UNIT-II

Measures of Central Tendency

The point around which the observations concentrate in general in the central part of the data is called central value of the data and the tendency of the observations to concentrate around a central point is known as Central Tendency.

Objects of Statistical Average:

- To get a single value that describes the characteristics of the entire group¬
- To facilitate comparison

Functions of Statistical Average:

- Gives information about the whole group
- Becomes the basis of future planning and actions
- Provides a basis for analysis
- Traces mathematical relationships
- Helps in decision making

Requisites of an Ideal Average:

- Simple and rigid definition
- Easy to understand
- Simple and easy to compute
- Based on all observations
- Least affected by extreme values
- Least affected by fluctuations of sampling
- Capable of further algebric treatment

ARITHMETIC MEAN

Arithmetic Mean of a group of observations is the quotient obtained by dividing the sum of all observations by their number. It is the most commonly used average or measure of the central tendency applicable only in case of quantitative data. Arithmetic mean is also simply called "mean".

Arithmetic mean is denoted by .

Merits of Arithmetic Mean:

- It is rigidly defined.
- It is easy to calculate and simple to follow.
- It is based on all the observations.

- It is readily put to algebraic treatment.
- It is least affected by fluctuations of sampling.
- It is not necessary to arrange the data in ascending or descending order.

Demerits of Arithmetic Mean:

- The arithmetic mean is highly affected by extreme values.
- It cannot average the ratios and percentages properly.
- It cannot be computed accurately if any item is missing.
- The mean sometimes does not coincide with any of the observed value.
- It cannot be determined by inspection.
- It cannot be calculated in case of open ended classes.

Methods of Calculating Arithmetic Mean:

- Direct Method
- Short cut method
- Step deviation method

Use of Arithmetic Mean:

Arithmetic Mean is recommended in following situation:

- When the frequency distribution is symmetrical.
- When we need a stable average.
- When other measures such as standard deviation, coefficient of correlation are to be computed later.

MEDIAN (M)

The median is that value of the variable which divides the group into two equal parts, one part comprising of all values greater and other of all values less than the median. For calculation of median the data has to be arranged in either ascending or descending order. Median is denoted by M.

Merits of Median:

- It is easily understood and easy to calculate.
- It is rigidly defined.
- It can sometimes be located by simple inspection and can also be computed graphically.
- It is positional average therefore not affected at all by extreme observations.
- It is only average to be used while dealing with qualitative data like intelligence, honesty etc.

- It is especially useful in case of open end classes since only the position and not the value of items must be known.
- It is not affected by extreme values.

Demerits of Median:

- For calculation, it is necessary to arrange data in ascending or descending order.
- Since it is a positional average, its value is not determined by each and every observation.
- It is not suitable for further algebric treatment.
- It is not accurate for large data.
- The value of median is more affected by sampling fluctuations than the value of the arithmetic mean.

Uses of Median:

The use of median is recommended in the following situations:

- When there are open-ended classes provided it does not fall in those classes.
- When exceptionally large or small values occur at the ends of the frequency distribution.
- When the observation cannot be measured numerically but can be ranked in order.
- To determine the typical value in the problems concerning distribution of wealth etc.

MODE (Z)

Mode is the value which occurs the greatest number of times in the data. The word mode has been derived from the French word 'La Mode' which implies fashion. The Mode of a distribution is the value at the point around which the items tend to be most heavily concentrated. It may be regarded as the most typical of a series of values. Mode is denoted by Z.

Merits of Mode:

- It is easy to understand and simple to calculate.
- It is not affected by extreme large or small values.
- It can be located only by inspection in ungrouped data and discrete frequency distribution.
- It can be useful for qualitative data.
- It can be computed in open-end frequency table.
- It can be located graphically.

Demerits of Mode:

- It is not well defined.
- It is not based on all the values.

- It is suitable for large values and it will not be well defined if the data consists of small number of values.
- It is not capable of further mathematical treatment.
- Sometimes, the data has one or more than one mode and sometimes the data has no mode at all.

Uses of Mode:

The use of mode is recommended in the following situations:

- When a quick approximate measure of central tendency is desired.
- When the measure of central tendency should be the most typical value.

GEOMETRIC MEAN (G.M)

The geometric mean also called geometric average is the nth root of the product of n non-negative quantities. Geometric Mean is denoted by G.M.

Properties of Geometric Mean:

- The geometric mean is less than arithmetic mean, G.M<A.M
- The product of the items remains unchanged if each item is replaced by the geometric mean.
- The geometric mean of the ratio of corresponding observations in two series is equal to the ratios their geometric means.
- The geometric mean of the products of corresponding items in two series.

Merits of Geometric Mean:

- It is rigidly defined and its value is a precise figure.
- It is based on all observations.
- It is capable of further algebraic treatment.
- It is not much affected by fluctuation of sampling.
- It is not affected by extreme values.

Demerits of Geometric Mean:

- It cannot be calculated if any of the observation is zero or negative.
- Its calculation is rather difficult.
- It is not easy to understand.
- It may not coincide with any of the observations.

Uses of Geometric Mean:

- Geometric Mean is appropriate when:
- Large observations are to be given less weight.

- We find the relative changes such as the average rate of population growth, the average rate of intrest etc.
- Where some of the observations are too small and/or too large.
- Also used for construction of Index Numbers.

HARMONIC MEAN (H.M)

Harmonic mean is another measure of central tendency. Harmonic mean is also useful for quantitative data. Harmonic mean is quotient of "number of the given values" and "sum of the reciprocals of the given values". It is denoted by H.M.

Merits of Harmonic Mean:

- It is based on all observations.
- It not much affected by the fluctuation of sampling.
- It is capable of algebraic treatment.
- It is an appropriate average for averaging ratios and rates.
- It does not give much weight to the large items and gives greater importance to small items.

Demerits of Harmonic Mean:

- Its calculation is difficult.
- It gives high weight-age to the small items.
- It cannot be calculated if any one of the items is zero.
- It is usually a value which does not exist in the given data.

Uses of Harmonic Mean:

- Harmonic mean is better in computation of average speed, average price etc. under certain conditions.

Mean

Question 1

If the mean of 5 observation x, x + 2, x + 4, x + 6 and x + 8 is 11, find the value of x.

Solution 1

$$\text{Mean} = \frac{\text{Sum of observations}}{\text{Total number of observations}}$$

$$\Rightarrow 11 = \frac{x + (x+2) + (x+4) + (x+6) + (x+8)}{5}$$

$$\Rightarrow 55 = 5x + 20$$

$$\Rightarrow 5x = 35$$

$$\Rightarrow x = 7$$

Question 2

If the mean of 25 observations is 27 and each observation is decreased by 7, what will be the new mean?

Solution 2

Mean of 25 observations = 27

$\Rightarrow$ Sum of 25 observations = $27 \times 25 = 675$

When each observation is decreased by 7,

New sum = $675 - 25 \times 7 = 675 = 175 = 500$

Thus, new mean = $\frac{500}{25} = 20$

Question 3

Compute the mean of the following data:

Class	1-3	3-5	5-7	7-9
Frequency	12	22	27	19

Solution 3

We have

Class interval	Frequency f_i	Mid-value x_i	$f_i \times x_i$
1 – 3	12	2	24
3 – 5	22	4	88
5 – 7	27	6	162
7 – 9	19	8	152
	$\Sigma f_i = 80$		$\Sigma f_i x_i = 426$

$$\therefore \text{Mean} = \frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{426}{80} = 5.325$$

<u>Question 4</u>

Find the mean, using direct method:

Class	Frequency
0 - 10	**7**
10 - 20	**5**
20- 30	**6**
30 - 40	**12**
40 - 50	**8**
50 - 60	**2**

Solution 4

We have

Class	Frequency f_i	Mid Value x_i	$f_i x_i$
0-10	7	5	35
10-20	5	15	75
20-30	6	25	150
30-40	12	35	420
40-50	8	45	360
50-60	2	55	110
	$\Sigma f_i = 40$		$\Sigma f_i x_i = 1150$

$$\therefore \text{Mean } \bar{x} = \frac{\Sigma(f_i \times x_i)}{\Sigma f_i} = \frac{1150}{40} = 28.75$$

Question 5

Find the mean, using direct method:

Class	Frequency
25 - 35	6
35 - 45	10
45 - 55	8
55 - 65	12
65 - 75	4

Solution 5
We have

Class	Frequency f_i	Mid - value x_i	$f_i x_i$
25 - 35	6	30	180
35 - 45	10	40	400
45 - 55	8	50	400
55 - 65	12	60	720
65 - 75	4	70	280
	$\Sigma f_i x_i = 40$		$\Sigma f_i x_i = 1980$

$\therefore$ Mean, $\bar{x} = \dfrac{\Sigma (f_i x_i)}{\Sigma f_i} = \dfrac{1980}{40} = 49.5$

Question 6

Find the mean, using direct method:

Class	Frequency
0 - 100	6
100 - 200	9
200 - 300	15
300 - 400	12
400 - 500	8

Solution 6
We have

Class	Frequency f_i	Mid Value x_i	$f_i x_i$
0 - 100	6	50	300
100 - 200	9	150	1350
200 - 300	15	250	3750
300 - 400	12	350	4200
400 - 500	8	450	3600
	$\Sigma f_i = 50$		$\Sigma f_i x_i = 13200$

$\therefore$ Mean, $\bar{x} = \frac{\Sigma(f_i x_i)}{\Sigma f_i} = \frac{13200}{40} = 264$

Question 7

Using an appropriate method, find the mean of the following frequency distribution:

Class interval	**84-90**	**90-96**	**96-102**	**102-108**	**108-114**	**114-120**
Frequency	**8**	**10**	**16**	**23**	**12**	**11**

Which method did you use, and why?

Solution 7

Class interval	Frequency f_i	Mid-value x_i	$u_i = \frac{x_i - A}{h} = \frac{x_i - 99}{6}$	$f_i \times u_i$
84 – 90	8	87	–2	–16
90 – 96	10	93	–1	–20
96 – 102	16	99 = A	0	0
102 – 108	23	105	1	23
108 – 114	12	111	2	24
114 – 120	11	117	3	33
	$\Sigma f_i = 80$			$\Sigma f_i u_i = 44$

Thus, $A = 99$, $h = 6$, $\Sigma f_i = 80$ and $\Sigma f_i u_i = 44$

$$\text{Mean} = A + \left\{h \times \frac{\Sigma f_i u_i}{\Sigma f_i}\right\}$$

$$= 99 + \left\{6 \times \frac{44}{80}\right\}$$

$$= 99 + 3.3$$

$$= 102.3$$

Since the values of x_i's f_i's are larger, we use step-deviation method.

Question 8

If the mean of the following frequency distribution is 24, find the value of p.

