A big thanks to

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Complex Number.

Cartesion eqⁿ

- Complex number = $x + iy$ → real; y → imaginary; $i = \sqrt{-1}$

denoted as

$$z = x + iy.$$

* Conjugate complex number

$$\bar{z} = x - iy.$$

→ imaginary part ki ~~value~~ sign change करना है

* $z_1 = x_1 + iy_1$, $z_2 = x_2 + iy_2$ } equal → if & only if $x_1 = x_2$, $y_1 = y_2$.

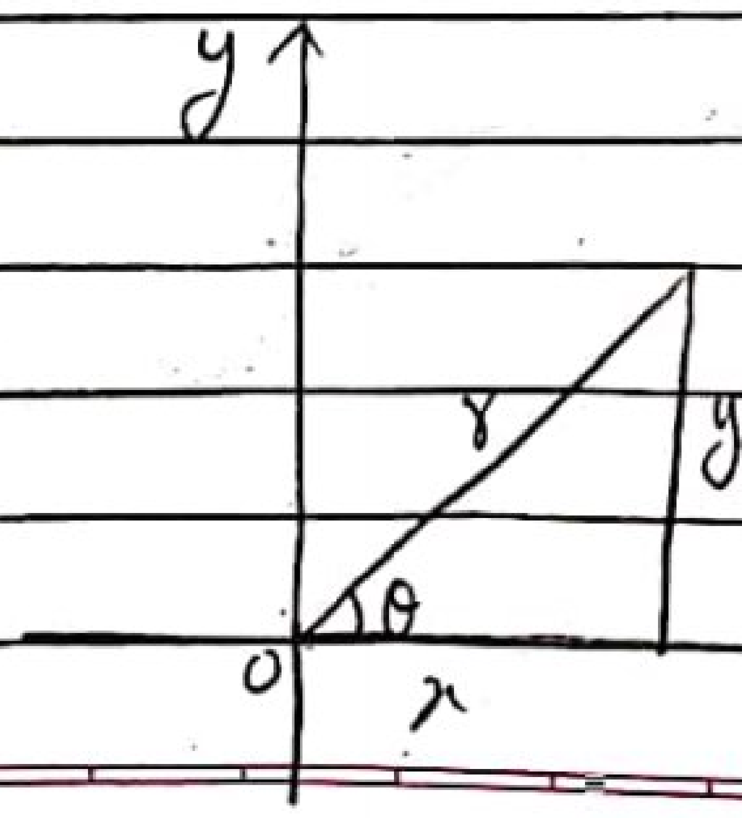
* Power of i

$$i = \sqrt{-1}$$
$$i^2 = -1$$
$$i^3 = i^2 \times i = -i$$
$$i^4 = -1 \times -1 = 1.$$
$$i^5 = 1 \times i = i$$
$$i^6 = i^4 \times i^2 = 1 \times -1 = -1$$

Modulus & Argument of complex number

$x = r\cos\theta$

$y = r\sin\theta$



$\boxed{r = \pm\sqrt{x^2 + y^2}}$ {Pyth...}

↳ modulus of z → $|z|$

$\boxed{\theta = \tan^{-1}(y/x)}$

↳ argument or amplitude.

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Polar form

$$Z = r(\cos\theta + i\sin\theta)$$

* Exponential form

$$Z = re^{i\theta}$$

* Corollary

$$\frac{1}{Z} = \cos\theta - i\sin\theta.$$

* Conjugate form

$$\bar{Z} = x - iy.$$

$$\bar{Z} = r(\cos\theta - i\sin\theta).$$

$$\bar{Z} = re^{-i\theta}.$$

De Moivre's Theorem & its ~~collaries~~ corollaries

$$(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$$

* corollaries

1) $(\cos\theta - i\sin\theta)^n = \cos n\theta - i\sin n\theta$

$$= (\cos\theta + i\sin\theta)^{-n}$$

$$= \frac{1}{(\cos\theta + i\sin\theta)^n}$$

2) $z^n = \cos n\theta + i\sin n\theta$

$z^{-n} = \cos n\theta - i\sin n\theta$

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Q.1 Simplify $\frac{(\cos 3\theta + i\sin 3\theta)(\cos\theta - i\sin\theta)}{(\cos 5\theta - i\sin 5\theta)}$

Applying De moiver's THM.

$$= \frac{(\cos\theta + i\sin\theta)^3 (\cos\theta + i\sin\theta)^{-1}}{(\cos\theta + i\sin\theta)^{-5}}$$

$$= \frac{(\cos\theta + i\sin\theta)^3 (\cos\theta + i\sin\theta)^5}{\cos\theta + i\sin\theta}$$

$$= \boxed{(\cos\theta + i\sin\theta)^7}$$

or

$$= \boxed{\cos 7\theta + i\sin 7\theta}$$

Q.2 if $z = \frac{1}{\sqrt{2}} + i\frac{1}{\sqrt{2}}$ & $\bar{z}$ is the conjugate of z, Prove that $z^{10} + \bar{z}^{10} = 0$.

Soln

$$z = \frac{1}{\sqrt{2}} + i\frac{1}{\sqrt{2}}$$

$$z = \cos\frac{\pi}{4} + i\sin\frac{\pi}{4}$$

$$\bar{z} = \cos\frac{\pi}{4} - i\sin\frac{\pi}{4}$$

$$z^{10} = \left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)^{10}$$

By De moiver's Theorem

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$= \cos 10\frac{\pi}{4} + i\sin 10\frac{\pi}{4}$

$Z^{10} = \underset{\to 0}{\cos 5\frac{\pi}{2}} + i\sin 5\frac{\pi}{2}.$

$Z^{10} = i\sin 5\frac{\pi}{2}. \quad \text{--- (i)}$

$\bar{Z}^{10} = \underset{\to 0}{\cos 5\frac{\pi}{2}} - i\sin 5\frac{\pi}{2}.$

$\bar{Z}^{10} = \cancel{\cos} - i\sin 5\frac{\pi}{2}. \quad \text{--- (ii)}$

(i + ii)

$Z^{10} + \bar{Z}^{10} = i\sin 5\frac{\pi}{2} - i\sin 5\frac{\pi}{2}.$

$Z^{10} + Z^{-10} = 0 = R.H.S.$

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3) Find the modulus & principal value of the argument $\frac{(1+i\sqrt{3})^{16}}{(\sqrt{3}-i)^{17}}$.

Sol^n

Consider $(1+i\sqrt{3})^{16} =$

$= \left[2\left(\frac{1}{2} + i\frac{\sqrt{3}}{2}\right)\right]^{16}$ {× & ÷ by 2.

$= 2^{16}\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)^{16}$

$2^{16}\left(\cos 16\frac{\pi}{3} + i\sin 16\frac{\pi}{3}\right)$.

Consider $(\sqrt{3}-i)^{17}$

$= ~~2^{16}(\cos\frac{\pi}{6}~~ \; 2^{17}\left(\frac{\sqrt{3}}{2} - \frac{1}{2}\right)^{17}$

$= 2^{17}\left(\cos\frac{\pi}{6} - i\sin\frac{\pi}{6}\right)$.

$$\frac{(1+i\sqrt{3})^{16}}{(\sqrt{3}-i)^{17}} = \frac{2^{16}\left[\cos\frac{16\pi}{3} + i\sin\frac{16\pi}{3}\right]}{2^{17}\left(\cos\frac{\pi}{6} - i\sin\frac{\pi}{6}\right)^{17}}$$

$$= \frac{1}{2}\left[\cos\frac{16\pi}{3} + i\sin\frac{16\pi}{3}\right]\left[\cos\frac{\pi}{6} - i\sin\frac{\pi}{6}\right]^{-17}$$

$$= \frac{1}{2}\left[\cos\frac{16\pi}{3} + i\sin\frac{16\pi}{3}\right]\left[\cos\left(-\frac{17\pi}{6}\right) - i\sin-\frac{17\pi}{6}\right].$$

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$$= \frac{1}{2}\left[\cos\frac{16\pi}{3} + i\sin\frac{16\pi}{3}\right]\left[\cos\frac{17\pi}{6} + i\sin\frac{17\pi}{6}\right]$$

$$= \frac{1}{2}\left[\cos\pi\left(\frac{16}{3}+\frac{17}{6}\right) + i\sin\pi\left(\frac{16}{3}+\frac{17}{6}\right)\right].$$

$$= \frac{1}{2}\left[\cos\pi\left(\frac{49}{6}\right) + i\sin\pi\left(\frac{49}{6}\right)\right].$$

$$= \frac{1}{2}\left[\cos\frac{49\pi}{6} + i\sin\frac{49\pi}{6}\right]$$

$$= \frac{1}{2}\left[\cos\left(8\pi+\frac{\pi}{6}\right) + i\sin\left(8\pi+\frac{\pi}{6}\right).\right]$$

$$= \frac{1}{2}\left[\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}.\right]$$

= By comparing $z = r(\cos\theta + i\sin\theta)$.

