

# **Calculus II Problem Set**

## **Part 2: Series Convergence and Divergence**

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# 1 Practice Problems

For each of the following series, determine whether it converges or diverges. State the test you used and show all necessary work.

1.  $\sum_{n=1}^{\infty} \frac{n}{2n+1}$
2.  $\sum_{n=1}^{\infty} \frac{1}{n^2+n}$
3.  $\sum_{n=1}^{\infty} 5 \left(\frac{2}{3}\right)^n$
4.  $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$
5.  $\sum_{n=2}^{\infty} \frac{1}{n \ln(n)}$
6.  $\sum_{n=1}^{\infty} \frac{n^2-1}{n^2+1}$
7.  $\sum_{n=1}^{\infty} \frac{1}{n^e}$
8.  $\sum_{n=1}^{\infty} \frac{2^n}{3^n-1}$
9.  $\sum_{n=1}^{\infty} \left(1 + \frac{1}{n}\right)^n$
10.  $\sum_{n=1}^{\infty} \frac{3}{\sqrt[3]{n}}$
11.  $\sum_{n=0}^{\infty} \frac{(-1)^n \pi^n}{4^n}$
12.  $\sum_{n=1}^{\infty} \frac{2+\sin(n)}{n^2}$
13.  $\sum_{n=1}^{\infty} \frac{n+5}{5n-1}$
14.  $\sum_{n=1}^{\infty} n e^{-n^2}$
15.  $\sum_{n=1}^{\infty} \frac{1}{n^{1.001}}$
16.  $\sum_{n=1}^{\infty} \frac{3n^2+n}{n^4+n^2}$
17.  $\sum_{n=1}^{\infty} \arctan(n)$
18.  $\sum_{n=1}^{\infty} \frac{1}{2n+3}$
19.  $\sum_{n=1}^{\infty} 3 \left(\frac{5}{4}\right)^n$
20.  $\sum_{n=1}^{\infty} \frac{1}{n^3+e^n}$
21.  $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^3+1}}$
22.  $\sum_{n=1}^{\infty} \frac{1}{n\sqrt{n}}$
23.  $\sum_{n=1}^{\infty} \frac{\ln(n)}{n^2}$
24.  $\sum_{n=1}^{\infty} \frac{n!}{100^n}$
25.  $\sum_{n=1}^{\infty} \frac{1}{n^\pi}$
26.  $\sum_{n=1}^{\infty} \frac{4+3^n}{2^n}$
27.  $\sum_{n=1}^{\infty} \frac{1}{1+(0.5)^n}$
28.  $\sum_{n=1}^{\infty} \frac{1}{n(n+1)(n+2)}$
29.  $\sum_{n=1}^{\infty} \frac{n}{e^n}$
30.  $\sum_{n=1}^{\infty} \frac{1}{n^{0.99}}$
31.  $\sum_{n=1}^{\infty} \cos(1/n)$
32.  $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}+1}$
33.  $\sum_{n=0}^{\infty} e^{-n}$
34.  $\sum_{n=1}^{\infty} \frac{n^3}{n^5+2}$
35.  $\sum_{n=1}^{\infty} \frac{5}{n}$
36.  $\sum_{n=1}^{\infty} \frac{n}{\sqrt{n^2+1}}$
37.  $\sum_{n=1}^{\infty} \frac{\sin^2(n)}{n^3}$
38.  $\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2}$
39.  $\sum_{n=1}^{\infty} \frac{1}{3^n+n}$
40.  $\sum_{n=1}^{\infty} \frac{1}{n \cdot 2^n}$
41.  $\sum_{n=1}^{\infty} \sqrt[n]{2}$
42.  $\sum_{n=1}^{\infty} \frac{1}{n^2+4}$
43.  $\sum_{n=1}^{\infty} \frac{2^n}{5^n+3^n}$
44.  $\sum_{n=1}^{\infty} \frac{1}{\sqrt[4]{n^3}}$
45.  $\sum_{n=1}^{\infty} \frac{3n}{n+2}$
46.  $\sum_{n=1}^{\infty} \frac{\sqrt{n}}{n^2+1}$
47.  $\sum_{n=1}^{\infty} \frac{1}{n!}$

