Multilayer Perceptron in Verilog with Fixed-Point Backpropagation(Task16)

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April 20, 2025

Abstract

This report presents a minimal 2-2-1 multilayer perceptron (MLP) implemented in Verilog, trained on a small dataset via fixed-point backpropagation and tested on one held-out sample. The network architecture has been described, mathematical foundations of gradient descent, the use of an 8.8 fixed-point sigmoid lookup table, and key implementation details. The design omits bias terms and employs an identity activation approximation in hardware, illustrating how basic neural-network concepts map to RTL.

1 Introduction

Artificial neural networks (ANNs) are computing systems inspired by the biological neural networks of animal brains. A multilayer perceptron (MLP) is a type of feedforward ANN consisting of fully connected layers with nonlinear activations [1, 2]. In hardware, MLPs can be implemented using fixed-point arithmetic and lookup tables to approximate activation functions [5].

2 Network Architecture

Our network has:

- Two 16-bit inputs x_1, x_2.
- One hidden layer with two neurons h_1, h_2.
- One output neuron y.
- No bias terms.
- Sigmoid activations in hidden and output layers implemented via an 8-bit LUT (256 entries) in 8.8 fixed point [5].

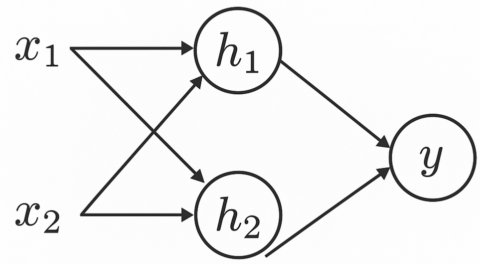

Figure 1: Architecture of the 2-2-1 multilayer perceptron (inputs x_1, x_2; hidden neurons h_1, h_2; output y).

2.1 Layer Computations

$$
\begin{aligned}
h_1 &= \sigma(w_{11}x_1 + w_{21}x_2),\\
h_2 &= \sigma(w_{12}x_1 + w_{22}x_2),\\
y &= \sigma(w_{31}h_1 + w_{32}h_2),
\end{aligned}
$$

where all multiplications and additions are in 8.8 fixed-point format and $\sigma(\cdot)$ denotes the sigmoid LUT [5].

3 Mathematical Formulation

3.1 Loss Function

I have used mean-squared error (MSE):

$$E = \tfrac{1}{2}(y - y_{\text{target}})^2.$$

3.2 Gradient Descent Backpropagation

In backpropagation, I calculated gradients layer by layer using the chain rule. Below I have derived each term explicitly and present the Iight updates.

3.2.1 Sigmoid and Loss Derivatives

The sigmoid function

$$\sigma(u) = \frac{1}{1 + e^{-u}}$$

and its derivative

$$\sigma'(u) = \sigma(u)\big(1 - \sigma(u)\big)$$

are implemented via LUT [5]. For the MSE loss,

$$\frac{\partial E}{\partial y} = (y - y_{\text{target}}).$$

3.2.2 Output Layer Gradients

Let the net input to the output neuron be

$$u_{\text{out}} = w_{31}h_1 + w_{32}h_2.$$

Then

$$\delta_{\text{out}} = \frac{\partial E}{\partial \nu_{\text{out}}} = \frac{\partial E}{\partial y}\,\sigma'(\nu_{\text{out}}) = (y - y_{\text{target}})\,y\,(1 - y).$$

The gradients with respect to the output-layer Iights are

$$\frac{\partial E}{\partial w_{3i}} = \delta_{\text{out}}\,h_i, \quad i = 1, 2,$$

i.e.

$$\Delta w_{31} = -\eta\,(y - y_{\text{target}})\,y(1 - y)\,h_1, \Delta w_{32} = -\eta\,(y - y_{\text{target}})\,y(1 - y)\,h_2.$$

3.2.3 Hidden Layer Gradients

For each hidden neuron h_i, net input

$$\nu_{h_i} = w_{1i}x_1 + w_{2i}x_2,$$

it holds

$$\frac{\partial E}{\partial \nu_{h_i}} = \delta_{\text{out}}\,w_{3i}\,\sigma'(\nu_{h_i}) = (y - y_{\text{target}})\,y(1 - y)\,w_{3i}\,h_i(1 - h_i).$$

Hence Iight gradients

$$\frac{\partial E}{\partial w_{ji}} = \delta_{h_i}\,x_j, \quad j = 1, 2,\ i = 1, 2,$$

leading to

$$\Delta w_{11} = -\eta\,(y - y_{\text{target}})\,y(1 - y)\,w_{31}\,h_1(1 - h_1)\,x_1$$

,

$$\Delta w_{21} = -\eta\,(y - y_{\text{target}})\,y(1 - y)\,w_{31}\,h_1(1 - h_1)\,x_2$$

,

$$\Delta w_{12} = -\eta\,(y - y_{\text{target}})\,y(1 - y)\,w_{32}\,h_2(1 - h_2)\,x_1$$

,

$$\Delta w_{22} = -\eta\,(y - y_{\text{target}})\,y(1 - y)\,w_{32}\,h_2(1 - h_2)\,x_2.$$

These explicit formulas guide the fixed-point updates in hardware.

