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Statistical Arbitrage with Momentum Using Machine Learning

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Abstract

In this paper machine learning is used to investigate statistical arbitrage in China stock market. We use HS300 index constituent stocks to construct pairs trading. The daily and monthly momentums in these stocks are used as new input factors to forecast the stock price. We develop a trading approach to find that random forest (RF) outperform deep neural net (DNN), XGBoost, support vector machine(SVM) and LSTM from January 2013 to August 2017.

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1. Introduction

It is a very difficult task to predict price change of stocks. Statistical arbitrage is based on the mean reversion principle. If the price fluctuation process of an asset is a stable time series, when the price of the asset fluctuates in a short time, its price will return to the mean state in the next period of time due to the effect of the equilibrium mechanism. Statistical arbitrage is to find a pair of stocks with high convergence in the stock market. The price fluctuations of the two stocks have strong similarities. When the price of one stock rises, the price of the other stock rises at the same time. In one certain period, the prices of the two stocks may deviate to a certain extent due to the random factors of their respective companies.

The literature on statistical arbitrage mostly uses the traditional mathematical modeling methods such as time series and stochastic control to find the pairing combination and solve the optimal transaction signal. It is expected that machine learning algorithms can extract information from data more effectively. Huck (2009) developed a

statistical arbitrage strategy using the integration of neural network and a multi criteria decision-making. His method consists of three steps: prediction, ranking and trading. Huck (2010) further improved this method through multistep predictions. Takeuchi and Lee (2013) developed an enhanced momentum strategy for the CRSP stock market from 1965 to 2009. Moritz and Zimmermann (2014) tested the statistical arbitrage strategy based on random forest for the CRSP stock market data from 1968 to 2012, and found that the average monthly risk adjusted excess return was 2%. When the feature data set includes 86 features of companies, the return increases to 2.28% monthly. Krauss et al. (2017) applied deep neural network, gradient enhancement tree and random forest to S&P 500 index from 1992 to 2015. Using revenue based characteristics, they found that the combination of the above methods can produce 0.45% revenue per day (before transaction costs). Fischer and Krauss (2018)use the LSTM network for the same prediction task. Huck (2019) show that these technical indicators have the ability to generate trading signals for portfolios with significant reversal effect and short holding period (one to five days).

Moreover, The momentum effect means that assets that have performed well in the past will often perform better in the future. The momentum effect was discovered by Jegadeesh and Titman (1993,2001). Rouwenhorst (1998) conducted an empirical study on the stocks of 2000 listed companies in twelve European countries and found that momentum effect exists significantly in the European market. Schiereck et al. (1999) concluded that the momentum strategy in the German market is better in the medium and long term. Chui's (2000)show that momentum strategies can outperform the market in Asian stock markets. Pedro et al. (2015) show that momentum strategies can achieve an excess return of more than 10% in several major global stock markets.

Therefore, this paper selects HS300constituent stocks to investigate statistical arbitrage using machine learning algorithms. We take the momentum factor as the input feature to explore whether machine learning can make full use of the momentum factor of stock price to successfully predict the rise and fall of stocks. The key task of the employed machine learning methods is to accurately predict whether a stock outperforms HS300 index as a benchmark.

The remainder of this paper is organized as follows. Section 2 briefly covers the data sample, software packages, and our methodology, i.e., the generation of training and trading sets, the construction of input sequences, the model architecture and training as well as the forecasting and trading steps. Section 3 presents the results and discusses our most relevant findings. Finally, Section 4concludes.

2. Data and Methodology

2.1. Data and features

We take 84 stocks from HS 300 Index in Shanghai and Shenzhen Exchange markets. The data is from Wind data over a 9-year period, from January 2, 2011 to December 31, 2019. This paper uses the sliding window method to generate the training set and trading set.

The construction of input features mainly refers to the processing method (Takeuchi and Lee, 2013). First, extract the daily returns of the stocks in the past 21 days, and then extract the monthly returns of the stocks in the following 12 months. Next, use the daily returns of the past 21 days to calculate the cumulative returns, and use the monthly returns of the following 12 months to calculate the 12 monthly cumulative returns. After calculating all the daily momentum factors and monthly momentum factors, it is necessary to calculate the quantile of each momentum factor in all 84 stocks. The quantile as each momentum factor is used as the input feature. In this way, a total of 33 characteristic variables are constructed.