48.  $\sum_{n=1}^{\infty} \frac{1}{5n+1}$
49.  $\sum_{n=1}^{\infty} (-1)^n$
50.  $\sum_{n=1}^{\infty} \frac{n+1}{n^2+1}$
51.  $\sum_{n=1}^{\infty} \frac{1}{4n^2-1}$
52.  $\sum_{n=1}^{\infty} \frac{e^{1/n}}{n^2}$
53.  $\sum_{n=1}^{\infty} \frac{3^n}{4^{n-1}}$
54.  $\sum_{n=1}^{\infty} \frac{n^2}{n^3+1}$
55.  $\sum_{n=1}^{\infty} \frac{5n^3-3n}{n^2(n-2)(n^2+5)}$
56.  $\sum_{n=1}^{\infty} \frac{1}{n^{1+1/n}}$
57.  $\sum_{n=1}^{\infty} \frac{2}{n^{0.85}}$
58.  $\sum_{n=1}^{\infty} n \sin\left(\frac{1}{n^3}\right)$
59.  $\sum_{n=1}^{\infty} \frac{1}{n^2-n}$
60.  $\sum_{n=1}^{\infty} \frac{3}{2n-\sqrt{n}}$
61.  $\sum_{n=1}^{\infty} \ln(n)$
62.  $\sum_{n=1}^{\infty} \frac{1}{5^n}$
63.  $\sum_{n=1}^{\infty} \frac{n^2-1}{3n^4+1}$
64.  $\sum_{n=1}^{\infty} \frac{1}{\cosh(n)}$
65.  $\sum_{n=1}^{\infty} \frac{n^n}{n!}$
66.  $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n(n+1)}}$
67.  $\sum_{n=1}^{\infty} \frac{1}{3\sqrt[5]{n^4}}$
68.  $\sum_{n=1}^{\infty} \frac{n-1}{n^3}$
69.  $\sum_{n=1}^{\infty} \frac{2+(-1)^n}{n\sqrt{n}}$
70.  $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^2+1}}$
71.  $\sum_{n=1}^{\infty} \frac{1}{n^3}$
72.  $\sum_{n=1}^{\infty} \frac{1}{1000}$
73.  $\sum_{n=1}^{\infty} \frac{\ln(n)}{n^3}$
74.  $\sum_{n=1}^{\infty} \frac{1}{2+3^n}$
75.  $\sum_{n=1}^{\infty} \frac{\cos(n\pi)}{\sqrt{n}}$
76.  $\sum_{n=1}^{\infty} \frac{1}{n^4+n^2}$
77.  $\sum_{n=1}^{\infty} \frac{4^n}{3^n+5^n}$
78.  $\sum_{n=1}^{\infty} \frac{\sqrt[3]{n}}{\sqrt{n^3+4n+3}}$
79.  $\sum_{n=1}^{\infty} \frac{1}{n\cdot\sqrt[3]{n}}$
80.  $\sum_{n=1}^{\infty} \frac{1}{n \ln(n^2)}$
81.  $\sum_{n=1}^{\infty} \frac{n}{n^2-1}$
82.  $\sum_{n=1}^{\infty} \frac{1+\cos(n)}{e^n}$
83.  $\sum_{n=1}^{\infty} n^{-2/3}$
84.  $\sum_{n=1}^{\infty} \frac{n^2+1}{n^2}$
85.  $\sum_{n=1}^{\infty} \frac{n}{(n+1)2^{n+1}}$
86.  $\sum_{n=1}^{\infty} \frac{e^n}{n^n}$
87.  $\sum_{n=1}^{\infty} \frac{2^{n-1}}{5^n}$
88.  $\sum_{n=1}^{\infty} \frac{3^n+2^n}{6^n}$
89.  $\sum_{n=1}^{\infty} \frac{1}{n^{1.1}}$
90.  $\sum_{n=1}^{\infty} \frac{3}{\sqrt{n^2+4}}$
91.  $\sum_{n=1}^{\infty} \frac{1}{1+\sqrt{n}}$
92.  $\sum_{n=1}^{\infty} \frac{n^2}{2n^2+1}$
93.  $\sum_{n=1}^{\infty} \frac{\ln(n)}{n}$
94.  $\sum_{n=1}^{\infty} \frac{5}{2^n}$
95.  $\sum_{n=1}^{\infty} \frac{1}{n+n \cos^2(n)}$
96.  $\sum_{n=1}^{\infty} \frac{n}{n^3+1}$
97.  $\sum_{n=1}^{\infty} \frac{n}{3n-1}$
98.  $\sum_{n=1}^{\infty} \frac{1}{4+n^2}$
99.  $\sum_{n=1}^{\infty} \frac{1}{3^n}$

100.  $\sum_{n=1}^{\infty} \frac{1}{(2n-1)(2n+1)}$
101.  $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}+n^2}$
102.  $\sum_{n=1}^{\infty} \frac{3n-1}{2n+1}$
103.  $\sum_{n=1}^{\infty} \frac{1}{\sqrt[3]{n^5}}$
104.  $\sum_{n=1}^{\infty} \frac{1}{2+\sqrt{n}}$
105.  $\sum_{n=1}^{\infty} \frac{1}{1+e^n}$
106.  $\sum_{n=1}^{\infty} \frac{1}{n^2-3}$
107.  $\sum_{n=1}^{\infty} \frac{n+4^n}{n+6^n}$
108.  $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n(n+1)(n+2)}}$
109.  $\sum_{n=1}^{\infty} \frac{1}{n^{0.5}}$
110.  $\sum_{n=1}^{\infty} \frac{1}{n(1+\ln n)}$
111.  $\sum_{n=1}^{\infty} \frac{3n+2}{n^3+1}$
112.  $\sum_{n=1}^{\infty} \frac{n}{\sqrt{n^3+2n}}$
113.  $\sum_{n=1}^{\infty} \frac{\ln(n+1)}{n+1}$
114.  $\sum_{n=1}^{\infty} \frac{1}{n^4}$
115.  $\sum_{n=1}^{\infty} \frac{n}{n!}$
116.  $\sum_{n=1}^{\infty} \frac{n^2+1}{2n^3+n}$
117.  $\sum_{n=1}^{\infty} \frac{1}{n^{1/3}}$
118.  $\sum_{n=1}^{\infty} \frac{2n}{3n^2-4}$
119.  $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}+\sqrt[3]{n}}$
120.  $\sum_{n=1}^{\infty} \frac{2n^2-1}{n^2}$
121.  $\sum_{n=1}^{\infty} \frac{\pi}{n^2}$
122.  $\sum_{n=1}^{\infty} \frac{1}{n \cdot 3^n}$
123.  $\sum_{n=1}^{\infty} \frac{1}{2n}$
124.  $\sum_{n=1}^{\infty} \frac{1}{n^2+2n+2}$
125.  $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^3}}$
126.  $\sum_{n=1}^{\infty} \frac{n-1}{n^2}$
127.  $\sum_{n=1}^{\infty} \frac{\sqrt{n}+1}{n\sqrt{n}+n}$
128.  $\sum_{n=1}^{\infty} \frac{n^2-5n}{n^3+n+1}$
129.  $\sum_{n=1}^{\infty} \frac{1}{\sqrt[3]{n}+1}$
130.  $\sum_{n=1}^{\infty} \frac{n+1}{n^3+1}$
131.  $\sum_{n=1}^{\infty} \frac{1}{5n-1}$
132.  $\sum_{n=1}^{\infty} \frac{1}{n^{1.0001}}$
133.  $\sum_{n=1}^{\infty} \frac{\arctan(n)}{n^2}$
134.  $\sum_{n=1}^{\infty} \frac{e^n+1}{ne^n+1}$
135.  $\sum_{n=1}^{\infty} \frac{1}{n(n+3)}$
136.  $\sum_{n=1}^{\infty} \frac{1}{n^{1/n}}$
137.  $\sum_{n=1}^{\infty} \frac{2+(-1)^n}{n^2}$
138.  $\sum_{n=1}^{\infty} \frac{1}{3^n-n}$
139.  $\sum_{n=1}^{\infty} \frac{2^n+3^n}{4^n+5^n}$
140.  $\sum_{n=1}^{\infty} \frac{n!}{n^n}$
141.  $\sum_{n=1}^{\infty} \frac{1}{n^{4/3}}$
142.  $\sum_{n=1}^{\infty} \frac{1}{(n+1)\ln(n+1)}$
143.  $\sum_{n=1}^{\infty} \frac{n}{n^2-n+1}$
144.  $\sum_{n=1}^{\infty} \frac{n^3}{e^{n^4}}$
145.  $\sum_{n=1}^{\infty} \frac{1}{2n^2+n}$
146.  $\sum_{n=1}^{\infty} \frac{3^n}{2^{2n}}$
147.  $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}+2^n}$
148.  $\sum_{n=1}^{\infty} \frac{\sqrt[3]{n^2+1}}{n^2}$
149.  $\sum_{n=1}^{\infty} \frac{1}{2n-1}$
150.  $\sum_{n=1}^{\infty} \frac{n}{3^n}$
151.  $\sum_{n=1}^{\infty} \frac{1}{n(n-1)}$

