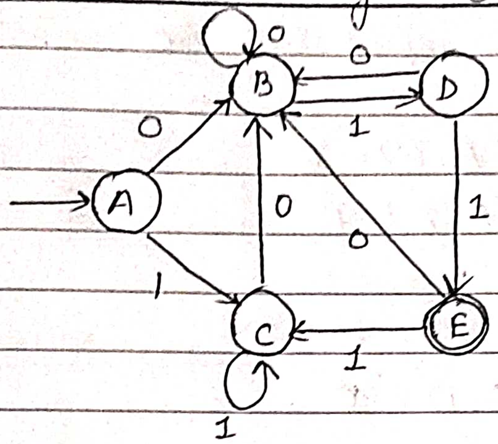
Minimization of DFA

(Table Filling Method – Myhill Nerode Theorem)

Let us understand by an example :-

Example



Step-1 First draw the table of Q×Q size here it's 5×5. let say (P, Q), here P belongs to set of final states and Q does not belong to F

	A	B	C	D	E
A					
B					
C					
D					
E					

Solution Mark all pairs where P ∈ F and Q ∉ F

	A	B	C	D	E
A					
B	✓				
C		✓			
D	✓	✓	✓		
E	✓	✓	✓	✓	

check for BA if any state is final
similarly check for all CA, CB, DA, DB, DC
Now when come EA, EB, EC, ED all consist of final states 'E' so mark ✓

Step-3 If there are any unmarked pairs (P, Q) such that $[\delta(P, x), \delta(Q, x)]$ is marked, then mark $[P, Q]$ where x is an input symbol

Repeat until no more markings can be done :-

Now the unmarked pairs here are, BA, CA, CB, DA, DB, DC
Find transition function each input state i.e.

Iteration 1

$(B, A) - \delta(B, 1) = D \qquad \delta(B, 0) = B$
$\quad - \delta(A, 1) = C \qquad \delta(A, 0) = B$

$(C, A) - \delta(C, 0) = B \qquad \delta(C, 1) = C$
$\quad \delta(A, 0) = B \qquad \delta(A, 1) = C$

$(C, B) - \delta(C, 0) = B \qquad \delta(C, 1) = C$
$\quad \delta(B, 0) = B \qquad \delta(B, 1)\ D$

$(D, A) - \delta(D, 0) = B \qquad \left.\begin{array}{l}\delta(D, 1) = \underline{E} \\ \delta(A, 1) = \underline{C}\end{array}\right\}$ mark DA
$\quad \delta(A, 0) = B$

$(D, B) - \delta(D, 0) = B \qquad \left.\begin{array}{l}\delta(D, 1) = \underline{E} \\ \delta(B, 1) = \underline{D}\end{array}\right\}$ mark DB
$\quad \delta(B, 0) = B$

$(D, C) = \delta(D, 0) = B \qquad \left.\begin{array}{l}\delta(D, 1) = \underline{E} \\ \delta(C, 1) = \underline{C}\end{array}\right\}$ mark DC
$\quad \delta(C, 0) = B$

Iteration 2

Check for unmarked pairs in iteration 1 i.e BA, CA, CB
again find transition function for each input.

$(B, A) - \delta(B, 0) = B \qquad \left.\begin{array}{l}\delta(B, 1) = \underline{D} \\ \delta(A, 1) = \underline{C}\end{array}\right\}$ since CD is marked in iteration 1 mark DA
$\quad \delta(A, 0) = B$

(C,A) — $\delta(C,0) = B$ $\quad \delta(C,1) = C$
$\delta(A,0) = B$ $\quad \delta(A,1) = C$

(C,B) — $\delta(C,0) = B$ $\quad \delta(C,1) = \underline{C}$
$\delta(B,0) = B$ $\quad \delta(B,1) = \underline{D}$ } mark as DC is already marked in iteration 1.

Iteration 3 Check for last i.e CA.

$(C,A) = \delta(C,0) = B$ $\quad \delta(C,1) = C$ * can not be marked as
$\delta(A,0) = B$ $\quad \delta(A,1) = C$ CC, BB is no pair marked.

Construction of DFA :-

All states marked are seperate state but A,C will be combined as it is unmarked state.

{A, C}, {B}, {D}, {E}