SEGNALI

SEGNALE: VARIAZIONE NEL TEMPO DI UNA GRANDEZZA FISICA ALLA QUALE È ASSOCIATA UNA INFORMAZIONE

- SEGNALI DETERMINISTICI: DESCRIVIBILI DA UN'EQUAZIONE
- SEGNALI CASUALI

- SEGNALI PERIODICI
- SEGNALI APERIODICI

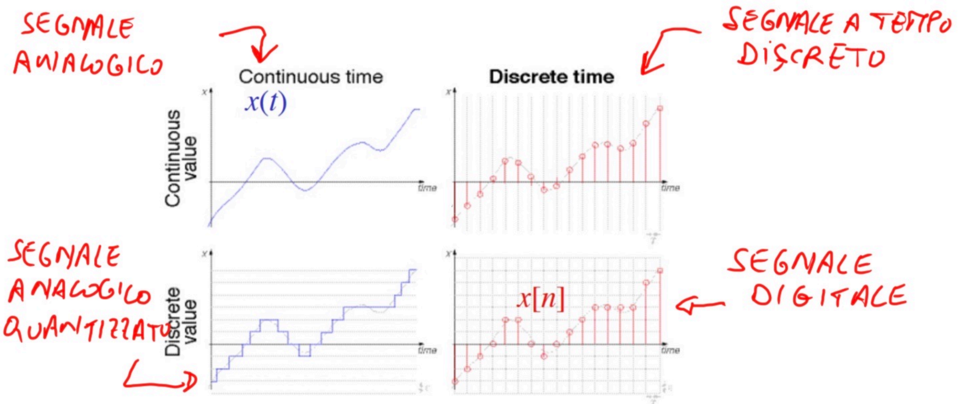
- SEGNALI REALI
- SEGNALI COMPLESSI (COMODI POICHÉ RAPPRESENTABILI SU PIANO

QUASI TUTTI I SEGNALI SONO A TEMPO CONTINUO E SONO IDICATI CON "$x(t)$", $t \in \mathbb{R}$

POI CI SONO I SEGNALI A TEMPO DISCRETO E SONO INDICATI CON "$x[n]$", $n \in \mathbb{Z}$

E SONO LE SUCCESSIONI

SERVE PERÒ DISCRETIZZARE I SEGNALI A TEMPO CONTINUO PER POTERLI STUDIARE.



Per comodità faremo di solito riferimento a segnali a TD

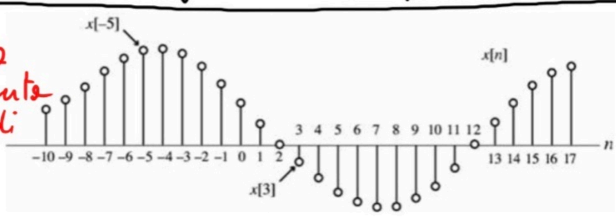
QUANTIZZARE È IN FUNZIONE DEL VALORE.

DISCRETIZZARE È IN FUNZIONE DEL TEMPO.

L'OPERAZIONE INVERSA È DETTA INTERPOLAZIONE

SEQUENZE <=> Segnali a Tempo Discreto

Le sequenze non è definita per valori di n non interi



n <=> Tempo Discreto

SOMMA / SOTTRAZIONE TRA SEQUENZE

$$y[n] = x_1[n] + x_2[n]$$

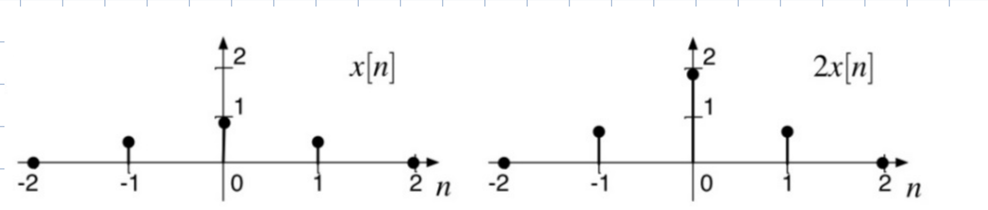
$$x_1 = \{1, \underset{\uparrow}{2}, 3\} \qquad x_1 = \{1, \underset{\uparrow}{2}, 3\} \quad +$$

$$x_2 = \{\underset{\uparrow}{0.5}, -1, 2\} \quad = \quad x_2 = \{0, \underset{\uparrow}{0.5}, -1, 2\} \quad =$$

$$1, 2.5, 2$$

MOLTIPLICARE PER UNA COSTANTE:

$$y[n] = A \cdot x[n]$$



SI PARLA DI AMPLIFICAZIONE

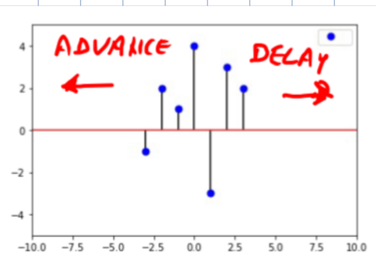
MOLTIPLICAZIONE TRA SEQUENZE:

$$y[n] = x_1[n] \cdot x_2[n]$$

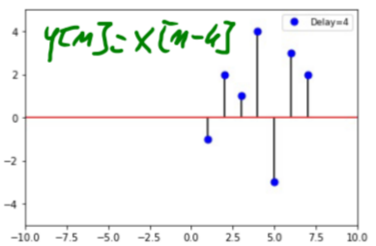
DETTA MODULAZIONE

SLITTAMENTO TEMPORALE

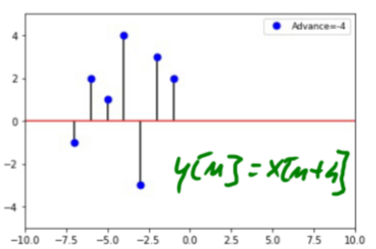
TIME SHIFT



$$y[n] = x[n-N]$$

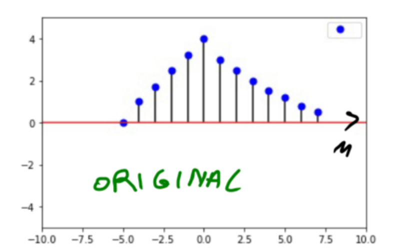


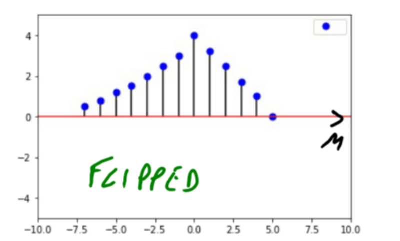
RITARDO $y[n] = x[n-N]$ SE $N > 0$



ANTICIPO $y[n] = x[n+N]$ SE $N < 0$

INVERSIONE TEMPORALE

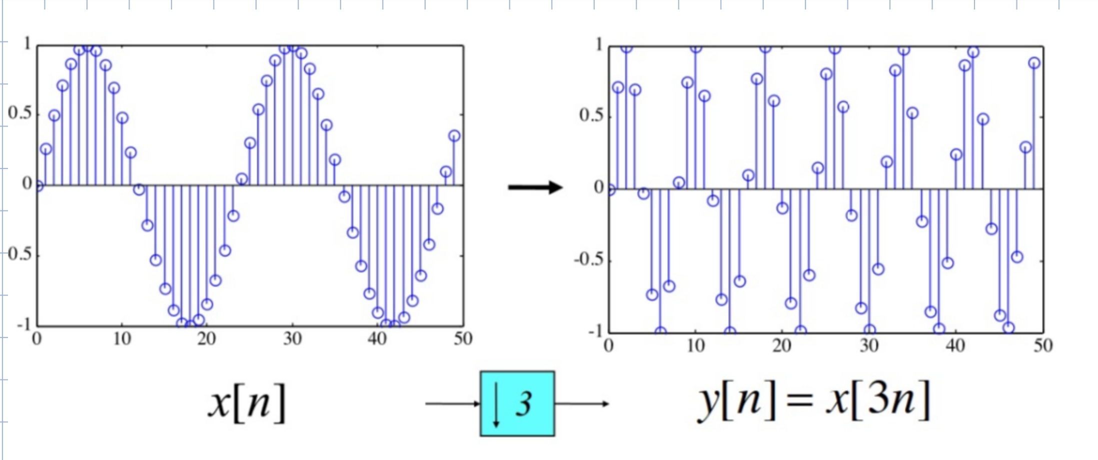




$$y[n] = x[-n]$$

SOTTOCAMPIONAMENTO (DOWNSAMPLING)

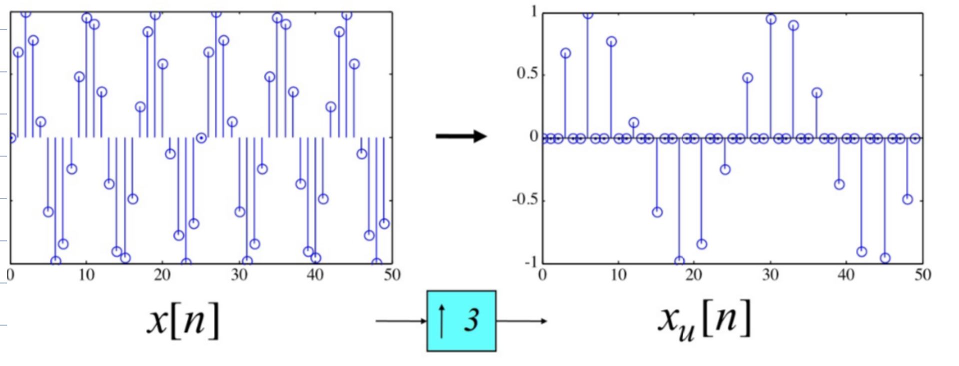
$$y[n] = x[Mn] \qquad M \in \mathbb{N}$$



UTILIZZATO PER LA COMPRESSIONE

SOVRACAMPIONAMENTO (UPSAMPLING)

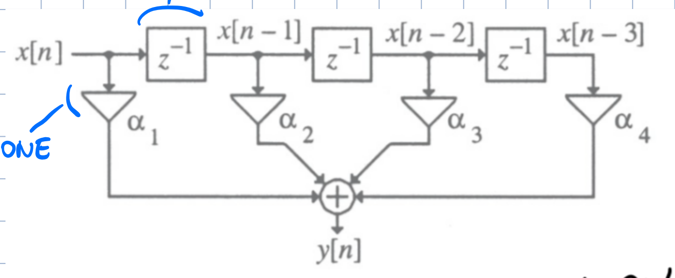
$$y[n] = x\left[\frac{n}{M}\right] \qquad M \in \mathbb{N}$$



QUESTE OPERAZIONI SI POSSONO COMBINARE

RITARDO UNITARIO

AMPLIFICAZIONE



$$y[n] = \alpha_1 \cdot x[n] + \alpha_2 \cdot x[n-1] + \alpha_3 \cdot x[n-2] + \alpha_4 \cdot x[n-3]$$

CLASSIFICAZIONE DELLE SEQUENZE

- DURATA

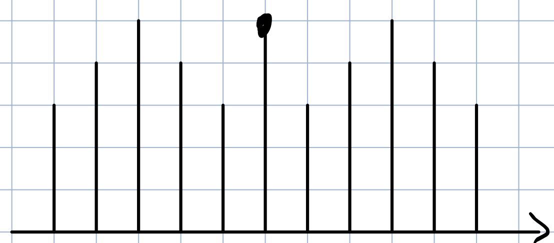
FINITI	INFINITI
$n_1 < n < n_2$	$[n_1, +\infty)$
	$(-\infty, n_1]$
	$(-\infty, +\infty)$

- **CAUSALITÀ**

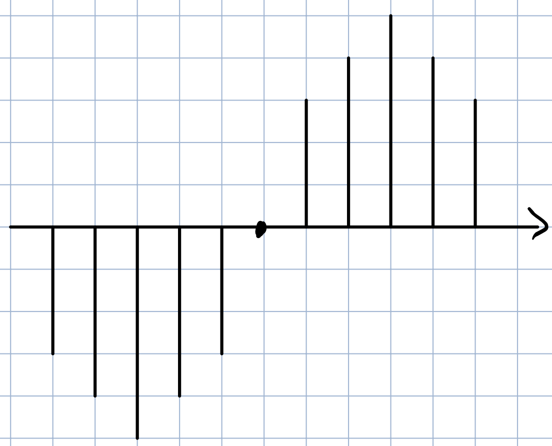
  - CAUSALE $x[n] = 0 \quad \forall \; n > 0$
  - ANTICAUSALE $x[n] = 0 \quad \forall \; n < 0$
  - BILATERE $x[n] = m \quad \forall \; n, m \in \mathbb{N}$

- **PARITÀ**

  - PARI $x[n] = x[-n]$



  - DISPARI $x[n] = -x[-n]$



OGNI SEGNALE REALE PUÒ ESSERE DIVISO IN UNA PARTE PARI E UNA DISPARI.

$$x[n] = \underset{\text{EVEN}}{x_e[n]} + \underset{\text{ODD}}{x_o[n]}$$

$$x_e[n] = \frac{1}{2}\left(x[n] + x[-n]\right)$$

$$x_o[n] = \frac{1}{2}\left(x[n] - x[-n]\right)$$

$$x_e[-n] = \frac{1}{2}\left(x[-n] + x[n]\right) = x_e[n]$$

$$-x_o[-n] = \frac{1}{2}\left(-x[-n] + x[n]\right) = x_o[n]$$

$$x_e[n] + x_o[n] = \frac{1}{2}\left(x[n] + x[-n]\right) + \frac{1}{2}\left(x[n] - x[-n]\right)$$

$$= \frac{2x[n]}{2} = x[n]$$

$$\sum_{n=-\infty}^{\infty} |x[n]| < \infty$$

ASSOLUTAMENTE SOMMABILE

$$\sum_{n=-\infty}^{+\infty} |x[n]|^2 < \infty$$

QUADRATICAMENTE SOMMABILE

$$|x[n]| \leq B \qquad B \in \mathbb{R} \quad \forall n \in \mathbb{N}$$

SEQUENZA LIMITATA

ENERGIA DI UNA SEQUENZA

$$E = \sum_{n=-\infty}^{+\infty} |x[n]|^2$$

SE UN SEGNALE È FINITO, L'ENERGIA È FINITA.

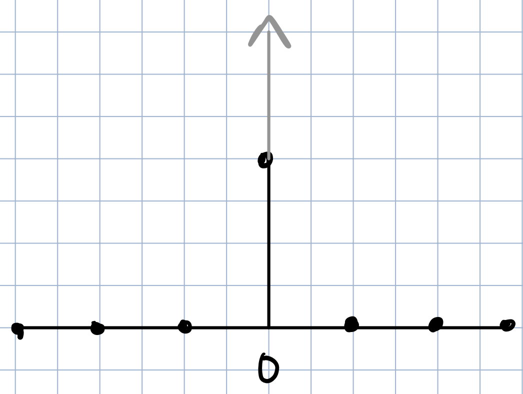
$$P = \lim_{N \to +\infty} \frac{1}{2N+1} \cdot \sum_{n=-N}^{N} |x[n]|^2$$

SE $\begin{cases} E \text{ finita} \\ P = 0 \end{cases}$ $\begin{cases} E = \infty \\ P \neq 0 \end{cases}$

SEQUENZE ELEMENTARI

IMPULSO UNITARIO ($\delta[n]$)

$$\delta[n] = \begin{cases} 1 & n = 0 \\ 0 & \forall n \neq 0 \end{cases}$$

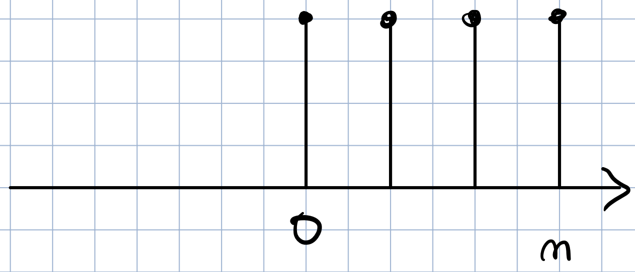


$$\delta[n-k] = \begin{cases} 1 & n = k \\ 0 & n \neq k \end{cases}$$

$$\boxed{x[n] = \sum_{k=-\infty}^{\infty} x[k]\, \delta[n-k]}$$

GRADINO UNITARIO

$$u[n] = \begin{cases} 1 & n \geq 0 \\ 0 & n < 0 \end{cases}$$

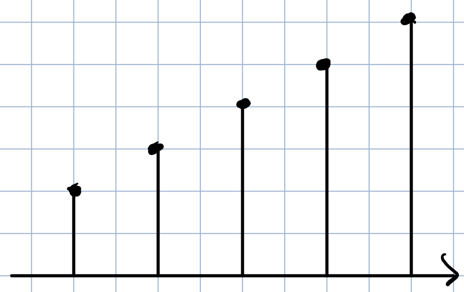


$$u[n] = \sum_{k=0}^{+\infty} \delta[n-k]$$ INTEGRALE NEL DISCRETO

$$\delta[n] = u[n] - u[n-1]$$ DERIVATA NEL DISCRETO

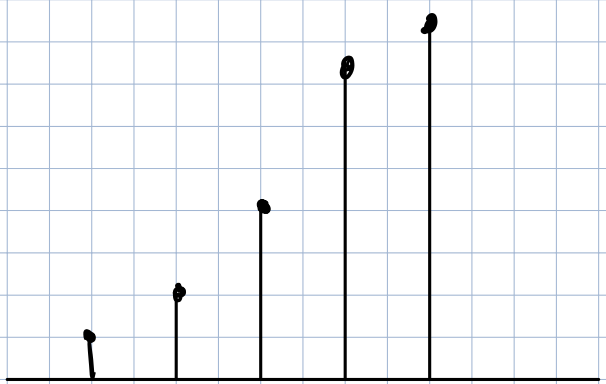
SEQUENZA A RAMPA

$$x[n] = n \cdot u[n] = \begin{cases} 0 & n < 0 \\ n & n \geq 0 \end{cases}$$



SEQUENZA ESPONENZIALE

$$x[n] = A\,\alpha^n$$



$|\alpha| > 1$

$|\alpha| < 1$

NUMERI COMPLESSI

$$z = x + Jy$$

$$z = |z| e^{J\theta} \qquad \rho = |z| = \sqrt{x^2 + y^2}$$

$$\theta = \begin{cases} \arccos\left(\frac{x}{|z|}\right) & y \geq 0 \\ -\arccos\left(\frac{x}{|z|}\right) & y < 0 \end{cases}$$

PERIODICITA'
PARI A π

$$x = \rho \cos\theta$$
$$y = \rho \sin\theta$$

$$\rho e^{i\theta} = \rho\left(\cos\theta + i\sin\theta\right)$$
$$e^{J\theta} = \cos\theta + i\sin\theta$$

$$\cos\theta = \frac{e^{J\theta} + e^{J\theta}}{2}$$

$$\sin\theta = \frac{e^{J\theta} - e^{-J\theta}}{2J}$$

① SOMMA NEI COMPLESSI

$$z_1 + z_2 = \left((x_1 + x_2), i(y_1 + y_2)\right)$$

② PRODOTTO

$$z_1 \cdot z_2 = (x_1 + Jy_1)(x_2 + Jy_2) = x_1 x_2 + Jx_1 y_2 + Jx_2 y_1 - y_1 y_2$$

$$= x_1 x_2 - y_1 y_2 + J(x_1 y_2 + x_2 y_1)$$

$$|z_1| e^{J\theta} \cdot |z_2| e^{J\theta} = |z_1||z_2| e^{J(\theta_1+\theta_2)}$$

③ CONIUGAZIONE

$z = x + Jy$ $\qquad$ $\bar{z} = x - Jy$

$$z \cdot \bar{z} = |z|^2$$

④ DIVISIONE

$$\frac{z_1}{z_2} = \frac{z_1 \cdot \bar{z}_2}{z_2 \cdot \bar{z}_2} = \frac{z_1 \cdot \bar{z}_2}{|z_2|^2}$$

⑤ POTENZA

$$z^n = |z|^n \cdot e^{J\theta n}$$

⑥ RADICI DELL'UNITA'

$$w^n = 1$$

INDUZIONE:

$n = 2$

$$w^2 = |w|^2 \cdot e^{j\theta 2} = 1 = 1 \cdot e^{j0}$$

$$|w|^2 = 1 \qquad 2\theta = 0 + 2k\pi \qquad k \in \mathbb{Z}$$

$$\theta = k\pi$$

k VA DA 0 A $n-1$

$$w^3 = 1 \qquad |w|^3 \cdot e^{j3\theta} = 1 \cdot e^{j0}$$

$$|w|^3 = 1 \qquad 3\theta = 2k\pi$$

$$\theta = \frac{2k\pi}{3}$$

$$w = \left\{ 1,\ e^{j\frac{2}{3}\pi},\ e^{j\frac{4}{3}\pi} \right\}$$

SINUSOIDE

$$x[n] = \underbrace{A}_{\text{AMPIEZZA}} \cos(\underbrace{\omega_0}_{\text{PULSAZIONE}} n + \underbrace{\phi}_{\text{FASE INIZIALE}})$$

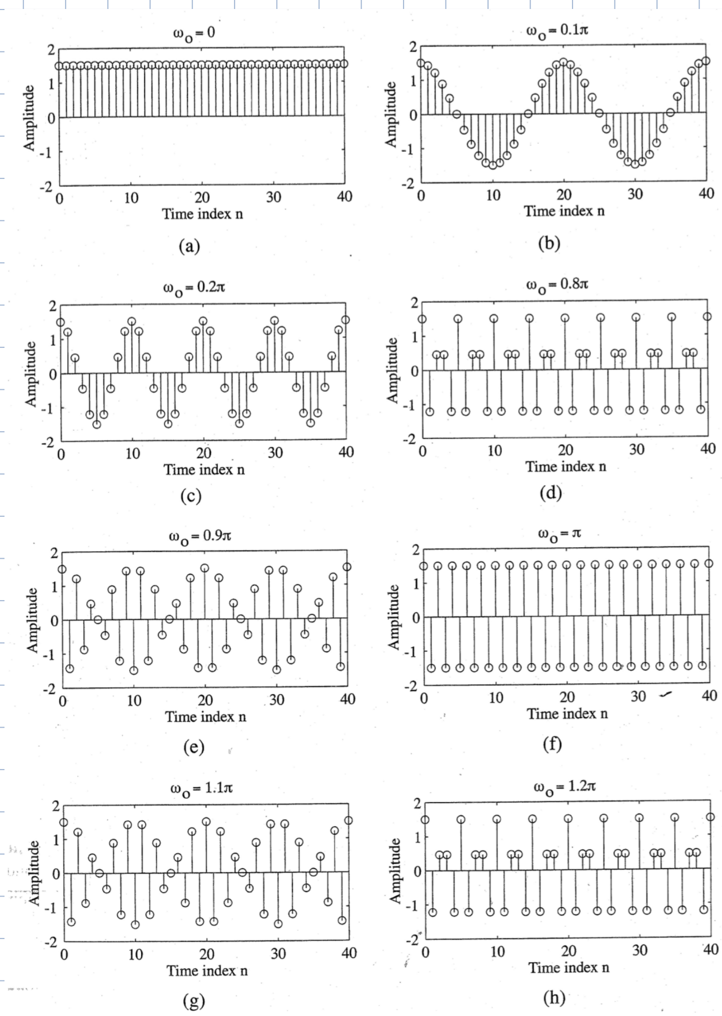
$$\frac{\omega}{2\pi} = f_0 \quad \text{FREQUENZA}$$

DUE SEQUENZE CHE DIFFERISCONO DI UN'INTERO SONO UGUALI:

$$A \cdot \cos[2\pi(\underbrace{f_0}_{\text{SEQUENZA}} + \underbrace{k}_{\text{INTERO}})n] = A \cos(2\pi \cdot f_0 \cdot n + 2\pi k \cdot n)$$

$$= A \cos(2\pi \cdot f_0 \cdot n) = x[n]$$

AL VARIARE DI ω_0 OTTENIAMO DIVERSE FORME:



$$x[n+N] = A\cos(\omega_0(n+N)) = A\cos(\omega_0 \cdot n + \omega_0 \cdot N)$$

$$= x[n]$$

$$\omega_0 \cdot N = 2\pi \cdot M$$

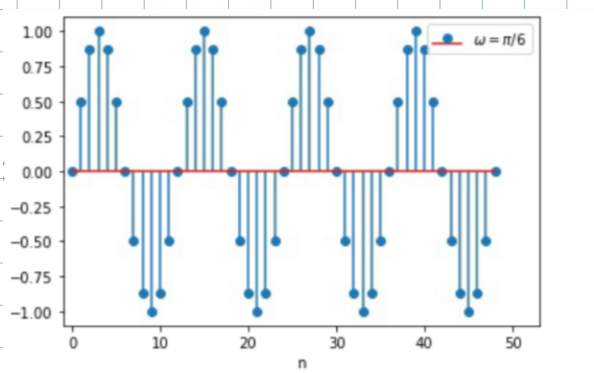
$$f_0 = \frac{\omega_0}{2\pi} = \frac{M}{N}$$

NUMERO RAZIONALE

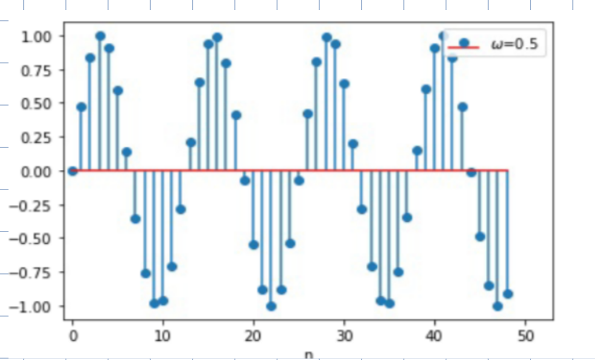
"

PERIODICA

SINUSOIDE PERIODICA



SINUSOIDE NON PERIODICA



SEQUENZA COMPLESSA

$$x[n] = \underbrace{A \; \alpha}_{\text{NUMERI COMPLESSI}}^{n}$$

$$A = |A| \cdot e^{J\phi}$$

$$\alpha = e^{(\sigma + J\omega_0)}$$

σ SERVE A DECIDERE IL CAMBIAMENTO NEL TEMPO

DETERMINA L'AMPIEZZA

$$x[n] = |A| \underbrace{e^{\sigma n}}_{\text{SCALA IN BASE AL TEMPO}} \cdot \underbrace{e^{J(n \cdot \omega_0 + \phi)}}_{\text{DETERMINA LA ROTAZIONE}}$$

$$x[n] = |A| e^{\sigma n} \cdot \left[\cos(\omega_0 \cdot n + \phi) + J \sin(\omega_0 n + \phi)\right]$$

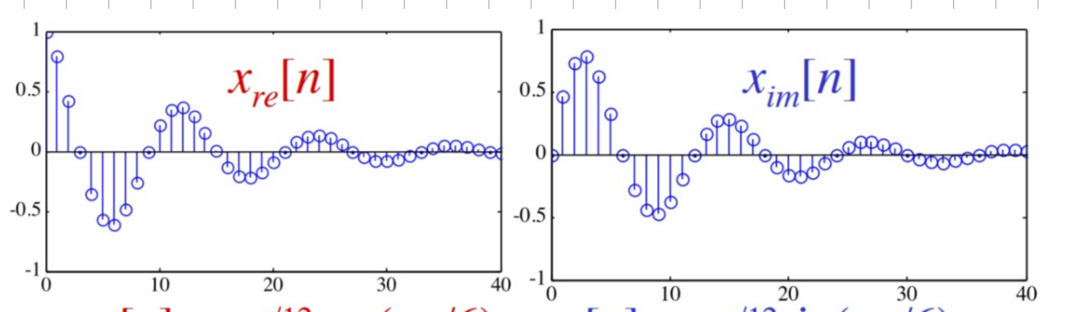
(EQUIVALENTE)

1

$$x[n] = e^{-\frac{1}{12}} \cdot e^{J\frac{\pi}{6} n}$$

$A = 1$ $\quad \omega_0 = \frac{\pi}{6}$

$\phi = 0$ $\quad \sigma = -\frac{1}{12}$

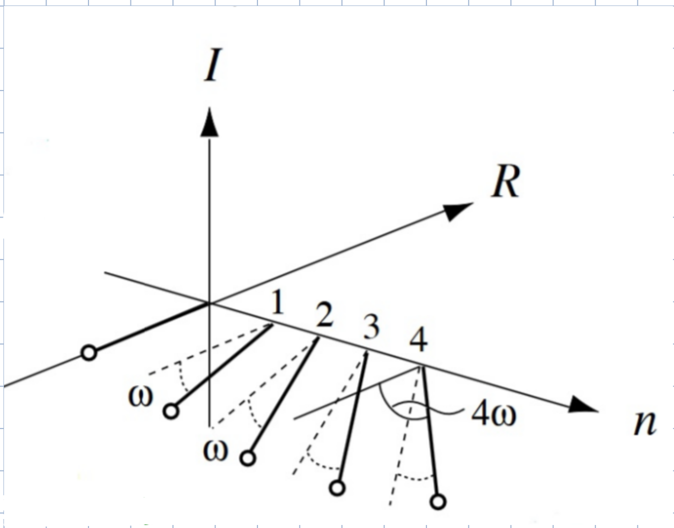


$x_{re}[n] = e^{-n/12}\cos(\pi n/6)$ $\quad x_{im}[n] = e^{-n/12}\sin(\pi n/6)$

σ FA ABBASSARE IL VALORE AL CRESCERE DEL TEMPO

2

$$x[n] = A \cdot e^{j(\omega_0 \cdot n)}$$



LA VELOCITÀ DELLA ROTAZIONE È DETERMINATA DA ω_0

10/10/22

1

2) Determinare le componenti pari e dispari della sequenza (il campione sottolineato è nell'origine):

$$x[n] = \{2, -2, \underline{4}, 1, -3\}$$

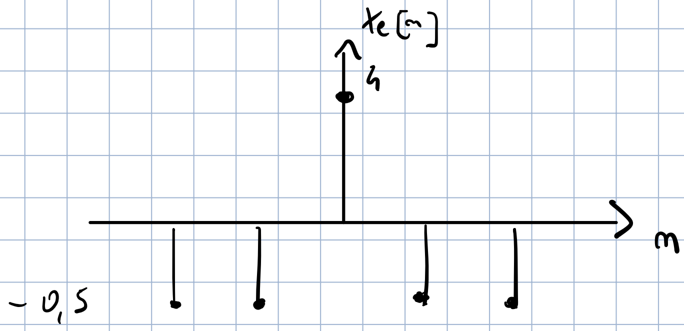
[$x_e[n] = \{-0.5, -0.5, \underline{4}, -0.5, -0.5\}$; $x_o[n] = \{2.5, -1.5, \underline{0}, 1.5, -2.5\}$]

PARI: $x_e[n] = \frac{1}{2}\left(x[n] + x[-n]\right)$

$x[-n] = \{-3, 1, 4, -2, 2\}$

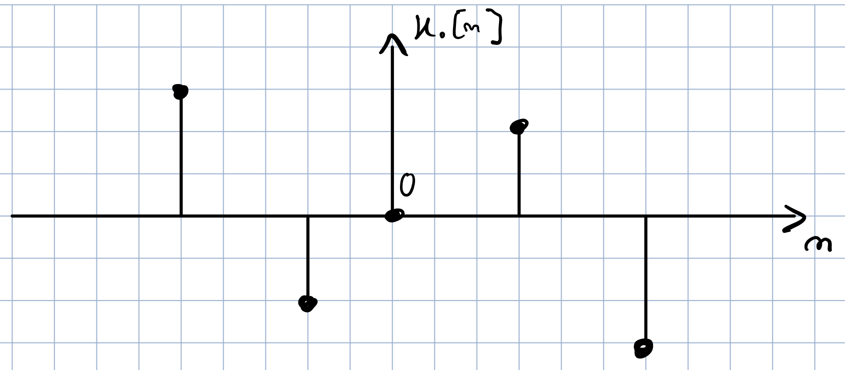
$x_e[n] = \frac{1}{2}\left(-1, -1, 8, -1, -1\right) =$

$= \left(-0.5, -0.5, 4, -0.5, -0.5\right)$



DISPARI: $x_o[n] = \frac{1}{2}\left\{x[n] - x[-n]\right\} =$

$= \frac{1}{2}\left\{5, -3, 0, 3, -5\right\} = \left\{2.5, -1.5, 0, 1.5, -2.5\right\}$



SI POTREBBE CALCOLARE L'ENERGIA:

$$E = \sum_{n=-\infty}^{\infty} |x[n]|^2$$

MA SERVE PRIMA FARE UN RIPASSO.