Class	0-10	10-20	20-30	30-40	40-50
Frequency	3	4	p	3	2

Solution 8

We have,

Class interval	Frequency f_i	Mid-value x_i	$f_i \times x_i$
$0-10$	3	5	15
$10-20$	4	15	60
$20-30$	p	25	25p
$30-40$	3	35	105
$40-50$	2	45	90
	$\Sigma f_i = 12+p$		$\Sigma f_i x_i = 270+25p$

$$\text{Now, Mean} = \frac{\Sigma f_i x_i}{\Sigma f_i}$$

$$\Rightarrow 24 = \frac{270+25p}{12+p}$$

$$\Rightarrow 24(12+p) = 270+25p$$

$$\Rightarrow 288+24p = 270+25p$$

$$\Rightarrow p = 18$$

Question 9

The following distribution shows the daily pocket allowance of children of a locality. If the mean pocket allowance is Rs.18, find the missing frequency f.

Daily pocket allowance (in)	**11-13**	**13-15**	**15-17**	**17-19**	**19-21**	**21-23**	**23-25**
Number of children	**7**	**6**	**9**	**13**	**f**	**5**	**4**

Solution 9

We have,

Class interval	Frequency f_i	Mid-value x_i	$f_i \times x_i$
11 – 13	7	12	84
13 – 15	6	14	84
15 – 17	9	16	144
17 – 19	13	18	234
19 – 21	f	20	20f
21 – 23	5	22	110
23 – 25	4	24	96
	$\sum f_i = 44 + f$		$\sum f_i x_i = 752 + 20f$

Now, Mean $= \dfrac{\sum f_i x_i}{\sum f_i}$

$\Rightarrow 18 = \dfrac{752 + 20f}{44 + f}$

$\Rightarrow 18(44 + f) = 752 + 20f$

$\Rightarrow 792 + 18f = 752 + 20f$

$\Rightarrow 2f = 40$

$\Rightarrow f = 20$

Question 10

If the mean of the following frequency distribution is 54, find the value of p.

Class	**0-20**	**20-40**	**40-60**	**60-80**	**80-100**
Frequency	**7**	**p**	**10**	**9**	**13**

We have,

Class interval	Frequency f_i	Mid-value x_i	$f_i \times x_i$
0 – 20	7	10	70
20 – 40	p	30	30p
40 – 60	10	50	500
60 – 80	9	70	630
80 – 100	13	90	1170
	$\sum f_i = 39 + p$		$\sum f_i x_i = 2370 + 30p$

Now, Mean $= \dfrac{\sum f_i x_i}{\sum f_i}$

$\Rightarrow 54 = \dfrac{2370 + 30p}{39 + p}$

$\Rightarrow 54(39 + p) = 2370 + 30p$

$\Rightarrow 27(39 + p) = 1185 + 15p$

$\Rightarrow 1053 + 27p = 1185 + 15p$

$\Rightarrow 12p = 132$

$\Rightarrow p = 11$

Question 11

The mean of the following data is 42. Find the missing frequencies x and y if the sum of frequencies is 100.

Class interval	0-10	10-20	20-30	30-40	40-50	50-60	60-70	70-80
Frequency	7	10	x	13	y	10	14	9

Solution 11

We have,

Class interval	Frequency f_i	Mid-value x_i	$f_i \times x_i$
0 – 10	7	5	35
10 – 20	10	15	150
20 – 30	x	25	25x
30 – 40	13	35	455
40 – 50	y	45	45y
50 – 60	10	55	550
60 – 70	14	65	910
70 – 80	9	75	675
	$\Sigma f_i = 63 + x + y = 100$		$\Sigma f_i x_i = 2775 + 25x + 45y$

Now, Mean $= \dfrac{\Sigma f_i x_i}{\Sigma f_i}$

$\Rightarrow 42 = \dfrac{2775 + 25x + 45y}{100}$

$\Rightarrow 4200 = 2775 + 25x + 45y$

$\Rightarrow 25x + 45y = 1425$

$\Rightarrow 5x + 9y = 285 \qquad \text{....(i)}$

Also, $63 + x + y = 100$

$\Rightarrow x + y = 37$

$\Rightarrow x = 37 - y$

Substituting in (i), we have

$5(37 - y) + 9y = 285$

$\Rightarrow 185 - 5y + 9y = 285$

$\Rightarrow 4y = 100$

$\Rightarrow y = 25$

$\Rightarrow x = 37 - y = 37 - 25 = 12$

Hence, $x = 12$ and $y = 25$

Question 12

The daily expenditure of 100 families is given below. Calculate f_1 and f_2 if the mean daily expenditure is Rs.188.

Expenditure (in Rs.)	140-160	160-180	180-200	200-220	220-240
Number of families	5	25	f_1	f_2	5

Solution 12

We have,

Class interval	Frequency f_i	Mid-value x_i	$f_i \times x_i$
140 – 160	5	150	750
160 – 180	25	170	4250
180 – 200	f_1	190	$190f_1$
200 – 220	f_2	210	$210f_2$
220 – 240	5	230	1150
	$\Sigma f_i = 35 + f_1 + f_2 = 100$		$\Sigma f_i x_i = 6150 + 190f_1 + 210f_2$

Now, Mean $= \dfrac{\Sigma f_i x_i}{\Sigma f_i}$

$\Rightarrow 188 = \dfrac{6150 + 190f_1 + 210f_2}{100}$

$\Rightarrow 18800 = 6150 + 190f_1 + 210f_2$

$\Rightarrow 1880 = 615 + 19f_1 + 21f_2$

$\Rightarrow 19f_1 + 21f_2 = 1265 \qquad \ldots.(i)$

Also, $35 + f_1 + f_2 = 100$

$\Rightarrow f_1 + f_2 = 65$

$\Rightarrow f_1 = 65 - f_2$

Substituting in (i), we have

$19(65 - f_2) + 21f_2 = 1265$

$\Rightarrow 1235 - 19f_2 + 21f_2 = 1265$

$\Rightarrow 2f_2 = 30$

$\Rightarrow f_2 = 15$

$\Rightarrow f_1 = 65 - f_2 = 65 - 15 = 50$

Hence, $f_1 = 50$ and $f_2 = 15$

Question 13

The mean of the following frequency distribution is 57.6 and the sum of the observations is 50.

Class	Frequency
0 - 20	**7**
20 - 40	f_1
40 - 60	**12**
60 - 80	f_2
80 - 100	**8**
100 - 120	**5**

Find f_1 and f_2.

Solution 13

We have

$7 + f_1 + 12 + f_2 + 8 + 5 = 50$

$\Rightarrow f_2 = 18 - f_1$

Class	Frequency f_i	Mid Value x_i	$f_i x_i$
0 - 20	7	10	70
20 - 40	f_1	30	$30 f_1$
40 - 60	12	50	600
60 - 80	$f_2 = 18 - f_1$	70	$1260 - 70 f_1$
80 - 100	8	90	720
100 - 120	5	110	550
	$\Sigma f_i = 50$		$\Sigma f_i x_i = 3200 - 40 f_1$

$$\therefore \text{Mean}, \bar{x} = \frac{\Sigma(f_i \times x_i)}{\Sigma f_i} = \frac{3200 - 40f_1}{50} = 57.6$$

$\Rightarrow 3200 - 40f_1 = 2880 \Rightarrow 40f_1 = 320$

$\Rightarrow \quad f_1 = 8$

Thus, $f_1 = 8$ and $f_2 = (18 - 8) = 10$

Question 14

During a medical check-up, the number of heart beats per minute of 30 patients were recorded and summarized as follows:

Number of heart-beats per minute	**65-68**	**68-71**	**71-74**	**74-77**	**77-80**	**80-83**	**83-86**
Number of patients	**2**	**4**	**3**	**8**	**7**	**4**	**2**

Find the mean heartbeats per minute for these patients, choosing a suitable method.

Solution 14

Class interval	Frequency f_i	Mid-value x_i	Deviation $d_i = x_i - 75.5$	$f_i \times d_i$
65 – 68	2	66.5	–9	–18
68 – 71	4	69.5	–6	–24
71 – 74	3	72.5	–3	–9
74 – 77	8	75.5 = A	0	0
77 – 80	7	78.5	3	21
80 – 83	4	81.5	6	24
83 – 86	2	84.5	9	18
	$\Sigma f_i = 30$			$\Sigma f_i d_i = 12$

Thus, $A = 75.5$, $\Sigma f_i = 30$ and $\Sigma f_i d_i = 12$

$$\text{Mean} = A + \frac{\Sigma f_i d_i}{\Sigma f_i}$$

$$= 75.5 + \frac{12}{30}$$

$$= 75.5 + 0.4$$

$$= 75.9$$

Thus, the mean heartbeats per minute for these patient is 75.9.

Question 15

Find the mean, using assumed mean method:

Marks	No, of students
0 - 10	12
10 - 20	18
20 -30	27
30 - 40	20
40 - 50	17
50 - 60	6

Solution 15

We have, Let A = 25 be the assumed mean

Marks	Frequency f_i	Mid value x_i	Deviation $d_i = (x_i - 25)$	$(f_i \times d_i)$
0 - 10	12	5	-20	-240
10 - 20	18	15	-10	-180
20 - 30	27	25 = A	0	0
30 - 40	20	35	10	200
40 - 50	17	45	20	340
50 - 60	6	55	30	180
	$\Sigma f_i = 100$			$\Sigma (f_i \times d_i) = 300$

$$\bar{x} = A + \frac{\Sigma (f_i \times x_i)}{\Sigma f_i} = \left(25 + \frac{300}{100}\right) = (25 + 3) = 28$$

Hence mean = 28

Question 16

Find the mean, using assumed mean method:

Class	Frequency
100 - 120	10
120 - 140	20
140 - 160	30
160 - 180	15
180 - 200	5

Solution 16

Let the assumed mean be 150, h = 20

Marks	Frequency f_i	Mid value x_i	Deviation $d_i = x_i - 150$	f_i di
100 - 120 120 - 140 140 - 160 160 - 180 180 - 200	10 20 30 15 5	110 130 150=A 170 190	-40 -20 0 20 40	-400 -400 0 300 200
	$\Sigma f_i = 80$			Σf_i di=-300

$$\bar{x} = A + \frac{\Sigma(f_i \times d_i)}{\Sigma f_i} = \left(150 + \frac{(-300)}{80}\right) = (150 - 3.75) = 146.25$$

Hence, Mean = 146.25

Question 17

Find the mean, using assumed mean method:

Class	Frequency
0 - 20	20
20 - 40	35
40 - 60	52
60 - 80	44
80 - 100	38
100 - 120	31

Solution 17

Let A = 50 be the assumed mean, we have

Marks	Frequency f_i	Mid value x_i	Deviation $d_i = (x_i - 50)$	$f_i \times d_i$
0 - 20	20	10	-40	-800
20 - 40	35	30	-20	-700
40 - 60	52	50 = A	0	0
60 - 80	44		20	880
80 - 100	38	70	40	1520
100 - 120	31	90	60	1860
		110		
	$\sum f_i = 220$			$\sum f_i \times d_i = 2760$

$$\therefore \bar{x} = A + \frac{\sum (f_i \times d_i)}{\sum f_i}$$

$$= 50 + \frac{2760}{220}$$

$$= 50 + 12.55$$

$$= 62.55$$

<u>Question 18</u>

The following table gives the literacy rate (in percentage) in 40 cities. Find the mean literacy rate, choosing a suitable method.

