

Summer 2016


Comparison of Option Price from Black-Scholes Model to Actual Values

Matthew J. Krznaric

University of Akron, mjk117@zips.uakron.edu

Please take a moment to share how this work helps you [through this survey](#). Your feedback will be important as we plan further development of our repository.

Follow this and additional works at: http://ideaexchange.uakron.edu/honors_research_projects

 Part of the [Applied Statistics Commons](#), [Finance and Financial Management Commons](#), and the [Statistical Methodology Commons](#)

Recommended Citation

Krznaric, Matthew J., "Comparison of Option Price from Black-Scholes Model to Actual Values" (2016). *Honors Research Projects*. 396.

http://ideaexchange.uakron.edu/honors_research_projects/396

This Honors Research Project is brought to you for free and open access by The Dr. Gary B. and Pamela S. Williams Honors College at IdeaExchange@UAkron, the institutional repository of The University of Akron in Akron, Ohio, USA. It has been accepted for inclusion in Honors Research Projects by an authorized administrator of IdeaExchange@UAkron. For more information, please contact mjon@uakron.edu, uapress@uakron.edu.

Comparison of Option Price from Black-Scholes Model to Actual Values

1. Introduction

With regard to finance, an option can be described as a contract in which the seller promises that the buyer has the right, but not the obligation, to buy or sell a security at a certain price up until, or at, its expiration date. Two of the most commonly seen options, American and European, have a fundamental difference in the styles concerning when the option can be applied, or exchanged for the stock price. With American-style options, the buyer has the right to exercise the contract at any time up until the expiration date. European-style options differ in that these types of contracts can only be exercised when the contract reaches its expiration date.

One of the most well-known models for computing theoretical European option prices is known as the Black-Scholes Formula. The model was introduced in 1973 in a paper, titled “The Pricing of Options and Corporate Liabilities”, that was published in the *Journal of Political Economy* by economists Fischer Black, Myron Scholes, and Robert Merton. While this model is useful, it is based on the following market assumptions that may hinder its accuracy, such as:

- 1) The short-rate interest rate and volatility are known and constant through time.
- 2) No transaction costs or services associated with buying or selling the option.
- 3) The options are European-style options which can only be exercised on the expiration date.
- 4) The returns on the underlying stock prices are normally distributed.
- 5) The Black-Scholes model assumes that markets are perfectly liquid and it is possible to purchase or sell any amount of stock or options or their fractions at any given time.

Due to the impractical assumptions, the Black-Scholes formula encompasses certain limitations in its ability to predict option prices. One limitation of this model is the use of a constant risk-free interest rate, although there in the actual market, interest rates can change rapidly in certain periods. Another possible concern with the model is that exchange traded options are generally American options, which can be exercised at any time up until expiration and thus would be harder to accurately price. However, most individuals exercise options near their expiration dates in order to use as much value of the option as possible. Despite certain limitations of the Black-Scholes formula, the model is one of the most used techniques for pricing options, and economists continue to amend the model in order to make it more realistic.

In this paper, I will analyze the price movement of 480 stocks in S&P500 during the 2014 year, to determine the effectiveness of the Black-Scholes formula for pricing call options.

2. Methodology

To simulate the S&P500 data with the Black-Scholes model, the following formulas were used to produce results. Explained in section 3, the volatility used was determined by a graph of the standard deviation of log returns instead of through calculating it by the formulas below.

$$\log return = \ln\left[\frac{S_t}{S_{t-1}}\right]$$

$$volatility_{\log return} = SD(\log return)$$

Once the volatility was determined, as well as other variables explained below in this section, I used the Black-Scholes formulas to calculate call options.