$r = \frac{1}{2}$ $\theta = \frac{\pi}{6}$

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Q.4) Show that $(4n)^{th}$ power of $\frac{1+7i}{(2-i)^2}$ is equal to $(-4)^n$ where n is the positive integer

consider $\frac{1+7i}{(2-i)^2}$

$(2-i)^2 = 2^2 - 2\times 2\, i + i^2$
$= 4 - 4i - 1$
$= 3 - 4i$

$= \frac{1+7i}{3-4i}$

$= \frac{1+7i}{3-4i} \times \frac{3+4i}{3+4i}$

$\frac{3+4i+21i+28i^2}{(3)^2 - (4i)^2}$

$\frac{3+4i+21i+28(-1)}{9+16}$

$= \frac{-25+25i}{25} = i-1$

$= -1+i$
$= -(1-i)$

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$= \sqrt{2}\left(\frac{1}{\sqrt{2}} - i\frac{1}{\sqrt{2}}\right)$ ~~x~~ $\times$ e $\div$ by $\sqrt{2}$

$= (-\sqrt{2})\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$

$\left[\frac{1+7i}{(2-i)^2}\right]^{4n} = (-\sqrt{2})^{4n}\left[\cos\frac{\pi}{4} - i\sin\frac{\pi}{4}\right]^{4n}$

By demoiver's Theorem

$(+\sqrt{2})^{4n}\left(\cos 4n\frac{\pi}{4} - i\sin\frac{\pi}{4}4n\right)$

$= (+\sqrt{2})^{4n}(\cos n\pi - i\sin n\pi)$

$= 4^n(\cos n\pi - i\sin \pi)$

$= 4^n(-1)^n$

$= -4^n = R.H.S.$

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Q. if α, β are the roots of the equation $x^2 - 2\sqrt{3}x + 4 = 0$ Prove that $\alpha^3 + \beta^3 = 0$

$x^2 - 2\sqrt{3}x + 4 = 0$

compar with $ax^2 + bx + c = 0$

$a = 1$, $b = -2\sqrt{3}$, $c = 4$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-2\sqrt{3}) \pm \sqrt{4\times 3 - 4\times 1\times 4}}{2\times 1}$$

$$= \frac{2\sqrt{3} \pm \sqrt{12-16}}{2}$$

$$= \frac{\cancel{2}\sqrt{3} \pm \cancel{2}i}{\cancel{2}}$$

$$= \sqrt{3} \pm i$$

$\times$ & $\div$ by 2

$$= 2\left(\frac{\sqrt{3}}{2} \pm \frac{1}{2}i\right).$$

$$= 2\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$$

$$\alpha = 2\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$$

$$\beta = 2\left(\cos\frac{\pi}{6} - i\sin\frac{\pi}{6}\right)$$

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$$\alpha^3 + \beta^3 = 2^3\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)^3 +$$
$$2^3\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)^3$$

applyin de moiver's Theorem.

$$= 2^3\left(\cos\frac{3\pi}{6} + i\sin\frac{3\pi}{6}\right) + 2^3\left(\cos\frac{3\pi}{6} - i\sin\frac{3\pi}{6}\right).$$

$$= 2^3\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right) + 2^3\left(\cos\frac{\pi}{2} - i\sin\frac{\pi}{2}\right)$$

$$= 2^3(0+i) + 2^3(0-i).$$

$$= 8i - 8i$$

$$= 0$$

$$= R.H.S$$

Expansion of $\sin n\theta$, $\cos n\theta$ in the power of $\sin\theta$ & $\cos\theta$

1) By De-Moivre's Theorem.

$$(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$$

2) Binomial Theorem

$$(\cos\theta + i\sin\theta)^n = {}^nC_0 \cos^n\theta (i\sin\theta)^0 + {}^nC_1 \cos^{n-1}(i\sin\theta) + {}^nC_2 \cos^{n-2}\theta (i\sin\theta)^2 + {}^nC_3 \cos^{n-3}\theta (i\sin\theta)^3 + \cdots + {}^nC_n (i\sin\theta)^n.$$

$$= {}^nC_0 \cos^n\theta + {}^nC_1 \cos^{n-1}\theta \, i\sin\theta - {}^nC_2 \cos^{n-2}\theta \sin^2\theta - {}^nC_3 \cos^{n-3}\theta \, i\sin^3\theta + \cdots + {}^nC_n i^n \sin^n\theta$$

$$= ({}^nC_0 \cos^n\theta - {}^nC_2 \cos^{n-2}\theta \sin^2\theta + \cdots) + i({}^nC_1 \cos^{n-1}\theta \sin\theta - {}^nC_3 \cos^{n-3} \sin\theta + \cdots)$$

real part
1) alternative + & -
2) Even power

imaginary part
1) alternative + & -
2) Odd power

Equating both

$$\cos n\theta + i\sin n\theta = ({}^nC_0 \cos^n\theta - {}^nC_2 \cos^{n-2}\theta \sin^2\theta + \cdots) + i({}^nC_1 \cos^{n-1}\theta \sin\theta - {}^nC_3 \cos^{n-3} \sin^3\theta \cdots)$$

comparing both.

$$\boxed{\begin{aligned} \cos\theta &= {}^nC_0 \cos^n\theta - {}^nC_2 \cos^{n-2}\theta \sin^2\theta + \cdots \\ \sin\theta &= {}^nC_1 \cos^{n-1}\theta \sin\theta - {}^nC_3 \cos^{n-3}\theta \sin^3\theta + \cdots \end{aligned}}$$

* Pascal's Triangle

1	0
1 1	1
1 2 1	2
1 3 3 1	3
1 4 6 4 1	4
1 5 10 10 5 1	5
1 6 15 20 15 6 1	6
1 7 21 35 35 21 7 1	7
1 8 28 56 70 56 28 8 1	8

n = odd imaginary part with cos

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Q. S.T $\sin 5\theta = 5\sin\theta - 20\sin^3\theta + 16\sin^5\theta$.
& $\cos 5\theta = 5\cos\theta - 20\cos^3\theta + 16\cos^5\theta$.

By Demoivre's Theorem.

$$(\cos\theta + i\sin\theta)^5 = \cos 5\theta + i\sin 5\theta \quad -(i)$$

By Bionomial theorem

$(\cos\theta + i\sin\theta)^5 = 1(\cos\theta)^5(i\sin\theta)^0 + 5(\cos\theta)^4 (i\sin\theta)^1 + 10(\cos\theta)^3(i\sin\theta)^2 + 10(\cos\theta)^2(i\sin\theta)^3 + 5(\cos\theta)(i\sin\theta)^4 + 1(\cos\theta)^0(i\sin\theta)^5$.

$= \cos^5\theta + 5\cos^4\theta \cdot i\sin\theta + 10\cos^3\theta(-1)\sin^2\theta + 10\cos^2\theta(-i)\sin^3\theta + 5\cos\theta(1)\sin^4\theta + 1 i\sin^5\theta$.

$= \cos^5\theta - 10\cos^3\theta \cdot \sin^2\theta + 5\cos\theta \cdot \sin^4\theta + 5\cos^4\theta \cdot i\sin\theta - 10\cos^2\theta i\sin^3\theta + i\sin^5\theta$

$= (\cos^5\theta - 10\cos^3\theta\sin^2\theta + 5\cos\theta \cdot \sin^4\theta) + i(5\cos^4\theta\sin\theta - 10\cos^2\theta\sin^3\theta + \sin^5\theta)$ $-(ii)$

Equating b.s.

$\cos 5\theta + i\sin 5\theta = \cos^5\theta - 10\cos^3\theta\sin^2\theta + 5\cos\theta\sin^4\theta) + i(5\cos^4\theta\sin\theta - 10\cos^2\theta\sin^3\theta + \sin^5\theta)$

$\cos 5\theta = \cos^5\theta - 10\cos^3\theta\sin^2\theta + 5\cos\theta \cdot \sin^4\theta)$

$\sin 5\theta = 5\cos^4\theta\sin\theta - 10\cos^2\theta\sin^3\theta + \sin^5\theta)$

$\hookrightarrow (\sin^2\theta + \cos^2\theta = 1)$

$= 5(1-\sin^2\theta)^2\sin\theta - 10(1-\sin^2\theta)\sin^3\theta + \sin^5\theta$

$= 5(1^2 - 2\sin^2\theta + (\sin^2\theta)^2)\sin\theta + (-10 + 10\sin^2\theta)\sin^3\theta$
$+ \sin^5\theta.$

$= \cancel{5 - 10\sin^2\theta + 5\sin^4\theta.\sin\theta - 10 + 10\sin^2\theta.\sin^3\theta}$
$\cancel{+\sin^5\theta}.$

$= \cancel{5 - 10\sin^2\theta + 5\sin^5\theta - 10 + 10\sin^5\theta + \sin^5\theta}.$

$= 5\sin\theta(1 - 2\sin^2\theta + \sin^4\theta) - 10\sin^3\theta + 10\sin^5\theta$
$+ \sin^5\theta.$

$= 5\sin\theta - 10\sin^3\theta + 5\sin^5\theta - 10\sin^3\theta + 10\sin^5\theta$
$+ \sin^5\theta$

