Calculus II Problem Set
Part 1: Sequences

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Contents

1 Finding the n-th Term

For each of the following sequences, find a formula for the general term a_n, assuming the pattern of the first few terms continues. Assume n begins with 1.

Problem 1.1. $1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots$

Problem 1.2. $5, 8, 11, 14, \ldots$

Problem 1.3. $\frac{1}{2}, -\frac{4}{3}, \frac{9}{4}, -\frac{16}{5}, \ldots$

Problem 1.4. $2, \frac{3}{4}, \frac{4}{9}, \frac{5}{16}, \ldots$

Problem 1.5. $1, 0, 1, 0, 1, 0, \ldots$

Problem 1.6. $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \ldots$

Problem 1.7. $0, 3, 8, 15, 24, \ldots$

Problem 1.8. $-\frac{2}{3}, \frac{4}{9}, -\frac{8}{27}, \frac{16}{81}, \ldots$

Problem 1.9. $\frac{1}{1}, \frac{2}{3}, \frac{3}{5}, \frac{4}{7}, \ldots$

Problem 1.10. $1, -1, 1, -1, 1, \ldots$

Problem 1.11. $2, 6, 12, 20, 30, \ldots$ *Hint: Look at the factors of each term.*

Problem 1.12. $\cos(0), \cos(\pi), \cos(2\pi), \cos(3\pi), \ldots$

Problem 1.13. $\frac{1}{2}, \frac{2}{5}, \frac{3}{10}, \frac{4}{17}, \ldots$

Problem 1.14. $1, \frac{1}{2}, \frac{1}{6}, \frac{1}{24}, \frac{1}{120}, \ldots$

Problem 1.15. $1, 5, 9, 13, 17, \ldots$

Problem 1.16. $\frac{\sqrt{1}}{3}, \frac{\sqrt{2}}{4}, \frac{\sqrt{3}}{5}, \frac{\sqrt{4}}{6}, \ldots$

Problem 1.17. $10, 5, \frac{5}{2}, \frac{5}{4}, \frac{5}{8}, \ldots$

Problem 1.18. $\frac{5}{1}, \frac{8}{3}, \frac{11}{5}, \frac{14}{7}, \ldots$

Problem 1.19. $\{0.9, 0.99, 0.999, 0.9999, \ldots\}$ *Hint: Express each term as* $1 - \ldots$

Problem 1.20. $0, \frac{1}{2}, 0, \frac{1}{2}, 0, \frac{1}{2}, \ldots$

Problem 1.21. $\frac{\ln 1}{1}, \frac{\ln 2}{2}, \frac{\ln 3}{3}, \frac{\ln 4}{4}, \ldots$

Problem 1.22. $5, -25, 125, -625, \ldots$

Problem 1.23. $\frac{1}{e}, \frac{2}{e^2}, \frac{3}{e^3}, \frac{4}{e^4}, \ldots$

Problem 1.24. $1, -\frac{1}{8}, \frac{1}{27}, -\frac{1}{64}, \ldots$

Problem 1.25. $1, 3, 1, 3, 1, 3, \ldots$

2 Convergence and Divergence of Sequences

Determine whether the sequence converges or diverges. If it converges, find the limit.

Problem 2.1. $a_n = \frac{3n^2-1}{10n+5n^2}$

Problem 2.2. $a_n = \frac{n}{n+1}$

Problem 2.3. $a_n = n\sin\left(\frac{1}{n}\right)$

Problem 2.4. $a_n = (-1)^n\frac{n}{n+1}$

Problem 2.5. $a_n = \frac{\ln(n)}{n}$

Problem 2.6. $a_n = \cos(n\pi)$

Problem 2.7. $a_n = \frac{n!}{2^n}$

Problem 2.8. $a_n = \frac{3^n}{n!}$

Problem 2.9. $a_n = \arctan(n)$

Problem 2.10. $a_n = \left(1+\frac{1}{n}\right)^n$

Problem 2.11. $a_n = \sqrt{n+1}-\sqrt{n}$

Problem 2.12. $a_n = \frac{\sin(n)}{n}$

Problem 2.13. $a_n = \frac{(-1)^n}{n^2}$

Problem 2.14. $a_n = ne^{-n}$

Problem 2.15. $a_n = \frac{n^3}{n^3+1}$

Problem 2.16. $a_n = n^{1/n}$

Problem 2.17. $a_n = \frac{2n+1}{1-3n}$

Problem 2.18. $a_n = \frac{4n^2-3}{3n^2+n+1}$

Problem 2.19. $a_n = \frac{\sqrt{n}}{\ln(n)}$

Problem 2.20. $a_n = \cos\left(\frac{2}{n}\right)$

Problem 2.21. $a_n = \frac{n^2}{2n-1}$

Problem 2.22. $a_n = (-1)^{n+1}\frac{1}{\sqrt{n}}$

Problem 2.23. $a_n = \tanh(n)$

Problem 2.24. $a_n = \frac{(n+1)!}{n!}$

Problem 2.25. $a_n = \frac{\cos^2(n)}{2^n}$

3 Solutions

3.1 Solutions for Section 1: Finding the n-th Term

1.1 The terms are the reciprocals of the natural numbers. $a_n = \frac{1}{n}$

1.2 This is an arithmetic sequence with first term $a_1 = 5$ and common difference $d = 3$. The formula is $a_n = a_1 + (n-1)d = 5 + (n-1)3 = 3n + 2$.