$$d_1 = \frac{\ln \frac{S}{K} + \left(r - \delta + \frac{1}{2}\sigma^2\right)t}{\sigma\sqrt{t}}$$

$$d_2 = d_1 - \sigma\sqrt{t}$$

$$C = Se^{-\delta t}N(d_1) - Ke^{-rt}N(d_2)$$

In this equation, S refers to the stock price, δ the dividend yield, t the time until expiration, N the Normal Distribution CDF, K the strike price, σ the volatility, and r the risk free rate. In this analysis, we set the dividends equal to zero. The annual volatility used in the method is equal to $\sigma = 0.1847$, which is equal to the standard deviation at the last day of the 240 trading days, obtained from the actual data. Detail of this analysis is shown in the next section. For the risk-free rate r, we will use a rate taken from a 10-year treasury bond during the 2014 year because U.S. treasury bonds are generally used in place of a risk-free rate. Our value of “r” used will be $r = 0.03$, which was the highest interest rate of the Treasury bond during 2014. The option prices were calculated using the 240 trading days and an initial stock price of $S=100$. Below is an example of how to calculate a call option at a strike price of $K = 90$:

$$d_1 = \frac{\ln \frac{100}{90} + \left(0.03 - 0 + \frac{1}{2}(0.1847)^2\right) * 240}{0.1847 * \sqrt{240}} = 3.9836$$

$$d_2 = 3.9836 - 0.1847 * \sqrt{240} = 1.1218$$

$$\begin{aligned} C &= (100)e^{-(0)(240)}N(3.9836) - (90)e^{-(0.03)(240)}N(1.1218) \\ &= 12.48869 \end{aligned}$$

The following are the Black-Scholes results of different call option prices using strike prices of $k = 90, 95, 100, 105,$ and 110 . On the right, Figure 2, is a plot of the call option price from the Black-Scholes model when using a strike price equal of $k = 100$, for each trading day starting from $t = 1/240$ to $t = 240/240$. The call option price increases as the expiration date is further away. This is because the further the expiration date, the larger the anticipated move of the price. Therefore, the call price must go up to reflect this possibility of profit.

	Black-Scholes
K	Call Prices
90	12.4887
95	8.6940
100	5.6632
105	3.5771
110	2.2267

Figure 1

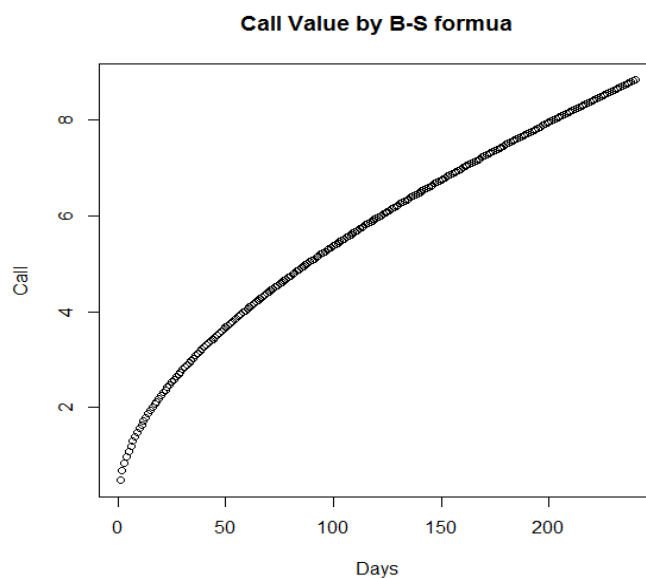


Figure 2

3. Analysis in R

To compare the Black-Scholes method to market results, I performed analysis in R to compute call option values using the actual data. The historical S&P500 prices were obtained from the Yahoo! Finance database for 240 trading days between the dates January 1, 2014 and December 12, 2014. I removed any stock indexes that did not have prices for the entire duration of the 240 days of analysis, resulting in 480 stocks remaining after the omissions. The data was structured in a matrix format, with the columns representing the stock number and the rows under each stock showing the close prices based on different days throughout the time period.

As stated above, the time period used for the analysis included 240 trading days from January 2, 2014 to December 12, 2014. Once the data was loaded, the daily log returns for each stock were able to be calculated over different periods from the starting point x_t and different dates during the 2014 year. Each log return was calculated using the following formula, $\log[D.ad[x_{\text{some date}},i]] - \log[D.ad[x_t,i]]$, where “i” represents the stock number used and x_t the starting date of January 2nd, 2014. Each log return was calculated to determine how the stock has moved since its starting date x_t , with each stock starting at zero.

