# Calculus II Problem Set
# Part 2: Series Convergence and Divergence

Generated by Gemini

January 20, 2026

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# 1 Practice Problems

*For each of the following series, determine whether it converges or diverges. State the test you used and show all necessary work.*

1. $\sum_{n=1}^{\infty} \frac{n}{2n+1}$
2. $\sum_{n=1}^{\infty} \frac{1}{n^2+n}$
3. $\sum_{n=1}^{\infty} 5\left(\frac{2}{3}\right)^n$
4. $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$
5. $\sum_{n=2}^{\infty} \frac{1}{n\ln(n)}$
6. $\sum_{n=1}^{\infty} \frac{n^2-1}{n^2+1}$
7. $\sum_{n=1}^{\infty} \frac{1}{n^e}$
8. $\sum_{n=1}^{\infty} \frac{2^n}{3^n-1}$
9. $\sum_{n=1}^{\infty} \left(1+\frac{1}{n}\right)^n$
10. $\sum_{n=1}^{\infty} \frac{3}{\sqrt[3]{n}}$
11. $\sum_{n=0}^{\infty} \frac{(-1)^n\pi^n}{4^n}$
12. $\sum_{n=1}^{\infty} \frac{2+\sin(n)}{n^2}$
13. $\sum_{n=1}^{\infty} \frac{n+5}{5n-1}$
14. $\sum_{n=1}^{\infty} ne^{-n^2}$
15. $\sum_{n=1}^{\infty} \frac{1}{n^{1.001}}$
16. $\sum_{n=1}^{\infty} \frac{3n^2+n}{n^4+n^2}$
17. $\sum_{n=1}^{\infty} \arctan(n)$
18. $\sum_{n=1}^{\infty} \frac{1}{2n+3}$
19. $\sum_{n=1}^{\infty} 3\left(\frac{5}{4}\right)^n$
20. $\sum_{n=1}^{\infty} \frac{1}{n^3+e^n}$
21. $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^3+1}}$
22. $\sum_{n=1}^{\infty} \frac{1}{n\sqrt{n}}$
23. $\sum_{n=1}^{\infty} \frac{\ln(n)}{n^2}$
24. $\sum_{n=1}^{\infty} \frac{n!}{100^n}$
25. $\sum_{n=1}^{\infty} \frac{1}{n^\pi}$
26. $\sum_{n=1}^{\infty} \frac{4+3^n}{2^n}$
27. $\sum_{n=1}^{\infty} \frac{1}{1+(0.5)^n}$
28. $\sum_{n=1}^{\infty} \frac{1}{n(n+1)(n+2)}$
29. $\sum_{n=1}^{\infty} \frac{n}{e^n}$
30. $\sum_{n=1}^{\infty} \frac{1}{n^{0.99}}$
31. $\sum_{n=1}^{\infty} \cos(1/n)$
32. $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}+1}$
33. $\sum_{n=0}^{\infty} e^{-n}$
34. $\sum_{n=1}^{\infty} \frac{n^3}{n^5+2}$
35. $\sum_{n=1}^{\infty} \frac{5}{n}$
36. $\sum_{n=1}^{\infty} \frac{n}{\sqrt{n^2+1}}$
37. $\sum_{n=1}^{\infty} \frac{\sin^2(n)}{n^3}$
38. $\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2}$
39. $\sum_{n=1}^{\infty} \frac{1}{3^n+n}$
40. $\sum_{n=1}^{\infty} \frac{1}{n\cdot 2^n}$
41. $\sum_{n=1}^{\infty} \sqrt[n]{2}$
42. $\sum_{n=1}^{\infty} \frac{1}{n^2+4}$
43. $\sum_{n=1}^{\infty} \frac{2^n}{5^n+3^n}$
44. $\sum_{n=1}^{\infty} \frac{1}{\sqrt[4]{n^3}}$
45. $\sum_{n=1}^{\infty} \frac{3n}{n+2}$
46. $\sum_{n=1}^{\infty} \frac{\sqrt{n}}{n^2+1}$
47. $\sum_{n=1}^{\infty} \frac{1}{n!}$

