

# Calculus II Problem Set

## Part 1: Sequences

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# Contents

<b>1</b>	<b>Finding the n-th Term</b>	<b>1</b>
<b>2</b>	<b>Convergence and Divergence of Sequences</b>	<b>2</b>
<b>3</b>	<b>Solutions</b>	<b>3</b>
3.1	Solutions for Section 1: Finding the n-th Term . . . . .	3
3.2	Solutions for Section 2: Convergence and Divergence . . . . .	4

# 1 Finding the n-th Term

For each of the following sequences, find a formula for the general term  $a_n$ , assuming the pattern of the first few terms continues. Assume  $n$  begins with 1.

**Problem 1.1.**  $1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \dots$

**Problem 1.2.**  $5, 8, 11, 14, \dots$

**Problem 1.3.**  $\frac{1}{2}, -\frac{4}{3}, \frac{9}{4}, -\frac{16}{5}, \dots$

**Problem 1.4.**  $2, \frac{3}{4}, \frac{4}{9}, \frac{5}{16}, \dots$

**Problem 1.5.**  $1, 0, 1, 0, 1, 0, \dots$

**Problem 1.6.**  $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \dots$

**Problem 1.7.**  $0, 3, 8, 15, 24, \dots$

**Problem 1.8.**  $-\frac{2}{3}, \frac{4}{9}, -\frac{8}{27}, \frac{16}{81}, \dots$

**Problem 1.9.**  $\frac{1}{1}, \frac{2}{3}, \frac{3}{5}, \frac{4}{7}, \dots$

**Problem 1.10.**  $1, -1, 1, -1, 1, \dots$

**Problem 1.11.**  $2, 6, 12, 20, 30, \dots$  *Hint: Look at the factors of each term.*

**Problem 1.12.**  $\cos(0), \cos(\pi), \cos(2\pi), \cos(3\pi), \dots$

**Problem 1.13.**  $\frac{1}{2}, \frac{2}{5}, \frac{3}{10}, \frac{4}{17}, \dots$

**Problem 1.14.**  $1, \frac{1}{2}, \frac{1}{6}, \frac{1}{24}, \frac{1}{120}, \dots$

**Problem 1.15.**  $1, 5, 9, 13, 17, \dots$

**Problem 1.16.**  $\frac{\sqrt{1}}{3}, \frac{\sqrt{2}}{4}, \frac{\sqrt{3}}{5}, \frac{\sqrt{4}}{6}, \dots$

**Problem 1.17.**  $10, 5, \frac{5}{2}, \frac{5}{4}, \frac{5}{8}, \dots$

**Problem 1.18.**  $\frac{5}{1}, \frac{8}{3}, \frac{11}{5}, \frac{14}{7}, \dots$

**Problem 1.19.**  $\{0.9, 0.99, 0.999, 0.9999, \dots\}$  *Hint: Express each term as  $1 - \dots$*

**Problem 1.20.**  $0, \frac{1}{2}, 0, \frac{1}{2}, 0, \frac{1}{2}, \dots$

**Problem 1.21.**  $\frac{\ln 1}{1}, \frac{\ln 2}{2}, \frac{\ln 3}{3}, \frac{\ln 4}{4}, \dots$

**Problem 1.22.**  $5, -25, 125, -625, \dots$

**Problem 1.23.**  $\frac{1}{e}, \frac{2}{e^2}, \frac{3}{e^3}, \frac{4}{e^4}, \dots$

**Problem 1.24.**  $1, -\frac{1}{8}, \frac{1}{27}, -\frac{1}{64}, \dots$

**Problem 1.25.**  $1, 3, 1, 3, 1, 3, \dots$

## 2 Convergence and Divergence of Sequences

*Determine whether the sequence converges or diverges. If it converges, find the limit.*

