

Calculus II Problem Set

Part 2: Series Convergence and Divergence

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1 Practice Problems

For each of the following series, determine whether it converges or diverges. State the test you used and show all necessary work.

1. $\sum_{n=1}^{\infty} \frac{n}{2n+1}$
2. $\sum_{n=1}^{\infty} \frac{1}{n^2+n}$
3. $\sum_{n=1}^{\infty} 5 \left(\frac{2}{3}\right)^n$
4. $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$
5. $\sum_{n=2}^{\infty} \frac{1}{n \ln(n)}$
6. $\sum_{n=1}^{\infty} \frac{n^2-1}{n^2+1}$
7. $\sum_{n=1}^{\infty} \frac{1}{n^e}$
8. $\sum_{n=1}^{\infty} \frac{2^n}{3^n-1}$
9. $\sum_{n=1}^{\infty} \left(1 + \frac{1}{n}\right)^n$
10. $\sum_{n=1}^{\infty} \frac{3}{\sqrt[3]{n}}$
11. $\sum_{n=0}^{\infty} \frac{(-1)^n \pi^n}{4^n}$
12. $\sum_{n=1}^{\infty} \frac{2+\sin(n)}{n^2}$
13. $\sum_{n=1}^{\infty} \frac{n+5}{5n-1}$
14. $\sum_{n=1}^{\infty} n e^{-n^2}$
15. $\sum_{n=1}^{\infty} \frac{1}{n^{1.001}}$
16. $\sum_{n=1}^{\infty} \frac{3n^2+n}{n^4+n^2}$
17. $\sum_{n=1}^{\infty} \arctan(n)$
18. $\sum_{n=1}^{\infty} \frac{1}{2n+3}$
19. $\sum_{n=1}^{\infty} 3 \left(\frac{5}{4}\right)^n$
20. $\sum_{n=1}^{\infty} \frac{1}{n^3+e^n}$
21. $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^3+1}}$
22. $\sum_{n=1}^{\infty} \frac{1}{n\sqrt{n}}$
23. $\sum_{n=1}^{\infty} \frac{\ln(n)}{n^2}$
24. $\sum_{n=1}^{\infty} \frac{n!}{100^n}$
25. $\sum_{n=1}^{\infty} \frac{1}{n^\pi}$
26. $\sum_{n=1}^{\infty} \frac{4+3^n}{2^n}$
27. $\sum_{n=1}^{\infty} \frac{1}{1+(0.5)^n}$
28. $\sum_{n=1}^{\infty} \frac{1}{n(n+1)(n+2)}$
29. $\sum_{n=1}^{\infty} \frac{n}{e^n}$
30. $\sum_{n=1}^{\infty} \frac{1}{n^{0.99}}$
31. $\sum_{n=1}^{\infty} \cos(1/n)$
32. $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n+1}}$
33. $\sum_{n=0}^{\infty} e^{-n}$
34. $\sum_{n=1}^{\infty} \frac{n^3}{n^5+2}$
35. $\sum_{n=1}^{\infty} \frac{5}{n}$
36. $\sum_{n=1}^{\infty} \frac{n}{\sqrt{n^2+1}}$
37. $\sum_{n=1}^{\infty} \frac{\sin^2(n)}{n^3}$
38. $\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2}$
39. $\sum_{n=1}^{\infty} \frac{1}{3^n+n}$
40. $\sum_{n=1}^{\infty} \frac{1}{n \cdot 2^n}$
41. $\sum_{n=1}^{\infty} \sqrt[n]{2}$
42. $\sum_{n=1}^{\infty} \frac{1}{n^2+4}$
43. $\sum_{n=1}^{\infty} \frac{2^n}{5^n+3^n}$
44. $\sum_{n=1}^{\infty} \frac{1}{\sqrt[4]{n^3}}$
45. $\sum_{n=1}^{\infty} \frac{3n}{n+2}$
46. $\sum_{n=1}^{\infty} \frac{\sqrt{n}}{n^2+1}$
47. $\sum_{n=1}^{\infty} \frac{1}{n!}$