Let $P^S = (P^S_t)_{t \in T}$ represent the price of stock s ,where $s \in \{1, ...n\}$. Then the return is defined as

$$R_{t,m}^{S} = \frac{P_{t}^{S}}{P_{t-m}^{S}} - 1$$
 (1)

For the daily returns, define the range of period $m \in \{1, 2, 3...19, 20, 21\}$, the range of period $m \in \{42, ...252, 273\}$ for the monthly returns. A binary variable

$$Y_{s\mid t+l} \in \{0,1\} \tag{2}$$

is constructed for a stock s to represent the rise and fall trend. If the return of stock s exceeds the median return of all stocks, the binary variable is $Y_{c|r+1} = 1$ (Category 1), otherwise $Y_{c|r+1} = 0$ (category 0).

In order to scientifically and reasonably evaluate the real return level of statistical arbitrage strategy, we use some classical performance evaluation indicators, including annualized return, return volatility, Sharp ratio, Sortino ratio, etc. On each trading day t+1, the probability that the return of stock S exceeds the median return of all

stocks at t is $p_{t+1|t}^S$. We can find the undervalued stocks at the top of the ranking of the probability, the

overvalued stocks at the bottom of the ranking. Thus we buy the stocks with the highest rising probability and sell the stocks with the highest falling probability.

2.2. Methodology

According to the empirical research of Moritz and Zimmermann (2014), Krauss et al. (2017), Fischer and Krauss (2018), Huck (2019), etc., the random forest algorithm has a good performance in financial sequence prediction. The random forest algorithm is not a single machine learning algorithm, but an integrated algorithm based on decision tree model. Its estimator is a decision tree. The performance of each decision tree in classification function determines the effect of random forest classification and prediction. In the process of decision tree growth, the selection of features follows the principle of minimum information purity.

SVM is based on statistical learning theory, with extremely strict theoretical basis, based on the minimum principle of VC dimensional theory and structural risk, and introduces the nuclear function, allowing its algorithm to map high-dimensional space, but avoid complex the calculation and effectively overcomes the problem of disaster. Since these more significant advantages, it is also applied in many fields and has achieved good results. Although SVM theory and algorithm have had a large extent development and progress through such problems, on some issues, such as training speed, nuclear function, calculation storage capacity, etc. Because of these advantages, SVM can be well applied to pattern recognition, probability density function estimation, time series prediction, regression estimation, etc.

Gradient boosting is one of the most powerful technologies for building prediction models. It is a representative algorithm of boosting in integrated algorithms. The integration algorithm constructs multiple weak evaluators on the data and summarizes the modeling results of all weak evaluators to obtain better regression or classification performance than a single model. The weak evaluator is defined as performing at least better than random guess is a better model, that is, any model with a prediction accuracy of no less than 50%. There are many ways to integrate different weak evaluators. For example, the bagging method of establishing multiple parallel independent weak evaluators at one time. There are also methods like the lifting method, which build weak evaluators one by one and gradually accumulate multiple weak evaluators after many iterations. The most famous lifting algorithms include AdaBoost and gradient lifting tree. XGBoost is developed from gradient lifting tree. Unlike traditional GBDT, the traditional GBDT only uses the first order countdown information when optimizing, while XGBoost performs the two order Taylor expansion for the loss function.

Deep neural network is composed of input layer, one or more hidden layers and output layers. The dimension of input layer and input feature is equal. The output layer is a classification or regression layer to match the output space. All layers are composed of neurons, the basic unit of this model. In the classical feed architecture, each neuron is fully connected with all neurons in the previous layer, and each neuron represents a certain weight. Moreover, the input layer and hidden layer of the neural network have bias units, which are used as the activation threshold of neurons in the subsequence layer.

RNN is a cyclic network structure with the ability to maintain information. The cyclic network module in RNN transmits information from the upper layer of the network to the lower layer. The output of the hidden layer of the network module at each time depends on the information of the previous time. The chain attribute of RNN shows that it is closely related to sequence annotation. In the training of RNN, there are problems of gradient explosion and

disappearance, and RNN is difficult to keep memory for a long time. LSTM network is an extension of RNN and is specially designed to avoid long-term dependency problems. The repetitive neural network module of LSTM has different structures, which is different from the naive RNN. There are four neural network layers that interact in a special way.

3. Performance analysis

The research method of this paper is mainly divided into four steps. Firstly, the data set in a research cycle is divided into two parts, the training set and the trading set. The training set is used to train the machine learning model, and the trading set is used to verify the prediction effect of the model. The second step is to generate the input characteristics and output characteristics. The third step is to train the random forest model on the training set and determine the optimal parameter setting of the random forest model. The fourth step is to use the random forest to predict on the trading set, rank the stocks according to the prediction results, and long the stocks with the highest rising probability and short the stocks with the highest falling probability.