$$\sum_{n=0}^{\infty} q^n = \frac{1}{1-q} \quad \text{SE} \quad |q|<1$$

$$S = \sum_{n=0}^{N} q^n = 1 + q + q^2 + \dots q^N$$

$$S \cdot q = q + q^2 + q^3 \dots q^{N+1}$$

$$S - Sq = S(1-q) = 1 + q - q + \dots q^N - q^N + q^{N+1} = 1 + q^{N+1}$$

$$S = \frac{1-q^{N+1}}{1-q} \xrightarrow[N \to +\infty]{} \frac{1}{1-q}$$

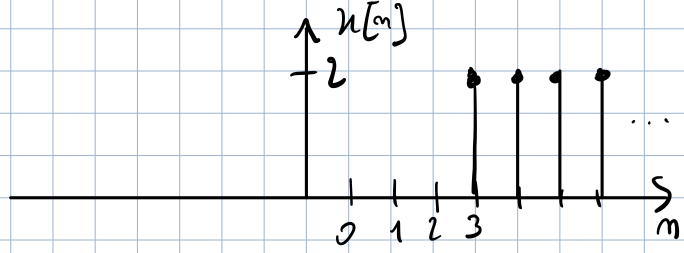
SE n PARTE DA 1:

$$\sum_{n=1}^{\infty} q^n = \sum_{n=0}^{\infty} q^n - q^0 = \frac{1}{1-q} - 1$$

TORNANDO ALL' ENERGIA:

2 CALCOLARE ENERGIA E POTENZA MEDIA DELLA SEQUENZA:

$$x[n] = 2u[n-3]$$



$$E = \sum_{n=-\infty}^{\infty} |x[n]|^2 = \sum_{n=3}^{\infty} |2u[n-3]|^2 = 4\sum_{n=3}^{\infty} 1 = \infty$$

NON È UN SEGNALE DI ENERGIA

$$P = \lim_{N\to\infty} \frac{1}{2N+1} \cdot \sum_{-N}^{N} |2u[n-3]|^2 =$$

$$= \lim_{N\to\infty} \frac{1}{2N+1} \cdot 4\sum_{3}^{N} 1 = \lim_{N\to+\infty} \frac{4(N-3)}{2N+1} =$$

$$= \lim_{N\to+\infty} \frac{4N-12}{2N+1} = \frac{\frac{1}{N}\left(4-\frac{3}{N}\right)}{\frac{1}{N}\left(2-\frac{1}{N}\right)} = \frac{4}{2} = 2$$

SEGNALE DI POTENZA

3 CALCOLARE ENERGIA E POTENZA MEDIA DELLA SEQUENZA:

$$x[n] = \left(\frac{1}{4}\right)^n \cdot u[n]$$

$$E = \sum_{-\infty}^{\infty} |x[n]|^2 = \sum_{n=0}^{\infty} \left|\left(\frac{1}{4}\right)^n\right|^2 = \sum_{n=0}^{\infty} \left(\frac{1}{16}\right)^n = \frac{1}{1-\frac{1}{16}} = \frac{16}{15}$$

$$P = \lim_{N\to+\infty} \frac{1}{2N+1} \cdot \sum_{n=0}^{\infty} \left(\frac{1}{16}\right)^n = \lim_{N\to+\infty} \underbrace{\frac{1}{2N+1}}_{\downarrow\, 0} \cdot \underbrace{\frac{1-\left(\frac{1}{16}\right)^{N+1}}{1-\frac{1}{16}}}_{\downarrow\, 0} = 0$$

4 CALCOLARE ENERGIA E POTENZA MEDIA DELLA SEQUENZA:

$$x[n] = \begin{cases} \left(\frac{1}{3}\right)^n & n \geq 0 \quad E_1 \quad P_1 \\ (4)^n & n \leq 0 \quad E_2 \quad P_2 \end{cases}$$

$$E_1 = \sum_{n=0}^{\infty} \left|\left(\frac{1}{3}\right)^n\right|^2 = \frac{1}{1-\frac{1}{9}} = \frac{1}{8}$$

$$E_2 = \sum_{n=-\infty}^{0} |4^n|^2 = \sum_{n=-\infty}^{0} |16|^n$$

PONIAMO $m = -n$

$$= \sum_{m=0}^{\infty} \left(\frac{1}{16}\right)^m = \frac{1}{1-\frac{1}{16}} = \frac{15}{16}$$

$$E = E_1 + E_2 = \frac{1}{8} + \frac{15}{16} = \frac{143}{120}$$

$$P = \lim_{N \to +\infty} \frac{1}{2N+1} \sum_{n=0}^{\infty} \frac{1-\left(\frac{1}{3}\right)^{N+1}}{1-\frac{1}{3}} + \lim_{N \to +\infty} \frac{1}{2N+1} \sum_{n=0}^{\infty} \frac{1-\left(\frac{1}{16}\right)^{N+1}}{1-\frac{1}{16}} =$$

$$= 0$$

SISTEMI A TEMPO DISCRETO

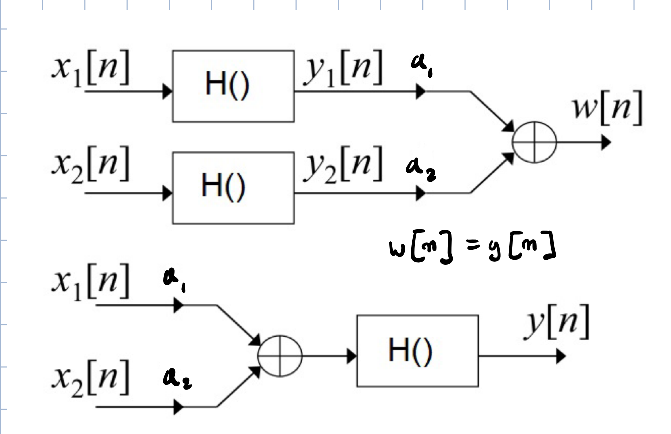
$$x[n] \to \boxed{H(\,)} \to y[n]$$

$$y[n] = H(x[n])$$

CLASSIFICAZIONE E PROPRIETA':

① LINEARITA'

$$y_1[n] = H(x_1[n])$$
$$y_2[n] = H(x_2[n])$$



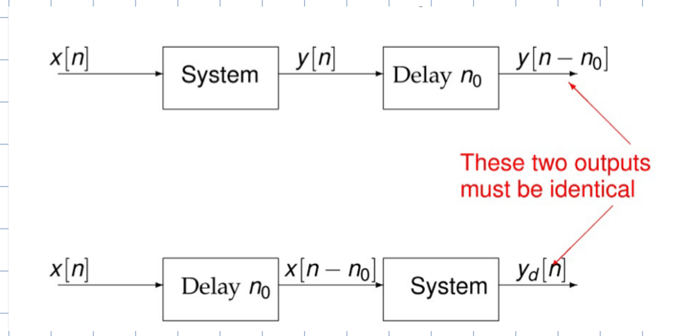
$$H(a_1 x_1[n] + a_2 x_2[n]) = a_1 y_1[n] + a_2 y_2[n]$$

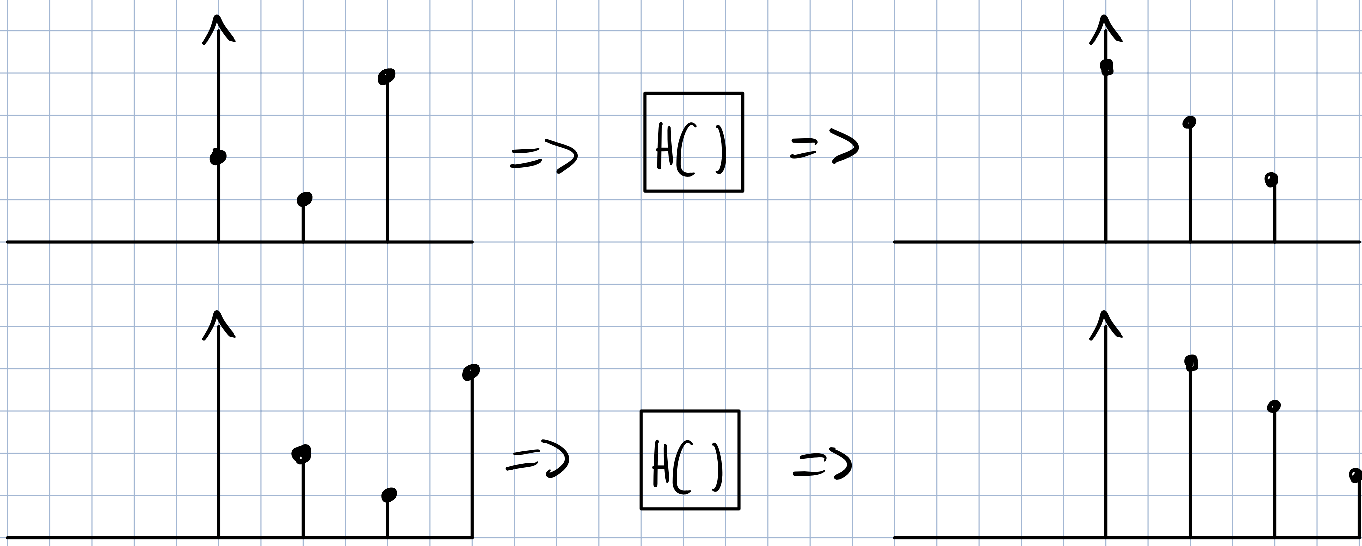
② INVARIANZA TEMPORALE

IL SISTEMA SI COMPORTA ALLO STESSO MODO AL VARIARE NEL TEMPO

$$H(x[n]) = y[n]$$

$$H(x[n-n_0]) = y[n-n_0]$$





③ CAUSALITÀ

SE I CAMPIONI UTILIZZATI SONO SOLO PRESENTI O PASSATI.

④ STABILITÀ BIBO (BOUNDED INPUT BOUNDED OUTPUT)

SE FORNIAMO AL SISTEMA INPUT LIMITATO E L'OUTPUT RIMANE LIMITATO.

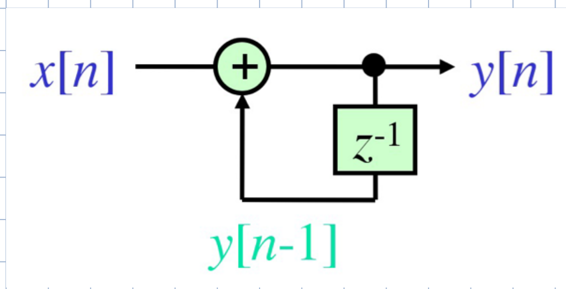
$$|x[n]| < B_x \quad \forall n \Rightarrow |y[n]| < B_y$$

$$B_x, B_y < \infty$$

È UN SISTEMA/FILTRO

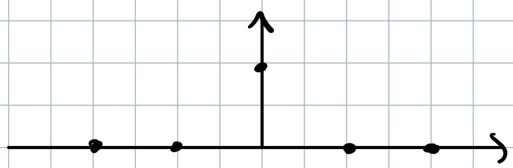
ACCUMULATORE

$$y[n] = \sum_{\ell=-\infty}^{n} x[\ell]$$



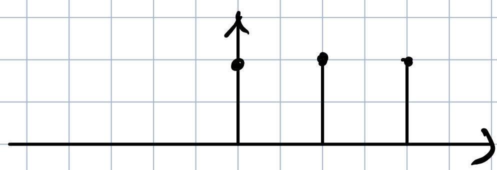
$$y[n] = \sum_{\ell=-\infty}^{n-1} x[\ell] + x[n] = y[n-1] + x[n]$$

$$x[n] = \delta[n]$$



INPUT

OUTPUT $\quad y[n] = u[n]$



VERIFICHIAMO LA LINEARITÀ DELL'ACCUMULATORE:

$$y_1[n] = \sum_{\ell=-\infty}^{n} x_1[\ell] \qquad y_2[n] = \sum_{\ell=-\infty}^{n} x_2[\ell]$$

$$\sum_{\ell=-\infty}^{n} x_1[\ell] + \sum_{\ell=-\infty}^{n} x_2[\ell] = w[n]$$

$$y[n] = \sum_{\ell=-\infty}^{n} \left(x_1[\ell] + x_2[\ell] \right) = \sum_{\ell=-\infty}^{n} x_1[\ell] + \sum_{\ell=-\infty}^{n} x_2[\ell]$$

$y[n] = w[n] \implies$ È UN SISTEMA LINEARE

STUDIAMO L' INVARIANZA TEMPORALE

$$y[n] = \sum_{l=-\infty}^{n-n_0} x[l]$$

$k = l - n_0$

$$y_d[n] = \sum_{l=-\infty}^{n} x[l-n_0] = \sum_{k=-\infty}^{n-n_0} x[k] = y[n-n_0]$$

$y[n] = y_d[n] \Rightarrow$ É INVARIANTE

VERIFICHIAMO CAUSALITA'

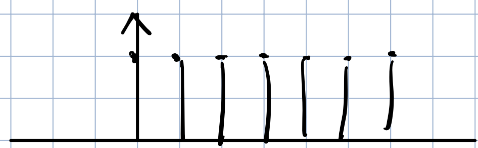
$$y[n] = \sum_{l=-\infty}^{n} x[l]$$

SI USA PRESENTE E PASSATO $\Rightarrow$ CAUSALE

STABILITA' BIBO

NON É STABILE ED É EVIDENTE CON SEGNALI COME

$x[n] = u[n]$



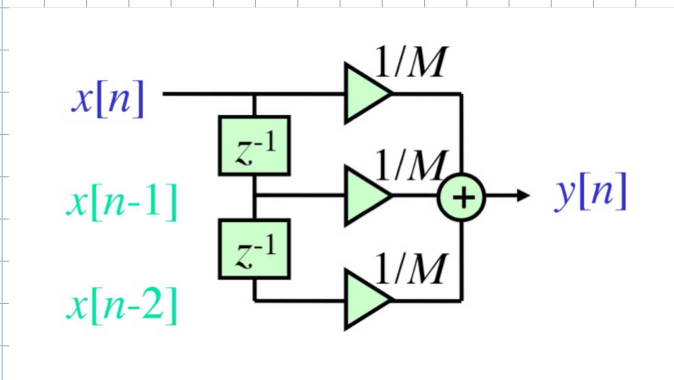
$$y[n] = \sum_{l=-\infty}^{n} u[l] = \begin{cases} 0 & n < 0 \\ n+1 & n \geq 0 \end{cases}$$

L'OUTPUT DIVERGE

È UN SISTEMA

FILTRO MEDIA MOBILE (MOVING AVARAGE FILTER)

$$y[n] = \frac{1}{M} \sum_{l=0}^{M-1} x[n-l]$$



È LINEARE?

$$y_1[n] = \frac{1}{M} \sum_{l=0}^{M-1} x_1[n-l]$$

$$y_2[n] = \frac{1}{M} \sum_{l=0}^{M-1} x_2[n-l]$$

$$y[n] = y_1[n] + y_2[n] =$$

$$= \frac{1}{M} \left(\sum_{l=0}^{M-1} x_1[n-l] + \sum_{l=0}^{M-1} x_2[n-l] \right)$$

$$w[n] = \frac{1}{M} \sum_{l=0}^{M-1} \left(x_1[n-l] + x_2[n-l] \right)$$

$$= \frac{1}{M} \left(\sum_{l=0}^{M-1} x_1[n-l] + \sum_{l=0}^{M-1} x_2[n-l] \right)$$

$y[n] = w[n] \implies$ È LINEARE

INVARIANZA TEMPORALE

$$y[n-n_0] = \frac{1}{M}\sum_{l=0}^{M-1} x[n-l]$$

$\Rightarrow$ È INVARIANTE

$$y_d[n] = \frac{1}{M}\sum_{l=0}^{M-1} x[n-l-n_0]$$

LA VERSIONE DEL FILTRO $y(n) = \frac{1}{M}\sum_{-\left(\frac{M-1}{2}\right)}^{\left(\frac{M-1}{2}\right)} x[n+l]$ NON È CAUSALE

VERIFICHIAMO CAUSALITA'

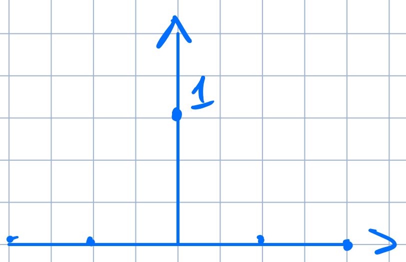
$$y[n] = \frac{1}{M}\sum_{l=0}^{M-1} x[n-l]$$ SI USA PRESENTE E PASSATO

$\Rightarrow$ CAUSALE

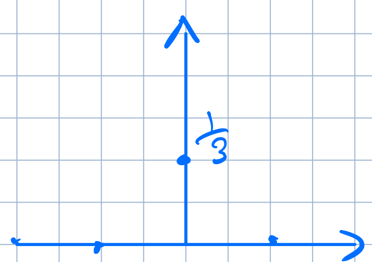
STABILITA' BIBO ?

È CHIARAMENTE STABILE PERCHÉ SI TRATTA DI UNA MEDIA.

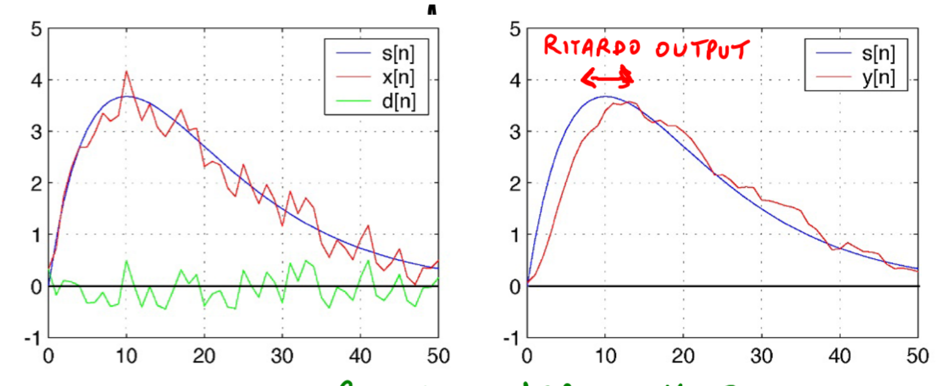
$M = 3$ $\quad d[n] = x[n]$



$\Rightarrow$

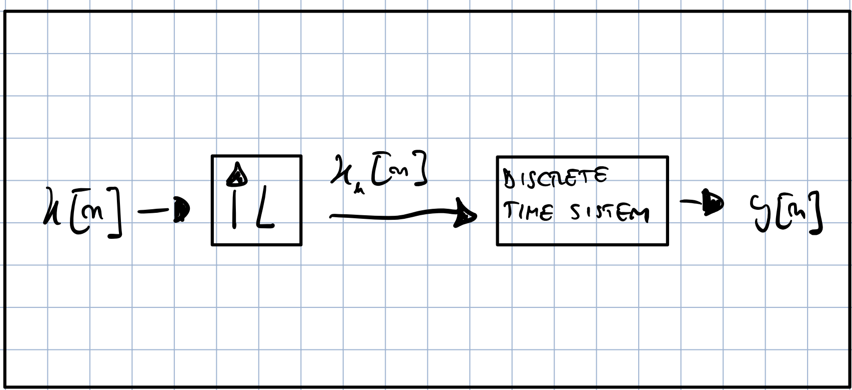


ESEMPIO DI APPLICAZIONE



INTERPOLATORE LINEARE

$$y[n] = x_u[n] + \frac{1}{2}\left(x_u[n-1] + x_u[n+1]\right)$$



USA LA SOMMA QUINDI É STABILE

É NON CAUSALE

É STABILE PERCHÉ FA LA MEDIA TRA DUE SEGNALI

NON É INVARIANTE PERCHÉ:

$$\boxed{y[n] = \begin{cases} x\left[\frac{n}{L}\right] \\ 0 \end{cases}}$$ UPSAMPLING

$n = m \cdot L \quad , \quad m = 0, \pm 1, \pm 2 \ldots$

$$y[n-n_0] = \begin{cases} x\left[\frac{n-n_0}{L}\right] \\ 0 \end{cases}$$

$$y_d[n] = \begin{cases} x\left[\frac{n}{L} - n_0\right] \\ 0 \end{cases}$$

$$y[n-n_0] \neq y_d[n]$$

LA CHIEDE ALL'ESAME

SISTEMI LTI LINEARI A TEMPO INVARIANTE

IL SISTEMA VIENE DESCRITTO DALLA RISPOSTA IMPULSIVA

$$\delta[n] \to \boxed{LTI} \to h[n]$$

SAPPIAMO CHE $x[n] = \sum_{m=-\infty}^{\infty} x[m] \cdot \delta[n-m]$

APPLICANDO L' INVARIANZA TEMPORALE:

$$\delta[n-m] \to \boxed{LTI} \to h[n-m]$$

APPLICHIAMO LA LINEARITA'

$$x[m]\,\delta[n-m] \to \boxed{LTI} \to x[m]\,h[n-m]$$

$$\sum_{m=-\infty}^{\infty} x[m]\,\delta[n-m] \to \boxed{LTI} \to \sum_{m=-\infty}^{\infty} x[m]\,h[n-m]$$

È DETTA SOMMA DI CONVOLUZIONE

$$\boxed{y[n] = x[n] * h[n]}$$

I SISTEMI VENGONO CLASSIFICATI IN BASE ALLA RISPOSTA IMPULSIVA:

- FIR FINITA
- IIR INFINITA

NEGLI LTI:

① CAUSALITA'

$$h[n] \Rightarrow h[n] = 0 \quad \forall n < 0$$

$$y[n] = \sum_{m=-\infty}^{\infty} h[m]x[n-m] = \cancel{\sum_{m=-\infty}^{-1} h[m]x[n-m]}$$

RIMANE SOLO NELLA PARTE CAUSALE DELLA SEQ.

$$= \sum_{m=0}^{\infty} h[m]x[n-m]$$

② STABILITA'

$$\sum_{m=-\infty}^{\infty} |h[n]| < \infty$$

EQUAZIONE DELLA DIFFERENZA FINITA

$$\sum_{k=0}^{N} a_k \cdot y[n-k] = \sum_{m=0}^{M} b_m \cdot x[n-m] \qquad \forall n > 0$$

$$\boxed{y[n] = \sum_{m=0}^{M} b_m \cdot x[n-m] - \sum_{k=0}^{N} a_k \cdot y[n-k]} \qquad \text{IIR}$$

$$\boxed{y[n] = \sum_{m=0}^{M} b_m \, x[n-m]} \qquad \text{FIR}$$

$$h[n] = \sum_{m=0}^{M} b_m \, x[n-m] = \underline{\{b_0, b_1, b_2 \dots b_M\}}$$

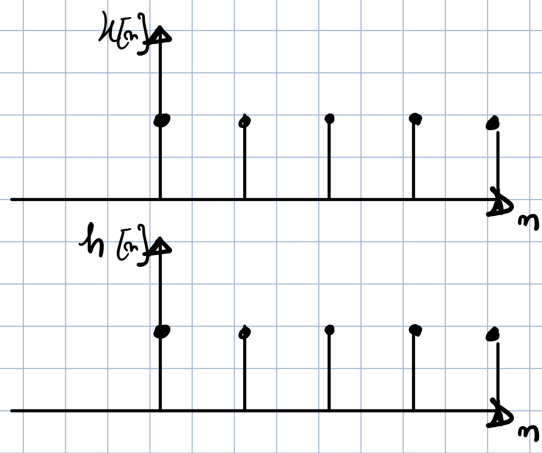
QUESTO È PALESEMENTE SOMMABILE

STABILE => ASSOLUTAMENTE SOMMABILI

CONVOLUZIONE

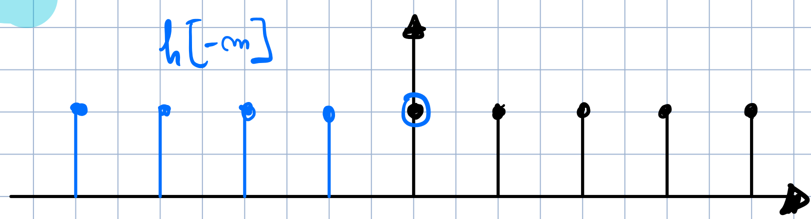
$x[n] = \{ \underset{\uparrow}{1}, 1, 1, 1, 1 \}$

$h[n] = \{ \underset{\uparrow}{1}, 1, 1, 1, 1 \}$



$$y[n] = x[n] * h[n] = \sum_{m=-\infty}^{\infty} x[m] \cdot h[n-m]$$

METODO GRAFICO



$y[0] = 1 \cdot 1 = 1$

$y[1] = 1 \cdot 1 + 1 \cdot 1 = 2$

$y[2] = 1 \cdot 1 + 1 \cdot 1 + 1 \cdot 1 = 3$

$y[3] = 4$

$y[4] = 5$

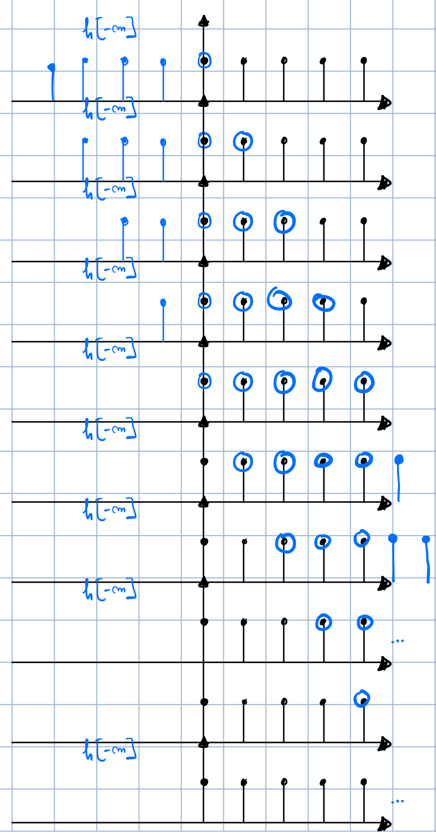
$y[5] = 4$

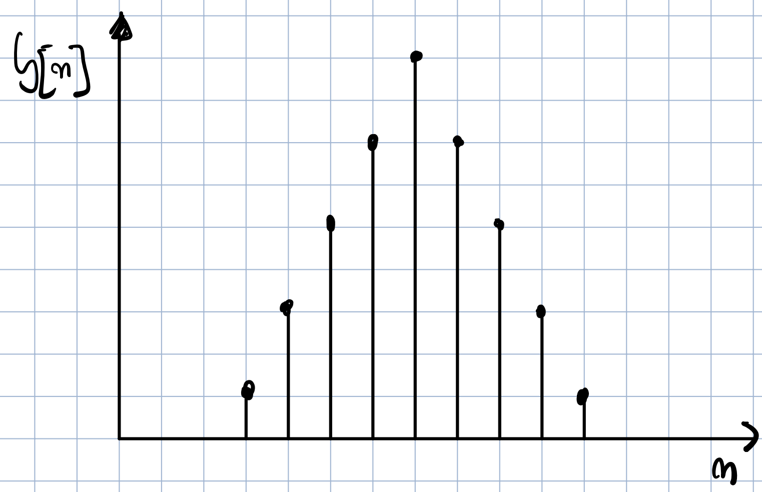
$y[6] = 3$

$y[7] = 2$

$y[8] = 1$

$y[9] = 0$





METODO TABELLARE

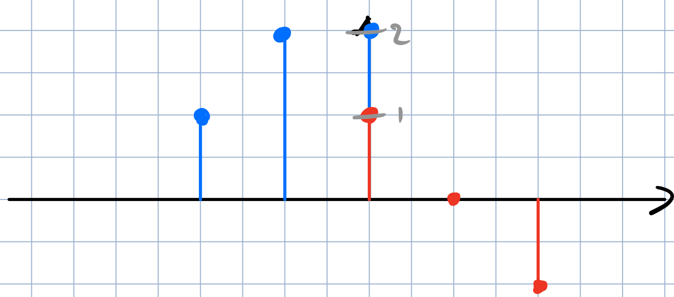
h[n] \ x[n]	1	1	1	1	1
1	1	1	1	1	1
1	1	1	1	1	1
1	1	1	1	1	1
1	1	1	1	1	1
1	1	1	1	1	1

$$y[n] = \{1,2,3,4,5,4,3,2,1\}$$

ESERCIZIO: CONVOLUZIONE CON METODO GRAFICO E TABELLARE

$$x_1[n] = \{1,0,-1\}$$

$$x_2[n] = \{2,2,1\}$$



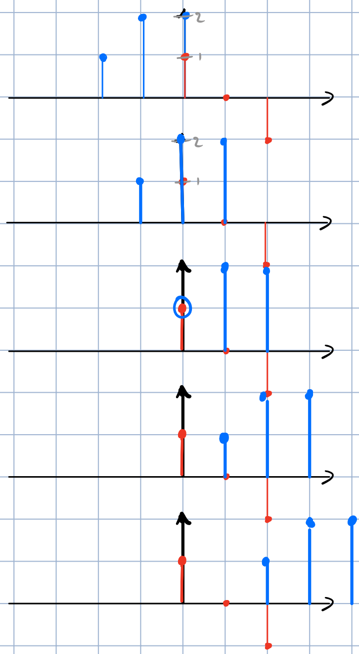
$y[0] = 1\cdot 2 = 2$

$y[1] = 1\cdot 2 + 0\cdot 2 = 2$

$y[2] = 1\cdot 1 + 0\cdot 2 + -1\cdot 2 = -1$

$y[3] = 0\cdot 1 + 2\cdot -1 = -2$

$y[4] = -1\cdot 1 = -1$



	2	2	1
1	2	2	1
0	0	0	0
-1	-2	-2	-1

$y[n] = \{2, 2, -1, -2, -1\}$

$y[n] = \{2, 2, -1, -2, -1\}$

24/10/22

(1)

$y[n] = x[2n] + x[n-1]$

• NON CAUSALE ($x[2n]$)

LINEARITA':

$y_1[n] = x_1[2n] + x_1[n-1]$

$y_2[n] = x_2[2n] + x_2[n-1]$

$w[n] = x_1[2n] + x_1[n-1] + x_2[2n] + x_2[n-1]$

$x[n] = x_1[n] + x_2[n]$ PASSANDO A $y[n]$ $\Rightarrow$

$y[n] = x_1[2n] + x_1[n-1] + x_2[2n] + x_2[n-1]$

$y[n] = w[n]$

$\Rightarrow$ IL SISTEMA È LINEARE

- INVARIANZA TEMPORALE

RITARDO OUTPUT SOTTRAENDO AD n, n_0

$$y[n-n_0] = x[2n-n_0] + x[n-1-n_0]$$

RITARDO INPUT SOSTITUENDO $n-n_0$ a n.

$$y_d[n] = x[2n-2n_0] + x[n-n_0-1]$$

$\Rightarrow$ NON È INVARIANTE TEMPORALMENTE

- STABILE BIBO

È UNA SOMMA QUINDI È STABILE BIBO

(2) $y[n] = n \cdot x^2[n]$

- NON LINEARE
($x^2[n]$)

$$y_1[n] = n \cdot x_1^2[n]$$
$$y_2[n] = n \cdot x_2^2[n]$$

$\rightarrow \oplus \rightarrow w[n] = n\left(x_1^2[n] + x_2^2[n]\right)$

$$y[n] = n\left(x_1[n] + x_2[n]\right)^2 = n\left(x_1^2[n] + 2x_1[n]x_2[n] + x_2^2[n]\right)$$

$$y[n] \neq w[n]$$

- INVARIANZA TEMPORALE

$$y[n-n_0] = (n-n_0) \cdot x^2[n-n_0]$$

$$y_d[n] = n \cdot x^2[n-n_0]$$

SONO DIVERSI QUINDI $\Rightarrow$ NON È A TEMPO INVARIANTE

- CAUSALE PERCHÉ USA SOLO CAMPIONI PRESENTI

- STABILE BIBO

ESSENDO MOLTIPLICATO PER n, SE $n \to +\infty$ L'OUTPUT DIVERGE

(3) $y[n] = 3 \cdot x[n] + 5$

- LINEARITÀ

$y_1[n] = 3 \cdot x_1[n] + 5$
$y_2[n] = 3 \cdot x_2[n] + 5$ $\xrightarrow{\oplus}$ $w[n] = 3\left(x_1[n] + x_2[n] + 10\right)$

$y[n] = 3\left(x_1[n] + x_2[n]\right) + 5$

$y[n] \neq w[n]$ $\Rightarrow$ NON LINEARE

- CAUSALE OPERA SU n

- STABILE PRODOTTO E SOMMA PER COSTANTI $\Rightarrow$ STABILE

- INVARIANZA TEMPORALE:

$y[n-n_0] = 3 \cdot x[n-n_0] + 5$
$\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad \|$ $\Rightarrow$ È A TEMPO INVARIANTE
$y_t[n] = 3 \cdot x[n-n_0] + 5$

RELAZIONE INGRESSO - USCITA

① $y[n] = \frac{1}{2}\left(x[2n] + x[2n-1]\right)$

$x[n] = \{\underset{\uparrow}{2}, 3, 2, 1\}$

$y[0] = \frac{1}{2}(2+0) = 1$

$y[1] = \frac{1}{2}(2+3) = \frac{5}{2}$

$y[2] = \frac{1}{2}(0+1) = \frac{1}{2}$

$y[n] = \{1, \frac{5}{2}, \frac{1}{2}\}$

② $y[n] - \frac{1}{2}y[n-1] = \frac{1}{4}x[n] + \frac{1}{4}x[n-1]$

$x[n] = \begin{cases} 0 & n < 0 \\ 1 & n \geq 0 \end{cases}$

SUPPONIAMO CHE IL SISTEMA SIA SCARICO $(y[n-1] = 0)$

È RICORSIVO

$y[n] = \frac{1}{4}\left(x[n] + x[n-1]\right) + \frac{1}{2}y(n-1)$

$y[0] = \frac{1}{4}(1+0) + \frac{1}{2}\cdot 0 = \frac{1}{4}$

$y[1] = \frac{1}{4}(1+1) + \frac{1}{2}\cdot\frac{1}{4} = \frac{1}{2} + \frac{1}{8} = \frac{5}{8}$

$y[2] = \frac{1}{4}(1+1) + \frac{1}{2}\cdot\frac{5}{8} = \frac{1}{2} + \frac{5}{16} = \frac{13}{16}$

$y[3] = \frac{1}{4}(1+1) + \frac{1}{2}\cdot\frac{13}{16} = \frac{1}{2} + \frac{13}{32} = \frac{29}{32}$

$y[n] = \left\{ \frac{1}{4}, \frac{5}{8}, \frac{13}{16}, \frac{29}{32} \right\}$

SPAZIO METRICO

INSIEME IN CUI POSSIAMO CALCOLARE LA DISTANZA TRA GLI ELEMENTI.

