **Big Theta (Θ)**

$f(n) = \Theta(g(n))$

$g(n)$ is both "tight" upper bound and lower bound of function $f(n)$

$f(n) = \Theta(g(n))$

iff

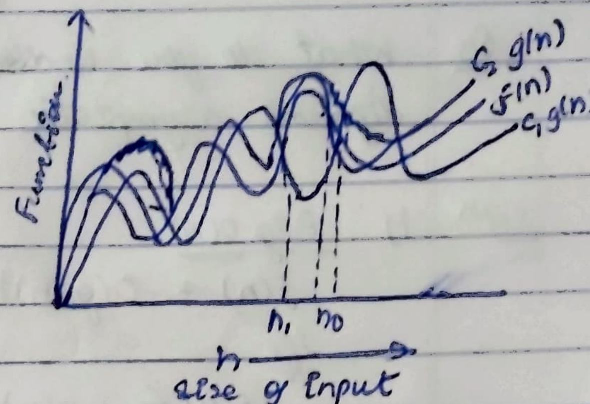
$c_1 g(n) \leq f(n) \leq c_2 g(n)$

$\forall\ n \geq \max(n_1, n_2)$

for some constant $c_1 > 0$ & $c_2 > 0$

Ex:- $3n+2 = \Theta(n)$ as $3n+2 \geq 3n$ &

$3n+2 \leq 4n$ for n, $K_1 = 3$, $K_2 = 4$, & $n_0 = 2$



(iv) **Small o(θ)**

$f(n) = o(g(n))$

$g(n)$ is upper bound of function $f(n)$

$f(n) = o(g(n))$

when $f(n) < c\,g(n)$

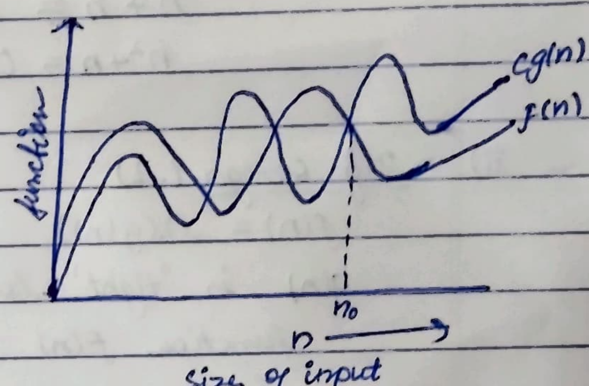
$\forall\ n > n_0$

& $\forall$ constants, $c > 0$

Ex:- $f(n) = n^2$

$g(n) = n^3$

$n^2 = o(n^3)$



v) **Small Omega (ω)**

$f(n) = \omega(g(n))$

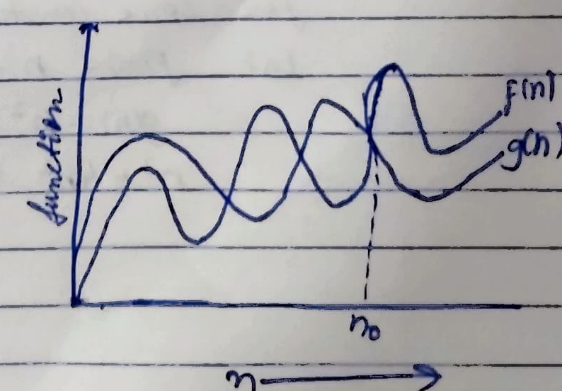
$g(n)$ is lower bound of func^n $f(n)$

$f(n) = \omega(g(n))$ when

$f(n) > c \cdot g(n)$

$\forall\ n > n_0$

& $\forall$ constants, $c > 0$



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Q2 What should be time complexity of -
for (i = 1 to n) { i = i * 2; }

Soln

```
for (i=1 to n)
{
    i = i * 2;        O(1)
}
```

$$i = \overbrace{1, 2, 4, \ldots\ldots n}^{K}$$

$a = 1$, $r = \frac{b_2}{b_1} = 2$

G.P K^{th} value, $t_K = ar^{K-1}$

$t_K = 2^{K-1}$

$t_K = \frac{2^K}{2}$ $\{t_K = n\}$

$2n = 2^K$

$\log_2(2n) = K \log_2 2$ $\{\log_a a = 1\}$

$K = \log_2 2n$

$K = \log_2 2 + \log_2 n$ $\{\log ab = \log a + \log b\}$

$K = 1 + \log_2 n$

$O(1 + \log_2 n)$

$\Rightarrow O(\log n)$

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Q3 $T(n) = \{3T(n-1)$ if $n>0$, otherwise $1\}$

