

EXTERNAL

Cboe American-Style Options Implied Volatility Calculation Methodology

This document details a methodology used to construct a volatility surface from American settlement style options. This volatility surface can be used to generate intra-day and end-of-day indicative prices for existing or hypothetical option series.

Implied Volatility Calculation Method

Discrete Dividends and Interest Rates

The discrete dividend is collected from Cboe/Hanweck. The Cboe/Hanweck applies a hybrid analyst+algorithm system for predicting future dividend payments and dates associated with an array of securities across the global market. The Cboe/Hanweck incorporates a database of corporate actions to build up dividend payments patterns, identifying changes in shares in issue (through rights issues, consolidations, bonus issues, and subdivisions), and recording various types of disbursements.

The algorithmic calculations process follows a series of rules and involves:

- Inspecting dividend history.
- Incorporating changes to the issued share capital.
- Identifying patterns.
- Calculating individual forward amounts and dates two fiscal years ahead.

Where a clear pattern cannot be identified (e.g., due to a recent change in the fiscal year-end date, a resumption or cut of dividend, or alteration in the dividend payment pattern); the security is flagged to the analysts, who perform individual market research to forecast the data.

The analysts maintain and monitor the history of the accuracy of forecasts vs. actual outcomes to control and improve their forecasts' quality. For more information about Cboe/Hanweck, visit <https://www.hanweck.com/>.

The risk-free interest rates are yields based on U.S. Treasury yield curve rates (commonly referred to as “Constant Maturity Treasury” rates), to which a cubic spline is applied to derive yields on the expiration dates.

The forward price is calculated as:

$$F_T(\text{Divs to Expiry } T) = \sum_{i, t_i=1}^{t_i \leq T} D_i e^{r_i(T-t_i)}$$

Where:

$F_t()$ = Forward Value at time t ;
 D_t = Dividend payment of value D_t at time t ; and
 r_t = risk free rate for period t_i to T ;

Calculation of Implied Volatility

The implied volatility (V) for each maturity and strike level is calculated by equating the Cox-Ross-Rubinstein Binomial Option Price to the observed option price from the market and the dividends/interest rates calculated in the section above. For each maturity,

$$|Model Price_i - Option Price_i| < Tolerance$$

Smile Interpolation and Extrapolation

To calculate the implied volatility for a non-listed strike and a non-listed maturity, we utilize two different techniques for smile interpolation and extrapolation: (i) the spline interpolation for smile interpolation and (ii) the classical SABR model for smile extrapolation.

Spline Interpolation

The spline interpolation is used for the strike range $K- \leq K \leq K+$, and the expiration range $T- \leq T \leq T+$. We use a cubic spline interpolation in total implied variance space. This is a method to interpolate unknown values between given points using piecewise third order polynomials. The idea is to fit a piecewise function of the form:

$$f(x) = \begin{cases} f_1(x) & x_1 \leq x < x_2 \\ f_2(x) & x_2 \leq x < x_3 \\ \dots & \dots \\ f_{n-1}(x) & x_{n-1} \leq x < x_n \end{cases}$$

$x = K$
 $f(x) = V(K)$

For each f_i is a third-order polynomial defined by:

$$f_i(x) = \alpha_i x^3 + \beta_i x^2 + \gamma_i x + \delta_i$$

$$f_i(x_i) = \alpha_i x_i^3 + \beta_i x_i^2 + \gamma_i x_i + \delta_i = f_{i-1}(x_i)$$

Next, we make the first and second derivatives agree at each adjacent points.

$$f'_i(x) = 3\alpha_i(x - x_i)^2 + 2\beta_i(x - x_i) + \gamma_i = f'_{i-1}(x)$$

$$f''_i(x) = 6\alpha_i(x - x_i) + 2\beta_i = f''_{i-1}(x)$$

Now, we have $4(n-1)$ coefficients and $4(n-1)$ equations, therefore, from basic algebra, we could derive the coefficients, and the volatility between known implied volatility.