Literacy rate (%)	45-55	55-65	65-75	75-85	85-95
Number of cities	4	11	12	9	4

Solution 18

We have,

Class interval	Frequency f_i	Mid-value x_i	$f_i \times x_i$
45 – 55	4	50	200
55 – 65	11	60	660
65 – 75	12	70	840
75 – 85	9	80	720
85 – 95	4	90	360
	$\Sigma f_i = 40$		$\Sigma f_i x_i = 2780$

$$\text{Now, Mean} = \frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{2780}{40} = 69.5$$

Thus, the mean literacy rate is 69.5%.

Question 19

Find the mean of the following frequency distribution using step-deviation method.

Class	0-10	10-20	20-30	30-40	40-50
Frequency	7	10	15	8	10

Solution 19

Class interval	Frequency f_i	Mid-value x_i	$u_i = \frac{x_i - A}{h} = \frac{x_i - 25}{10}$	$f_i \times u_i$
0 – 10	7	5	−2	−14
10 – 20	10	15	−1	−10
20 – 30	15	25 = A	0	0
30 – 40	8	35	1	8
40 – 50	10	45	2	20
	$\Sigma f_i = 50$			$\Sigma f_i u_i = 4$

Thus, $A = 25$, $h = 10$, $\Sigma f_i = 50$ and $\Sigma f_i u_i = 4$

$$\text{Mean} = A + \left\{h \times \frac{\Sigma f_i u_i}{\Sigma f_i}\right\}$$

$$= 25 + \left\{10 \times \frac{4}{50}\right\}$$

$$= 25 + 0.8$$

$$= 25.8$$

Question 20

Find the mean of the following data, using step-deviation method:

Class	5-15	15-25	25-35	35-45	45-55	55-65	65-75
Frequency	6	10	16	15	24	8	7

Solution 20

Class interval	Frequency f_i	Mid-value x_i	$u_i = \frac{x_i - A}{h} = \frac{x_i - 40}{10}$	$f_i \times u_i$
5 – 15	6	10	–3	–18
15 – 25	10	20	–2	–20
25 – 35	16	30	–1	–16
35 – 45	15	40 = A	0	0
45 – 55	24	50	1	24
55 – 65	8	60	2	16
65 – 75	7	70	3	21
	$\sum f_i = 86$			$\sum f_i u_i = 7$

Thus, $A = 40$, $h = 10$, $\sum f_i = 86$ and $\sum f_i u_i = 7$

$$\text{Mean} = A + \left\{h \times \frac{\sum f_i u_i}{\sum f_i}\right\}$$

$$= 40 + \left\{10 \times \frac{7}{86}\right\}$$

$$= 40 + 0.81$$

$$= 40.81$$

Question 21

The weights of tea in 70 packets are shown in the following table:

Weight (in grams)	200-201	201-202	202-203	203-204	204-205	205-206

Number of packets	13	27	18	10	1	1

Find the mean weight of packets using step-deviation method.

Solution 21

Class interval	Frequency f_i	Mid-value x_i	$u_i = \frac{x_i - A}{h} = \frac{x_i - 203.5}{1}$	$f_i \times u_i$
200 - 201	13	200.5	-3	-39
201 - 202	27	201.5	-2	-54
202 - 203	18	202.5	-1	-18
203 - 204	10	203.5 = A	0	0
204 - 205	1	204.5	1	1
205 - 206	1	205.5	2	2
	$\sum f_i = 70$			$\sum f_i u_i = -108$

Thus, $A = 203.5$, $h = 1$, $\sum f_i = 70$ and $\sum f_i u_i = -108$

$$\text{Mean} = A + \left\{h \times \frac{\sum f_i u_i}{\sum f_i}\right\}$$

$$= 203.5 + \left\{1 \times \frac{-108}{70}\right\}$$

$$= 203.5 - 1.54$$

$$= 201.96$$

Question 22

Find the mean of the following frequency distribution using a suitable method:

Class	20-30	30-40	40-50	50-60	60-70
Frequency	25	40	42	33	10

Solution 22

Class interval	Frequency f_i	Mid-value x_i	Deviation $d_i = x_i - 45$	$f_i \times d_i$
20 – 30	25	25	−20	−500
30 – 40	40	35	−10	−400
40 – 50	42	45 = A	0	0
50 – 60	33	55	10	330
60 – 70	10	65	20	200
	$\sum f_i = 150$			$\sum f_i d_i = -370$

Thus, $A = 45$, $\sum f_i = 150$ and $\sum f_i d_i = -370$

$$\text{Mean} = A + \frac{\sum f_i d_i}{\sum f_i}$$

$$= 45 + \frac{(-370)}{150}$$

$$= 45 - 2.47$$

$$= 42.53$$

Question 23

In a annual examination marks (out of 90) obtained by students of class X in mathematics are given below:

Marks obtained	0-15	15-30	30-45	45-60	60-75	75-90
Number of students	2	4	5	20	9	10

Find the mean marks.

Solution 23

Class interval	Frequency f_i	Mid-value x_i	$u_i = \frac{x_i - A}{h} = \frac{x_i - 52.5}{15}$	$f_i \times u_i$
0 – 15	2	7.5	–3	–6
15 – 30	4	22.5	–2	–8
30 – 45	5	37.5	–1	–5
45 – 60	20	52.5 = A	0	0
60 – 75	9	67.5	1	9
75 – 90	10	82.5	2	20
	$\sum f_i = 50$			$\sum f_i u_i = 10$

Thus, $A = 52.5$, $h = 15$, $\sum f_i = 50$ and $\sum f_i u_i = 10$

$$\text{Mean} = A + \left\{h \times \frac{\sum f_i u_i}{\sum f_i}\right\}$$

$$= 52.5 + \left\{15 \times \frac{10}{50}\right\}$$

$$= 52.5 + 3$$

$$= 55.5$$

Thus, the mean marks are 55.5.

Question 24

Find the arithmetic mean of the following frequency distribution using step-deviation method:

Age (in year)	18-24	24-30	30-36	36-42	42-48	48-54
Number of workers	6	8	12	8	4	2

Solution 24

Class interval	Frequency f_i	Mid-value x_i	$u_i = \frac{x_i - A}{h} = \frac{x_i - 39}{6}$	$f_i \times u_i$
18 – 24	6	21	–3	–18
24 – 30	8	27	–2	–16
30 – 36	12	33	–1	–12
36 – 42	8	39 = A	0	0
42 – 48	4	45	1	4
48 – 54	2	51	2	4
	$\sum f_i = 40$			$\sum f_i u_i = -38$

Thus, $A = 39$, $h = 6$, $\sum f_i = 40$ and $\sum f_i u_i = -38$

$$\text{Mean} = A + \left\{h \times \frac{\sum f_i u_i}{\sum f_i}\right\}$$

$$= 39 + \left\{6 \times \frac{-38}{40}\right\}$$

$$= 39 - 5.7$$

$$= 33.3$$

<u>Question 25</u>

Find the arithmetic mean of each of the following frequency distribution using step-deviation method:

Class	Frequency
500 - 520	14
520 - 540	9
540 - 560	5
560 - 580	4
580 - 600	3
600 - 620	5

Solution 25

Let h = 20 and assume mean = 550, we prepare the table given below:

Age	Frequency f_i	Mid value x_i	$u_i = \left(\frac{x_i - 550}{20}\right)$	$(f_i \times u_i)$
500 - 520	14	510	-2	-27
520 - 540	9	530	-1	-9
540 - 560	5	550 = A	0	0
560 - 580	4	570	1	4
580 - 600	3	590	2	6
600 - 620	5	610	3	15
	$\Sigma f_i = 40$			$\Sigma(f_i \times u_i) = -12$

Thus, A = 550, h = 20, and $\Sigma f_i = 40$, $\Sigma(f_i \times u_i) = -12$

$$\therefore \text{Mean}, \bar{x} = A + \left[h \times \frac{\Sigma(f_i \times u_i)}{\Sigma f_i}\right]$$

$$= 550 + \left(20 \times \frac{-12}{40}\right)$$

$$= 550 - 6 = 544$$

Hence the mean of the frequency distribution is 544

Question 26

Find the mean age from the following frequency distribution:

Age(in years)	No. of persons
25 - 29	4
30 - 34	14
35 - 39	22
40 - 44	16
45 - 49	6
50 - 54	5
55 - 59	3

Hint: change the given series to the exclusive series

Solution 26
The given series is an inclusive series, making it an exclusive series, we have

Class	Frequency f_i	Mid value x_i	$u_i = \left(\frac{x_i - 42}{5}\right)$	$(f_i \times u_i)$
24.5 - 29.5	4	27	-3	-12
29.5 - 34.5	14	32	-2	-28
34.5 - 39.5	22	37	-1	-22
39.5 - 44.5	16	42 = A	0	0
44.5 - 49.5	6	47	1	6
49.5 - 54.5	5	52	2	10
54.5 - 59.5	3	57	3	9
	$\sum f_i = 70$			$\Sigma(f_i \times u_i) = -37$

Thus, A = 42, h = 5, $\sum f_i$ = 70 and $\Sigma(f_i \times u_i) = -37$

$$\therefore \text{Mean, } \bar{x} = A + \left[h \times \frac{\Sigma(f_i \times u_i)}{\Sigma f_i}\right]$$

$$= 42 + \left(5 \times \frac{-37}{70}\right)$$

$$= 42 - 2.64$$

$$= 39.36 \text{ years}$$

Hence, Mean = 39.36 years

<u>Question 27</u>

The following table shows the age distribution of patients of malaria in a village during a particular month:

Age(in years)	No. of cases
5 - 14	6
15 - 24	11
24 - 34	21
35 - 44	23
45 - 54	14
55 - 64	5

Find the average age of the patients.