1.3 Signs are alternating: $(-1)^{n+1}$. Numerators are squares: n^2. Denominators are $n + 1$. So, $a_n = \frac{(-1)^{n+1} n^2}{n+1}$.

1.4 Numerators are $n + 1$. Denominators are squares: n^2. So, $a_n = \frac{n+1}{n^2}$.

1.5 The terms alternate between 1 and 0. We can write this using cosine or with $(-1)^n$. A simple form is $a_n = \frac{1+(-1)^{n+1}}{2}$.

1.6 This is a geometric sequence with first term $a_1 = 1/2$ and common ratio $r = 1/2$. The formula is $a_n = a_1 r^{n-1} = \frac{1}{2}\left(\frac{1}{2}\right)^{n-1} = \left(\frac{1}{2}\right)^n = \frac{1}{2^n}$.

1.7 The terms are one less than the perfect squares. $a_n = n^2 - 1$.

1.8 This is a geometric sequence with ratio $r = -2/3$. The first term is $-2/3$. So, $a_n = \left(-\frac{2}{3}\right)^n$.

1.9 Numerators are n. Denominators are consecutive odd numbers, which can be written as $2n - 1$. So, $a_n = \frac{n}{2n-1}$.

1.10 The terms alternate between 1 and -1. $a_n = (-1)^{n+1}$ or $a_n = (-1)^{n-1}$.

1.11 The terms are $1 \cdot 2, 2 \cdot 3, 3 \cdot 4, 4 \cdot 5, 5 \cdot 6, \ldots$. The formula is $a_n = n(n + 1)$.

1.12 The arguments of cosine are $0, \pi, 2\pi, \ldots$. This is $(n-1)\pi$. The terms are $1, -1, 1, -1, \ldots$. So, $a_n = \cos((n-1)\pi) = (-1)^{n-1}$.

1.13 Numerators are n. Denominators are one more than the squares, n^2+1. So, $a_n = \frac{n}{n^2+1}$.

1.14 The denominators are factorials: $1!, 2!, 3!, \ldots$. So, $a_n = \frac{1}{n!}$.

1.15 Arithmetic sequence with $a_1 = 1$ and $d = 4$. $a_n = 1 + (n-1)4 = 4n - 3$.

1.16 Numerators are $\sqrt{n}$. Denominators are $n + 2$. So, $a_n = \frac{\sqrt{n}}{n+2}$.

1.17 Geometric sequence with $a_1 = 10$ and $r = 1/2$. $a_n = 10\left(\frac{1}{2}\right)^{n-1}$.

1.18 Numerators are an arithmetic sequence $3n + 2$. Denominators are an arithmetic sequence of odd numbers $2n - 1$. So, $a_n = \frac{3n+2}{2n-1}$.

1.19 $a_1 = 1 - 0.1$, $a_2 = 1 - 0.01$, $a_3 = 1 - 0.001$. This can be written as $a_n = 1 - \frac{1}{10^n}$.

1.20 The sequence is $\frac{1-(-1)^n}{4}$.

1.21 $a_n = \frac{\ln n}{n}$.

1.22 Geometric, with $a_1 = 5, r = -5$. $a_n = 5(-5)^{n-1} = (-1)^{n-1}5^n$.

1.23 $a_n = \frac{n}{e^n}$.

1.24 The signs alternate $(-1)^{n+1}$. The denominators are cubes n^3. $a_n = \frac{(-1)^{n+1}}{n^3}$.

1.25 The sequence is $2 + (-1)^{n+1}$.

3.2 Solutions for Section 2: Convergence and Divergence

2.1 Divide numerator and denominator by n^2: $\lim_{n\to\infty} \frac{3-1/n^2}{10/n+5} = \frac{3-0}{0+5} = \frac{3}{5}$. **Converges to 3/5**.

2.2 Divide by n: $\lim_{n\to\infty} \frac{1}{1+1/n} = \frac{1}{1+0} = 1$. **Converges to 1**.

2.3 Rewrite as $\frac{\sin(1/n)}{1/n}$. Let $x = 1/n$. As $n \to \infty$, $x \to 0$. The limit is $\lim_{x\to 0} \frac{\sin x}{x} = 1$. **Converges to 1**.