SABR Model

The SABR model extrapolates the smile for $0 \leq K(T) \leq K(T) -$ (left extrapolation) and $K(T) + < K(T)$ (right extrapolation).

The SABR model was introduced as a simple class of stochastic volatility processes for the underlying. Given traded and liquid options, we fit the SABR model on the observed smile and estimate the parameters. Using these parameters, we can estimate implied volatility to price at various points on the volatility surface.

The SABR model assumes that the forward rate and the instantaneous volatility are driven by two correlated Brownian motions:

$$\begin{aligned} df_t &= \alpha_t F_t^\beta dW_t^1 \\ d\alpha_t &= v\alpha_t dW_t^2 \\ E[dW_t^1 dW_t^2] &= \rho dt \end{aligned}$$

where:

- α is the instantaneous vol;
- v is the vol of vol;
- ρ is the correlation between the Brownian motions driving the F and the v ;
- β is the leverage effect;

The formula that the implied volatility must satisfy is:

$$|SABR \text{ Implied Vol} - \text{Observed Implied Vol}| < \text{Tolerance}$$

$$\begin{aligned} SABR \text{ Implied Vol}(T, F, K, \alpha, \beta, v, \rho) \\ = \frac{\alpha z}{\chi(z) F K^{(1-\beta)/2}} \left(1 + \frac{(1-\beta)^2 (\ln F/K)^2}{24} + \frac{(1-\beta)^4 (\ln F/K)^4}{1920} \right)^{-1} \\ \times \left(1 + \left(\frac{(1-\beta)^2 (\alpha)^2}{24(FK)^{1-\beta}} + \frac{\alpha \beta v \rho}{4(FK)^{(1-\beta)/2}} + \frac{(2-3\rho^2)v^2}{24} \right) T \right), \end{aligned}$$

Where:

$$z = \frac{v \ln F}{\alpha} (FK)^{\frac{1-\beta}{2}}, \chi(z) = \ln \left(\frac{\sqrt{1-2\rho z + z^2} + z - \rho}{1-\rho} \right)$$

Given the initial value of the forward, F , the model is governed by 4 parameters, α, β, v, ρ .

Document Information <DO NOT EDIT>

SharePoint-populated data:

Major Version #	1
Last Revised Date	4/15/2021
Last Review Date	4/15/2021

Disclaimer

Copyright © 2020 Cboe Global Indices, LLC (“CGI”). All rights reserved. Cboe®, Cboe Global Markets®, Cboe Volatility Index®, VIX® and TYVIX® are registered trademarks and Cboe Global IndicesSM, SRVIXSM, XSPSM, BuyWriteSM and PutWriteSM are service marks of CGI and its affiliates (each a “Cboe Company” and collectively “Cboe”). Cboe trademarks may only be used with written permission, subject to Cboe’s trademark usage guidelines as published from time to time. This document does not constitute an offer of services in jurisdictions where CGI does not have the necessary licenses. Except for certain custom index calculation services, all information provided by CGI is impersonal and not tailored to the needs of any person, entity or group of persons. All indices are calculated by CGI without regard to the needs of any investor, investment fund or other financial product, or any issuer of any financial product. CGI receives compensation in connection with licensing indices to third parties and providing custom calculation services. Past performance of an index is not an indication or guarantee of future results. Indices may be calculated using data from markets operated by Cboe Companies.

