Algorithms for Non-Linear Programming

This repository contains implementations of various algorithms for solving non-linear programming problems.

Getting Started

Install Python 3.10

```
sudo apt-get install python3.10-full
```

Install Venv Package

```
sudo apt-get install python3-venv
```

Create a virtual environment

```
python3.10 -m venv env
```

Activate the virtual environment

```
source env/bin/activate
```

Install the required packages

```
pip install -r requirements.txt
```

How to Use

Run the specific problem script, for example:

```
python problem1.py
```

or run the main script to execute all problems:

```
python run_all_problems.py
```

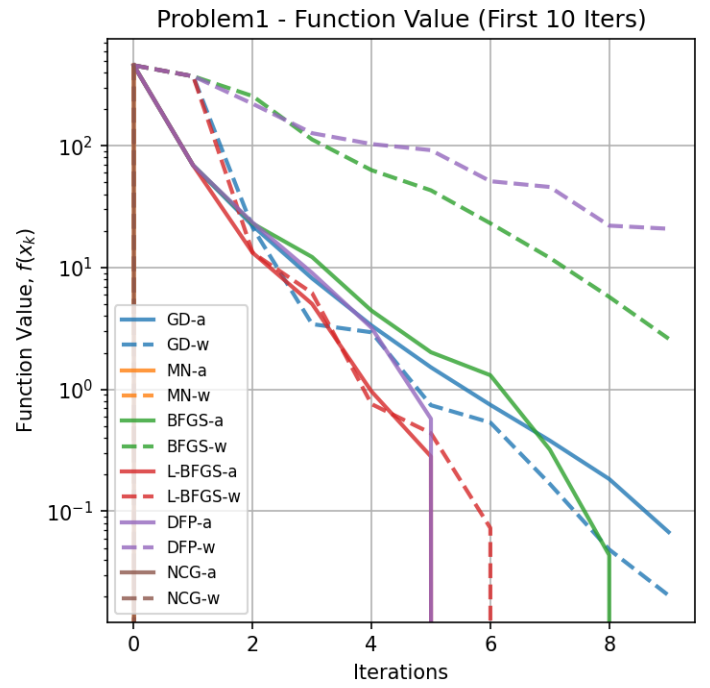To convert the markdown file to a PDF, use the following command:
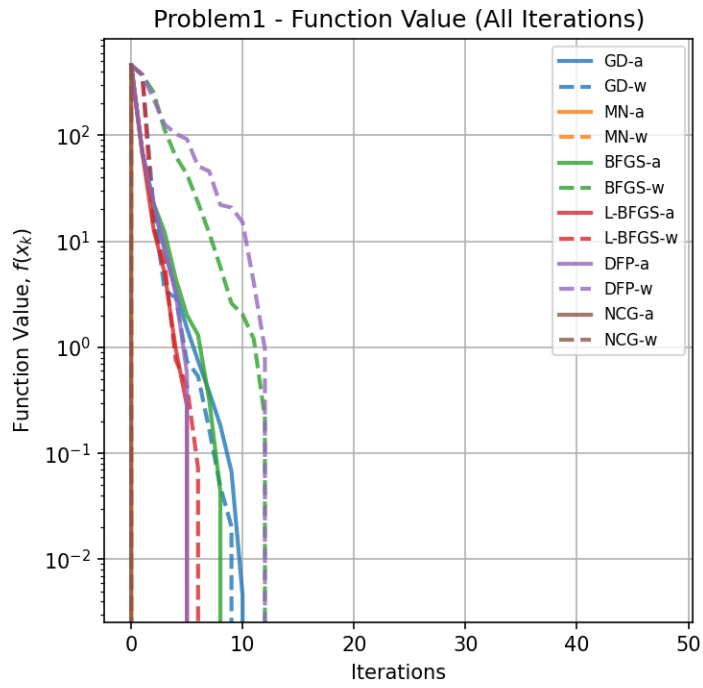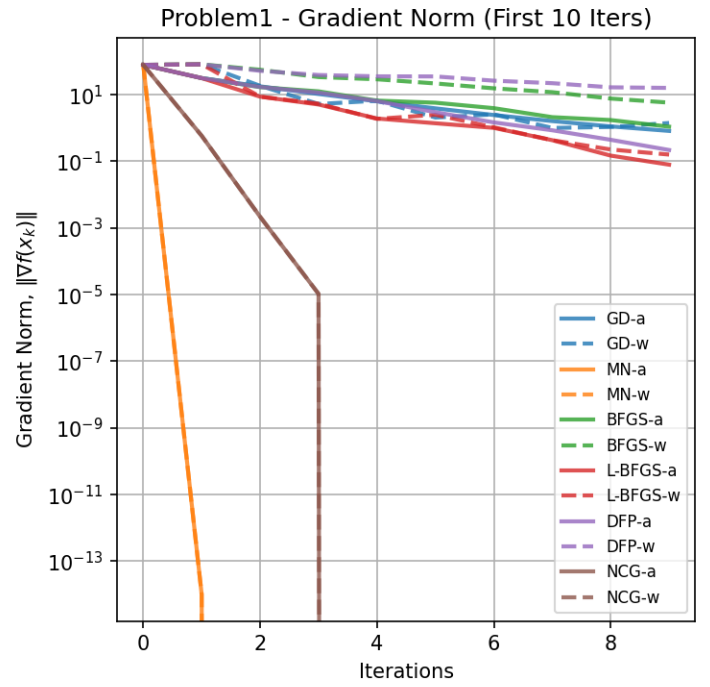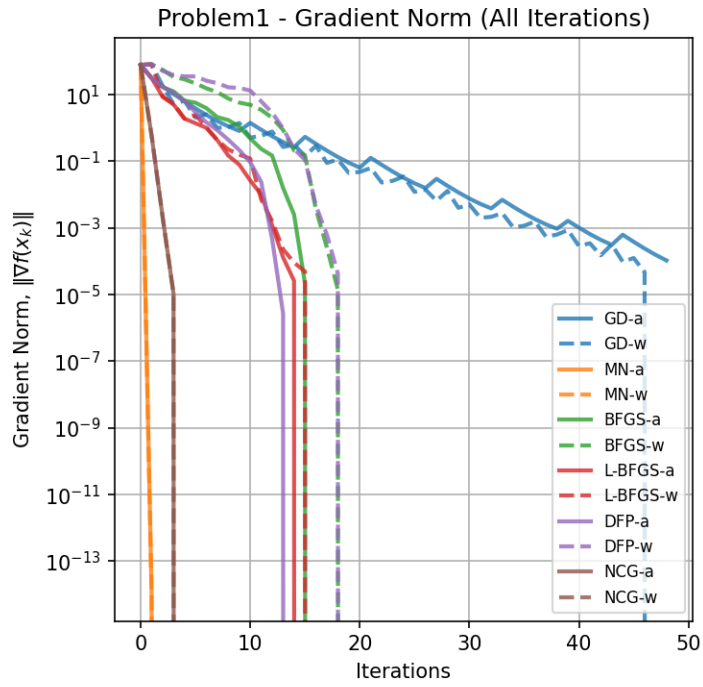
```
pandoc README.md -o README.pdf -V geometry:margin=0.5in
```

