



Eliminating ϵ - Transitions

NFA with ϵ or ϵ -NFA can be converted to NFA or NFA without ϵ or null move.

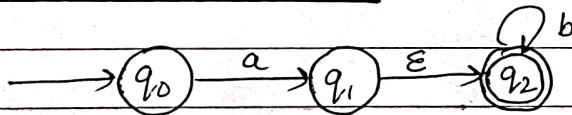
1. Epsilon closure :-

- * For each state q in the NFA, find the set of all states reachable from q by following zero or more epsilon transitions.
- * This set is called the epsilon closure of q , denoted as ϵ -closure (q).

2. Remove ϵ transition :-

- * Create a NFA without epsilon transition
- * The states of the new NFA will be sets of states from the original NFA, representing the epsilon closure of each state.
- * For each state S in the new NFA and each input symbol a :
 - Find the set of states (T) that can be reached from any state in S by following a transition on a .
 - Add a transition from S to ϵ -closure (T) on symbol a in the new NFA.

Example Convert ϵ -NFA to NFA



First find ϵ -closure for q_0, q_1, q_2

Solution

$$\epsilon\text{-closure}(q_0) = \{q_0\}$$

$$\epsilon\text{-closure}(q_1) = \{q_1, q_2\}$$

$$\epsilon\text{-closure}(q_2) = \{q_2\}$$



Now find δ' transition for each input :-

$$\begin{aligned}\delta'(q_0, a) &= \epsilon\text{-closure}(\delta(\delta^+(q_0, \epsilon), a)) \\ &= \epsilon\text{-closure}(\delta(\epsilon\text{-closure}(q_0, a))) \\ &= \epsilon\text{-closure}(\delta(q_0, a)) \\ &= \epsilon\text{-closure}(q_1) \\ &= \{q_1, q_2\}\end{aligned}$$

$$\begin{aligned}\delta'(q_0, b) &= \epsilon\text{-closure}(\delta(\epsilon\text{-closure}(q_0, b))) \\ &= \epsilon\text{-closure}(\delta(q_0, b)) \\ &= \emptyset\end{aligned}$$

$$\begin{aligned}\delta'(q_1, a) &= \epsilon\text{-closure}(\delta(\epsilon\text{-closure}(q_1, a))) \\ &= \epsilon\text{-closure}(\delta(q_1, q_2), a) \\ &= \epsilon\text{-closure}(\delta(q_1, a) \cup \delta(q_2, a)) \\ &= \epsilon\text{-closure}(\emptyset \cup \emptyset) \\ &= \emptyset\end{aligned}$$

$$\begin{aligned}\delta'(q_1, b) &= \epsilon\text{-closure}(\delta(\epsilon\text{-closure}(q_1, b))) \\ &= \epsilon\text{-closure}(\delta(q_1, q_2), b) \\ &= \epsilon\text{-closure}(\delta(q_1, b) \cup \delta(q_2, b)) \\ &= \epsilon\text{-closure}(\emptyset \cup q_2) \\ &= q_2\end{aligned}$$

$$\begin{aligned}\delta'(q_2, a) &= \epsilon\text{-closure}(\delta(\epsilon\text{-closure}(q_2, a))) \\ &= \epsilon\text{-closure}(\delta(q_2, a)) \\ &= \emptyset\end{aligned}$$

$$\begin{aligned}\delta'(q_2, b) &= \epsilon\text{-closure}(\delta(\epsilon\text{-closure}(q_2, b))) \\ &= \epsilon\text{-closure}(\delta(q_2, b)) \\ &= q_2\end{aligned}$$



δ' transitions :-

$$\delta'(q_0, a) = q_0, q_1$$

$$\delta'(q_0, b) = \phi$$

$$\delta'(q_1, a) = \phi$$

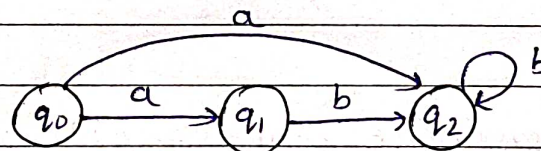
$$\delta'(q_1, b) = q_2$$

$$\delta'(q_2, a) = \phi$$

$$\delta'(q_2, b) = q_2$$

Transition Table :-

States	a	b
$\rightarrow q_0$	$\{q_1, q_2\}$	ϕ
* q_1	ϕ	$\{q_2\}$
* q_2	ϕ	$\{q_2\}$



* State q_1 and q_2 becomes the final state as ϵ -closure of q_1 and q_2 contain the final state q_2 . The NFA can be shown by the following transition diagram shown above.