Option Mechanics: Put-Call Parity and Synthetic Collars

Comprehensive Study Guide

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1 Conceptual Foundations

1.1 The Governing Law: Put-Call Parity

Put-Call Parity is the fundamental no-arbitrage condition in options pricing. It establishes a rigid mathematical relationship between the prices of European call options, put options, the underlying asset, and a risk-free bond.

The fundamental equation is:

$$C + PV(K) = P + S$$

Where:

- C: Price of the Call option.
- P: Price of the Put option.
- S: Current spot price of the underlying asset (Stock).
- $PV(K)$: Present Value of the strike price K (often written as Ke^{-rt}).

Intuition A portfolio consisting of a **Long Call** and a **Risk-Free Bond** (equal to the strike price) must have the exact same payoff at expiration as a portfolio consisting of a **Long Put** and the **Underlying Stock**. If they did not, an arbitrageur could buy the cheaper portfolio and sell the expensive one for a risk-free profit.

1.2 Defining "Synthetic" Positions

In options trading, a **synthetic** position is a strategy that replicates the risk and reward profile of a specific instrument using a combination of *other* instruments.

Synthetics allow traders to create the payoff of an option without actually trading that specific option. This is derived directly by rearranging the Put-Call Parity equation.

1.2.1 Synthetic Long Call

Rearranging parity to solve for C:

$$C = P + S - PV(K)$$

Disregarding the bond component for the directional exposure, the **Synthetic Long Call** is constructed via:

$$\text{Synthetic Call} = \text{Long Stock } (S) + \text{Long Put } (P)$$

Intuition: This is often called a "Married Put."

- If S rises, you profit from the shares (unlimited upside).
- If S crashes, the Put option gains value, offsetting the stock loss (limited downside).
- This creates the exact same payoff profile as buying a Call option.

1.2.2 Synthetic Long Put

Rearranging parity to solve for P:

$$P = C - S + PV(K)$$

The **Synthetic Long Put** is constructed via:

$$\text{Synthetic Put} = \text{Long Call } (C) + \text{Short Stock } (-S)$$

Intuition: This is often called a "Married Call."

- You are short the stock (profiting if it drops).
- You hold a Long Call to protect against the price rising indefinitely.
- This replicates the payoff of a Long Put (profit on downside, capped risk on upside).

2 The Standard Collar Strategy

A standard collar is a hedging strategy used to protect a long stock position against downside risk while financing that protection by capping upside potential.

The position consists of three parts:

1. **Long Stock** ($+S$): The asset being protected.
2. **Long Put** ($+P_L$): Out-of-the-money (OTM) put at Strike K_L (Floor).
3. **Short Call** ($-C_H$): Out-of-the-money (OTM) call at Strike K_H (Ceiling).

$$\text{Collar Payoff} = S + P_L - C_H$$

Where $K_L < S < K_H$.

3 Deriving Collars via Put-Call Parity

Using Put-Call Parity, we can prove that a Collar is mathematically equivalent to other structures involving vertical spreads and bonds. We will derive two variations: one using two calls, and one using two puts.

3.1 Variation 1: The Collar Using Two Calls

Objective: Express the Collar equation using only Call options and bonds.
We must replace the **Long Put (P_L)** in the standard collar with its synthetic equivalent.
Step 1: Parity at the Lower Strike (K_L)

$$P_L = C_L - S + PV(K_L)$$

Step 2: Substitution Substitute this expression for P_L into the Standard Collar equation:

$$\text{Collar} = \underbrace{S}_{\text{Stock}} + \underbrace{(C_L - S + PV(K_L))}_{\text{Synthetic Put}} - \underbrace{C_H}_{\text{Short Call}}$$

Step 3: Simplification The Long Stock ($+S$) and the Short Stock component inside the synthetic put ($-S$) cancel each other out.

$$\text{Collar} = C_L - C_H + PV(K_L)$$

Result: A Collar is synthetically equivalent to a **Bull Call Spread** plus a **Risk-Free Bond**.

- Long Call at K_L.
- Short Call at K_H.
- Long Bond (Present Value of K_L).

3.2 Variation 2: The Collar Using Two Puts

Objective: Express the Collar equation using only Put options and bonds.
We must replace the **Short Call ($-C_H$)** in the standard collar with its synthetic equivalent.
Step 1: Parity at the Higher Strike (K_H)

$$C_H - P_H = S - PV(K_H)$$

Multiplying by -1 to solve for the Short Call ($-C_H$):

$$-C_H = PV(K_H) - P_H - S$$

Step 2: Substitution Substitute this expression for $-C_H$ into the Standard Collar equation:

$$\text{Collar} = \underbrace{S}_{\text{Stock}} + \underbrace{P_L}_{\text{Long Put}} + \underbrace{(PV(K_H) - P_H - S)}_{\text{Synthetic Short Call}}$$

Step 3: Simplification The Long Stock ($+S$) cancels out with the Short Stock component ($-S$).

$$\text{Collar} = P_L - P_H + PV(K_H)$$

Result: A Collar is synthetically equivalent to a **Bull Put Spread** plus a **Risk-Free Bond**.

- Long Put at K_L (Lower Strike).
- Short Put at K_H (Higher Strike).
- Long Bond (Present Value of K_H).

4 Summary of Equivalences

By rigorously applying Put-Call Parity, we conclude that the risk profile of holding a stock with a collar is identical to holding a Bull Spread combined with a zero-coupon bond.

Strategy	**Components**	**Equation**
Standard Collar	Stock + Long Put (K_L) + Short Call (K_H)	$S + P_L - C_H$
Synthetic via Calls	Bull Call Spread + Bond (K_L)	$(C_L - C_H) + PV(K_L)$
Synthetic via Puts	Bull Put Spread + Bond (K_H)	$(P_L - P_H) + PV(K_H)$

Table 1: Mathematical Equivalence of Collar Strategies