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| 152. $\sum_{n=1}^{\infty} \frac{n-2}{n}$             | 166. $\sum_{n=1}^{\infty} \frac{1}{n^3-1}$        |
| 153. $\sum_{n=1}^{\infty} \frac{1}{\sqrt[3]{n^2-1}}$ | 167. $\sum_{n=1}^{\infty} \frac{2n+3}{n^2+3n+2}$  |
| 154. $\sum_{n=1}^{\infty} \frac{1}{n^5}$             | 168. $\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$       |
| 155. $\sum_{n=1}^{\infty} \frac{1}{n+\sqrt{n}}$      | 169. $\sum_{n=1}^{\infty} \frac{1}{n^2+n+1}$      |
| 156. $\sum_{n=1}^{\infty} \frac{\ln(n)}{\sqrt{n}}$   | 170. $\sum_{n=1}^{\infty} \frac{n}{n^2+1}$        |
| 157. $\sum_{n=1}^{\infty} \frac{2}{n}$               | 171. $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^2+n}}$ |
| 158. $\sum_{n=1}^{\infty} \frac{1}{n^2-4n+5}$        | 172. $\sum_{n=1}^{\infty} \frac{1}{n^{1.5}}$      |
| 159. $\sum_{n=1}^{\infty} \frac{1}{2\sqrt{n}}$       | 173. $\sum_{n=1}^{\infty} \frac{1}{2n+1}$         |
| 160. $\sum_{n=1}^{\infty} \frac{\cos(1/n)}{n^2}$     | 174. $\sum_{n=1}^{\infty} \frac{3^n}{5^n}$        |
| 161. $\sum_{n=1}^{\infty} \frac{1}{(n+1)^3}$         | 175. $\sum_{n=1}^{\infty} \frac{1}{n+\ln n}$      |
| 162. $\sum_{n=1}^{\infty} \frac{1}{n \cdot 2^n + 1}$ | 176. $\sum_{n=1}^{\infty} \frac{n^2}{n^3+n+1}$    |
| 163. $\sum_{n=1}^{\infty} \frac{n+2}{n+1}$           | 177. $\sum_{n=1}^{\infty} \frac{2}{n\sqrt{n}}$    |
| 164. $\sum_{n=1}^{\infty} \frac{1}{n^2}$             | 178. $\sum_{n=1}^{\infty} \frac{n+1}{n2^n}$       |
| 165. $\sum_{n=1}^{\infty} \frac{1}{n}$               |   |