It is not possible to invest directly in an index. Exposure to an asset class represented by an index may be available through investment funds or other financial products based on that index (each of the foregoing, an “Indexed Financial Product”). Indexed Financial Products may be traded on markets operated by Cboe Companies and/or third-party companies, for which such companies derive fees. Cboe does not sponsor, endorse, sell, promote or manage any investment fund or other financial product (whether or not based on an index) that is offered by a third party. There is no assurance that any Indexed Financial Product will accurately track index performance or provide positive investment returns. No Cboe Company is an investment adviser or tax advisor, and no representation is made regarding the advisability or tax consequences of investing in, holding or selling any investment fund or other investment product. A decision to invest in, hold or sell any investment fund or other financial product should not be made in reliance on any of the statements set forth in this document. Prospective investors are advised to make an investment in, hold or sell investment funds or other financial products only after carefully considering the associated risks and tax consequences as detailed in an offering memorandum or similar document that is prepared by or on behalf of the issuer of the investment fund or other financial product, with the advice of a qualified professional investment adviser and tax advisor. Inclusion of a security within an index is not a recommendation to buy, sell, or hold such security, nor should it be considered investment advice. Indices are calculated by CGI using information described in published index methodologies, and the information may include market data from exchanges owned and operated by other Cboe Companies, and/or estimated values where information is not representative or is unavailable.

This document has been prepared solely for informational purposes based upon information generally available to the public and from sources believed to be reliable. No values or other content contained in this document (including without limitation, index values or information, ratings, credit-related analyses and data, research, valuations and models) or any part thereof (“Content”) may be modified, reverse-engineered, reproduced or distributed in any form or by any means, or stored in a database or retrieval system, without the prior written permission of Cboe. Content shall not be used for any unlawful or unauthorized purposes. Cboe and its third-party data providers, including licensors of index values and trademarks (including the Index Providers listed below) used in connection with CGI indices and Indexed Financial Products (collectively, the “Cboe Parties”) do not

guarantee the accuracy, completeness, timeliness or availability of Content, index values or the data used to calculate index values. The Cboe Parties are not responsible for any errors or omissions, regardless of the cause, or for the results obtained from the use of the Content or index values. THE CONTENT AND INDICES ARE PROVIDED "AS IS" WITHOUT WARRANTY OF ANY KIND, EITHER EXPRESS OR IMPLIED, INCLUDING, WITHOUT LIMITATION, ANY WARRANTY WITH RESPECT MERCHANTABILITY OR FITNESS FOR A PARTICULAR PURPOSE. USERS OF CONTENT AND/OR THE INDICES BEAR ALL RISKS OF LOSS. IN NO EVENT SHALL THE CBOE PARTIES BE RESPONSIBLE OR LIABLE TO ANY PERSON OR ENTITY FOR ANY DAMAGES OF ANY KIND, INCLUDING, BUT NOT LIMITED TO DIRECT, INDIRECT, SPECIAL, INCIDENTAL, CONSEQUENTIAL, PUNITIVE, LOST PROFITS OR LOST OPPORTUNITIES, WHETHER OR NOT A CBOE PARTY HAS BEEN ADVISED OF THE POSSIBILITY OF SUCH DAMAGES, ARISING OUT OF DELAYS, ERRORS OR OMISSIONS IN CONTENT, USE OR INABILITY TO USE CONTENT OR INDICES, OR ANY OTHER REASON. THE FOREGOING SHALL APPLY REGARDLESS OF WHETHER A CLAIM ARISES IN CONTRACT, TORT, NEGLIGENCE, STRICT LIABILITY, OR OTHERWISE.

Certain activities of various Cboe Companies and/or divisions and business units within Cboe Companies are kept separate in order to avoid conflicts of interest and preserve the independence and objectivity of their respective activities. As a result, certain information is maintained by one Cboe company, division or business unit that is not known to other Cboe Companies, divisions or business units. CGI has established policies and procedures to maintain the confidentiality of certain non-public information received in connection with the administration of indices.

Cboe Companies (other than CGI) operate markets for the trading of financial products and provide services to many organizations, including issuers of securities, investment advisers, broker-dealers, investment banks, other financial institutions and financial intermediaries, and accordingly may provide or receive fees or other economic benefits to or from those organizations. These organizations may also be CGI index licensees or customers, and CGI may include securities of or relating to such organizations in indices.