This enables one to easily determine whether the stock moves up or down in log scale throughout different time periods tested. Log scale provides results similar to those estimated by an arithmetic scale: for example a 5% increase in stock price over six months will be represented by a value of $e^{(0.5)} = 1.0513$ or 5.13% increase in the adjusted close price. The following graphs show the distribution of the log returns for each stock price over the 240 trading days in the 2014 year. On the left, Figure 3 plots log returns of each of the stocks together, with the white line representing the average return of the S&P500 data. Figure 4 on the right displays a full-scale look at the average values across 2014.

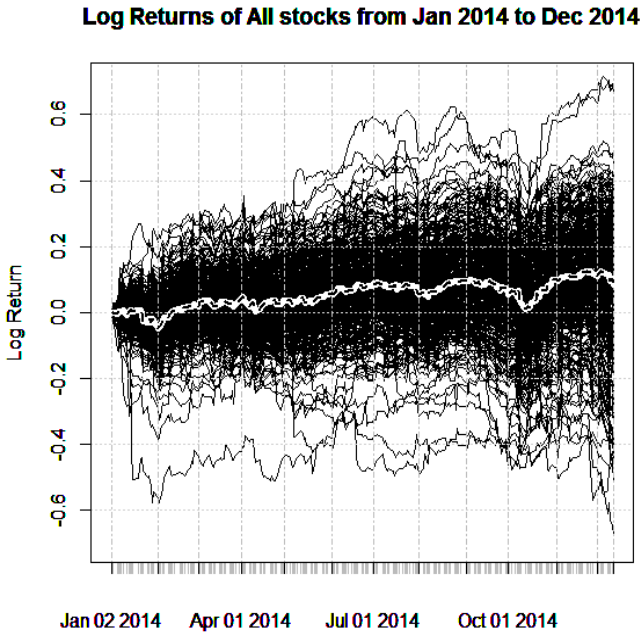


Figure 3

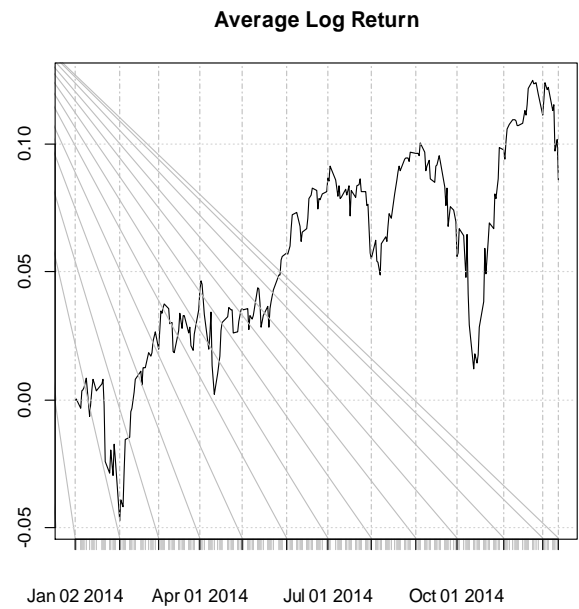


Figure 4

Once the daily log returns of the stock prices were calculated, I was able to analyze the standard deviation of the log returns over the 2014 year. The following graph depicts these values, with the red line indicating 0.1847 times the square root of days, which assumes the volatility was constant at 0.1847 over the 240 trading days. This volatility of $\sigma = 0.1847$ represents the value used in Section 2 in the Black-Scholes formulas.