**Problem 2.1.**  $a_n = \frac{3n^2-1}{10n+5n^2}$

**Problem 2.2.**  $a_n = \frac{n}{n+1}$

**Problem 2.3.**  $a_n = n \sin\left(\frac{1}{n}\right)$

**Problem 2.4.**  $a_n = (-1)^n \frac{n}{n+1}$

**Problem 2.5.**  $a_n = \frac{\ln(n)}{n}$

**Problem 2.6.**  $a_n = \cos(n\pi)$

**Problem 2.7.**  $a_n = \frac{n!}{2^n}$

**Problem 2.8.**  $a_n = \frac{3^n}{n!}$

**Problem 2.9.**  $a_n = \arctan(n)$

**Problem 2.10.**  $a_n = \left(1 + \frac{1}{n}\right)^n$

**Problem 2.11.**  $a_n = \sqrt{n+1} - \sqrt{n}$

**Problem 2.12.**  $a_n = \frac{\sin(n)}{n}$

**Problem 2.13.**  $a_n = \frac{(-1)^n}{n^2}$

**Problem 2.14.**  $a_n = ne^{-n}$

**Problem 2.15.**  $a_n = \frac{n^3}{n^3+1}$

**Problem 2.16.**  $a_n = n^{1/n}$

**Problem 2.17.**  $a_n = \frac{2n+1}{1-3n}$

**Problem 2.18.**  $a_n = \frac{4n^2-3}{3n^2+n+1}$

**Problem 2.19.**  $a_n = \frac{\sqrt{n}}{\ln(n)}$

**Problem 2.20.**  $a_n = \cos\left(\frac{2}{n}\right)$

**Problem 2.21.**  $a_n = \frac{n^2}{2n-1}$

**Problem 2.22.**  $a_n = (-1)^{n+1} \frac{1}{\sqrt{n}}$

**Problem 2.23.**  $a_n = \tanh(n)$

**Problem 2.24.**  $a_n = \frac{(n+1)!}{n!}$

**Problem 2.25.**  $a_n = \frac{\cos^2(n)}{2^n}$

### 3 Solutions

#### 3.1 Solutions for Section 1: Finding the n-th Term

- 1.1** The terms are the reciprocals of the natural numbers.  $a_n = \frac{1}{n}$
- 1.2** This is an arithmetic sequence with first term  $a_1 = 5$  and common difference  $d = 3$ . The formula is  $a_n = a_1 + (n - 1)d = 5 + (n - 1)3 = 3n + 2$ .
- 1.3** Signs are alternating:  $(-1)^{n+1}$ . Numerators are squares:  $n^2$ . Denominators are  $n + 1$ . So,  $a_n = \frac{(-1)^{n+1}n^2}{n+1}$ .
- 1.4** Numerators are  $n + 1$ . Denominators are squares:  $n^2$ . So,  $a_n = \frac{n+1}{n^2}$ .
- 1.5** The terms alternate between 1 and 0. We can write this using cosine or with  $(-1)^n$ . A simple form is  $a_n = \frac{1+(-1)^{n+1}}{2}$ .
- 1.6** This is a geometric sequence with first term  $a_1 = 1/2$  and common ratio  $r = 1/2$ . The formula is  $a_n = a_1 r^{n-1} = \frac{1}{2} \left(\frac{1}{2}\right)^{n-1} = \left(\frac{1}{2}\right)^n = \frac{1}{2^n}$ .
- 1.7** The terms are one less than the perfect squares.  $a_n = n^2 - 1$ .
- 1.8** This is a geometric sequence with ratio  $r = -2/3$ . The first term is  $-2/3$ . So,  $a_n = \left(-\frac{2}{3}\right)^n$ .
- 1.9** Numerators are  $n$ . Denominators are consecutive odd numbers, which can be written as  $2n - 1$ . So,  $a_n = \frac{n}{2n-1}$ .
- 1.10** The terms alternate between 1 and -1.  $a_n = (-1)^{n+1}$  or  $a_n = (-1)^{n-1}$ .
- 1.11** The terms are  $1 \cdot 2, 2 \cdot 3, 3 \cdot 4, 4 \cdot 5, 5 \cdot 6, \dots$ . The formula is  $a_n = n(n + 1)$ .
- 1.12** The arguments of cosine are  $0, \pi, 2\pi, \dots$ . This is  $(n-1)\pi$ . The terms are  $1, -1, 1, -1, \dots$ . So,  $a_n = \cos((n-1)\pi) = (-1)^{n-1}$ .
- 1.13** Numerators are  $n$ . Denominators are one more than the squares,  $n^2 + 1$ . So,  $a_n = \frac{n}{n^2+1}$ .
- 1.14** The denominators are factorials:  $1!, 2!, 3!, \dots$ . So,  $a_n = \frac{1}{n!}$ .
- 1.15** Arithmetic sequence with  $a_1 = 1$  and  $d = 4$ .  $a_n = 1 + (n - 1)4 = 4n - 3$ .
- 1.16** Numerators are  $\sqrt{n}$ . Denominators are  $n + 2$ . So,  $a_n = \frac{\sqrt{n}}{n+2}$ .
- 1.17** Geometric sequence with  $a_1 = 10$  and  $r = 1/2$ .  $a_n = 10 \left(\frac{1}{2}\right)^{n-1}$ .
- 1.18** Numerators are an arithmetic sequence  $3n + 2$ . Denominators are an arithmetic sequence of odd numbers  $2n - 1$ . So,  $a_n = \frac{3n+2}{2n-1}$ .
- 1.19**  $a_1 = 1 - 0.1, a_2 = 1 - 0.01, a_3 = 1 - 0.001$ . This can be written as  $a_n = 1 - \frac{1}{10^n}$ .