Solution 27

The given series is an inclusive series making it an exclusive series,we get

class	Frequency f_i	Mid value x_i	$u_i = \left(\frac{x_i - 29.5}{10}\right)$	$(f_i \times u_i)$
4.5 - 14.5	6	9.5	-2	-12
14.5 - 24.5	11	19.5	-1	-11
24.5 - 34.5	21	29.5=A	0	0
34.5 - 44.5	23	39.5	1	23
44.5 - 54.5	14	49.5	2	28
54.5 - 64.5	5	59.5	3	15
	$\Sigma f_i = 80$			$\Sigma(f_i \times u_i) = 43$

Thus, A = 29.5, h = 10, $\Sigma f_i = 80$ and $\Sigma(f_i \times u_i) = 43$

$$\therefore \text{Mean}, \bar{x} = A + \left[h \times \frac{\Sigma(f_i \times u_i)}{\Sigma f_i}\right]$$

$$= 29.5 + \left(\frac{43}{80} \times 10\right)$$

$$= 29.5 + 5.37$$

$$= 34.87 \text{ years}$$

Hence, Mean = 34.87 years

Question 28

Weight of 60 eggs were recorded as given below:

Weight (in grams)	75-79	80-84	85-89	90-94	95-99	100-104	105-109

Number of eggs	4	9	13	17	12	3	2

Calculate their mean weight to the nearest gram.

Solution 28

The given series is an inclusive series.

Making it an exclusive series, we get

Class interval	Frequency f_i	Mid-value x_i	$u_i = \frac{x_i - A}{h} = \frac{x_i - 92}{5}$	$f_i \times u_i$
74.5 – 79.5	4	77	–3	–12
79.5 – 84.5	9	82	–2	–18
84.5 – 89.5	13	87	–1	–13
89.5 – 94.5	17	92 = A	0	0
94.5 – 99.5	12	97	1	12
99.5 – 104.5	3	102	2	6
104.5 – 109.5	2	107	3	6
	$\sum f_i = 60$			$\sum f_i u_i = -19$

Thus, $A = 92$, $h = 5$, $\sum f_i = 60$ and $\sum f_i u_i = -19$

$$\text{Mean} = A + \left\{h \times \frac{\sum f_i u_i}{\sum f_i}\right\}$$

$$= 92 + \left\{5 \times \frac{-19}{60}\right\}$$

$$= 92 - 1.58$$

$$= 90.42$$

$$= 90$$

Question 29

The following table shows the marks scored by 80 students in an examination:

Marks	Less than 5	Less than 10	Less than 15	Less than 20	Less than 25	Less than 30	Less than 35	Less than 40

Number of students	3	10	25	49	65	73	78	80

Calculate the mean marks correct to 2 decimal places.

Solution 29

The given data can be written as follows:

Class interval	Frequency f_i	Mid-value x_i	$u_i = \frac{x_i - A}{h} = \frac{x_i - 22.5}{5}$	$f_i \times u_i$
0 – 5	3	2.5	−4	−12
5 – 10	7	7.5	−3	−21
10 – 15	15	12.5	−2	−30
15 – 20	24	17.5	−1	−24
20 – 25	16	22.5 = A	0	0
25 – 30	8	27.5	1	8
30 – 35	5	32.5	2	10
35 – 40	2	37.5	3	6
	$\sum f_i = 80$			$\sum f_iu_i = -63$

Thus, $A = 22.5$, $h = 5$, $\sum f_i = 80$ and $\sum f_iu_i = -63$

$$\text{Mean} = A + \left\{h \times \frac{\sum f_iu_i}{\sum f_i}\right\}$$

$$= 22.5 + \left\{5 \times \frac{-63}{80}\right\}$$

$$= 22.5 - 3.9375$$

$$= 18.5625$$

$$= 18.56$$

Median

Question 1

In a hospital, the ages of diabetic patients were recorded as follows. Find the median age.

Age (in years)	0-15	15-30	30-45	45-60	60-75

Number of patients	5	20	40	50	25

Solution 1

Class interval	Frequency	Cumulative frequency
0 – 15	5	5
15 – 30	20	25
30 – 45	40	65
45 – 60	50	115
60 – 75	25	140

Here, $N = 140 \Rightarrow \frac{N}{2} = 70$

The cumulative frequency just greater than 70 is 115.

Hence, median class is 45 – 60.

$\therefore$ $l = 45$, $h = 15$, $f = 50$, cf = cf of preceding class = 65

$$\text{Now, Median} = l + \left\{ h \times \frac{\left(\frac{N}{2} - cf\right)}{f} \right\}$$

$$= 45 + \left\{15 \times \frac{(70 - 65)}{50}\right\}$$

$$= 45 + \left\{15 \times \frac{5}{50}\right\}$$

$$= 45 + 1.5$$

$$= 46.5$$

Thus, the median age of diabetic patients is 46.5 years.

Question 2

Compute the median from the following data:

Marks	No. of students
0 - 7	3
7 - 14	4
14 - 21	7

21 - 28	11
28 - 35	0
35 - 42	16
42 - 49	9

Solution 2
We prepare the frequency table, given below

Marks	No. of students f_i	C.F.
0 - 7	3	3
7 - 14	4	7
14 - 21	7	14
21 - 28	11	25
28 - 35	0	25
35 - 42	16	41
42 - 49	9	50
	$N = \sum f_i = 50$	

Now, $N = 50 \Rightarrow \left(\frac{N}{2}\right) = 25$

The cumulative frequency is 25 and corresponding class is 21 - 28.
Thus, the median class is 21 - 28

l = 21, h = 7, f = 11, c = C.F. preceding class 21 - 28 is 14 and $\frac{N}{2} = 25$

$$\text{Median} = l + \left[h \times \frac{\left(\frac{N}{2} - c\right)}{f}\right] = 21 + \left[7 \times \frac{(25 - 14)}{11}\right]$$

$$= (21 + 7) = 28$$

Hence the median is 28.

Question 3

The following table shows the daily wages of workers in a factory:

Daily wages	No. of workers

0 - 100	40
100 - 200	32
200 - 300	48
300 - 400	22
400 - 500	8

Find the median daily wage income of the workers.

Solution 3

We prepare the frequency table given below:

Daily wages	Frequency f_i	C.F.
0 - 100	40	40
100 - 200	32	72
200 - 300	48	120
300 - 400	22	142
400 - 500	8	150
	N = Σf_i = 150	

Now, N = 150, therefore $\left(\frac{N}{2}\right) = 75$

The cumulative frequency just greater than 75 is 120 and corresponding class is 200 - 300.

Thus, the median class is 200 - 300

l = 200, h = 100, f = 48

c = C.F. preceding median class = 72 and $\left(\frac{N}{2}\right) = 75$

$$\text{Median}, m_e = l + \left[h \times \frac{\left(\frac{N}{2} - c\right)}{f}\right] = 200 + \left(100 \times \frac{(75-72)}{48}\right)$$

$$= 200 + 6.25 = 206.25$$

Hence the median of daily wages is Rs. 206.25.

Hence the median is 28.

Question 4

Calculate the median from the following frequency distribution:

Class	Frequency

5 - 10	5
10 - 15	6
15 - 20	15
20 - 25	10
25 - 30	5
30 - 35	4
35- 40	2
40 - 45	2

Solution 4

We prepare the frequency table, given below:

Class	Frequency f_i	C.F
5 - 10	5	5
10 - 15	6	11c
15 - 20	15f	26
20 - 25	10	36
25 - 30	5	41
30 - 35	4	45
35- 40	2	47
40 - 45	2	49
	$\sum f_i = 49$	

Now, N = 49 $\Rightarrow \frac{N}{2} = \frac{49}{2} = 24.5$

The cumulative frequency just greater than 24.5 is 26 and corresponding class is 15 - 20.

Thus, the median class is 15 - 20

$\therefore$ l = 15, h = 5, f = 15

c = CF preceding median class = 11 and $\left(\frac{N}{2}\right) = 24.5$

$$\text{Median } m_e = l + \left[h \times \frac{\left(\frac{N}{2} - c\right)}{f}\right] = 15 + \left(5 \times \frac{(24.5 - 11)}{15}\right)$$

$$= 15 + \left(5 \times \frac{13.5}{15}\right) = 15 + 4.5 = 19.5$$

Median of frequency distribution is 19.5

Question 5

Given below is the number of units of electricity consumed in a week in a certain locality:

Consumption (in units)	No. of consumers
65 - 85	4
85 - 105	5
105 - 125	13
125 - 145	20
145 - 165	14
165 - 185	7
185 - 205	4

Calculate the median.

Solution 5

We prepare the cumulative frequency table as given below:

Consumption	Frequency f_i	C.F
65 - 85	4	4
85 - 105	5	9
105 - 125	13	22
125 - 145	20	42
145 - 165	14	56
165 - 185	7	63
185 - 205	4	67
	$N = \sum f_i = 67$	

Now, N = 67 $\Rightarrow \left(\frac{N}{2}\right) = \frac{67}{2} = 33.5$

The cumulative frequency just greater than 33.5 is 42 and the corresponding class 125 - 145.

Thus, the median class is 125 - 145

$\therefore$ l = 125, h = 20, $f_i = 20$ and c = CF preceding the median class = 22, $\frac{N}{2}$ = 33.5

$$\text{Median} = m_e = l + \left[h \times \frac{\left(\frac{N}{2} - c\right)}{f}\right] = 125 + \left[20 \times \frac{(33.5 - 22)}{20}\right]$$

$$= (125 + 11.5) = 136.5$$

Hence median of electricity consumed is 136.5

Question 6

Calculate the median from the following data:

Height(in cm)	No. of boys
135 - 140	6
140 - 145	10
145 - 150	18
150 - 155	22
155 - 160	20
160 - 165	15
165 - 170	6
170 - 175	3

Solution 6

Frequency table is given below:

Height	Frequency f_i	C.F
135 - 140	6	6
140 - 145	10	16
145 - 150	18	34
150 - 155	22	56
155 - 160	20	76
160 - 165	15	91
165 - 170	6	97
170 - 175	3	100
	$N = \sum f_i = 100$	

N = 100, $\left(\frac{N}{2}\right) = 50$

The cumulative frequency just greater than 50 is 56 and the corresponding class is 150 - 155

Thus, the median class is 150 - 155

l = 150, h = 5, f = 22, c = C.F.preceding median class = 34

$$\text{Median } m_e = l + \left[h \times \frac{\left(\frac{N}{2} - c\right)}{f} \right]$$

$$= 150 + \left(5 \times \frac{(50 - 34)}{22} \right)$$

$$= 150 + 3.64 = 153.64$$

Hence, Median = 153.64

Question 7

Calculate the missing frequency from the following distribution, it being given that the median of the distribution is 24.

Class	Frequency f_i
0 - 10	5
10 - 20	25
20 - 30	x
30 - 40	18
40 - 50	7

Solution 7

The frequency table is given below. Let the missing frequency be x

Class	Frequency f_i	C.F
0 - 10	5	5
10 - 20	25	30
20 - 30	x	30 + x
30 - 40	18	48 + x
40 - 50	7	55 + x

Median = 24 $\Rightarrow$ Median class is 20 - 30

$$\left(\frac{N}{2}\right) = \left(\frac{55+x}{2}\right) = 27.5 + \frac{x}{2}$$

l = 20, h = 10, f = x, c = C.F. preceding median class = 30

$$\text{Median} = l + \left[h \times \frac{\left(\frac{N}{2} - c\right)}{f}\right]$$

$$\Rightarrow 24 = 20 + \left[10 \times \frac{\left(27.5 + \frac{x}{2} - 30\right)}{x}\right]$$

$$24 = 20 + \left[10 \times \frac{\left(\frac{x}{2} - 2.5\right)}{x}\right]$$

$$24x = 20x + 5x - 25$$

$$0 = x - 25 \quad \therefore x = 25$$

Hence, the missing frequency is 25.