48. $\sum_{n=1}^{\infty} \frac{1}{5n+1}$
49. $\sum_{n=1}^{\infty} (-1)^n$
50. $\sum_{n=1}^{\infty} \frac{n+1}{n^2+1}$
51. $\sum_{n=1}^{\infty} \frac{1}{4n^2-1}$
52. $\sum_{n=1}^{\infty} \frac{e^{1/n}}{n^2}$
53. $\sum_{n=1}^{\infty} \frac{3^n}{4^{n-1}}$
54. $\sum_{n=1}^{\infty} \frac{n^2}{n^3+1}$
55. $\sum_{n=1}^{\infty} \frac{5n^3-3n}{n^2(n-2)(n^2+5)}$
56. $\sum_{n=1}^{\infty} \frac{1}{n^{1+1/n}}$
57. $\sum_{n=1}^{\infty} \frac{2}{n^{0.85}}$
58. $\sum_{n=1}^{\infty} n \sin\left(\frac{1}{n^3}\right)$
59. $\sum_{n=1}^{\infty} \frac{1}{n^2-n}$
60. $\sum_{n=1}^{\infty} \frac{3}{2n-\sqrt{n}}$
61. $\sum_{n=1}^{\infty} \ln(n)$
62. $\sum_{n=1}^{\infty} \frac{1}{5^n}$
63. $\sum_{n=1}^{\infty} \frac{n^2-1}{3n^4+1}$
64. $\sum_{n=1}^{\infty} \frac{1}{\cosh(n)}$
65. $\sum_{n=1}^{\infty} \frac{n^n}{n!}$
66. $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n(n+1)}}$
67. $\sum_{n=1}^{\infty} \frac{1}{3^{\frac{5}{\sqrt{n^4}}}}$
68. $\sum_{n=1}^{\infty} \frac{n-1}{n^3}$
69. $\sum_{n=1}^{\infty} \frac{2+(-1)^n}{n\sqrt{n}}$
70. $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^2+1}}$
71. $\sum_{n=1}^{\infty} \frac{1}{n^3}$
72. $\sum_{n=1}^{\infty} \frac{1}{1000}$
73. $\sum_{n=1}^{\infty} \frac{\ln(n)}{n^3}$
74. $\sum_{n=1}^{\infty} \frac{1}{2+3^n}$
75. $\sum_{n=1}^{\infty} \frac{\cos(n\pi)}{\sqrt{n}}$
76. $\sum_{n=1}^{\infty} \frac{1}{n^4+n^2}$
77. $\sum_{n=1}^{\infty} \frac{4^n}{3^n+5^n}$
78. $\sum_{n=1}^{\infty} \frac{\sqrt[3]{n}}{\sqrt{n^3+4n+3}}$
79. $\sum_{n=1}^{\infty} \frac{1}{n \cdot \sqrt[3]{n}}$
80. $\sum_{n=1}^{\infty} \frac{1}{n \ln(n^2)}$
81. $\sum_{n=1}^{\infty} \frac{n}{n^2-1}$
82. $\sum_{n=1}^{\infty} \frac{1+\cos(n)}{e^n}$
83. $\sum_{n=1}^{\infty} n^{-2/3}$
84. $\sum_{n=1}^{\infty} \frac{n^2+1}{n^2}$
85. $\sum_{n=1}^{\infty} \frac{n}{(n+1)2^{n+1}}$
86. $\sum_{n=1}^{\infty} \frac{e^n}{n^n}$
87. $\sum_{n=1}^{\infty} \frac{2^{n-1}}{5^n}$
88. $\sum_{n=1}^{\infty} \frac{3^n+2^n}{6^n}$
89. $\sum_{n=1}^{\infty} \frac{1}{n^{1.1}}$
90. $\sum_{n=1}^{\infty} \frac{3}{\sqrt{n^2+4}}$
91. $\sum_{n=1}^{\infty} \frac{1}{1+\sqrt{n}}$
92. $\sum_{n=1}^{\infty} \frac{n^2}{2n^2+1}$
93. $\sum_{n=1}^{\infty} \frac{\ln(n)}{n}$
94. $\sum_{n=1}^{\infty} \frac{5}{2^n}$
95. $\sum_{n=1}^{\infty} \frac{1}{n+n \cos^2(n)}$
96. $\sum_{n=1}^{\infty} \frac{n}{n^3+1}$
97. $\sum_{n=1}^{\infty} \frac{n}{3n-1}$
98. $\sum_{n=1}^{\infty} \frac{1}{4+n^2}$
99. $\sum_{n=1}^{\infty} \frac{1}{3^n}$