LA DISTANZA HA LE SEGUENTI PROPRIETÀ:

- $d(x,y) > 0$
- $d(x,y) = 0 \iff x = y$
- $d(x,y) = d(y,x)$
- $d(x,y) \leq d(x,z) + d(y,z)$

SPAZIO COMPLETO

INSIEME DI ELEMENTI IL CUI LIMITE È ALL'INTERNO DELLO SPAZIO STESSO.

SPAZIO VETTORIALE

- UN CAMPO K (I CUI ELEMENTI SONO SCALARI)

$$\begin{cases} \mathbb{R} : \text{SPAZIO VETTORIALE REALE} \\ \mathbb{C} : \text{SPAZIO VETTORIALE COMPLESSO} \end{cases}$$

- UN INSIEME DI ELEMENTI (VETTORI)
- SOMMA
- MOLTIPLICAZIONE PER SCALARE

PROPRIETÀ SOMMA:

- COMMUTATIVA: $\forall \bar{x}, \bar{y} \in V \quad \bar{x} + \bar{y} = \bar{y} + \bar{x}$
- ASSOCIATIVA: $\forall \bar{x}, \bar{y}, \bar{z} \in V \quad (\bar{x} + \bar{y}) \bar{z} = \bar{x} + (\bar{y} + \bar{z})$
- ELEMENTO NEUTRO: $\exists\, 0 \in V : \forall \bar{x} \in V \quad \bar{x} + 0 = \bar{x}$
- ELEMENTO OPPOSTO: $\exists\, \bar{x}, \bar{y} \in V : \quad \bar{x} + \bar{y} = 0$

PROPRIETÀ DEL PRODOTTO CON SCALARE

- DISTRIBUTIVA: $\forall \bar{x}, \bar{y} \in V, \forall a \in K \quad a(\bar{x} + \bar{y}) = a\bar{x} + a\bar{y}$
- ASSOCIATIVA: $\forall a, b \in K, \forall \bar{x} \in V \quad (a+b)\bar{x} = a\bar{x} + b\bar{x}$
- ELEMENTO NEUTRO: $\exists\, 1 \in K : \forall \bar{x} \in V \quad 1 \cdot \bar{x} = \bar{x}$

ESEMPIO DI SPAZIO VETTORIALE È QUELLO EUCLIDEO $(\mathbb{R}^n)$

ALTRI ESEMPI: $\mathbb{C}^n$, $K^{m \times n}$, $K[x]$

SPAZIO NORMATO

SE POSSIAMO DEFINIRE LA LUNGHEZZA DI OGNI VETTORE.

$|\bar{X}|$ DETTA NORMA $\quad 0 \leq |\bar{x}| \leq \infty$

- $|\lambda \bar{x}| = \lambda |\bar{x}| \quad \lambda \in K$
- $|\bar{x} + \bar{y}| \leq |\bar{x}| + |\bar{y}|$

$d(x, y) = |\bar{x} - \bar{y}|$

$$|\bar{x}| = \sqrt{\sum_{i=1}^{n} x_i^{\,p}} \qquad \bar{X} = [x_1, x_2 \dots, x_n]$$

$d(\bar{x}, \bar{y}) = |\bar{x} - \bar{y}|_p$ $\quad p = 2 \Rightarrow \quad d = \sqrt{\sum_{i=1}^{n} (x_i - y_i)^2}$

SPAZIO DI HILBERT

LO SPAZIO DEI SEGNALI È DI HILBERT

VIENE DEFINITO IL PRODOTTO SCALARE $\langle \bar{x}, \bar{y} \rangle$

PROPRIETA'

- HERMITIANA o COMMUTATIVA : $\langle \bar{x}, \bar{y} \rangle = \langle \bar{y}, \bar{x} \rangle^*$ — COMPLESSO CONIUGATO
- $\langle a \cdot \bar{x} + b \bar{y}, z \rangle = a \langle \bar{x}, \bar{z} \rangle + b \langle \bar{y}, \bar{z} \rangle$
- $\langle \bar{x}; \bar{x} \rangle \geq 0$
- $\langle \bar{x}, \bar{x} \rangle = 0 \iff \bar{x} = 0$

$$|\bar{x}| = \sqrt{\langle \bar{x}; \bar{x} \rangle}$$

$$|\langle \bar{x}, \bar{y} \rangle| = |\bar{x}| |\bar{y}| \cos\theta$$

VETTORI ORTOGONALI $\quad \langle \bar{x}, \bar{y} \rangle = 0$

VETTORI PARALLELI $\quad \langle \bar{x}, \bar{y} \rangle = |\bar{x}| |\bar{y}|$

BASE ORTONORMALE

$(u_i, u_j) \quad \langle u_i, u_j \rangle = 0 \quad \forall \; i \neq j$

$|u_i| = 1 \quad \forall i$

$\langle u_i, u_j \rangle = \delta_{ij}$ — DELTA DI (KRONAKER)? $\begin{cases} 1 & i = j \\ 0 & i \neq j \end{cases}$

$$\overline{X} = (x_1, x_2 \dots x_n) = \sum_{i=1}^{n} x_i \cdot \overline{v}_i$$

SEGNALI DI ENERGIA A TEMPO DISCRETO

$$\overline{X} = \{x_i\} \equiv i = 1, 2, \dots \infty$$

$$\sum_{n=0}^{\infty} |x[n]|^2 < \infty$$

$$\{\delta[n-k], k \in \mathbb{Z}\} \qquad x[n] = \sum_{k=-\infty}^{\infty} x[k] \cdot \delta[n-k]$$

SEGNALI DI ENERGIA A TEMPO CONTINUO

$$\overline{X} \equiv f(t) \qquad \|f(t)\|^2 = \int \overline{X}^2(t)\, dt$$

$$\langle x(t), y(t) \rangle = \int_{-\infty}^{\infty} x(t)\, y^*(t)\, dt$$

$$\|x(t)\|^2 = \langle x(t), x(t) \rangle = \int_{-\infty}^{\infty} |x(t)|^2 dt$$

TRASFORMATA DI FOURIER A TEMPO DISCRETO

DOBBIAMO PASSARE AL DOMINIO DELLA FREQUENZA GRAZIE ALLA TRASFORMATA:

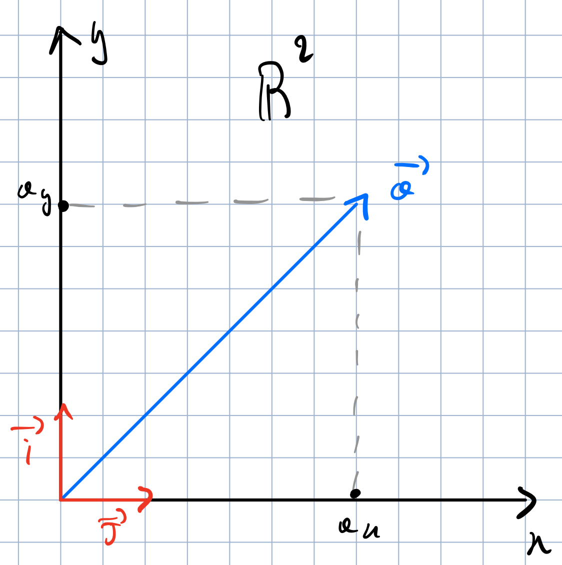
$$X\left(e^{j\omega}\right) = \boxed{\sum_{n=-\infty}^{\infty} x[n] \cdot \boxed{e^{-j\omega n}}}$$

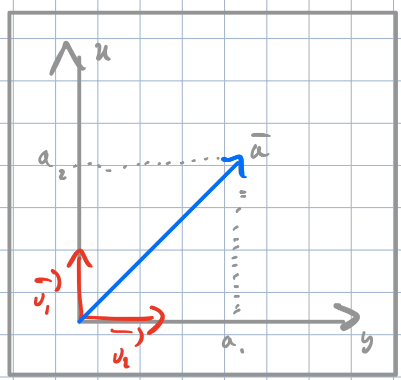
FUNZIONE CONTINUA

VETTORE $\overline{X} = x_i \cdot \overline{v}_j$

BASE ORTONORMALE

DTFT È L'ACRONIMO PER INDICARLA





$$\overline{a} = a_1 \overline{v}_1 + a_2 \overline{v}_2 = \sum a_i \cdot \overline{v}$$

$$\langle \overline{v}_1, v_2 \rangle = \delta_{1,2}$$

$$|\overline{v}_1| = |\overline{v}_2| = 1$$

$$\begin{cases} a_1 = \langle \overline{a}, \overline{v}_1 \rangle \\ a_2 = \langle \overline{a}, \overline{v}_2 \rangle \end{cases}$$

$$a_1 = |\overline{a}| \cdot \cos\theta$$

$$a_2 = |\overline{a}| \cdot \sin\theta$$

$$\langle f(x), g(x) \rangle = \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} f(x) \cdot g^*(x)\, dx$$

$$\boxed{x[n] = \frac{1}{2\pi} \int_{-\pi}^{\pi} X\left(e^{j\omega}\right) \cdot e^{j\omega n} \cdot d\omega}$$

TRASFORMATA INVERSA DI FOURIER

(IDTFT)

DIMOSTRIAMO CHE e^{jw} È UN' INSIEME DI BASI ORTONORMALI,

$$\left\{ \frac{e^{jnw}}{2\pi} \right\} \quad n = 0, \pm 1, \pm 2 \dots \quad n \in \mathbb{Z}$$

(W SAREBBE θ)
(J SAREBBE i)

$$e^{\pm jnw} = \cos(nw) \pm j\sin(nw)$$

PRODOTTO SCALARE TRA DUE VETTORI PER DIMOSTRARE CHE SONO BASI ORTONORMALI:

$$\frac{1}{2\pi}\int_{-\pi}^{\pi} e^{jnw} \cdot e^{-jmw}\, dw = \frac{1}{2}\int_{-\pi}^{\pi} e^{jw(n-m)}\, dw =$$

$$= \frac{1}{2\pi}\int_{-\pi}^{\pi} \left[\cos(n-m)w + j \cdot \sin(n \cdot m) \cdot w\right] dw =$$

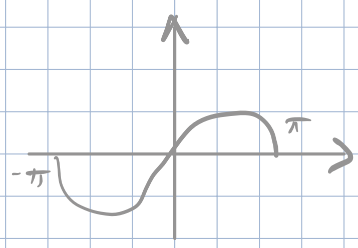
SE $n - m = 0 \Rightarrow n = m$

$$= \frac{1}{2\pi}\int_{-\pi}^{\pi} 1 \cdot dw = 1$$ ← QUANDO SONO PARALLELI

$$\frac{1}{2\pi}\int_{-\pi}^{\pi} \left[\cos(n-m)w + j \cdot \sin(n \cdot m) \cdot w\right] dw =$$

SE $n \neq m$

DATO CHE STIAMO FACENDO LA MEDIA, IL SIN = 0



$$\frac{1}{2\pi}\int_{-\pi}^{\pi}\cos(n-m)\cdot w\,dw = \frac{1}{2\pi}\left[\frac{\sin(n-m)w}{(n-m)}\right]_{-\pi}^{\pi} = 0$$

$\blacksquare$

PULSAZIONE:

$$w = 2\pi f = \frac{2\pi}{T} \qquad \hat{w} = \frac{w}{f_s} \qquad \hat{f} = \frac{f}{f_s}$$

FREQUENZA

FREQUENZA DI SAMPLING

$$X(e^{jw+2\pi}) = \sum_{n=-\infty}^{\infty} x[n]\cdot e^{-j(w+2\pi)n} = \sum_{n=-\infty}^{\infty} X[n]\cdot e^{-jwn}\cdot \cancel{e^{-j2\pi n}} =$$

$\cos(2\pi) + j\sin(2\pi)$

$$= \sum_{n=-\infty}^{\infty} X[n]\cdot e^{-jwn}$$

$$X(e^{jw}) = X_{re}(e^{jw}) + jX_{im}(e^{jw})$$

$$X(e^{jw}) = |X(e^{jw})|\cdot e^{j\theta(w)} \qquad \theta(w)$$

SPETTRO DI AMPIEZZA

FASE

$$\theta(w)\begin{cases}\arctan\dfrac{X_{im}}{X_{re}} & x_{re} > 0\\[2ex] \arctan\dfrac{X_{im}}{X_{re}} \pm K & x_{re} > 0\end{cases}$$

PROPRIETA':

$x[n] \in \mathbb{R}$

$$X^*(e^{jw}) = \sum_{n=-\infty}^{\infty} x^*[n] \cdot e^{jwn} =$$

$$= \sum_{n=-\infty}^{\infty} x[n] \cdot e^{-j(-w)\cdot n} =$$

$$= X(e^{-jw})$$

ESSENDO UNA SOMMATORIA INFINITA, NON TUTTI I SEGNALI AMMETTONO TRASFORMATA DI FOURIER:

CONDIZIONI NECESSARIE:

- $x[n]$ DEVE ESSERE ASSOLUTAMENTE SOMMABILE

$$X_N(e^{jw}) = \sum_{n=-N}^{N} x[n] \cdot e^{-jwn}$$

$$\lim_{N\to+\infty} \left| X(e^{jw}) - X_N(e^{jw}) \right| = 0$$

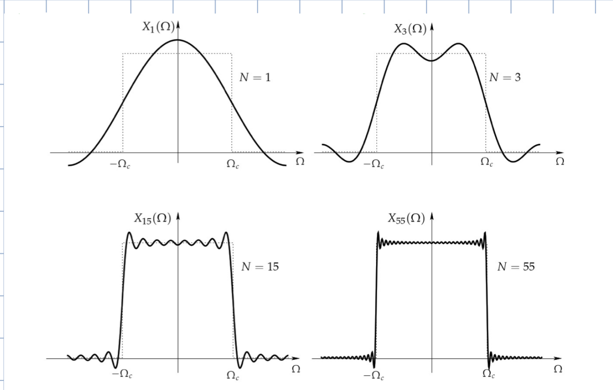
CONVERGENZA UNIFORME

- $\sum_{n=\infty}^{\infty} |x[n]|^2 < \infty$ AMMETTE TRASFORMATA MA PUÒ ESSERE DISCONTINUA.

NEI PUNTI DI DISCONTINUITÀ

DI CREARE DELLE OSCILLAZIONI

FENOMENO DI GHIBS



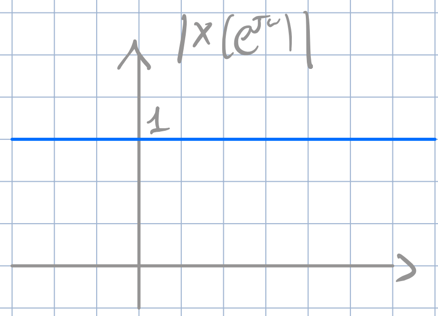
TFT NOTEVOLI

IMPULSO UNITARIO

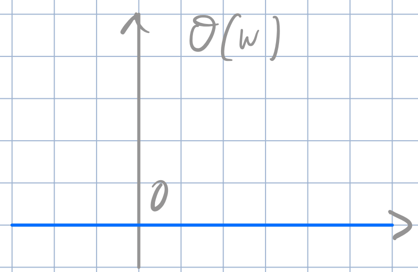
$$\delta[n] = \begin{cases} 1 & n = 0 \\ 0 & n \neq 0 \end{cases}$$

É ASSOLUTAMENTE SOMMABILE

$$X(e^{jw}) = \sum_{n=-\infty}^{\infty} \delta[n] \cdot e^{-jw} = 1$$



$$\delta[n] \longleftrightarrow 1$$



SEGNALE ESPONENZIALE

$$x[n] = e^{jw_0 n}$$

$$x[n] = e^{jw_0 n} = \frac{1}{2\pi} \int_{-\pi}^{\pi} \underset{2\pi \cdot \delta(w - w_0)}{\underline{X(e^{jw})}} \cdot e^{jwn} \, dw =$$

$$= \frac{1}{2\pi} \sum_{k=-\infty}^{-\infty} 2\pi \, \delta(w - w_0 + 2\pi k) \longleftrightarrow e^{jw_0 n}$$

SEQUENZA ESPONENZIALE CAUSALE

$$x[n] = \alpha^n \cdot u[n] \qquad |\alpha| < 1$$

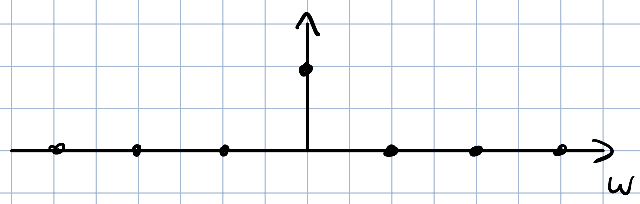
$$\sum_{+\infty}^{-\infty} \left| \alpha^n \cdot u[n] \right| = \sum_{n=0}^{\infty} |\alpha^n| = \frac{1}{1 - |\alpha|} < \infty$$

ESISTE LA TFT :

$$X(e^{jw}) = \sum_{n=0}^{\infty} \alpha^n \cdot e^{jwn} = \sum_{n=0}^{\infty} (\alpha \cdot e^{jw})^n = \frac{1}{1-\alpha \cdot e^{-jw}}$$

DELTA DI DIRAK

$\delta(w)$



$$\int_{-\infty}^{+\infty} \delta(w)\, dw = 1$$

È L'EQUIVALENTE NEL CONTINUO DELL'IMPULSO UNITARIO
(O DELTA DI KRONAKER)

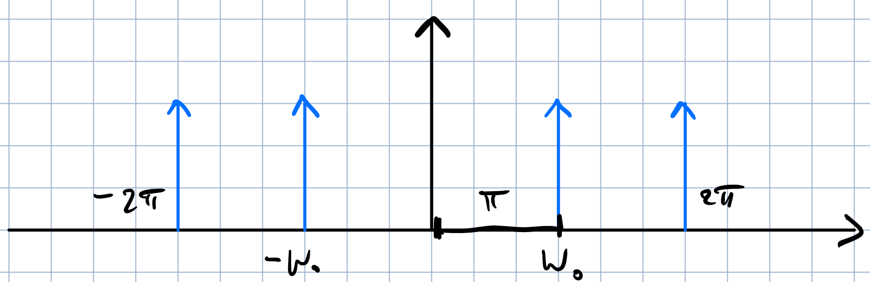
SEQUENZE QUADRATICAMENTE SOMMABILI

$$x[n] = \sin(w_0 \cdot n) = \frac{e^{jw_0 n} - e^{-jw_0 n}}{2j}$$

$$X(e^{jw}) = \sum_{n=-\infty}^{\infty} \frac{e^{jw_0 n} - e^{-jw_0 n}}{2j} \cdot e^{-jwn} =$$

$$= \frac{1}{2}j \cdot DTFT\left(e^{jw_0 n}\right) + \frac{1}{2}j\, DTFT\left(\cdot\, e^{jw_0 n}\right) =$$

$$= \frac{2\pi}{2j} \sum_{n=-\infty}^{\infty} \delta(w - w_0 + 2\pi) - \frac{2\pi}{2j} \sum_{n=-\infty}^{\infty} \delta(w - w_0 + 2\pi)$$



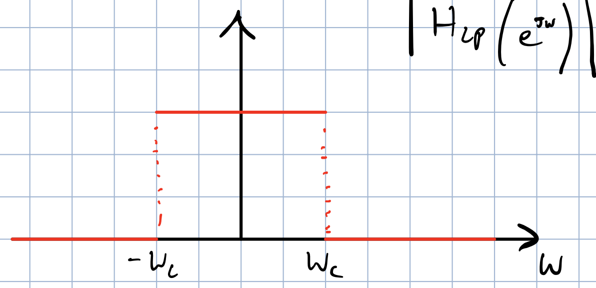
CONTROLLARE

$$\sin(w_0 n) \longleftrightarrow j\pi\left(\delta(w + w_0) - \delta(w + w_0)\right)$$

TRASFORMATA DEL FILTRO PASSA-BASSO (LOW PASS FILTER)

È UN FILTRO IDEALE.

$$H_{LP}(e^{jw}) = \begin{cases} 1 & 0 < |w| \leq w_c \\ 0 & -w_c < |w| \leq w_c \end{cases}$$

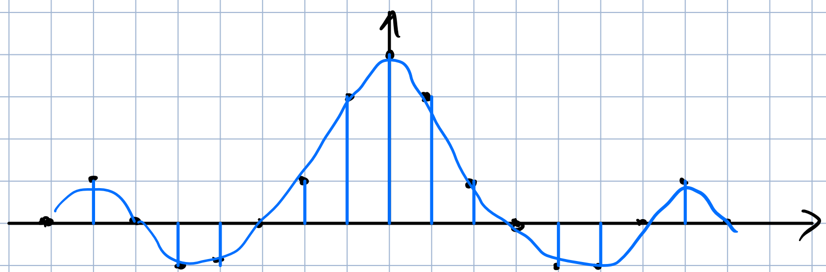


FACCIO L'ANTITRASFORMATA

$$h_{LP}[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_{LP}(e^{jw}) \cdot e^{jwn}\, dw = \frac{1}{2\pi}\int_{-w_c}^{w_c} e^{jwn}\, dw = \frac{1}{2\pi}\left[\frac{e^{jwn}}{jn}\right]_{-w_c}^{w_c}$$

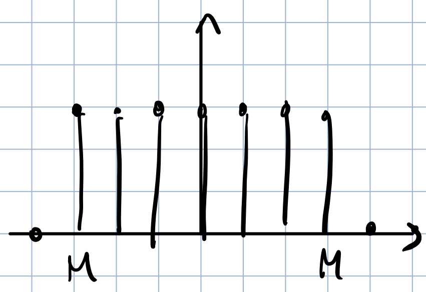
$$= \frac{1}{2\pi}\left(\frac{e^{jw_cn} - e^{-jw_cn}}{jn}\right) = \frac{\sin(w_c n)}{\pi n}$$

$$h_{LP}[n] = \frac{w_c}{2\pi} \cdot \text{sinc}(w_c \cdot n)$$



NELLA REALTÀ NON POTENDO FARE SOMME INFINITE,

ORA DETERMINAMO LA DTFT DI UN SEGNALE BEN DEFINITO NEL DOMINIO DEL TEMPO DISCRETO



$L = 2M+1$

$$rect[n] = \begin{cases} 1 & |n| \leq M \\ 0 & |n| > M \end{cases}$$

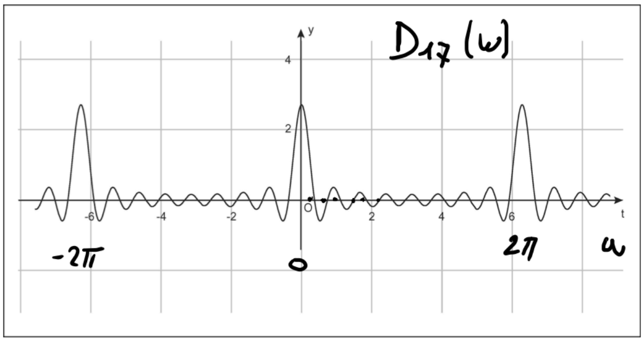
$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n] \cdot e^{-j\omega n} = \sum_{n=-M}^{M} e^{-j\omega n} = e^{j\omega M} \sum_{n=0}^{2M} e^{-j\omega n}$$

$$= e^{j\omega M} \left(\frac{1 - e^{-j\omega(2M+1)}}{1 - e^{-j\omega}} \right) =$$

$$= \frac{e^{j\omega M} \cdot e^{-j\omega \frac{2M+1}{2}} \left(e^{j\omega\left(\frac{2M+1}{2}\right)} - e^{-j\omega\left(\frac{2M+1}{2}\right)} \right)}{e^{-j\frac{\omega}{2}} \left(e^{j\omega/2} - e^{-j\frac{\omega}{2}} \right)} =$$

$$= \frac{\sin\left(\frac{\omega}{2}(2M+1)\right)}{\sin\left(\frac{\omega}{2}\right)} = L\, D_L(\omega)$$

$$D_L = \frac{\sin\left(\frac{\omega L}{2}\right)}{L \sin\left(\frac{\omega}{2}\right)}$$



PROPRIETÀ:

- $$e^{j\omega n_0} X(e^{j\omega}) = |X(e^{j\omega})| \cdot e^{-j\theta} \cdot e^{j\omega n_0} =$$

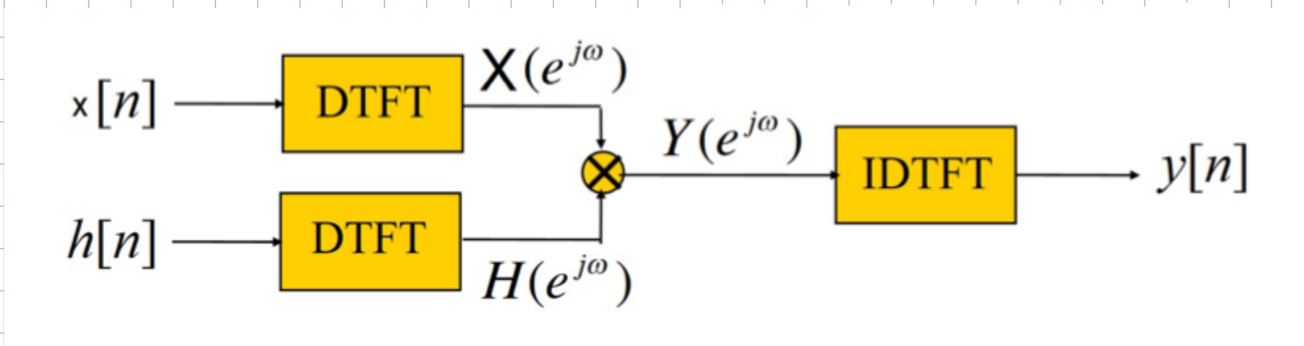
$$|X(e^{j\omega})| \, e^{-j(\theta-\omega n_0)}$$

- $$y[n] = \underbrace{x[n] * h[n]}_{}$$

RISPOSTA IMPULSIVA

CONVOLUZIONE

$$Y(e^{j\omega}) = X(e^{j\omega}) \cdot H(e^{j\omega})$$



DIM:

$$y[n] = \sum_{k=-\infty}^{\infty} x[k] \cdot h[n-k]$$

$$Y(e^{j\omega}) = \sum_{n=-\infty}^{\infty} \left(\sum_{k=-\infty}^{\infty} x[k] \cdot h[\underbrace{n-k}_{m=n-k}] \right) \cdot e^{-j\omega n} =$$

$$= \sum_{m=-\infty}^{\infty} \left(\sum_{k=-\infty}^{\infty} x[k] \cdot h[m] \cdot e^{-j\omega(m+k)} \right) =$$

$$= \sum_{k=-\infty}^{\infty} x[k] \underbrace{\left(\sum_{m=-\infty}^{\infty} h[m] \cdot e^{-j\omega m} \right)}_{H(e^{j\omega})} e^{-j\omega k} = H(e^{j\omega}) \cdot \sum_{k=-\infty}^{\infty} x[k] \cdot e^{-j\omega k} =$$

$$= X(e^{j\omega}) \cdot H(e^{j\omega}) = Y(e^{j\omega})$$

$$y[n] = \text{IDTFT}\{Y(e^{j\omega})\}$$

- **TEOREMA DI PARSEVAL**

$$\sum_{m=-\infty}^{\infty} x[m] \cdot h^*[m] = \frac{1}{2\pi} \cdot \int_{-\pi}^{\pi} X(e^{j\omega}) \cdot H^*(e^{j\omega}) \, d\omega$$

LA FORMA CHE CI INTERESSA È L'APPLICAZIONE ALL'ENERGIA:

$$\sum_{m=-\infty}^{\infty} x[m] \cdot x^*[m] = \sum_{m=-\infty}^{\infty} |x[m]|^2 = \underbrace{\frac{1}{2\pi} \int_{-\pi}^{\pi} |X(e^{j\omega})|^2 d\omega}$$

DENSITÀ DI ENERGIA NEL MONDO DELLA FREQUENZA

NELL'ESEMPIO DI PRIMA (LOW PASS FILTER)

$$E = \sum_{m=-\infty}^{\infty} \left| \frac{\sin c(\omega_c n)}{n\pi} \right|^2$$

$$E = \frac{1}{2\pi} \int_{-\pi}^{\pi} |X(e^{j\omega})|^2 d\omega = \frac{1}{2\pi} \int_{-\omega_c}^{\omega_c} 1 \, d\omega = \frac{2\omega_c}{2\pi} = \frac{\omega_c}{\pi}$$

RISPOSTA IN FREQUENZA

$$H(e^{j\omega}) = DTFT\{h[n]\}$$

LA TRASFORMATA DI FOURIER DELLA RISPOSTA IMPULSIVA

IN UN SISTEMA LTI:

$$x[n] = e^{j\omega n}$$

$$y[n] = x[n] * h[n]$$

$$\sum_{m=-\infty}^{\infty} h[m] \cdot e^{j\omega n} \cdot e^{-j\omega m} = \left(\sum_{m=-\infty}^{\infty} h[m] \cdot e^{-j\omega m} \right) e^{j\omega n}.$$

$$\Rightarrow \boxed{y[n] = H(e^{j\omega}) \cdot \underbrace{e^{j\omega n}}_{x[n]}}$$

RISPOSTA IN FREQUENZA

ESSENDO $H(e^{j\omega})$ UN NUMERO COMPLESSO, CAUSERÀ:

- MODIFICA IN AMPIEZZA : $|H(e^{j\omega})|$
- MODIFICA DI FASE : $\theta(\omega)$ (INTRODURRE UN RITARDO)

$$x[n] = e^{j\omega_0 n}$$

$$y[n] = H(e^{j\omega}) \cdot e^{j\omega_0 n} = \overbrace{|H(e^{j\omega})| \cdot e^{j\theta}}^{\text{FORMA POLARE}} \cdot e^{j\omega_0 n}$$

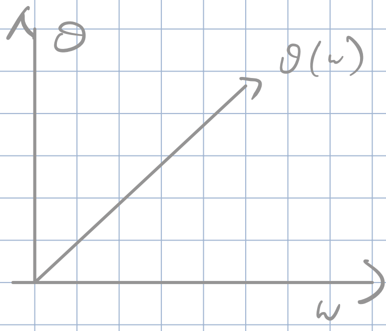
$$= |H(e^{j\omega})| \cdot e^{j\omega_0 (n + \theta/\omega_0)}$$

$$\underbrace{\tau}_{\text{TAU}}{}_{p} = \frac{-\theta}{\omega_0}$$

SE ABBIAMO PIÙ COMPONENTI: $\tau_g = -\frac{d\theta}{\omega}$

I FILTRI SONO DETTI A FASE LINEARE: $\theta(\omega) = -k\omega$

$$\tau_g = -\frac{d\theta}{d\omega} = \frac{d}{d\omega}(-k\omega) = k$$



$$y[n] = H(e^{j\omega_0 n}) \cdot e^{j\omega_0 n} = |H(e^{j\omega_0 n})| \cdot e^{j\theta\omega_0} \cdot e^{j\omega_0 n} =$$

VARIAZIONE IN AMPIEZZA

$$= H(e^{j\omega_0 n})\, e^{j\omega_0 (n + \frac{\theta}{m})}$$

RITARDO

LA FASE DEL SISTEMA DIPENDE LINEARMENTE DALLA FREQUENZA:

$$\theta(\omega) = -k\omega \qquad \tau_g = -\frac{d\theta(\omega)}{d\omega} = k$$

$$h[n] = \frac{1}{5}\{\underset{\uparrow}{1}, 1, 1, 1, 1\}$$

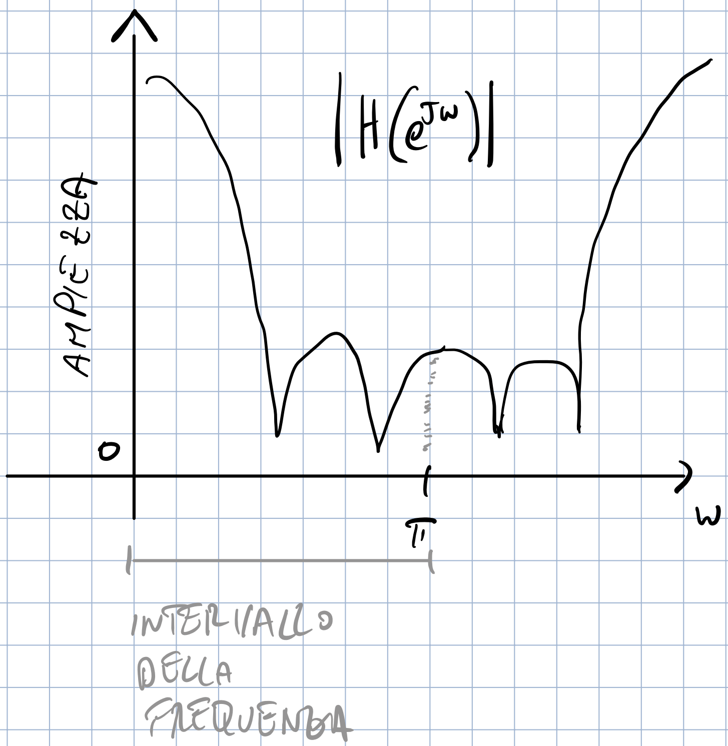
$$H(e^{j\omega}) = \sum_{n=-\infty}^{\infty} h[n] \cdot e^{-j\omega n} = \frac{1}{5}\sum_{n=0}^{4} e^{-j\omega n} =$$

$$= \frac{1}{5} e^{-j\omega 0} + \frac{1}{5} e^{-j\omega 1} + \frac{1}{5} e^{-j\omega 2} + \frac{1}{5} e^{-j\omega 3} + \frac{1}{5} e^{-j\omega 4} =$$

$$= \frac{1}{5} e^{-j2\omega} \left(e^{-j2\omega} + e^{-j\omega} + e^{j0} + e^{-j\omega} + e^{-j2\omega} \right) =$$

$$= \underbrace{\frac{1}{5} e^{-j2\omega}}_{\text{FASE}} \underbrace{\left(1 + 2\cos\omega + 2\cos(2\omega) \right)}_{\text{AMPIEZZA}} = H(e^{j\omega})$$

È UN FILTRO PASSA-BASSO CHE TAGLIA LE ALTE FREQUENZE.



