

Homework 7.5 Strategy for Integration

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1 Integration Problems and Solutions

1.1 Problem 1

- (a) Evaluate the integral: $\int \frac{3x}{1+x^2} dx$

Solution

To evaluate the integral of $(3x)/(1+x^2)$, u-substitution is used. Let $u = 1 + x^2$, so $du = 2x dx$. The integral becomes:

$$\int \frac{3}{2u} du = \frac{3}{2} \int \frac{1}{u} du$$

This evaluates to: **Answer:** $\frac{3}{2} \ln |1 + x^2| + C$

- (b) Evaluate the integral: $\int \frac{3}{1+x^2} dx$

Solution

The integral of $3/(1+x^2)$ is a standard form. It evaluates to: **Answer:** $3 \tan^{-1}(x) + C$

- (c) Evaluate the integral: $\int \frac{3}{1-x^2} dx$

Solution

For the integral of $3/(1-x^2)$, partial fraction decomposition is applied. The expression can be rewritten as:

$$\frac{3}{2} \left[\int \frac{1}{1-x} dx + \int \frac{1}{1+x} dx \right]$$

This integrates to $-(\frac{3}{2}) \ln |1-x| + (\frac{3}{2}) \ln |1+x| + C$, which can be simplified to: **Answer:** $\frac{3}{2} \ln \left| \frac{1+x}{1-x} \right| + C$

1.2 Problem 2

- (a) Evaluate the integral: $\int 7x\sqrt{x^2-1} dx$

Solution

To solve the integral of $7x(x^2-1)$, let $u = x^2 - 1$, which gives $du = 2x dx$. The integral becomes:

$$\frac{7}{2} \int \sqrt{u} du$$

This evaluates to: **Answer:** $\frac{7}{3}(x^2-1)^{3/2} + C$

- (b) Evaluate the integral: $\int \frac{7}{x\sqrt{x^2-1}} dx$

Solution

The integral of $7/(x(x^2-1))$ is a standard form related to inverse trigonometric functions. The solution is:

Answer: $7 \sec^{-1} |x| + C$

- (c) Evaluate the integral: $\int \frac{7\sqrt{x^2-1}}{x} dx$

Solution

For the integral of $7(x^2-1)/x$, a trigonometric substitution is appropriate. Let $x = \sec(\theta)$, so $dx = \sec(\theta) \tan(\theta) d\theta$. The integral simplifies to:

$$7 \int \tan^2(\theta) d\theta = 7 \int (\sec^2(\theta) - 1) d\theta$$

This integrates to $7(\tan(\theta) - \theta) + C$. Substituting back gives: **Answer:** $7(\sqrt{x^2-1} - \sec^{-1}(x)) + C$

1.3 Problem 3

- (a) Evaluate the integral: $\int \frac{2 \ln(x)}{x} dx$

Solution

To integrate $2 \ln(x)/x$, use u-substitution with $u = \ln(x)$, so $du = (1/x) dx$. The integral becomes:

$$2 \int u du$$

which results in: **Answer:** $(\ln(x))^2 + C$

- (b) Evaluate the integral: $\int 2 \ln(2x) dx$

Solution

The integral of $2 \ln(2x)$ requires integration by parts. Let $u = \ln(2x)$ and $dv = 2 dx$. Then $du = (1/x) dx$ and $v = 2x$. The formula $\int u dv = uv - \int v du$ gives:

$$2x \ln(2x) - \int 2 dx$$

which evaluates to: **Answer:** $2x \ln(2x) - 2x + C$

- (c) Evaluate the integral: $\int 2x \ln |x| dx$

Solution

For the integral of $2x \ln |x|$, integration by parts is also used. Let $u = \ln |x|$ and $dv = 2x dx$. Then $du = (1/x) dx$ and $v = x^2$. This leads to:

$$x^2 \ln |x| - \int x dx$$

which results in: **Answer:** $x^2 \ln |x| - \frac{x^2}{2} + C$

1.4 Problem 4

- (a) Evaluate the integral: $\int 2 \sin^2(x) dx$

Solution

To evaluate the integral of $2\sin^2(x)$, the half-angle identity $\sin^2(x) = (1 - \cos(2x))/2$ is used. The integral becomes:

$$\int (1 - \cos(2x)) dx$$

which evaluates to: **Answer:** $x - \frac{1}{2} \sin(2x) + C$

(b) Evaluate the integral: $\int 2 \sin^3(x) dx$

Solution

For the integral of $2\sin^3(x)$, rewrite $\sin^3(x)$ as $\sin(x)(1 - \cos^2(x))$. Using u-substitution with $u = \cos(x)$ and $du = -\sin(x) dx$, the integral becomes:

$$-2 \int (1 - u^2) du$$

This evaluates to: **Answer:** $-2(\cos(x) - \frac{1}{3} \cos^3(x)) + C$

(c) Evaluate the integral: $\int 2 \sin(2x) dx$

Solution

The integral of $2\sin(2x)$ is a straightforward integration. It evaluates to: **Answer:** $-\cos(2x) + C$