Question 8

The median of the following data is 16. Find the missing frequencies a and b if the total of frequencies is 70.

Class	0-5	5-10	10-15	15-20	20-25	25-30	30-35	35-40
Frequencies	12	a	12	15	b	6	6	4

Solution 8

Class interval	Frequency f_i	Cumulative frequency cf
0 – 5	12	12
5 – 10	a	12 + a
10 – 15	12	24 + a
15 – 20	15	39 + a
20 – 25	b	39 + a + b
25 – 30	6	45 + a + b
30 – 35	6	51 + a + b
35 – 40	4	55 + a + b = 70

Here, $N = 70 \Rightarrow \frac{N}{2} = 35$

Median is 16, which lies in the class 15 – 20.

Hence, median class is 15 – 20.

$\therefore$ l = 15, h = 5, f = 15, cf = cf of preceding class = 24 + a

$$\text{Now, Median} = l + \left\{ h \times \frac{\left(\frac{N}{2} - cf\right)}{f} \right\}$$

$$\therefore\ 16 = 15 + \left\{ 5 \times \frac{(35 - 24 - a)}{15} \right\}$$

$$\Rightarrow 16 = 15 + \left\{ \frac{11 - a}{3} \right\}$$

$$\Rightarrow 1 = \frac{11 - a}{3}$$

$$\Rightarrow 3 = 11 - a$$

$$\Rightarrow a = 8$$

Now, $55 + a + b = 70$

$$\Rightarrow 55 + 8 + b = 70$$

$$\Rightarrow 63 + b = 70$$

$$\Rightarrow b = 7$$

Hence, the missing frequencies are a = 8 and b = 7.

Question 9

In the following data the median of the runs scored by 60 top batsmen of the world in one-day international cricket matches is 5000. Find the missing frequencies x and y.

Runs scored	2500-3500	3500-4500	4500-5500	5500-6500	6500-7500	7500-8500
Number of batsmen	5	x	y	12	6	2

Solution 9

Class interval	Frequency f_i	Cumulative frequency cf
2500 – 3500	5	5
3500 – 4500	x	5 + x
4500 – 5500	y	5 + x + y
5500 – 6500	12	17 + x + y
6500 – 7500	6	23 + x + y
7500 – 8500	2	25 + x + y = 60

Here, $N = 60 \Rightarrow \frac{N}{2} = 30$

Median is 5000, which lies in the dass 4500 – 5500.

Hence, median class is 4500 – 5500.

$\therefore$ l = 4500, h = 1000, f = y, cf = cf of preceding dass = 5 + x

$$\text{Now, Median} = l + \left\{h \times \frac{\left(\frac{N}{2} - cf\right)}{f}\right\}$$

$$\therefore 5000 = 4500 + \left\{1000 \times \frac{(30 - 5 - x)}{y}\right\}$$

$$\Rightarrow 500 = 1000 \times \frac{25 - x}{y}$$

$$\Rightarrow 0.5 = \frac{25 - x}{y}$$

$\Rightarrow 0.5y = 25 - x$

$\Rightarrow x + 0.5y = 25 \qquad ....(i)$

Also, $25 + x + y = 60$

$\Rightarrow x + y = 35 \qquad ....(ii)$

Subtracting (i) from (ii), we get

$0.5y = 10$

$$\Rightarrow y = \frac{10}{0.5} = 20$$

$\Rightarrow x = 35 - 20 = 15$

Hence, the missing frequencies are x = 15 and y = 20.

Note: Answer not matching with back answer.

Question 10

If the median of the following frequency distribution is 32.5, find the value of f_1 and f_2.

Class Interval	Frequency
0 – 10	f_1
10 – 20	5
20 – 30	9
30 – 40	12
40 – 50	f_2
50 – 60	3
60 – 70	2
Total	40

Solution 10

Let f_1 and f_2 be the frequencies of class intervals 0 - 10 and 40 - 50

$f_1 + 5 + 9 + 12 + f_2 + 3 + 2 = 40$

$\Rightarrow f_1 + f_2 = 9$

Median is 32.5 which lies in 30 - 40, so the median class is 30 - 40

l = 30, h = 10, f = 12, N = 40 and $c = f_1 + 5 + 9 = (f_1 + 14)$

$$\text{Now, median} = l + \left[h \times \frac{\left(\frac{N}{2} - c\right)}{f} \right]$$

$$\Rightarrow \qquad 32.5 = \left[30 + \left(10 \times \frac{20 - f_1 - 14}{12} \right) \right]$$

$$\Rightarrow \qquad = \left[30 + \left(10 \times \frac{6 - f_1}{12} \right) \right]$$

$$\Rightarrow \qquad = \left[30 + \left(\frac{30 - 5f_1}{6} \right) \right]$$

$$\frac{30 - 5f_1}{6} = 2.5$$

$$30 - 5f_1 = 15$$

$$5f_1 = 15 \Rightarrow f_1 = 3$$

$$f_1 = 3 \text{ and } f_2 = (9 - 3) = 6$$

Question 11

Calculate the median for the following data:

Age(in years)	Frequency
19 - 25	35
26 - 32	96
33 - 39	68
40 - 46	102
47 - 53	35
54 - 60	4

Solution 11

The given series is of inclusive form. Converting it into exclusive form and preparing the cumulative frequency table, we get

Class	Frequency f_i	C.F
18.5 - 25.5	35	35
25.5 - 32.5	96	131
32.5 - 39.5	68	199
39.5 - 46.5	102	301
46.5 - 53.5	35	336
53.5 - 60.5	4	340
	$\Sigma f_i = N = 340$	

$$N = 340 \Rightarrow \frac{N}{2} = 170$$

The cumulative frequency just greater than 170 is 199 and the corresponding class is 32.5 - 39.5.

$\therefore$ Median class is 32.5 - 39.5

$\therefore$ l = 32.5, h = 7, f = 68, c = C.F. of preceding median class = 131

$$\text{Median } m_e = l + \left[h \times \frac{\left(\frac{N}{2} - c\right)}{f} \right] = 32.5 + \left[7 \times \frac{(170 - 131)}{68} \right]$$

$$= 32.5 + 4 = 36.5$$

Hence median is 36.5 years

Question 12

Find the median wages for the following frequencies distribution:

Wages per day (in Rs)	Frequency
61 - 70	5
71 - 80	15
81 - 90	20
91 - 100	30
101 - 110	20
111 - 120	8

Solution 12

Given series is in inclusive form converting it into exclusive form and preparing the cumulative frequency table, we get

Wages per day (in Rs)	Frequency f_i	C.F
60.5 - 70.5	5	5
70.5 - 80.5	15	20
80.5 - 90.5	20	40
90.5 - 100.5	30	70
100.5 - 110.5	20	90
110.5 - 120.5	8	98
	$\sum f_i = N = 98$	

$$N = 98 \Rightarrow \frac{N}{2} = 49$$

The cumulative frequency just greater than 49 is 70 and corresponding class is 90.5 - 100.5.

$\therefore$ median class is 90.5 - 100.5

$\therefore$ l = 90.5, h = 10, f = 30, c = CF preceding median class = 40

$$\text{Median} = l + \left[h \times \frac{\left(\frac{N}{2} - c\right)}{f}\right] = 90.5 + \left[10 \times \frac{49 - 40}{30}\right]$$

$$= 90.5 + 3 = \text{Rs}\,93.50$$

Hence, Median = Rs 93.50

Question 13

Find the median from the following table:

Class	Frequency
1 - 5	7
6 - 10	10
11 - 15	16
16 - 20	32
21 - 25	24
26 - 30	16
31 - 35	11
35 - 40	5
40 - 45	2

Solution 13

The given series is converted from inclusive to exclusive form and preparing the cumulative frequency table, we get

Marks	Frequency f_i	C.F
0.5 - 5.5	7	7
5.5 - 10.5	10	17
10.5 - 15.5	16	33
15.5 - 20.5	32	65
20.5 - 25.5	24	89
25.5 - 30.5	16	105
30.5 - 35.5	11	116
35.5 - 40.5	5	121
40.5 - 45.5	2	123
	$\Sigma f_i = N = 123$	

$$N = 123 \Rightarrow \frac{N}{2} = \frac{123}{2} = 61.5$$

The cumulative frequency just greater than 61.5 is 65.

$\therefore$ The corresponding median class is 15.5 - 20.5.

Then the median class is 15.5 - 20.5

$\therefore$ $l = 15.5$, $h = 5$, $f = 32$, $c =$ C.F. preceding median class $= 33$

$$\text{Median} = l + \left[h \times \frac{\left(\frac{N}{2} - c\right)}{f}\right] = 15.5 + \left[5 \times \frac{(61.5 - 33)}{32}\right]$$

$$= 15.5 + 4.45 = 19.95$$

Hence, Median = 19.95

Question 14

Find the median from the following data:

Marks	No. of students
Below 10	12
Below 20	32
Below 30	57
Below 40	80
Below 50	92
Below 60	116
Below 70	164
Below 80	200

Solution 14

Marks	Frequency f_i	C.F
0 - 10	12	12
10 - 20	20	32
20 - 30	25	57
30 - 40	23	80
40 - 50	12	92
50 - 60	24	116
60 - 70	48	164
70 - 80	36	200
	N = $\sum f_i = 200$	

$N = 200 = \frac{N}{2} = 100$

The cumulative frequency just greater than 100 is 116 and the corresponding class is 50 - 60.

Thus the median class is 50 - 60

$\therefore$ l = 50, h = 10, f = 24, c = C.F. preceding median class = 92, $\frac{N}{2}$ = 100

$$\text{Median} = l + \left[h \times \frac{\left(\frac{N}{2} - c\right)}{f} \right]$$

$$= 50 + \left[10 \times \frac{(100 - 92)}{24} \right]$$

$$= 50 + \left[10 \times \frac{84}{24} \right]$$

$$= 50 + 3.33 = 53.33$$

Hence, Median = 53.33

Mode

Question 1

Find the mode of the following frequency distribution:

Marks	10-20	20-30	30-40	40-50	50-60
Frequency	12	35	45	25	13

Solution 1

As the class 30 – 40 has the maximum frequency,

so it is the modal class.