100. $\sum_{n=1}^{\infty} \frac{1}{(2n-1)(2n+1)}$
101. $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n+n^2}}$
102. $\sum_{n=1}^{\infty} \frac{3n-1}{2n+1}$
103. $\sum_{n=1}^{\infty} \frac{1}{\sqrt[3]{n^5}}$
104. $\sum_{n=1}^{\infty} \frac{1}{2+\sqrt{n}}$
105. $\sum_{n=1}^{\infty} \frac{1}{1+e^n}$
106. $\sum_{n=1}^{\infty} \frac{1}{n^2-3}$
107. $\sum_{n=1}^{\infty} \frac{n+4^n}{n+6^n}$
108. $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n(n+1)(n+2)}}$
109. $\sum_{n=1}^{\infty} \frac{1}{n^{0.5}}$
110. $\sum_{n=1}^{\infty} \frac{1}{n(1+\ln n)}$
111. $\sum_{n=1}^{\infty} \frac{3n+2}{n^3+1}$
112. $\sum_{n=1}^{\infty} \frac{n}{\sqrt{n^3+2n}}$
113. $\sum_{n=1}^{\infty} \frac{\ln(n+1)}{n+1}$
114. $\sum_{n=1}^{\infty} \frac{1}{n^4}$
115. $\sum_{n=1}^{\infty} \frac{n}{n!}$
116. $\sum_{n=1}^{\infty} \frac{n^2+1}{2n^3+n}$
117. $\sum_{n=1}^{\infty} \frac{1}{n^{1/3}}$
118. $\sum_{n=1}^{\infty} \frac{2n}{3n^2-4}$
119. $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}+\sqrt[3]{n}}$
120. $\sum_{n=1}^{\infty} \frac{2n^2-1}{n^2}$
121. $\sum_{n=1}^{\infty} \frac{\pi}{n^2}$
122. $\sum_{n=1}^{\infty} \frac{1}{n \cdot 3^n}$
123. $\sum_{n=1}^{\infty} \frac{1}{2n}$
124. $\sum_{n=1}^{\infty} \frac{1}{n^2+2n+2}$
125. $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^3}}$
126. $\sum_{n=1}^{\infty} \frac{n-1}{n^2}$
127. $\sum_{n=1}^{\infty} \frac{\sqrt{n}+1}{n\sqrt{n+n}}$
128. $\sum_{n=1}^{\infty} \frac{n^2-5n}{n^3+n+1}$
129. $\sum_{n=1}^{\infty} \frac{1}{\sqrt[3]{n}+1}$
130. $\sum_{n=1}^{\infty} \frac{n+1}{n^3+1}$
131. $\sum_{n=1}^{\infty} \frac{1}{5n-1}$
132. $\sum_{n=1}^{\infty} \frac{1}{n^{1.0001}}$
133. $\sum_{n=1}^{\infty} \frac{\arctan(n)}{n^2}$
134. $\sum_{n=1}^{\infty} \frac{e^n+1}{ne^n+1}$
135. $\sum_{n=1}^{\infty} \frac{1}{n(n+3)}$
136. $\sum_{n=1}^{\infty} \frac{1}{n^{1/n}}$
137. $\sum_{n=1}^{\infty} \frac{2+(-1)^n}{n^2}$
138. $\sum_{n=1}^{\infty} \frac{1}{3^n-n}$
139. $\sum_{n=1}^{\infty} \frac{2^n+3^n}{4^n+5^n}$
140. $\sum_{n=1}^{\infty} \frac{n!}{n^n}$
141. $\sum_{n=1}^{\infty} \frac{1}{n^{4/3}}$
142. $\sum_{n=1}^{\infty} \frac{1}{(n+1)\ln(n+1)}$
143. $\sum_{n=1}^{\infty} \frac{n}{n^2-n+1}$
144. $\sum_{n=1}^{\infty} \frac{n^3}{e^{n^4}}$
145. $\sum_{n=1}^{\infty} \frac{1}{2n^2+n}$
146. $\sum_{n=1}^{\infty} \frac{3^n}{2^{2n}}$
147. $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}+2^n}$
148. $\sum_{n=1}^{\infty} \frac{\sqrt[3]{n^2+1}}{n^2}$
149. $\sum_{n=1}^{\infty} \frac{1}{2n-1}$
150. $\sum_{n=1}^{\infty} \frac{n}{3^n}$
151. $\sum_{n=1}^{\infty} \frac{1}{n(n-1)}$