3.1. Performance for different algorithms

During the trading period, the cumulative returns of the five algorithms are shown in Figure 1. We see the trends, and find that the cumulative profit of the random forest algorithm is much higher than that of the other four algorithms. The performance of LSTM algorithm is close to that of SVM algorithm. During the whole trading period, the trend of its cumulative income is better than that of the HS300 index. XGBoost algorithm performs worse than the HS300 index in the early stage, performs better in the late trading period, and finally outperforms the HS300 index. During the whole empirical period, the cumulative return of DNN algorithm was poor, and finally failed to exceed the HS300 index. In addition, in order to explore the profitability of the machine learning algorithm, the proportion of the trading days with the returns greater than 0 before transaction cost and the proportion of the after the transaction cost are respectively counted, as shown in Table 1.

The proportion of RAF, SVM and XGBoost is higher than that of the HS300 index, while the ratio of DNN algorithm to LSTM algorithm is slightly lower than that of the HS300 index. After transaction costs, only RAF and XGBoost algorithms have a higher proportion than the HS300 index, while the other three algorithms have a lower proportion than HS300 Index. Through the above analysis, it can be found that most machine learning has the ability to predict China's stock market. In addition, RAF algorithm performs better than the other four algorithms.

According to the accuracy performance of the random forest model in the training set, the number of estimators is 39, the Gini coefficient used in the branching standard, the maximum depth of the random forest is 20, the maximum feature is 20, the minimum segmented leaf node is 30, and the minimum number of leaf nodes is 25. Since the random forest model adopted this time uses 39 estimators, and each estimator has more branches and a large width.



Figure 1. The cumulative returns of different algorithms (before transaction costs)

3.2. Profitability over time

we display strategy performance over time from January 2013 to December 2019. The transaction cost is 1.5 ‰. The evaluation indicators of the profitability in the three stages are shown in Table 1. From January 2013 to May 2015, the profitability of the statistical strategy is better than that of the HS300index in the same period with a daily average return of 0.0036, while a daily average return of the HS300index is 0.0012. In addition, the annual return of the statistical arbitrage strategy is 1.2438, while the annual return of the HS300index is 0.3143. The alpha value of the strategy is 1.4862. In terms of risk, the annualized volatility of the statistical arbitrage strategy is 1.2483, while the annualized volatility of the HS300index is 0.2216. The Sharp ratio of the statistical arbitrage strategy 2.1295, while the Sharp ratio of the HS300index in the same period is 1.3451. According to the above analysis, although the volatility of the strategy is higher than that of the HS300index, the statistical arbitrage strategy without transaction cost is much better than that of the HS300index, After transaction costs, the fluctuation relationship between return and risk of the statistical arbitrage strategy based on momentum factor and random forest is roughly equivalent to that of the HS300index.

From May 2015 to August 2017, the profitability of the statistical arbitrage is better than that of the HS300index, regardless of whether the transaction cost is deducted, with a daily average return of 0.0031, a daily average return of 0.0016 after transaction costs, and a daily average return of the HS300index is -0.0002 in the same period. In addition, during this period, the annualized return of statistical arbitrage strategy is 0.9331, while the annualized return after transaction cost is 0.3242, while the annualized return of the HS300index is -0.0851. Compared with the HS300index, the alpha value of statistical arbitrage strategy is 1.1764, and the alpha value after transaction costs is 0.4909. In terms of risk, the annualized volatility of the statistical arbitrage strategy is 0.4902, the annualized volatility after transaction costs is 0.4895, and the annualized volatility of the HS300index is 0.2818. The Sharp ratio of statistical arbitrage strategy is 1.5893, the Sharp ratio after transaction costS is 1.5893, while the Sharp ratio of the HS300index is -0.1729. Considering the risk and return at the same time, not only the statistical arbitrage strategy without transaction costs is much better than the HS300 index, but also after transaction costs is still much better than the HS300index.