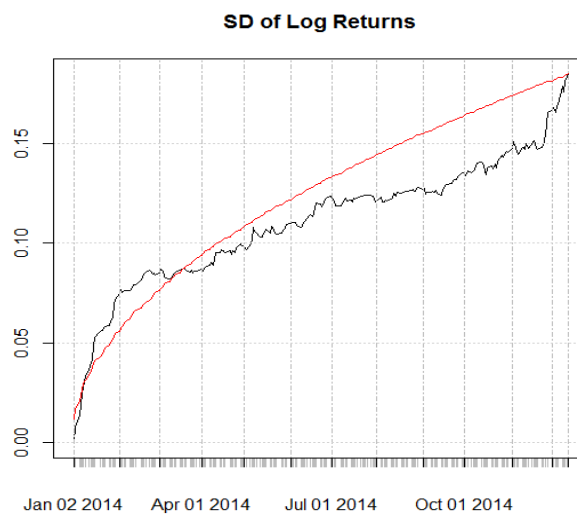


Figure 5

Once the daily log returns of the adjusted close stock prices were analyzed, I was able to plot histograms depicting the distribution of log returns at different periods of time, such as 30, 60, and 90 days. I took snapshots of the graph with all stocks above, Figure 3, and analyzed how the log return distributions developed over time, with the following graphs depicting the distributions of the prices at four different months in the year. The histograms show that as more trading days are used in the analysis, the distributions become more log-normal and the data is better at predicting the mean change of the stocks over the 240 trading days. These results hold true to the Black-Scholes method assuming that the returns will be lognormally distributed.

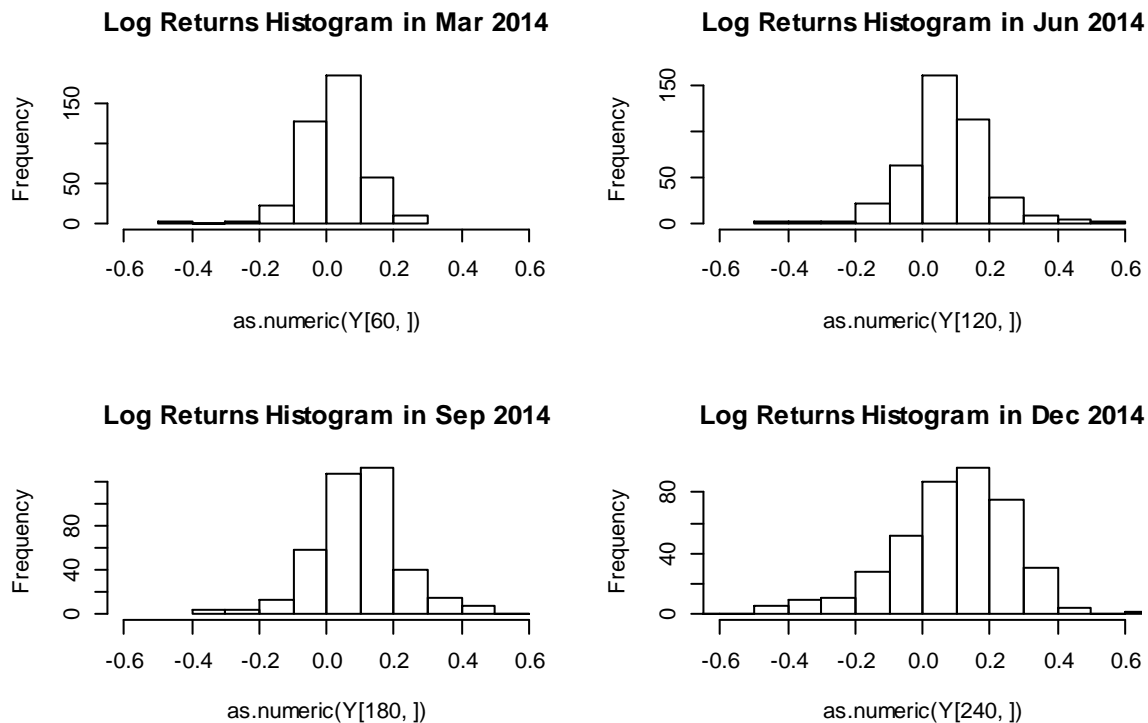


Figure 6

Once the histograms were created, I was able to create a call option graph that plots the mean log return values over the 240 trading days used. We can plot the trend of the price data over the time period, starting from a value of zero: this will enable us to see how the call option increases or decreases over the 2014 year. Option values can also be created using different dates

throughout the year to see how the stock price data changes as more days are used in the analysis. For example, the following graph illustrates how a call option would be priced on the last trading day used, December 12th, 2014.