1.20 The sequence is  $\frac{1-(-1)^n}{4}$ .

1.21  $a_n = \frac{\ln n}{n}$ .

1.22 Geometric, with  $a_1 = 5, r = -5$ .  $a_n = 5(-5)^{n-1} = (-1)^{n-1}5^n$ .

1.23  $a_n = \frac{n}{e^n}$ .

1.24 The signs alternate  $(-1)^{n+1}$ . The denominators are cubes  $n^3$ .  $a_n = \frac{(-1)^{n+1}}{n^3}$ .

1.25 The sequence is  $2 + (-1)^{n+1}$ .

## 3.2 Solutions for Section 2: Convergence and Divergence

2.1 Divide numerator and denominator by  $n^2$ :  $\lim_{n \rightarrow \infty} \frac{3-1/n^2}{10/n+5} = \frac{3-0}{0+5} = \frac{3}{5}$ . **Converges to  $3/5$ .**

2.2 Divide by  $n$ :  $\lim_{n \rightarrow \infty} \frac{1}{1+1/n} = \frac{1}{1+0} = 1$ . **Converges to 1.**

2.3 Rewrite as  $\frac{\sin(1/n)}{1/n}$ . Let  $x = 1/n$ . As  $n \rightarrow \infty$ ,  $x \rightarrow 0$ . The limit is  $\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$ . **Converges to 1.**

2.4 The term  $\frac{n}{n+1}$  approaches 1, but the  $(-1)^n$  factor causes the sequence to oscillate between values close to 1 and -1. It does not approach a single value. **Diverges.**

2.5 Use L'Hôpital's Rule on the function  $f(x) = \frac{\ln x}{x}$ .  $\lim_{x \rightarrow \infty} \frac{\ln x}{x} = \lim_{x \rightarrow \infty} \frac{1/x}{1} = 0$ . **Converges to 0.**

2.6 The sequence is  $-1, 1, -1, 1, \dots$ . It oscillates and does not approach a single limit. **Diverges.**

2.7 Factorials grow faster than exponentials. The terms  $a_n$  will grow without bound.  $\lim_{n \rightarrow \infty} \frac{n!}{2^n} = \infty$ . **Diverges.**

2.8 Factorials grow faster than exponentials. The denominator grows much faster than the numerator.  $\lim_{n \rightarrow \infty} \frac{3^n}{n!} = 0$ . **Converges to 0.**

2.9 As  $n$  approaches infinity, the argument of arctan goes to infinity.  $\lim_{n \rightarrow \infty} \arctan(n) = \frac{\pi}{2}$ . **Converges to  $\pi/2$ .**

2.10 This is a standard limit form.  $\lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^n = e$ . **Converges to e.**