## 2 Solutions

1. **Diverges.** Test for Divergence:  $\lim_{n \rightarrow \infty} \frac{n}{2n+1} = \frac{1}{2} \neq 0$ .
2. **Converges.** Limit Comparison Test with  $b_n = \frac{1}{n^2}$ .  $\lim_{n \rightarrow \infty} \frac{1/(n^2+n)}{1/n^2} = \lim_{n \rightarrow \infty} \frac{n^2}{n^2+n} = 1$ . Since  $\sum \frac{1}{n^2}$  converges (p-series,  $p = 2 > 1$ ), the series converges.
3. **Converges.** Geometric Series Test with  $a = 10/3$  and  $r = 2/3$ . Since  $|r| < 1$ , it converges. Sum is  $\frac{a}{1-r} = \frac{10/3}{1-2/3} = 10$ .
4. **Diverges.** p-Series Test with  $p = 1/2 \leq 1$ .
5. **Diverges.** Integral Test. Let  $f(x) = \frac{1}{x \ln x}$ .  $\int_2^\infty \frac{1}{x \ln x} dx = [\ln(\ln x)]_2^\infty = \infty$ .
6. **Diverges.** Test for Divergence:  $\lim_{n \rightarrow \infty} \frac{n^2-1}{n^2+1} = 1 \neq 0$ .
7. **Converges.** p-Series Test since  $p = e \approx 2.718 > 1$ .
8. **Converges.** Limit Comparison Test with  $b_n = (\frac{2}{3})^n$ .  $\lim_{n \rightarrow \infty} \frac{2^n/(3^n-1)}{(2/3)^n} = \lim_{n \rightarrow \infty} \frac{3^n}{3^n-1} = 1$ . Since  $\sum (\frac{2}{3})^n$  is a convergent geometric series, the series converges.
9. **Diverges.** Test for Divergence:  $\lim_{n \rightarrow \infty} (1 + \frac{1}{n})^n = e \neq 0$ .
10. **Diverges.** p-Series Test with  $p = 1/3 \leq 1$ . The 3 is a constant multiple.
11. **Converges.** Geometric Series Test with  $r = -\pi/4$ . Since  $|r| = \pi/4 < 1$ , it converges.
12. **Converges.** Direct Comparison Test. Since  $1 \leq 2 + \sin(n) \leq 3$ , we have  $\frac{2+\sin(n)}{n^2} \leq \frac{3}{n^2}$ .  $\sum \frac{3}{n^2}$  converges (p-series,  $p = 2 > 1$ ), so the smaller series converges.
13. **Diverges.** Test for Divergence:  $\lim_{n \rightarrow \infty} \frac{n+5}{5n-1} = \frac{1}{5} \neq 0$ .
14. **Converges.** Integral Test. Let  $f(x) = xe^{-x^2}$ .  $\int_1^\infty xe^{-x^2} dx = [-\frac{1}{2}e^{-x^2}]_1^\infty = \frac{1}{2e}$ . Since the integral converges, the series converges.
15. **Converges.** p-Series Test with  $p = 1.001 > 1$ .
16. **Converges.** Limit Comparison Test with  $b_n = \frac{1}{n^2}$ .  $\lim_{n \rightarrow \infty} \frac{(3n^2+n)/(n^4+n^2)}{1/n^2} = 3$ . Converges because  $\sum \frac{1}{n^2}$  converges.
17. **Diverges.** Test for Divergence:  $\lim_{n \rightarrow \infty} \arctan(n) = \frac{\pi}{2} \neq 0$ .
18. **Diverges.** Limit Comparison Test with  $b_n = \frac{1}{n}$ .  $\lim_{n \rightarrow \infty} \frac{1/(2n+3)}{1/n} = \frac{1}{2}$ . Diverges because the harmonic series  $\sum \frac{1}{n}$  diverges.
19. **Diverges.** Geometric Series Test with  $r = 5/4$ . Since  $|r| \geq 1$ , it diverges.
20. **Converges.** Direct Comparison Test with  $b_n = \frac{1}{e^n}$ .  $\frac{1}{n^3+e^n} < \frac{1}{e^n}$ .  $\sum (\frac{1}{e})^n$  is a convergent geometric series.

21. **Converges.** Limit Comparison Test with  $b_n = \frac{1}{n^{3/2}}$ .  $\lim_{n \rightarrow \infty} \frac{\frac{1}{\sqrt{n^3+1}}}{\frac{1}{n^{3/2}}} = 1$ . Converges because  $\sum \frac{1}{n^{3/2}}$  converges ( $p = 3/2 > 1$ ).
22. **Converges.** p-Series Test,  $a_n = \frac{1}{n^{3/2}}$ ,  $p = 3/2 > 1$ .
23. **Converges.** Integral Test. Use integration by parts for  $\int \frac{\ln x}{x^2} dx$ . It converges. Alternatively, Direct Comparison: for  $n > e$ ,  $\ln n < n^{1/2}$ , so  $\frac{\ln n}{n^2} < \frac{n^{1/2}}{n^2} = \frac{1}{n^{3/2}}$ , which converges.
24. **Diverges.** Test for Divergence. Factorial grows faster than exponential.  $\lim_{n \rightarrow \infty} \frac{n!}{100^n} = \infty$ .
25. **Converges.** p-Series Test,  $p = \pi > 1$ .
26. **Diverges.**  $a_n = (\frac{4}{2})^n + (\frac{3}{2})^n = 2^n + (1.5)^n$ . Test for Divergence:  $\lim_{n \rightarrow \infty} a_n = \infty$ .
27. **Diverges.** Test for Divergence:  $\lim_{n \rightarrow \infty} \frac{1}{1+(0.5)^n} = \frac{1}{1+0} = 1 \neq 0$ .
28. **Converges.** Limit Comparison Test with  $b_n = \frac{1}{n^3}$ .
29. **Converges.** Integral Test.  $\int_1^\infty xe^{-x} dx = 2/e$ . The series converges.
30. **Diverges.** p-Series Test with  $p = 0.99 \leq 1$ .
31. **Diverges.** Test for Divergence:  $\lim_{n \rightarrow \infty} \cos(1/n) = \cos(0) = 1 \neq 0$ .
32. **Diverges.** Limit Comparison Test with  $b_n = \frac{1}{\sqrt{n}}$ .
33. **Converges.** Geometric Series with  $r = 1/e < 1$ .
34. **Converges.** Limit Comparison Test with  $b_n = \frac{1}{n^2}$ .
35. **Diverges.** This is 5 times the harmonic series. Diverges.
36. **Diverges.** Test for Divergence:  $\lim_{n \rightarrow \infty} \frac{n}{\sqrt{n^2+1}} = 1 \neq 0$ .
37. **Converges.** Direct Comparison Test with  $b_n = \frac{1}{n^3}$ .  $\frac{\sin^2 n}{n^3} \leq \frac{1}{n^3}$ .
38. **Converges.** Integral Test. Let  $u = \ln x$ .  $\int_2^\infty \frac{dx}{x(\ln x)^2} = \int_{\ln 2}^\infty u^{-2} du$ , which converges.
39. **Converges.** Direct Comparison Test with  $b_n = \frac{1}{3^n}$ .
40. **Converges.** Direct Comparison Test with  $b_n = \frac{1}{2^n}$ .
41. **Diverges.** Test for Divergence:  $\lim_{n \rightarrow \infty} 2^{1/n} = 2^0 = 1 \neq 0$ .
42. **Converges.** Integral Test.  $\int_1^\infty \frac{1}{x^2+4} dx$  converges.
43. **Converges.** Limit Comparison Test with  $b_n = (\frac{2}{5})^n$ .
44. **Diverges.** p-Series Test with  $p = 3/4 \leq 1$ .