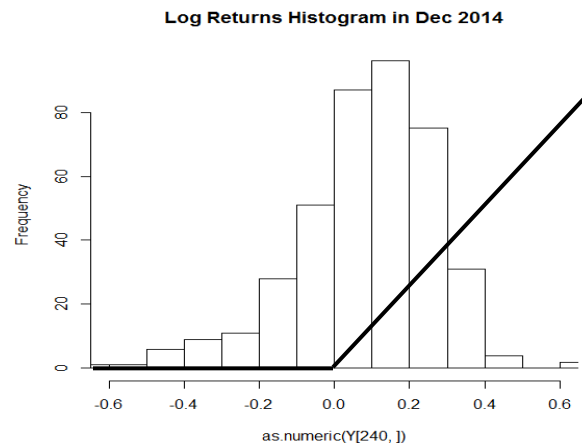


Figure 7

Call option prices can be altered by changing the strike price k , or the price at which the owner of the option can buy the underlying security, helping to determine how decisions would be affected by different prices. For example, the stock price may be above one strike price value, meaning the individuals should exercise the option, but below others used in the analysis. Comparing different strike prices placed with options is thus vital because they will make the difference between choosing whether or not to exercise a call option in a given situation. By calculating the call value for each trading day along the 2014 year, I was able to estimate the overall call option value to be placed for managing against the change in adjusted stock price. Once the call options were created, I was able to compare the results to the plot estimates computed earlier through the Black-Scholes formula, showing the difference between market and theoretical pricing. Overall, this will enable the reader to see if the Black-Scholes method differs from the marketplace due to the “ideal conditions” of both the stock market and options that the approach relies heavily on.

The following graph shows the plot of the main call value over the 240 trading days using the actual data, constructed through the R program in this section:

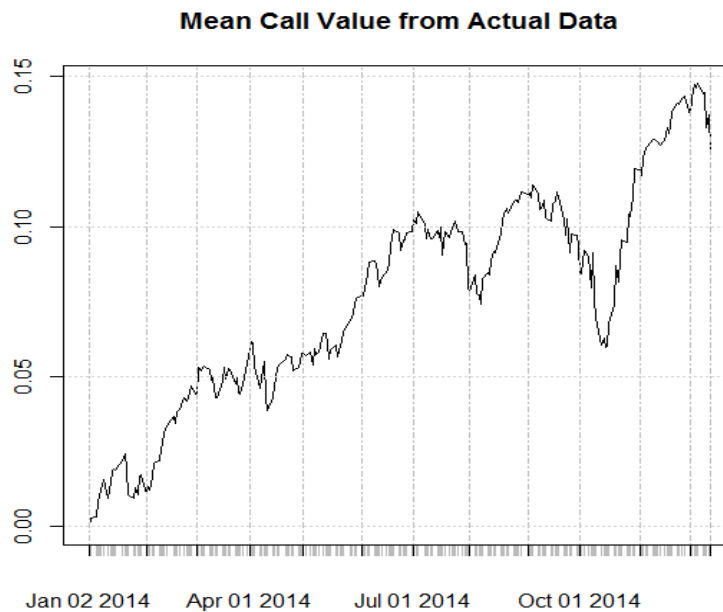


Figure 8

This plot shows what “correct” call price would have been on the day of 01/02/2014, for a randomly picked stock in SP500, with the strike price equal to spot price, for various times-to-expiration. For example, if you bought a call option on Jan 02, 2014 with strike of 100 of an underlying stock of spot price 100, and time-to-expiration of 1 year, then the average call option would have gained a log-value of 0.13, seen from the end of the plot above. This is $e^{(0.13)} = 1.139$. Therefore, the call option should have priced at 13.9% of the stock price, which is \$13.9. If the market brokers were somehow able to predict the future distribution of all stocks, then this would have been the “fair” price of the call option.

4. Comparisons and Conclusions

To compare the results received from the Black-Scholes method to the call value seen from the analysis created in R, both sets of values needed to be on the same scale. Thus, log returns were created for the Black-Scholes numbers through the formula $(\log(S+\text{Call})-\log(S))$. Once both sets of data were calculated as log returns, they could be compared by imposing them on the same graph, seen below.

