Logic Programming Worksheet I

Propositional Logic

1. Let p stand for the proposition "I bought a lottery ticket" and q for "I won the jackpot". Express the following as natural English sentences:
   (a) $\neg p$
   (b) $p \vee q$
   (c) $p \wedge q$
   (d) $p \Rightarrow q$
   (e) $\neg p \Rightarrow \neg q$
   (f) $\neg p \vee (p \wedge q)$
2. Formalise the following in terms of atomic propositions r, b, and w, first making clear how they correspond to the English text.
   (a) Berries are ripe along the path, but rabbits have not been seen in the area.
   (b) Rabbits have not been seen in the area, and walking on the path is safe, but berries are ripe along the path.
   (c) If berries are ripe along the path, then walking is safe if and only if rabbits have not been seen in the area.
   (d) It is not safe to walk along the path, but rabbits have not been seen in the area and the berries along the path are ripe.
   (e) For walking on the path to be safe, it is necessary but not sufficient that berries not be ripe along the path and for rabbits not to have been seen in the area.
   (f) Walking is not safe on the path whenever rabbits have been seen in the area and berries are ripe along the path.
3. Formalise these statements and determine (with truth tables or otherwise) whether they are consistent (i.e. if there are some assumptions on the atomic propositions that make it true): "The system is in a multiuser state if and only if it is operating normally. If the system is operating normally, the kernel is functioning. Either the kernel is not functioning or the system is in interrupt mode. If the system is not in multiuser state, then it is in interrupt mode. The system is not in interrupt mode."
4. When is a propositional formula P *valid*? When is P *satisfiable*?
5. For each of the following propositions, construct a truth table and state whether the proposition is valid or satisfiable. (For brevity, you can just write one truth table with many columns.)
   (a) $p \wedge \neg p$
   (b) $p \vee \neg p$
   (c) $(p \vee \neg q) \Rightarrow q$
   (d) $(p \vee q) \Rightarrow (p \wedge q)$
   (e) $(p \Rightarrow q) \Leftrightarrow (\neg q \Rightarrow \neg p)$
   (f) $(p \Rightarrow q) \Rightarrow (q \Rightarrow p)$
6. For each of the following propositions, construct a truth table and state whether the proposition is valid or satisfiable.

(a) $p \Rightarrow (\neg q \vee r)$

(b) $\neg p \Rightarrow (q \Rightarrow r)$

(c) $(p \Rightarrow q) \vee (\neg p \Rightarrow r)$

(d) $(p \Rightarrow q) \wedge (\neg p \Rightarrow r)$

(e) $(p \Leftrightarrow q) \vee (\neg q \Leftrightarrow r)$

(f) $(\neg p \Leftrightarrow \neg q) \Leftrightarrow (q \Leftrightarrow r)$

7. Formalise the following and, by writing truth tables for the premises and conclusion, determine whether the arguments are valid.

   (a) Either John isn't stupid and he is lazy, or he's stupid.
   John is stupid.
   Therefore, John isn't lazy.

   (b) The butler and the cook are not both innocent
   Either the butler is lying or the cook is innocent
   Therefore, the butler is either lying or guilty

8. Use truth tables to determine which of the following are equivalent to each other:

   (a) P

   (b) $\neg P$

   (c) $P \Rightarrow F$

   (d) $P \Rightarrow T$

   (e) $F \Rightarrow P$

   (f) $T \Rightarrow P$

   (g) $\neg\neg P$

9. Use truth tables to determine which of the following are equivalent to each other:

   (a) $(P \wedge Q) \vee (\neg P \wedge \neg Q)$

   (b) $\neg P \vee Q$

   (c) $(P \vee \neg Q) \wedge (Q \vee \neg P)$

   (d) $\neg(P \vee Q)$

   (e) $(Q \wedge P) \vee \neg P$

10. Imagine that a logician puts four cards on the table in front of you. Each card has a number on one side and a letter on the other. On the uppermost faces, you can see E, K, 4, and 7. He claims that if a card has a vowel on one side, then it has an even number on the other. How many cards do you have to turn over to check this?

11. Give a truth-table definition of the ternary boolean operation **if** P **then** Q **else** R.

Predicate Logic

1. Formalise the following statements in predicate logic, making clear what your atomic predicate symbols stand for and what the domains of any variables are.
   (a) Anyone who has forgiven at least one person is a saint.
   (b) Nobody in the calculus class is smarter than everybody in the discrete maths class.
   (c) Anyone who has bought a Rolls Royce with cash must have a rich uncle.
   (d) If anyone in the college has the measles, then everyone who has a friend in the college will have to be quarantined.
   (e) Everyone likes Mary, except Mary herself.
   (f) Jane saw a bear, and Roger saw one too.
   (g) Jane saw a bear, and Roger saw it too.
   (h) If anyone can do it, Jones can.
   (i) If Jones can do it, anyone can.
2. Translate the following into idiomatic English.
   (a) $\forall\, x.(\mathrm{H}(x) \wedge \forall\, y.\neg \mathrm{M}(x, y)) \Rightarrow \mathrm{U}(x)$ where $\mathrm{H}(x)$ means x is a man, $\mathrm{M}(x, y)$ means x is married to y, $\mathrm{U}(x)$ means x is unhappy, and x and y range over people.
   (b) $\exists\, z.\mathrm{P}(z, x) \wedge \mathrm{S}(z, y) \wedge \mathrm{W}(y)$ where $\mathrm{P}(z, x)$ means z is a parent of x, $\mathrm{S}(z, y)$ means z and y are siblings, $\mathrm{W}(y)$ means y is a woman, and x, y, and z range over people.
3. State whether the following are true or false, where x, y and z range over the integers.
   (a) $\forall\, x.\exists\, y.(2x - y = 0)$
   (b) $\exists\, y.\forall\, x.(2x - y = 0)$
   (c) $\forall\, x.\exists\, y.(x - 2y = 0)$
   (d) $\forall\, x.x < 10 \Rightarrow \forall\, y.(y < x \Rightarrow y < 9)$
   (e) $\exists\, y.\exists\, z.y + z = 100$
   (f) $\forall\, x.\exists\, y.(y > x \wedge \exists\, z.y + z = 100)$
4. What changes above if x, y and z range over the reals?
5. Formalise the following (over the real numbers):
   (a) Negative numbers don't have square roots
   (b) Every positive number has exactly two square roots