$= 5\sin\theta - 20\sin^3\theta + 16\sin^5\theta.$

$\boxed{\sin 5\theta = 5\sin\theta - 20\sin^3\theta + 16\sin^5\theta.}$

$\cos 5\theta = \cos^5\theta - 10\cos^3\theta\sin^2\theta + 5\cos\theta.\sin^4\theta$
$= \cos^5\theta - 10\cos^3\theta(1 - \cos^2\theta) + 5\cos\theta.(1 - \cos^2\theta)^2$

$= \cos^5\theta - 10\cos^3\theta(1 - \cos^2\theta) + 5\cos\theta(1^2 - 2\cos^2\theta$
$+ (\cos^2\theta)^2.$

$= \cos^5\theta - 10\cos^3\theta + 10\cos^5\theta + 5\cos\theta - 10\cos^3\theta$
$5\cos^5\theta.$

$\cancel{= \cos^5\theta - 20\cos^3\theta}$

$\boxed{\cos 5\theta = 5\cos\theta - 20\cos^3\theta + 16\cos^5\theta.}$

Q Show that $\frac{\sin 5\theta}{\sin\theta} = 16\cos^4\theta - 12\cos^2\theta + 1$

$\frac{\sin 5\theta}{\sin\theta} = \frac{5\sin\theta - 20\sin^3\theta + 16\sin^5\theta}{\sin\theta}$

$= 5 - 20\sin^2\theta + 16\sin^4\theta$

$= 5 - 20(1-\cos^2\theta) + 16(1-\cos^2\theta)^2 =$

$= 5 - 20 + 20\cos^2\theta + 16(1^2 - 2\cos^2\theta + \cos^4\theta)$

$= 5 - 20 + 20\cos^2\theta + 16 - 32\cos^2\theta + 16\cos^4\theta$

$\frac{\sin 5\theta}{\sin\theta} = 16\cos^4\theta - 12\cos^2\theta + 1$ = RHS

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Q. Use De moivre's theorem to S.T.

$$\tan 5\theta = \frac{5\tan\theta - 10\tan^3\theta + \tan^5\theta}{1-10\tan^2\theta + 5\tan^4\theta}.$$

$$\tan 5\theta = \frac{\sin 5\theta}{\cos 5\theta}$$

$$= \frac{5\cos^4\theta\sin\theta - 10\cos^2\theta\sin^3\theta + \sin^5\theta}{\cos^5\theta - 10\cos^3\theta\sin^2\theta + 5\cos\theta\sin^4\theta}$$

Devide by $\cos^5\theta$.

$$\frac{\frac{5\sin\theta}{\cos\theta} - \frac{10\sin^3\theta}{\cos^3\theta} + \frac{\sin^5\theta}{\cos^5\theta}}{\frac{\cos^5\theta}{\cos^5\theta} - \frac{10\sin^2\theta}{\cos^2\theta} + \frac{5\sin^4\theta}{\cos^4\theta}} =$$

$$= \frac{5\tan\theta - 10\tan^3\theta + \tan^5\theta}{1 - 10\tan^2\theta + 5\tan^4\theta}.$$

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Q. Show that

$$\tan 7\theta = \frac{7\tan\theta - 35\tan^3\theta + 21\tan^5\theta - \tan^7\theta}{1 - 21\tan^2\theta + 35\tan^4\theta - 7\tan^6\theta}$$

$$\tan 7\theta = \frac{\sin 7\theta}{\cos 7\theta}$$

By Demoivre's Theorem

$\cancel{\cos 7\theta +}$ $(\cos\theta + i\sin\theta)^7 = \cos 7\theta + i\sin 7\theta$

By B.T

$(\cos\theta + i\sin\theta)^7 = 1(\cos\theta)^7(i\sin\theta)^0 + 7(\cos\theta)^6(i\sin\theta)^1$
$21(\cos\theta)^5(i\sin\theta)^2 + 35(\cos\theta)^4(i\sin\theta)^3 + 35(\cos\theta)^3$
$(i\sin\theta)^4 + 21(\cos\theta)^2(i\sin)^5 + 7(\cos\theta)^1(i\sin)^6$
$+ 1(\cos\theta)^0(i\sin\theta)^7$.

$= \cos^7\theta + 7\cos^6\theta\, i\sin\theta + 21\cos^5\theta(-1)\sin^2\theta$
$+ 35\cos^4\theta(-i)\sin^3\theta + 35\cos^3\theta\sin^4\theta + 21\cos^2\theta$
$i\sin^5\theta + 7\cos\theta(-1)\sin^6\theta + (-i)\sin^7\theta$.

$= \cos^7\theta - 21\cos^5\theta\sin^2\theta + 35\cos^3\theta\sin^4\theta$
$- 7\cos\theta\sin^6\theta + 7\cos^6\theta\, i\sin\theta - 35\cos^4\theta\, i\sin^3\theta$
$+ 21\cos^2\theta\, i\sin^5\theta - i\sin^7\theta$.

$= \cos^7\theta - 21\cos^5\theta\sin^2\theta + 35\cos^3\theta\sin^4\theta - 7\cos\theta$
$\sin^6\theta + i(7\cos^6\theta\sin\theta - 35\cos^4\theta\sin^3\theta +$
$21\cos^2\theta\sin^5\theta - \sin^7\theta)$

$\cos 7\theta + i\sin 7\theta = \cos^7\theta - 21\cos^5\theta\sin^2\theta + 35\cos^3\theta$
$\sin^4\theta - 7\cos\theta\sin^6\theta + i(7\cos^6\theta\sin\theta -$
$35\cos^4\theta\sin^3\theta + 21\cos^2\theta\sin^5\theta -$
$\sin^7\theta)$.

comparing b.s.

$$\cos 7\theta = \cos^7\theta - 21\cos^5\theta\,\sin^2\theta + 35\cos^3\theta\,\sin^4\theta - 7\cos\,\sin^6\theta$$

$$\sin 7\theta = 7\cos^6\theta\,\sin\theta - 35\cos^4\theta\,\sin^3\theta + 21\cos^2\theta\,\sin^5\theta - \sin^7\theta.$$

$$\frac{\sin 7\theta}{\cos 7\theta} = \frac{\cos^7\theta - 21\cos^5\theta\sin^2\theta + 35\cos^3\theta\sin^4\theta - 7\cos\theta\sin^6\theta}{7\cos^6\theta\sin\theta - 35\cos^4\theta\sin^3\theta + 21\cos^2\theta\sin^5\theta - \sin^7\theta}$$

$\div$ b.s by $\cos^7\theta$.

$$= \frac{\dfrac{\cos^7\theta}{\cos^7\theta} - \dfrac{21\cos^5\theta\sin^2\theta}{\cos^7\theta} + \dfrac{35\cos^3\theta\sin^4\theta}{\cos^7\theta} - \dfrac{7\cos\theta\sin^6\theta}{\cos^7\theta}}{\dfrac{7\cos^6\theta\sin\theta}{\cos^7\theta} - \dfrac{35\cos^4\theta\sin^3\theta}{\cos^7\theta} + \dfrac{21\cos^2\theta\sin^5\theta}{\cos^7\theta} - \dfrac{\sin^7\theta}{\cos^7\theta}}$$

$$= \frac{1 - 21\tan^2\theta + 35\tan^4\theta - 7\tan^6\theta}{7\tan\theta - 35\tan^3\theta + 21\tan^5\theta - \tan^7\theta}.$$

~~$\div$ by $\sin^7\theta$.~~

Expansion of $\sin^n\theta$, $\cos^n\theta$ in terms of cosine & sine multiple of θ

(i) if $x = \cos\theta + i\sin\theta$ then

$\frac{1}{x} = \cos\theta - i\sin\theta$ --- corollary.

$$\boxed{x + \frac{1}{x} = 2\cos\theta}$$

$$\boxed{x - \frac{1}{x} = 2i\sin\theta}$$

(ii) If $x^n = \cos n\theta + i\sin n\theta$ then

$\frac{1}{x^n} = \cos n\theta - i\sin n\theta$ — corollary.

$$\boxed{x^n + \frac{1}{x^n} = 2\cos n\theta}$$

$$\boxed{x^n - \frac{1}{x^n} = 2i\sin n\theta}$$

Steps

1) $x + \frac{1}{x} = 2\cos\theta$

$x - \frac{1}{x} = 2i\sin\theta.$

2) Binomial Theorem

3) $x^n + \frac{1}{x^n} = 2\cos n\theta$

$x^n - \frac{1}{x^n} = 2i\sin n\theta$

B.T लगाते समय दोनो term के बीच (−) हो तो alternate signs.

