Programming Assignment **3**

Name: Jesus David Gutierrez Moreno

Methods: Broyden-Fletcher-Goldfarb-Shanno (BFGS), Low-memory BFGS (L-BFGS), Newton-CG.

Problem 1	Final f(x) value	N. of Iterations	CPU time Seconds	Termination Criteria	Modifications relevant?	Convergence rate (start, middle, end)
BFGS	2.75e-17	35	1.65e-02	Convergence		Linear, then superlinear
L-BFGS	6.55e-26	41	2.65e-02	Convergence	Yes, although more iterations were required, it achieved a lower f(x) value	Linear, then superlinear
Newton-CG	2.20e-22	23	1.33e-02	Convergence	Significantly faster than BFGS variants	Linear, then superlinear
Problem 2						
BFGS	1.12e-17	47	1.45e-02	Convergence		Linear, superlinear, linear
L-BFGS	2.43e-16	54	1.65e-02	Convergence	Similar performance	Linear, superlinear, linear
Newton-CG	6.97e-17	45	1.09-02	Convergence	Yes better performance in terms of convergence	Linear, superlinear, linear
Problem 3						
BFGS	3.99	87	2.523	Convergence	Got stuck on a worse local minima	Superlinear, sublinear
L-BFGS	3.99	86	3.36e-02	Convergence	No, although **significantly** faster, got stuck on a worse local minima	Superlinear, sublinear
Newton-CG	6.06e-17	23	7.91e-03	Convergence	Yes, faster and found global minima	Linear
Problem 4						
BFGS	9.04e-17	210	1.61e-01	Convergence		Sublinear, superlinear
L-BFGS	2.49e-16	74	4.02e-02	Convergence	Significantly faster	Linear
Newton-CG	1.39e-16	172	1.09e-01	Convergence		Sublinear, quadratic
Problem 5						
BFGS	2.55e-16	440	2.66e-01	Max iterations		Sublinear, superlinear
L-BFGS	3.91e-15	543	1.78e-01	Convergence	Faster	Sublinear, superlinear
Newton-CG	1.62e-16	20	1.30e-02	Max Iterations		linear
Problem 6						
BFGS	8.73e+02	1206	50	CPU time exceeded	Failed to minimize	Linear, sublinear
L-BFGS	3.99	5020	2.375e+01	CPU time exceeded	No, still failed to minimize	Linear, superlinear
Newton-CG	1.80e-16	19	5.615e-01	Convergence	Yes, Faster and achieved local minima	Linear, sublinear, linear
Problem 7						
BFGS	2.36e+05	4	6.441e+01	CPU time exceeded	Failed to minimize	sublinear
L-BFGS	9.85e+03	245	5.018e+01	CPU time exceeded	Failed to minimize	Sublinear
Newton-CG	5.56e-17	18	4.176e+01	Convergence	Achieved local minima	linear
Problem 8						
BFGS	6.09e-18	16	8.791e-03	Convergence		Superlinear
L-BFGS	1.11e-17	20	4.502e-03	Convergence		Superlinear
Newton-CG	4.26e-23	9	5.727e-03	Convergence	Similar performance, however less iterations and closer to local minima	Superlinear
Problem 9						
BFGS	1.46e-16	17	4.33e-03	Convergence		Linear, superlinear
L-BFGS	1.26e-17	20	3.608e-03	Convergence	Faster	Linear, sublinear, linear
Newton-CG	1.32e-18	8	1.682e-03	Convergence		superlinear

Problem 10						
BFGS	5.63e-09	72	1.934e-02	Convergence		Linear, sublinear, linear
L-BFGS	8.43e-10	55	1.403e-02	Convergence	Faster	linear
Newton-CG	2.77e-08	18	4.454e-03	Convergence		linear

It can be observed that all Quasi-Newton methods have much better performance than the steepest descent method, as expected. Although the Newton method offers quadratic convergence close to the local minima, by using quasi-newton methods, we gain speed in terms of computation per iteration. Newton-CG seems to perform best for this set of problems and manages to solve all the problems, especially 6 and 7, which were not possible for the Newton method alone. Adding CG search solves the plain Newton step, achieving a better performance than the modified Newton, as expected. This can be observed just by looking at the number of iterations and computation time.