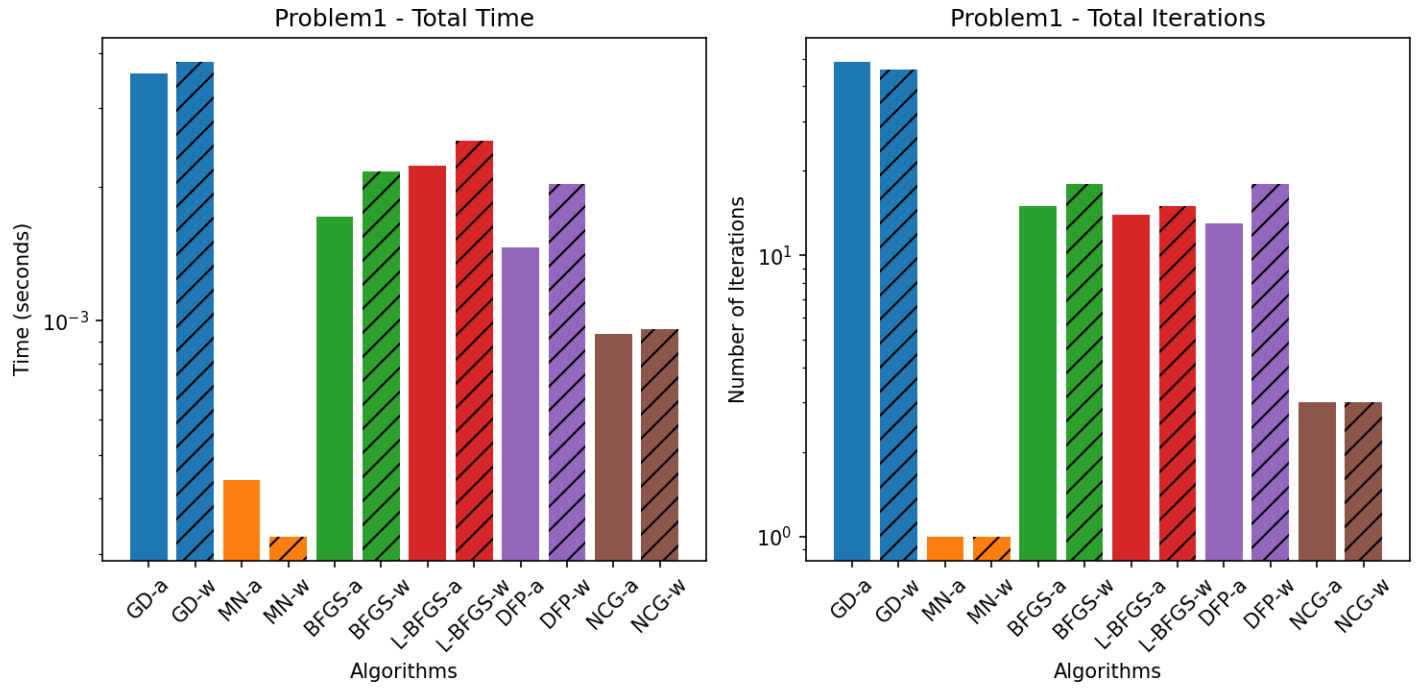## Results

Plots for each problem will be saved in the `figures` directory. Data for each problem will be saved in the `data` directory.
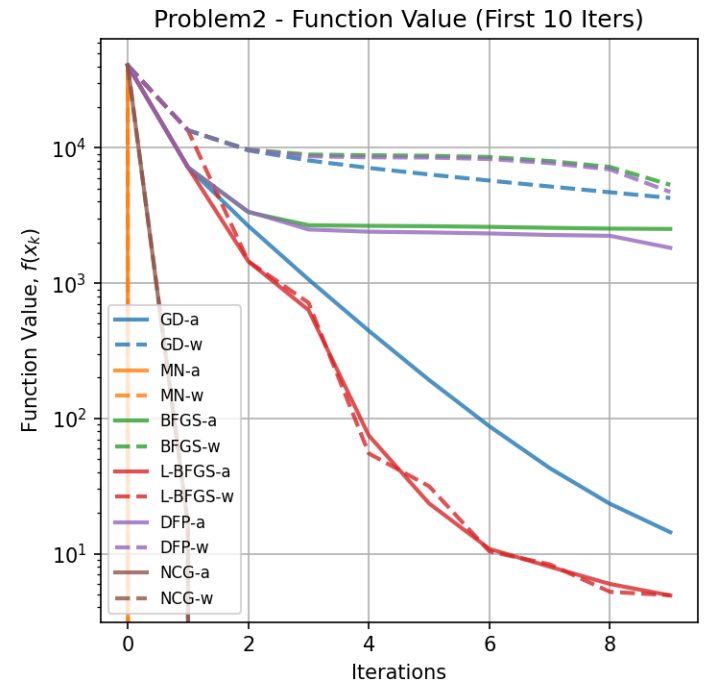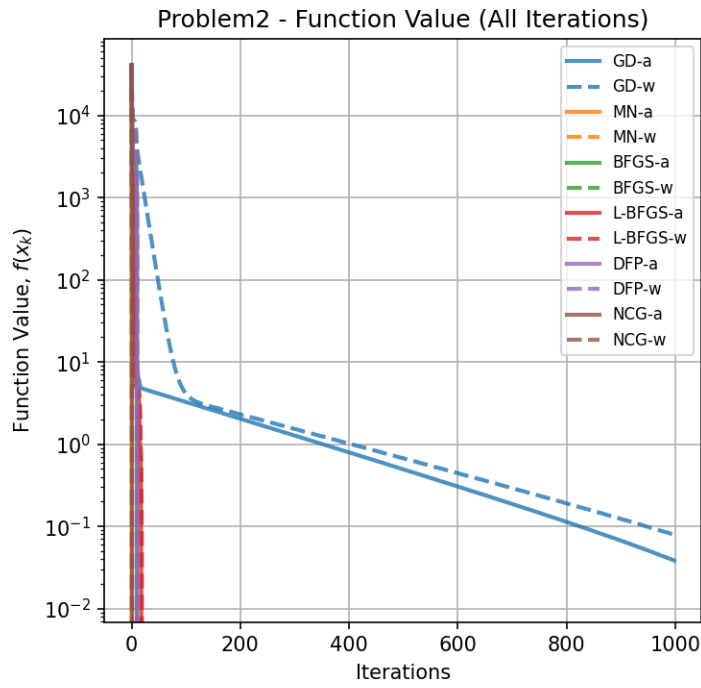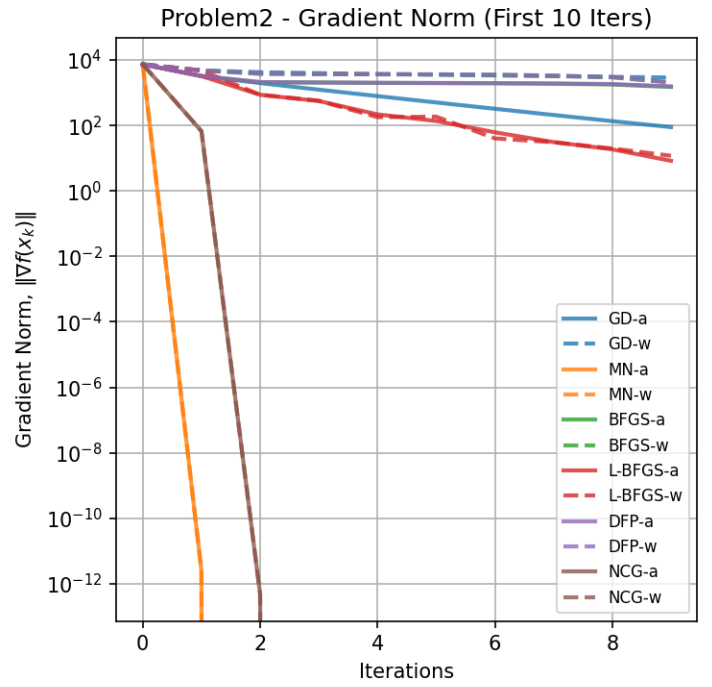
Problem 1