45. **Diverges.** Test for Divergence:  $\lim_{n \rightarrow \infty} \frac{3n}{n+2} = 3 \neq 0$ .
46. **Converges.** Limit Comparison Test with  $b_n = \frac{1}{n^{3/2}}$ .
47. **Converges.** Direct Comparison Test with  $b_n = \frac{1}{n^2}$  for  $n \geq 2$ . Or Ratio Test.
48. **Diverges.** Limit Comparison Test with  $b_n = \frac{1}{n}$ .
49. **Diverges.** Test for Divergence. The terms are  $-1, 1, -1, 1, \dots$ , which does not approach 0.
50. **Diverges.** Limit Comparison Test with  $b_n = \frac{1}{n}$ .
51. **Converges.** Limit Comparison Test with  $b_n = \frac{1}{n^2}$ .  $\lim_{n \rightarrow \infty} \frac{\frac{1}{(4n^2-1)}}{1/n^2} = \lim_{n \rightarrow \infty} \frac{n^2}{4n^2-1} = \frac{1}{4}$ . Since the limit is a finite positive number and  $\sum \frac{1}{n^2}$  converges (p-series,  $p = 2 > 1$ ), the series converges.
52. **Converges.** Limit Comparison Test with  $b_n = \frac{1}{n^2}$ .  $\lim_{n \rightarrow \infty} \frac{e^{1/n}/n^2}{1/n^2} = \lim_{n \rightarrow \infty} e^{1/n} = e^0 = 1$ . Since  $\sum \frac{1}{n^2}$  converges, the series converges.
53. **Converges.** Geometric Series. The series can be written as  $\sum_{n=1}^{\infty} 4 \left(\frac{3}{4}\right)^n$ . The ratio is  $r = 3/4$ . Since  $|r| < 1$ , it converges.
54. **Diverges.** Limit Comparison Test with  $b_n = \frac{1}{n}$ .  $\lim_{n \rightarrow \infty} \frac{n^2/(n^3+1)}{1/n} = \lim_{n \rightarrow \infty} \frac{n^3}{n^3+1} = 1$ . Since the harmonic series  $\sum \frac{1}{n}$  diverges, the series diverges.
55. **Converges.** Limit Comparison Test. The dominant term in the numerator is  $5n^3$ . In the denominator, it's  $n^2 \cdot n \cdot n^2 = n^5$ . So we compare with  $b_n = \frac{n^3}{n^5} = \frac{1}{n^2}$ . The limit of the ratio is 5. Since  $\sum \frac{1}{n^2}$  converges, the series converges.
56. **Diverges.** Limit Comparison Test with  $b_n = \frac{1}{n}$ .  $\lim_{n \rightarrow \infty} \frac{\frac{1}{n^{1+1/n}}}{1/n} = \lim_{n \rightarrow \infty} \frac{n}{n \cdot n^{1/n}} = \lim_{n \rightarrow \infty} \frac{1}{n^{1/n}} = \frac{1}{1} = 1$ . Since  $\sum \frac{1}{n}$  diverges, the series diverges.
57. **Diverges.** p-Series Test. This is  $2 \sum \frac{1}{n^{0.85}}$ . Since  $p = 0.85 \leq 1$ , it diverges.
58. **Converges.** Limit Comparison Test. For small  $x$ ,  $\sin(x) \approx x$ . As  $n \rightarrow \infty$ ,  $1/n^3 \rightarrow 0$ . So  $a_n \approx n(\frac{1}{n^3}) = \frac{1}{n^2}$ . We compare with  $b_n = \frac{1}{n^2}$ .  $\lim_{n \rightarrow \infty} \frac{n \sin(1/n^3)}{1/n^2} = \lim_{n \rightarrow \infty} \frac{\sin(1/n^3)}{1/n^3} = 1$ . Since  $\sum \frac{1}{n^2}$  converges, the series converges.
59. **Converges.** Limit Comparison Test with  $b_n = \frac{1}{n^2}$  (for  $n \geq 2$ ). The limit of the ratio is 1, so it converges.
60. **Diverges.** Limit Comparison Test with  $b_n = \frac{1}{n}$ .  $\lim_{n \rightarrow \infty} \frac{\frac{3}{(2n-\sqrt{n})}}{1/n} = \frac{3}{2}$ . Since the harmonic series diverges, this series diverges.
61. **Diverges.** Test for Divergence:  $\lim_{n \rightarrow \infty} \ln(n) = \infty \neq 0$ .
62. **Converges.** Geometric Series with  $r = 1/5 < 1$ .