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Q. Expand $\sin^7\theta$ in the series of sine multiple of θ

$x - \frac{1}{x} = 2i\sin\theta$

$(2i\sin\theta)^7 = \left(x - \frac{1}{x}\right)^7$

$2^7 i^7 \sin^7\theta = 1\,x^7\left(\frac{1}{x}\right)^0 - 7x^6\left(\frac{1}{x}\right)^1 + 21x^5\left(\frac{1}{x}\right)^2$

$- 35x^4\left(\frac{1}{x}\right)^3 + 35x^3\left(\frac{1}{x}\right)^4 + 21x^2\left(\frac{1}{x}\right)^5 + 7x\left(\frac{1}{x}\right)^6$

$- x^0\left(\frac{1}{x}\right)^7$

$= x^7 - \frac{7\cancel{x^6}}{x} + \frac{21x^5}{x^2} - \frac{35x^4}{x^3} + \frac{35x^3}{x^4} - \frac{21x^2}{x^5}$

$+ \frac{7x}{x^6} - \frac{1}{x^7}$

$= x^7 - 7x^5 + 21x^3 - 35x + \frac{35}{x} - \frac{21}{x^3} + \frac{7}{x^5} - \frac{1}{x^7}$

$= x^7 - \frac{1}{x^7} - 7x^5 + \frac{7}{x^5} + 21x^3 - \frac{21}{x^3} - 35x + \frac{35}{x}$

$= \left(x^7 - \frac{1}{x^7}\right) - 7\left(x^5 - \frac{7}{x^5}\right) + 21\left(x^3 - \frac{21}{x^3}\right) - 35\left(x + \frac{1}{x}\right)$

$2^7 i^7 \sin^7\theta = 2i\sin 7\theta - 7(2i\sin 5\theta) + 21(2i\sin 3\theta) - 35(2i\sin\theta)$

$2^{\cancel{7}\,6}\, i^{\cancel{7}\,6} \sin^7\theta = \cancel{2i}(\sin 7\theta - 7\sin 5\theta + 21\sin 3\theta - 35\sin\theta)$

$\sin^7\theta = \frac{-1}{64}\left[\sin 7\theta - 7\sin 5\theta + 21\sin 3\theta - 35\sin\theta\right]$

Q. Expand $\cos^7\theta$ in a series of cosine multiple of θ

$$x + \frac{1}{x} = 2\cos\theta$$

$$(2\cos\theta)^7 = \left(x + \frac{1}{x}\right)^7$$

$$2^7\cos^7\theta = x^7\left(\frac{1}{x}\right)^0 + 7x^6\left(\frac{1}{x}\right)^1 + 21x^5\left(\frac{1}{x}\right)^2 + 35x^4\left(\frac{1}{x}\right)^3$$
$$+ 35x^3\left(\frac{1}{x}\right)^4 + 21x^2\left(\frac{1}{x}\right)^5 + 7x^1\left(\frac{1}{x}\right)^6 + x^0\left(\frac{1}{x}\right)^7$$

$$= x^7 + \frac{7x^6}{x} + \frac{21x^5}{x^2} + \frac{35x^4}{x^3} + \frac{35x^3}{x^4} + \frac{21x^2}{x^5} + \frac{7x}{x^6} + \frac{1}{x^7}$$

$$= x^7 + 7x^5 + 21x^3 + 35x + \frac{35}{x} + \frac{21}{x^3} + \frac{7}{x^5} + \frac{1}{x^7}$$

$$= \left(x^7 + \frac{1}{x^7}\right) + \left(7x^5 + \frac{7}{x^5}\right) + \left(21x^3 + \frac{21}{x^3}\right) + \left(35x + \frac{35}{x}\right)$$

$$= 2\cos 7\theta + 7\left(x^5 + \frac{1}{x^5}\right) + 21\left(x^3 + \frac{1}{x^3}\right) + 35\left(x + \frac{1}{x}\right)$$

$$2^7\cos^7\theta = 2\cos 7\theta + 7(2\cos 5\theta) + 21(2\cos 3\theta) + 35(2\cos\theta)$$

$$2^6\cos^7\theta = \cos 7\theta + 7\cos 5\theta + 21\cos 3\theta + 35\cos\theta$$

$$\boxed{\cos^7\theta = \frac{1}{64}\left[\cos 7\theta + 7\cos 5\theta + 21\cos 3\theta + 35\cos\theta\right]}$$

Q. S.T $\sin^5\theta = \frac{1}{16}(\sin 5\theta - 5\sin 3\theta + 10\sin\theta)$

$(2i\sin\theta)^5 = \left(x - \frac{1}{x}\right)^5$

$2^5 i \sin^5\theta = x^5 - 5x^4 \cdot \frac{1}{x} + 10x^3 \cdot \frac{1}{x^2} - 10x^2 \frac{1}{x^3} + 5x \cdot \frac{1}{x^4} - \frac{1}{x^5}$

$= x^5 - 5x^3 + 10x - 10\frac{1}{x} + 5 \cdot \frac{1}{x^3} - \frac{1}{x^5}$

$= x^5 - \frac{1}{x^5} - 5x^3 + \frac{5}{x^3} + 10x - \frac{10}{x}$

$= 2i\sin 5\theta - 5(2i\sin 3\theta) + 10(2i\sin\theta)$

$= 2i(\sin 5\theta - 5\sin 3\theta + 10\sin\theta)$

$\sin^5\theta = \frac{2i}{2^5 i}(\sin 5\theta - 5\sin 3\theta + 10\sin\theta)$

$\sin^5\theta = \frac{1}{16}(\sin 5\theta - 5\sin 3\theta + 10\sin\theta)$

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Q. if $\sin^4\theta \cos^3\theta = a_1\cos\theta + a_3\cos 3\theta + a_5\cos 5\theta + a_7\cos 7\theta$ prove that

$$a_1 + 9a_3 + 25a_5 + 49a_7 = 0$$

$$x - \frac{1}{x} = 2i\sin\theta$$

$$x + \frac{1}{x} = 2\cos\theta$$

$$(2i\sin\theta)^4(2\cos\theta)^3 = \left(x-\frac{1}{x}\right)^4\left(x+\frac{1}{x}\right)^3$$

$$= \left(x-\frac{1}{x}\right)\left[\left(x-\frac{1}{x}\right)\left(x+\frac{1}{x}\right)\right]^3$$

$$= \left(x-\frac{1}{x}\right)\left[x^2-\frac{1}{x^2}\right]^3$$

$$(2^4\times 1\sin^4\theta)\times 2^3\cos^3\theta = \left(x-\frac{1}{x}\right)\left(x^2-\frac{1}{x^2}\right)^3$$

$$2^7\sin^4\theta\cos^3\theta = \left(x-\frac{1}{x}\right)\left(x^2-\frac{1}{x^2}\right)^3$$

$$= \left(x-\frac{1}{x}\right)\left[(x^2)^3 - \left(\frac{1}{x^2}\right)^3 - 3(x^2)^2\frac{1}{x^2} + 3x^2\left(\frac{1}{x^2}\right)^2\right]$$

$$= \left(x-\frac{1}{x}\right)\left[x^8 - \frac{1}{x^8} - \frac{3x^4}{x^2} + \frac{3x^2}{x^4}\right]$$

$$= \left(x-\frac{1}{x}\right)\left[x^8 - \frac{1}{x^8} - 3x^2 + \frac{3}{x^2}\right]$$

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$$= x^7 - \frac{1}{x^5} - 3x^3 + \frac{3}{x} - x^5 + \frac{1}{x^7} + 3x - \frac{3}{x^3}$$

$$= x^7 + \frac{1}{x^7} - x^5 - \frac{1}{x^5} - 3x^3 - \frac{3}{x^3} + 3x + \frac{3}{x}.$$

$$= x^7 + \frac{1}{x^7} - 1\left(x^5 + \frac{1}{x^5}\right) - 3\left(x^3 + \frac{1}{x^3}\right) + 3\left(x + \frac{1}{x}\right)$$

$$2^7 \sin^4\theta . \cos^3\theta = 2\cos 7\theta - 2\cos 5\theta - 3\cos 3\theta + 3\cos\theta.$$

$$\sin^4\theta . \cos^3\theta = \frac{1}{2^6}(\cos 7\theta - \cos 5\theta - 3\cos 3\theta + 3\cos\theta)$$

$$\sin^4\theta . \cos^3\theta = \frac{1}{2^6}(\cos\theta + \cos 3\theta + \cos 5\theta + \cos 7\theta)$$

$$= \frac{3\cos\theta}{2^6} - \frac{3\cos 3\theta}{2^6} + \frac{1\cos 5\theta}{2^6} + \frac{1\cos 7\theta}{2^6}$$

$$a_1 = \frac{3}{2^6}, \quad a_3 = \frac{-3}{2^6}, \quad a_5 = \frac{1}{2^6}, \quad a_7 = \frac{1}{2^6}$$

$$\frac{3}{2^6} + 9 \times \frac{(-3)}{2^6} + 25\frac{1}{2^6} + 49\frac{1}{2^6}$$

$$\frac{3}{64} + \left(\frac{-27}{64}\right) + \frac{25}{64} + \frac{49}{64} = \frac{-52+52}{64} = 0$$