2.11 Multiply by the conjugate:  $a_n = (\sqrt{n+1} - \sqrt{n}) \frac{\sqrt{n+1} + \sqrt{n}}{\sqrt{n+1} + \sqrt{n}} = \frac{n+1-n}{\sqrt{n+1} + \sqrt{n}} = \frac{1}{\sqrt{n+1} + \sqrt{n}}$ . The limit of this expression as  $n \rightarrow \infty$  is 0. **Converges to 0.**

2.12 Use the Squeeze Theorem. Since  $-1 \leq \sin(n) \leq 1$ , we have  $-\frac{1}{n} \leq \frac{\sin(n)}{n} \leq \frac{1}{n}$ . Since  $\lim_{n \rightarrow \infty} (-\frac{1}{n}) = 0$  and  $\lim_{n \rightarrow \infty} (\frac{1}{n}) = 0$ , the limit of  $a_n$  is also 0. **Converges to 0.**

2.13 The absolute value is  $|a_n| = \frac{1}{n^2}$ , which goes to 0. Therefore, the sequence itself must go to 0. **Converges to 0.**

- 2.14** Rewrite as  $\frac{n}{e^n}$ . Use L'Hôpital's Rule on  $f(x) = \frac{x}{e^x}$ .  $\lim_{x \rightarrow \infty} \frac{x}{e^x} = \lim_{x \rightarrow \infty} \frac{1}{e^x} = 0$ .  
**Converges to 0.**
- 2.15** Divide by  $n^3$ :  $\lim_{n \rightarrow \infty} \frac{1}{1+1/n^3} = 1$ . **Converges to 1.**
- 2.16** This is a standard limit. Let  $y = n^{1/n}$ . Then  $\ln y = \frac{\ln n}{n}$ . We already know  $\lim_{n \rightarrow \infty} \frac{\ln n}{n} = 0$ . So,  $\lim \ln y = 0$ , which means  $\lim y = e^0 = 1$ . **Converges to 1.**
- 2.17** Divide by  $n$ :  $\lim_{n \rightarrow \infty} \frac{2+1/n}{1/n-3} = \frac{2}{-3}$ . **Converges to -2/3.**
- 2.18** Divide by  $n^2$ :  $\lim_{n \rightarrow \infty} \frac{4-3/n^2}{3+1/n+1/n^2} = \frac{4}{3}$ . **Converges to 4/3.**
- 2.19** Polynomials (even roots) grow faster than logarithms. The limit is  $\infty$ . **Diverges.**
- 2.20** As  $n \rightarrow \infty$ ,  $2/n \rightarrow 0$ . Since cosine is continuous,  $\lim_{n \rightarrow \infty} \cos\left(\frac{2}{n}\right) = \cos(0) = 1$ .  
**Converges to 1.**
- 2.21** The degree of the numerator (2) is greater than the degree of the denominator (1). The limit is  $\infty$ . **Diverges.**
- 2.22** The absolute value  $|a_n| = \frac{1}{\sqrt{n}}$  goes to 0, so the sequence converges to 0. **Converges to 0.**
- 2.23**  $\tanh(n) = \frac{e^n - e^{-n}}{e^n + e^{-n}}$ . Divide by  $e^n$ :  $\frac{1 - e^{-2n}}{1 + e^{-2n}}$ . As  $n \rightarrow \infty$ ,  $e^{-2n} \rightarrow 0$ . The limit is  $\frac{1-0}{1+0} = 1$ .  
**Converges to 1.**
- 2.24**  $a_n = \frac{(n+1) \cdot n!}{n!} = n + 1$ . The limit is  $\infty$ . **Diverges.**
- 2.25** Use the Squeeze Theorem.  $0 \leq \cos^2(n) \leq 1$ , so  $0 \leq \frac{\cos^2(n)}{2^n} \leq \frac{1}{2^n}$ . Since  $\lim_{n \rightarrow \infty} 0 = 0$  and  $\lim_{n \rightarrow \infty} \frac{1}{2^n} = 0$ , the sequence converges to 0. **Converges to 0.**