	Iters	Time	Convergence	Func Evals	Grad Evals	Hess Evals
GD-a	49	0.00627708	Converged. Gradient norm is below tolerance.	190	99	0
GD-w	46	0.00667191	Converged. Gradient norm is below tolerance.	202	47	0
MN-a	1	0.000771761	Converged. Gradient norm is below tolerance.	3	4	2
MN-w	1	0.000727892	Converged. Gradient norm is below tolerance.	3	3	2
BFGS-a	15	0.00304484	Converged. Gradient norm is below tolerance.	44	47	0
BFGS-w	18	0.00391817	Converged. Gradient norm is below tolerance.	49	38	0
L-BFGS-a	14	0.00389934	Converged. Gradient norm is below tolerance.	32	43	0
L-BFGS-w	15	0.00442886	Converged. Gradient norm is below tolerance.	33	31	0
DFP-a	13	0.00260139	Converged. Gradient norm is below tolerance.	40	41	0
DFP-w	18	0.00356841	Converged. Gradient norm is below tolerance.	41	38	0
NCG-a	3	0.00164366	Converged. Gradient norm is below tolerance.	7	8	4
NCG-w	3	0.00169659	Converged. Gradient norm is below tolerance.	7	5	4

Problem1 - Gradient Norm (All Iterations)

Problem1 - Gradient Norm (First 10 Iters)

Problem1 - Function Value (All Iterations)
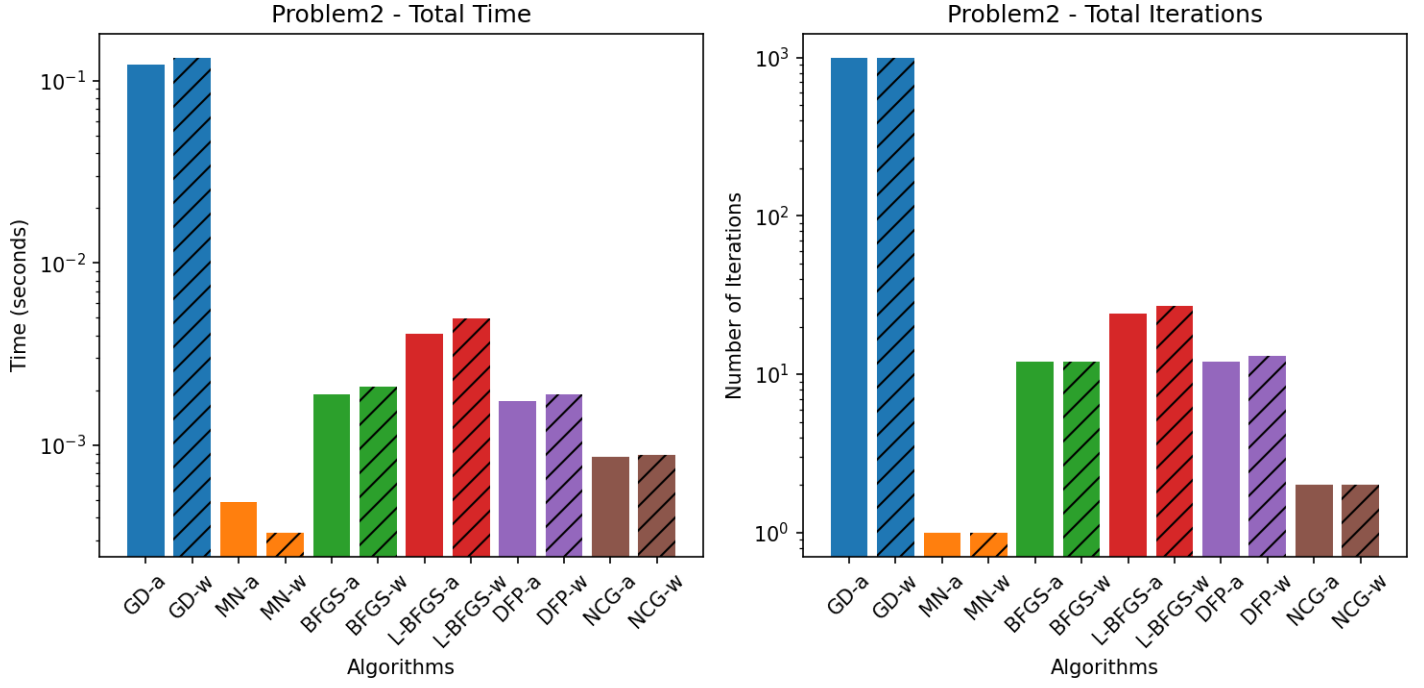
Problem1 - Function Value (First 10 Iters)

2

Problem1 - Total Time / Problem1 - Total Iterations

Problem 2

	Iters	Time	Convergence	Func Evals	Grad Evals	Hess Evals
GD-a	999	0.225948	Failed. Maximum iterations reached.	8722	2000	0
GD-w	999	0.238851	Failed. Maximum iterations reached.	10965	1000	0
MN-a	1	0.000850439	Converged. Gradient norm is below tolerance.	3	4	2
MN-w	1	0.000647545	Converged. Gradient norm is below tolerance.	3	3	2
BFGS-a	12	0.00360751	Converged. Gradient norm is below tolerance.	81	38	0
BFGS-w	12	0.00393152	Converged. Gradient norm is below tolerance.	100	26	0
L-BFGS-a	24	0.0086751	Converged. Gradient norm is below tolerance.	56	73	0
L-BFGS-w	27	0.00937366	Converged. Gradient norm is below tolerance.	67	55	0
DFP-a	12	0.00339293	Converged. Gradient norm is below tolerance.	71	38	0
DFP-w	13	0.00374603	Converged. Gradient norm is below tolerance.	81	28	0
NCG-a	2	0.00157499	Converged. Gradient norm is below tolerance.	5	6	3
NCG-w	2	0.00161791	Converged. Gradient norm is below tolerance.	5	4	3

Problem2 - Gradient Norm (All Iterations)

Problem2 - Gradient Norm (First 10 Iters)

Problem2 - Function Value (All Iterations)

Problem2 - Function Value (First 10 Iters)