Hence proved

Q. Using De-moivre's Theorem

$$\cos^6\theta + \sin^6\theta = \frac{1}{8}(3\cos 4\theta + 5)$$

$$x + \frac{1}{x} = 2\cos\theta$$

$$x - \frac{1}{x} = 2i\sin\theta$$

$$(2\cos\theta)^6 = \left(x + \frac{1}{x}\right)^6$$

$$= 1x^6\left(\frac{1}{x}\right)^0 + 6x^5\left(\frac{1}{x}\right)^1 + 15x^4\left(\frac{1}{x}\right)^2$$

$$+ 20x^3\left(\frac{1}{x}\right)^3 + 15x^2\left(\frac{1}{x}\right)^4 + 6x^1\left(\frac{1}{x}\right)^5$$

$$+ 1x^0\left(\frac{1}{x}\right)^6$$

$$= x^6 + 6x^4 + 15x^2 + 20 + \frac{15}{x^2} + \frac{6}{x^4} + \frac{1}{x^6}$$

$$= x^6 + \frac{1}{x^6} + 6x^4 + \frac{1}{x^4} + 15x^2 + \frac{15}{x^2} + 20$$

$$2^6\cos^6\theta = x^6 + \frac{1}{x^6} + 6x^4 + \frac{1}{x^4} + 15x^2 + \frac{15}{x^2} + 20$$

$$(2i\sin\theta)^6 = \left(x + \frac{1}{x}\right)^6$$

$$= 1x^6\left(\frac{1}{x}\right)^0 + 6x^5\left(\frac{1}{x}\right)^1 + 15x^4\left(\frac{1}{x}\right)^2 + 20x^3\left(\frac{1}{x}\right)^3$$

$$+ 15(x)^2\left(\frac{1}{x}\right)^4 + 6x^1\left(\frac{1}{x}\right)^5 + 1x^0\frac{1}{x^6}$$

$$= x^6 - 6x^4 + 15x^2 - 20 + \frac{15}{x^2} - \frac{6}{x^4} + \frac{1}{x^6}$$

$$= x^6 + \frac{1}{x^6} - 6\left(x^4 + \frac{1}{x^4}\right) + 15\left(x^2 + \frac{1}{x^2}\right) - 20$$

$$2^6(-1)\sin^6\theta = x^6 + \frac{1}{x^6} - 6\left(x^4 + \frac{1}{x^4}\right) + 15\left(x^2 + \frac{1}{x^2}\right) - 20.$$

$$2^6\sin^6\theta = -1\left(x^6 + \frac{1}{x^6}\right) + 6\left(x^4 + \frac{1}{x^4}\right) - 15\left(x^2 + \frac{1}{x^2}\right) + 20.$$

$$2^6(\sin^6 + \cos^6\theta) = -\left(\cancel{x^6 + \frac{1}{x^6}}\right) + 6x^4 + \frac{6}{x^4} \cancel{- 15x^2 - \frac{15}{x^2}} + 20\Big) + \Big[\left(\cancel{x^6 + \frac{1}{x^6}}\right) + 6x^4 + \frac{6}{x^4} + \cancel{15x^2 + \frac{15}{x^2}} + 20\Big].$$

$$\cancel{2^6(\sin^6\theta + \cos^6\theta) = 2x^6 + \frac{2}{x^6} + 12x^4 + \frac{12}{x^4} + 40}.$$

$$2^6\sin^6\theta\cos^6\theta = 12x^4 + \frac{12}{x^4} + 40.$$

$$= 12\left(x^4 + \frac{1}{x^4}\right) + 40.$$

$$= 12(2\cos 4\theta) + 40.$$

$$\sin^6\theta\cos^6\theta = \frac{1}{\cancel{64}\ \cancel{16}\ 8}\left(\cancel{24}\ 3\cos 4\theta\right) + \cancel{40}\ 5.$$

$$\boxed{\sin^6\theta\cos^6\theta = \frac{1}{8}\,(3\cos 4\theta + 5)}$$

Roots of complex number

- De-Moivre's Theorem

$$\cos\theta = \cos(2k\pi+\theta)$$
$$\sin\theta = \sin(2k\pi+\theta)$$
where k is a integer

$$(\cos\theta + i\sin\theta)^{1/n} = \left[\cos(2\pi k+\theta) + i\sin(2k\pi+\theta)\right]^{1/n}$$

$$= \left[\cos\frac{(2\pi k+\theta)}{n} + i\sin\frac{(2k\pi+\theta)}{n}\right]$$

Putting the value of $k = 0, 1, 2, 3, 4 \ldots n$ we can find n roots.

$n = 6 \rightarrow 0, 1, 2, 3, 4, 5.$

Note :- $\cos 0 + i\sin 0 = 1$

$\cos\pi + i\sin\pi = -1$

$\cos\frac{\pi}{2} + i\sin\frac{\pi}{2} = i$

$\cos\frac{\pi}{2} - i\sin\frac{\pi}{2} = -i$

$\sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right) = 1+i$

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Q. Find the cube roots of unity if ω is the complex cube roots of unity prove that $(1-\omega)^6 = -27$

Sol^n

$$x^3 = 1$$

$$x^3 = (\cos 0 + i \sin 0)$$

$$x = (\cos 0 + i \sin 0)^{1/3}$$

$$\cos\theta = \cos(2k\pi + 0)$$

$$\cos 0 = \cos(2k\pi + 0)$$
$$= \cos(2k\pi)$$

$$x = (\cos(2k\pi) + i \sin 2k\pi)^{1/3}$$

$$x = \left[\cos(2k\pi) + i\sin(2k\pi)\right]^{1/3}$$

By DMT,

$$x = \left[\cos\left(\frac{2k\pi}{3}\right) + i\sin\left(\frac{2k\pi}{3}\right)\right]$$

$k = 0, 1, 2.$

for $k = 0$,

$$x = \left[\cos\left(\frac{2\times 0\times\pi}{3}\right) + i\sin\left(\frac{2\times 0\times \pi}{3}\right)\right]$$

$$x = \cos 0 + i \sin 0.$$

$$\boxed{x = 1}$$

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for $K=1$
Let $x_1 = \omega$

$x_1 = [\cos \frac{2\times 1 \times \pi}{3} + i\sin \frac{2\times 1\times \pi}{3}]$

$\boxed{\omega = x_1 = [\cos \frac{2\pi}{3} + i\sin \frac{2\pi}{3}]}$

$= -\frac{1}{2} + i\frac{\sqrt{3}}{2}$

$x_1 = -\frac{1}{2} + \frac{\sqrt{3}}{2} i$

for $K = 2$

$x_2 = \cos\left(\frac{2\times 2\times \pi}{3}\right) + i\sin\left(\frac{2\times 2\times \pi}{3}\right)$

$= \cos\left(\frac{4\pi}{3}\right) + i\sin\left(\frac{4\pi}{3}\right)$

$\cancel{= -\frac{1}{2} - i\frac{\sqrt{3}}{2}}$

$\omega^2 = \left[\cos\left(\frac{2\pi}{3}\right) + i\sin\left(\frac{2\pi}{3}\right)\right]^2$

Let's

$= 1 + \omega + \omega^2$

$= 1 + \cos\frac{2\pi}{3} + i\sin\left(\frac{2\pi}{3}\right) + \left(\cos\left(\frac{2\pi}{3}\right) + i\sin\frac{2\pi}{3}\right)^2$

$= \left[1 + \cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3} + \cos\frac{4\pi}{3} + i\sin\frac{4\pi}{3}\right]$

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$$= 1 + \cos\left(\pi - \frac{\pi}{3}\right) + i\sin\left(\pi - \frac{\pi}{3}\right) + \cos\left(\pi + \frac{\pi}{3}\right)$$
$$+ i\sin\left(\pi + \frac{\pi}{3}\right)$$

$$\cancel{= 1 + \cos\left(\frac{\pi}{3}\right) + i\sin\left(\frac{\pi}{3}\right) - \cos\frac{\pi}{3} + i\sin}$$

$$= 1 - \cos\frac{\pi}{3} + \cancel{i\sin\frac{\pi}{3}} - \cos\frac{\pi}{3} - \cancel{i\sin\frac{\pi}{3}}$$

$$= 1 - \frac{1}{2} - \frac{1}{2}.$$

$$\boxed{1 + \omega + \omega^2 = 0}$$

acc to question.