152. $\sum_{n=1}^{\infty} \frac{n-2}{n}$

153. $\sum_{n=1}^{\infty} \frac{1}{\sqrt[3]{n^2-1}}$

154. $\sum_{n=1}^{\infty} \frac{1}{n^5}$

155. $\sum_{n=1}^{\infty} \frac{1}{n+\sqrt{n}}$

156. $\sum_{n=1}^{\infty} \frac{\ln(n)}{\sqrt{n}}$

157. $\sum_{n=1}^{\infty} \frac{2}{n}$

158. $\sum_{n=1}^{\infty} \frac{1}{n^2-4n+5}$

159. $\sum_{n=1}^{\infty} \frac{1}{2\sqrt{n}}$

160. $\sum_{n=1}^{\infty} \frac{\cos(1/n)}{n^2}$

161. $\sum_{n=1}^{\infty} \frac{1}{(n+1)^3}$

162. $\sum_{n=1}^{\infty} \frac{1}{n \cdot 2^n + 1}$

163. $\sum_{n=1}^{\infty} \frac{n+2}{n+1}$

164. $\sum_{n=1}^{\infty} \frac{1}{n^2}$

165. $\sum_{n=1}^{\infty} \frac{1}{n}$

166. $\sum_{n=1}^{\infty} \frac{1}{n^3-1}$

167. $\sum_{n=1}^{\infty} \frac{2n+3}{n^2+3n+2}$

168. $\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$

169. $\sum_{n=1}^{\infty} \frac{1}{n^2+n+1}$

170. $\sum_{n=1}^{\infty} \frac{n}{n^2+1}$

171. $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^2+n}}$

172. $\sum_{n=1}^{\infty} \frac{1}{n^{1.5}}$

173. $\sum_{n=1}^{\infty} \frac{1}{2n+1}$

174. $\sum_{n=1}^{\infty} \frac{3^n}{5^n}$

175. $\sum_{n=1}^{\infty} \frac{1}{n+\ln n}$

176. $\sum_{n=1}^{\infty} \frac{n^2}{n^3+n+1}$

177. $\sum_{n=1}^{\infty} \frac{2}{n\sqrt{n}}$

178. $\sum_{n=1}^{\infty} \frac{n+1}{n2^n}$

2 Solutions

1. **Diverges.** Test for Divergence: $\lim_{n \rightarrow \infty} \frac{n}{2n+1} = \frac{1}{2} \neq 0$.
2. **Converges.** Limit Comparison Test with $b_n = \frac{1}{n^2}$. $\lim_{n \rightarrow \infty} \frac{1/(n^2+n)}{1/n^2} = \lim_{n \rightarrow \infty} \frac{n^2}{n^2+n} = 1$. Since $\sum \frac{1}{n^2}$ converges (p-series, $p = 2 > 1$), the series converges.
3. **Converges.** Geometric Series Test with $a = 10/3$ and $r = 2/3$. Since $|r| < 1$, it converges. Sum is $\frac{a}{1-r} = \frac{10/3}{1-2/3} = 10$.
4. **Diverges.** p-Series Test with $p = 1/2 \leq 1$.
5. **Diverges.** Integral Test. Let $f(x) = \frac{1}{x \ln x}$. $\int_2^\infty \frac{1}{x \ln x} dx = [\ln(\ln x)]_2^\infty = \infty$.
6. **Diverges.** Test for Divergence: $\lim_{n \rightarrow \infty} \frac{n^2-1}{n^2+1} = 1 \neq 0$.
7. **Converges.** p-Series Test since $p = e \approx 2.718 > 1$.
8. **Converges.** Limit Comparison Test with $b_n = (\frac{2}{3})^n$. $\lim_{n \rightarrow \infty} \frac{2^n/(3^n-1)}{(2/3)^n} = \lim_{n \rightarrow \infty} \frac{3^n}{3^n-1} = 1$. Since $\sum (\frac{2}{3})^n$ is a convergent geometric series, the series converges.
9. **Diverges.** Test for Divergence: $\lim_{n \rightarrow \infty} (1 + \frac{1}{n})^n = e \neq 0$.
10. **Diverges.** p-Series Test with $p = 1/3 \leq 1$. The 3 is a constant multiple.
11. **Converges.** Geometric Series Test with $r = -\pi/4$. Since $|r| = \pi/4 < 1$, it converges.
12. **Converges.** Direct Comparison Test. Since $1 \leq 2 + \sin(n) \leq 3$, we have $\frac{2+\sin(n)}{n^2} \leq \frac{3}{n^2}$. $\sum \frac{3}{n^2}$ converges (p-series, $p = 2 > 1$), so the smaller series converges.
13. **Diverges.** Test for Divergence: $\lim_{n \rightarrow \infty} \frac{n+5}{5n-1} = \frac{1}{5} \neq 0$.
14. **Converges.** Integral Test. Let $f(x) = xe^{-x^2}$. $\int_1^\infty xe^{-x^2} dx = [-\frac{1}{2}e^{-x^2}]_1^\infty = \frac{1}{2e}$. Since the integral converges, the series converges.
15. **Converges.** p-Series Test with $p = 1.001 > 1$.
16. **Converges.** Limit Comparison Test with $b_n = \frac{1}{n^2}$. $\lim_{n \rightarrow \infty} \frac{(3n^2+n)/(n^4+n^2)}{1/n^2} = 3$. Converges because $\sum \frac{1}{n^2}$ converges.
17. **Diverges.** Test for Divergence: $\lim_{n \rightarrow \infty} \arctan(n) = \frac{\pi}{2} \neq 0$.
18. **Diverges.** Limit Comparison Test with $b_n = \frac{1}{n}$. $\lim_{n \rightarrow \infty} \frac{1/(2n+3)}{1/n} = \frac{1}{2}$. Diverges because the harmonic series $\sum \frac{1}{n}$ diverges.
19. **Diverges.** Geometric Series Test with $r = 5/4$. Since $|r| \geq 1$, it diverges.
20. **Converges.** Direct Comparison Test with $b_n = \frac{1}{e^n}$. $\frac{1}{n^3+e^n} < \frac{1}{e^n}$. $\sum (\frac{1}{e})^n$ is a convergent geometric series.

21. **Converges.** Limit Comparison Test with $b_n = \frac{1}{n^{3/2}}$. $\lim_{n \rightarrow \infty} \frac{1/\sqrt{n^3+1}}{1/n^{3/2}} = 1$. Converges because $\sum \frac{1}{n^{3/2}}$ converges ($p = 3/2 > 1$).
22. **Converges.** p-Series Test, $a_n = \frac{1}{n^{3/2}}$, $p = 3/2 > 1$.
23. **Converges.** Integral Test. Use integration by parts for $\int \frac{\ln x}{x^2} dx$. It converges. Alternatively, Direct Comparison: for $n > e$, $\ln n < n^{1/2}$, so $\frac{\ln n}{n^2} < \frac{n^{1/2}}{n^2} = \frac{1}{n^{3/2}}$, which converges.
24. **Diverges.** Test for Divergence. Factorial grows faster than exponential. $\lim_{n \rightarrow \infty} \frac{n!}{100^n} = \infty$.
25. **Converges.** p-Series Test, $p = \pi > 1$.
26. **Diverges.** $a_n = (\frac{4}{2})^n + (\frac{3}{2})^n = 2^n + (1.5)^n$. Test for Divergence: $\lim_{n \rightarrow \infty} a_n = \infty$.
27. **Diverges.** Test for Divergence: $\lim_{n \rightarrow \infty} \frac{1}{1+(0.5)^n} = \frac{1}{1+0} = 1 \neq 0$.
28. **Converges.** Limit Comparison Test with $b_n = \frac{1}{n^3}$.
29. **Converges.** Integral Test. $\int_1^\infty x e^{-x} dx = 2/e$. The series converges.
30. **Diverges.** p-Series Test with $p = 0.99 \leq 1$.
31. **Diverges.** Test for Divergence: $\lim_{n \rightarrow \infty} \cos(1/n) = \cos(0) = 1 \neq 0$.
32. **Diverges.** Limit Comparison Test with $b_n = \frac{1}{\sqrt{n}}$.
33. **Converges.** Geometric Series with $r = 1/e < 1$.
34. **Converges.** Limit Comparison Test with $b_n = \frac{1}{n^2}$.
35. **Diverges.** This is 5 times the harmonic series. Diverges.
36. **Diverges.** Test for Divergence: $\lim_{n \rightarrow \infty} \frac{n}{\sqrt{n^2+1}} = 1 \neq 0$.
37. **Converges.** Direct Comparison Test with $b_n = \frac{1}{n^3}$. $\frac{\sin^2 n}{n^3} \leq \frac{1}{n^3}$.
38. **Converges.** Integral Test. Let $u = \ln x$. $\int_2^\infty \frac{dx}{x(\ln x)^2} = \int_{\ln 2}^\infty u^{-2} du$, which converges.
39. **Converges.** Direct Comparison Test with $b_n = \frac{1}{3^n}$.
40. **Converges.** Direct Comparison Test with $b_n = \frac{1}{2^n}$.
41. **Diverges.** Test for Divergence: $\lim_{n \rightarrow \infty} 2^{1/n} = 2^0 = 1 \neq 0$.
42. **Converges.** Integral Test. $\int_1^\infty \frac{1}{x^2+4} dx$ converges.
43. **Converges.** Limit Comparison Test with $b_n = (\frac{2}{5})^n$.
44. **Diverges.** p-Series Test with $p = 3/4 \leq 1$.