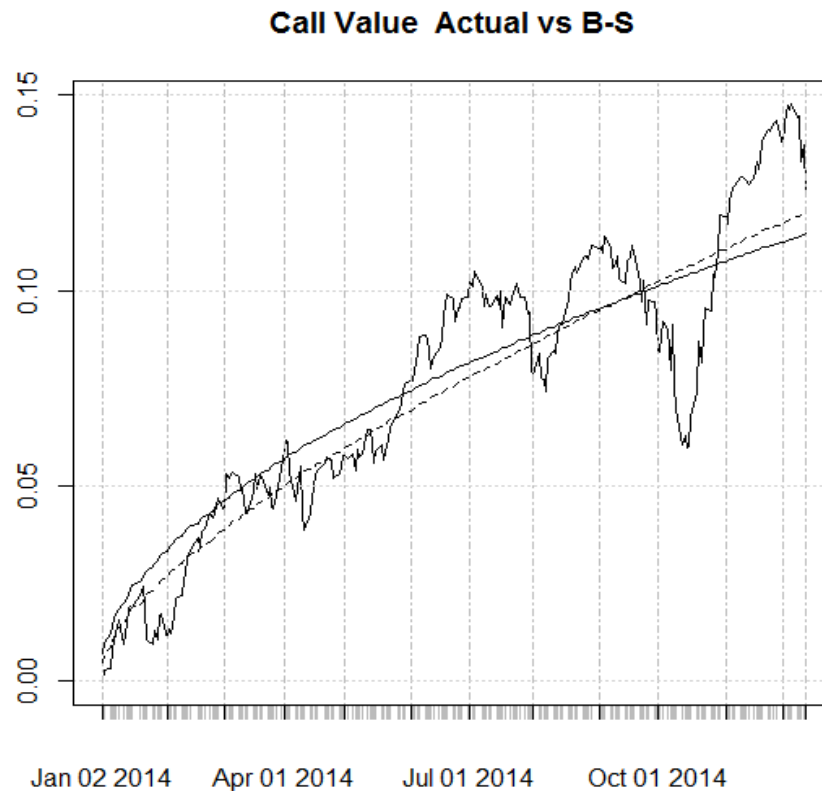


Figure 9

The bold, curved line is the option value obtained from B-S formula with $r=.03$ and $\sigma = 0.1847$. First thing to note is that even though we have used Black-Scholes formula with “peek-into-the-future” annual volatility value of .184, the formula significantly undervalues the option compared to what actually happened. If we were to guess the ball-park figure of the call option, we must set the annual volatility to $\sigma = 0.27$, which is shown in the solid line. However, we know that the volatility at the end of the year was .184, so this is a misrepresentation of the

market, just to match the call option price. The other attempt with the Black-Scholes model was using a risk free rate of 10%, an increase from the original 3% of the Treasury bond. This is shown by the dashed line. It remarkably resembles the call value line given by the actual market. This means that seen from Black-Scholes formula, market have moved as if the risk-free interest rate was 10%, with volatility of 18.4%.

Regardless of which curved line considered, the Black-Scholes method is not an accurate way of modeling the real data. While the lines follow the overall trend of an increase in option value over the 240 trading days, neither one predicts the changes in volatility at certain points in time. For example a significant drop in value between September and October can be observed in the S&P500 data, but neither model accounts for the decrease in price. The Black-Scholes method fails to reproduce drops at all in the data, a disadvantage when using this technique for analyzing stock data. Due to assuming a constant volatility in price over the time period, using Black-Scholes can lead to failing to identify large changes in the data.

Due to these differences between the Black-Scholes prices and those of the actual stocks, the conclusion can be made that the model is not too accurate in pricing call options. Alterations of the market assumptions, especially concerning constant volatility and known interest rates, should be made to compensate for some of the limitations of the Black-Scholes model. However, the Black-Scholes method can be of value for identifying an option's overall distribution over time and is a decent starting point for pricing stock options.