$$\boxed{(1 + \omega^2) = -\omega}$$

$$(1-\omega)^6 = [(1-\omega)^2]^3$$

$$= (1 - 2\omega + \omega^2)^3$$
$$= (1 + \omega^2 - 2\omega)^3$$
$$= (-\omega - 2\omega)^3$$
$$= (-3\omega)^3$$
$$= -27\omega^3$$
$$= -27 \times 1$$

$$\boxed{(1-\omega)^6 = -27}$$

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Q if ω is a complex ~~unity~~ cube root of unity prove that $1+\omega+\omega^2=0$ &
$$\frac{1}{1+2\omega}+\frac{1}{2+\omega}+\frac{1}{1+\omega}=0.$$

$$1+\omega+\omega^2=0.$$

$$L.H.S=\frac{1}{1+2\omega}+\frac{1}{2+\omega}-\frac{1}{1+\omega}$$

$$=\frac{(2+\omega)(1+\omega)+(1+2\omega)\cdot(1+\omega)-(1+2\omega)(2+\omega)}{(1+2\omega)(2+\omega)(1+\omega)}$$

$$\cancel{=\frac{(2+2\omega+\omega+\omega^2)+(1+\omega+2\omega+2\omega^2)+(2+\omega+4\omega+2\omega^2)}{(2+\omega+4\omega+2\omega^2)(1+\omega)}}$$

$$\cancel{=\frac{5+11\omega+5\omega^2}{2+2\omega+\omega+\omega^2+4\omega+4\omega^2+2\omega^2+2\omega^3}}$$

$$\cancel{=\frac{5+11\omega+5\omega^2}{2+7\omega+7\omega^2+2\omega^3}}$$

$$=\frac{(2+2\omega+\omega+\omega^2)+(1+\omega+2\omega+2\omega^2)-(2+\omega+4\omega+2\omega^2)}{(1+2\omega)(2+\omega)(1+\omega)}$$

$$=\frac{(2+3\omega+\omega^2)+(1+3\omega+2\omega^2)-(2+5\omega+2\omega^2)}{(1+2\omega)(2+\omega)(1+\omega)}$$

$$=\frac{\cancel{2}+3\omega+\omega^2+1+3\omega+\cancel{2\omega^2}-\cancel{2}+5\omega+\cancel{2\omega^2}}{(1+2\omega)(2+\omega)(1+\omega)}$$

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$$= \frac{1+\omega+\omega^2}{(1+2\omega)(2+\omega)(1+\omega)}$$

$$= \frac{0}{(1+2\omega)(2+\omega)(1+\omega)} = \underline{\underline{0}} = \underline{\underline{R.H.S}}$$

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Q $x^6+1=0$

$x^6+1=0$

$x^6=-1$

$x^6 = \cos\pi + i\sin\pi$

$x = (\cos\pi + i\sin\pi)^{1/6}$

$\cos\theta = \cos(2k\pi+\theta)$

$\cos\pi = \cos(2k\pi+\pi)$

$= \cos(2k+1)\pi$

$\sin(\pi) = \sin(2k\pi+\pi)$

$= \sin(2k+1)\pi$

$x = (\cos(2k+1)\pi + i\sin(2k+1)\pi)^{1/6}$

By De-MT Theorem.

$x = \cos\frac{(2k+1)\pi}{6} + i\sin\frac{(2k+1)\pi}{6}$

$k = 0, 1, 2, 3, 4, 5,$

for $k=0$.

$x_0 = \cos\frac{\pi}{6} + i\sin\frac{\pi}{6}$

$= \frac{\sqrt{3}}{2} + i\frac{1}{2}$

for $k=1$

$x_1 = \cos\frac{\pi}{2} + i\sin\frac{\pi}{2} = i$

for $k=2$

$$x_2 = \cos 5\frac{\pi}{6} + i\sin 5\frac{\pi}{6}.$$

$$= -\frac{\sqrt{3}}{2} + i\sin\frac{1}{2}.$$

for $k=3$

$$x_3 = \cos 7\frac{\pi}{6} + i\sin 7\frac{\pi}{6}$$

$$= -\frac{\sqrt{3}}{2} - i\frac{1}{2}.$$

for $k=4$

$$x_4 = \cos\frac{9\pi}{6} + i\sin\frac{9\pi}{6}.$$

$$= \cos\frac{3\pi}{2} + i\sin\frac{3\pi}{2}.$$

$$= -i$$

for $k=5.$

$$x_5 = \cos\frac{11\pi}{6} + i\sin\frac{11\pi}{6}.$$

$$= \cos\frac{11\pi}{6} + i\sin\frac{\pi 11}{6}$$

$$x_5 = +\frac{\sqrt{3}}{2} - \frac{1}{2}i$$

$$\pm i,\ \frac{\sqrt{3}\pm i}{2},\ \frac{-\sqrt{3}\pm i}{2}$$

4) $x^6 - i = 0$

$x^6 - i = 0$

$x^6 = i$

$x^6 = \cos\frac{\pi}{2} + i\sin\frac{\pi}{2}$

$x = \left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)^{1/6}$

~~By DMT~~

$\cancel{= \left(\cos\frac{\pi}{2}\times\frac{1}{6} + i\sin\frac{\pi}{2}\times\frac{1}{6}\right)}$

$\cancel{= \left(\cos\frac{\pi}{12} + i\sin\frac{\pi}{12}\right)}$

$\cancel{= \cos\pi + \pi}$

$\cos\theta = \cos(2k\pi + \theta)$

$\cos\frac{\pi}{2} = \cos\left(2k\pi + \frac{\pi}{2}\right)$

$= \cos\left(\frac{2\times 2\times k\pi}{2} + \frac{\pi}{2}\right)$

$= \cos\frac{4k\pi}{2} + \frac{\pi}{2}$

$= \cos(4k+1)\frac{\pi}{2}$

$\sin\theta = \sin(4k+1)\frac{\pi}{2}$

$x = \left(\cos(4k+1)\frac{\pi}{2} + i\sin(4k+1)\frac{\pi}{2}\right)^{1/6}$

By DMT

$x = \cos(4k+1)\frac{\pi}{2} + i\sin(4k+1)\frac{\pi}{12}$

$k = 0, 1, 2, 3, 4, 5.$

for $k = 0$

$$x_0 = \cos(4\times 0+1)\frac{\pi}{12} + i\sin(4\times 0+1)\frac{\pi}{12}$$

$$x_0 = \cos\frac{\pi}{12} + i\sin\frac{\pi}{12} \quad \text{--- } 1^{st}$$

$k = 1$

$$x_1 = \cos\frac{5\pi}{12} + i\sin\frac{5\pi}{12}. \quad \text{--- } 2^{nd}$$

$k = 2$

$$x_2 = \cos\frac{\cancel{9}\,\pi}{\cancel{12}\,4} + i\sin\frac{\cancel{9}\,\pi}{\cancel{12}\,4} \quad \text{--- } 3^{rd}$$

$$= \cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}$$

$$x_2 = -\frac{1}{\sqrt{2}} + i\frac{1}{\sqrt{2}} \quad \text{--- } \cancel{4^{th}}$$

$\cancel{x}$ $k = 3$

$$x_3 = \cos\frac{13\pi}{12} + i\sin\frac{13\pi}{12} \quad \text{--- } 4^{th}$$

$k = 4$

$$x_4 = \cos\frac{17\pi}{12} + i\sin\frac{17\pi}{12} \quad 5^{th}$$

$$x_5 = \cos\frac{21\pi}{12} + i\sin\frac{21\pi}{12} \quad 6^{th}$$

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Thus the roots are

$\cos\frac{\pi}{12} + i\sin\frac{\pi}{12}$; $\cos\frac{5\pi}{12} + i\sin\frac{5\pi}{12}$

; $-\frac{1}{\sqrt{2}} + i\frac{1}{\sqrt{2}}$; $\cos\frac{13\pi}{12} + i\sin\frac{13\pi}{12}$.

; $\cos\frac{17\pi}{12} + i\sin\frac{17\pi}{12}$; $\cos\frac{21\pi}{12} + i\sin\frac{21\pi}{12}$.

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Q find the roots common to $x^4+1=0$ & $x^6-i=0$

$x^4+1=0$
$x^4=-1$
$x^4=\cos\pi+i\sin\pi$

$\cos\theta = 2k\pi+\theta$
$\cos\pi = 2k\pi+\pi$
$\cos\pi = (2k+1)\pi$
$\sin\pi = (2k+1)\pi$

$x^4 = \cos(2k+1)\pi + i\sin(2k+1)\pi$

$x = [(\cos(2k+1)\pi + i\sin(2k+1)\pi]^{1/4}$

$x = \cos(2k+1)\frac{\pi}{4} + i\sin(2k+1)\frac{\pi}{4}$

$k = 0,1,2,3$
for $k=0$,
$x_0 = \cos\frac{\pi}{4} + i\sin\frac{\pi}{4}$
$x_0 = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i$ 1st.

