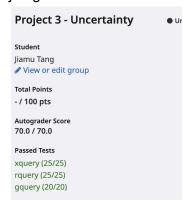
## Jiamu Tang jtang41

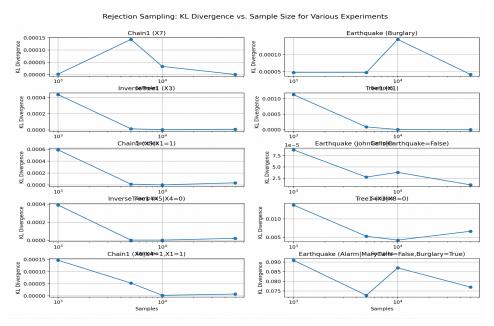


## Q1:

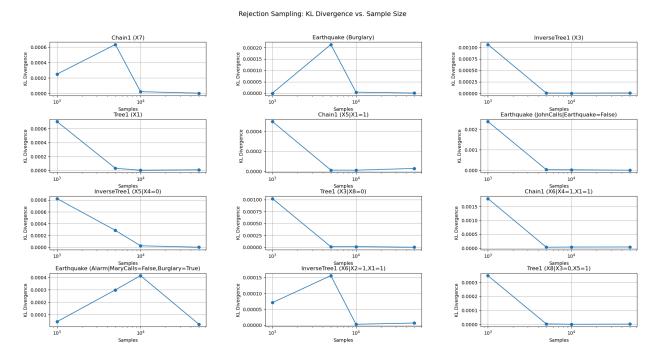
Based on my experiments, I found that rejection sampling with about 5000 to 10000 samples does a great job approximating the exact distribution in many Bayesian networks—most cases show nearly zero KL divergence. That said, some queries, especially in more complex networks like the earthquake model, still have higher KL divergence, which means these cases are tougher and might need more samples or a different approach. To improve the high divergence in the earthquake case, i tried increase the sampling number to a higher number there still have no improvement. Thus, I modified the function to incorporate early rejection, where each variable—including those with evidence—is sampled normally, and its value is immediately checked against the evidence; if a mismatch is found, the trial is aborted early. This change is made to avoid the unnecessary computation of sampling the remaining variables when a sample is destined to be rejected, thereby reducing the overall runtime even though it doesn't entirely resolve the inefficiency when evidence is rare using this improved method helped a lot and Fine-tuning the sample size is also key for reliable results after the improvment. Here is what i used:

def rejection\_sampling(network, query\_var, evidence, num\_samples=50000)

Here is the plot before the improved rejection method



Here is the plot after implementing the early rejection method: (big improvement in three earthquake cases!!!!!):



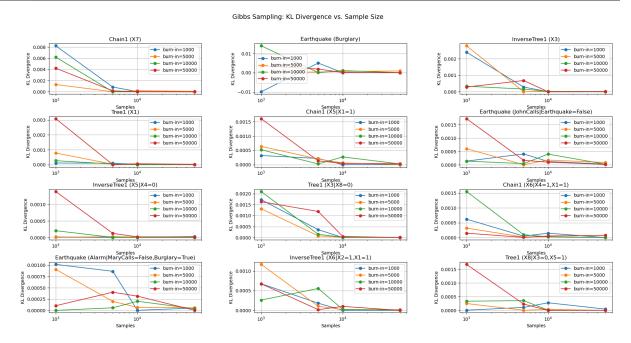
Q2:

From my experiments, I found that Gibbs sampling gets "close" to the true distribution with roughly 5,000 to 10,000 samples—especially when I use a moderate burn-in period of about 1,000 to 5,000 iterations. While sometimes even 1,000 samples with minimal burn-in can be acceptable, more samples tend to make the estimates more stable and less variable. Compared to rejection sampling, Gibbs sampling is much more efficient because it doesn't waste time on

discarded samples once the chain converges. So, allowing a burn-in period is definitely helpful for achieving more robust and reliable answers.

And i use: 10000,10000





Q3:

In my experiments, networks like Chain1 and InverseTree1 and Tree1 converged quickly—using around 5000–10,000 samples, the KL divergence was almost zero, which suggests that their structure and probability values are relatively simple. However, the Earthquake network, especially for the query "Alarm|MaryCalls=False, Burglary=True," was more challenging. Even with increased sample sizes and different burn-in periods, the KL divergence remained noticeably higher, indicating that the extreme probability values and strong conditional dependencies require more careful tuning. Similarly, the Tree1 query "X3|X8=0" was sensitive to the burn-in period, showing that network topology also affects convergence. Overall, these results highlight that both network structure and the nature of the evidence can significantly impact the performance of sampling methods, making cases like the Earthquake network and certain Tree1 queries worth further investigation.