$\therefore\ x_k = 30,\ h = 10,\ f_k = 45,\ f_{k-1} = 35$ and $f_{k+1} = 25$

Now,

$$\text{Mode} = x_k + h\left\{\frac{(f_k - f_{k-1})}{(2f_k - f_{k-1} - f_{k+1})}\right\}$$

$$= 30 + 10\left\{\frac{45 - 35}{2(45) - 35 - 25}\right\}$$

$$= 30 + 10 \times \frac{10}{30}$$

$$= 30 + 3.33$$

$$= 33.33$$

Question 2

Compute the mode of the following data:

Class	0-20	20-40	40-60	60-80	80-100
Frequency	25	16	28	20	5

Solution 2

As the class 40 – 60 has the maximum frequency,
so it is the modal class.

$\therefore\ x_k = 40,\ h = 20,\ f_k = 28,\ f_{k-1} = 16$ and $f_{k+1} = 20$

Now,

$$\text{Mode} = x_k + h\left\{\frac{(f_k - f_{k-1})}{(2f_k - f_{k-1} - f_{k+1})}\right\}$$

$$= 40 + 20\left\{\frac{28 - 16}{2(28) - 16 - 20}\right\}$$

$$= 40 + 20 \times \frac{12}{20}$$

$$= 40 + 12$$

$$= 52$$

Question 3

Heights of students of Class X are given in the following frequency distribution:

Height (in cm)	150-155	155-160	160-165	165-170	170-175
Number of students	15	8	20	12	5

Find the modal height.
Also, find the mean height. Compare and interpret the two measures of central tendency.

Solution 3

As the class 160 – 165 has the maximum frequency,
so it is the modal class.

$\therefore\ x_k = 160,\ h = 5,\ f_k = 20,\ f_{k-1} = 8,\ f_{k+1} = 12$

Now,

$$\text{Mode} = x_k + h\left\{\frac{(f_k - f_{k-1})}{(2f_k - f_{k-1} - f_{k+1})}\right\}$$

$$= 160 + 5\left\{\frac{20 - 8}{2(20) - 8 - 12}\right\}$$

$$= 160 + 5 \times \frac{12}{20}$$

$$= 160 + 3$$

$$= 163$$

Thus, the modal height is 163 cm, which means maximum number of students have height 163 cm.

Class interval	Frequency f_i	Mid-value x_i	$u_i = \frac{x_i - A}{h} = \frac{x_i - 162.5}{5}$	$f_i \times u_i$
150 – 155	15	152.5	−2	−30
155 – 160	8	157.5	−1	−8
160 – 165	20	162.5 = A	0	0
165 – 170	12	167.5	1	12
170 – 175	5	172.5	2	10
	$\sum f_i = 60$			$\sum f_i u_i = -16$

Thus, $A = 162.5,\ h = 5,\ \sum f_i = 60$ and $\sum f_i u_i = -16$

$$\text{Mean} = A + \left\{h \times \frac{\sum f_i u_i}{\sum f_i}\right\}$$

$$= 162.5 + \left\{5 \times \frac{-16}{60}\right\}$$

$$= 162.5 - 1.33$$

$$= 161.17$$

Thus, the mean height is 161.17 cm, which means on an average, the height of a student in a class is 161.17 cm.

Question 4

Find the mode of the following distribution:

Class interval	Frequency
10 - 14	8
14 - 18	6
18 - 22	11
22 - 26	20
26 - 30	25
30 - 34	22
34 - 38	10
38 - 42	4

Solution 4

As the class 26 - 30 has maximum frequency so it is modal class

$x_k = 26,\ f_k = 25,\ f_{k-1} = 20,\ f_{k+1} = 22,\ h = 4$

$$\text{Mode,}\quad m_0 = x_k + \left[h \times \frac{(f_k - f_{k-1})}{(2f_k - f_{k-1} - f_{k+1})}\right]$$

$$= 26 + \left(\frac{(25-20)}{(2\times 25 - 20 - 22)} \times 4\right)$$

$$= 26 + \frac{5}{2} = 26 + 2.5 = 28.5$$

Hence, mode = 28.5

Question 5

Given below is the distribution of total household expenditure of 200 manual workers in a city:

Expenditure	No. of manual workers
1000 - 1500	24
1500 - 2000	40
2000 - 2500	31
2500 - 3000	28
3000 - 3500	32
3500 - 4000	23
4000 - 4500	17
4500 - 5000	5

Find the average expenditure done by maximum number of manual workers.

Solution 5

As the class 1500 - 2000 has maximum frequency, so it os modal class

$x_k = 1500,\ f_k = 40,\ f_{k-1} = 24,\ f_{k+1} = 31,\ h = 500$

$$\text{Mode, } m_0 = x_k + \left[h \times \frac{(f_k - f_{k-1})}{(2f_k - f_{k-1} - f_{k+1})}\right]$$

$$= 1500 + \left[500 \times \frac{(40 - 24)}{(2 \times 40 - 24 - 31)}\right]$$

$$= 1500 + \left(500 \times \frac{16}{25}\right)$$

$$= 1500 + 320 = \text{Rs } 1820$$

Hence the average expenditure done by maximum number of workers = Rs. 1820

Question 6

Calculate the mode from the following data:

Monthly salary (in Rs)	No. of employees
0 - 5000	90
5000- 10000	150
10000 - 15000	100
15000 - 20000	80
20000 - 25000	70
25000 - 30000	10

Solution 6

As the class 5000 - 10000 has maximum frequency, so it is modal class

$x_k = 5000,\ f_k = 150,\ f_{k-1} = 90,\ f_{k+1} = 100 \text{ and } h = 5000$

$$\text{Mode, } m_0 = x_k + \left[h \times \frac{(f_k - f_{k-1})}{(2f_k - f_{k-1} - f_{k+1})}\right]$$

$$= 5000 + \left[5000 \times \frac{(150 - 90)}{(300 - 90 - 100)}\right]$$

$$= 5000 + 2727.27$$

$$= \text{Rs.} 7727.27$$

Hence, mode = Rs. 7727.27

Question 7

Compute the mode from the following data:

Age (in years)	No. of patients
0 - 5	6
5 - 10	11
10 - 15	18
15 - 20	24
20 - 25	17
25 - 30	13
30 - 35	5

Solution 7

As the class 15 - 20 has maximum frequency so it is modal class.

$x_k = 15,\ f_k = 24,\ f_{k-1} = 18,\ f_{k+1} = 17$ and $h = 5$

$$\text{Mode, } m_0 = x_k + \left[h \times \frac{(f_k - f_{k-1})}{(2f_k - f_{k-1} - f_{k+1})}\right]$$

$$= 15 + \left[5 \times \frac{(24 - 18)}{(48 - 18 - 17)}\right]$$

$$= (15 + 2.30) = 17.3 \text{ years}$$

Hence mode = 17.3 years

Question 8

Compute the mode from the following series:

Size	Frequency
45 - 55	7
55 - 65	12
65 - 75	17
75 - 85	30
85 - 95	32
95 - 105	6
105 - 115	10

Solution 8

As the class 85 - 95 has the maximum frequency it is modal class

$x_k = 85,\ f_k = 32,\ f_{k-1} = 30,\ f_{k+1} = 6$ and $h = 10$

$$\text{Mode},\quad m_0 = x_k + \left[h \times \frac{(f_k - f_{k-1})}{(2f_k - f_{k-1} - f_{k+1})}\right]$$

$$= 85 + \left[10 \times \frac{(32-30)}{(64-30-6)}\right]$$

$$= 85 + \frac{5}{7} = 85 + 0.71 = 85.71$$

Hence, mode = 85.71

Question 9

Compute the mode of the following data:

Class Interval	Frequency
1 - 5	3
6 - 10	8
11 - 15	13
16 - 20	18
21 - 25	28
26 - 30	20
31 - 35	13
36 - 40	8
41 - 45	6
46 - 50	4

Solution 9

The given series is converted from inclusive to exclusive form and on preparing the frequency table, we get

Class	Frequency
0.5 - 5.5	3
5.5- 10.5	8
10.5 - 15. 5	13
15.5 - 20.5	18
20.5 - 25. 5	28
25.5 - 30.5	20
30.5 - 35.5	13
35.5 - 40.5	8
40.5 - 45.5	6
45.5 - 50.5	3

As the class 20.5 - 25.5 has maximum frequency, so it is modal class

$x_k = 20.5,\ f_k = 28,\ f_{k-1} = 18,\ f_{k+1} = 20$ and $h = 5$

$$\text{Mode, } m_0 = x_k + \left[h \times \frac{(f_k - f_{k-1})}{(2f_k - f_{k-1} - f_{k+1})}\right]$$

$$= 20.5 + \left[5 \times \frac{(28-18)}{(56-18-20)}\right]$$

$$= 20.5 + \left[\frac{5 \times 10}{18}\right]$$

$$= 20.5 + 2.78$$

$$= 23.28$$

Hence, mode = 23.28

Question 10

The age wise participation of students in the Annual Function of a school is shown in the following distribution.

Age (in years)	5-7	7-9	9-11	11-13	13-15	15-17	17-19
Number of students	x	15	18	30	50	48	x

Find the missing frequencies when the sum of frequencies is 181. Also, find the mode of the data.

Solution 10

Sum of the frequencies = 181

$\Rightarrow x + 15 + 18 + 30 + 50 + 48 + x = 181$

$\Rightarrow 2x + 161 = 181$

$\Rightarrow 2x = 20$

$\Rightarrow x = 10$

Thus, the missing frequencies are 10 and 10.

As the class 13 – 15 has the maximum frequency, so it is the modal class.