$x_1 = \cos 3\frac{\pi}{4} + i\sin 3\frac{\pi}{4}$.
$= -\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i$
$x_2 = \cos 5\frac{\pi}{4} + \sin 5\frac{\pi}{4}$

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$x_2 = -\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}j$

$x_3 = \cos\frac{7\pi}{4} + j\sin\frac{7\pi}{4}$

$= \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}j$

$\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}j \; ; \; -\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}j \; ; \; -\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}j$

$; \; \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}j$

$x^6 - j = 0.$

$x^6 = j$

$x^6 = \cos\frac{\pi}{2} + j\sin\frac{\pi}{2}.$

$= \cos(2k\pi + \theta)$

$\cos\frac{\pi}{2} = \cos\left(2k\pi + \frac{\pi}{2}\right)$

$= \cos\left(2 \times 2k\frac{\pi}{2} + \frac{\pi}{2}\right)$

$= \cos(4k+1)\frac{\pi}{2}.$

$\sin\frac{\pi}{2} = \sin(4k+1)\frac{\pi}{2}.$

$x^6 = \cos(4k+1)\frac{\pi}{2} + j\sin(4k+1)\frac{\pi}{2}$

$x = \left(\cos(4k+1)\frac{\pi}{2} + j\sin(4k+1)\frac{\pi}{2}\right)^{1/6}.$

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By DMT.

$$x = \cos(4k+1)\frac{\pi}{2}\times\frac{1}{6} + i\sin(4k+1)\frac{\pi}{2}\cdot\frac{1}{6}.$$

$$x = \cos(4k+1)\frac{\pi}{12} + i\sin(4k+1)\frac{\pi}{12}.$$

for $k = 0, 1, 2, 3, 4, 5$

for $k=0$

$$x_0 = \cos\frac{\pi}{12} + i\sin\frac{\pi}{12}.$$

for $k=1$

$$x_1 = \cos\frac{5\pi}{12} + i\sin\frac{5\pi}{12}.$$

for $k=2$.

$$x_2 = \cos\frac{9\pi}{12} + i\sin\frac{9\pi}{12}$$

$$= \cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}.$$

$$= -\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i$$

for $k=3$: $x_3 = \cos\frac{13\pi}{12} + i\sin\frac{13\pi}{12}.$

for $k=4$

$$x_4 = \cos\frac{17\pi}{12} + i\sin\frac{17\pi}{12}$$

$$x_5 = \cos\frac{21\pi}{12} + i\sin\frac{21\pi}{12}$$

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$\cos\frac{\pi}{12} + i\sin\frac{\pi}{12}$; $\cos\frac{5\pi}{12} + i\sin\frac{5\pi}{12}$;

$-\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i$; $\cos\frac{13\pi}{12} + i\sin\frac{13\pi}{12}$

; $\cos\frac{17\pi}{12} + i\sin\frac{17\pi}{12}$; $\cos\frac{21\pi}{12} + i\sin\frac{21\pi}{12}$

Common root b/w $x^4+1=0$ & $x^6-i=0$ is

$\boxed{-\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i}$ & $\boxed{\cos\frac{7\pi}{4} + i\sin\frac{7\pi}{4}}$

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Q6) Show that the roots of the eqn $(x+1)^6 + (x-1)^6 = 0$ are given by $i \cot \frac{(2k+1)\pi}{12}$

$k = 0, 1, 2, 3, 4, 5.$

Variation ↳ Show that the roots of the eqn $(x+1)^7 + (x-1)^7 = 0$ are given by

$\pm i \cot \frac{r\pi}{7} \quad r = 1, 2, 3$

$(x+1)^7 - (x-1)^7 = 0$

$(x+1)^7 = (x-1)^7$

$\frac{(x+1)^7}{(x-1)^7} = 1$

$\cos 0 + i \sin 0 = 1.$

$\cos \theta = \cos(2k\pi + \theta)$

$= \cos(2k\pi + 0)$

$= \cos 2k\pi$

$\sin \theta = \sin 2k\pi.$

$\frac{(x+1)^7}{(x-1)^7} = \cos 2k\pi + i \sin 2k\pi$

$\left(\frac{x+1}{x-1}\right)^7 = \cos 2k\pi + i \sin 2k\pi.$

$\frac{x+7}{x-7} = (\cos 2k\pi + i \sin 2k\pi)^{\frac{1}{7}}.$

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$$\frac{x+1}{x-1} = \cos 2k\pi \times \frac{1}{7} + i \sin 2k\pi \times \frac{1}{7}$$

$$\cancel{\frac{x+1}{x-1} = \cos 14k\pi + i\sin 14k\pi}$$

Let $\theta = \frac{2k\pi}{7}$

$$\frac{x+1}{x-1} = \frac{\cos\theta + i\sin\theta}{1}$$

Comp & Dividen $\left\{ \frac{N+D}{D+N} \right\}$

$$\frac{x+1+x-1}{(x-1)-(x+1)} = \frac{\cos\theta + i\sin\theta + 1}{1-(\cos\theta + i\sin\theta)}$$

$$\frac{2x}{-2} = \frac{\cos\theta + i\sin\theta + 1}{1-\cos\theta - i\sin\theta}$$

$$-x = \frac{2\cos^2\theta/2 + i\sin\theta}{2\sin^2\theta/2 - i\sin\theta}$$

$$-x = \frac{2\cos^2\theta/2 + i2\sin\frac{\theta}{2}\cos\theta/2}{2\sin^2\theta/2 - i2\sin\frac{\theta}{2}\cos\theta/2}$$

$$= \frac{2\cos\theta/2(\cos\theta/2 + i\sin\theta/2)}{2\sin\theta/2(\sin\theta/2 - i\cos\theta/2)}$$

$$x = \cot\theta/2 \frac{(\cos\theta/2 + i\sin\theta/2)}{-\sin\theta/2 + i\cos\theta/2}$$

$\times$ by $\frac{i}{i}$

$$= \cot\theta/2 \frac{(\cos\theta/2 + i\sin\theta/2)}{-[\cos(\frac{\pi}{2}+\theta/2) + i\sin(\frac{\pi}{2}+\theta/2)]}$$

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$$x = \cot\theta_{/2}\left[\cos\theta_{/2} + i\sin\theta_{/2}\right] + \cos\left(\frac{\pi}{2}+\frac{\theta}{2}\right) + i\sin\left(\frac{\pi}{2}+\frac{\theta}{2}\right)$$

$$x = \cot\theta_{/2}\left[\cos\theta_{/2} + i\sin\theta_{/2}\right]\left[\cos\left(\frac{\pi}{2}+\frac{\theta}{2}\right) + i\sin\left(\frac{\pi}{2}+\frac{\theta}{2}\right)\right]$$

~~By DMT~~ By corollary

$$x = \cot\theta_{/2}\left[\cos\theta_{/2} + i\sin\theta_{/2}\right]\left[\cos\left(\frac{\pi}{2}+\frac{\theta}{2}\right) - i\sin\left(\frac{\pi}{2}+\frac{\theta}{2}\right)\right]$$

$$= \cot\theta_{/2}\left[\cos\frac{\theta}{2}\cdot\cos\left(\frac{\pi}{2}+\frac{\theta}{2}\right) - \cos\frac{\theta}{2}\cdot i\sin\left(\frac{\pi}{2}+\frac{\theta}{2}\right) + i\sin\frac{\theta}{2}\cdot\cos\left(\frac{\pi}{2}+\frac{\theta}{2}\right) - i^2\sin\frac{\theta}{2}\cdot\sin\left(\frac{\pi}{2}+\frac{\theta}{2}\right)\right]$$

$$= \cot\frac{\theta}{2}\left[\cos\frac{\theta}{2}\cdot\cos\left(\frac{\pi}{2}+\frac{\theta}{2}\right) + \sin\frac{\theta}{2}\cdot\sin\left(\frac{\pi}{2}+\frac{\theta}{2}\right) - \cos\frac{\theta}{2}\cdot i\sin\left(\frac{\pi}{2}+\frac{\theta}{2}\right) + i\sin\frac{\theta}{2}\cdot\cos\left(\frac{\pi}{2}+\frac{\theta}{2}\right)\right]$$

$$= \cot\frac{\theta}{2}\left[\cos\left(\cancel{\frac{\theta}{2}} - \frac{\pi}{2} + \cancel{\frac{\theta}{2}}\right) + i\left(\cos\frac{\theta}{2}\sin\left(\frac{\pi}{2}+\frac{\theta}{2}\right) - \sin\frac{\theta}{2}\cdot\cos\left(\frac{\pi}{2}+\frac{\theta}{2}\right)\right)\right]$$

$$= \cot\frac{\theta}{2}\left[\cos\left(\frac{\pi}{2}\right) + i\sin\left(\frac{\pi}{2} + \cancel{\frac{\theta}{2}} + \cancel{\frac{\theta}{2}}\right)\right]$$

$$= \cot\frac{\theta}{2}\left[\cos\left(\frac{\pi}{2}\right) + i\sin\left(\frac{\pi}{2}\right)\right]$$

$$= \cot\frac{\theta}{2}\left[0 + i\right]$$

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$x = \cot\frac{\theta}{2}\ (-i)$

$x = i\cot\frac{\theta}{2}$

$\boxed{x = i\cot\frac{\theta}{2}}$

$x = i\cot\frac{2k\pi}{2\cdot 7}$

$\boxed{x = i\cot\frac{k\pi}{7}}$

$k = 0, 1, 2, 3, 4, 5, 6,$

$\cancel{r}\ r = 1, 2, 3$

$x = \pm i\cot\frac{r\pi}{7} \rightarrow r = 1, 2, 3.$

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7) Solve the eqⁿ $z^3 = (z+1)^3$ & show that the the real part of all roots is $-1/2$

$z^3 = (z+1)^3$

$\frac{z^3}{(z+1)^3} = 1$

$\left(\frac{z}{z+1}\right)^3 = 1$

$\frac{z}{z+1} = 1^{1/3}$

$= \cos 0 + i \sin 0.$

$\cos 0 = \cos 2k\pi + 0$ ... i k = n
$= \cos 2k\pi + 0$
$= \cos 2k\pi$
$\sin 0 = \sin 2k\pi.$

$\frac{z}{z+1} = (\cos 2k\pi + i \sin 2k\pi)^{1/3}$

By DMT.