References

"BLACK - SCHOLES -- OPTION PRICING MODELS." *BLACK - SCHOLES -- OPTION*

PRICING MODELS. Bradley University, n.d. Web. 20 Apr. 2016.

Ray, Sarbapriya. "A Close Look into Black-Scholes Option Pricing Model."

Worldsciencepublisher.org. World Science Publisher, 2012. Web. 20 Mar. 2016.

Thomsett, Michael. "9 Serious Flaws In The Black Scholes Pricing Model." *Linkedin.com*. N.p.,

21 June 2015. Web. 20 Mar. 2016.

Winston, Rory. "The Research Kitchen." *The Research Kitchen*. N.p., 28 Apr. 2008. Web. 10

Apr. 2016.

R Code

```
rm(list=ls())

load("E:\\SP500_2000to2015.Rdata")

library(quantmod)

D.ad <- D.ad[,-66] #- remove Stock "BMC"

ls()

Tkr=1 #- Choose the 1st column

plot(D.ad[,Tkr], main="Stock Prices from Jan. 2000 to Dec. 2015", ylab="Stock Value")

plot(log(D.ad[,Tkr]), main="Log of Stock Prices", ylab="Log Value")

plot(diff(log(D.ad[,Tkr])), main= "Daily Log Returns From 2000-2015")

#Calculating log returns of different stocks

log_returns <- diff(log(D.ad),lag=1)

dim(log_returns)

log_returns[3500:3600, 1]

log_returns[3520:3530, 1]

log_returns[3522, 1]      #- So row number 3522 = 2014-01-02

#- 1. try to find what row number is 2015-12-31

log_returns[3925:4025, 1]

log_returns[4025, 1] # Row number 4025 = 2015-12-31
```

#- Find col number for stock = FB

```
which(colnames(D.ad)== "FB")
```

```
plot(log_returns[3522:3822, "FB"], main="FB Log Returns from Jan 2014 to Mar 2015")
```

#- Plot log-return for FB

```
plot(log_returns[3522:3822, 480], main="FB Log Returns from Jan 2014 to Mar 2015") #- This  
will do the same
```

```
plot(log(D.ad[3522:3822, 480]), main="FB Log Price from Jan 2014 to Mar 2015") #- Plot log-  
price for FB
```

```
plot(log(D.ad[3522:3822, 480])-log(as.numeric(D.ad[3522, 480]))+1, main="FB Log Price from  
Jan 2014 to Mar 2015, Starting at 1" ) #- Same plot, starting at 1
```

```
#-----
```

#Produce quarterly histogram

```
Y = matrix(0, 240, 1)
```

```
for(i in 1:480) {
```

```
  X = log(D.ad[3522:3761, i])
```

```
  X0 = log(as.numeric(D.ad[3522, i]))
```

```
  if (sum(is.na(X))==0 & min(X) > 3) #- plot only stock that has no NA, and price above 3
```

```
  {
```

```
    Y = cbind(Y,X-X0)
```

#- plot only stock that has no NA, and price above 3

```
plot(X-X0,ylim=c(-.7,.7), main="Log Returns of All stocks from Jan 2014 to Dec 2014",  
ylab="Log Return")  
  
}  
  
par(new=T)  
  
}
```

Y = Y[,-1] #- remove the first column of zeros

dim(Y)

M = xts(rowMeans(Y), order.by=index(Y[,1]))

lines(M, lwd=5, col="white")

lines(M, lwd=2, lty=2)

#-- Plotting Average Log Return over 2014 year -----

dev.new()

plot(M, main="Average Log Return")

#-- Determining Log Return Standard Deviation -----

S = xts(apply(Y,1,sd), order.by=index(Y[,1]))

dev.new()

plot(S, main="SD of Log Returns")

t=1:240

S[240] = 0.1847 - standard deviation at end of 240 days

lines(xts(.1847/sqrt(240)*sqrt(t), order.by=index(Y)), main="SD of Log Returns", col="red")

plot(Y[,12], main="Log Return Over 2014 year") **#- each column is just a log-return of each stock**

plot(Y[,13], main="Log Return Over 2014 year") **#- each column is just a log-return of each stock**

plot(Y[,14], main="Log Return Over 2014 year") **#- each column is just a log-return of each stock**