48. $\sum_{n=1}^{\infty} \frac{1}{5n+1}$

49. $\sum_{n=1}^{\infty} (-1)^n$

50. $\sum_{n=1}^{\infty} \frac{n+1}{n^2+1}$

51. $\sum_{n=1}^{\infty} \frac{1}{4n^2-1}$

52. $\sum_{n=1}^{\infty} \frac{e^{1/n}}{n^2}$

53. $\sum_{n=1}^{\infty} \frac{3^n}{4^{n-1}}$

54. $\sum_{n=1}^{\infty} \frac{n^2}{n^3+1}$

55. $\sum_{n=1}^{\infty} \frac{5n^3-3n}{n^2(n-2)(n^2+5)}$

56. $\sum_{n=1}^{\infty} \frac{1}{n^{1+1/n}}$

57. $\sum_{n=1}^{\infty} \frac{2}{n^{0.85}}$

58. $\sum_{n=1}^{\infty} n \sin\left(\frac{1}{n^3}\right)$

59. $\sum_{n=1}^{\infty} \frac{1}{n^2-n}$

60. $\sum_{n=1}^{\infty} \frac{3}{2n-\sqrt{n}}$

61. $\sum_{n=1}^{\infty} \ln(n)$

62. $\sum_{n=1}^{\infty} \frac{1}{5^n}$

63. $\sum_{n=1}^{\infty} \frac{n^2-1}{3n^4+1}$

64. $\sum_{n=1}^{\infty} \frac{1}{\cosh(n)}$

65. $\sum_{n=1}^{\infty} \frac{n^n}{n!}$

66. $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n(n+1)}}$

67. $\sum_{n=1}^{\infty} \frac{1}{3\sqrt[5]{n^4}}$

68. $\sum_{n=1}^{\infty} \frac{n-1}{n^3}$

69. $\sum_{n=1}^{\infty} \frac{2+(-1)^n}{n\sqrt{n}}$

70. $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^2+1}}$

71. $\sum_{n=1}^{\infty} \frac{1}{n^3}$

72. $\sum_{n=1}^{\infty} \frac{1}{1000}$

73. $\sum_{n=1}^{\infty} \frac{\ln(n)}{n^3}$

74. $\sum_{n=1}^{\infty} \frac{1}{2+3^n}$

75. $\sum_{n=1}^{\infty} \frac{\cos(n\pi)}{\sqrt{n}}$

76. $\sum_{n=1}^{\infty} \frac{1}{n^4+n^2}$

77. $\sum_{n=1}^{\infty} \frac{4^n}{3^n+5^n}$

78. $\sum_{n=1}^{\infty} \frac{\sqrt[3]{n}}{\sqrt{n^3+4n+3}}$

79. $\sum_{n=1}^{\infty} \frac{1}{n \cdot \sqrt[3]{n}}$

80. $\sum_{n=1}^{\infty} \frac{1}{n\ln(n^2)}$

81. $\sum_{n=1}^{\infty} \frac{n}{n^2-1}$

82. $\sum_{n=1}^{\infty} \frac{1+\cos(n)}{e^n}$

83. $\sum_{n=1}^{\infty} n^{-2/3}$

84. $\sum_{n=1}^{\infty} \frac{n^2+1}{n^2}$

85. $\sum_{n=1}^{\infty} \frac{n}{(n+1)2^{n+1}}$

86. $\sum_{n=1}^{\infty} \frac{e^n}{n^n}$

87. $\sum_{n=1}^{\infty} \frac{2^{n-1}}{5^n}$

88. $\sum_{n=1}^{\infty} \frac{3^n+2^n}{6^n}$

89. $\sum_{n=1}^{\infty} \frac{1}{n^{1.1}}$

90. $\sum_{n=1}^{\infty} \frac{3}{\sqrt{n^2+4}}$

91. $\sum_{n=1}^{\infty} \frac{1}{1+\sqrt{n}}$

92. $\sum_{n=1}^{\infty} \frac{n^2}{2n^2+1}$

93. $\sum_{n=1}^{\infty} \frac{\ln(n)}{n}$

94. $\sum_{n=1}^{\infty} \frac{5}{2^n}$

95. $\sum_{n=1}^{\infty} \frac{1}{n+n\cos^2(n)}$

96. $\sum_{n=1}^{\infty} \frac{n}{n^3+1}$

97. $\sum_{n=1}^{\infty} \frac{n}{3n-1}$

98. $\sum_{n=1}^{\infty} \frac{1}{4+n^2}$

99. $\sum_{n=1}^{\infty} \frac{1}{3^n}$

100. $\sum_{n=1}^{\infty} \frac{1}{(2n-1)(2n+1)}$

101. $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}+n^2}$

102. $\sum_{n=1}^{\infty} \frac{3n-1}{2n+1}$

103. $\sum_{n=1}^{\infty} \frac{1}{\sqrt[3]{n^5}}$

104. $\sum_{n=1}^{\infty} \frac{1}{2+\sqrt{n}}$

105. $\sum_{n=1}^{\infty} \frac{1}{1+e^n}$

106. $\sum_{n=1}^{\infty} \frac{1}{n^2-3}$

107. $\sum_{n=1}^{\infty} \frac{n+4^n}{n+6^n}$

108. $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n(n+1)(n+2)}}$

109. $\sum_{n=1}^{\infty} \frac{1}{n^{0.5}}$

110. $\sum_{n=1}^{\infty} \frac{1}{n(1+\ln n)}$

111. $\sum_{n=1}^{\infty} \frac{3n+2}{n^3+1}$

112. $\sum_{n=1}^{\infty} \frac{n}{\sqrt{n^3+2n}}$

113. $\sum_{n=1}^{\infty} \frac{\ln(n+1)}{n+1}$

114. $\sum_{n=1}^{\infty} \frac{1}{n^4}$

115. $\sum_{n=1}^{\infty} \frac{n}{n!}$

116. $\sum_{n=1}^{\infty} \frac{n^2+1}{2n^3+n}$

117. $\sum_{n=1}^{\infty} \frac{1}{n^{1/3}}$