$\therefore\ x_k = 13,\ h = 2,\ f_k = 50,\ f_{k-1} = 30$ and $f_{k+1} = 48$

Now,

$$\text{Mode} = x_k + h\left\{\frac{(f_k - f_{k-1})}{(2f_k - f_{k-1} - f_{k+1})}\right\}$$

$$= 13 + 2\left\{\frac{50 - 30}{2(50) - 30 - 48}\right\}$$

$$= 13 + 2 \times \frac{20}{22}$$

$$= 13 + 1.81$$

$$= 14.81$$

Mean, Median, Mode of Grouped Data

Question 1

Find the mean, mode and median of the following data:

Class	Frequency
0 - 10	5
10 - 20	10
20 - 30	18
30 - 40	30
40 - 50	20
50 - 60	12
60 - 70	5

Solution 1

Let assumed mean be 35, h = 10, now we have

Class	Frequency f_i	Mid-value x_i	$u_i = \frac{x_i - A}{h}$	C.F	$f_i u_i$
0-10	5	5	-3	5	-15
10-20	10	15	-2	15	-20
20-30	18	25	-1	33	-18
30-40	30	35 = A	0	63	0
40-50	20	45	1	83	20
50-60	12	55	2	95	24
60-70	5	65	3	100	15
	N = 100				$\Sigma f_i u_i = 6$

(i)Mean $\bar{x} = A + h\left(\frac{\Sigma f_i u_i}{N}\right)$

$$= 35 + 10 \times \left(\frac{6}{100}\right) = 35 + 0.6 = 35.6$$

(ii)N = 100, $\frac{N}{2} = 50$

Cumulative frequency just after 50 is 63

∴ Median class is 30 - 40

∴ l = 30, h = 10, N = 100, c = 33, f = 30

$$\therefore \text{Median } M_e = l + h\left(\frac{\frac{N}{2} - c}{f}\right) = 30 + 10\left(\frac{50 - 33}{30}\right)$$

$$= 30 + 10\left(\frac{17}{30}\right) = 30 + 5.67 = 35.67$$

(iii)Mode = 3 × median - 2 × mean

= 3 × 35.67 - 2 × 35.6 = 107.01 - 71.2

= 35.81

Thus, Mean = 35.6, Median = 35.67 and Mode = 35.81

Question 2

Find the mean, median and mode of the following data:

Class	0-20	20-40	40-60	60-80	80-100	100-120	120-140
Frequency	6	8	10	12	6	5	3

Solution 2

We prepare the following table:

Class interval	Frequency f_i	Mid-value x_i	$f_i \times x_i$	Cumulative frequency
0 – 20	6	10	60	6
20 – 40	8	30	240	14
40 – 60	10	50	500	24
60 – 80	12	70	840	36
80 – 100	6	90	540	42
100 – 120	5	110	550	47
120 – 140	3	130	390	50
	$\Sigma f_i = 50$		$\Sigma f_i x_i = 3120$	

<u>Mean :</u>

$$\text{Mean} = \frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{3120}{50} = 62.4$$

<u>Median :</u>

$$N = 50 \Rightarrow \frac{N}{2} = 25$$

The cumulative frequency just greater than 25 is 36.

Hence, median class is 60 – 80.

$\therefore$ l = 60, h = 20, f = 12, cf = cf of preceding class = 24

$$\text{Now, Median} = l + \left\{ h \times \frac{\left(\frac{N}{2} - cf\right)}{f} \right\} = 60 + \left\{ 20 \times \frac{25 - 24}{12} \right\} = 60 + 1.67 = 61.67$$

<u>Mode :</u>

$$\begin{aligned} \text{Mode} &= 3\text{Median} - 2\text{Mean} \\ &= 3 \times 61.67 - 2 \times 62.4 \\ &= 185.01 - 124.8 \\ &= 60.21 \end{aligned}$$

Question 3

Find the mean, median and mode of the following data:

Class	0-50	50-100	100-150	150-200	200-250	250-300	300-350
Frequency	2	3	5	6	5	3	1

Solution 3

We prepare the following table:

Class interval	Frequency f_i	Mid-value x_i	$f_i \times x_i$	Cumulative frequency
0 – 50	2	25	50	2
50 – 100	3	75	225	5
100 – 150	5	125	625	10
150 – 200	6	175	1050	16
200 – 250	5	225	1125	21
250 – 300	3	275	825	24
300 – 350	1	325	325	25
	$\Sigma f_i = 25$		$\Sigma f_i x_i = 4225$	

Mean :

$$\text{Mean} = \frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{4225}{25} = 169$$

Median :

$$N = 25 \Rightarrow \frac{N}{2} = 12.5$$

The cumulative frequency just greater than 12.5 is 16.

Hence, median class is 150 – 200.

$\therefore\ l = 150,\ h = 50,\ f = 6,\ cf = cf \text{ of preceding class} = 10$

$$\text{Now, Median} = l + \left\{ h \times \frac{\left(\frac{N}{2} - cf\right)}{f} \right\} = 150 + \left\{ 50 \times \frac{12.5 - 10}{6} \right\} = 150 + 20.83 = 170.83$$

Mode :

$$\begin{aligned} \text{Mode} &= 3\text{Median} - 2\text{Mean} \\ &= 3 \times 170.83 - 2 \times 169 \\ &= 512.49 - 338 \\ &= 174.49 \end{aligned}$$

Question 4

Find the mode, median and mean for the following data:

Marks obtained	25-35	35-45	45-55	55-65	65-75	75-85
Number of students	7	31	33	17	11	1

Solution 4

We prepare the following table:

Class interval	Frequency f_i	Mid-value x_i	$f_i \times x_i$	Cumulative frequency
25 – 35	7	30	210	7
35 – 45	31	40	1240	38
45 – 55	33	50	1650	71
55 – 65	17	60	1020	88
65 – 75	11	70	770	99
75 – 85	1	80	80	100
	$\sum f_i = 100$		$\sum f_i x_i = 4970$	

Mean :

$$\text{Mean} = \frac{\sum f_i x_i}{\sum f_i} = \frac{4970}{100} = 49.70$$

Median :

$$N = 100 \Rightarrow \frac{N}{2} = 50$$

The cumulative frequency just greater than 50 is 71

Hence, median class is 45 – 55.

$\therefore$ l = 45, h = 10, f = 33, cf = cf of preceding class = 38

$$\text{Now, Median} = l + \left\{ h \times \frac{\left(\frac{N}{2} - cf\right)}{f} \right\} = 45 + \left\{ 10 \times \frac{50 - 38}{33} \right\} = 45 + 3.64 = 48.64$$

Mode :

$$\begin{aligned} \text{Mode} &= 3\text{Median} - 2\text{Mean} \\ &= 3 \times 48.64 - 2 \times 49.70 \\ &= 145.92 - 99.4 \\ &= 46.52 \end{aligned}$$

Question 5

A survey regarding the heights of 50 girls of a class was conducted and the following data was obtained:

Height in cm	No. of girls

120 - 130	2
130 - 140	8
140 - 150	12
150 - 160	20
160 - 170	8
Total	50

Find the mean, Median and mode of the above data.

Solution 5

Let the assumed mean A be 145.Class interval h = 10.

Class	Frequency f_i	Mid-Value x_i	$u_i = \left(\frac{x_i - A}{h}\right)$	$f_i u_i$	C.F.
120-130	2	125	-2	-4	2
130-140	8	135	-1	-8	10
140-150	12	145=A	0	0	22
150-160	20	155	1	20	42
160-170	8	165	2	16	50
	N = 50			$\Sigma f_i u_i = 24$	

(i)Mean $\bar{x} = A + h\left(\frac{\Sigma f_i u_i}{N}\right) = 145 + 10 \times \left(\frac{24}{50}\right)$

$$= 145 + 4.8 = 149.8$$

(ii)N = 50, $\therefore \frac{N}{2} = \frac{50}{2} = 25$

Cumulative frequency just after 25 is 42

Corresponding median class is 150 - 160

Cumulative frequency before median class, c = 22

Median class frequency f = 20

$$\text{Median } M_e = l + h\left(\frac{\frac{N}{2} - c}{f}\right) = 150 + 10 \times \left(\frac{25 - 22}{20}\right)$$

$$= 150 + \frac{10 \times 3}{20} = 150 + 1.5 = 151.5$$

(iii)Mode = 3 median - 2 mean
= 3 151.5 - 2 149.8 = 454.5 - 299.6
= 154.9

Thus, Mean = 149.8, Median = 151.5, Mode = 154.9

Question 6

The following table gives the daily income of 50 workers of a factory:

Daily income(in Rs)	No. of workers
100 - 120	12
120 - 140	14
140 - 160	8
160 - 180	6
180 - 200	10

Find the mean, mode and median of the above data

Solution 6

Class	Frequency f_i	Mid-value x_i	$u_i = \left(\frac{x_i - A}{h}\right)$	$f_i u_i$	C.F.
100-120	12	110	-2	-24	12
120-140	14	130	-1	-14	26
140-160	8	150= A	0	0	34
160-180	6	170	1	6	40
180-200	10	190	2	20	50
	N = 50			$\Sigma f_i u_i = -12$	

Let assumed mean A = 150 and h = 20

(i)Mean $= A + h\left(\frac{\Sigma f_i u_i}{N}\right) = 150 + 20 \times \left(\frac{-12}{50}\right)$

$$= 150 - \frac{24}{5} = 150 - 4.8 = 145.2$$

(ii) $N = 50,\ \frac{N}{2} = \frac{50}{2} = 25$

Cumulative frequency just after 25 is 26

∴ Corresponding frequency median class is 120 - 140

So, l = 120, f = 14, $\frac{N}{2} = 25,$ h = 20, c = 12

$$\therefore \quad \text{Median} = l + h\left(\frac{\frac{N}{2} - c}{f}\right) = 120 + 20\left(\frac{25-12}{14}\right)$$

$$= 120 + \frac{20 \times 13}{14} = 120 + \frac{130}{7} = 120 + 18.6 = 138.6$$

(iii)Mode = 3 Median - 2 Mode

= 3 138.6 - 2 145.2

= 415.8 - 190.4

= 125.4

Hence, Mean = 145.2, Median = 138.6 and Mode = 125.4

Question 7

The table below shows the daily expenditure on food of 30 households in a locality:

Daily expenditure	No. of households
100 - 150	6
150 - 200	7
200 - 250	12
250 - 300	3
300 - 350	2

Find the mean and median daily expenditure on food

Solution 7

Class	Frequency f_i	Mid-value x_i	$u_i = \left(\frac{x_i - A}{h}\right)$	$f_i u_i$	C.F.

100-150	6	125	-2	-12	6
150-200	7	175	-1	-7	13
200-250	12	225	0	0	25
250-300	3	275	1	3	28
300-350	2	325	2	4	30
	N = 30			$\Sigma f_i u_i = -12$	

Let assumed mean = 225 and h = 50

(i)Mean = $A + h\left(\frac{\Sigma f_i u_i}{N}\right) = 225 + 50\left(\frac{-12}{30}\right) = 225 - 20 = 205$

(ii) $\frac{N}{2} = \frac{30}{2} = 15$

Cumulative frequency just after 15 is 25

$\therefore$ Corresponding class interval is 200 - 250

$\therefore$ Median class is 200 - 250

Cumulative frequency c just before this class = 13

So, $l = 200, f = 12, \frac{N}{2} = 15, h = 50, c = 13$

$$\therefore \text{Median} = l + h\left(\frac{\frac{N}{2} - c}{f}\right) = 200 + 50\left(\frac{15 - 13}{12}\right)$$

$$= 200 + \frac{50 \times 2}{12} = 200 + \frac{25}{3} = 200 + 8.33 = 208.33$$

Hence, Mean = 205 and Median = 208.33

Question 1

Write the median class of the following distribution:

Class	0-10	10-20	20-30	30-40	40-50	50-60	60-70
Frequency	4	4	8	10	12	8	4

Solution 1

Class interval	Frequency	Cumulative frequency
0 – 10	4	4
10 – 20	4	8
20 – 30	8	16
30 – 40	10	26
40 – 50	12	38
50 – 60	8	46
60 – 70	4	50

Here, $N = 50 \Rightarrow \frac{N}{2} = 25$

The cumulative frequency just greater than 25 is 26.

Hence, median dass is 30 – 40.