4
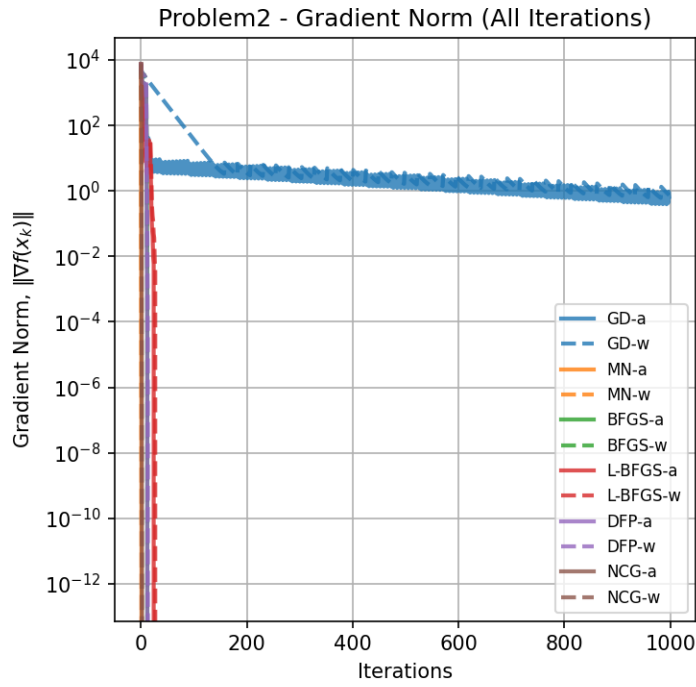
Problem 3

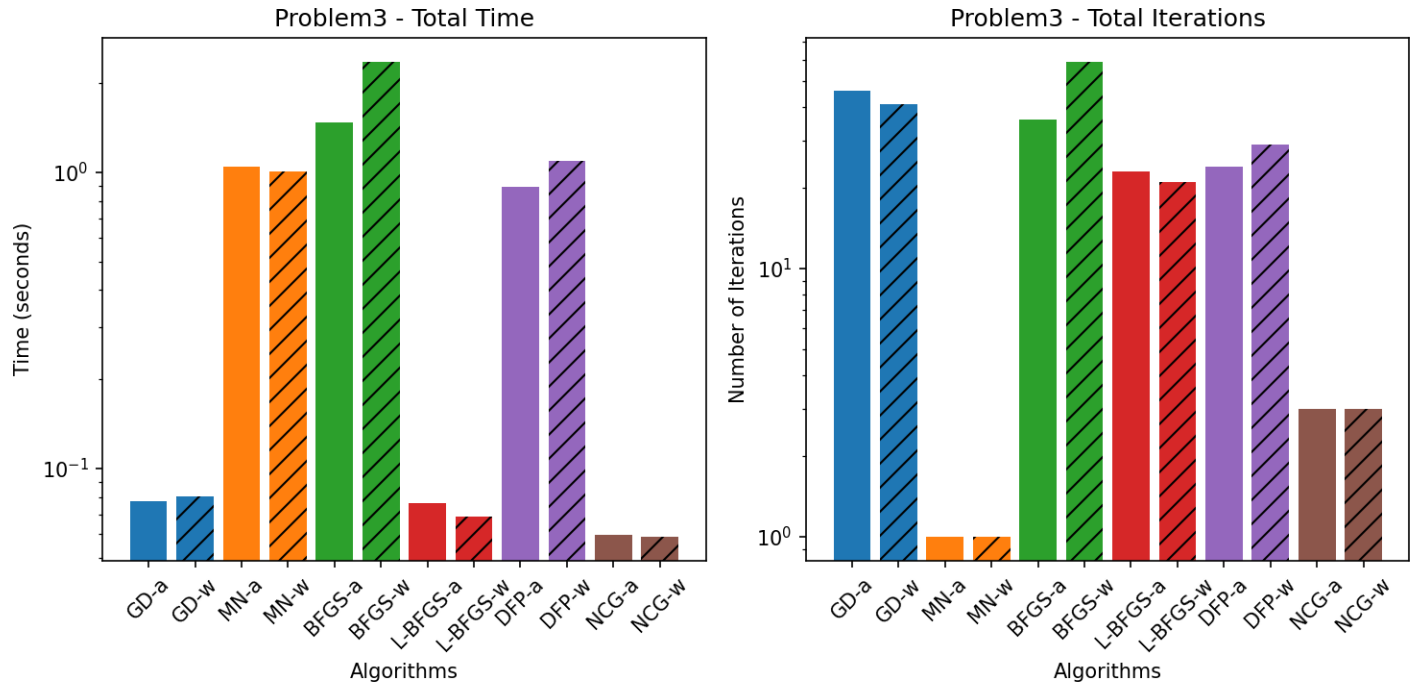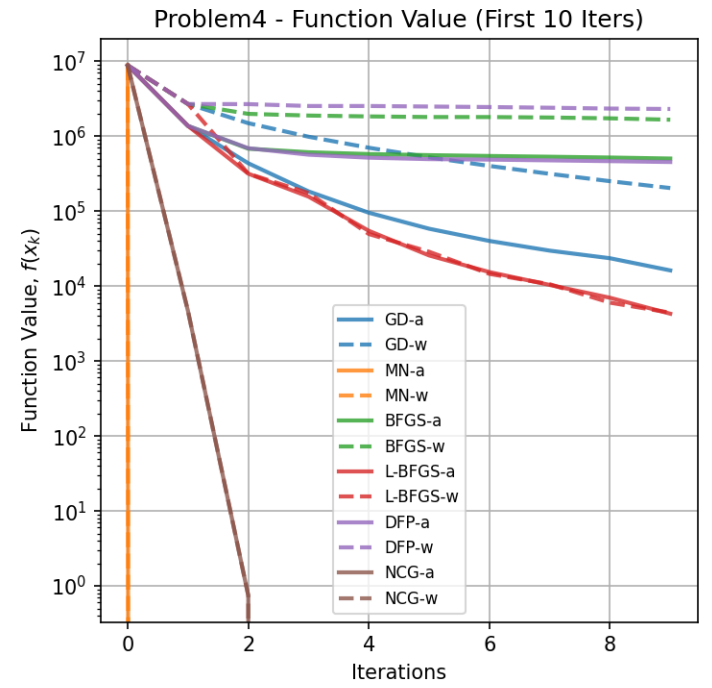|	Iters	Time	Convergence	Func Evals	Grad Evals	Hess Evals
GD-a	46	0.149904	Converged. Gradient norm is below tolerance.	178	93	0
GD-w	41	0.159244	Converged. Gradient norm is below tolerance.	180	42	0
MN-a	1	2.87684	Converged. Gradient norm is below tolerance.	3	4	2
MN-w	1	2.54326	Converged. Gradient norm is below tolerance.	3	3	2
BFGS-a	36	2.36012	Converged. Gradient norm is below tolerance.	111	110	0
BFGS-w	59	4.97915	Converged. Gradient norm is below tolerance.	179	120	0
L-BFGS-a	23	0.164352	Converged. Gradient norm is below tolerance.	51	70	0
L-BFGS-w	21	0.137408	Converged. Gradient norm is below tolerance.	46	43	0
DFP-a	24	1.62924	Converged. Gradient norm is below tolerance.	74	74	0
DFP-w	29	1.9033	Converged. Gradient norm is below tolerance.	88	60	0
NCG-a	3	0.284889	Converged. Gradient norm is below tolerance.	7	8	4
NCG-w	3	0.129295	Converged. Gradient norm is below tolerance.	7	5	4

Problem3 - Gradient Norm (All Iterations)

Problem3 - Gradient Norm (First 10 Iters)

Problem3 - Function Value (All Iterations)

Problem3 - Function Value (First 10 Iters)