From August 2017 to December 2019, although the cumulative rate of return of the statistical arbitrage strategy eventually exceeds the HS300 index before transaction costs, the statistical arbitrage strategy fluctuates greatly and is inferior to the HS300 index in. The average daily return of the statistical arbitrage strategy is 0.0009, the average daily return after transaction costs is -0.0006, while the average daily return of the HS300 index in the same period is 0.0001. In addition, during this period, the annualized return of the statistical arbitrage strategy is 0.1448, while the annualized return after transaction costs is -0.2158, while the annualized return of the HS300index is 0.0184. Compared with the HS300index, the alpha value of the statistical arbitrage strategy is 0.2401, and the alpha value after transaction cost is -0.1505. In terms of risk, the annualized volatility of the statistical arbitrage strategy is 0.4133, the annualized volatility after transaction costs is 0.4895, and the annualized volatility of the HS300index in the same period is 0.4127. The Sharp ratio of the statistical arbitrage strategy based on momentum factors and random forest is 0.5337, the Sharp ratio after transaction cost is -0.3823, while the Sharp ratio of the HS300 index in the same period is -0.1908. The fluctuation relationship between return and risk of the statistical arbitrage strategy after transaction costs is far worse than the HS300index. Therefore, it can be found that the statistical arbitrage strategy can more accurately predict the rise and fall of the market from August 2017 to December 2019, but the prediction ability is not enough to gain from the market.

Through the above empirical analysis of the statistical arbitrage strategy based on momentum factor and random forest, the following conclusions can be summarized,: first, the statistical arbitrage strategy can obtain a return far exceeding the HS300index from January 2013 to August 2017, and from August 2017 to December 2019, The statistical arbitrage strategy can more accurately predict the rise and fall of the market, but the prediction ability is not enough to make profits from the market. The return after transaction costs is worse than that of HS300index. Second, the profit of statistical arbitrage strategy continues to decline with the extension of time, which may be due to the optimal parameters selected by the parameters of random forest in the first research cycle.

3.3. Robustness test

A common method to test the robustness of machine learning algorithm is to change the accuracy of the algorithm and observe whether the experimental results change when the dimension of parameter setting changes, so as to test whether the model is stable. This paper changes the classification accuracy of the model by changing the number of estimators in the random forest, and observes the change trend of cumulative return and Sharp ratio with the number of estimators. As shown in Table 1, when the estimators of the statistical arbitrage policy based on momentum factor and random forest is 37-41, the daily average return, annualized return, annualized volatility, alpha value, beta value, Sharp ratio, sortino ratio and calmar ratio of the strategy are at a very stable level. When the number of estimators is more than 41 or less than 37, the daily average return, annualized return, annualized volatility, alpha value, beta value, Sharp ratio, Sortino ratio and calmar ratio of the strategy begin to fluctuate greatly. Therefore, it can be found that when the number of estimators is 37 to 41, the random forest model constructed in this paper is in a stable state.

Table 1 Performance with different stages

Time	13/01-15/05	15/05- 17/08	17/08- 19/12	13/01- 15/05	15/05- 17/08	17/08- 19/12	13/01- 15/05	15/05- 17/08	17/08- 19/12
	Before transaction cost			After transaction cost			The HS300index		
Mean return	0.0036	0.0031	0.0009	0.0021	0.0016	-0.0006	0.0012	-0.0002	0.0001
Max	0.1238	0.1846	0.1075	0.1222	0.1829	0.1059	0.0461	0.0671	0.0595
Quartile 1	0.0187	0.0157	0.0145	0.0171	0.0141	0.0130	0.0080	0.0063	0.0068
Median	0.0049	0.0033	0.0009	0.0033	0.0018	-0.0006	0.0005	0.0008	0.0001
Quartile 3	-0.0104	-0.0098	-0.0135	-0.0119	-0.0113	-0.0150	-0.0061	-0.0049	-0.0064
Min	-0.1206	-0.1356	-0.0928	-0.1219	-0.1369	-0.0941	-0.0770	-0.0875	-0.0584
Standard deviation	0.0266	0.0309	0.0260	0.0266	0.0308	0.0260	0.0140	0.0178	0.0124
Skewness Kurtosis	-0.5212 2.8906	0.3569 5.7621	-0.0143 1.5208	-0.5212 2.8906	0.3569 5.7621	-0.0143 1.5208	-0.2538 3.0785	-1.0446 5.8067	-0.0563 2.7703
Annualized return	1.2438	0.9331	0.1448	0.5371	0.3242	-0.2158	0.3143	-0.0851	0.0184
Annualized volatility	0.4222	0.4902	0.4133	0.4216	0.4895	0.4127	0.2216	0.2818	0.1971
Cumulative returns	5.0248	3.3262	0.3505	1.5993	0.8665	-0.4173	0.8357	-0.1793	0.0413
Alpha	1.4862	1.1764	0.2401	0.7032	0.4909	-0.1505	0.0000	0.0000	0.0000
Beta	-0.0447	-0.0057	0.1399	-0.0447	-0.0057	0.1397	1.0000	1.0000	1.0000
Sharpe ratio	2.1295	1.5893	0.5337	1.2328	0.8171	-0.3823	1.3451	-0.1729	0.1908
Downside risk	0.2885	0.3185	0.2852	0.2988	0.3285	0.2971	0.1472	0.2195	0.1387
Sortino ratio	3.1167	2.4461	0.7734	1.7395	1.2177	-0.5309	2.0240	-0.2220	0.2713
Maximum drawdown	-0.2594	-0.4588	-0.4797	-0.2856	-0.4730	-0.5796	-0.2482	-0.4670	-0.3246
Calmar ratio	4.7951	2.0338	0.3018	1.8805	0.6855	-0.3723	1.2667	-0.1822	0.0566
Omega ratio	1.4437	1.3600	1.0955	1.2382	1.1718	0.9367	1.2673	0.9648	1.0339