Question 2

What is the lower limit of the modal class of the following frequency distribution?

Age (in years)	0-10	10-20	20-30	30-40	40-50	50-60
Number of patients	16	13	6	11	27	18

Solution 2

Class having maximum frequency is the modal class.

Here, maximum frequency = 27

Hence, the modal class is 40 - 50.

Thus, the lower limit of the modal class is 40.

Question 3

The monthly pocket money of 50 students of a class are given in the following distribution:

Monthly pocket money (in Rs.)	0-50	50-100	100-150	150-200	200-250	250-300
Number of students	2	7	8	30	12	1

Find the modal class and also give class mark of the modal class.

Solution 3

Class having maximum frequency is the modal class.

Here, maximum frequency = 30

Hence, the modal class is 150 – 200

$$\therefore \text{ Class-mark of the modal class} = \frac{150+200}{2} = \frac{350}{2} = 175$$

Question 4

A data has 25 observations arranged in a descending order. Which observation represents the median?

Solution 4

Number of observations = n = 25 (odd)

$$\text{Hence, median} = \text{Value of } \left(\frac{n+1}{2}\right)^{th} \text{ observation}$$

$$= \text{Value of } \left(\frac{25+1}{2}\right)^{th} \text{ observation}$$

$$= \text{Value of } 13^{th} \text{ observation}$$

Question 5

For a cetain distribution, mode and median were found to be 1000 and 1250 respectively. Find mean for this distribution using an empirical relation.

Solution 5

Mode = 1000

Median = 1250

Empirical Relationship :

Mode = 3Median − 2Mean

$$\Rightarrow \text{Mean} = \frac{3\text{Median} - \text{Mode}}{2} = \frac{3 \times 1250 - 1000}{2} = \frac{3750 - 1000}{2} = \frac{2750}{2} = 1375$$

Question 6

In a class test, 50 students obtained marks as follows:

Marks obtained	0-20	20-40	40-60	60-80	80-100
Number of students	4	6	25	10	5

Find the modal class and the median class.

Solution 6

Class having maximum frequency is the modal class.

Here, maximum frequency = 25

Hence, the modal class is 40 − 60

Class interval	Frequency	Cumulative frequency
0 − 2	4	4
20 − 40	6	10
40 − 60	25	35
60 − 80	10	45
80 − 100	5	50

Here, $N = 50 \Rightarrow \frac{N}{2} = 25$

The cumulative frequency just greater than 25 is 35.

Hence, the median class is 40 − 60.

Question 7

Find the class marks of classes 10-25 and 35-55.

Solution 7

$$\text{Class-mark of dass } 10-25 = \frac{10+25}{2} = \frac{35}{2} = 17.5$$

$$\text{Class-mark of dass } 35-55 = \frac{35+55}{2} = \frac{90}{2} = 45$$

Question 8

While calculating the mean of a given data by the assumed-mean method, the following values were obtained:

$A = 25, \ \sum f_i d_i = 110, \sum f_i = 50.$

Find the mean.

Solution 8

$A = 25, \sum f_i = 50 \text{ and } \sum f_i d_i = 110$

$$\text{Mean} = A + \frac{\sum f_i d_i}{\sum f_i}$$

$$= 25 + \frac{110}{50}$$

$$= 25 + 2.2$$

$$= 27.2$$

Question 9

The distributions X and Y with total number of observations 36 and 64, and mean 4 and 3 respectively are combined. What is the mean of the resulting distribution X + Y?

Solution 9

For distribution X:

Total number of observations = 36

Mean = 4

$\Rightarrow$ Sum of observations = 36 x 4 = 144

For distribution Y:

Total number of observations = 64

Mean = 3

$\Rightarrow$ Sum of observations = 36 x 4 = 192

For distribution X + Y:

Total number of observations = 36 + 64 = 100

Sum of observations = 144 + 192 = 336

$$\Rightarrow \text{Mean} = \frac{336}{100} = 3.36$$

Question 10

In a frequency distribution table with 12 classes, the class-width is 2.5 and the lowest class boundary is 8.1, then what is the upper class boundary of the highest class?

Solution 10

Number of classes = 12

Class-width = 2.5

Lowest class boundary = 8.1

Thus, Upper class boundary of the highest class

= Lowest class boundary + Class-width x Number of classes

$= 8.1 + 2.5 \times 12$

$= 8.1 + 30$

$= 38.1$

Question 11

The observations 29, 32, 48, 50, x, x + 2, 72, 78, 84, 95 are arranged in ascending order. What is the value of x if the median of the data is 63?

Solution 11

Observations in ascending order:

29, 32, 48, 50, x, x + 2, 72, 78, 84, 95

$\Rightarrow$ Number of observations = N = 10 (even)

$$\Rightarrow \text{Median} = \frac{\text{Value of } \left(\frac{N}{2}\right)^{th} \text{ observation} + \text{Value of } \left(\frac{N}{2}+1\right)^{th} \text{ observation}}{2}$$

$$\Rightarrow 63 = \frac{\text{Value of } \left(\frac{10}{2}\right)^{th} \text{ observation} + \text{Value of } \left(\frac{10}{2}+1\right)^{th} \text{ observation}}{2}$$

$\Rightarrow 126 = \text{Value of } 5^{th} \text{ observation} + \text{Value of } 6^{th} \text{ observation}$

$\Rightarrow 126 = x + (x + 2)$

$\Rightarrow 126 = 2x + 2$

$\Rightarrow 2x = 124$

$\Rightarrow x = 62$

Question 12

The median of 19 observations is 30. Two more observations are made and the values of these are 8 and 32. Find the median of the 21 observations taken together.

Solution 12

Total number of observations = N = 19 (odd)

$\Rightarrow$ Median = Value of $\left(\frac{N+1}{2}\right)^{th}$ observation = Value of 10^{th} observation

Given, median = 30

$\Rightarrow 10^{th}$ observation is 30.

Now, two values 8 and 32 are added.

Since 8 < 30 and 32 > 30, each one of these two will go on

either side of median.

Hence, median is not affected.

$\Rightarrow$ Median = 30

Question 13

If the median of $\frac{x}{5}, \frac{x}{4}, \frac{x}{2}$, x and $\frac{x}{3}$, when x > 0, is 8,

find the value of x.

Solution 13

Data in ascending order:

$\frac{x}{5}, \frac{x}{4}, \frac{x}{3}, \frac{x}{2}, x$

$\Rightarrow$ Numberof observations = 5 (odd)

$\Rightarrow$ Median = Value of $\left(\frac{5+1}{2}\right)^{th}$ observation

$\Rightarrow 8$ = Value of 3^{rd} observation

$\Rightarrow 8 = \frac{x}{3}$

$\Rightarrow x = 24$

Question 14

What is the cumulative frequency of the modal class of the following distribution?

Class	3-6	6-9	9-12	12-15	15-18	18-21	21-24
Frequency	7	13	10	23	4	21	16

Solution 14

Class interval	Frequency	Cumulative frequency
3 – 6	7	7
6 – 9	13	20
9 – 12	10	30
12 – 15	23	53
15 – 18	4	57
18 – 21	21	78
21 – 24	16	94

Here, maximum frequency = 23

Hence, Modal class is 12 – 15.

Thus, the cumulative frequency of modal class is 53.

Question 15

Find the mode of the given data:

Class interval	0-20	20-40	40-60	60-80
Frequency	15	6	18	10

Solution 15

As the class 40 – 60 has the maximum frequency, so it is the modal class.

$\therefore\ x_k = 40,\ h = 20,\ f_k = 18,\ f_{k-1} = 6$ and $f_{k+1} = 10$

Now,

$$\text{Mode} = x_k + h\left\{\frac{(f_k - f_{k-1})}{(2f_k - f_{k-1} - f_{k+1})}\right\}$$

$$= 40 + 20\left\{\frac{18 - 6}{2(18) - 6 - 10}\right\}$$

$$= 40 + 20 \times \frac{12}{20}$$

$$= 40 + 12$$

$$= 52$$

HARMONIC MEAN

1. Find the harmonic mean of the following data {8, 9, 6, 11, 10, 5} ?

Solution:

Given data: {8, 9, 6, 11, 10, 5}

So Harmonic mean = 6 / 1/ 8+1/9+1/6+1/11+1/10+1/5

H =6/0.7936= 7.560

Harmonic mean(H) = 7.560

2. Calculate the harmonic mean for the following data:

x	1	3	5	7	9	11
f	2	4	6	8	10	12

Solution:

The calculation for the harmonic mean is shown in the below table:

x	f	1/x	f/x
1	2	1	2
3	4	0.333	1.332
5	6	0.2	1.2
7	8	0.143	1.144
9	10	0.1111	1.111

11	12	0.091	1.092
	N =42		Σ f/x = 7.879

The formula for weighted harmonic mean is

$HM_w = N / [(f_1/x_1) + (f_2/x_2) + (f_3/x_3) + \ldots.(f_n/x_n)]$

$HM_w = 42 / 7.879$

$HM_w = 5.331$

Therefore, the harmonic mean, HM_w is 5.331.

3.The table given below represent the frequency-distribution of ages for Standard 1st students.

Ages	4	5	6	7
Number of Students	10	6	8	12

Find the Harmonic Mean of the given class.

Solution:

Here the data given are distributed data. So the ages are the variables and the number of student is considered as the frequency.

Ages (x)	Number of Students (f)	f/x
4	10	2.5
5	6	1.2
6	8	1.33

7	12	0.58
Total	∑f=28	∑(fx)= 5.6

So Harmonic mean = ∑f / ∑(f/x)=285.6 = 5 years.

Therefore, Harmonic mean (H) = 5 years

Geometric Mean

1 : Find the geometric mean of the following data.

Weight of ear head x (g)	**Log x**
45	**1.653**
60	**1.778**
48	**1.681**
100	**2.000**
65	**1.813**
Total	**8.925**

Solution: Here n=5

GM = Antilog∑logx/n

= Antilog 8.925/5

= Antilog 1.785

= 60.95

Therefore the G.M of the given data is 60.95

2: Find the geometric mean of the following grouped data for the frequency distribution of weights.

Weights of ear heads (g)	**No of ear heads (f)**
60-80	**22**
80-100	**38**
100-120	**45**

120-140	**35**
140-160	**20**
Total	**160**

Solution:

Weights of ear heads (g)	**No of ear heads (f)**	**Mid x**	**Log x**	**f log x**
60-80	22	70	1.845	40.59
80-100	38	90	1.954	74.25
100-120	45	110	2.041	91.85
120-140	35	130	2.114	73.99
140-160	20	150	2.716	43.52
Total	**160**			**324.2**

From the given data, n = 160

We know that the G.M for the grouped data is

$$GM = \text{Antilog} \sum f\log x / n$$

GM = Antilog (324.2 /160)

GM = Antilog (2.02625)

GM = 106.23

Therefore, the G.M = 106.23