Cboe has licensed a number of trademarks from third parties from whom it has also licensed indices ("Index Providers") which may be included in this document or in other materials related to CGI indices and Cboe Indexed Financial Products, as follows:

- Standard & Poor's®, S&P®, S&P 100®, S&P 500®, Standard & Poor's 500®, SPDR®, Standard & Poor's Depositary Receipts®, Standard & Poor's 500, 500, Standard & Poor's 100, 100, Standard & Poor's SmallCap 600, S&P SmallCap 600, S&P 500 Dividend Index, Standard & Poor's Super Composite 1500, S&P Super Composite 1500, Standard & Poor's 1500 and S&P 1500 are trade names or trademarks of Standard & Poor's Financial Services, LLC. Any products that have an S&P Index or indices as their underlying interest are not sponsored, endorsed, sold or promoted by S&P OPCO LLC ("Standard & Poor's").
- Russell, Russell 1000®, Russell 2000®, Russell 3000® and Russell MidCap® are registered trademarks of the Frank Russell Company, used under license.
- FTSE® and the FTSE indices are trademarks and service marks of FTSE International Limited, used under license.
- Dow Jones®, Dow Jones Industrial Average®, DJIA® and Dow Jones Indexes are trademarks or service marks of Dow Jones Trademark Holdings, LLC ("Dow Jones"), have been licensed for use for certain purposes by the Cboe Companies. Derivative indices created, used and distributed by Cboe Companies and any investment products based thereon are not sponsored, endorsed, sold or promoted by Dow Jones, and Dow Jones makes no

representation regarding the advisability of investing in any investment product that is based on any such derivative indices.

- The Nasdaq-100 Index®, Nasdaq-100®, The Nasdaq National Market®, Nasdaq®, Nasdaq-100 Shares and Nasdaq-100 Trust are trademarks or service marks of The Nasdaq Stock Market, Inc. (with which its affiliates are the "Corporations"). These marks are licensed for use by Cboe Exchange, Inc. in connection with the trading of products based on the Nasdaq-100 Index. The CBOE Nasdaq-100 Volatility Index (the "Volatility Index") and CBOE Nasdaq-100 BuyWrite Index (the "BuyWrite Index") are not derived, maintained, published, calculated or disseminated by the Corporations. Neither the Volatility Index, the BuyWrite Index nor any Product based on the Volatility Index or BuyWrite Index has been passed on by the Corporations as to its legality or suitability. Such Products are not issued, endorsed, sold, or promoted by the Corporations. THE CORPORATIONS MAKE NO WARRANTIES AND BEAR NO LIABILITY WITH RESPECT TO THE VOLATILITY INDEX OR BUYWRITE INDEX.
- CME® and CBOT® are trademarks of CME Group, Inc. (CME). Cboe has, with the permission of CME, used the CME trademark in the Cboe/CME FX Euro Volatility IndexSM, Cboe/CME FX British Pound Volatility IndexSM, and Cboe/CME FX Yen Volatility IndexSM, and the CBOT trademark in Cboe/CBOT 10-year U.S. Treasury Note Volatility Index.
- MSCI and the MSCI index names are service marks of MSCI Inc. ("MSCI") or its affiliates and have been licensed for use by the Cboe Companies. Any derivative indexes and any financial products based on the derivative indexes ("Products") are not sponsored, guaranteed or endorsed by MSCI, its affiliates or any other party involved in, or related to, making or compiling such MSCI index. Neither MSCI, its affiliates nor any other party involved in, or related to, making or compiling any MSCI index makes any representations regarding the advisability of investing in such Products; or any warranty, express or implied; or bears any liability as to the results to be obtained by any person or any entity from the use of any such MSCI index or any data included therein. No purchaser, seller or holder of any Product, or any other person or entity, should use or refer to any MSCI trade name, trademark or service mark to sponsor, endorse, market or promote this security without first contacting MSCI to determine whether MSCI's permission is required.

v20200724