Sol^n $T(n) = 3T(n-1)$ —— ①

Put $n = n-1$ in ①

$T(n-1) = 3T(n-2)$ —— ②

Put value of $T(n-1)$ from ② to ①

$T(n) = 3[3T(n-2)] \Rightarrow 9T(n-2)$ —— ③

Put $n = n-2$ in ①

$T(n) = 3T(n-3)$ —— ④

Put value of $T(n-2)$ from ④ to ③

$T(n) = 9[3T(n-3)]$

$T(n) = 27T(n-3)$

By generalizing, $T(n) = 3^k T(n-k)$ —— ⑤

Let $n-k = 1$

$k = n-1$

Put value of k in ⑤

T(n) value of k in ⑤

$T(n) = 3^{n-1} T(n-n+1)$

$T(n) = 3^{n-1} T(1)$

$T(n) = \frac{3^n}{3} \times 1$

$O(3^n)$

Q4 $T(n) = \{2T(n-1) - 1$ if $n>0$, otherwise $1\}$

Sol^n $T(n) = 2T(n-1) - 1$ —— ①

Put $n = n-1$ in ①

$T(n-1) = 2T(n-2) - 1$ —— ②

Put value of $T(n-1)$ from ② to ①

$T(n) = 2[2T(n-2) - 1] - 1$

$T(n) = 4T(n-2) - 2 - 1$ —— ③

Put $n = n-2$ in ①

$T(n-2) = 2T(n-3) - 1$ —— ④

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Put value of $T(n-2)$ from ④ to ③

$T(n) = 4[2T(n-3)-1]-2-1$

$T(n) = 8T(n-3)-4-2-1$ —— ⑤

$T(n) = 8T(n-3)-7$

By generalizing, we get.

$T(n) = 2^k T(n-k) - (2^k-1)$ —— ⑥

Let $n-k=1$

$k = n-1$

Put k in ⑥

$T(n) = 2^{n-1} T(n-n+1) - (2^{n-1}-1)$

$= 2^{n-1}T(1) - 2^{n-1} - 1$

$= 2^{n-1} - 2^{n-1} + 1$

$= 1$

$= O(1)$

Q5 What should be time complexity of -

```
void function (int n) {
    int i, ~~j, k~~, count = 0;
    for (i=1; i*i<=n; i++)
        count++
}
```

Solⁿ As $i^2 \le n$

$O \le \sqrt{n}$

$i = 1, 2, 3, 4, \ldots, \sqrt{n}$

$\sum_{i=1} 1+2+3+4+ \ldots + \sqrt{n}$

$T(n) = \frac{\sqrt{n} \times (\sqrt{n}+1)}{2}$

$T(n) = \frac{n \times \sqrt{n}}{2}$

$T(n) = O(n)$

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Q6 What should be time complexity of -

```
int i=1, s=1;
while (s<=n) {
    i++; s=s+1;
    printf("#");
}
```

Sol^n

$i = 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \ldots\ldots$

$s = 1 + 3 + 6 + 10 + 15 + \ldots\ldots$

sum of $s = 1 + 3 + 6 + 10 + \ldots\ldots + n$ — ①

Also $s = 1 + 3 + 6 + 10 + \ldots\ldots + T_{n-1} + T_n$ — ②

$0 = 1 + 2 + 3 + 4 + \ldots\ldots + n - T_n$

$T_k = 1 + 2 + 3 + 4 + \ldots\ldots + k$

$T_k = \frac{1}{2} k (k+1)$

for k iterations.

$1 + 2 + 3 + \ldots\ldots + k <= n$

$\Rightarrow \frac{k(k+1)}{2} <= n$

$\frac{k^2+k}{2} <= n$

$O(k^2) <= n$

$k = O(\sqrt{n})$

$k = O(\sqrt{n})$

$\underline{T(n) = O(\sqrt{n})}$

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Q7 Time complexity of -

```
void function (int n) {
   int i, j, k, count = 0;
   for (i = n/2 ; i <= n; i++)
      for (j = 1 ; j <= n ; j = j*2)
         for (k = 1 ; k <= n ; k = k*2)
            count++
}
```

Sol^n

For $k = k^2$

$k = 1, 2, 4, 8, \text{-----}, n$

G.P $\Rightarrow$ $a = 1$, $r = 2$

$$= \frac{a(r^n - 1)}{r-1}$$

$$= \frac{1(2^k - 1)}{1}$$

$n \Rightarrow 2^k - 1$

$\log n = k$

i	j	k
1	$\log n$	$\log n * \log n$
2	$\log n$	$\log n * \log n$
⋮	⋮	⋮
n	$\log n$	$\log n * \log n$