Problem3 - Total Time

Problem3 - Total Iterations
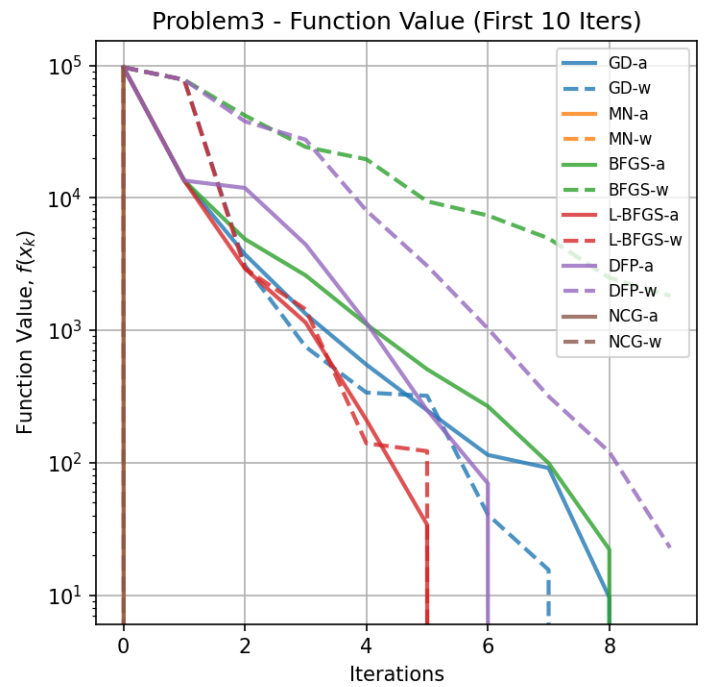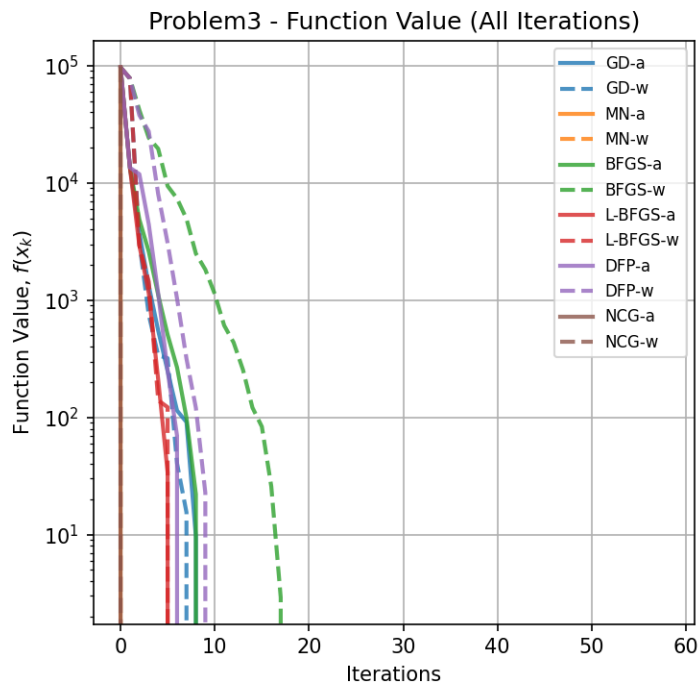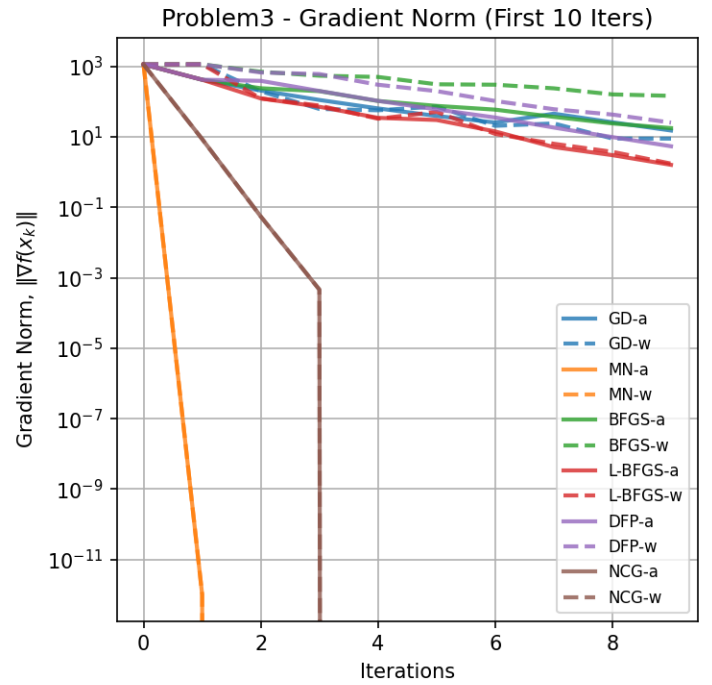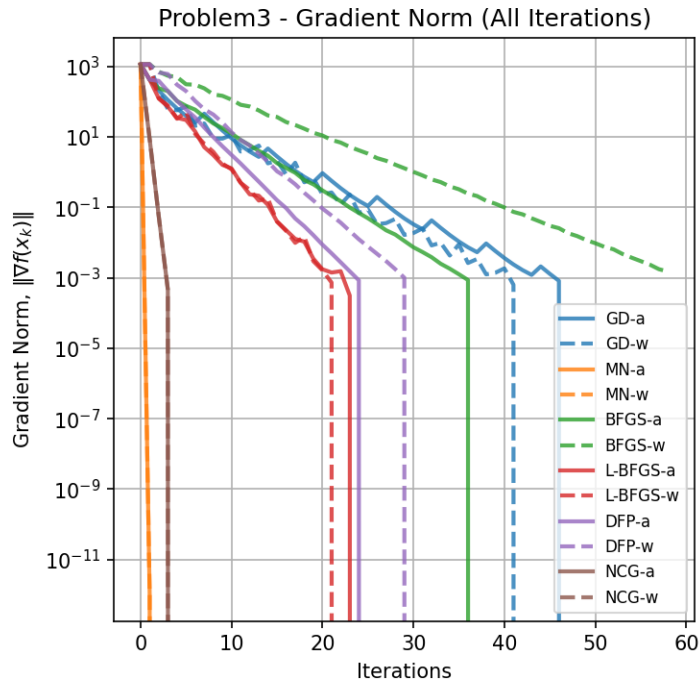
Problem 4

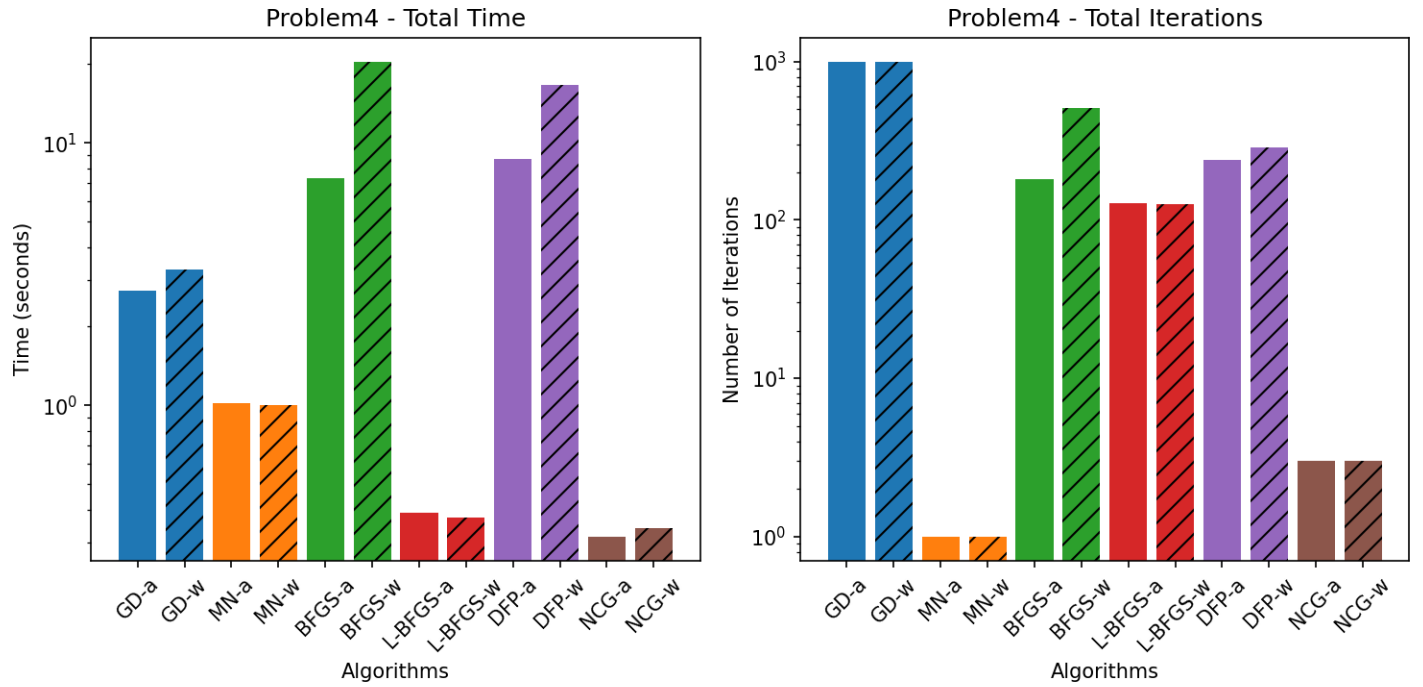|	Iters	Time	Convergence	Func Evals	Grad Evals	Hess Evals
GD-a	999	5.64754	Failed. Maximum iterations reached.	8722	2000	0
GD-w	999	8.46339	Failed. Maximum iterations reached.	10965	1000	0
MN-a	1	3.11792	Converged. Gradient norm is below tolerance.	3	4	2
MN-w	1	2.79129	Converged. Gradient norm is below tolerance.	3	3	2
BFGS-a	181	22.8544	Converged. Gradient norm is below tolerance.	1451	545	0
BFGS-w	508	39.9129	Converged. Gradient norm is below tolerance.	4779	1018	0
L-BFGS-a	127	1.54507	Converged. Gradient norm is below tolerance.	267	382	0
L-BFGS-w	126	0.781778	Converged. Gradient norm is below tolerance.	267	253	0
DFP-a	240	15.2997	Converged. Gradient norm is below tolerance.	1268	722	0
DFP-w	288	17.783	Converged. Gradient norm is below tolerance.	1750	578	0
NCG-a	3	0.533978	Converged. Gradient norm is below tolerance.	7	8	4
NCG-w	3	0.468502	Converged. Gradient norm is below tolerance.	7	5	4