ESERCIZI :

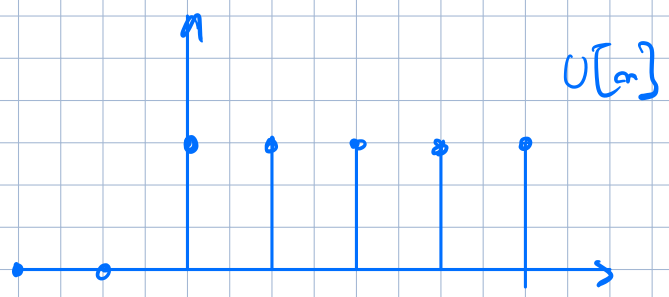
① $x[n] = \delta[n - \ \ ]$

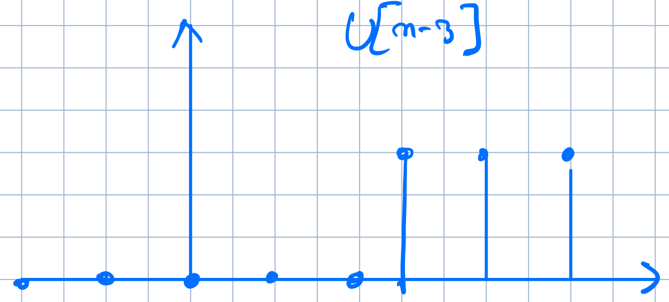
OPERAZIONE DI TIME SHIFT

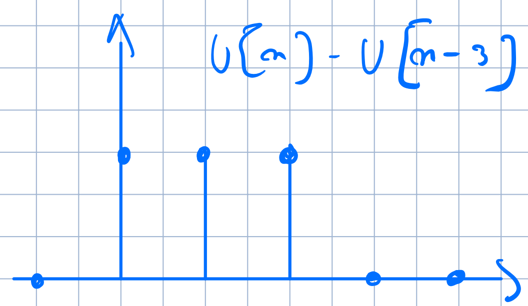
$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} \delta[n - n_0]\, e^{-j\omega n} = e^{-j\omega n_0}$$

$$\delta[n] \xleftrightarrow{DTFT} 1$$

② $X[n] = U[n] - U[n-3]$







$$= \sum_{n=-\infty}^{\infty} \delta[n] + \delta[n-1] + \delta[n-3] = 1 - e^{-jw} - e^{-2jw}$$

③ $X[n] = \left(\frac{1}{3}\right)^n \cdot U[n-2] = \left(\frac{1}{3}\right)^2 \left(\frac{1}{3}\right)^{n-2} U[n-2] =$

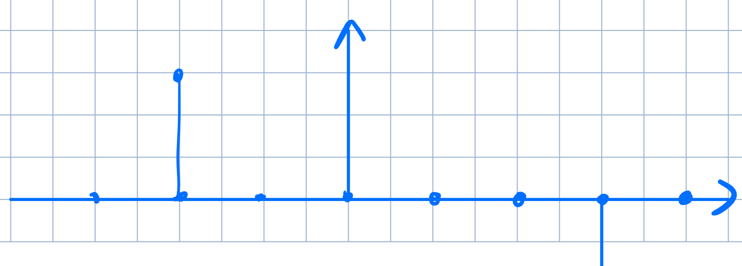
PROPRIETA'

$X[n] = \alpha^n \cdot U[n]$

$$\Rightarrow X(e^{jw}) = \left(\frac{1}{3}\right)^2 \sum_{n=-\infty}^{\infty} \left(\frac{1}{3}\right)^{n-2} U[n-2]\, e^{-jwn} =$$

$$= \left(\frac{1}{3}\right)^2 e^{-j2w} \qquad \boxed{\frac{1}{1-\frac{1}{3}e^{-jw}}}$$

④ $X[n] = \delta[n+3] - \delta[n-3]$



$$X(e^{jw}) = \sum_{n=-\infty}^{\infty} x[n] \cdot e^{-jwn} = \sum_{n=-\infty}^{\infty} \left(\delta[n+3] + \delta[n-3]\right) \cdot e^{-jwn} =$$

$$= e^{j3w} - e^{-j3w} = 2j\sin(3w)$$

(5) $x[n] = 2^n \cdot u[n]$

$$X(e^{jw}) = \sum_{n=-\infty}^{\infty} 2^n \cdot u[n] \cdot e^{-jw} = \sum_{n=0}^{-\infty} 2^n e^{-jwn} = \sum_{m=0}^{\infty} 2^{-m} \cdot e^{jwm}$$

$m = -n$

$$= \sum_{m=0}^{\infty} \left(\frac{1}{2} e^{jw}\right)^m = \frac{1}{1 - \frac{1}{2} e^{jw}}$$

(6) $x[n] = \left(\frac{1}{4}\right)^{-n} \cdot u[-n]$

$$x[-n] \xleftrightarrow{DTFT} X(e^{-jw})$$

$$s[n] = \left(\frac{1}{4}\right)^n \cdot u[n]$$

$$S(e^{jw}) = \sum_{n=-\infty}^{\infty} \left(\frac{1}{4}\right)^n \cdot u[n] \cdot e^{-jwn} = \sum_{n=0}^{\infty} \left(\frac{1}{4}\right)^n \cdot e^{-jwn}$$

$$= \frac{1}{1 - \frac{1}{4} e^{-jw}}$$

$$\Rightarrow X(e^{jw}) = \frac{1}{1 - \frac{1}{4} e^{jw}}$$

(7) $x_1[n] = \{\underset{\uparrow}{0}, -1, 2\}$ $x_2[n] = \{\underset{\uparrow}{1}, 3\}$

$$y[n] = x_1[n] * x_2[n] \overset{DTFT}{\Longleftrightarrow} Y(e^{j\omega}) = X_1(e^{j\omega}) \cdot X_2(e^{j\omega})$$

DTFT

$$X_1(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x_1[n] \cdot e^{-j\omega} = e^{-j\omega} + 2e^{-j2\omega}$$

$$X_2(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x_2[n] \cdot e^{-j\omega} = 1 + 3e^{-j\omega}$$

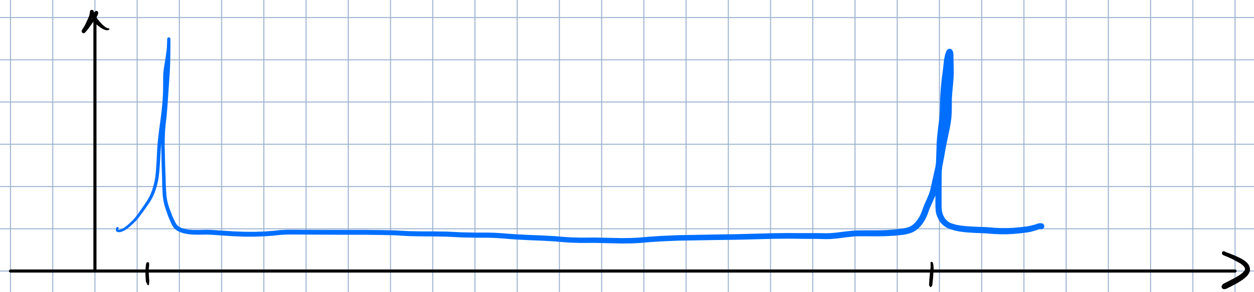
$$Y(e^{j\omega}) = \left(e^{-j\omega} + 2e^{-j2\omega}\right)\left(1 + 3e^{-j\omega}\right) = e^{-j\omega} - e^{-2j\omega} + 6e^{-3j\omega}$$

FACCIO ANTITRASFORMATA PER TORNARE AL DOMINIO DEL TEMPO.

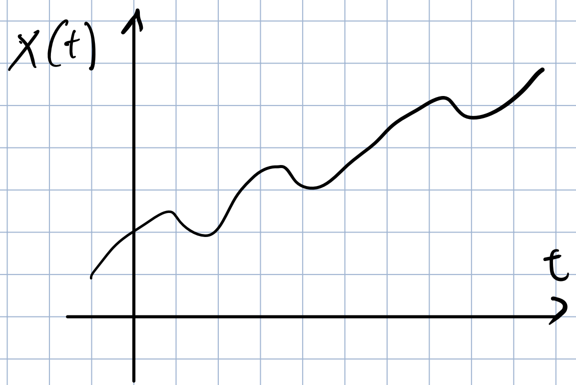
$$Y(e^{j\omega}) = e^{-j\omega} - e^{-2j\omega} + 6e^{-3j\omega}$$

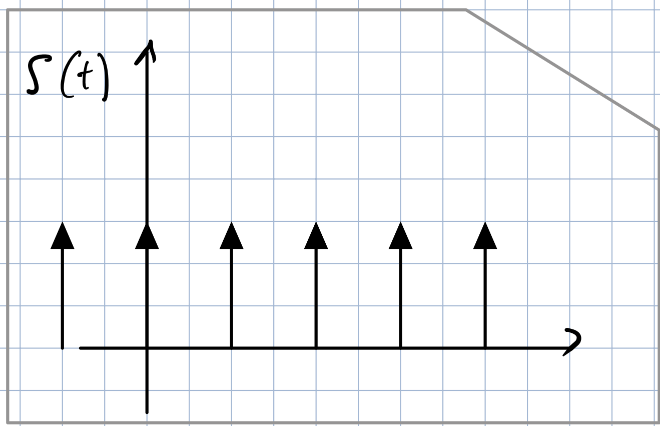
$$y[n] = -\delta[n-1] - \delta[n-2] + 6\delta[n-3]$$

LA TRASFORMATA DEL SIN NEL MONDO DELLA FREQUENZA CORRISPONDE ALLA DELTA DI DIRAC.



FREQUENZA NORMALIZZATA = $\frac{\text{FREQUENZA}}{\text{SAMPLING RATE}}$



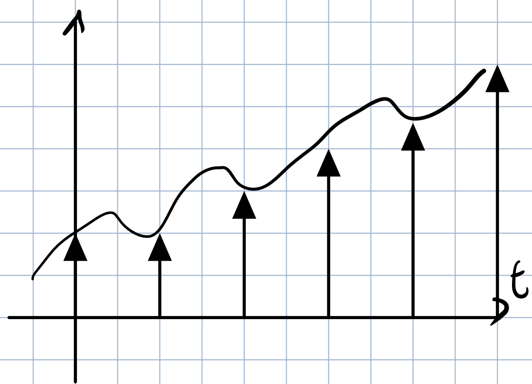


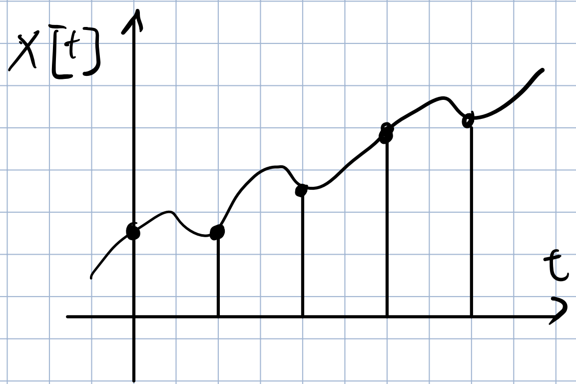
DIRAC COMB

$$\delta(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT_s)$$

$$x_s(t) = x(t) \cdot \delta(t)$$

$$x_s(t) = \sum_{n=-\infty}^{\infty} x(nT_s)\,\delta(t - nT_s)$$

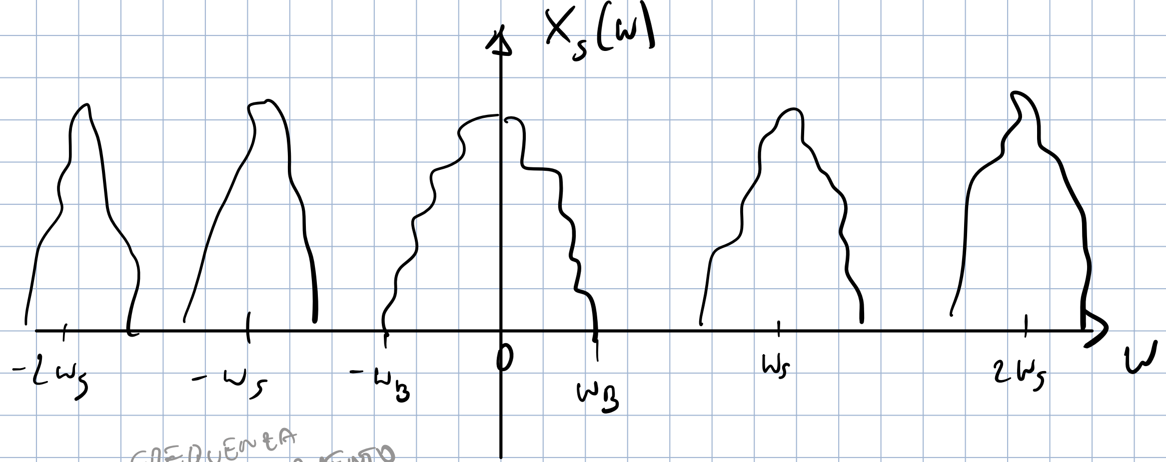




$$X_s(\omega) = \frac{1}{2\pi}\left[X(\omega) * D(\omega)\right]$$

$$D(\omega) = \omega_s \sum_{m=-\infty}^{\infty} \delta(\omega - m\omega_s)$$

$$X_s(\omega) = \frac{\omega_s}{2\pi} \sum_{m=-\infty}^{\infty} X(\omega - m\omega_s) = \frac{1}{T_s}\left[X(\omega) + X(\omega - \omega_s) + \dots\right]$$



FREQUENZA CAMPIONAMENTO

$$\omega_s - \omega_B > \omega_B$$

FREQUENZA DI BANDA

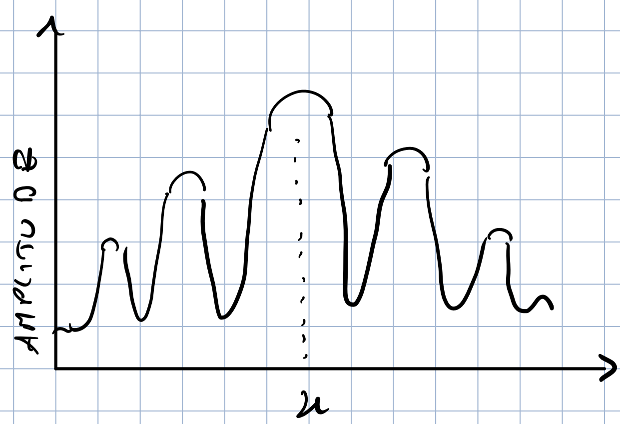
$$\boxed{\omega_s > 2\omega_B}$$

PREVIENE L'ALIASING

$$X_r(\omega) = X_s(\omega) H_s(\omega)$$

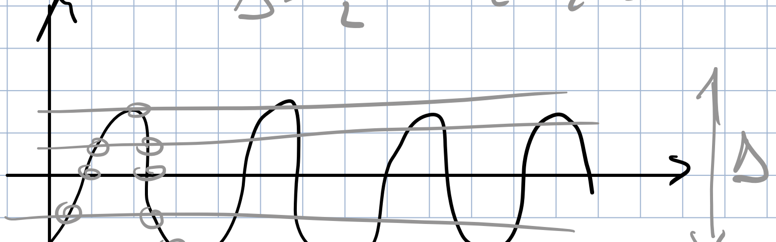
$$\Downarrow$$

$$x_r(t) = x_s(t) * h_s(t)$$



QUANTIZZAZIONE

$\Delta = \frac{2V}{L}$ $L = 2^m$ bit



CON LA QUANTIZZAZIONE SI PERDE INFORMAZIONE

$$x_q[n] = x[n] + e_q[n]$$

DTFT DISCRETE TIME FOURIER TRANSFORM

$$X\left(e^{jw}\right) = \sum_{n=-\infty}^{\infty} x[n] \cdot e^{-jwn}$$

$$x[n] = \frac{1}{2} \int_{-\pi}^{\pi} X\left(e^{jw}\right) \cdot e^{jwn} \, dw$$

DFT (DISCRETE FOURIER TRANSFORM)

$$x[n] = x[n+N] \qquad N \in \mathbb{N}$$

$$X[K] = \sum_{n=0}^{N-1} x[n] \cdot e^{-jK\frac{2\pi}{N}n}$$

INDICE FREQUENZA DISCRETA

INDICE TEMPO DISCRETO

DIMOSTRIAMO CHE È PERIODICA:

$$X[k+N] = \sum_{n=0}^{N+1} x[n] \cdot e^{-j(k+N)\frac{2\pi}{N}n} =$$

$$= \sum_{n=0}^{N+1} x[n] \cdot e^{-jk\frac{2\pi}{N}n} \overbrace{e^{-j\cancel{N}\frac{2\pi}{\cancel{N}}n}}^{1} = \sum_{n=0}^{N+1} x[n] \cdot e^{-jk\frac{2\pi}{N}n} = X[k]$$

ANTITRASFORMATA:

$$x[n] = \frac{1}{N} \sum_{k=0}^{N-1} X[k] \cdot e^{jk\frac{2\pi}{N}n}$$

DIMOSTRIAMO CHE È PERIODICA:

$$x[n+N] = \frac{1}{N}\sum_{k=0}^{N} X[k]\cdot e^{jk\frac{2\pi}{N}(n+N)} =$$

$$= \frac{1}{N}\sum_{k=0}^{N} X[k]\cdot e^{jk\frac{2\pi}{N}n}\cdot \cancel{e^{jk\frac{2\pi}{N}N}} = \frac{1}{N}\sum_{k=0}^{N} X[k]\cdot e^{jk\frac{2\pi}{N}n} = x[n]$$

ESTENSIONE PERIODICA DEL SEGNALE (PERCHÉ LA TRASFORMATA FUNZIONA SOLO CON SEGNALI PERIODICI)

$$\tilde{x}[n] = \sum_{p=-\infty}^{\infty} x[n-pN]$$

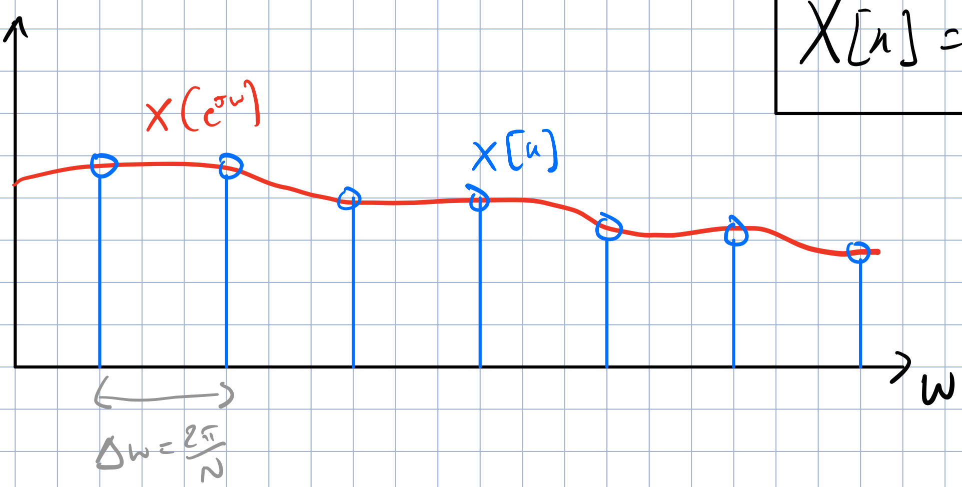
RELAZIONE TRA DFT E DTFT

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]\cdot e^{-j\omega n}$$

$$X[k] = X(e^{j\omega}) \iff \omega = k\frac{2\pi}{N}$$

$$X[k] = \sum_{n=0}^{N-1} x[n]\cdot e^{-jk\frac{2\pi}{N}n}$$

$$\boxed{X[k] = X(e^{j\omega})\Big|_{\omega=k\frac{2\pi}{N}}}$$



15/11/22

$$x[n] = 1 \quad 0 \le n \le 7$$

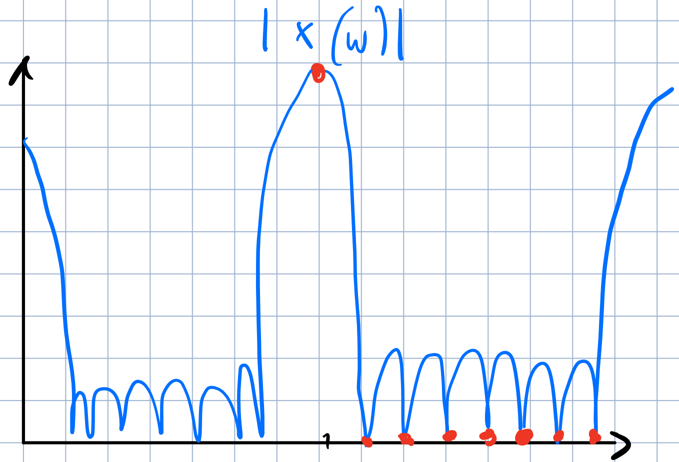
DTFT $\{x[n]\}$

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]\, e^{-j\omega n} = \sum_{n=0}^{7} e^{-j\omega n} = \frac{1-e^{-j8\omega}}{1-e^{-j\omega}} =$$

$$= \frac{e^{-j\frac{8\omega}{2}}\left(e^{j\frac{\omega 8}{2}} - e^{-j\frac{\omega}{2}}\right)}{e^{-j\frac{\omega}{2}}\left(e^{j\frac{\omega}{2}} - e^{-j\frac{\omega}{2}}\right)} = e^{-j\frac{7\omega}{2}} \cdot \frac{\sin(4\omega)}{\sin(\omega/2)}$$

DFT $\{x[n]\}$

$$X[k] = \sum_{n=0}^{7} x[n] \cdot e^{-jk\frac{2\pi}{8}n} = \begin{cases} 8 & k=0 \\ 0 & k=1 \dots 7 \end{cases}$$



UTILIZZANDO IL ZERO PADDING

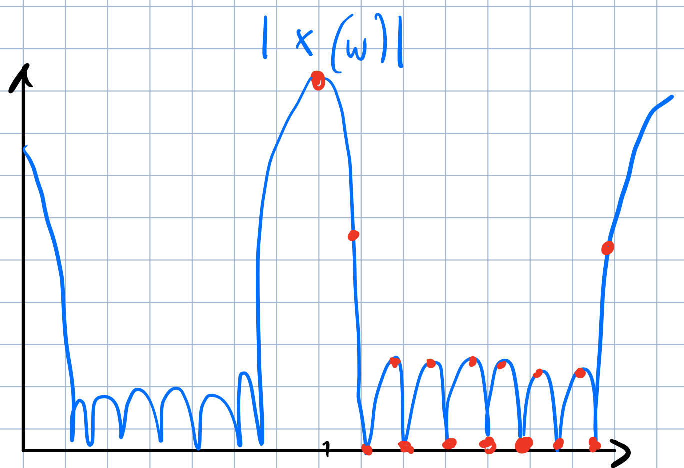
AUMENTIAMO N DUNQUE DIMINUENDO $\Delta\omega$.

IN MODO DA MIGLIORARE L' APPROSSIMAZIONE.

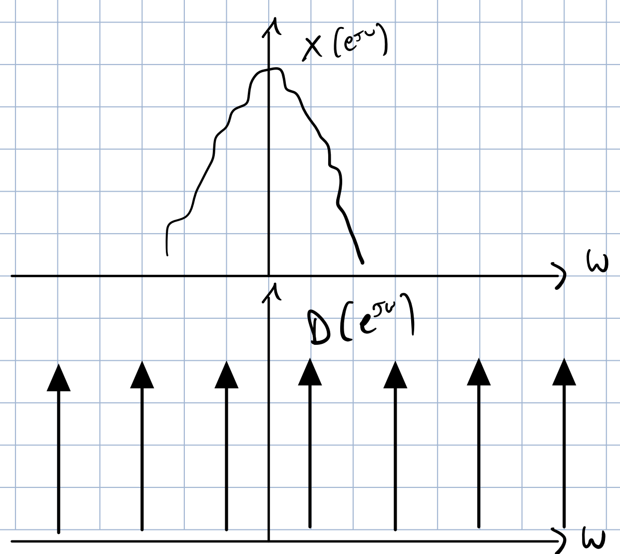
$$x_e[n] = \begin{cases} 1 & 0 \le n \le 7 \\ 0 & 8 \le n \le 15 \end{cases}$$

IN QUESTO MODO $\Delta\omega$ È DIMINUITA MA GLI ZERI NON HANNO IMPATTO SULLA DFT

$$X_e[k] \sum_{n=0}^{15} x_e[n] \cdot e^{-j\frac{2\pi}{16}k} = e^{\frac{-j16\omega}{2}} \cdot \frac{\sin(8\omega)}{\sin(\omega/2)}$$



LA TRASFORMATA DISCRETA DI FOURIER È QUINDI UN CAMPIONAMENTO DELLA TRASFORMATA A TEMPO DISCRETO.