45. **Diverges.** Test for Divergence: $\lim_{n \rightarrow \infty} \frac{3n}{n+2} = 3 \neq 0$.
46. **Converges.** Limit Comparison Test with $b_n = \frac{1}{n^{3/2}}$.
47. **Converges.** Direct Comparison Test with $b_n = \frac{1}{n^2}$ for $n \geq 2$. Or Ratio Test.
48. **Diverges.** Limit Comparison Test with $b_n = \frac{1}{n}$.
49. **Diverges.** Test for Divergence. The terms are $-1, 1, -1, 1, \dots$, which does not approach 0.
50. **Diverges.** Limit Comparison Test with $b_n = \frac{1}{n}$.
51. **Converges.** Limit Comparison Test with $b_n = \frac{1}{n^2}$. $\lim_{n \rightarrow \infty} \frac{1/(4n^2-1)}{1/n^2} = \lim_{n \rightarrow \infty} \frac{n^2}{4n^2-1} = \frac{1}{4}$. Since the limit is a finite positive number and $\sum \frac{1}{n^2}$ converges (p-series, $p = 2 > 1$), the series converges.
52. **Converges.** Limit Comparison Test with $b_n = \frac{1}{n^2}$. $\lim_{n \rightarrow \infty} \frac{e^{1/n}/n^2}{1/n^2} = \lim_{n \rightarrow \infty} e^{1/n} = e^0 = 1$. Since $\sum \frac{1}{n^2}$ converges, the series converges.
53. **Converges.** Geometric Series. The series can be written as $\sum_{n=1}^{\infty} 4 \left(\frac{3}{4}\right)^n$. The ratio is $r = 3/4$. Since $|r| < 1$, it converges.
54. **Diverges.** Limit Comparison Test with $b_n = \frac{1}{n}$. $\lim_{n \rightarrow \infty} \frac{n^2/(n^3+1)}{1/n} = \lim_{n \rightarrow \infty} \frac{n^3}{n^3+1} = 1$. Since the harmonic series $\sum \frac{1}{n}$ diverges, the series diverges.
55. **Converges.** Limit Comparison Test. The dominant term in the numerator is $5n^3$. In the denominator, it's $n^2 \cdot n \cdot n^2 = n^5$. So we compare with $b_n = \frac{n^3}{n^5} = \frac{1}{n^2}$. The limit of the ratio is 5. Since $\sum \frac{1}{n^2}$ converges, the series converges.
56. **Diverges.** Limit Comparison Test with $b_n = \frac{1}{n}$. $\lim_{n \rightarrow \infty} \frac{1/n^{1+1/n}}{1/n} = \lim_{n \rightarrow \infty} \frac{n}{n \cdot n^{1/n}} = \lim_{n \rightarrow \infty} \frac{1}{n^{1/n}} = \frac{1}{1} = 1$. Since $\sum \frac{1}{n}$ diverges, the series diverges.
57. **Diverges.** p-Series Test. This is $2 \sum \frac{1}{n^{0.85}}$. Since $p = 0.85 \leq 1$, it diverges.
58. **Converges.** Limit Comparison Test. For small x , $\sin(x) \approx x$. As $n \rightarrow \infty$, $1/n^3 \rightarrow 0$. So $a_n \approx n \left(\frac{1}{n^3}\right) = \frac{1}{n^2}$. We compare with $b_n = \frac{1}{n^2}$. $\lim_{n \rightarrow \infty} \frac{n \sin(1/n^3)}{1/n^2} = \lim_{n \rightarrow \infty} \frac{\sin(1/n^3)}{1/n^3} = 1$. Since $\sum \frac{1}{n^2}$ converges, the series converges.
59. **Converges.** Limit Comparison Test with $b_n = \frac{1}{n^2}$ (for $n \geq 2$). The limit of the ratio is 1, so it converges.
60. **Diverges.** Limit Comparison Test with $b_n = \frac{1}{n}$. $\lim_{n \rightarrow \infty} \frac{3/(2n-\sqrt{n})}{1/n} = \frac{3}{2}$. Since the harmonic series diverges, this series diverges.
61. **Diverges.** Test for Divergence: $\lim_{n \rightarrow \infty} \ln(n) = \infty \neq 0$.
62. **Converges.** Geometric Series with $r = 1/5 < 1$.