Problem4 - Total Time

Problem4 - Total Iterations
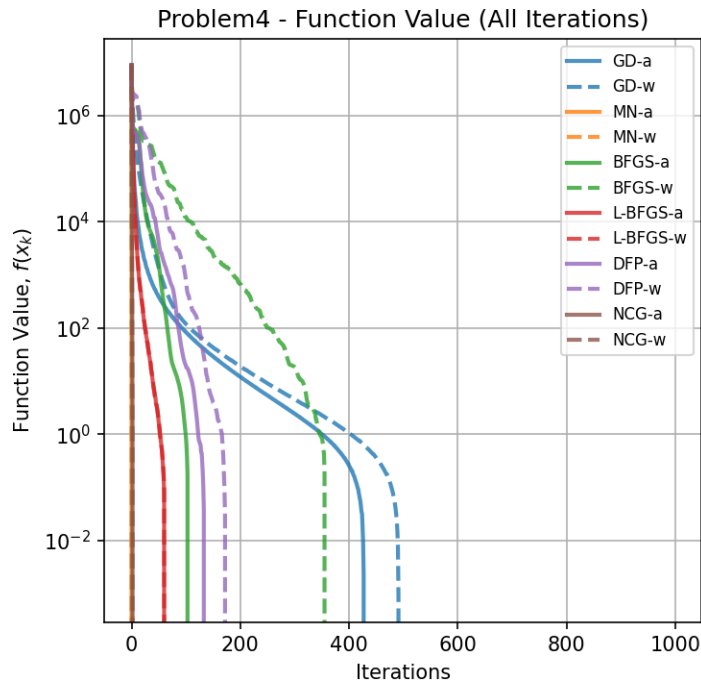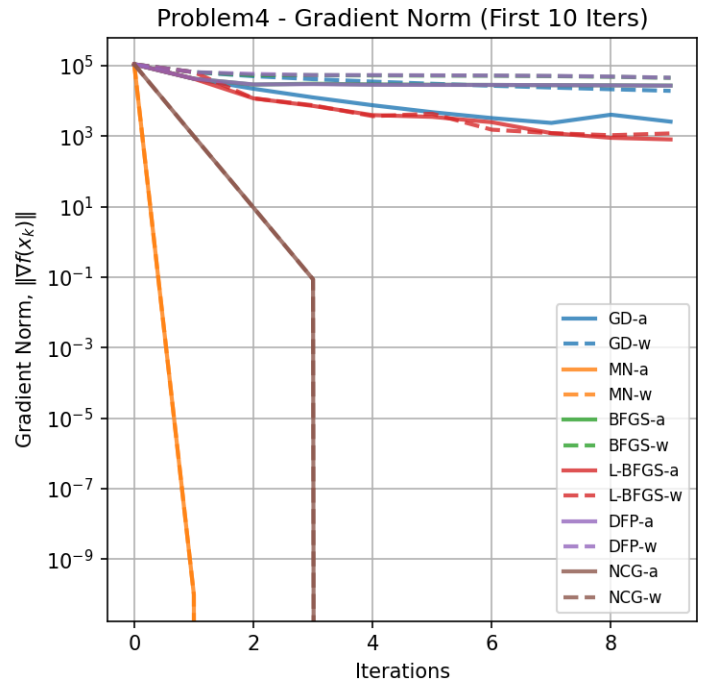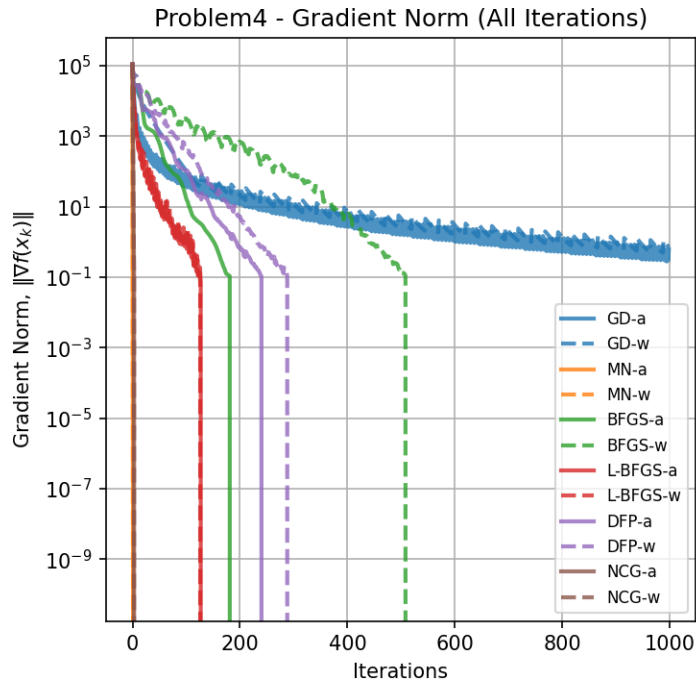
Problem 5