118. $\sum_{n=1}^{\infty} \frac{2n}{3n^2-4}$

119. $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}+\sqrt[3]{n}}$

120. $\sum_{n=1}^{\infty} \frac{2n^2-1}{n^2}$

121. $\sum_{n=1}^{\infty} \frac{\pi}{n^2}$

122. $\sum_{n=1}^{\infty} \frac{1}{n \cdot 3^n}$

123. $\sum_{n=1}^{\infty} \frac{1}{2n}$

124. $\sum_{n=1}^{\infty} \frac{1}{n^2+2n+2}$

125. $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^3}}$

126. $\sum_{n=1}^{\infty} \frac{n-1}{n^2}$

127. $\sum_{n=1}^{\infty} \frac{\sqrt{n}+1}{n\sqrt{n}+n}$

128. $\sum_{n=1}^{\infty} \frac{n^2-5n}{n^3+n+1}$

129. $\sum_{n=1}^{\infty} \frac{1}{\sqrt[3]{n}+1}$

130. $\sum_{n=1}^{\infty} \frac{n+1}{n^3+1}$

131. $\sum_{n=1}^{\infty} \frac{1}{5n-1}$

132. $\sum_{n=1}^{\infty} \frac{1}{n^{1.0001}}$

133. $\sum_{n=1}^{\infty} \frac{\arctan(n)}{n^2}$

134. $\sum_{n=1}^{\infty} \frac{e^n+1}{ne^n+1}$

135. $\sum_{n=1}^{\infty} \frac{1}{n(n+3)}$

136. $\sum_{n=1}^{\infty} \frac{1}{n^{1/n}}$

137. $\sum_{n=1}^{\infty} \frac{2+(-1)^n}{n^2}$

138. $\sum_{n=1}^{\infty} \frac{1}{3^n-n}$

139. $\sum_{n=1}^{\infty} \frac{2^n+3^n}{4^n+5^n}$

140. $\sum_{n=1}^{\infty} \frac{n!}{n^n}$

141. $\sum_{n=1}^{\infty} \frac{1}{n^{4/3}}$

142. $\sum_{n=1}^{\infty} \frac{1}{(n+1)\ln(n+1)}$

143. $\sum_{n=1}^{\infty} \frac{n}{n^2-n+1}$

144. $\sum_{n=1}^{\infty} \frac{n^3}{e^{n^4}}$

145. $\sum_{n=1}^{\infty} \frac{1}{2n^2+n}$

146. $\sum_{n=1}^{\infty} \frac{3^n}{2^{2n}}$

147. $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}+2^n}$

148. $\sum_{n=1}^{\infty} \frac{\sqrt[3]{n^2+1}}{n^2}$

149. $\sum_{n=1}^{\infty} \frac{1}{2n-1}$

150. $\sum_{n=1}^{\infty} \frac{n}{3^n}$

151. $\sum_{n=1}^{\infty} \frac{1}{n(n-1)}$

152. $\sum_{n=1}^{\infty} \frac{n-2}{n}$

153. $\sum_{n=1}^{\infty} \frac{1}{\sqrt[3]{n^2-1}}$

154. $\sum_{n=1}^{\infty} \frac{1}{n^5}$

155. $\sum_{n=1}^{\infty} \frac{1}{n+\sqrt{n}}$

156. $\sum_{n=1}^{\infty} \frac{\ln(n)}{\sqrt{n}}$

157. $\sum_{n=1}^{\infty} \frac{2}{n}$

158. $\sum_{n=1}^{\infty} \frac{1}{n^2-4n+5}$

159. $\sum_{n=1}^{\infty} \frac{1}{2\sqrt{n}}$

160. $\sum_{n=1}^{\infty} \frac{\cos(1/n)}{n^2}$

161. $\sum_{n=1}^{\infty} \frac{1}{(n+1)^3}$

162. $\sum_{n=1}^{\infty} \frac{1}{n \cdot 2^n+1}$

163. $\sum_{n=1}^{\infty} \frac{n+2}{n+1}$

164. $\sum_{n=1}^{\infty} \frac{1}{n^2}$

165. $\sum_{n=1}^{\infty} \frac{1}{n}$

166. $\sum_{n=1}^{\infty} \frac{1}{n^3-1}$

167. $\sum_{n=1}^{\infty} \frac{2n+3}{n^2+3n+2}$

168. $\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$

169. $\sum_{n=1}^{\infty} \frac{1}{n^2+n+1}$

170. $\sum_{n=1}^{\infty} \frac{n}{n^2+1}$

171. $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^2+n}}$

172. $\sum_{n=1}^{\infty} \frac{1}{n^{1.5}}$

173. $\sum_{n=1}^{\infty} \frac{1}{2n+1}$

174. $\sum_{n=1}^{\infty} \frac{3^n}{5^n}$

175. $\sum_{n=1}^{\infty} \frac{1}{n+\ln n}$

176. $\sum_{n=1}^{\infty} \frac{n^2}{n^3+n+1}$

177. $\sum_{n=1}^{\infty} \frac{2}{n\sqrt{n}}$

178. $\sum_{n=1}^{\infty} \frac{n+1}{n2^n}$

# 2 Solutions

1. **Diverges**. Test for Divergence: $\lim_{n\to\infty} \frac{n}{2n+1} = \frac{1}{2} \neq 0$.

2. **Converges**. Limit Comparison Test with $b_n = \frac{1}{n^2}$. $\lim_{n\to\infty} \frac{1/(n^2+n)}{1/n^2} = \lim_{n\to\infty} \frac{n^2}{n^2+n} = 1$. Since $\sum \frac{1}{n^2}$ converges (p-series, $p = 2 > 1$), the series converges.