$\Rightarrow O(n * \log n * \log n)$

$\Rightarrow O(n \log^2 n)$

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Q6 What should

int i= 1

while

i++

pr

}

Sol^n i = 1

s = 1+

Sum

Also

O=

T_k

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Q7 Time complexity of -

```
void function (int n) {
  int i, j, k, count = 0;
  for (i = n/2 ; i <= n; i++)
    for (j = 1 ; j <= n ; j = j * 2)
      for (k = 1; k <= n ; k = k * 2)
        count++
}
```

Sol^n

For $k = k^2$

$k = 1, 2, 4, 8, \text{------}, n$

$G.P \Rightarrow a = 1, r = 2$

$= \frac{a(r^n - 1)}{r - 1}$

$= \frac{1(2^k - 1)}{1}$

$n \Rightarrow 2^k - 1$

$\log n = k$

i	j	k
1	log n	log n * log n
2	log n	log n * log n
n	log n	log n * log n

$\Rightarrow O(n * \log n * \log n)$

$\Rightarrow O(n \log^2 n)$

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Q8 Time complexity of -

```
function (int n) {
    if (n == 1) return;
        for (i= 1 to n) {
            for (j= 1 to n) {
                printF ("*");
            }
        }
        function (n-3);
    }
```

Soln For (i= 1 to n)

we get j= n times every turn.

$\because i \times j = n^2$

Now, $T(n) = n^2 + T(n-3);$

$T(n-3) = (n^2 3)^2 + T(n-6);$ K times.

$T(n-6) = (n^3 6)^2 + T(n-5);$

$T(1) = 1;$

Now subs. each value in $T(n)$

$T(n) = n^2 + (n-3)^2 + (n-6)^2 + ---- + 1$

Let,

$n - 3k = 1$

$k = (n-1)/3$

Total terms $= k+1$

$T(n) = n^2 + (n-3)^2 + (n-6)^2 + ----- + 1$

$T(n) \approx k n^2$

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$T(n) \approx \frac{(n-1)}{3} \times n^2$

$\therefore T(n) = O(n^3)$

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Q9 Time complexity of -

```
void function (int n) {
   for (i=1 to n) {
      for (j=1; j<=n ; j=j+1]
         printf("*")
   }
}
```

Solⁿ For :- $i=1$ $\quad j=1+2+\cdots\cdots(n \geq j+1)$

$i=2$ $\quad j=1+3+5+\cdots\cdots(n \geq j+1)$

$i=3$ $\quad j=1+4+7+\cdots\cdots(n \geq j+1)$

$\vdots$

m^{th} term of A.P is

$T(m) = a+d \times m$

$T(m) = 1+d \times m$

$(n-1)/d = m$

for $i=1$ $\quad (n-1)/1$ times

$i=2$ $\quad (n-2)/2$ times

$i=3$ $\quad (n-1)/3$ times

$\vdots$

$i=n-1$ $\quad 1$

we get,

$\cancel{T(m) = a+d \times m}$ $T(n)$

$\cancel{T(m) = 1+d \times m}$

$T(n) = i_1 j_1 + i_2 j_2 + \cdots\cdots i_n + j_{n-1}$

$= \frac{(n-1)}{1} + \frac{(n-2)}{2} + \frac{(n-3)}{3} + \cdots\cdots + 1$

$= n + \frac{n}{2} + \frac{n}{3} + \cdots\cdots + \frac{n}{n-1} - n * 1$

$= n\left[1+\frac{1}{2}+\frac{1}{3}+\cdot\frac{1}{n-1}\right] - n + 1$

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$= n \times \log n - n + 1$

Since $\int \frac{1}{x} = \log x$

$T(n) = O(n \log n)$

Q10 For the functions n^k and c^n, what is the asymptotic relationship between these functions?
Assume that k>=1 and c>1 are constants. Find out the value of c and n0 for which relation holds.

Soln As given n^k & c^n
relation between n^k and c^n is

$n^k = O(c^n)$

$n^k \leq a(c^n)$

$\forall\ n \geq n_0$ &

constant, $a > 0$

for $n_0 = 1$

$c = 2$

$\Rightarrow 1^k \leq a^{2 \cdot 1}$

$\Rightarrow n_0 = 1$ & $c = 2$

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