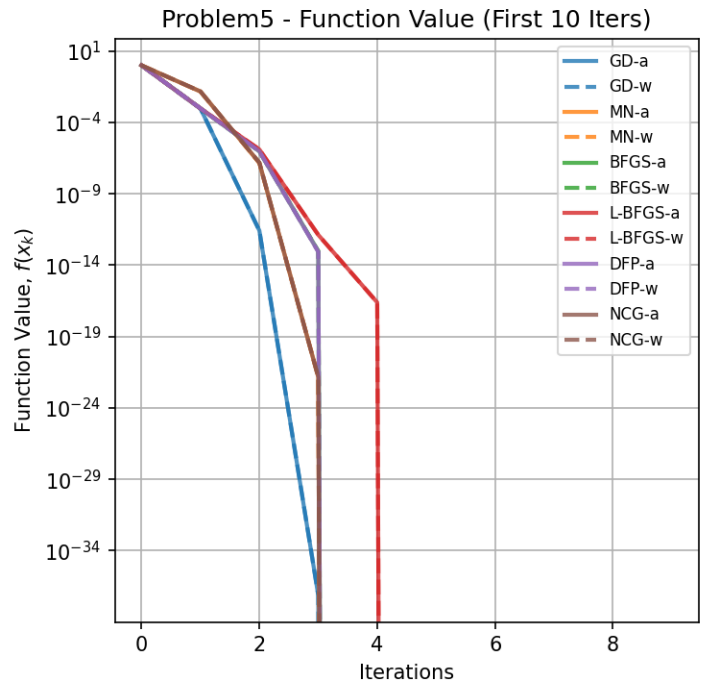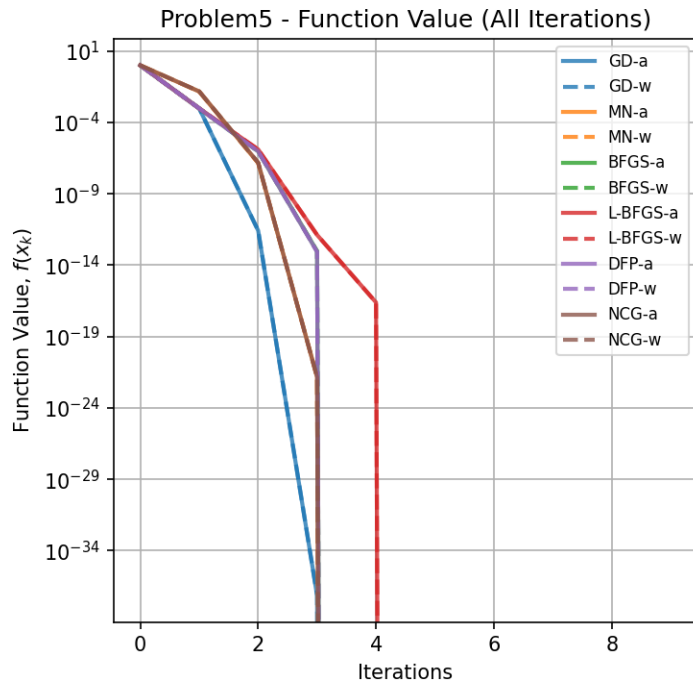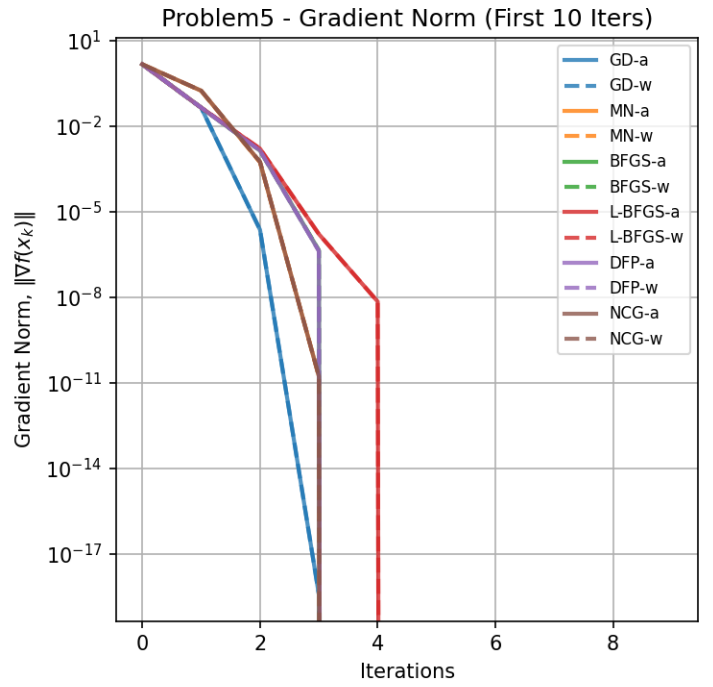|	Iters	Time	Convergence	Func Evals	Grad Evals	Hess Evals
GD-a	3	0.000770092	Converged. Gradient norm is below tolerance.	7	7	0
GD-w	3	0.000644922	Converged. Gradient norm is below tolerance.	7	4	0
MN-a	3	0.0023911	Converged. Gradient norm is below tolerance.	7	8	4
MN-w	3	0.00140238	Converged. Gradient norm is below tolerance.	7	5	4
BFGS-a	3	0.000858784	Converged. Gradient norm is below tolerance.	7	11	0
BFGS-w	3	0.00101233	Converged. Gradient norm is below tolerance.	7	8	0
L-BFGS-a	4	0.00116229	Converged. Gradient norm is below tolerance.	9	13	0
L-BFGS-w	4	0.00128317	Converged. Gradient norm is below tolerance.	9	9	0
DFP-a	3	0.000855923	Converged. Gradient norm is below tolerance.	7	11	0
DFP-w	3	0.000930071	Converged. Gradient norm is below tolerance.	7	8	0
NCG-a	3	0.000947952	Converged. Gradient norm is below tolerance.	7	8	4
NCG-w	3	0.00102019	Converged. Gradient norm is below tolerance.	7	5	4

Problem5 - Gradient Norm (All Iterations)

Problem5 - Gradient Norm (First 10 Iters)

Problem5 - Function Value (All Iterations)

Problem5 - Function Value (First 10 Iters)