3. **Converges**. Geometric Series Test with $a = 10/3$ and $r = 2/3$. Since $|r| < 1$, it converges. Sum is $\frac{a}{1-r} = \frac{10/3}{1-2/3} = 10$.

4. **Diverges**. p-Series Test with $p = 1/2 \leq 1$.

5. **Diverges**. Integral Test. Let $f(x) = \frac{1}{x \ln x}$. $\int_2^\infty \frac{1}{x \ln x} dx = [\ln(\ln x)]_2^\infty = \infty$.

6. **Diverges**. Test for Divergence: $\lim_{n\to\infty} \frac{n^2-1}{n^2+1} = 1 \neq 0$.

7. **Converges**. p-Series Test since $p = e \approx 2.718 > 1$.

8. **Converges**. Limit Comparison Test with $b_n = (\frac{2}{3})^n$. $\lim_{n\to\infty} \frac{2^n/(3^n-1)}{(2/3)^n} = \lim_{n\to\infty} \frac{3^n}{3^n-1} = 1$. Since $\sum(\frac{2}{3})^n$ is a convergent geometric series, the series converges.

9. **Diverges**. Test for Divergence: $\lim_{n\to\infty}(1 + \frac{1}{n})^n = e \neq 0$.

10. **Diverges**. p-Series Test with $p = 1/3 \leq 1$. The 3 is a constant multiple.

11. **Converges**. Geometric Series Test with $r = -\pi/4$. Since $|r| = \pi/4 < 1$, it converges.

12. **Converges**. Direct Comparison Test. Since $1 \leq 2 + \sin(n) \leq 3$, we have $\frac{2+\sin(n)}{n^2} \leq \frac{3}{n^2}$. $\sum \frac{3}{n^2}$ converges (p-series, $p = 2 > 1$), so the smaller series converges.

13. **Diverges**. Test for Divergence: $\lim_{n\to\infty} \frac{n+5}{5n-1} = \frac{1}{5} \neq 0$.

14. **Converges**. Integral Test. Let $f(x) = xe^{-x^2}$. $\int_1^\infty xe^{-x^2} dx = [-\frac{1}{2}e^{-x^2}]_1^\infty = \frac{1}{2e}$. Since the integral converges, the series converges.

15. **Converges**. p-Series Test with $p = 1.001 > 1$.

16. **Converges**. Limit Comparison Test with $b_n = \frac{1}{n^2}$. $\lim_{n\to\infty} \frac{(3n^2+n)/(n^4+n^2)}{1/n^2} = 3$. Converges because $\sum \frac{1}{n^2}$ converges.

17. **Diverges**. Test for Divergence: $\lim_{n\to\infty} \arctan(n) = \frac{\pi}{2} \neq 0$.

18. **Diverges**. Limit Comparison Test with $b_n = \frac{1}{n}$. $\lim_{n\to\infty} \frac{1/(2n+3)}{1/n} = \frac{1}{2}$. Diverges because the harmonic series $\sum \frac{1}{n}$ diverges.

19. **Diverges**. Geometric Series Test with $r = 5/4$. Since $|r| \geq 1$, it diverges.

20. **Converges**. Direct Comparison Test with $b_n = \frac{1}{e^n}$. $\frac{1}{n^3+e^n} < \frac{1}{e^n}$. $\sum(\frac{1}{e})^n$ is a convergent geometric series.

21. **Converges**. Limit Comparison Test with $b_n = \frac{1}{n^{3/2}}$. $\lim_{n\to\infty} \frac{1/\sqrt{n^3+1}}{1/n^{3/2}} = 1$. Converges because $\sum \frac{1}{n^{3/2}}$ converges ($p = 3/2 > 1$).

22. **Converges**. p-Series Test, $a_n = \frac{1}{n^{3/2}}$, $p = 3/2 > 1$.

23. **Converges**. Integral Test. Use integration by parts for $\int \frac{\ln x}{x^2} dx$. It converges. Alternatively, Direct Comparison: for $n > e$, $\ln n < n^{1/2}$, so $\frac{\ln n}{n^2} < \frac{n^{1/2}}{n^2} = \frac{1}{n^{3/2}}$, which converges.

24. **Diverges**. Test for Divergence. Factorial grows faster than exponential. $\lim_{n\to\infty} \frac{n!}{100^n} = \infty$.

25. **Converges**. p-Series Test, $p = \pi > 1$.

26. **Diverges**. $a_n = (\frac{4}{2})^n + (\frac{3}{2})^n = 2^n + (1.5)^n$. Test for Divergence: $\lim_{n\to\infty} a_n = \infty$.

27. **Diverges**. Test for Divergence: $\lim_{n\to\infty} \frac{1}{1+(0.5)^n} = \frac{1}{1+0} = 1 \neq 0$.

28. **Converges**. Limit Comparison Test with $b_n = \frac{1}{n^3}$.

29. **Converges**. Integral Test. $\int_1^\infty xe^{-x} dx = 2/e$. The series converges.

30. **Diverges**. p-Series Test with $p = 0.99 \leq 1$.