- 63. Converges.** Limit Comparison Test with  $b_n = \frac{n^2}{n^4} = \frac{1}{n^2}$ . The limit of the ratio is  $1/3$ . Converges.
- 64. Converges.** Use  $a_n = \frac{2}{e^n + e^{-n}}$ . Direct Comparison Test with  $b_n = \frac{2}{e^n}$ . Since  $e^n + e^{-n} > e^n$ , we have  $\frac{2}{e^n + e^{-n}} < \frac{2}{e^n}$ .  $\sum 2(1/e)^n$  is a convergent geometric series.
- 65. Diverges.** Test for Divergence. Since  $n^n$  grows much faster than  $n!$ ,  $\lim_{n \rightarrow \infty} \frac{n^n}{n!} = \infty$ .
- 66. Diverges.** Limit Comparison Test with  $b_n = \frac{1}{n}$ . The denominator  $\sqrt{n(n+1)}$  behaves like  $\sqrt{n^2} = n$ .
- 67. Diverges.** p-Series Test. This is  $\frac{1}{3} \sum \frac{1}{n^{4/5}}$ . Since  $p = 4/5 \leq 1$ , it diverges.
- 68. Converges.** Limit Comparison Test with  $b_n = \frac{n}{n^3} = \frac{1}{n^2}$ .
- 69. Converges.** Direct Comparison Test.  $1 \leq 2 + (-1)^n \leq 3$ . So,  $\frac{2+(-1)^n}{n^{3/2}} \leq \frac{3}{n^{3/2}}$ . Since  $\sum \frac{3}{n^{3/2}}$  converges (p-series,  $p = 3/2 > 1$ ), the series converges.
- 70. Diverges.** Limit Comparison Test with  $b_n = \frac{1}{n}$ .
- 71. Converges.** p-Series Test with  $p = 3 > 1$ .
- 72. Diverges.** Test for Divergence:  $\lim_{n \rightarrow \infty} \frac{1}{1000} = \frac{1}{1000} \neq 0$ .
- 73. Converges.** Direct Comparison Test. For  $n \geq 2$ ,  $\ln(n) < n$ . So  $\frac{\ln(n)}{n^3} < \frac{n}{n^3} = \frac{1}{n^2}$ . Since  $\sum \frac{1}{n^2}$  converges, this series converges.
- 74. Converges.** Direct Comparison Test with  $b_n = \frac{1}{3^n}$ .
- 75. Diverges.** Note: Original problem  $\frac{\cos(n\pi)}{\sqrt{n}}$  is an alternating series. This replacement is solvable by methods in this section. Direct Comparison Test: Since  $-1 \leq \cos(n) \leq 1$ , the numerator  $2 + \cos(n)$  is always between 1 and 3. Thus,  $a_n = \frac{2+\cos(n)}{\sqrt{n}} \geq \frac{1}{\sqrt{n}}$ . Since  $\sum \frac{1}{\sqrt{n}}$  diverges (p-series,  $p = 1/2 \leq 1$ ), the series diverges.
- 76. Converges.** Direct Comparison Test with  $b_n = \frac{1}{n^4}$ .
- 77. Converges.** Limit Comparison Test with  $b_n = \frac{4^n}{5^n} = (\frac{4}{5})^n$ , a convergent geometric series.
- 78. Converges.** Limit Comparison Test. Dominant terms give  $\frac{n^{1/3}}{\sqrt{n^3}} = \frac{n^{1/3}}{n^{3/2}} = \frac{1}{n^{7/6}}$ . Since  $p = 7/6 > 1$ ,  $\sum \frac{1}{n^{7/6}}$  converges, so the series converges.
- 79. Converges.** p-Series Test.  $a_n = \frac{1}{n^{4/3}}$ .  $p = 4/3 > 1$ .
- 80. Diverges.**  $a_n = \frac{1}{2n \ln n}$ . Integral Test.  $\frac{1}{2} \int_2^\infty \frac{1}{x \ln x} dx = \frac{1}{2} [\ln(\ln x)]_2^\infty = \infty$ .
- 81. Diverges.** Limit Comparison Test with  $b_n = \frac{1}{n}$  (for  $n \geq 2$ ).
- 82. Converges.** Direct Comparison Test.  $0 \leq 1 + \cos(n) \leq 2$ . So  $a_n \leq \frac{2}{e^n}$ . Converges.