2.4 The term $\frac{n}{n+1}$ approaches 1, but the $(-1)^n$ factor causes the sequence to oscillate between values close to 1 and -1. It does not approach a single value. **Diverges**.

2.5 Use L'Hôpital's Rule on the function $f(x) = \frac{\ln x}{x}$. $\lim_{x\to\infty} \frac{\ln x}{x} = \lim_{x\to\infty} \frac{1/x}{1} = 0$. **Converges to 0**.

2.6 The sequence is $-1, 1, -1, 1, \ldots$. It oscillates and does not approach a single limit. **Diverges**.

2.7 Factorials grow faster than exponentials. The terms a_n will grow without bound. $\lim_{n\to\infty} \frac{n!}{2^n} = \infty$. **Diverges**.

2.8 Factorials grow faster than exponentials. The denominator grows much faster than the numerator. $\lim_{n\to\infty} \frac{3^n}{n!} = 0$. **Converges to 0**.

2.9 As n approaches infinity, the argument of arctan goes to infinity. $\lim_{n\to\infty} \arctan(n) = \frac{\pi}{2}$. **Converges to** $\pi/2$.

2.10 This is a standard limit form. $\lim_{n\to\infty} \left(1 + \frac{1}{n}\right)^n = e$. **Converges to e**.

2.11 Multiply by the conjugate: $a_n = (\sqrt{n+1} - \sqrt{n})\frac{\sqrt{n+1}+\sqrt{n}}{\sqrt{n+1}+\sqrt{n}} = \frac{n+1-n}{\sqrt{n+1}+\sqrt{n}} = \frac{1}{\sqrt{n+1}+\sqrt{n}}$. The limit of this expression as $n \to \infty$ is 0. **Converges to 0**.

2.12 Use the Squeeze Theorem. Since $-1 \leq \sin(n) \leq 1$, we have $-\frac{1}{n} \leq \frac{\sin(n)}{n} \leq \frac{1}{n}$. Since $\lim_{n\to\infty}(-\frac{1}{n}) = 0$ and $\lim_{n\to\infty}(\frac{1}{n}) = 0$, the limit of a_n is also 0. **Converges to 0**.

2.13 The absolute value is $|a_n| = \frac{1}{n^2}$, which goes to 0. Therefore, the sequence itself must go to 0. **Converges to 0**.

2.14 Rewrite as $\frac{n}{e^n}$. Use L'Hôpital's Rule on $f(x) = \frac{x}{e^x}$. $\lim_{x\to\infty} \frac{x}{e^x} = \lim_{x\to\infty} \frac{1}{e^x} = 0$. **Converges to 0**.

2.15 Divide by n^3: $\lim_{n\to\infty} \frac{1}{1+1/n^3} = 1$. **Converges to 1**.

2.16 This is a standard limit. Let $y = n^{1/n}$. Then $\ln y = \frac{\ln n}{n}$. We already know $\lim_{n\to\infty} \frac{\ln n}{n} = 0$. So, $\lim \ln y = 0$, which means $\lim y = e^0 = 1$. **Converges to 1**.

2.17 Divide by n: $\lim_{n\to\infty} \frac{2+1/n}{1/n-3} = \frac{2}{-3}$. **Converges to -2/3**.

2.18 Divide by n^2: $\lim_{n\to\infty} \frac{4-3/n^2}{3+1/n+1/n^2} = \frac{4}{3}$. **Converges to 4/3**.

2.19 Polynomials (even roots) grow faster than logarithms. The limit is ∞. **Diverges**.

2.20 As $n \to \infty$, $2/n \to 0$. Since cosine is continuous, $\lim_{n\to\infty} \cos\left(\frac{2}{n}\right) = \cos(0) = 1$. **Converges to 1**.

2.21 The degree of the numerator (2) is greater than the degree of the denominator (1). The limit is ∞. **Diverges**.

2.22 The absolute value $|a_n| = \frac{1}{\sqrt{n}}$ goes to 0, so the sequence converges to 0. **Converges to 0**.

2.23 $\tanh(n) = \frac{e^n - e^{-n}}{e^n + e^{-n}}$. Divide by e^n: $\frac{1-e^{-2n}}{1+e^{-2n}}$. As $n \to \infty$, $e^{-2n} \to 0$. The limit is $\frac{1-0}{1+0} = 1$. **Converges to 1**.

2.24 $a_n = \frac{(n+1)\cdot n!}{n!} = n + 1$. The limit is ∞. **Diverges**.

2.25 Use the Squeeze Theorem. $0 \le \cos^2(n) \le 1$, so $0 \le \frac{\cos^2(n)}{2^n} \le \frac{1}{2^n}$. Since $\lim_{n\to\infty} 0 = 0$ and $\lim_{n\to\infty} \frac{1}{2^n} = 0$, the sequence converges to 0. **Converges to 0**.