FREQUENZA:

$$X_s[k] = X(e^{j\omega}) \cdot D(e^{j\omega})$$

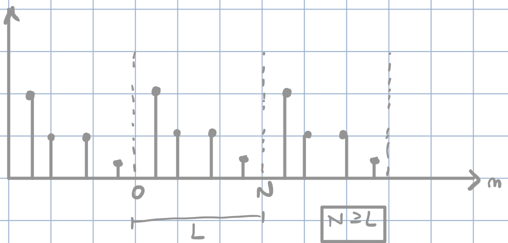
1D FT

TEMPO DISCRETO

$$x_s[n] = x[n] * d[n]$$

$$x_s[n] = x[n] * \sum_{l=-\infty}^{\infty} \delta[n-lN] = \sum_{l=-\infty}^{\infty} x[n-lN]$$

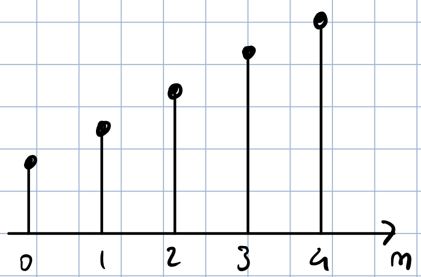
OTTENIAMO UNA VERSIONE PERIODICA DEL SEGNALE



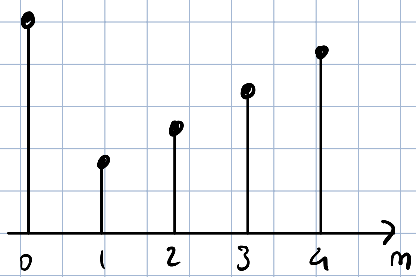
$\boxed{N \geq L}$ EQUIVALENTE DELLA CONDIZIONE DI NEIQUIST NEL TEMPO

ABBIAMO BISOGNO DI ADATTARE LE OPERAZIONI AI SEGNALI PERIODICI :

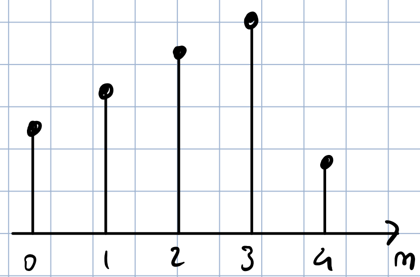
TIME SHIFT CIRCOLARE



$x[n]$



$x[\langle n-1\rangle_6] = x[\langle n+5\rangle_6]$

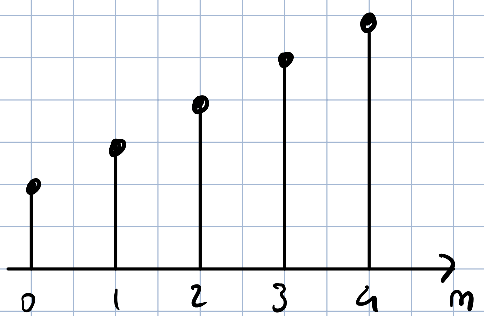


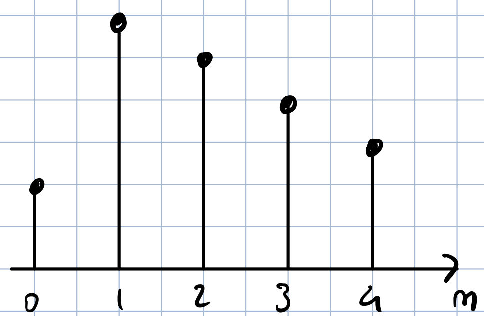
$x[\langle n-4\rangle_6] = x[\langle n+2\rangle_6]$

$$x[\langle n-n_0\rangle_N] = \begin{cases} x[n-n_0] & n_0 \leq n \leq N-1 \\ x[N+n-n_0] & 0 \leq n \leq n_0 \end{cases}$$

TIME REVERSAL CIRCOLARE

$$x[\langle -n\rangle_N] = \begin{cases} x[0] & n=0 \\ x[N-n] & 1 \leq n \leq N \end{cases}$$





CONVOLUZIONE LINEARE

$$y[n] = \underbrace{x[n]}_{N} * \underbrace{h[n]}_{M} = \sum_{m=-\infty}^{\infty} x[m] \cdot h[n-m]$$

$N+M-1$ TERMINI

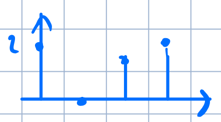
CONVOLUZIONE CIRCOLARE

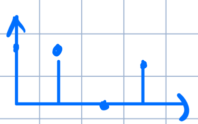
$$y[n] = \underbrace{x[n]}_{N} \underset{N}{\circledast} \underbrace{h[n]}_{N} = \sum_{m=0}^{N-1} x[m] \cdot h[\langle n-m \rangle_N]$$

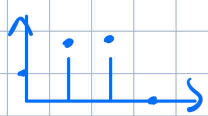
ESEMPIO:

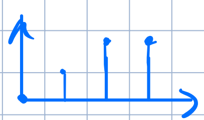
1

$$x[n] = \{\underset{\uparrow}{1}, 2, 0, 1\} \quad ; \quad h[n] = \{\underset{\uparrow}{2}, 2, 1, 0\}$$

$h[0-m] \Rightarrow$  $y_c[0] = 2\cdot 1 + 0\cdot 2 + 1\cdot 0 + 2\cdot 1 = 4$

$h[1-m] \Rightarrow$  $y_c[1] = 2\cdot 1 + 2\cdot 2 + 0\cdot 0 + 1\cdot 1 = 7$

$h[2-m] \Rightarrow$  $y_c[2] = 1\cdot 1 + 2\cdot 2 + 2\cdot 0 + 0\cdot 1 = 5$

$h[3-m] \Rightarrow$  $y_c[3] = 0\cdot 1 + 1\cdot 2 + 2\cdot 0 + 2\cdot 1 = 4$

$$y_c[n] = \{\underset{\uparrow}{4}, 7, 5, 4\}$$

② $y[n] = x[n] * h[n]$

$x[n] = \{1, 2, 0, 1\}$ $N=4$

$h[n] = \{2, 2, 1, 0\}$ $M=4$

$N+M-1 = 7$

	1	2	0	1
2	2	4	0	2
2	2	4	0	2
1	1	2	0	1
0	0	0	0	0

$y[n] = \{2, 6, 5, 4, 2, 1, 0\}$

$\Downarrow$

$y_c[n] = \{4, 7, 5, 4\}$

TEOREMA DI PARSEVAL

$$x[n] \circledast h[n] \xleftrightarrow{DFT} X[k]\,H[k]$$

$$X[k] = \sum_{n=0}^{N-1} x[n] \cdot e^{-jk\frac{2\pi}{N}n}$$

$W_N = e^{-j\frac{2\pi}{N}}$ TWIDDLE FACTOR

IN FORMA PIÚ COMPATTA:

$$X[k] = \sum_{n=0}^{N-1} x[n] \cdot W_N^{kn} \qquad k = 0, 1, \dots N-1$$

ANTI-TRASFORMATA:

$$x[n] = \sum_{n=0}^{N-1} X[k] \cdot W_N^{-kn} \qquad n = 0, 1, \dots N-1$$

IN FORMA MATRICIALE:

$$X = D_N\, x$$

$$\underbrace{\begin{bmatrix} X[0] \\ \vdots \\ X[N-1] \end{bmatrix}}_{X} = \underbrace{\begin{bmatrix} 1 & \cdots & 1 \\ \vdots & & W^{N-1} \\ 1 & W_N^{N-1} & W_N^{(N-1)^2} \end{bmatrix}}_{D_N} \underbrace{\begin{bmatrix} x[0] \\ \vdots \\ x[N-1] \end{bmatrix}}_{x}$$

ANTITRASFORMATA

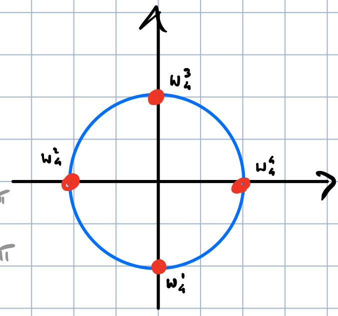
$$\boxed{D_N^{-1}} = \frac{1}{N} D_N^{*}$$

$W_4^1 = e^{-j\frac{2\pi}{4}} \quad -\frac{\pi}{2}$

$W_4^2 = e^{-j\cancel{\frac{2\pi}{4}}\cancel{2}} \quad -\pi$

$W_4^3 = e^{-j\frac{2\pi}{4}3} \quad -\frac{3}{2}\pi$

$W_4^4 = e^{-j\cancel{\frac{2\pi}{4}}\cancel{4}} \quad -2\pi$



DTFT 4 PUNTI

$$D_4 = \begin{bmatrix} (W_4^0)^0 & (W_4^0)^1 & (W_4^0)^2 & (W_4^0)^3 \\ (W_4^1)^0 & (W_4^1)^1 & (W_4^1)^2 & (W_4^1)^3 \\ (W_4^2)^0 & (W_4^2)^1 & (W_4^2)^2 & (W_4^2)^3 \\ (W_4^3)^0 & (W_4^3)^1 & (W_4^3)^2 & (W_4^3)^3 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & j \end{bmatrix}$$

CONVOLUZIONE LINEARE = CONVOLUZIONE CIRCOLARE

$x[n] \to$ LUNGHEZZA N

$h[n] \to$ LUNGHEZZA M

$x[n] * h[n] \to N+M-1$

NEL MONDO DELLA FREQUENZA È DIVERSO, QUINDI APPLICHIAMO IL ZERO PADDING

$$x_e[n] = \begin{cases} x[n] & 0 \le n \le M-1 \\ 0 & M \le n \le L-1 \end{cases}$$

$$h_e[n] = \begin{cases} h[n] & 0 \le n \le M-1 \\ 0 & M \le n \le L-1 \end{cases}$$

$$x[n] * h[n] = x_e[n] \circledast h_e[n]$$

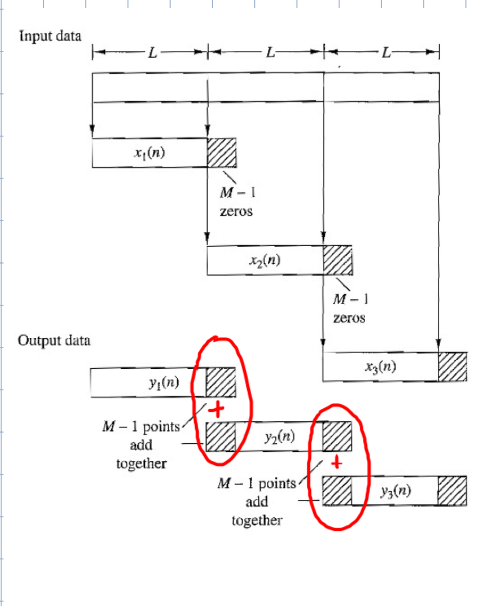
Questo metodo in real time introduce troppo ritardi, quindi sono utilizzate tecniche diverse per la convoluzione tra due sequenze di grandezza diversa.

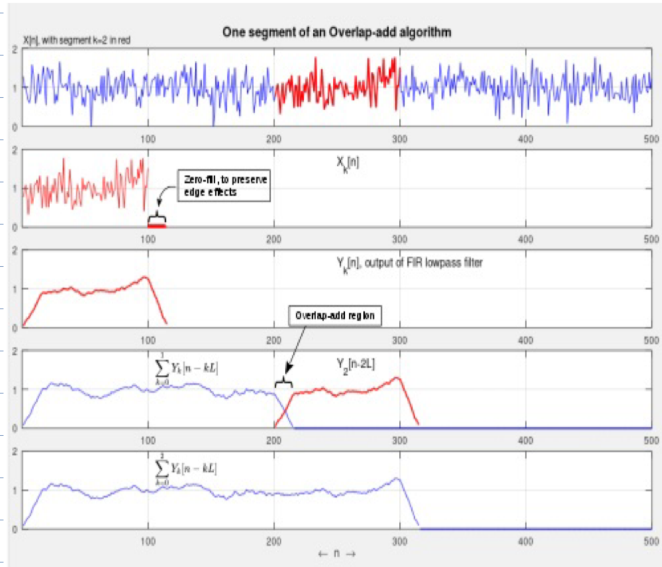
METODO OVERLAP AND ADD

Con questo metodo vengono preservati i bordi.

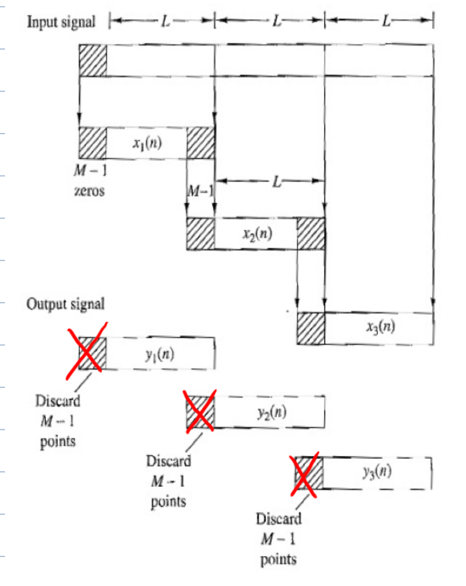
$$L + M - 1 = N$$

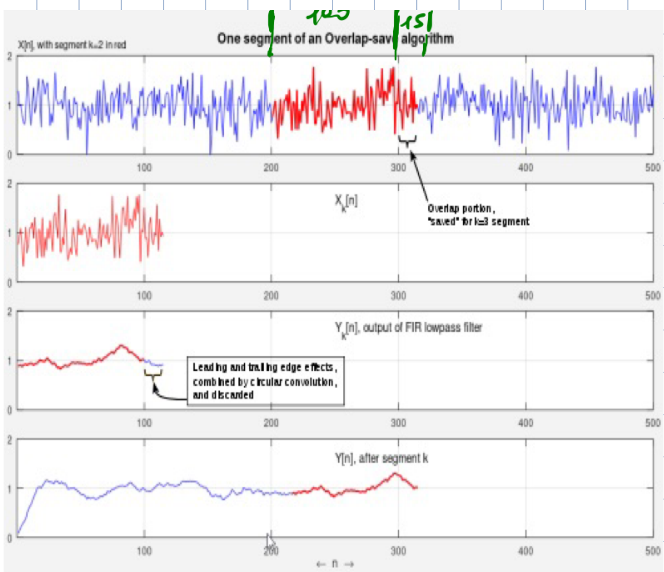
$$M + (L - 1) = N$$





METODO OVERLAP AND SAVE



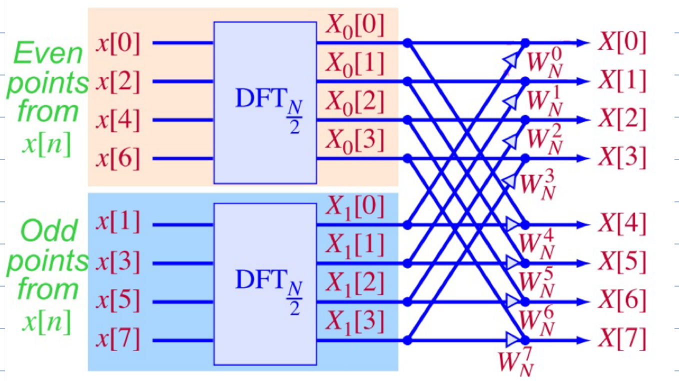


$$X[k] = \sum_{n=0}^{N-1} x[n] \cdot W_N^{kn}$$

$$X[k] = \sum_{m=0}^{\frac{N}{2}-1} \left(x[2m]\, W_N^{2mk} + X[2m+1] \cdot W_N^{(2m+1)k} \right) =$$

$W_N^{2m} = W_{\frac{N}{2}}^{m}$

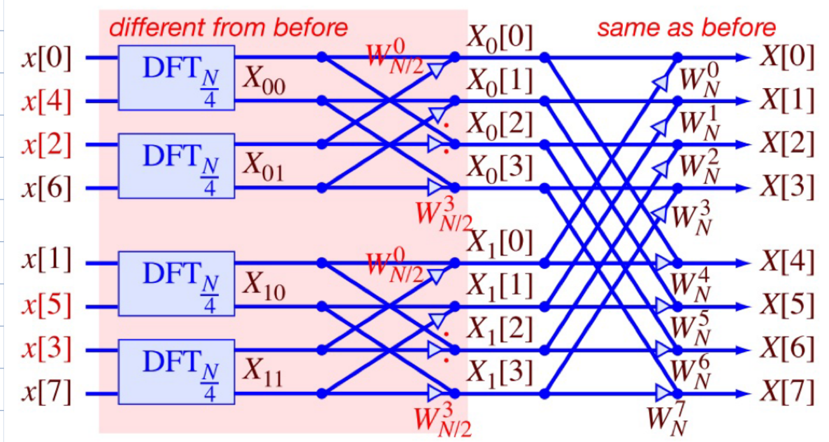
$$= \sum_{m=0}^{\frac{N}{2}-1} x[2m]\, W_N^{mk} + W_{\frac{N}{2}}^{k} \sum_{m=0}^{\frac{N}{2}-1} X[2m+1] \cdot W_N^{mk}$$



OPERAZIONI : $2\left(\frac{N}{2}\right)^2 = \frac{N^2}{2}$

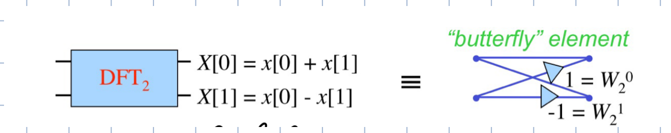
ABBIAMO GIA' DIMEZZATO LE OPERAZIONI.

POSSIAMO ANCORA RIDURRE :

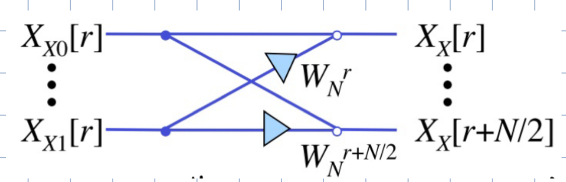


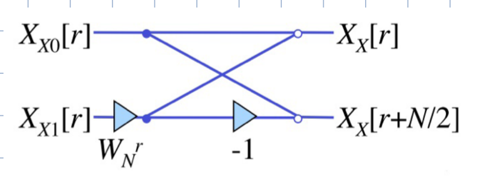
$$4\left(\frac{N}{4}\right)^2 = \frac{N^2}{4}$$

INFINE :



$$X[k] = \sum_{n=0}^{1} x[n] \, W_2^{kn}$$





$$W^{r+\frac{N}{2}} = W^r \cdot W^{\frac{N}{2}} = W_N^r \cdot e^{-j\frac{2\pi}{2}\cdot\frac{N}{2}} = -W_N^r$$

$$X_x[r] = X_{x_0}[r] + W_N^r \cdot x_1[r]$$

$$X_x[r+N/2] = X_{x_0}[r] + W_N^r x_1[r]$$

TRASFORMATA z

$$X(z) = \sum_{n=-\infty}^{\infty} x[n] \cdot z^{-n}$$

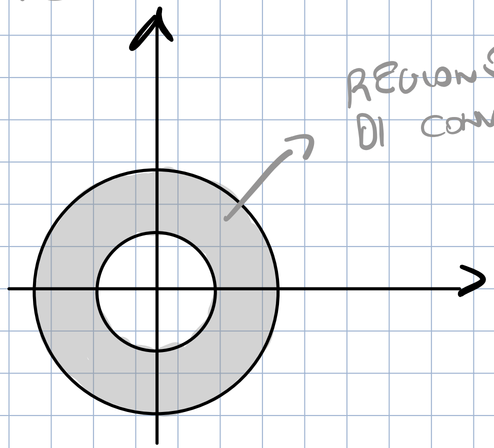
$$z = r \cdot e^{j\omega} \qquad X(z) = \sum_{n=-\infty}^{\infty} \left(x[n] \cdot r^{-n}\right) e^{-j\omega n}$$

SE $|z| = 1$ ABBIAMO LA TRASFORMATA:

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n] \cdot e^{-j\omega n} = X(z)\Big|_{z=e^{j\omega}}$$

$$\sum_{n=-\infty}^{\infty} \left| x[n] \cdot r^{-n} \right| < \infty \Rightarrow$$ DEFINISCE IL ROC

ROC:



REGIONE DI CONVERGENZA

PERCHÈ ESISTA LA TRASFORMATA LA CIRCONFERENZA UNITARIA DEVE ESSERE NEL ROC

ESEMPI

(1) $x[n] = \alpha^n \cdot u[n]$

TRASFORMATA Z:

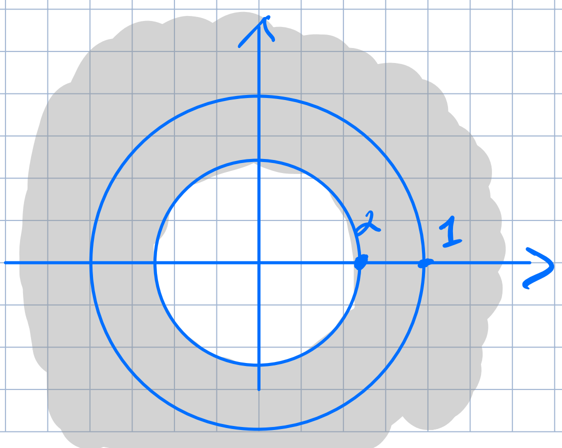
A CAUSA DEL GRADINO

$$X(z) = \sum_{n=-\infty}^{\infty} x[n] \cdot z^{-n} = \sum_{n=-\infty}^{\infty} \alpha^n \cdot u[n] \cdot z^{-n} = \sum_{n=0}^{\infty} \alpha^n \cdot z^{-n}$$

$$= \sum_{n=0}^{\infty} \left(\frac{\alpha}{z}\right)^n \Rightarrow X(z) = \frac{1}{1-\frac{\alpha}{z}} = \frac{z}{z-\alpha}$$

$$\left|\frac{\alpha}{z}\right| < 1 \qquad |z| > \alpha$$

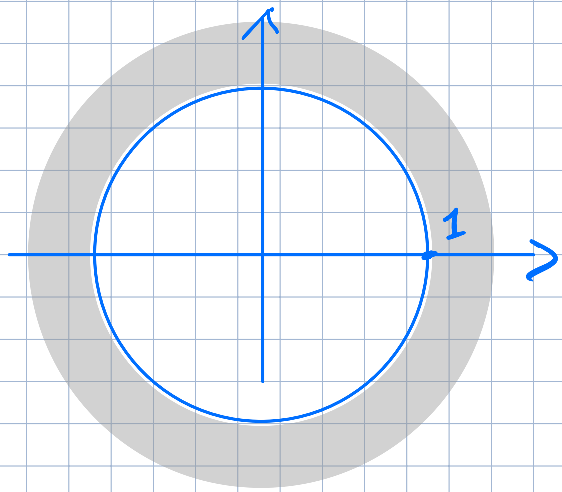
SE $|\alpha| < 1 \Rightarrow$ ESISTE LA DTFT



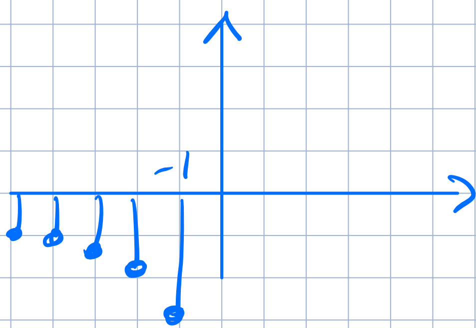
(2) $x[n] = u[n]$

$$X(z) = \sum_{n=-\infty}^{\infty} u[n] \cdot z^{-n} = \sum_{n=0}^{\infty} \left(\frac{1}{z}\right)^n$$

$\left|\frac{1}{z}\right| < 1 \qquad |z| > 1$



(3) $x[n] = -\alpha^n u[-n-1]$



$$X(z) = \sum_{n=-\infty}^{\infty} x[n] \cdot z^{-n} =$$

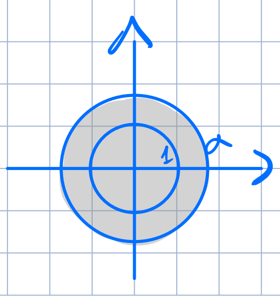
$-n = m$

$$\sum_{n=-\infty}^{\infty} -\alpha^n u[-n-1] \cdot z^{-n} = \sum_{n=-\infty}^{-1} \left(\frac{\alpha}{z}\right)^n = \sum_{m=1}^{\infty} -\left(\frac{z}{\alpha}\right)^m = \frac{-\alpha}{\alpha - z} + 1$$

$$= \frac{z}{z-\alpha}$$

$\left|\frac{z}{\alpha}\right| < 1 \qquad |z| < |\alpha|$

Se $\alpha > 1$ $\Rightarrow$ AMMETTE DTFT



NELLE SEQUENZE INFINITE QUANDO ABBIAMO SEGNALI CAUSALI LA REGIONE È ESTERNA, SE ANTICAUSALE È INTERNA, SENNÒ È UN' ANELLO.

DIMOSTRAZIONE:

$$\sum_{n=-\infty}^{\infty} \left| x[n] \cdot r^{-n} \right| = \sum_{n=-\infty}^{-1} \left| x[n] \cdot r^{-n} \right| + \sum_{n=0}^{\infty} \left| x[n] \cdot r^{-n} \right|$$

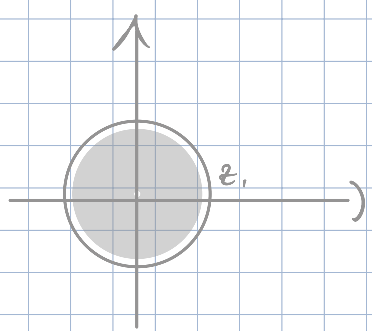
$n = -m$

$$= \underline{\sum_{m=1}^{\infty} \left| x[m] \cdot r^{m} \right|} + \underline{\sum_{n=0}^{\infty} \left| x[n] \cdot r^{-n} \right|} =$$

ANTICAUSALE:

SE z_1 È IL NUMERO PER CUI ABBIAMO CONVERGENZA

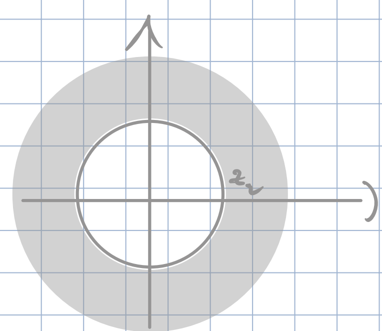
$$|z| < |z_1|$$



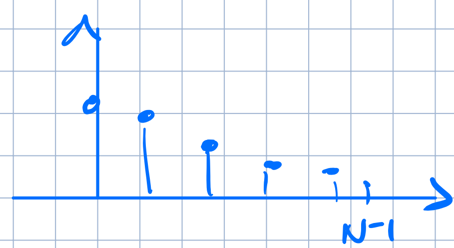
CAUSALE:

SE z_2 È IL NUMERO PER CUI ABBIAMO CONVERGENZA

$$|z| > |z_2|$$



4)
$$x[n] = \begin{cases} \alpha^n & 0 \le n \le N-1 \\ 0 & \text{ALTRIMENTI} \end{cases}$$



$$X(z) = \sum_{n=-\infty}^{\infty} x[n] \cdot z^{-n} = \sum_{n=0}^{N-1} (\alpha \cdot z^{-1})^n = \frac{1-\left(\frac{\alpha}{z}\right)^N}{1-\frac{\alpha}{z}}$$

$z \neq 0$

NELLE SEQUENZE FINITE

CAUSALI:

CONVERGE IN $|z| \neq 0$

ANTICAUSALI:

CONVERGE IN $|z| \neq \infty$

BILATERE

CONVERGE IN $0 < |z| < \infty$

POLI E ZERI

$$X(z) = \frac{N(z)}{D(z)} = \frac{b_0 + b_1 \cdot z^{-1} + b_2 \cdot z^{-2} + \ldots + b_M \cdot z^{-M}}{a_0 + a_1 \cdot z^{-1} + a_2 \cdot z^{-2} + \ldots + b_N \cdot z^{-N}}$$

RICORDANO LE EQUAZIONI ALLE DIFFERENZE

M = ZERI DEL NUMERATORI => ZERI DELLA Z T

N = ZERI DEL DENOMINATORE => POLI

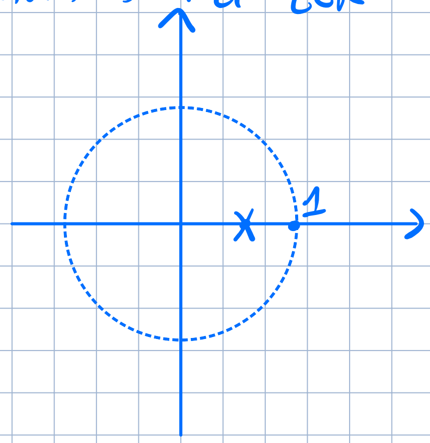
$$X(z) = \frac{z}{z-\alpha}$$

ZERO : $z = 0$

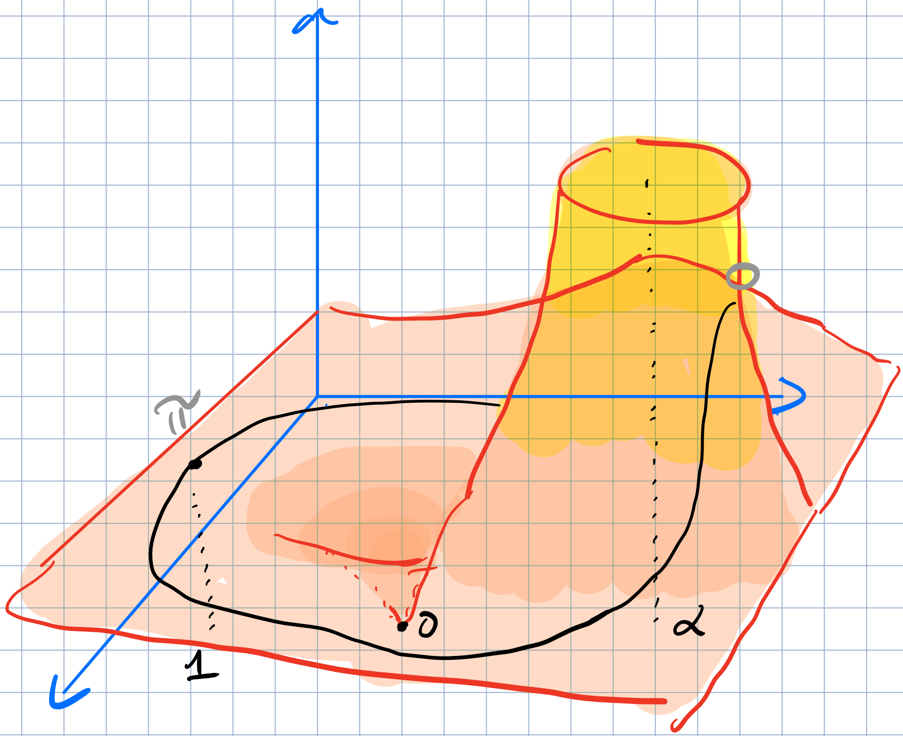
POLO : $z - \alpha = 0 \quad z = \alpha$

ESEMPI

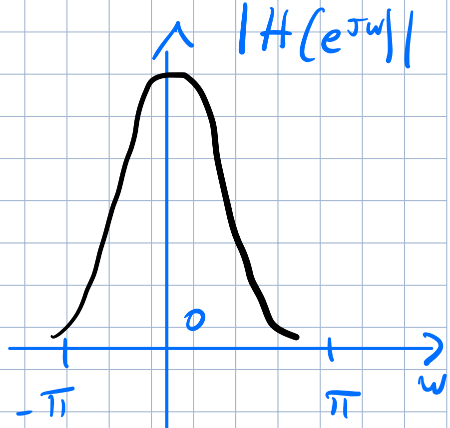
DIAGRAMMA POLI-ZERI



(1) $$X(z) = \frac{z}{z-\alpha} \qquad \alpha = 0.8$$



$\Rightarrow$



23/11

ESERCIZI

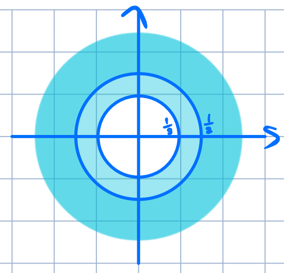
(1) $$x[n] = \left(\frac{1}{2}\right)^n \cdot u[n] + \left(-\frac{1}{3}\right)^n \cdot u[n]$$

$$X(z) = \sum_{n=0}^{\infty} \left(\frac{1}{2} \cdot z^{-1}\right)^n + \sum_{n=0}^{\infty} \left(-\frac{1}{3} \cdot z^{-1}\right)^n =$$

$$\left|\frac{1}{2z}\right| < 1 \qquad |z| > \frac{1}{2} \qquad \left|-\frac{1}{3z}\right| < 1 \qquad |z| > \left|\frac{1}{3}\right|$$

$$= \frac{1}{1-\frac{1}{2z}} + \frac{1}{1+\frac{1}{3z}} = \frac{z}{z-\frac{1}{2}} + \frac{z}{z+\frac{1}{3}}$$

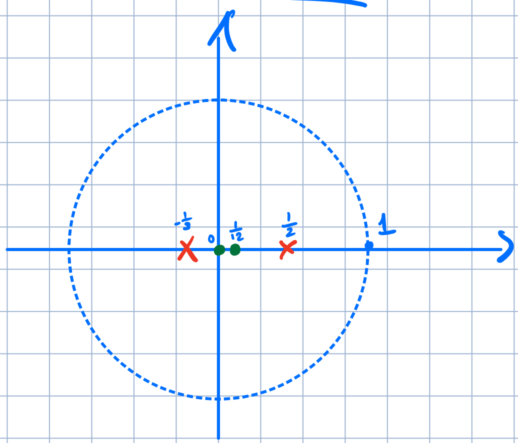
$$\text{ROC}: \quad \text{ROC}_1 \cap \text{ROC}_2 = |z| > \frac{1}{2}$$



$$\frac{z}{z-\frac{1}{2}} + \frac{z}{z+\frac{1}{3}} = \frac{z\left(z+\frac{1}{3}\right) + z\left(z-\frac{1}{2}\right)}{\left(z-\frac{1}{2}\right)\left(z+\frac{1}{3}\right)} =$$

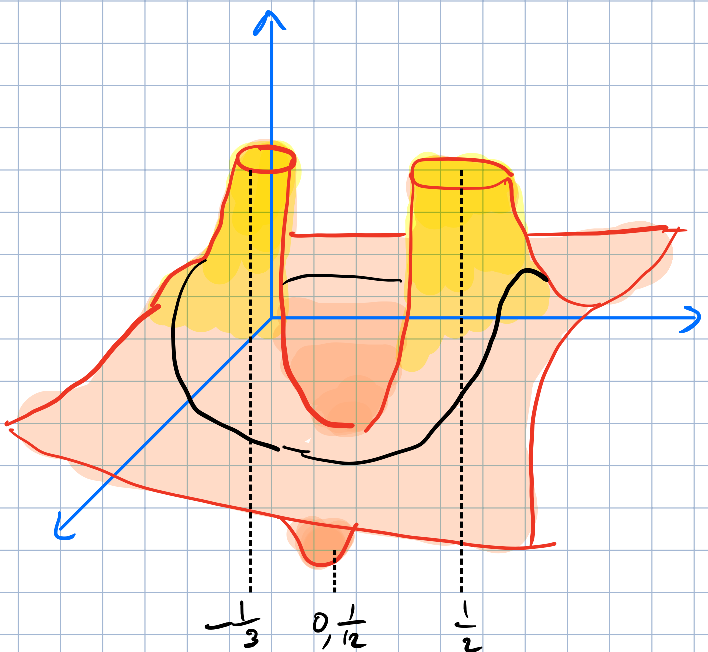
$$= \frac{2z\left(z-\frac{1}{12}\right)}{\left(z-\frac{1}{2}\right)\left(z+\frac{1}{3}\right)}$$

POLI - ZERI

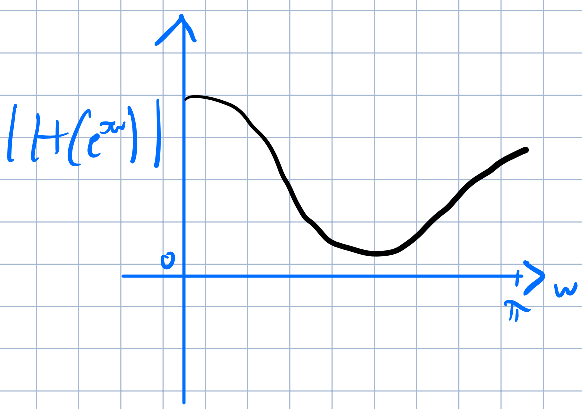


$$\boxed{\text{ZERI}} \begin{cases} z = 0 \\ z = \frac{1}{12} \end{cases}$$

$$\boxed{\text{POLI}} \begin{cases} z = \frac{1}{2} \\ z = -\frac{1}{3} \end{cases}$$