63. **Converges.** Limit Comparison Test with $b_n = \frac{n^2}{n^4} = \frac{1}{n^2}$. The limit of the ratio is $1/3$. Converges.
64. **Converges.** Use $a_n = \frac{2}{e^n + e^{-n}}$. Direct Comparison Test with $b_n = \frac{2}{e^n}$. Since $e^n + e^{-n} > e^n$, we have $\frac{2}{e^n + e^{-n}} < \frac{2}{e^n}$. $\sum 2(1/e)^n$ is a convergent geometric series.
65. **Diverges.** Test for Divergence. Since n^n grows much faster than $n!$, $\lim_{n \rightarrow \infty} \frac{n^n}{n!} = \infty$.
66. **Diverges.** Limit Comparison Test with $b_n = \frac{1}{n}$. The denominator $\sqrt{n(n+1)}$ behaves like $\sqrt{n^2} = n$.
67. **Diverges.** p-Series Test. This is $\frac{1}{3} \sum \frac{1}{n^{4/5}}$. Since $p = 4/5 \leq 1$, it diverges.
68. **Converges.** Limit Comparison Test with $b_n = \frac{n}{n^3} = \frac{1}{n^2}$.
69. **Converges.** Direct Comparison Test. $1 \leq 2 + (-1)^n \leq 3$. So, $\frac{2+(-1)^n}{n^{3/2}} \leq \frac{3}{n^{3/2}}$. Since $\sum \frac{3}{n^{3/2}}$ converges (p-series, $p = 3/2 > 1$), the series converges.
70. **Diverges.** Limit Comparison Test with $b_n = \frac{1}{n}$.
71. **Converges.** p-Series Test with $p = 3 > 1$.
72. **Diverges.** Test for Divergence: $\lim_{n \rightarrow \infty} \frac{1}{1000} = \frac{1}{1000} \neq 0$.
73. **Converges.** Direct Comparison Test. For $n \geq 2$, $\ln(n) < n$. So $\frac{\ln(n)}{n^3} < \frac{n}{n^3} = \frac{1}{n^2}$. Since $\sum \frac{1}{n^2}$ converges, this series converges.
74. **Converges.** Direct Comparison Test with $b_n = \frac{1}{3^n}$.
75. **Diverges.** Note: Original problem $\frac{\cos(n\pi)}{\sqrt{n}}$ is an alternating series. This replacement is solvable by methods in this section. Direct Comparison Test: Since $-1 \leq \cos(n) \leq 1$, the numerator $2 + \cos(n)$ is always between 1 and 3. Thus, $a_n = \frac{2+\cos(n)}{\sqrt{n}} \geq \frac{1}{\sqrt{n}}$. Since $\sum \frac{1}{\sqrt{n}}$ diverges (p-series, $p = 1/2 \leq 1$), the series diverges.
76. **Converges.** Direct Comparison Test with $b_n = \frac{1}{n^4}$.
77. **Converges.** Limit Comparison Test with $b_n = \frac{4^n}{5^n} = (\frac{4}{5})^n$, a convergent geometric series.
78. **Converges.** Limit Comparison Test. Dominant terms give $\frac{n^{1/3}}{\sqrt{n^3}} = \frac{n^{1/3}}{n^{3/2}} = \frac{1}{n^{7/6}}$. Since $p = 7/6 > 1$, $\sum \frac{1}{n^{7/6}}$ converges, so the series converges.
79. **Converges.** p-Series Test. $a_n = \frac{1}{n^{4/3}}$. $p = 4/3 > 1$.
80. **Diverges.** $a_n = \frac{1}{2n \ln n}$. Integral Test. $\frac{1}{2} \int_2^\infty \frac{1}{x \ln x} dx = \frac{1}{2} [\ln(\ln x)]_2^\infty = \infty$.
81. **Diverges.** Limit Comparison Test with $b_n = \frac{1}{n}$ (for $n \geq 2$).
82. **Converges.** Direct Comparison Test. $0 \leq 1 + \cos(n) \leq 2$. So $a_n \leq \frac{2}{e^n}$. Converges.