#-- Plotting Log Return Histograms over different intervals-----

par(mfrow = c(3,2))

hist(as.numeric(Y[1,]), xlim = c(-0.6,0.6), main="Jan 2014 (1Day)", xlab="Log Return")

hist(as.numeric(Y[60,]), xlim = c(-0.6,0.6), main="Mar 2014 (60 Days)", xlab="Log Return")

hist(as.numeric(Y[120,]), xlim = c(-0.6,0.6), main="May 2014 (120 Days)", xlab="Log Return")

hist(as.numeric(Y[180,]), xlim = c(-0.6,0.6), main="July 2014 (180 Days)", xlab="Log Return")

hist(as.numeric(Y[240,]), xlim = c(-0.6,0.6), main="Sep 2014 (240 Days)", xlab="Log Return")

par(mfrow=c(1,1))

```
#-- Calculating Call Price Over 2014 Year -----
Call.price=0
for (day in 1:240) {
  #M[day] = mean(as.numeric(Y[day,]))
  Call.price[day] = mean((as.numeric(Y[day,])>0)*as.numeric(Y[day,]))
  plot(Call.price, main="Call Price of Total Stocks over Time Period: Jan 2014 - Dec 2014")
}
Call.price=xts(Call.price, order.by=index(Y))
```

```
dev.new()
plot(Call.price, type="l", main="Mean Call Value from Actual Data")
```

```
mean(Call.price)
```

```
[1] 0.07629075
```

```
#-- Calculating Black-Scholes option values -----
```

```
BS <- function(S,K,r,si,T,del) {
  d1=(log(S/K) + (r-del+si^2/2)*T) / (si*sqrt(T))
  d2=(log(S/K) + (r-del-si^2/2)*T) / (si*sqrt(T))
  pnorm(d1)
  pnorm(d2)
  Call = S*exp(-del*T)*pnorm(d1) - K*exp(-r*T)*pnorm(d2)
  return(Call)
}
```

S = 100

K1 = 90

K2 = 95

K3 = 100

K4 = 105

K5 = 110

r = .03

si = .1847

T = (1:240)/240

del= .00

#####Calculating Black-Scholes values based on different strike prices#####

C1 <- BS(S,K1,r,si,T,del)

BS.c1 <- xts(log(S+C1)-log(S), order.by=index(Y))

mean(C1)

C2 <- BS(S,K2,r,si,T,del)

BS.c2 <- xts(log(S+C2)-log(S), order.by=index(Y))

mean(C2)

C3 <- BS(S,K3,r,si,T,del)

BS.c3 <- xts(log(S+C3)-log(S), order.by=index(Y))

mean(C3)

```
C4 <- BS(S,K4,r,si,T,del)
```

```
BS.c4 <- xts(log(S+C4)-log(S), order.by=index(Y))
```

```
mean(C4)
```

```
C5 <- BS(S,K5,r,si,T,del)
```

```
BS.c5 <- xts(log(S+C5)-log(S), order.by=index(Y))
```

```
mean(C5)
```

```
#####Comparing B-S to Actual Call Option#####
```

```
dev.new()
```

```
plot(Call.price, type="l", main="Call Value Actual vs B-S")
```

```
Kcall = 100
```

```
si=.27
```

```
Call <- BS(S,Kcall,r,si,T,del)
```

```
BS.call <- xts(log(S+Call)-log(S), order.by=index(Y))
```

```
lines(BS.call, lwd=1)
```

```
Call2 <- BS(S,Kcall,.1,.1845,T,del)
```

```
BS.call2 <- xts(log(S+Call2)-log(S), order.by=index(Y))
```

```
lines(BS.call2, lwd=1, lty=2)
```

```
##-- Black-Scholes Plot with Strike Price of K = 100----##
```

```
r1 = 0.03
```

```
si1 = 0.1847
```

```
CallBS <- BS(S,Kcall,r,si1,T,del)
```

```
plot(CallBS,main="Call Value by B-S formula", xlab="Days")
```