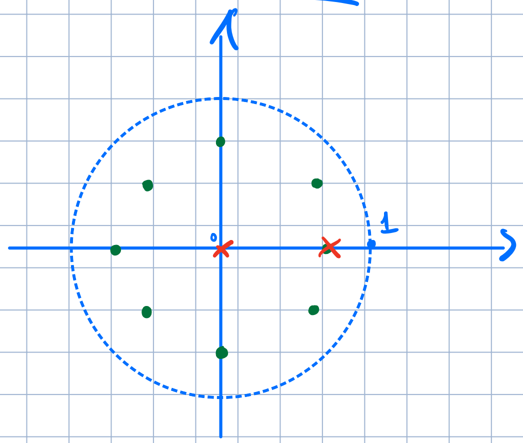
$\Rightarrow$



② $X(z) = \dfrac{z^N - \alpha^N}{z^{N-1}(z-\alpha)}$ FILTRO A MEDIA MOBILE

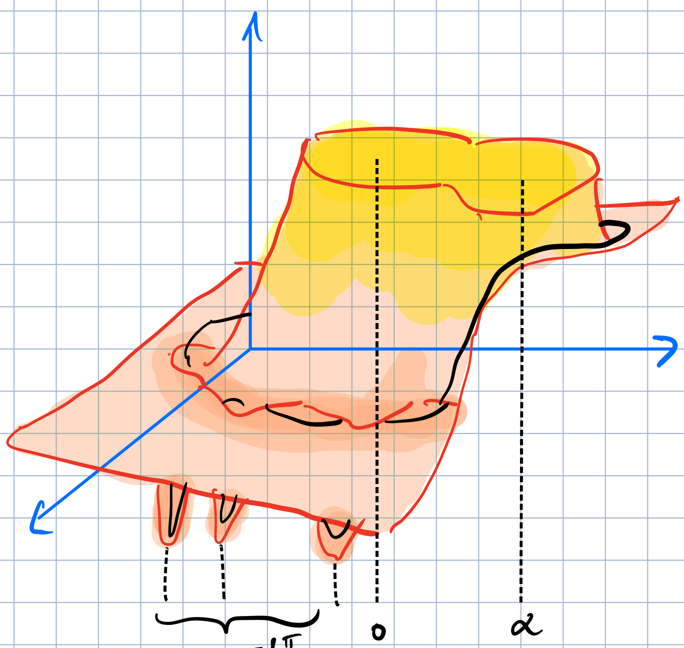
DIAMO UN VALORE AD α. (0,8)

POLI - ZERI

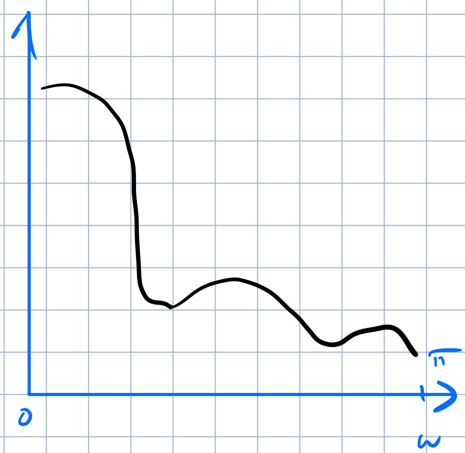


ZERI : $z^N - \alpha^N = 0$ $z_N = \alpha \cdot e^{j\frac{2\pi}{N}k}$

POLI : $z = \alpha$, $z = 0$ -> N-1 VOLTE



$\underbrace{\qquad\qquad}_{\alpha \cdot e^{j\frac{2\pi}{N}k}}$



TIME SHIFT TRASFORMATA Z

$$x[n-n_0] = z^{-n_0} \cdot X(z)$$

$n_0 > 0$ $\quad \frac{1}{z^{n_0}}$ $\quad z^{n_0} \neq 0$

$n_0 < 0$ $\quad z^{n_0}$ $\quad z^{n_0} \neq \infty$

TRASFORMATA DELLA DELTA

$$x[n] = \delta[n]$$

$$X(z) = \sum_{n=-\infty}^{\infty} \delta[n] \cdot z^{-n} = z^0 = 1$$ ROC $= \mathbb{C}$

$$\delta[n] \overset{z}{\longleftrightarrow} 1$$

SE APPLICHIAMO UN RITARDO:

$$x[n] = \delta[n-1]$$

$$X(z) = \sum_{n=-\infty}^{\infty} \delta[n-1] \cdot z^{-n} = z^{-1}$$ ROC $= \mathbb{C} \setminus \{0\}$

$n_0 = 1$

SE APPLICHIAMO UN ANTICIPO

$x[n] = \delta[n+1]$ $\quad X(z) = \sum_{n=-\infty}^{\infty} \delta[n+1] \cdot z^{-n} = z$ $\quad$ ROC $= z \neq \infty$

TRASFORMATA DI SEQUENZA RIBALTATA

$$x[-n] \longleftrightarrow X\left(\frac{1}{z}\right)$$

ESEMPIO

$$x[n] = \alpha^{-n} \cdot u[-n] \qquad X(z) = \frac{1}{1-\alpha z} \qquad |z| < |\alpha^{-1}|$$

CONVOLUZIONE ATTRAVERSO LA TRASFORMATA Z

$$x[n] * h[n] \xleftrightarrow{z} X_{(z)} \cdot H_{(z)}$$

STEPS:

1. TRASFORMATE
2. PRODOTTO
3. ANTITRASFORMATA

ESSENDOCI L'ANTITRASFORMATA È PIÙ FACILE USARE QUELLA DI FOURIER PER LA CONVOLUZIONE

ANTITRASFORMATA Z

$$x[n] = \frac{1}{2\pi j} \oint X(z) \cdot z^{n-1} \, dz$$

ESERCIZI

1

$$x[n] = 2\left(\frac{1}{2}\right)^n u[n] + 2\left(-\frac{3}{4}\right)^n \cdot u[n]$$

$$X(z) = \sum_{n=0}^{\infty} 2\left(\frac{1}{2}\right)^n \cdot z^{-n} + \sum_{n=0}^{\infty} 2\cdot\left(-\frac{3}{4}\right)^n \cdot z^{-n} =$$

$$= 2\left(\sum_{n=0}^{\infty}\left(\frac{1}{2}\cdot z^{-1}\right)^n + \sum_{n=0}^{\infty}\left(-\frac{3}{4}\cdot z^{-1}\right)^n\right) =$$

$$= 2\left(\frac{1}{1-\frac{1}{2z}} + \frac{1}{1+\frac{3}{4z}}\right) = 2\left(\frac{1+\frac{3}{4z}+1-\frac{1}{2z}}{\left(1-\frac{1}{2z}\right)\left(1+\frac{3}{4z}\right)}\right) =$$

$$= 2\left(\frac{8z+1}{\left(1-\frac{1}{2z}\right)\left(1+\frac{3}{4z}\right)}\right) = \frac{16z+2}{\left(1-\frac{1}{2z}\right)\left(1+\frac{3}{4z}\right)}$$

$ROC_1: |z| > \frac{1}{2}$

$ROC_2: |z| > \frac{3}{4}$

$ROC_1 \cap ROC_2 = |z| > \frac{3}{4}$

2

$$x[n] = \left(\frac{1}{2}\right)^n \cdot u[n+2] + 3^n \cdot u[n-1]$$

$$4\cdot\left(\frac{1}{2}\right)^{n+2} \cdot u[n+2]$$

$$X_1(z) = 4 \cdot z^2 \cdot \sum_{n=-\infty}^{\infty}\left(\frac{1}{2}\right)^n \cdot u[n] \cdot z^{-n} = 4z^2 \cdot \sum_{n=0}^{\infty}\left(\frac{1}{2}\right)^n \cdot z^{-n}$$

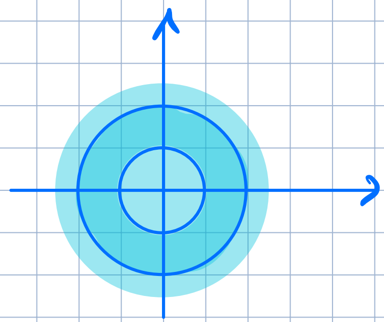
$$= \frac{4z^2}{1-\frac{1}{2z}} \qquad |z| > \frac{1}{2}$$

$$X_2(z) = \sum_{n=-\infty}^{-1} 3^n \cdot u[-n-1] \cdot z^{-n} = \sum_{n=-\infty}^{-1} \left(\frac{3}{z}\right)^n = \sum_{m=0}^{\infty} \left(\frac{z}{3}\right)^m - 1 =$$

$n = -m$

$$= \frac{1}{1-\frac{z}{3}} - 1 = \frac{z}{3-z} \qquad \left|\frac{z}{3}\right| < 1 \qquad |z| < 3$$

$$RoC_1 \cap RoC_2 = \frac{1}{2} < |z| < 3$$



③ $x[n] = \left(\frac{4}{5}\right)^n \cdot u[n]$

$h[n] = \delta[n] - \left(\frac{4}{5}\right) \delta[n-1]$

CALCOLARE $x[n] * h[n]$

$$X(z) = \sum_{n=0}^{\infty} \left(\frac{4}{5} \cdot z^{-1}\right)^n = \frac{1}{1-\frac{4}{5z}} \qquad |z| > \frac{4}{5}$$

$$H(z) = 1 - \frac{4}{5} \cdot z^{-1}$$

$$X(z) \cdot H(z) = \frac{1}{1-\frac{4}{5z}} \cdot \left(1 - \frac{4}{5} z^{-1}\right) = 1$$

TRASFORMATA INVERSA CON TABELLE:

$y[n] = \delta[n]$

④ $H(z) = 1 + z^{-1} + z^{-3}$

IZT : $h[n] = \delta[n] + \delta[n-1] + \delta[n-3]$

⑤ $H(z) = \dfrac{\frac{1}{2}z}{z^2 - z + \frac{1}{4}} = \dfrac{\frac{1}{2}z}{\left(z-\frac{1}{2}\right)^2} = \dfrac{\frac{1}{2}\cdot z^{-1}}{(1-.5\cdot z^{-1})^2} =$

TABELLE

$h[n] = n\left(\frac{1}{2}\right)^n \cdot u[n]$

Sequence	Transform	ROC
$\delta[n]$	1	All z
$u[n]$	$\frac{1}{1-z^{-1}}$	$\lvert z\rvert > 1$
$-u[-n-1]$	$\frac{1}{1-z^{-1}}$	$\lvert z\rvert < 1$
$\delta[n-m]$	z^{-m}	All z except 0 or ∞
$a^n u[n]$	$\frac{1}{1-az^{-1}}$	$\lvert z\rvert > \lvert a\rvert$
$-a^n u[-n-1]$	$\frac{1}{1-az^{-1}}$	$\lvert z\rvert < \lvert a\rvert$
$na^n u[n]$	$\frac{az^{-1}}{(1-az^{-1})^2}$	$\lvert z\rvert > \lvert a\rvert$
$-na^n u[-n-1]$	$\frac{az^{-1}}{(1-az^{-1})^2}$	$\lvert z\rvert < \lvert a\rvert$
$\begin{cases} a^n & 0 \le n \le N-1, \\ 0 & \text{otherwise} \end{cases}$	$\frac{1-a^N z^{-N}}{1-az^{-1}}$	$\lvert z\rvert > 0$
$\cos(\omega_0 n)u[n]$	$\frac{1-\cos(\omega_0)z^{-1}}{1-2\cos(\omega_0)z^{-1}+z^{-2}}$	$\lvert z\rvert > 1$
$r^n\cos(\omega_0 n)u[n]$	$\frac{1-r\cos(\omega_0)z^{-1}}{1-2r\cos(\omega_0)z^{-1}+r^2z^{-2}}$	$\lvert z\rvert > r$

FUNZIONE DI TRASFERIMENTO

ABBIAMO VISTO CHE:

$y[n] = x[n] * h[n]$

$\Updownarrow$

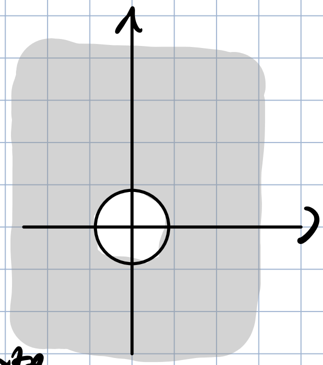
$Y(z) = X(z)\cdot H(z)$

$H(z) = \dfrac{Y(z)}{X(z)}$

DETTA FUNZIONE DI TRASFERIMENTO

SAPPIAMO CHE $h[n] = 0 \quad \forall n < 0$ NEI SISTEMI CAUSALI

$$H(z) = \sum_{n=-\infty}^{\infty} h[n] \cdot z^{-n} = \sum_{n=0}^{\infty} h[n] \cdot z^{-n}$$



SAPPIAMO CHE IL ROC È ESTERNO, IN QUESTO CASO IL POLO PIÙ LONTANO DAL CENTRO SI TROVERÀ SULLA CIRCONFERENZA IN QUANTO NON PUÒ TROVARSI NEL ROC.

STABILITÀ BIBO

$$\sum_{n=-\infty}^{\infty} |h[n]| < \infty$$

- SE $|z| = 1$ $\quad \sum_{n=-\infty}^{\infty} |h[n]| = \sum_{n=-\infty}^{\infty} |h[n] \cdot z^{-n}|$

RIPASSO

FILTRARE NEL DOMINIO DEL TEMPO VUOL DIRE

$$y[n] = x[n] * h[n]$$

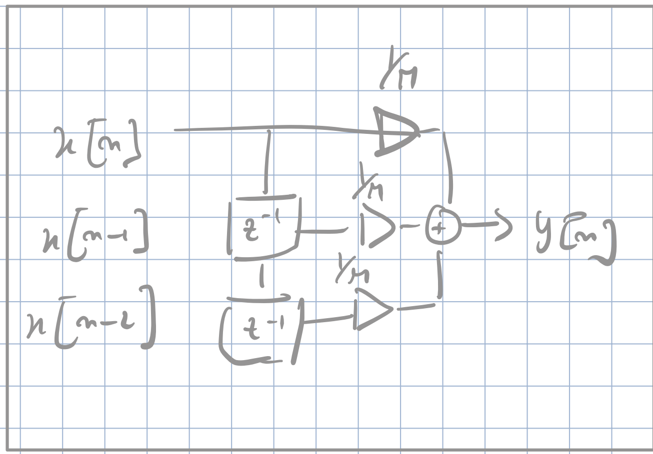
FARE LA CONVOLUZIONE.

TIPI DI FILTRI:

- FIR $\quad y[n] = \sum_{k=0}^{M} b_k \cdot x[n-k]$ $\quad$ M È IL NUMERO DI CAMPIONI
- IIR $\quad y[n] = \sum_{k=0}^{M} b_k \, x[n-k] - \sum_{k=1}^{N} a_k \cdot y[n-k]$ $\quad$ FORMA RICORSIVA

ESEMPIO

$$y[n] = \frac{1}{M} \sum_{k=0}^{M-1} x[n-k]$$



$$h[n] = \underbrace{\left\{ \frac{1}{M}, \frac{1}{M}, \dots \frac{1}{M} \right\}}_{M \text{ TERMINI}}$$

FILTRAGGIO NEL DOMINIO DELLA FREQUENZA

$$Y(e^{j\omega}) = X(e^{j\omega}) \cdot H(e^{j\omega})$$

$$+$$

IDTFT

PER IL FILTRO A MEDIA MOBILE

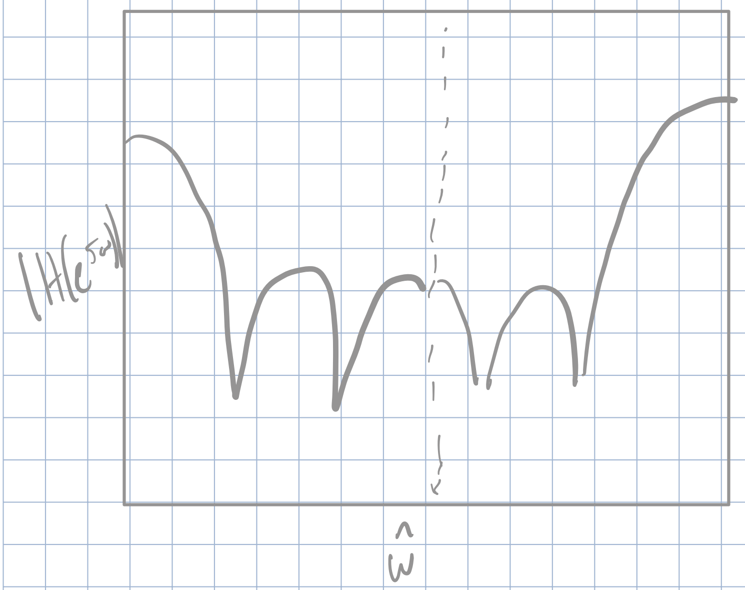
$$h[n] = \left\{ \frac{1}{5}, \frac{1}{5}, \frac{1}{5}, \frac{1}{5}, \frac{1}{5} \right\}$$

$$H(e^{j\omega}) = \sum_{n=-\infty}^{\infty} h[n] \cdot e^{-j\omega n} = \frac{1}{5} \sum_{n=0}^{4} e^{-j\omega n} =$$

$$= \frac{1}{5} \left(e^{-j\omega} + e^{j\omega} + e^{-j2\omega} + e^{-j3\omega} + e^{-j4\omega} \right)$$

$$= \frac{1}{5} e^{-j2\omega} \left[1 + 2\cos(\omega) + 2\cos(2\omega) \right]$$

DETERMINARE TIPO FILTRO



$\Rightarrow$ LOW PASS FILTER

FILTRI NEL DOMINIO Z

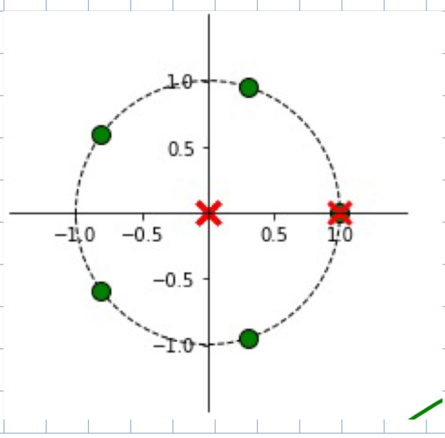
$$H(z) = \sum_{n=-\infty}^{\infty} h[n] \cdot z^{-n} \Rightarrow \frac{Y(z)}{X(z)}$$

PER IL FILTRO A MEDIA MOBILE

$$h[n] = \left\{ \frac{1}{M} \dots \frac{1}{M} \right\}$$

$$H(z) = \frac{1}{M} \sum_{n=0}^{M-1} z^{-n} = \frac{1}{M} \cdot \frac{1 - z^{-M}}{1 - z^{-1}} = \frac{z^{M}}{M} \cdot \frac{z^{M} - 1}{1 - z^{-1}}$$

$$= \frac{z^{M} - 1}{M \cdot z^{M-1} (z-1)}$$



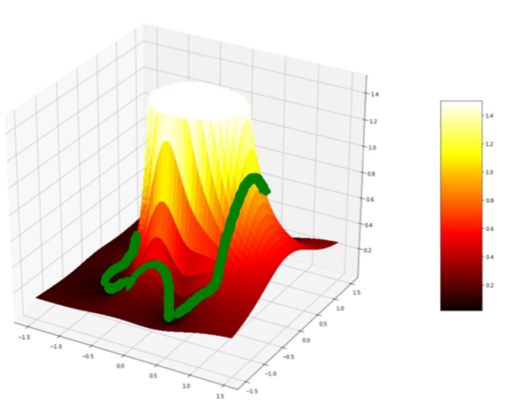
ZERI:

$$z^{M} - 1 = 0$$

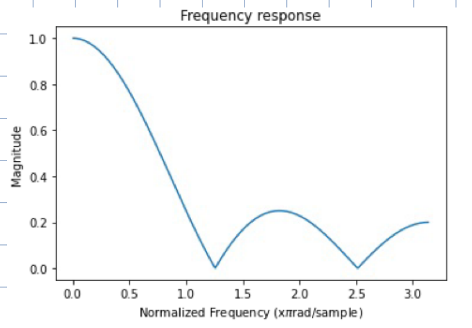
$$\Rightarrow z = e^{j\frac{2\pi}{M}k} \quad k = 0 .. M$$

POLI:

$$z = 1, \quad z = 0$$



$\Rightarrow$



ESERCIZI

$$y[n] - \frac{3}{4} y[n-1] + \frac{1}{8} y[n-2] = x[n] + 2x[n-1]$$

$$Y(z) = \left[1 - \frac{3}{4} \cdot z^{-1} + \frac{1}{8} \cdot z^{-2}\right] = X(z)\left(1 + 2z^{-1}\right)$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1+2z^{-1}}{1 - \frac{3}{4} \cdot z^{-1} + \frac{1}{8} z^{-2}} = \frac{z^2 \cdot \left(1 + 2z^{-1}\right)}{z^2 \cdot \left(1 - \frac{3}{4} \cdot z^{-1} + \frac{1}{8} \cdot z^{-2}\right)} =$$

$$= \frac{z^2 + 2z}{z^2 - \frac{3}{4} z + \frac{1}{8}}$$

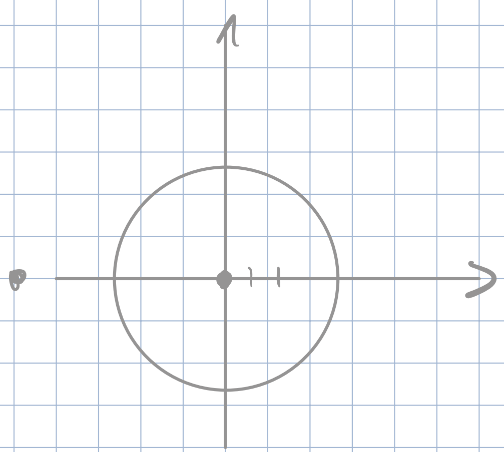
ZERI:

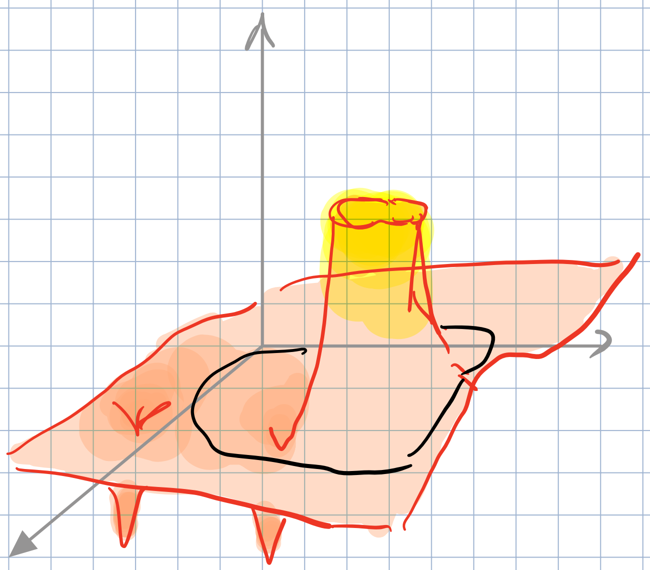
$$z^2 + 2z = \begin{cases} 0 \\ -2 \end{cases}$$

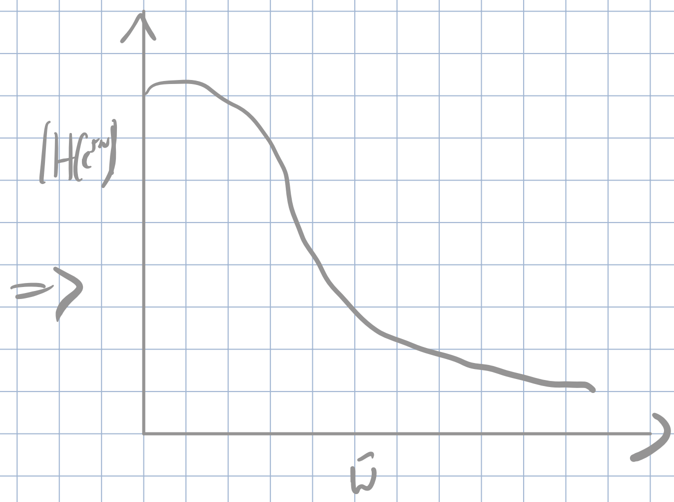
POLI:

$$z^2 - \frac{3}{4} z + \frac{1}{8} = 0$$

$$z = \frac{6 \pm \sqrt{36-32}}{16} \begin{cases} \frac{1}{4} \\ \frac{1}{2} \end{cases}$$

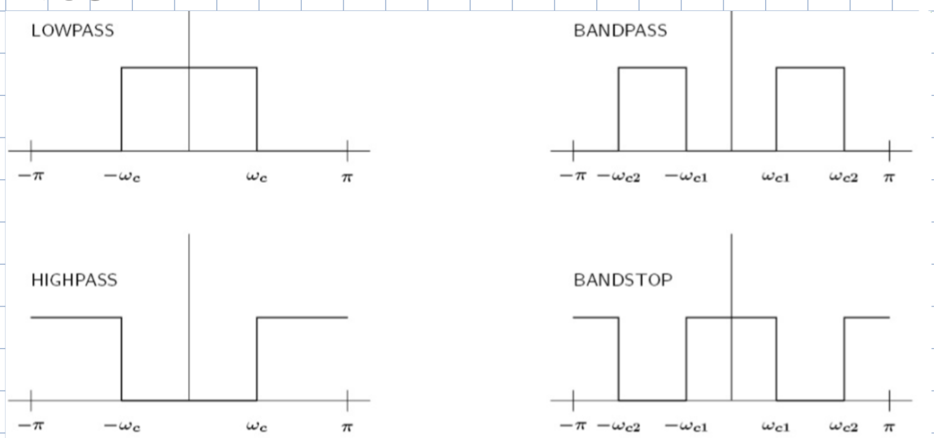






FILTRI

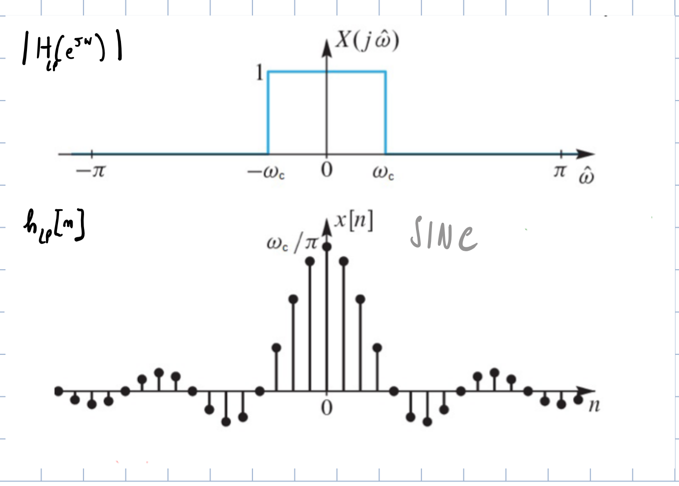
IDEALI :



STUDIAMO IL PASSABASSO POICHÈ DA ESSO POSSIAMO RICAVARE GLI ALTRI.

$$h_{lp}[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_{lp}(e^{j\omega}) \cdot e^{j\omega n}\, d\omega =$$

$$= \frac{1}{2\pi}\int_{-\omega_c}^{\omega_c} e^{j\omega n}\, d\omega = \frac{1}{2\pi} \cdot \frac{e^{j\omega n}}{jn}\Bigg|_{-\omega_c}^{\omega_c}$$

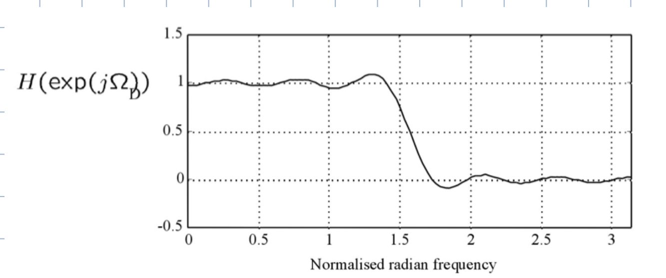


$$\boxed{h_{lp}[n] = \frac{\omega_c}{\pi}\, \mathrm{sinc}(\omega_c n)}$$

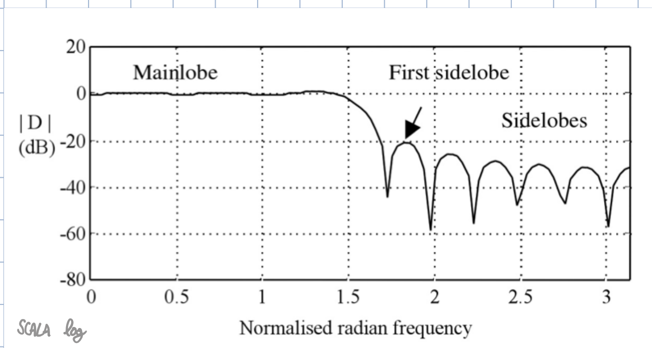
TAGLIAMO IL SINC E EFFETTUIAMO LO SHIFT PER RENDERLO CAUSALE.

$-\frac{M}{2} \to$ TIME SHIFT

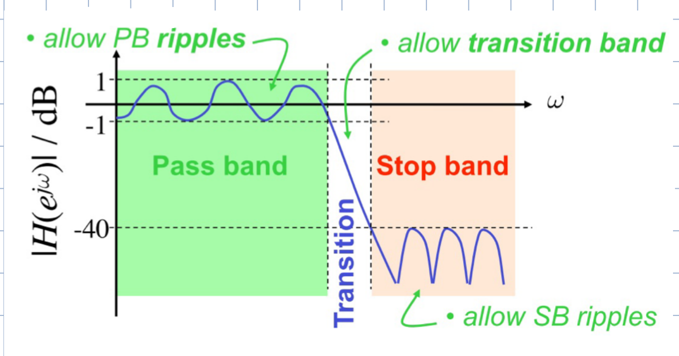
$$H(e^{j\omega}) = \frac{\omega_c}{\pi} \sum_{n=-M/2}^{M/2} \frac{\sin(n\cdot\omega_c)}{n\cdot\omega_c} \cdot e^{-j\omega n}$$



IL GRAFICO RISULTA OSCILLANTE A CAUSA DEL FENOMENO DI GIBBS



LA FINESTRA RETTANGOLARE CAUSA TAGLI DRASTICI. (PESSIMO FILTRO)



QUESTA È UNA VERSIONE REALISTICA E MIGLIORE DI UN FILTRO BAND-PASS E BAND-STOP.

Bel (unità di misura)

Da Wikipedia, l'enciclopedia libera.

Il **bel**, simbolo **B**, è un'unità di misura relativa con la quale una grandezza fisica (come la potenza di una radiazione o di un segnale) viene paragonata a un valore di riferimento su scala logaritmica. Nonostante non sia prevista all'interno del Sistema Internazionale, il suo uso è comunque tollerato. Viene spesso usato nel campo dell'acustica (potenza di un suono) o delle radiazioni elettromagnetiche (in particolare per indicare il guadagno o la perdita di un segnale radio).

Viene usato per esprimere il rapporto tra due grandezze omogenee.

Si calcola in questo modo:

$$L_{bel} = \log_{10}\left(\frac{x_1}{x_2}\right)$$

$$decibel \Rightarrow dB = 0,1B$$

$$\left(\frac{x_1}{x_2}\right) dB = 10\log_{10}\left(\frac{x_1}{x_2}\right)$$

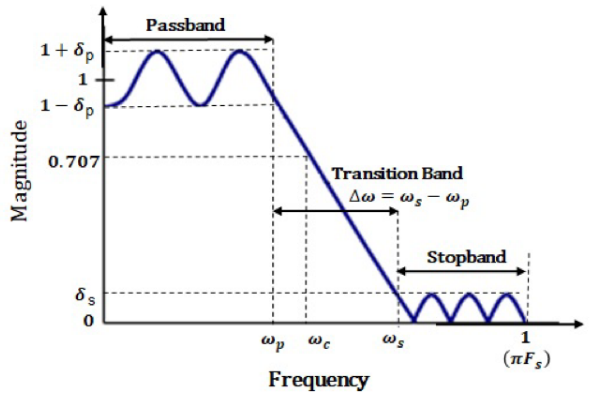
Uso del fattore 20 quando i dB sono riferiti alle potenze [modifica | modifica wikitesto]

In fisica e in ingegneria spesso si assume, senza neppure esplicitarlo, che i rapporti in dB che verranno calcolati siano sempre relativi a energie o potenze, anche partendo da altre grandezze da cui energie e potenze dipendono non linearmente come tensioni e correnti. Questo introduce nei calcoli un fattore 20 che può creare confusione.