$\frac{z}{z+1} = \cos \frac{2k\pi}{3} + i \sin \frac{2k\pi}{3}.$

Let $\frac{2k\pi}{3} = \theta.$

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$$\frac{z}{z+1} = \frac{\cos\theta + i\sin\theta}{1}$$

$$\frac{z}{z+1-z} = \frac{\cos\theta + i\sin\theta}{1-\cos\theta - i\sin\theta} \qquad \frac{D+N}{N-D}$$

$$z = \frac{\cos\theta + i\sin\theta}{2\sin^2\theta - i\sin\theta}$$

$$= \frac{\cos\theta + \cancel{i2\sin\theta_{/2}\cos\theta_{/2}}\ i\sin\theta}{2\sin^2\theta - i2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}$$

$$= \frac{\cos\theta + i\sin\theta}{2\cos^2\theta - i2\sin\theta_{/2}\cos\frac{\theta}{2}} \cdot \frac{(2i)\sin\theta_{/2}\cos\theta_{/2})}{(2i\sin\theta_{/2}\cos\theta_{/2})}$$

$$= \frac{(\cos\theta.\sin\theta_{/2} + i\cos\theta.\cos\theta_{/2} + i\sin\theta\sin\theta_{/2} - \sin\theta.\cos\theta_{/2})}{2\sin\theta_{/2}\underbrace{(\sin^2\theta_{/2} + \cos^2\theta_{/2})}_{1}}$$

$$= \frac{\cos\theta.\sin\theta_{/2} + i\cos\theta\cos\theta_{/2} + i\sin\theta\sin\theta_{/2} - \sin\theta.\cos\theta_{/2}}{2\sin\theta_{/2}}$$

$$= -\frac{[\sin\theta\cos\theta_{/2} - \cos\theta\sin\theta_{/2}] + i[\cos\theta.\cos\frac{\theta}{2} + \sin\theta.\sin\frac{\theta}{2}]}{2\sin\theta_{/2}}$$

$$= \frac{-[\sin(\theta + \frac{\theta}{2})] + i(\cos\theta - \frac{\theta}{2})}{2\sin\theta_{/2}}.$$

$$= \frac{-\sin\left(\frac{\theta}{2}\right) + i\cos\left(\frac{\theta}{2}\right)}{2\sin\theta/2}$$

$$= \frac{-\sin\theta/2}{2\sin\theta/2} + \frac{i\cos\left(\frac{\theta}{2}\right)}{2\sin\theta/2}.$$

$$= \frac{-1}{2} + \frac{i}{2}\cot\theta/2$$

Hence prove real part is $-\frac{1}{2}$.

Q. if $\alpha, \beta, \gamma, \delta$ are the roots of equations $x^4+x^3+x^2+x+1=0$, find & show that $(1-\alpha)(1-\beta)(1-\gamma)(1-\delta) = 5$.

$x^4+x^3+x^2+x+1=0$

$(x-1)(x^4+x^3+x^2+x+1)=0$

$\cancel{x^5+x^4+x^3+x^2+x-x^4-x^3-x^2-x-1}$

$x^5+\cancel{x^4}+\cancel{x^3}+\cancel{x^2}+\cancel{x}-\cancel{x^4}-\cancel{x^3}-\cancel{x^2}-\cancel{x}-1=0$

$x^5-1=0$

$x^5=1$

$x^5 = \cos 0 + i\sin 0$

$x^5 = \cos 2n\pi + i\sin 2n\pi$

$x = (\cos 2n\pi + i\sin 2n\pi)^{\frac{1}{5}}$.

By DMT

$x = \cos 2n\frac{\pi}{5} + \left[i\sin 2n\frac{\pi}{5}\right]$

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$n = 0, 1, 2, 3, 4, \cancel{5}.$

for $n = 0$.

$x_0 = \cos 0 + i \sin 0.$

$x_0 = 1.$

for $n = 1$

$x_1 = \cos \frac{2\pi}{5} + i \sin \frac{2\pi}{5}.$

$\cancel{x_1 = -1 + 0}.$

$n = 2$

$x_2 = \cos \frac{4\pi}{5} + i \sin \frac{4\pi}{5}.$

$n = 3$

$x_3 = \cos \frac{6\pi}{5} + i \sin \frac{6\pi}{5}$

$x_4 = \cos \frac{8\pi}{5} + i \sin \frac{8\pi}{5}.$

$\cancel{x_5 = \cos \frac{10\pi}{5} + i \sin \frac{10\pi}{5}}$

$(1-\alpha)(1-\beta)(1-\gamma)(1-\delta) = x^4 + x^3 + x^2 + x + 1$

$(1-\alpha)(1-\beta)(1-\gamma)(1-\delta) = 1^4 + 1^3 + 1^2 + 1 + 1$

$\boxed{(1-\alpha)(1-\beta)(1-\gamma)(1-\delta) = 5.}$

Use of Exponential form of complex Number.

$z = x + iy \longrightarrow$ cartesian

$z = r(\cos\theta + i\sin\theta) \longrightarrow$ Polar form

$z = re^{i\theta} \longrightarrow$ exponential form

NOTE :-

$1 = \cos 2n\pi + i\sin 2n\pi = e^{i2n\pi}$

$i = \cos\frac{\pi}{2} + i\sin\frac{\pi}{2} = e^{i\pi/2}$

$\sqrt{i} = \left[\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right]^{1/2} = \left[e^{i\pi/2}\right]^{1/2}$

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Q. Prove that i^i is real

$$i = \cos\frac{\pi}{2} + i\sin\frac{\pi}{2} = e^{i\pi/2}$$

$$i^i = \left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)^i = \left(e^{i\pi/2}\right)^i$$

$$i^i = \left(e^{i\pi/2}\right)^i$$

$$i^i = e^{i^2\pi/2}$$

$$\boxed{i^i = e^{-\pi/2}}$$

→ which is completely real number.

Q. Seperate real & imaginary part of z^z where $z = \frac{1}{2} + i\frac{\sqrt{3}}{2}$

$$z = \frac{1}{2} + i\frac{\sqrt{3}}{2}.$$

$$= \cos\frac{\pi}{3} + i\sin\frac{\pi}{3} = e^{i\pi/3}$$

$$z^z = \left(e^{i\pi/3}\right)^{\frac{1}{2} + i\frac{\sqrt{3}}{2}}$$

$$z^z = e^{i\pi/6 + i^2\frac{\sqrt{3}\pi}{2\times 3\sqrt{3}}}$$

$$z^z = \left(e^{i\pi/6 - \sqrt{3}\pi/6}\right)$$

$$z^z = e^{-\pi/2\sqrt{3}}\frac{\sqrt{3}}{2} + e^{-\pi/2\sqrt{3}}\, i\, \frac{1}{2}.$$

Comparing with $x + iy$

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$$x = e^{-\pi/2\sqrt{3}} \frac{\sqrt{3}}{2}$$

$$y = \frac{e^{-\pi/2\sqrt{3}}}{2}.$$

Q if $i^{i^{i^{\cdots\infty}}} = A+iB$ P.T $A^2+B^2 = e^{-\pi B}$ & ~~then~~ $\tan\left(\frac{\pi}{2}A\right) = B$.

$$i^{A+iB} = A+iB$$

$$i = \cos\frac{\pi}{2} + i\sin\frac{\pi}{2} = e^{i\pi/2}$$

$$i^{A+iB} = \left(e^{i\pi/2}\right)^{A+iB}$$

$$= e^{iA\pi/2 - \frac{\pi B}{2}}$$

$$= e^{iA\pi/2} \cdot e^{-\pi B/2} = e^{iA\pi/2} \cdot e^{-\pi B/2}$$

$$= e^{-\frac{\pi B}{2}} \cdot e^{iA\frac{\pi}{2}}$$

$$= e^{-\frac{\pi B}{2}} \left[\cos\left(\frac{A\pi}{2}\right) + i\sin\left(\frac{A\pi}{2}\right)\right]$$

$$= e^{-\pi B/2}\cos\left(\frac{A\pi}{2}\right) + i\, e^{-\pi B/2}\sin\frac{A\pi}{2}$$

compare $x+iy$.

$x = e^{-\pi B/2}\cos\frac{A\pi}{2}$, $y = i e^{-\pi B/2}\sin\left(\frac{A\pi}{2}\right)$

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$A = e^{-\pi B/2} \cos\left(\frac{A\pi}{2}\right)$

$B = e^{-\pi B/2} \sin\left(\frac{A\pi}{2}\right)$

$A^2 = e^{-\pi B} \cos^2\left(\frac{A\pi}{2}\right)$

$B^2 = e^{-\pi B} \sin^2\left(\frac{A\pi}{2}\right)$

$A^2 + B^2 = e^{-\pi B} \cos^2 \frac{A\pi}{2} + e^{-\pi B} \sin^2\left(\frac{A\pi}{2}\right)$

$= e^{-\pi B}\left(\cos^2 \frac{A\pi}{2} + \sin^2\left(\frac{A\pi}{2}\right)\right)$

$\boxed{A^2 + B^2 = e^{-\pi B}}$ H.P