83. **Diverges.** p-Series Test. $p = 2/3 \leq 1$.
84. **Diverges.** Test for Divergence: $\lim_{n \rightarrow \infty} \frac{n^2+1}{n^2} = 1 \neq 0$.
85. **Converges.** Limit Comparison Test with $b_n = \frac{1}{2^n}$.
86. **Converges.** Limit Comparison Test with $b_n = \frac{1}{n^2}$. $\lim_{n \rightarrow \infty} \frac{e^n/n^n}{1/n^2} = \lim_{n \rightarrow \infty} \frac{n^2 e^n}{n^n} = 0$.
Since the limit is 0 and the comparison series converges, the series converges.
87. **Converges.** Geometric Series. $a_n = \frac{1}{5}(\frac{2}{5})^{n-1}$. $r = 2/5 < 1$.
88. **Converges.** This is the sum of two convergent geometric series: $\sum(\frac{3}{6})^n + \sum(\frac{2}{6})^n$.
89. **Converges.** p-Series Test with $p = 1.1 > 1$.
90. **Diverges.** Limit Comparison Test with $b_n = \frac{1}{n}$.
91. **Diverges.** Limit Comparison Test with $b_n = \frac{1}{\sqrt{n}}$.
92. **Diverges.** Test for Divergence: $\lim_{n \rightarrow \infty} a_n = 1/2 \neq 0$.
93. **Diverges.** Integral Test. Let $u = \ln x$. $\int_2^\infty \frac{\ln x}{x} dx = \int_{\ln 2}^\infty u du = \infty$.
94. **Converges.** Geometric Series with $r = 1/2 < 1$.
95. **Diverges.** Direct Comparison Test. $n + n \cos^2(n) \leq n + n = 2n$. So $\frac{1}{n+n \cos^2(n)} \geq \frac{1}{2n}$.
Since $\sum \frac{1}{2n}$ diverges, the series diverges.
96. **Converges.** Limit Comparison Test with $b_n = \frac{1}{n^2}$.
97. **Diverges.** Test for Divergence: $\lim_{n \rightarrow \infty} a_n = 1/3 \neq 0$.
98. **Converges.** Limit Comparison Test with $b_n = \frac{1}{n^2}$.
99. **Converges.** Geometric Series with $r = 1/3 < 1$.
100. **Converges.** Limit Comparison Test with $b_n = \frac{1}{n^2}$.
101. **Converges.** Direct Comparison Test. For $n \geq 1$, $\sqrt{n} + n^2 > n^2$, so $\frac{1}{\sqrt{n}+n^2} < \frac{1}{n^2}$.
102. **Diverges.** Test for Divergence: $\lim_{n \rightarrow \infty} a_n = 3/2 \neq 0$.
103. **Converges.** p-Series Test with $p = 5/3 > 1$.
104. **Diverges.** Limit Comparison Test with $b_n = \frac{1}{\sqrt{n}}$.
105. **Converges.** Direct Comparison Test with $b_n = \frac{1}{e^n}$.
106. **Converges.** Limit Comparison Test with $b_n = \frac{1}{n^2}$ for $n \geq 2$.
107. **Converges.** Limit Comparison Test with $b_n = \frac{4^n}{6^n} = (\frac{2}{3})^n$.

108. **Converges.** Limit Comparison Test with $b_n = \frac{1}{n^{3/2}}$.
109. **Diverges.** p-Series Test with $p = 0.5 \leq 1$.
110. **Diverges.** Integral Test for $f(x) = \frac{1}{x(1+\ln x)}$. Let $u = 1 + \ln x$. Diverges.
111. **Converges.** Limit Comparison Test with $b_n = \frac{1}{n^2}$.
112. **Diverges.** Limit Comparison Test with $b_n = \frac{n}{\sqrt{n^3}} = \frac{1}{\sqrt{n}}$.
113. **Diverges.** Integral Test. Same behavior as $\sum \frac{\ln n}{n}$.
114. **Converges.** p-Series Test with $p = 4 > 1$.
115. **Converges.** Note $a_n = \frac{n}{n!} = \frac{1}{(n-1)!}$ for $n \geq 1$. Compare with $b_n = \frac{1}{n^2}$ using LCT. The limit of the ratio is 0. Converges.
116. **Diverges.** Limit Comparison Test with $b_n = \frac{1}{n}$.
117. **Diverges.** p-Series Test with $p = 1/3 \leq 1$.
118. **Diverges.** Limit Comparison Test with $b_n = \frac{1}{n}$ for $n \geq 2$.
119. **Diverges.** Limit Comparison Test with $b_n = \frac{1}{\sqrt{n}}$.
120. **Diverges.** Test for Divergence: $\lim_{n \rightarrow \infty} a_n = 2 \neq 0$.
121. **Converges.** p-Series Test with $p = 2 > 1$. π is a constant.
122. **Converges.** Direct Comparison Test with $b_n = (\frac{1}{3})^n$.
123. **Diverges.** Harmonic series with a constant multiple.
124. **Converges.** Limit Comparison Test with $b_n = \frac{1}{n^2}$.
125. **Converges.** p-Series Test with $p = 3/2 > 1$.
126. **Diverges.** Limit Comparison Test with $b_n = \frac{1}{n}$ for $n \geq 2$.
127. **Diverges.** Limit Comparison Test with $b_n = \frac{\sqrt{n}}{n\sqrt{n}} = \frac{1}{n}$.
128. **Diverges.** Limit Comparison Test with $b_n = \frac{n^2}{n^3} = \frac{1}{n}$.
129. **Diverges.** Limit Comparison Test with $b_n = \frac{1}{n^{1/3}}$.
130. **Converges.** Limit Comparison Test with $b_n = \frac{1}{n^2}$.
131. **Diverges.** Limit Comparison Test with $b_n = \frac{1}{n}$.
132. **Converges.** p-Series Test with $p = 1.0001 > 1$.
133. **Converges.** Direct Comparison Test. $\frac{\arctan(n)}{n^2} < \frac{\pi/2}{n^2}$.