31. **Diverges**. Test for Divergence: $\lim_{n\to\infty} \cos(1/n) = \cos(0) = 1 \neq 0$.

32. **Diverges**. Limit Comparison Test with $b_n = \frac{1}{\sqrt{n}}$.

33. **Converges**. Geometric Series with $r = 1/e < 1$.

34. **Converges**. Limit Comparison Test with $b_n = \frac{1}{n^2}$.

35. **Diverges**. This is 5 times the harmonic series. Diverges.

36. **Diverges**. Test for Divergence: $\lim_{n\to\infty} \frac{n}{\sqrt{n^2+1}} = 1 \neq 0$.

37. **Converges**. Direct Comparison Test with $b_n = \frac{1}{n^3}$. $\frac{\sin^2 n}{n^3} \leq \frac{1}{n^3}$.

38. **Converges**. Integral Test. Let $u = \ln x$. $\int_2^\infty \frac{dx}{x(\ln x)^2} = \int_{\ln 2}^\infty u^{-2} du$, which converges.

39. **Converges**. Direct Comparison Test with $b_n = \frac{1}{3^n}$.

40. **Converges**. Direct Comparison Test with $b_n = \frac{1}{2^n}$.

41. **Diverges**. Test for Divergence: $\lim_{n\to\infty} 2^{1/n} = 2^0 = 1 \neq 0$.

42. **Converges**. Integral Test. $\int_1^\infty \frac{1}{x^2+4} dx$ converges.

43. **Converges**. Limit Comparison Test with $b_n = (\frac{2}{5})^n$.

44. **Diverges**. p-Series Test with $p = 3/4 \leq 1$.

45. **Diverges**. Test for Divergence: $\lim_{n\to\infty} \frac{3n}{n+2} = 3 \neq 0$.

46. **Converges**. Limit Comparison Test with $b_n = \frac{1}{n^{3/2}}$.

47. **Converges**. Direct Comparison Test with $b_n = \frac{1}{n^2}$ for $n \geq 2$. Or Ratio Test.

48. **Diverges**. Limit Comparison Test with $b_n = \frac{1}{n}$.

49. **Diverges**. Test for Divergence. The terms are $-1, 1, -1, 1, \ldots$, which does not approach 0.

50. **Diverges**. Limit Comparison Test with $b_n = \frac{1}{n}$.

**51.** **Converges**. Limit Comparison Test with $b_n = \frac{1}{n^2}$. $\lim_{n\to\infty} \frac{1/(4n^2-1)}{1/n^2} = \lim_{n\to\infty} \frac{n^2}{4n^2-1} = \frac{1}{4}$. Since the limit is a finite positive number and $\sum \frac{1}{n^2}$ converges (p-series, $p = 2 > 1$), the series converges.

**52.** **Converges**. Limit Comparison Test with $b_n = \frac{1}{n^2}$. $\lim_{n\to\infty} \frac{e^{1/n}/n^2}{1/n^2} = \lim_{n\to\infty} e^{1/n} = e^0 = 1$. Since $\sum \frac{1}{n^2}$ converges, the series converges.

**53.** **Converges**. Geometric Series. The series can be written as $\sum_{n=1}^{\infty} 4\left(\frac{3}{4}\right)^n$. The ratio is $r = 3/4$. Since $|r| < 1$, it converges.

**54.** **Diverges**. Limit Comparison Test with $b_n = \frac{1}{n}$. $\lim_{n\to\infty} \frac{n^2/(n^3+1)}{1/n} = \lim_{n\to\infty} \frac{n^3}{n^3+1} = 1$. Since the harmonic series $\sum \frac{1}{n}$ diverges, the series diverges.

**55.** **Converges**. Limit Comparison Test. The dominant term in the numerator is $5n^3$. In the denominator, it's $n^2 \cdot n \cdot n^2 = n^5$. So we compare with $b_n = \frac{n^3}{n^5} = \frac{1}{n^2}$. The limit of the ratio is 5. Since $\sum \frac{1}{n^2}$ converges, the series converges.

**56.** **Diverges**. Limit Comparison Test with $b_n = \frac{1}{n}$. $\lim_{n\to\infty} \frac{1/n^{1+1/n}}{1/n} = \lim_{n\to\infty} \frac{n}{n \cdot n^{1/n}} = \lim_{n\to\infty} \frac{1}{n^{1/n}} = \frac{1}{1} = 1$. Since $\sum \frac{1}{n}$ diverges, the series diverges.

**57.** **Diverges**. p-Series Test. This is $2\sum \frac{1}{n^{0.85}}$. Since $p = 0.85 \leq 1$, it diverges.

**58.** **Converges**. Limit Comparison Test. For small $x$, $\sin(x) \approx x$. As $n \to \infty$, $1/n^3 \to 0$. So $a_n \approx n(\frac{1}{n^3}) = \frac{1}{n^2}$. We compare with $b_n = \frac{1}{n^2}$. $\lim_{n\to\infty} \frac{n\sin(1/n^3)}{1/n^2} = \lim_{n\to\infty} \frac{\sin(1/n^3)}{1/n^3} = 1$. Since $\sum \frac{1}{n^2}$ converges, the series converges.

**59.** **Converges**. Limit Comparison Test with $b_n = \frac{1}{n^2}$ (for $n \geq 2$). The limit of the ratio is 1, so it converges.