È ciò che accade ad esempio in elettronica ed elettrotecnica quando si devono trattare rapporti in dB fra due grandezze indicanti tensioni o correnti elettriche, per esprimere un'amplificazione di tensione o di corrente. Infatti in questi casi non si intende il rapporto fra le grandezze stesse, ma fra le potenze che le tensioni o le correnti svilupperebbero se applicate a una medesima impedenza. Quindi, essendo la potenza W proporzionale al quadrato della tensione V o della corrente I, sfruttando le proprietà dei logaritmi si ricavano ed utilizzano le formule seguenti:

$$\text{Rapporto di potenze}_{dB} = 10\log_{10}\left(\frac{W1}{W2}\right) = 10\log_{10}\left(\frac{V1}{V2}\right)^2 = 20\log_{10}\left(\frac{V1}{V2}\right)$$

$$\text{Rapporto di potenze}_{dB} = 10\log_{10}\left(\frac{W1}{W2}\right) = 10\log_{10}\left(\frac{I1}{I2}\right)^2 = 20\log_{10}\left(\frac{I1}{I2}\right)$$



PASSBAND RIPPLE

$$20\log_{10}(1+dp) = 0.2\,dB$$

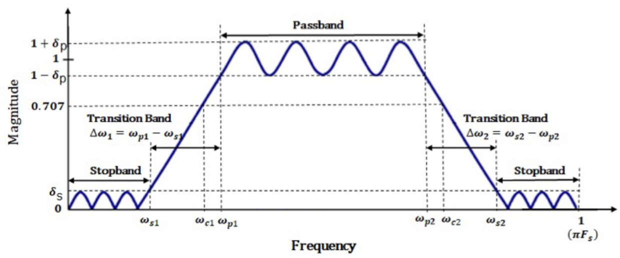
$$1+dp = 10^{0.2/20} = 1.023$$

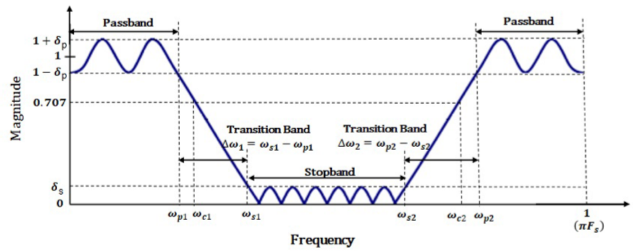
$$dP = 0.023$$

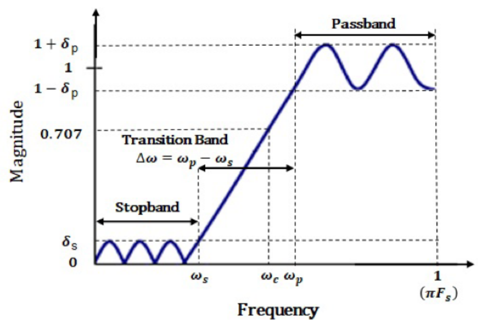
STOPBAND RIPPLE

$$20\log_{10}(dp) = 33\ dB$$

$$dp = 10^{-33/20} = 0{,}022$$







FREQUENZA DI TAGLIO

$$|H(e^{jw_c})|^2 = \frac{1}{2}|H_{max}(e^{jw})|$$

POTENZA DIMEZZATA

$$|H(e^{jw_c})| = \frac{1}{\sqrt{2}}|H_{max}(e^{jw})|$$

0,705 AMPIEZZA

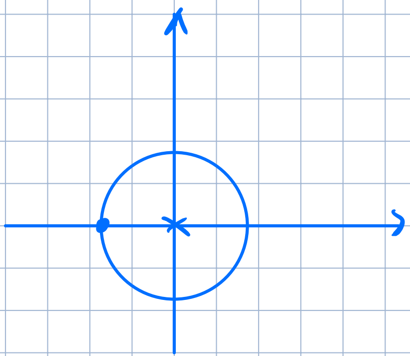
IN DECIBEL:

$$|H(e^{jw_c})| \overset{dB}{=} 20\log_{10}\frac{1}{\sqrt{2}} + |H_{max}(e^{jw})| = |H_{max}(e^{jw})| - 3dB$$

ESEMPI

① $h[n] = \left\{ \frac{1}{2}, \frac{1}{2} \right\}$ $\qquad y[n] = \frac{1}{2}x[n] + \frac{1}{2}x[n-1]$

$$H(z) = \frac{Y(z)}{X(z)} = \sum_{n=-\infty}^{\infty} h[n] \cdot z^{-n} = \frac{1}{2} + \frac{1}{2} \cdot z^{-1} = \frac{z+1}{2z}$$



$\Longrightarrow$ LOW PASS FILTER

$$H(e^{jw}) = \frac{1}{2} e^{-jw} \left(e^{jw/2} + e^{-jw/2} \right) = \underbrace{e^{-jw}}_{\text{FASE}} \cdot \underbrace{\cos\left(\frac{w}{2}\right)}_{\text{AMPIEZZA}}$$

$$\left| H(e^{jw}) \right| = \frac{1}{\sqrt{2}} = \cos\left(\frac{w_c}{2}\right)$$

$$w_c = 2\cos^{-1}\left(\frac{1}{\sqrt{2}}\right) = \frac{\pi}{4}$$

② $h[n] = \left\{ \frac{1}{2}, -\frac{1}{2} \right\}$

$$H(z) = \frac{1}{2} - \frac{1}{2} z^{-1} = \frac{z-1}{2z}$$

ZERO : $z = 1$

POLO : $z = 0$

$\Downarrow$

HIGH PASS

$$H(e^{jw}) = H_z \Big|_{e^{jw}} = \frac{1}{2} - \frac{1}{2} \cdot e^{-jw} =$$

$$= \frac{1}{2} e^{-\frac{jw}{2}} \cdot \left(e^{\frac{jw}{2}} - e^{-\frac{jw}{2}} \right) = j e^{-\frac{jw}{2}} \cdot \sin\left(\frac{w}{2}\right)$$

(3) COSTRUIRE UN FILTRO:

$w_1 = 0,1 \ rad/s \rightarrow$ SEGNALE

$w_2 = 0,7 \ rad/s \rightarrow$ NOISE

$$h[n] = \{\alpha, \beta, \alpha\}$$

$$H(e^{jw}) = \sum_{n=-\infty}^{\infty} h[n] \cdot e^{-jwn} = \alpha + \beta \cdot e^{-jw} + \alpha \cdot e^{-2jw} =$$

$$= e^{-jw}\left(\beta + \alpha\left(e^{jw} + e^{-jw}\right)\right) = e^{-jw} \cdot \left(\beta + 2\alpha\cos w\right)$$

$$|H(e^{jw})| = \begin{cases} 1 & w = w_1 \\ 0 & w = w_2 \end{cases}$$

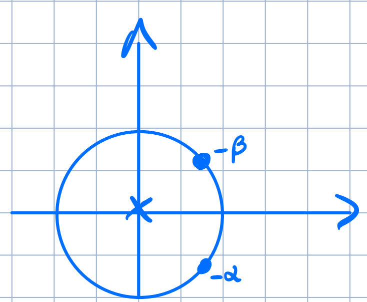
$$|H(e^{jw})| = |\beta + \alpha\cos w|$$

$$\begin{cases} \beta + 2\alpha\cos(0.1) = 1 \\ \beta + 2\alpha\cos(0.7) = 0 \end{cases} \Rightarrow \left\{ 2\alpha\left[\cos(0.1) - \cos(0.7)\right] = 1 \right.$$

$$\alpha = \frac{1}{2[\cos(0.1) - \cos(0.7)]} \qquad \beta = -\frac{\cancel{2}\cos(0.7)}{\cancel{2}(\cos(0.1) - \cos(0.7))}$$

$$H(z) = \alpha + \beta \cdot z^{-1} + \alpha \cdot z^{-2} = \alpha + \frac{\beta}{z} + \frac{\alpha}{z^2} =$$

$$= \frac{z^2 + \beta z + \alpha}{z^2} = \frac{z(z + \beta) + \alpha}{z^2}$$

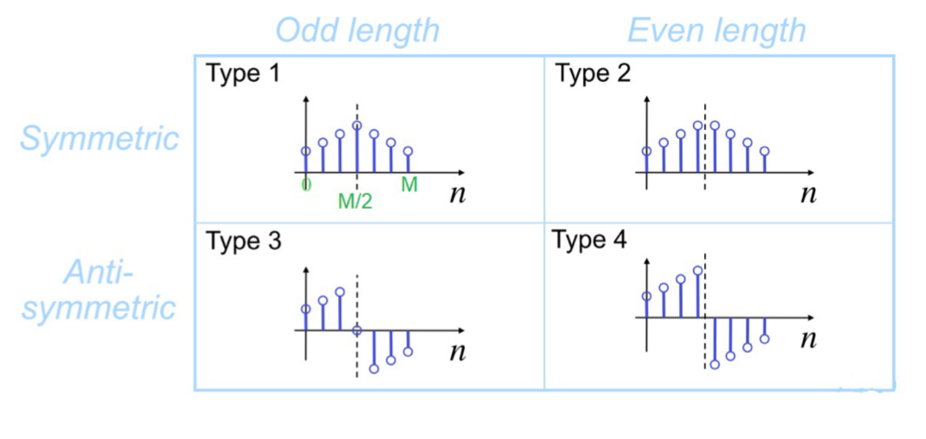


ZERI: $z = -\alpha$, $z = -\beta$

POLI: $z = 0$

FILTRO A FASE LINEARE (NON DISTORCENTE)

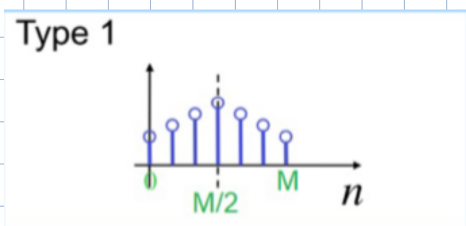
M ORDINE M+1 LUNGHEZZA



$\Rightarrow h[n] = h[M-n]$

$\Rightarrow h[n] = -h[M-n]$

• TIPO 1:



N DISPARI

M PARI

ESEMPIO

① $M = 8$ $N = 9$

$h[0] = h[8]$ $h[1] = h[7]$ $h[2] = h[6]$ $h[3] = h[5]$

$$H(z) = h[0] + h[1]z^{-1} + h[2]\cdot z^{-2} + h[3]\cdot z^{-3} + \ldots + h[8]\cdot z^{-8}$$

$$= h[0]\cdot(1+z^{-8}) + h[1](z^{-1}+z^{-7}) + h[2](z^{-2}+z^{-6}) + h[3](z^{-3}+z^{-5}) + h[4]\cdot z^{-4} =$$

$$= z^{-4}\left(h[0]\cdot(z^{4}+z^{-4}) + h[1](z^{3}-z^{-3}) + h[2](z^{2}-z^{-2}) + h[3](z+z^{-1}) + h[4]\right)$$

$$H(e^{j\omega}) = H(z)\big|_{e^{j\omega}}$$

$$H(e^{jw}) = e^{-j4w}\left(2h[0]\cos(4w)+2h[1]\cos(3w)+2h[2]\cdot\cos(2w)+2h[3]\cdot\cos(w)+h[4]\right)$$

IN GENERALE PER FILTRI DI QUESTO TIPO

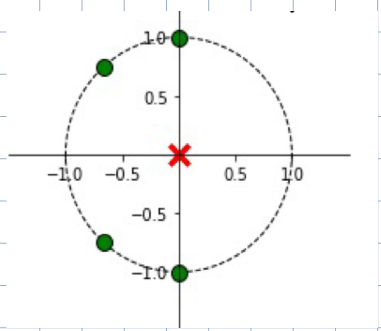
$$H(e^{jw}) = \underbrace{e^{-jw\frac{M}{2}}}_{\text{FASE}}\ \underbrace{\left(h\left[\tfrac{M}{2}\right] + 2\sum_{m=1}^{M/2} h\left[\tfrac{M}{2}-m\right]\cdot\cos(wm)\right)}_{\text{FUNZIONE REALE}}$$

$$\theta(w) = -4w = \begin{cases} -\frac{M}{2}w \rightarrow H_R(w)>0 \\ -\frac{M}{2}w+\pi \rightarrow H_R(w)<0 \end{cases}$$

$$\tau_g = -\frac{d\theta}{dw} = -\frac{d}{dw}\left(-\frac{M}{2}w\right) = \frac{M}{2}$$

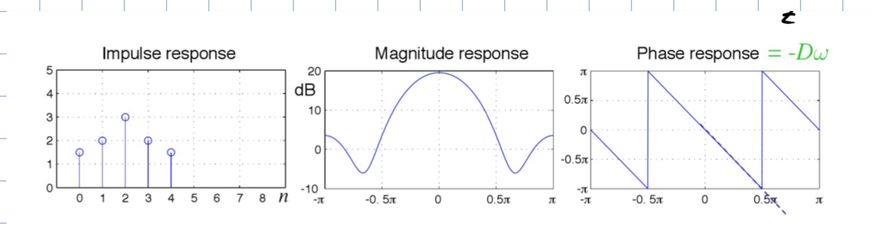
ESEMPIO

(1)



$$h[n] = \{1.5,\ 2,\ 3,\ 2,\ 1.5\}$$

$$H(z) = \sum_{n=0}^{M} h[n]\cdot z^{-n} + \sum_{n=0} h[M-n]\cdot z^{-n}$$



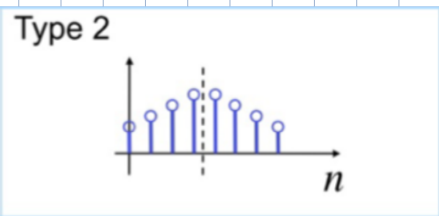
$$k = M - n$$

$$= \pm\sum_{k=M}^{0} h[k]\cdot z^{k-M} = \pm z^{-M}\sum_{k=0}^{M} h[k]\cdot z^{k}$$

$$H(z) = \pm z^{-M}\cdot H(z^{-1}) \qquad z_0 = r\cdot e^{\pm j\theta} \Rightarrow \frac{1}{z_0} = \frac{e^{\mp j\theta}}{r}$$

SE UNO DEGLI ZERI SI TROVA SULLA CIRCONFERENZA UNITARIA, IL SUO RECIPROCO SARÀ IL COMPLESSO CONIUGATO.

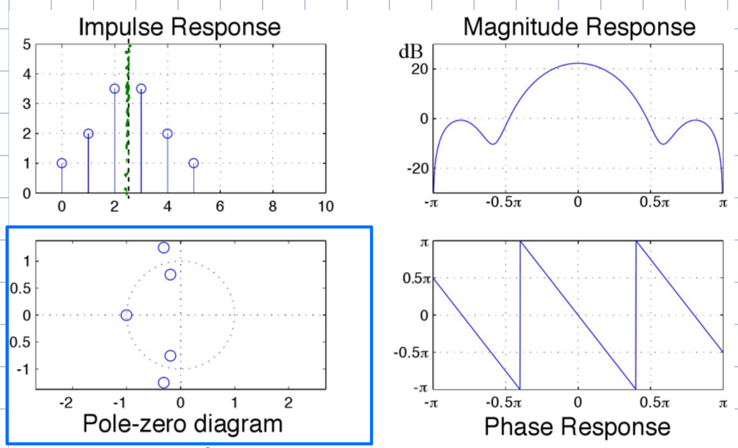
- TIPO 2:



$$H(e^{j\omega}) = e^{-j\omega\frac{M}{2}} \cdot \sum_{m=1}^{N/2} h[N/2 - m] \cdot \cos\left(\omega\left(m - \tfrac{1}{2}\right)\right)$$

$$\theta(\omega) = -\frac{M}{2}\omega \qquad \tau_g = \frac{M}{2}$$

ESEMPIO



$$h[n] = \{1, 2, 3.5, 3.5, 2, 1\}$$

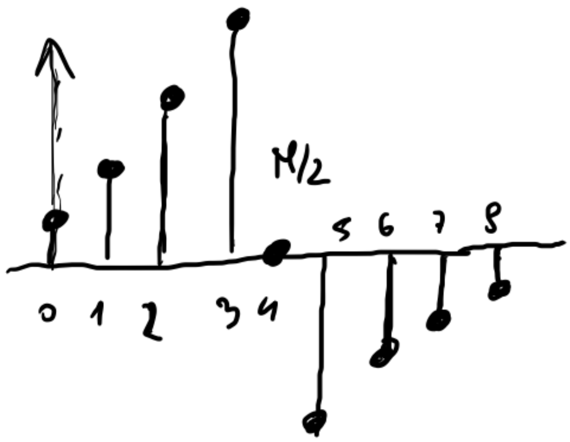
CI SONO 2 ZERI COINCIDENTI

$$H(z) = z^{-M}\left(H\left(\frac{1}{z}\right)\right)$$

$\boxed{\text{SE } z = -1}$

$$H(z^{-1}) = -1^{M}(H(-1)) \Rightarrow H(z^{-1}) = -H(z^{-1})$$

- **TIPO 3:** $h[n] = -h[M-n]$



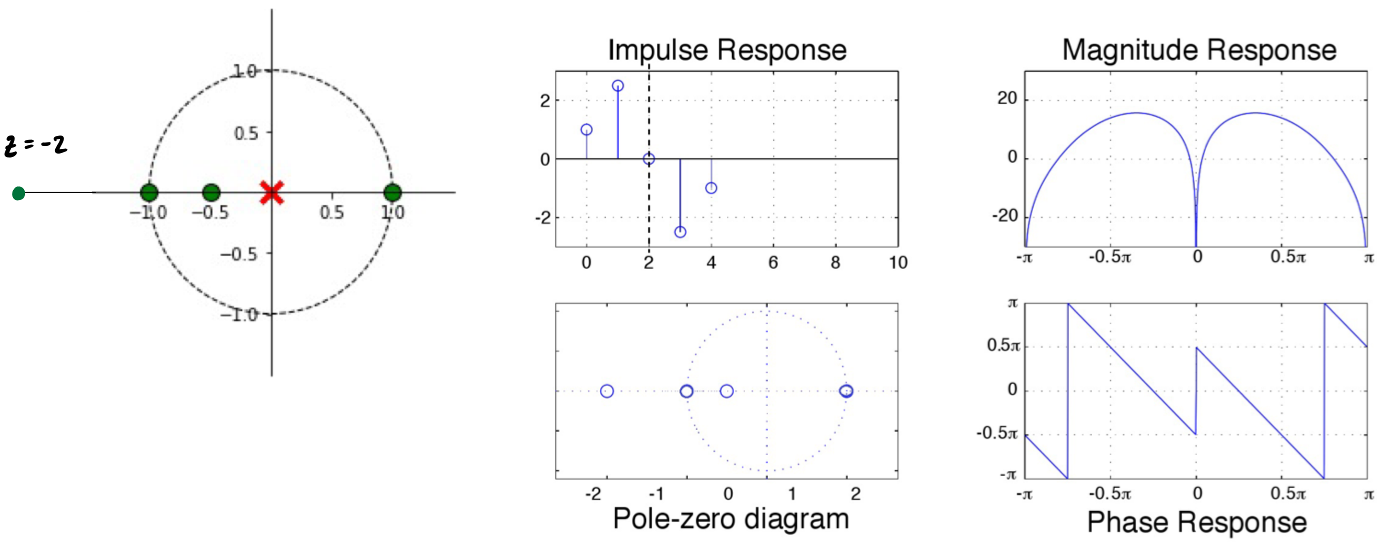
$$h[0] = -h[8]$$
$$h[1] = -h[7]$$
$$h[2] = -h[6]$$
$$h[3] = -h[5]$$
$$h[4] = -h[4] = 0$$

$$H(z) = h[0]\left(1-z^{-8}\right) + h[1]\left(z^{-1}-z^{-7}\right) + h[2]\left(z^{-2}-z^{-6}\right) + h[3]\left(z^{-3}-z^{-5}\right) + 0$$

ALLA FINE: $$H(e^{j\omega}) = j\, e^{-j\omega\frac{M}{2}} \left(2 \cdot \sum_{n=1}^{M/2} h\left[\frac{M}{2}-n\right] \cdot \sin(\omega n)\right)$$

ESEMPIO:

$$h[n] = \{1,\ 2.5,\ 0,\ -2.5,\ -1\}$$



$$H(z) = -z^{-M}\left(H\left(\tfrac{1}{z}\right)\right)$$

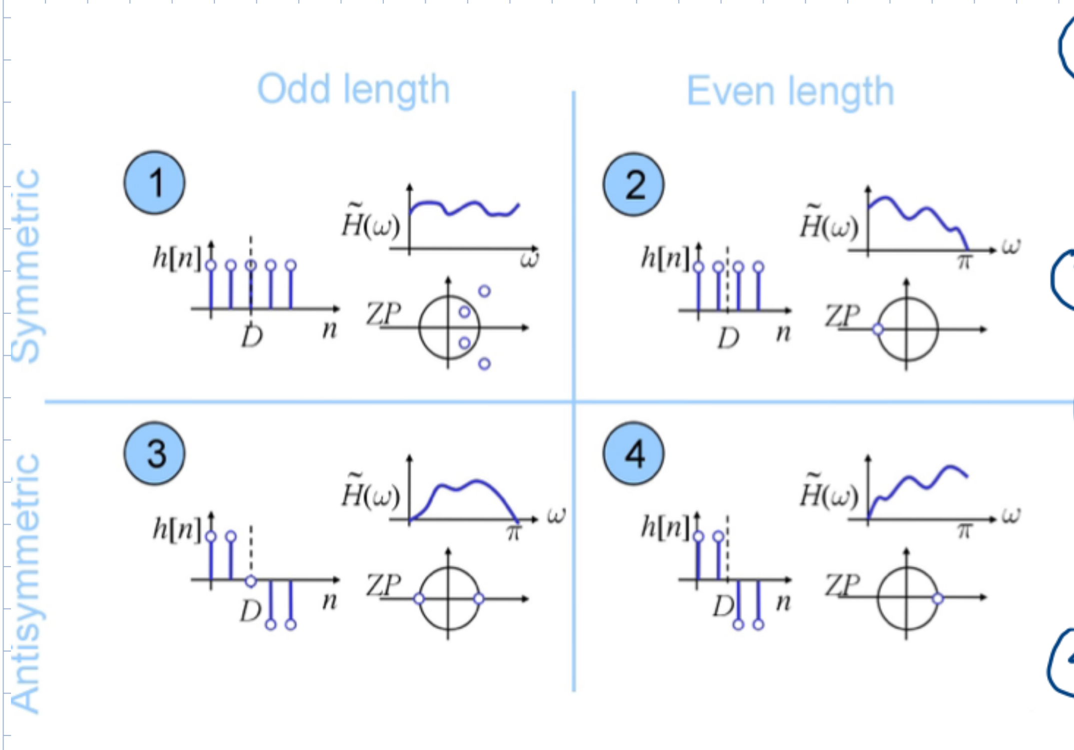
$\boxed{\text{SE } z=1}$ $$H(-1) = -(1^{-M})(H(1)) \Rightarrow H(1) = -H(1)$$

$$H(1-1) = -H(-1)$$

- **TIPO 4**

$$H(e^{j\omega}) = j \cdot e^{-j\omega\frac{M}{2}} \cdot \left(2\sum_{n=0}^{M/2} h\left[\tfrac{M+1}{2} - n\right] \cdot \sin\left(\omega\left(n - \tfrac{1}{2}\right)\right)\right)$$

SOMMARIO



(1) È il più versatile e può essere usato per qualsiasi tipo di filtro

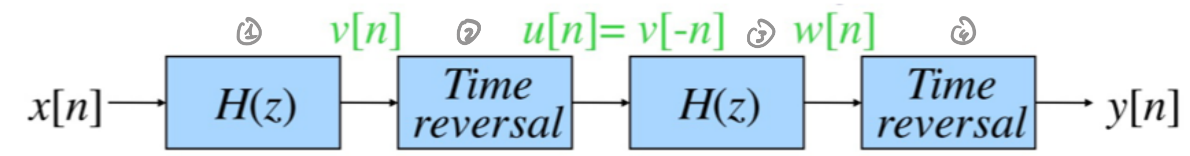
(2) No passa alto

(3) No passa alto
No passa basso
↳ indicato come passa banda

(4) No passa basso

FILTRO A FASE 0 (PER I FILTRI IIR)

A CAUSA DEL TIME REVERSAL È INDICATO SOLO PER I SEGNALI NON CAUSALI.



Tempo	Frequenza
$v[n] = x[n] * h[n]$	$V(e^{j\omega}) = H(e^{j\omega})\,X(e^{j\omega})$
$u[n] = v[-n]$	$U(e^{j\omega}) = V(e^{-j\omega}) = V^*(e^{j\omega})$
$w[n] = u[n] * h[n]$	$W(e^{j\omega}) = H(e^{j\omega})\,U(e^{j\omega})$
$y[n] = w[-n]$	$Y(e^{j\omega}) = W^*(e^{j\omega})$

① $X(e^{j\omega})H(e^{j\omega})$

② $X(e^{-j\omega})H(e^{-j\omega})$

③ $X(e^{-j\omega})H(e^{-j\omega})\cdot H(e^{j\omega}) = X(e^{-j\omega})\,|H(e^{j\omega})|^2$

④ $X(e^{j\omega})\cdot|H(e^{j\omega})|^2$

ESERCIZIO

① $y[n] = x[n] - \frac{1}{2}x[n-1] + 0.4\cdot x[n-2]$

TODO

② $y[n] = x[n] - x[n-2]$ FIR

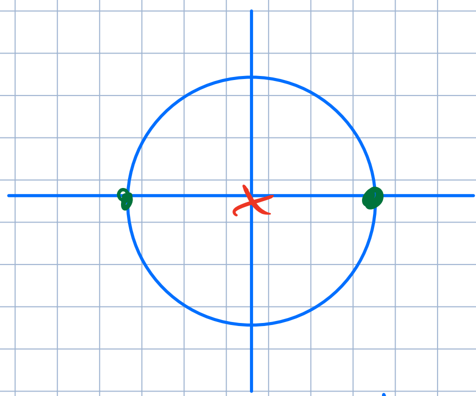
RISPOSTA QUANDO $x[n] = \cos\left(\frac{\pi}{4} n\right)$

$$Y(z) = X(z) - z^{-2} X(z) = X(z)\left(1 - z^{-2}\right)$$

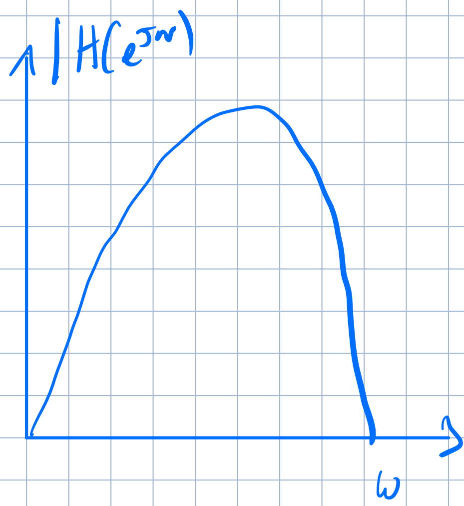
$$H(z) = \frac{Y(z)}{X(z)} = 1 - z^{-2} = \frac{z^2}{z^2} \cdot 1 - z^{-2} = \frac{z^2 - 1}{z^2}$$

ZERI $= \pm 1$

POLI $= 0$



$\Rightarrow$ PASSA-BANDA



$$H(e^{j\omega}) = H(z)\Big|_{e^{j\omega}} = 1 - e^{-j\omega 2}$$

COSA SUCCEDE A $\frac{\pi}{4}$

$$\Rightarrow 1 - e^{-j\frac{\pi}{2}} = 1 - (-j) = 1 + j$$

$H(e^{j\omega}) = 1 + j$ $\boxed{\sqrt{2} \cdot e^{j\frac{\pi}{4}}}$ IN FORMULA MODULO + ANGOLO

$$y[n] = \sqrt{2} \cdot \cos\left[\frac{\pi}{4} n + \frac{\pi}{4}\right]$$

FILTRI IIR

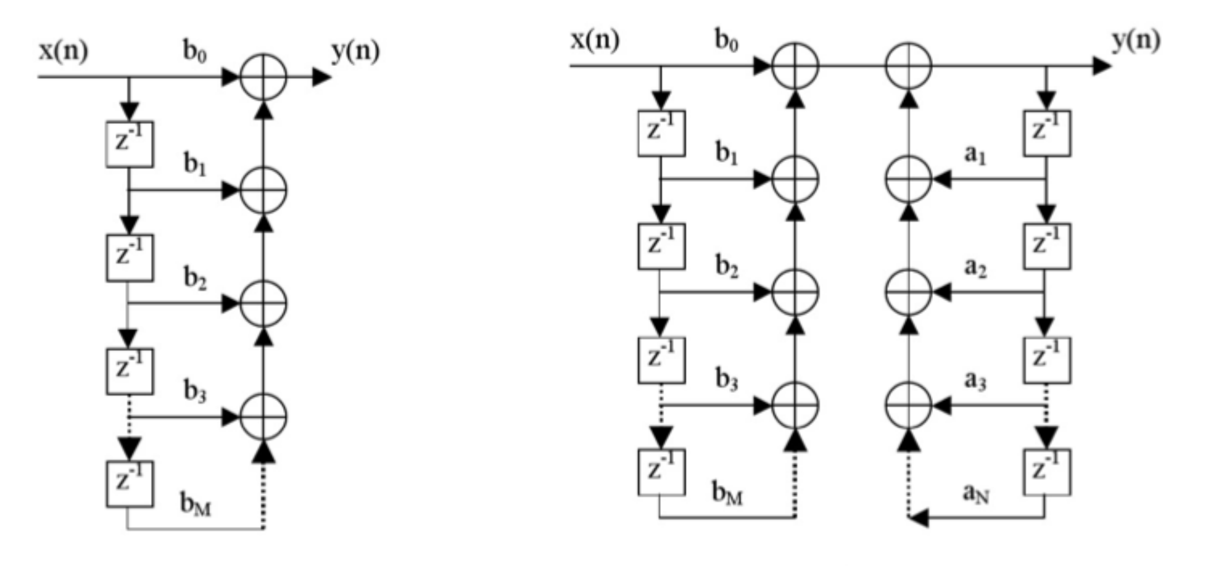
$$y[n] = \sum_{k=0}^{M} b_k \, x[n-k] - \sum_{k=1}^{N} a_k \cdot y[n-k]$$

$\boxed{a_0 = 1}$

$$\Rightarrow \sum_{k=0}^{M} b_k \cdot x[n-k] = \sum_{k=0}^{N} a_k \cdot y[n-k]$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{\sum_{k=0}^{M} b_k \cdot z^{-k}}{\sum_{k=0}^{M} a_k \cdot z^{k}}$$

PROBLEMA DI STABILITA' NEI FILTRI



Struttura di un sistema FIR | Struttura di un sistema IIR

	FIR	IIR
PRO	No ricorsivi - solo zeri ↳ Sono sempre stabili Possono avere fase lineare	Ricorsivi => zeri e poli ↳ non è garantita la stabilità Non hanno fase lineare
CONTRO	Hanno ordine elevato $N \geq 20 \to 200$ Unicamente digitali ↳ no controparte analogica	Ordine ridotto ↳ $N \simeq 1/10$ FIR Vengono ricavati da filtri analogici

FIR

NON RICORSIVI
↳ STABILI

POSSONO AVERE FASE LINEARE

HANNO ORDINE ELEVATO

UNICAMENTE DIGITALI

IIR

RICORSIVI
↳ NON GARANTITA STABILITÀ

NON HANNO FASE LINEARE

HANNO ORDINE RIDOTTO

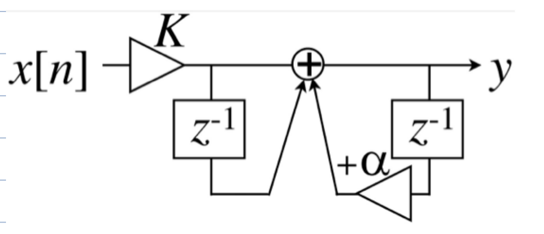
VENGONO RICAVATI DA FILTRI ANALOGICI

ESEMPIO

$$y[n] = k\,x[n] + k\,x[n-1] + \alpha\, y[n-1]$$

$$y[n] - \alpha\, y[n-1] = k\,x[n]$$

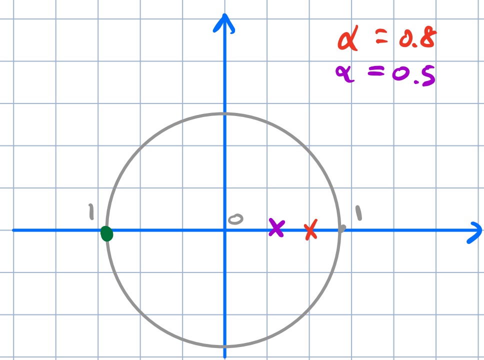
$$Y(z)\left(1-\alpha z^{-1}\right) = X(z)\left(1+z^{-1}\right)$$



FREQUENZA 0 VUOL DIRE LIVELLO DI CORRENTE CONTINUA:

$\omega = 0 \Rightarrow z = 1$

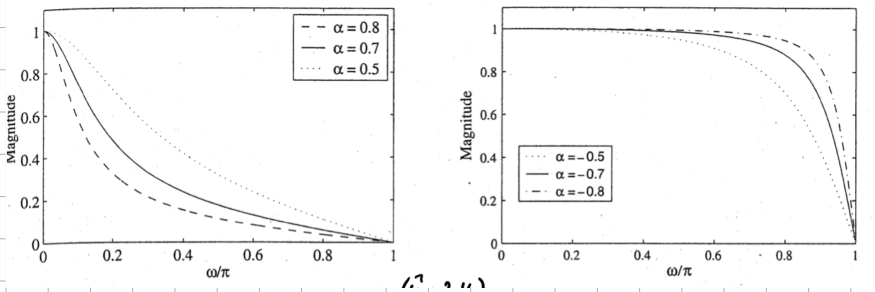
$$H(z) = k\left(\frac{1+z^{-1}}{1-\alpha z^{-1}}\right)$$

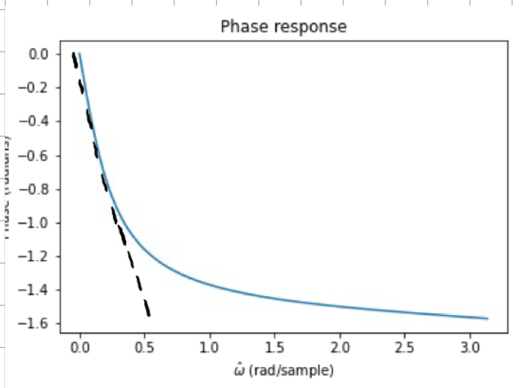


$$H(z) = k\left(\frac{1+z^{-1}}{1-\alpha \cdot z^{-1}}\right) \quad \begin{matrix}\omega=0\\ z=1\\ k=1\end{matrix} \Rightarrow H(1) = k\,\frac{1+1}{1-\alpha} = 1$$

$$\Rightarrow k = \frac{1-\alpha}{2} \Rightarrow H(z) = \frac{1-\alpha}{2}\cdot\frac{1+z^{-1}}{1-\alpha z^{-1}}$$

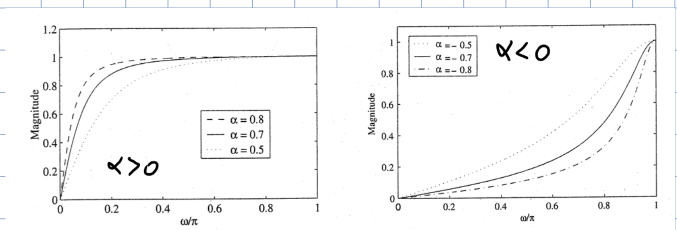
$$H(e^{j\omega}) = k\,\frac{1-e^{-j\omega}}{1+\alpha e^{-j\omega}}$$

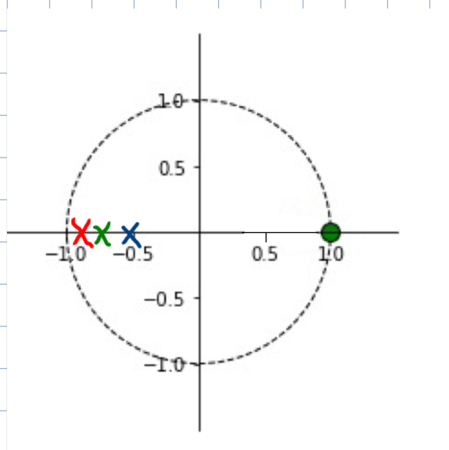


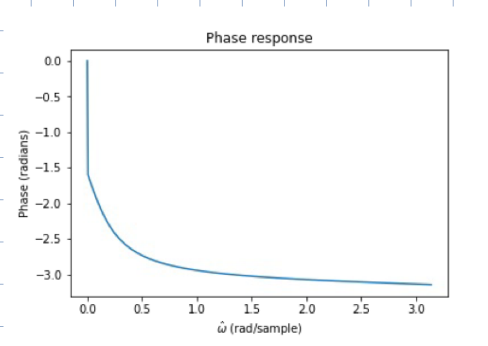


LA FASE NON È LINEARE MA IN ALCUNE REGIONI PUÒ ESSERLO

PER FARE UN PASSA ALTO POSSIAMO RIBALTARE GLI ZERI E POLI:





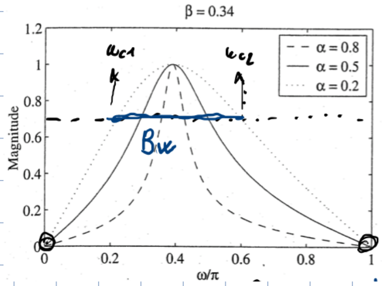


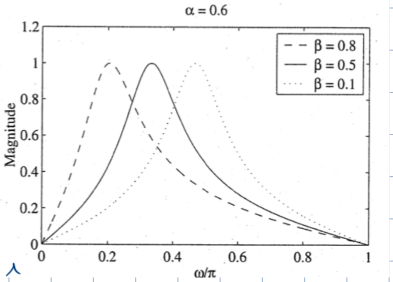
PER UN PASSABANDA SERVE PASSARE AL SECONDO ORDINE:

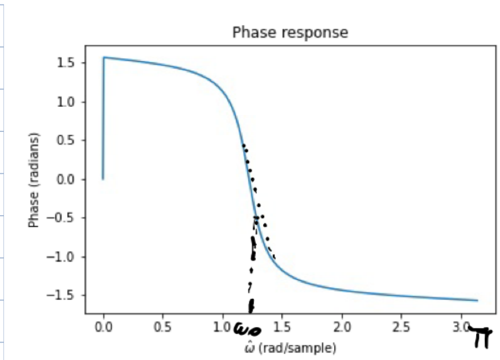
$$H(z) = \frac{1-\alpha}{2} \cdot \frac{1 - z^{-2}}{1 + \beta(1+\alpha) z^{-1} + \alpha \cdot z^{-2}}$$

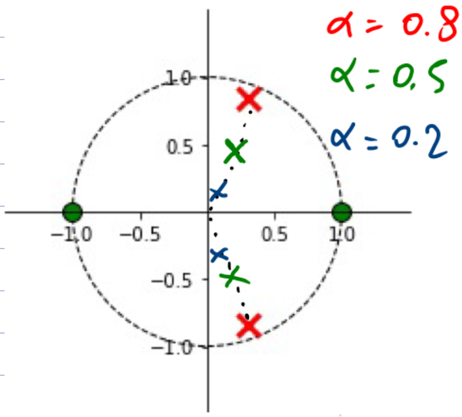
$$\omega_0 = \cos^{-1}(\beta)$$

$$B_W = \cos^{-1}\left(\frac{2\alpha}{1+\alpha^2}\right) = \omega_{c_2} - \omega_{c_1}$$



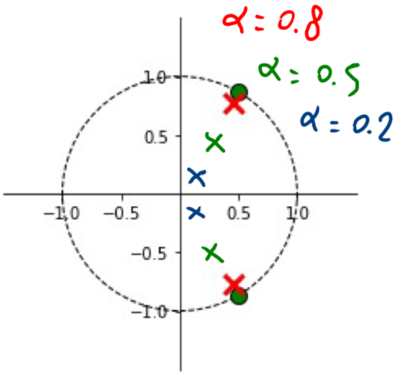


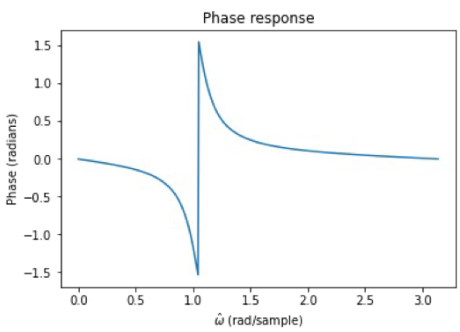


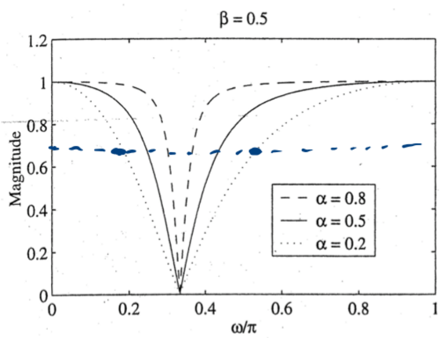


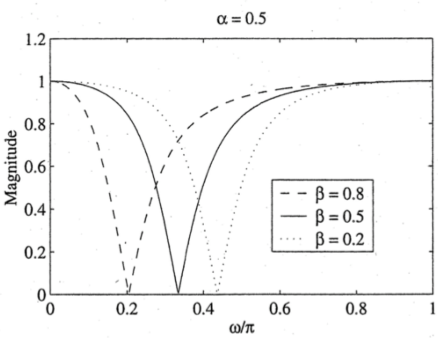
ELIMINA BANDA :

$$H(z) = \frac{1+\alpha}{2} \cdot \frac{1 - 2\beta \cdot z^{-1} + z^{-2}}{1 - \beta(1+\alpha) \cdot z^{-1} + \alpha \cdot z^{-2}}$$









ESERCIZIO

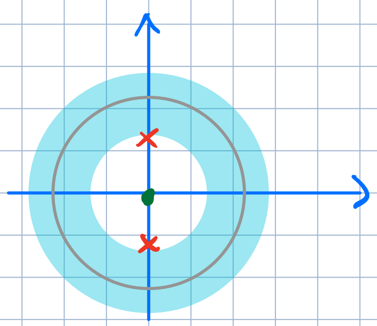
① $$H(z) = \frac{z^{-1}}{1+\frac{1}{4}z^{-2}}$$ CAUSALE

④ DISEGNARE POLI - ZERI :

$$\frac{z^2}{z^2}\,\frac{z^{-1}}{1+\frac{1}{4}z^{-2}} = \frac{z}{z^2+\frac{1}{4}}$$

ZERI: $z=0$

POLI: $\begin{cases} \frac{1}{2}j \\ -\frac{1}{2}j \end{cases}$



È STABILE POICHÈ I POLI SONO INTERNI ALLA CIRCONFERENZA

ROC

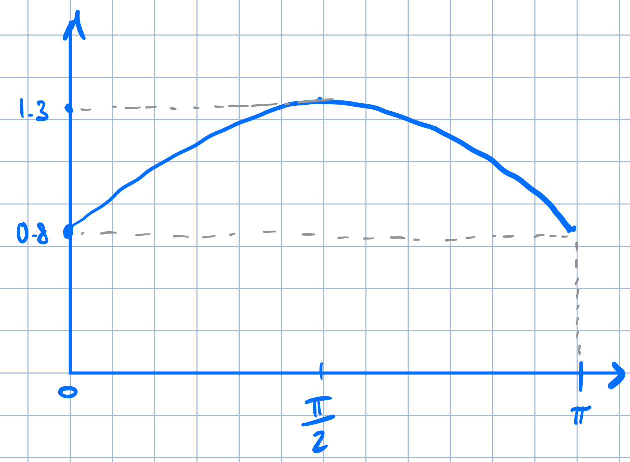
RISPOSTA IN FREQUENZA : (PERCHÉ LA CIRCONFERENZA È NEL ROC)

$$H(e^{jw}) = \frac{e^{-jw}}{1+\frac{1}{4}e^{-j2w}} = \frac{\cos w - j\sin w}{1+\frac{1}{4}[\cos(2w)-j\sin(2w)]}$$

GRAFICO AMPIEZZA:

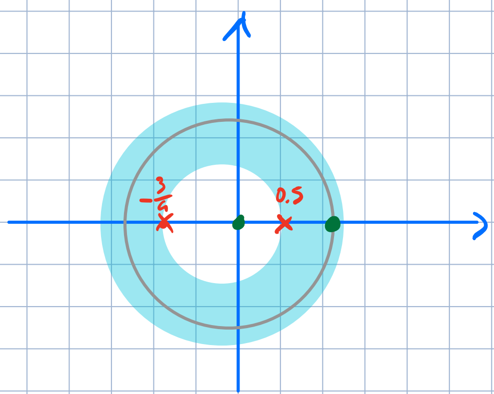
$w=0$ $$\left|\frac{1}{1+\frac{1}{4}}\right| = 0.8$$ $w=\pi$ 0.8

$w=\frac{\pi}{2}$ $$\left|\frac{-j}{1-\frac{1}{4}}\right| = 1.3$$



(2) $$H(z) = \frac{z^2 - z}{z^2 + \frac{1}{4}z - \frac{3}{8}}$$

ZERI: $z = \left\langle \begin{matrix} 0 \\ 1 \end{matrix} \right.$



POLI: $$\frac{-\frac{1}{4} \pm \sqrt{\frac{1}{8} + 4\frac{3}{8}}}{2}$$

$$= \frac{-\frac{1}{4} \pm \sqrt{\frac{13}{8}}}{2}$$

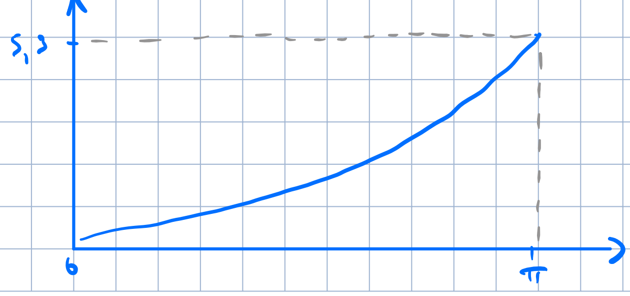
$$= -\frac{1}{8} \pm \sqrt{\frac{13}{32}}$$

RISPOSTA IN FREQUENZA:

$$H(e^{j\omega}) = \frac{1 - e^{-j\omega}}{1 + \frac{1}{4}e^{-j\omega} - \frac{3}{8}e^{-2j\omega}}$$

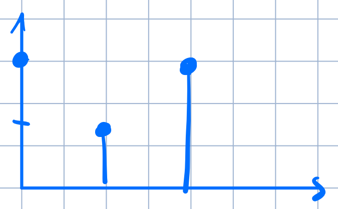
$w = 0$ $\quad 0$

$w = \pi$ $$\frac{1 - e^{-j\pi}}{1 + \frac{1}{4}e^{-j\pi} - \frac{3}{8}\cdot e^{-2j\pi}} = \frac{2}{1 + (\cos\pi - j\sin\pi)\left(\frac{1}{4} - \frac{3}{8}\right)} = 5,3$$



③ $h[n] = \{1, 0.5, \dots\}$

LUNGHEZZA MINIMA PER FILTRO DI TIPO 1:



$M = 2$

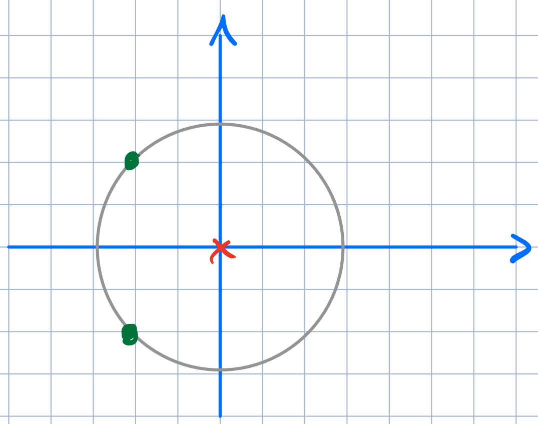
$N = 3$

$h[n] = \{1, 0.5, 1\}$

$$H(z) = 1 + 0,5 \cdot z^{-1} + z^{-2} = \frac{z^2}{z^2} \cdot \left(1 + 0,5 \cdot z^{-1} + z^{-2}\right) = \frac{z^2 + 0.5z + 1}{z^2}$$

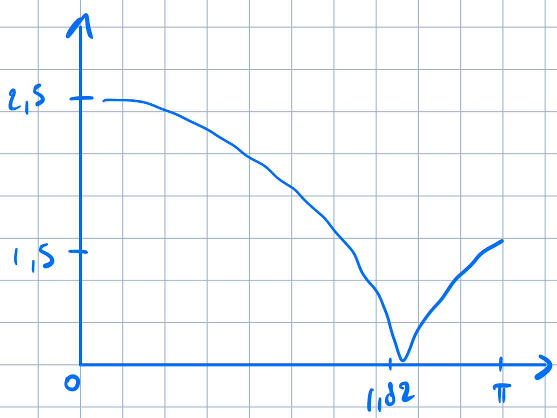
ZERI: $\dfrac{-\frac{1}{2} \pm \sqrt{\frac{1}{4} - 4}}{2} = \dfrac{-\frac{1}{2} \pm \sqrt{-15}}{2}$ $\quad z = 0.25 \pm j\,0.968$

POLI: $z = 0$



RISPOSTA IN FREQ:

$$H(e^{j\omega}) = 1 + \frac{1}{2} e^{-j\omega} + e^{-j2\omega}$$
$$= e^{-j\omega}\left(0.5 + 2\cos(\omega)\right)$$



$\omega = 0 \quad 2,5$

$\omega = \pi \quad 1,5$

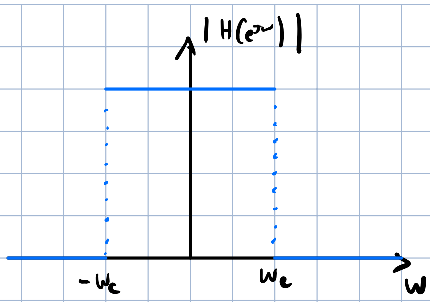
$\Rightarrow$ NOTCH

PROGETTAZIONE FILTRO FIR

WINDOWING

19/12

$$H_{LP}(e^{j\omega}) = \begin{cases} 1 & |\omega| < \omega_c \\ 0 & \text{ALTRIMENTI} \end{cases}$$

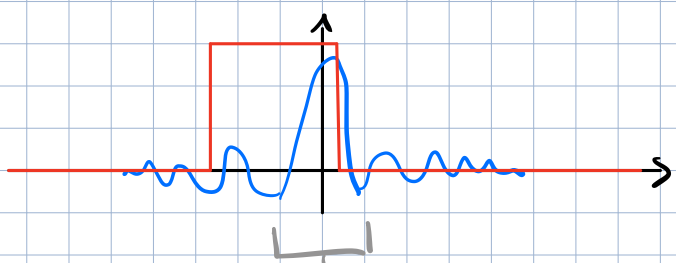


$$h_{LP}[n] = \frac{\omega_c}{\pi} \frac{\sin(\omega_c n)}{n \omega_c}$$

È INFINITA (SYNC, IL PASSA BASSO IDEALE)
È ANTICAUSALE

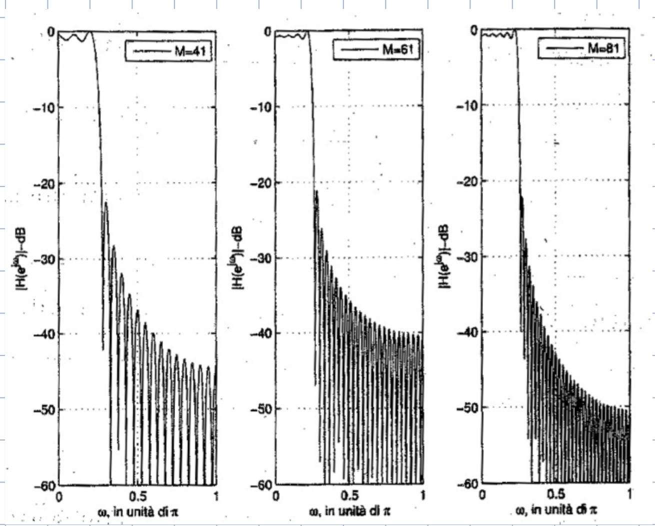
$$h_N[n] = \overbrace{W_N[n]}^{\text{FINESTRA}} \cdot h_{LP}[n]$$ (NEL TEMPO)

$$H_N(e^{j\omega}) = W_N(e^{j\omega}) * H_{LP}(e^{j\omega})$$ (NELLA FREQUENZA)



QUESTA AMPIEZZA DETERMINA LA GRANDEZZA DELLA REGIONE DI TRANSIZIONE

PER IL PRINCIPIO DI INDETERMINAZIONE DI HEISENBERG IL FILTRO IDEALE È IL PEGGIORE UTILIZZABILE.



ALL'AUMENTARE DEI CAMPIONI LA ZONA DI TRANSIZIONE DIMINUISCE MA LE OSCILLAZIONI AUMENTANO.

Some commonly used windows are shown in Figure 7.21.[6] These windows are defined by the following equations:

Rectangular

$$w[n] = \begin{cases} 1, & 0 \le n \le M, \\ 0, & \text{otherwise} \end{cases}$$

Bartlett (triangular)

$$w[n] = \begin{cases} 2n/M, & 0 \le n \le M/2, \\ 2 - 2n/M, & M/2 < n \le M, \\ 0, & \text{otherwise} \end{cases}$$

Hanning

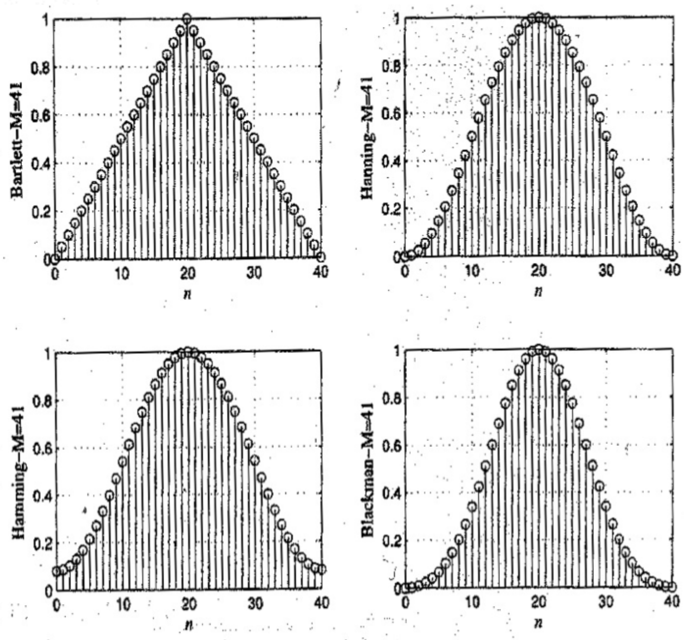
$$w[n] = \begin{cases} 0.5 - 0.5\cos(2\pi n/M), & 0 \le n \le M, \\ 0, & \text{otherwise} \end{cases}$$

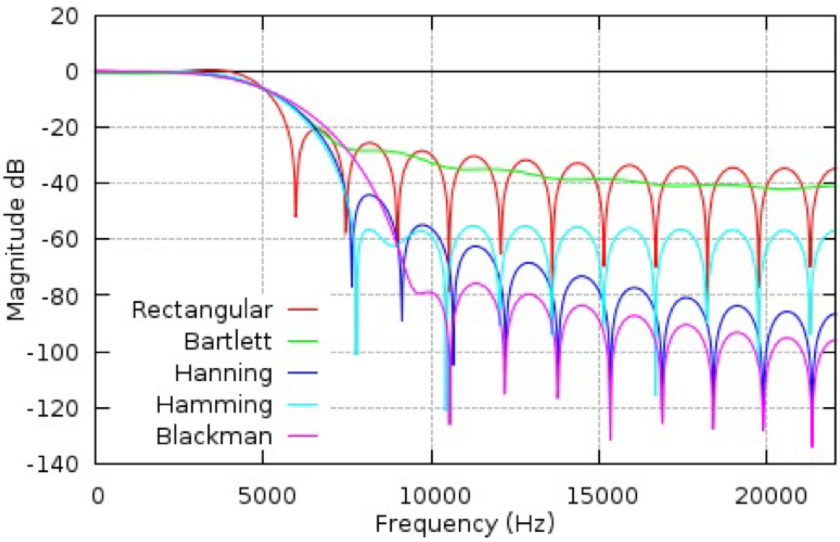
Hamming

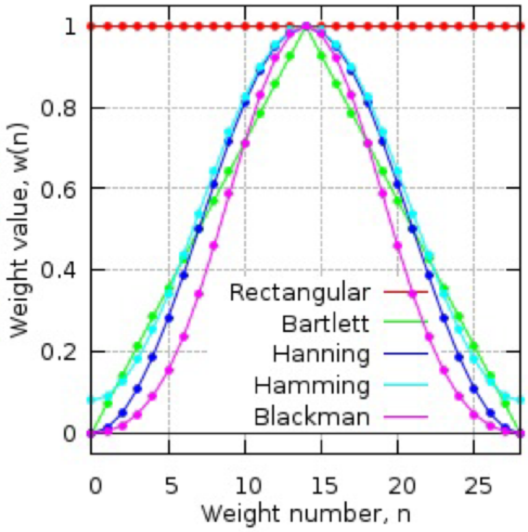
$$w[n] = \begin{cases} 0.54 - 0.46\cos(2\pi n/M), & 0 \le n \le M, \\ 0, & \text{otherwise} \end{cases}$$

Blackman

$$w[n] = \begin{cases} 0.42 - 0.5\cos(2\pi n/M) + 0.08\cos(4\pi n/M), & 0 \le n \le M, \\ 0, & \text{otherwise} \end{cases}$$

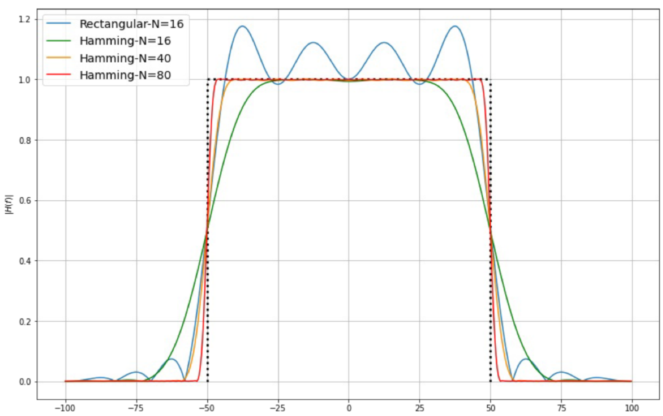




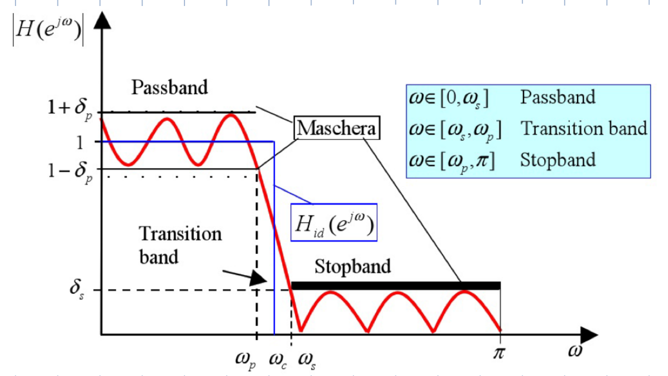


$\Delta W \cong \frac{\text{COSTANTE}}{N}$

Name of window function	Transition width/ sample frequency $\frac{\Delta w}{2\pi}$	Passband ripple (dB)	Main lobe relative to largest side lobe (dB)	Maximum stopband attenuation (dB)
Rectangular	0.9 / N	0.75	13	21
Hann(ing)	3.1/N	0.055	31	44
Hamming	3.3/N	0.019	41	53
Blackman	5.5/N	0.0017	57	74
Kaiser (β=4.54)	2.93/N	0.0274		50
Kaiser (β=8.96)	5.71/N	0.000275		90



PER PROGETTARE UN FILTRO TRAMITE WINDOWING DOBBIAMO DOTARCI DI:



- PASSBAND
- TRANSITION BAND
- STOPBAND

POI ANDIAMO A VEDERE NELLA TABELLA CHE FINESTRA SODDISFA I NOSTRI REQUISITI, OTTENIAMO I CAMPIONI E TESTIAMO.

NON È UN METODO "OTTIMO" DAL PUNTO DI VISTA STATISTICO DEI MINIMI QUADRATI $\Rightarrow$ POCO EFFICIENTE.

ESEMPIO

① • FIR DI TIPO 1

- $\delta_s = \delta_p = 0.01 \Rightarrow -20\log_{10}(0.01) = -40\,dB$
- PASSA BASSO
- ESTREMI BANDA DI TRANSIZIONE: $w_p = 0.2\pi \quad w_c = 0.3\pi \Rightarrow \Delta w = 0.1\pi$
  $\Rightarrow w_c = 0.25\pi$

PROVIAMO CON QUELLA DI HANN:

$$\Delta w = 2\pi \cdot \frac{3.1}{N} = 0.1\pi \Rightarrow N \cong 62 \quad (\text{ESSENDO TIPO } 1 \Rightarrow 63)$$

PROVIAMO CON KAISER :

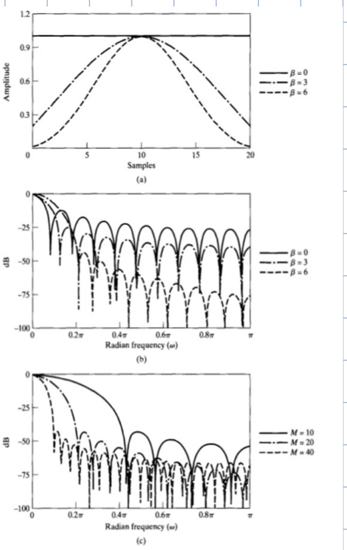


Figure 7.24 (a) Kaiser windows for $\beta = 0$, 3, and 6 and $M = 20$. (b) Fourier transforms corresponding to windows in (a). (c) Fourier transforms of Kaiser windows with $\beta = 6$ and $M = 10$, 20, and 40.

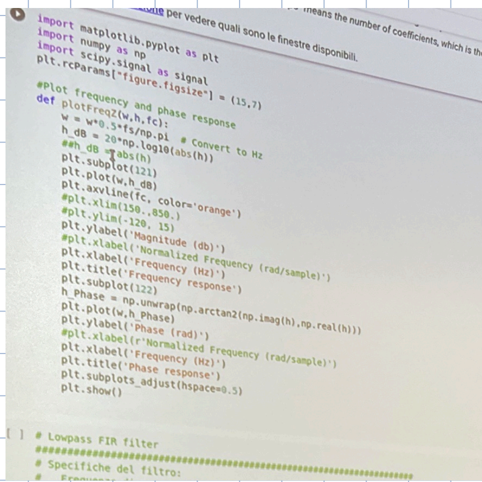
$$w[m] = \frac{I_0\left(\beta \cdot \sqrt{1-\left(\frac{2m-M}{M}\right)^2}\right)}{I_0(\beta)}$$

FUNZIONE DI BESSEL

DOVE $I_0 = 1 + \sum_{k=1}^{\infty} \left[\frac{(x/2)^k}{k!}\right]^2$

2

- LOW PASS
- $f_s = 8000$ Hz SAMPLING
- $f_c = 1750$ Hz
- $\Delta f = 500$ Hz
- $\delta_s = 50$ dB



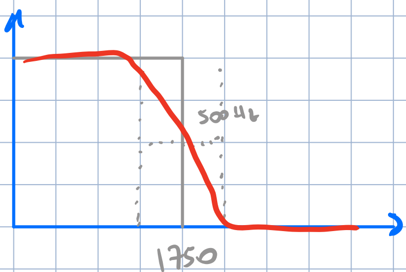
```
per vedere quali sono le finestre disponibili.        means the number of coefficients, which is th
import matplotlib.pyplot as plt
import numpy as np
import scipy.signal as signal
plt.rcParams["figure.figsize"] = (15,7)

#Plot frequency and phase response
def plotFreqZ(w,h,fc):
    w = w*0.5*fs/np.pi  # Convert to Hz
    h_dB = 20*np.log10(abs(h))
    ##h_dB = abs(h)
    plt.subplot(121)
    plt.plot(w,h_dB)
    plt.axvline(fc, color='orange')
    #plt.xlim(150.,850.)
    #plt.ylim(-120, 15)
    plt.ylabel('Magnitude (db)')
    #plt.xlabel('Normalized Frequency (rad/sample)')
    plt.xlabel('Frequency (Hz)')
    plt.title('Frequency response')
    plt.subplot(122)
    h_Phase = np.unwrap(np.arctan2(np.imag(h),np.real(h)))
    plt.plot(w,h_Phase)
    plt.ylabel('Phase (rad)')
    #plt.xlabel(r'Normalized Frequency (rad/sample)')
    plt.xlabel('Frequency (Hz)')
    plt.title('Phase response')
    plt.subplots_adjust(hspace=0.5)
    plt.show()

[ ] # Lowpass FIR filter
    ###########################################################################
    # Specifiche del filtro:
```

CON BLACKMAN :

$$N = 5.5 \frac{f_s}{\Delta f} = 89$$



CON $N = 89$ SI ARRIVA A -80dB