- 83. Diverges.** p-Series Test.  $p = 2/3 \leq 1$ .
- 84. Diverges.** Test for Divergence:  $\lim_{n \rightarrow \infty} \frac{n^2+1}{n^2} = 1 \neq 0$ .
- 85. Converges.** Limit Comparison Test with  $b_n = \frac{1}{2^n}$ .
- 86. Converges.** Limit Comparison Test with  $b_n = \frac{1}{n^2}$ .  $\lim_{n \rightarrow \infty} \frac{e^n/n^n}{1/n^2} = \lim_{n \rightarrow \infty} \frac{n^2 e^n}{n^n} = 0$ . Since the limit is 0 and the comparison series converges, the series converges.
- 87. Converges.** Geometric Series.  $a_n = \frac{1}{5} \left(\frac{2}{5}\right)^{n-1}$ .  $r = 2/5 < 1$ .
- 88. Converges.** This is the sum of two convergent geometric series:  $\sum \left(\frac{3}{6}\right)^n + \sum \left(\frac{2}{6}\right)^n$ .
- 89. Converges.** p-Series Test with  $p = 1.1 > 1$ .
- 90. Diverges.** Limit Comparison Test with  $b_n = \frac{1}{n}$ .
- 91. Diverges.** Limit Comparison Test with  $b_n = \frac{1}{\sqrt{n}}$ .
- 92. Diverges.** Test for Divergence:  $\lim_{n \rightarrow \infty} a_n = 1/2 \neq 0$ .
- 93. Diverges.** Integral Test. Let  $u = \ln x$ .  $\int_2^\infty \frac{\ln x}{x} dx = \int_{\ln 2}^\infty u du = \infty$ .
- 94. Converges.** Geometric Series with  $r = 1/2 < 1$ .
- 95. Diverges.** Direct Comparison Test.  $n + n \cos^2(n) \leq n + n = 2n$ . So  $\frac{1}{n+n \cos^2(n)} \geq \frac{1}{2n}$ . Since  $\sum \frac{1}{2n}$  diverges, the series diverges.
- 96. Converges.** Limit Comparison Test with  $b_n = \frac{1}{n^2}$ .
- 97. Diverges.** Test for Divergence:  $\lim_{n \rightarrow \infty} a_n = 1/3 \neq 0$ .
- 98. Converges.** Limit Comparison Test with  $b_n = \frac{1}{n^2}$ .
- 99. Converges.** Geometric Series with  $r = 1/3 < 1$ .
- 100. Converges.** Limit Comparison Test with  $b_n = \frac{1}{n^2}$ .
- 101. Converges.** Direct Comparison Test. For  $n \geq 1$ ,  $\sqrt{n} + n^2 > n^2$ , so  $\frac{1}{\sqrt{n}+n^2} < \frac{1}{n^2}$ .
- 102. Diverges.** Test for Divergence:  $\lim_{n \rightarrow \infty} a_n = 3/2 \neq 0$ .
- 103. Converges.** p-Series Test with  $p = 5/3 > 1$ .
- 104. Diverges.** Limit Comparison Test with  $b_n = \frac{1}{\sqrt{n}}$ .
- 105. Converges.** Direct Comparison Test with  $b_n = \frac{1}{e^n}$ .
- 106. Converges.** Limit Comparison Test with  $b_n = \frac{1}{n^2}$  for  $n \geq 2$ .
- 107. Converges.** Limit Comparison Test with  $b_n = \frac{4^n}{6^n} = \left(\frac{2}{3}\right)^n$ .

- 108. Converges.** Limit Comparison Test with  $b_n = \frac{1}{n^{3/2}}$ .
- 109. Diverges.** p-Series Test with  $p = 0.5 \leq 1$ .
- 110. Diverges.** Integral Test for  $f(x) = \frac{1}{x(1+\ln x)}$ . Let  $u = 1 + \ln x$ . Diverges.
- 111. Converges.** Limit Comparison Test with  $b_n = \frac{1}{n^2}$ .
- 112. Diverges.** Limit Comparison Test with  $b_n = \frac{n}{\sqrt{n^3}} = \frac{1}{\sqrt{n}}$ .
- 113. Diverges.** Integral Test. Same behavior as  $\sum \frac{\ln n}{n}$ .
- 114. Converges.** p-Series Test with  $p = 4 > 1$ .
- 115. Converges.** Note  $a_n = \frac{n}{n!} = \frac{1}{(n-1)!}$  for  $n \geq 1$ . Compare with  $b_n = \frac{1}{n^2}$  using LCT. The limit of the ratio is 0. Converges.
- 116. Diverges.** Limit Comparison Test with  $b_n = \frac{1}{n}$ .
- 117. Diverges.** p-Series Test with  $p = 1/3 \leq 1$ .
- 118. Diverges.** Limit Comparison Test with  $b_n = \frac{1}{n}$  for  $n \geq 2$ .
- 119. Diverges.** Limit Comparison Test with  $b_n = \frac{1}{\sqrt{n}}$ .
- 120. Diverges.** Test for Divergence:  $\lim_{n \rightarrow \infty} a_n = 2 \neq 0$ .
- 121. Converges.** p-Series Test with  $p = 2 > 1$ .  $\pi$  is a constant.
- 122. Converges.** Direct Comparison Test with  $b_n = (\frac{1}{3})^n$ .
- 123. Diverges.** Harmonic series with a constant multiple.
- 124. Converges.** Limit Comparison Test with  $b_n = \frac{1}{n^2}$ .
- 125. Converges.** p-Series Test with  $p = 3/2 > 1$ .
- 126. Diverges.** Limit Comparison Test with  $b_n = \frac{1}{n}$  for  $n \geq 2$ .
- 127. Diverges.** Limit Comparison Test with  $b_n = \frac{\sqrt{n}}{n\sqrt{n}} = \frac{1}{n}$ .
- 128. Diverges.** Limit Comparison Test with  $b_n = \frac{n^2}{n^3} = \frac{1}{n}$ .
- 129. Diverges.** Limit Comparison Test with  $b_n = \frac{1}{n^{1/3}}$ .
- 130. Converges.** Limit Comparison Test with  $b_n = \frac{1}{n^2}$ .
- 131. Diverges.** Limit Comparison Test with  $b_n = \frac{1}{n}$ .
- 132. Converges.** p-Series Test with  $p = 1.0001 > 1$ .
- 133. Converges.** Direct Comparison Test.  $\frac{\arctan(n)}{n^2} < \frac{\pi/2}{n^2}$ .