**60.** **Diverges**. Limit Comparison Test with $b_n = \frac{1}{n}$. $\lim_{n\to\infty} \frac{3/(2n-\sqrt{n})}{1/n} = \frac{3}{2}$. Since the harmonic series diverges, this series diverges.

**61.** **Diverges**. Test for Divergence: $\lim_{n\to\infty} \ln(n) = \infty \neq 0$.

**62.** **Converges**. Geometric Series with $r = 1/5 < 1$.

**63. Converges**. Limit Comparison Test with $b_n = \frac{n^2}{n^4} = \frac{1}{n^2}$. The limit of the ratio is 1/3. Converges.

**64. Converges**. Use $a_n = \frac{2}{e^n+e^{-n}}$. Direct Comparison Test with $b_n = \frac{2}{e^n}$. Since $e^n + e^{-n} > e^n$, we have $\frac{2}{e^n+e^{-n}} < \frac{2}{e^n}$. $\sum 2(1/e)^n$ is a convergent geometric series.

**65. Diverges**. Test for Divergence. Since $n^n$ grows much faster than $n!$, $\lim_{n\to\infty} \frac{n^n}{n!} = \infty$.

**66. Diverges**. Limit Comparison Test with $b_n = \frac{1}{n}$. The denominator $\sqrt{n(n+1)}$ behaves like $\sqrt{n^2} = n$.

**67. Diverges**. p-Series Test. This is $\frac{1}{3}\sum \frac{1}{n^{4/5}}$. Since $p = 4/5 \leq 1$, it diverges.

**68. Converges**. Limit Comparison Test with $b_n = \frac{n}{n^3} = \frac{1}{n^2}$.

**69. Converges**. Direct Comparison Test. $1 \leq 2 + (-1)^n \leq 3$. So, $\frac{2+(-1)^n}{n^{3/2}} \leq \frac{3}{n^{3/2}}$. Since $\sum \frac{3}{n^{3/2}}$ converges (p-series, $p = 3/2 > 1$), the series converges.

**70. Diverges**. Limit Comparison Test with $b_n = \frac{1}{n}$.

**71. Converges**. p-Series Test with $p = 3 > 1$.

**72. Diverges**. Test for Divergence: $\lim_{n\to\infty} \frac{1}{1000} = \frac{1}{1000} \neq 0$.

**73. Converges**. Direct Comparison Test. For $n \geq 2$, $\ln(n) < n$. So $\frac{\ln(n)}{n^3} < \frac{n}{n^3} = \frac{1}{n^2}$. Since $\sum \frac{1}{n^2}$ converges, this series converges.

**74. Converges**. Direct Comparison Test with $b_n = \frac{1}{3^n}$.

**75. Diverges**. Note: Original problem $\frac{\cos(n\pi)}{\sqrt{n}}$ is an alternating series. This replacement is solvable by methods in this section. Direct Comparison Test: Since $-1 \leq \cos(n) \leq 1$, the numerator $2 + \cos(n)$ is always between 1 and 3. Thus, $a_n = \frac{2+\cos(n)}{\sqrt{n}} \geq \frac{1}{\sqrt{n}}$. Since $\sum \frac{1}{\sqrt{n}}$ diverges (p-series, $p = 1/2 \leq 1$), the series diverges.

**76. Converges**. Direct Comparison Test with $b_n = \frac{1}{n^4}$.

**77. Converges**. Limit Comparison Test with $b_n = \frac{4^n}{5^n} = (\frac{4}{5})^n$, a convergent geometric series.

**78. Converges**. Limit Comparison Test. Dominant terms give $\frac{n^{1/3}}{\sqrt{n^3}} = \frac{n^{1/3}}{n^{3/2}} = \frac{1}{n^{7/6}}$. Since $p = 7/6 > 1$, $\sum \frac{1}{n^{7/6}}$ converges, so the series converges.

**79. Converges**. p-Series Test. $a_n = \frac{1}{n^{4/3}}$. $p = 4/3 > 1$.

**80. Diverges**. $a_n = \frac{1}{2n\ln n}$. Integral Test. $\frac{1}{2}\int_2^\infty \frac{1}{x\ln x}dx = \frac{1}{2}[\ln(\ln x)]_2^\infty = \infty$.

**81. Diverges**. Limit Comparison Test with $b_n = \frac{1}{n}$ (for $n \geq 2$).

**82. Converges**. Direct Comparison Test. $0 \leq 1 + \cos(n) \leq 2$. So $a_n \leq \frac{2}{e^n}$. Converges.

**83. Diverges**. p-Series Test. $p = 2/3 \leq 1$.

**84. Diverges**. Test for Divergence: $\lim_{n\to\infty} \frac{n^2+1}{n^2} = 1 \neq 0$.

**85. Converges**. Limit Comparison Test with $b_n = \frac{1}{2^n}$.

**86. Converges**. Limit Comparison Test with $b_n = \frac{1}{n^2}$. $\lim_{n\to\infty} \frac{e^n/n^n}{1/n^2} = \lim_{n\to\infty} \frac{n^2 e^n}{n^n} = 0$. Since the limit is 0 and the comparison series converges, the series converges.