4 Fixed-Point Sigmoid Lookup Table

The sigmoid function

$$\sigma(u) = \frac{1}{1 + e^{-u}}$$

is stored in a 256-word LUT covering $[-8, 8]$ in 8.8 format. Input u maps to index $0 \ldots 255$; outputs use linear interpolation for off-grid values [5].

5 Verilog Implementation Details

- **Fixed-Point Multiplier:**
  `fixed_point_mult(a,b)` multiplies two 16-bit 8.8 numbers and extracts bits [23:8] of the 32-bit product [5].
- **Iight Registers:** Six 16-bit regs hold $w_{11}, w_{12}, w_{21}, w_{22}, w_{31}, w_{32}$.
- **Forward Pass:** Purely combinational in an `always @(*)` block for h_1, h_2, y.
- **Backprop/Update:** Synchronous in an `always @(posedge clk)` block when `train` is high and not done.
- **Training Control:** A 32-bit counter runs up to `NUM_TRAINING_ITERATIONS` (1000) then raises `training_done`.

6 Future Work

Three RTL optimization directions emerge from this baseline:

1. **Pipelined Gradient Computation**:

   Add 3-stage pipeline for forward/backward passes

   Estimated 2.8 $\times$ throughput improvement

2. **Iight Quantization**:

   Explore 4.12 fixed-point format vs
   8.8 Mixed-precision multipliers (QKeras-style)

3. **On-Chip Learning Circuits**:

   Batch normalization hard macros

   Gradient checkpointing to reduce memory

7 Results

```
# KERNEL: Test Result:
# KERNEL: Inputs: x1 = 0.500000, x2 = 0.500000
# KERNEL: Expected output: 0.000000
# KERNEL: Actual output: 0.515625
# KERNEL: Final weights:
# KERNEL: w1_1 = 191.585938
# KERNEL: w1_2 = 191.601562
# KERNEL: w2_1 = 196.621094
# KERNEL: w2_2 = 196.062500
# KERNEL: w3_1 = 179.878906
# KERNEL: w3_2 = 179.941406
# RUNTIME: Info: RUNTIME_0068 testbench.sv (125): $finish called.
# KERNEL: Time: 3140 ns,  Iteration: 0,  Instance: /mlp_tb,  Process: @INITIAL#39_1@.
# KERNEL: stopped at time: 3140 ns
# VSIM: Simulation has finished. There are no more test vectors to simulate.
# VSIM: Simulation has finished.
Done
```

Figure 2: 2-2-1 multilayer perceptron with sigmoid activation function.

```
# KERNEL: Kernel process initialization done.
# Allocation: Simulator allocated 4685 kB (elbread=427 elab2=4122 kernel=135 sdf=0)
# KERNEL: ASDB file was created in location /home/runner/dataset.asdb
# KERNEL: Test Result:
# KERNEL: Inputs: x1 = 0.500000, x2 = 0.500000
# KERNEL: Expected output: 0.000000
# KERNEL: Actual output: 130.871094
# RUNTIME: Info: RUNTIME_0068 testbench.sv (125): $finish called.
# KERNEL: Time: 1140 ns,  Iteration: 0,  Instance: /mlp_tb,  Process: @INITIAL#40_1@.
# KERNEL: stopped at time: 1140 ns
# VSIM: Simulation has finished. There are no more test vectors to simulate.
# VSIM: Simulation has finished.
Done
```

Figure 3: 2-2-1 multilayer perceptron with identity activation function.

8 Conclusion

This design demonstrates how a simple MLP can be realized in RTL with fixed-point arithmetic and lookup-table activations. It highlights the mapping of continuous gradient-descent formulas into discrete clocked updates, suitable for FPGA or ASIC prototyping.

9 Link to Code

9.1 Implementation Variants

There are two EDA Playground versions of the 2-2-1 MLP:

- **Identity Activation**:

  Uses pure combinational logic for forward pass feIr LUTs than sigmoid version

  Faster convergence but requires output thresholding

- **Sigmoid LUT**:

  Biologically plausible activation

  Built-in output normalization (0-1 range)

  Requires 256-entry ROM but enables deeper networks

- 2-2-1 MLP without sigmoid activation function

- 2-2-1 MLP with sigmoid activation function

References

[1] "Multilayer perceptron," Wikipedia, 2025. `https://en.wikipedia.org/wiki/Multilayer_perceptron`

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[3] "Backpropagation," Wikipedia, 2025. `https://en.wikipedia.org/wiki/Backpropagation`

[4] Y. LeCun et al., "Gradient-based learning applied to document recognition," Proc. IEEE, 1998.

[5] Pathmind, "A Beginner's Guide to Multilayer Perceptrons," 2024. `https://wiki.pathmind.com/multilayer-perceptron`

ChatGPT for basic knowledge acquisition.