- 134. Diverges.** Limit Comparison Test with  $b_n = \frac{e^n}{ne^n} = \frac{1}{n}$ .
- 135. Converges.** Limit Comparison Test with  $b_n = \frac{1}{n^2}$ .
- 136. Diverges.** Test for Divergence:  $\lim_{n \rightarrow \infty} n^{1/n} = 1$ , so  $\lim_{n \rightarrow \infty} a_n = 1 \neq 0$ .
- 137. Converges.** Direct Comparison Test with  $b_n = \frac{3}{n^2}$ .
- 138. Converges.** Limit Comparison Test with  $b_n = \frac{1}{3^n}$ .
- 139. Converges.** Limit Comparison Test with  $b_n = \frac{3^n}{5^n} = (\frac{3}{5})^n$ .
- 140. Converges.** Direct Comparison Test. For  $n \geq 2$ ,  $a_n = \frac{1 \cdot 2 \cdots n}{n \cdot n \cdots n} = \frac{1}{n} \frac{2}{n} \cdots \frac{n}{n} \leq \frac{1}{n} \cdot \frac{2}{n} = \frac{2}{n^2}$ . Since  $\sum \frac{2}{n^2}$  converges, the series converges.
- 141. Converges.** p-Series Test with  $p = 4/3 > 1$ .
- 142. Diverges.** Integral Test. Let  $u = \ln(x+1)$ . Diverges.
- 143. Diverges.** Limit Comparison Test with  $b_n = \frac{1}{n}$ .
- 144. Converges.** Integral Test. Let  $u = x^4$ ,  $du = 4x^3dx$ . The integral  $\int_1^\infty x^3 e^{-x^4} dx$  converges.
- 145. Converges.** Limit Comparison Test with  $b_n = \frac{1}{n^2}$ .
- 146. Converges.** Geometric Series.  $a_n = (\frac{3}{4})^n$ .  $r = 3/4 < 1$ .
- 147. Converges.** Direct Comparison Test with  $b_n = \frac{1}{2^n}$ .
- 148. Converges.** Limit Comparison Test with  $b_n = \frac{n^{2/3}}{n^2} = \frac{1}{n^{4/3}}$ . Converges since  $p = 4/3 > 1$ .
- 149. Diverges.** Limit Comparison Test with  $b_n = \frac{1}{n}$ .
- 150. Converges.** Limit Comparison Test with convergent series  $b_n = \frac{1}{2^n}$ . The limit of the ratio is 0.
- 151. Converges.** Limit Comparison Test with  $b_n = \frac{1}{n^2}$  for  $n \geq 2$ .
- 152. Diverges.** Test for Divergence:  $\lim_{n \rightarrow \infty} \frac{n-2}{n} = 1 \neq 0$ .
- 153. Diverges.** Limit Comparison Test with  $b_n = \frac{1}{n^{2/3}}$  for  $n \geq 2$ .
- 154. Converges.** p-Series Test with  $p = 5 > 1$ .
- 155. Diverges.** Limit Comparison Test with  $b_n = \frac{1}{n}$ .
- 156. Diverges.** Direct Comparison Test. For  $n > e$ ,  $\ln(n) > 1$ , so  $\frac{\ln(n)}{\sqrt{n}} > \frac{1}{\sqrt{n}}$ . Since  $\sum \frac{1}{\sqrt{n}}$  diverges, the series diverges.
- 157. Diverges.** Constant multiple of the harmonic series.

- 158. Converges.** Limit Comparison Test with  $b_n = \frac{1}{n^2}$ .
- 159. Diverges.** p-Series Test with  $p = 1/2 \leq 1$ .
- 160. Converges.** Limit Comparison Test with  $b_n = \frac{1}{n^2}$ .  $\lim_{n \rightarrow \infty} \cos(1/n) = 1$ .
- 161. Converges.** Limit Comparison Test with  $b_n = \frac{1}{n^3}$ .
- 162. Converges.** Direct Comparison Test with  $b_n = \frac{1}{n \cdot 2^n}$ , which converges by comparison with  $(1/2)^n$ .
- 163. Diverges.** Test for Divergence:  $\lim_{n \rightarrow \infty} \frac{n+2}{n+1} = 1 \neq 0$ .
- 164. Converges.** p-Series Test with  $p = 2 > 1$ .
- 165. Diverges.** p-Series Test (Harmonic Series) with  $p = 1$ .
- 166. Converges.** Limit Comparison Test with  $b_n = \frac{1}{n^3}$  for  $n \geq 2$ .
- 167. Diverges.** Limit Comparison Test with  $b_n = \frac{1}{n}$ .
- 168. Converges.** Limit Comparison Test with  $b_n = \frac{1}{n^2}$ .
- 169. Converges.** Limit Comparison Test with  $b_n = \frac{1}{n^2}$ .
- 170. Diverges.** Limit Comparison Test with  $b_n = \frac{1}{n}$ .
- 171. Diverges.** Limit Comparison Test with  $b_n = \frac{1}{n}$ .
- 172. Converges.** p-Series Test with  $p = 1.5 > 1$ .
- 173. Diverges.** Limit Comparison Test with  $b_n = \frac{1}{n}$ .
- 174. Converges.** Geometric Series with  $r = 3/5 < 1$ .
- 175. Diverges.** Limit Comparison Test with  $b_n = \frac{1}{n}$ .
- 176. Diverges.** Limit Comparison Test with  $b_n = \frac{1}{n}$ .
- 177. Converges.** p-Series Test with  $p = 3/2 > 1$ .
- 178. Converges.** Limit Comparison Test with the convergent geometric series  $b_n = (\frac{1}{2})^n$ .