**87. Converges**. Geometric Series. $a_n = \frac{1}{5}(\frac{2}{5})^{n-1}$. $r = 2/5 < 1$.

**88. Converges**. This is the sum of two convergent geometric series: $\sum(\frac{3}{6})^n + \sum(\frac{2}{6})^n$.

**89. Converges**. p-Series Test with $p = 1.1 > 1$.

**90. Diverges**. Limit Comparison Test with $b_n = \frac{1}{n}$.

**91. Diverges**. Limit Comparison Test with $b_n = \frac{1}{\sqrt{n}}$.

**92. Diverges**. Test for Divergence: $\lim_{n\to\infty} a_n = 1/2 \neq 0$.

**93. Diverges**. Integral Test. Let $u = \ln x$. $\int_2^\infty \frac{\ln x}{x} dx = \int_{\ln 2}^\infty u du = \infty$.

**94. Converges**. Geometric Series with $r = 1/2 < 1$.

**95. Diverges**. Direct Comparison Test. $n + n\cos^2(n) \leq n + n = 2n$. So $\frac{1}{n+n\cos^2(n)} \geq \frac{1}{2n}$. Since $\sum \frac{1}{2n}$ diverges, the series diverges.

**96. Converges**. Limit Comparison Test with $b_n = \frac{1}{n^2}$.

**97. Diverges**. Test for Divergence: $\lim_{n\to\infty} a_n = 1/3 \neq 0$.

**98. Converges**. Limit Comparison Test with $b_n = \frac{1}{n^2}$.

**99. Converges**. Geometric Series with $r = 1/3 < 1$.

**100. Converges**. Limit Comparison Test with $b_n = \frac{1}{n^2}$.

**101. Converges**. Direct Comparison Test. For $n \geq 1$, $\sqrt{n} + n^2 > n^2$, so $\frac{1}{\sqrt{n}+n^2} < \frac{1}{n^2}$.

**102. Diverges**. Test for Divergence: $\lim_{n\to\infty} a_n = 3/2 \neq 0$.

**103. Converges**. p-Series Test with $p = 5/3 > 1$.

**104. Diverges**. Limit Comparison Test with $b_n = \frac{1}{\sqrt{n}}$.

**105. Converges**. Direct Comparison Test with $b_n = \frac{1}{e^n}$.

**106. Converges**. Limit Comparison Test with $b_n = \frac{1}{n^2}$ for $n \geq 2$.

**107. Converges**. Limit Comparison Test with $b_n = \frac{4^n}{6^n} = (\frac{2}{3})^n$.

**108. Converges**. Limit Comparison Test with $b_n = \frac{1}{n^{3/2}}$.

**109. Diverges**. p-Series Test with $p = 0.5 \leq 1$.

**110. Diverges**. Integral Test for $f(x) = \frac{1}{x(1+\ln x)}$. Let $u = 1 + \ln x$. Diverges.

**111. Converges**. Limit Comparison Test with $b_n = \frac{1}{n^2}$.

**112. Diverges**. Limit Comparison Test with $b_n = \frac{n}{\sqrt{n^3}} = \frac{1}{\sqrt{n}}$.

**113. Diverges**. Integral Test. Same behavior as $\sum \frac{\ln n}{n}$.

**114. Converges**. p-Series Test with $p = 4 > 1$.

**115. Converges**. Note $a_n = \frac{n}{n!} = \frac{1}{(n-1)!}$ for $n \geq 1$. Compare with $b_n = \frac{1}{n^2}$ using LCT. The limit of the ratio is 0. Converges.