1.5 Problem 5

Evaluate the integral: $\int \frac{\cos(x)}{3 - \sin(x)} dx$

Solution

To solve the integral of $\cos(x)/(3 - \sin(x))$, a u-substitution is performed. Let $u = 3 - \sin(x)$, so $du = -\cos(x) dx$. The integral becomes:

$$-\int \frac{1}{u} du$$

which evaluates to: **Answer:** $-\ln|3 - \sin(x)| + C$

1.6 Problem 6

Evaluate the integral: $\int \frac{x}{x^4 + 16} dx$

Solution

For the integral of $x/(x+16)$, a substitution is made. Let $u = x^2$, so $du = 2x dx$. The integral becomes:

$$\frac{1}{2} \int \frac{1}{u^2 + 16} du$$

This is a standard arctangent form, and the result is: **Answer:** $\frac{1}{8} \tan^{-1}\left(\frac{x^2}{4}\right) + C$

1.7 Problem 7

Evaluate the integral: $\int 5t \sin(t) \cos(t) dt$

Solution

To evaluate the integral of $5t \sin(t)\cos(t)$, the double angle identity $\sin(2t) = 2\sin(t)\cos(t)$ is used. The integral becomes:

$$\frac{5}{2} \int t \sin(2t) dt$$

Integration by parts is then applied, with $u = t$ and $dv = \sin(2t) dt$. This yields $du = dt$ and $v = -(\frac{1}{2})\cos(2t)$. The final result is: **Answer:** $\frac{5}{2} [-\frac{t}{2}\cos(2t) + \frac{1}{4}\sin(2t)] + C$

1.8 Problem 8

Evaluate the integral: $\int \frac{2x-3}{x^2+3x} dx$

Solution

For the integral of $(2x-3)/(x^2+3x)$, we can split the denominator by factoring: $x(x+3)$. However, a u -substitution is more direct. Let $u = x^2+3x$, then $du = (2x+3) dx$. The integral becomes $(2x-3)/(u) * (du/(2x+3))$. The integral of $(2x-3)/(x^2+3x)$ is: **Answer:** $\ln|x^2+3x| + C$

1.9 Problem 9

Evaluate the integral: $\int x \sec(x) \tan(x) dx$

Solution

To integrate $x \sec(x)\tan(x)$, integration by parts is the appropriate method. Let $u = x$ and $dv = \sec(x) \tan(x) dx$. Then $du = dx$ and $v = \sec(x)$. This gives:

$$x \sec(x) - \int \sec(x) dx$$

The final answer is: **Answer:** $x \sec(x) - \ln|\sec(x) + \tan(x)| + C$

1.10 Problem 10

Evaluate the integral: $\int 9\theta \tan^2(\theta) d\theta$

Solution

To evaluate the integral of $9 \tan^2(\theta)$, use the identity $\tan^2(\theta) = \sec^2(\theta) - 1$. The integral becomes:

$$9 \int \theta(\sec^2(\theta) - 1) d\theta = 9 \left[\int \theta \sec^2(\theta) d\theta - \int \theta d\theta \right]$$

The first part requires integration by parts with $u = \theta$ and $dv = \sec^2(\theta) d\theta$, leading to $\theta \tan(\theta) - \int \tan(\theta) d\theta$. The final result is: **Answer:** $9 \left[\theta \tan(\theta) + \ln|\cos(\theta)| - \frac{\theta^2}{2} \right] + C$

2 Problem Types and Techniques Used

- **U-Substitution:** This was a fundamental technique used in problems 1a, 2a, 3a, 4b, 5, 6, and 8.
- **Integration by Parts:** This method was essential for problems 3b, 3c, 7, 9, and 10.
- **Trigonometric Integrals:** Problems 4a, 4b, and 4c involved powers and multiples of trigonometric functions, requiring specific identities.
- **Trigonometric Substitution:** Problem 2c utilized this technique to simplify a radical expression.

- **Partial Fraction Decomposition:** Problem 1c was solved using this algebraic method.
- **Standard Integral Forms:** Problems 1b and 2b were recognizable as basic antiderivatives involving inverse trigonometric functions.

3 Algebraic Manipulations and Tricks

- **Trigonometric Identities:**
 - Half-angle identity: $\sin^2(x) = (1 - \cos(2x))/2$ (Problem 4a).
 - Pythagorean identity: $\sin^2(x) = 1 - \cos^2(x)$ (Problem 4b).
 - Double angle identity: $\sin(2t) = 2\sin(t)\cos(t)$ (Problem 7).
 - Identity for $\tan^2(\theta)$: $\tan^2(\theta) = \sec^2(\theta) - 1$ (Problem 10).
- **Completing the Square and Factoring:** While not explicitly used in the final solutions, these are often considered for rational functions and were part of the initial analysis for problems like 1c and 8.
- **Substitution before Integration by Parts:** In problem 7, a trigonometric identity was used to simplify the integrand before applying integration by parts.