Table 2 Robustness test

Estimators	36	37	38	39	40	41	42
Mean return	0.0018	0.0023	0.0025	0.0025	0.0023	0.0024	0.0021
Maximum	0.1846	0.2001	0.1846	0.1846	0.1846	0.1846	0.1846
Quartile 1	0.0155	0.0160	0.0155	0.0162	0.0157	0.0157	0.0158
Median	0.0029	0.0032	0.0032	0.0031	0.0033	0.0031	0.0028
Quartile 3	-0.0110	-0.0112	-0.0108	-0.0112	-0.0113	-0.0112	-0.0113
Minimum	-0.1411	-0.1356	-0.1356	-0.1356	-0.1356	-0.1356	-0.1356
Standard dev.	0.0290	0.0280	0.0274	0.0279	0.0282	0.0283	0.0283
Skewness	-0.1670	0.0528	0.0010	0.0156	-0.0345	0.1036	-0.0831

Kurtosis	4.4126	4.6122	4.1974	4.0709	4.0915	4.1059	3.9580
Annualized return	0.4011	0.6324	0.7078	0.7060	0.6229	0.6731	0.5254
Annualized volatility	0.4609	0.4441	0.4347	0.4434	0.4475	0.4489	0.4488
Alpha	0.5507	0.7977	0.8704	0.8784	0.7871	0.8409	0.6808
Beta	0.0527	0.0193	0.0341	0.0180	0.0374	0.0495	0.0384
Sharpe ratio	0.9636	1.3263	1.4496	1.4275	1.3069	1.3717	1.1664
Sortino ratio	1.3770	1.9570	2.1527	2.1255	1.9219	2.0520	1.6994
Max. drawdown	-0.5344	-0.4985	-0.4420	-0.4797	-0.4681	-0.4282	-0.4935
Calmar ratio	0.7507	1.2686	1.6013	1.4717	1.3306	1.5719	1.0647
Omega ratio	1.1927	1.2697	1.2973	1.2922	1.2649	1.2809	1.2326
Downside risk	0.3226	0.3010	0.2928	0.2978	0.3043	0.3001	0.3081

4. Conclusion

This paper constructs the input features from the perspective of momentum, and explores the profitability of the statistical arbitrage strategy based on momentum factors and random forest in China Stock Markets. Specifically, when we predict the rise and fall of a stock after one day, we first need to calculate 33 indicators of a stock's daily cumulative return and monthly cumulative return. After calculating all daily momentum factors and monthly momentum factors, we need to further calculate the quantile of each momentum factor in all 84 stocks. The quantile corresponding to each momentum factor is used as the input feature of the random forest to predict the rise and fall of the stock.

This paper finds that the statistical arbitrage strategy based on momentum factors and random forest can obtain a return far beyond the HS300index from January 2013 to August 2017. From August 2017 to December 2019, the statistical arbitrage strategy can more accurately predict the rise and fall of the market, but the prediction ability is not enough to make profits. In general, the statistical arbitrage strategy can obtain benefits beyond the market during the trading period. However, during the whole empirical period, this profitability continues to decline with the extension of trading time, but this decline in profitability is likely due to the change of the data set of the corresponding training set with the change of training cycle, The rule of information in the data set may change accordingly, and the random forest model is the best parameter selected according to the first training cycle. In addition, the empirical study of this paper shows that the statistical arbitrage strategy is better than the traditional momentum and reversal strategy. It covers the momentum information of the market in the past period, and contains more information about the market, so it has stronger profitability.

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