**116. Diverges**. Limit Comparison Test with $b_n = \frac{1}{n}$.

**117. Diverges**. p-Series Test with $p = 1/3 \leq 1$.

**118. Diverges**. Limit Comparison Test with $b_n = \frac{1}{n}$ for $n \geq 2$.

**119. Diverges**. Limit Comparison Test with $b_n = \frac{1}{\sqrt{n}}$.

**120. Diverges**. Test for Divergence: $\lim_{n\to\infty} a_n = 2 \neq 0$.

**121. Converges**. p-Series Test with $p = 2 > 1$. $\pi$ is a constant.

**122. Converges**. Direct Comparison Test with $b_n = (\frac{1}{3})^n$.

**123. Diverges**. Harmonic series with a constant multiple.

**124. Converges**. Limit Comparison Test with $b_n = \frac{1}{n^2}$.

**125. Converges**. p-Series Test with $p = 3/2 > 1$.

**126. Diverges**. Limit Comparison Test with $b_n = \frac{1}{n}$ for $n \geq 2$.

**127. Diverges**. Limit Comparison Test with $b_n = \frac{\sqrt{n}}{n\sqrt{n}} = \frac{1}{n}$.

**128. Diverges**. Limit Comparison Test with $b_n = \frac{n^2}{n^3} = \frac{1}{n}$.

**129. Diverges**. Limit Comparison Test with $b_n = \frac{1}{n^{1/3}}$.

**130. Converges**. Limit Comparison Test with $b_n = \frac{1}{n^2}$.

**131. Diverges**. Limit Comparison Test with $b_n = \frac{1}{n}$.

**132. Converges**. p-Series Test with $p = 1.0001 > 1$.

**133. Converges**. Direct Comparison Test. $\frac{\arctan(n)}{n^2} < \frac{\pi/2}{n^2}$.

**134. Diverges**. Limit Comparison Test with $b_n = \frac{e^n}{ne^n} = \frac{1}{n}$.

**135. Converges**. Limit Comparison Test with $b_n = \frac{1}{n^2}$.

**136. Diverges**. Test for Divergence: $\lim_{n\to\infty} n^{1/n} = 1$, so $\lim_{n\to\infty} a_n = 1 \neq 0$.

**137. Converges**. Direct Comparison Test with $b_n = \frac{3}{n^2}$.

**138. Converges**. Limit Comparison Test with $b_n = \frac{1}{3^n}$.

**139. Converges**. Limit Comparison Test with $b_n = \frac{3^n}{5^n} = (\frac{3}{5})^n$.

**140. Converges**. Direct Comparison Test. For $n \geq 2$, $a_n = \frac{1\cdot 2\cdots n}{n\cdot n\cdots n} = \frac{1}{n}\frac{2}{n}\cdots\frac{n}{n} \leq \frac{1}{n}\cdot\frac{2}{n} = \frac{2}{n^2}$. Since $\sum \frac{2}{n^2}$ converges, the series converges.

**141. Converges**. p-Series Test with $p = 4/3 > 1$.

**142. Diverges**. Integral Test. Let $u = \ln(x+1)$. Diverges.

**143. Diverges**. Limit Comparison Test with $b_n = \frac{1}{n}$.

**144. Converges**. Integral Test. Let $u = x^4$, $du = 4x^3dx$. The integral $\int_1^\infty x^3e^{-x^4}dx$ converges.

**145. Converges**. Limit Comparison Test with $b_n = \frac{1}{n^2}$.

**146. Converges**. Geometric Series. $a_n = (\frac{3}{4})^n$. $r = 3/4 < 1$.

**147. Converges**. Direct Comparison Test with $b_n = \frac{1}{2^n}$.

**148. Converges**. Limit Comparison Test with $b_n = \frac{n^{2/3}}{n^2} = \frac{1}{n^{4/3}}$. Converges since $p = 4/3 > 1$.

**149. Diverges**. Limit Comparison Test with $b_n = \frac{1}{n}$.

**150. Converges**. Limit Comparison Test with convergent series $b_n = \frac{1}{2^n}$. The limit of the ratio is 0.

**151. Converges**. Limit Comparison Test with $b_n = \frac{1}{n^2}$ for $n \geq 2$.

**152. Diverges**. Test for Divergence: $\lim_{n\to\infty} \frac{n-2}{n} = 1 \neq 0$.

**153. Diverges**. Limit Comparison Test with $b_n = \frac{1}{n^{2/3}}$ for $n \geq 2$.

**154. Converges**. p-Series Test with $p = 5 > 1$.

**155. Diverges**. Limit Comparison Test with $b_n = \frac{1}{n}$.

**156. Diverges**. Direct Comparison Test. For $n > e$, $\ln(n) > 1$, so $\frac{\ln(n)}{\sqrt{n}} > \frac{1}{\sqrt{n}}$. Since $\sum \frac{1}{\sqrt{n}}$ diverges, the series diverges.

**157. Diverges**. Constant multiple of the harmonic series.

**158. Converges**. Limit Comparison Test with $b_n = \frac{1}{n^2}$.

**159. Diverges**. p-Series Test with $p = 1/2 \leq 1$.

**160. Converges**. Limit Comparison Test with $b_n = \frac{1}{n^2}$. $\lim_{n\to\infty} \cos(1/n) = 1$.

**161. Converges**. Limit Comparison Test with $b_n = \frac{1}{n^3}$.

**162. Converges**. Direct Comparison Test with $b_n = \frac{1}{n \cdot 2^n}$, which converges by comparison with $(1/2)^n$.

**163. Diverges**. Test for Divergence: $\lim_{n\to\infty} \frac{n+2}{n+1} = 1 \neq 0$.

**164. Converges**. p-Series Test with $p = 2 > 1$.

**165. Diverges**. p-Series Test (Harmonic Series) with $p = 1$.

**166. Converges**. Limit Comparison Test with $b_n = \frac{1}{n^3}$ for $n \geq 2$.

**167. Diverges**. Limit Comparison Test with $b_n = \frac{1}{n}$.

**168. Converges**. Limit Comparison Test with $b_n = \frac{1}{n^2}$.

**169. Converges**. Limit Comparison Test with $b_n = \frac{1}{n^2}$.

**170. Diverges**. Limit Comparison Test with $b_n = \frac{1}{n}$.

**171. Diverges**. Limit Comparison Test with $b_n = \frac{1}{n}$.

**172. Converges**. p-Series Test with $p = 1.5 > 1$.

**173. Diverges**. Limit Comparison Test with $b_n = \frac{1}{n}$.

**174. Converges**. Geometric Series with $r = 3/5 < 1$.

**175. Diverges**. Limit Comparison Test with $b_n = \frac{1}{n}$.

**176. Diverges**. Limit Comparison Test with $b_n = \frac{1}{n}$.

**177. Converges**. p-Series Test with $p = 3/2 > 1$.

**178. Converges**. Limit Comparison Test with the convergent geometric series $b_n = (\frac{1}{2})^n$.