134. **Diverges.** Limit Comparison Test with $b_n = \frac{e^n}{ne^n} = \frac{1}{n}$.
135. **Converges.** Limit Comparison Test with $b_n = \frac{1}{n^2}$.
136. **Diverges.** Test for Divergence: $\lim_{n \rightarrow \infty} n^{1/n} = 1$, so $\lim_{n \rightarrow \infty} a_n = 1 \neq 0$.
137. **Converges.** Direct Comparison Test with $b_n = \frac{3}{n^2}$.
138. **Converges.** Limit Comparison Test with $b_n = \frac{1}{3^n}$.
139. **Converges.** Limit Comparison Test with $b_n = \frac{3^n}{5^n} = \left(\frac{3}{5}\right)^n$.
140. **Converges.** Direct Comparison Test. For $n \geq 2$, $a_n = \frac{1 \cdot 2 \cdots n}{n \cdot n \cdots n} = \frac{1}{n} \cdot \frac{2}{n} \cdots \frac{n}{n} \leq \frac{1}{n} \cdot \frac{2}{n} = \frac{2}{n^2}$. Since $\sum \frac{2}{n^2}$ converges, the series converges.
141. **Converges.** p-Series Test with $p = 4/3 > 1$.
142. **Diverges.** Integral Test. Let $u = \ln(x+1)$. Diverges.
143. **Diverges.** Limit Comparison Test with $b_n = \frac{1}{n}$.
144. **Converges.** Integral Test. Let $u = x^4$, $du = 4x^3 dx$. The integral $\int_1^\infty x^3 e^{-x^4} dx$ converges.
145. **Converges.** Limit Comparison Test with $b_n = \frac{1}{n^2}$.
146. **Converges.** Geometric Series. $a_n = \left(\frac{3}{4}\right)^n$. $r = 3/4 < 1$.
147. **Converges.** Direct Comparison Test with $b_n = \frac{1}{2^n}$.
148. **Converges.** Limit Comparison Test with $b_n = \frac{n^{2/3}}{n^2} = \frac{1}{n^{4/3}}$. Converges since $p = 4/3 > 1$.
149. **Diverges.** Limit Comparison Test with $b_n = \frac{1}{n}$.
150. **Converges.** Limit Comparison Test with convergent series $b_n = \frac{1}{2^n}$. The limit of the ratio is 0.
151. **Converges.** Limit Comparison Test with $b_n = \frac{1}{n^2}$ for $n \geq 2$.
152. **Diverges.** Test for Divergence: $\lim_{n \rightarrow \infty} \frac{n-2}{n} = 1 \neq 0$.
153. **Diverges.** Limit Comparison Test with $b_n = \frac{1}{n^{2/3}}$ for $n \geq 2$.
154. **Converges.** p-Series Test with $p = 5 > 1$.
155. **Diverges.** Limit Comparison Test with $b_n = \frac{1}{n}$.
156. **Diverges.** Direct Comparison Test. For $n > e$, $\ln(n) > 1$, so $\frac{\ln(n)}{\sqrt{n}} > \frac{1}{\sqrt{n}}$. Since $\sum \frac{1}{\sqrt{n}}$ diverges, the series diverges.
157. **Diverges.** Constant multiple of the harmonic series.

158. **Converges.** Limit Comparison Test with $b_n = \frac{1}{n^2}$.
159. **Diverges.** p-Series Test with $p = 1/2 \leq 1$.
160. **Converges.** Limit Comparison Test with $b_n = \frac{1}{n^2}$. $\lim_{n \rightarrow \infty} \cos(1/n) = 1$.
161. **Converges.** Limit Comparison Test with $b_n = \frac{1}{n^3}$.
162. **Converges.** Direct Comparison Test with $b_n = \frac{1}{n \cdot 2^n}$, which converges by comparison with $(1/2)^n$.
163. **Diverges.** Test for Divergence: $\lim_{n \rightarrow \infty} \frac{n+2}{n+1} = 1 \neq 0$.
164. **Converges.** p-Series Test with $p = 2 > 1$.
165. **Diverges.** p-Series Test (Harmonic Series) with $p = 1$.
166. **Converges.** Limit Comparison Test with $b_n = \frac{1}{n^3}$ for $n \geq 2$.
167. **Diverges.** Limit Comparison Test with $b_n = \frac{1}{n}$.
168. **Converges.** Limit Comparison Test with $b_n = \frac{1}{n^2}$.
169. **Converges.** Limit Comparison Test with $b_n = \frac{1}{n^2}$.
170. **Diverges.** Limit Comparison Test with $b_n = \frac{1}{n}$.
171. **Diverges.** Limit Comparison Test with $b_n = \frac{1}{n}$.
172. **Converges.** p-Series Test with $p = 1.5 > 1$.
173. **Diverges.** Limit Comparison Test with $b_n = \frac{1}{n}$.
174. **Converges.** Geometric Series with $r = 3/5 < 1$.
175. **Diverges.** Limit Comparison Test with $b_n = \frac{1}{n}$.
176. **Diverges.** Limit Comparison Test with $b_n = \frac{1}{n}$.
177. **Converges.** p-Series Test with $p = 3/2 > 1$.
178. **Converges.** Limit Comparison Test with the convergent geometric series $b_n = (\frac{1}{2})^n$.