Problem5 - Total Time

Problem5 - Total Iterations
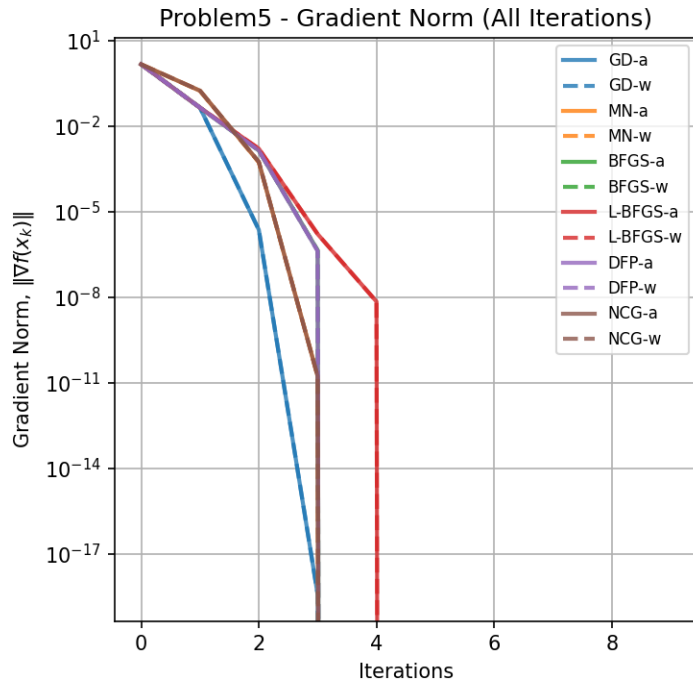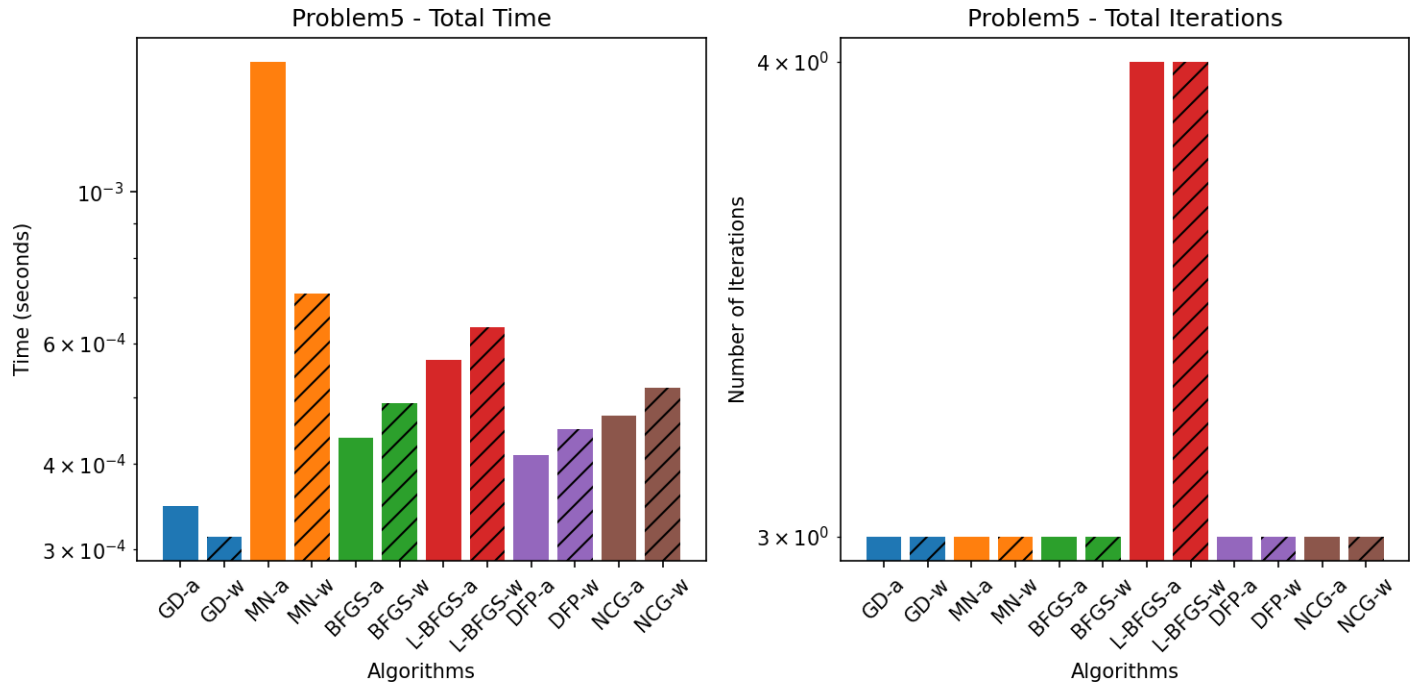
Problem 6

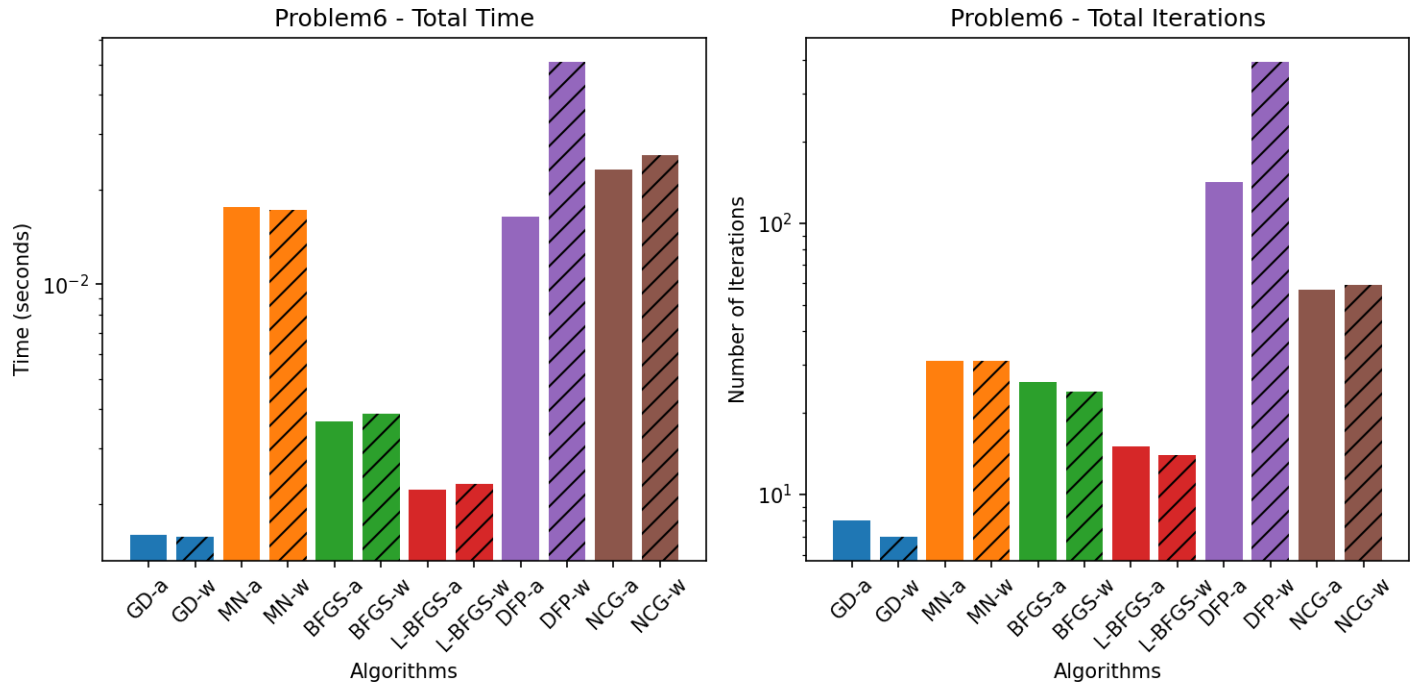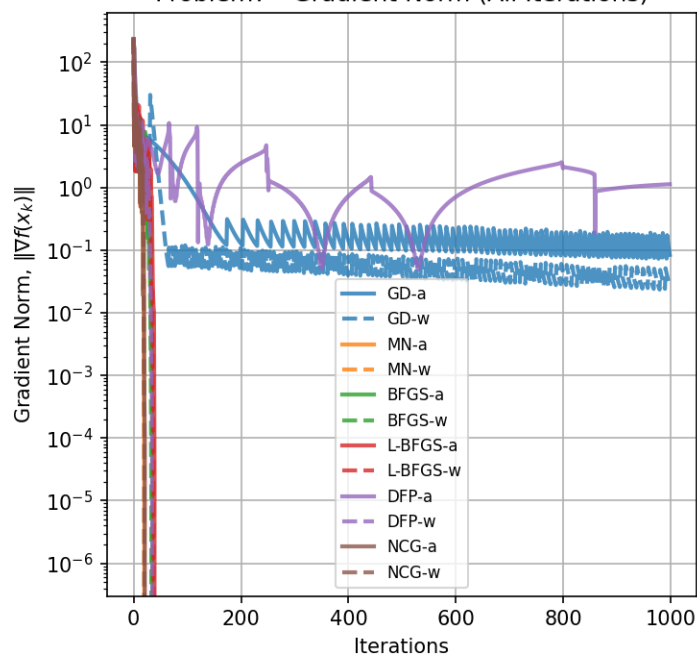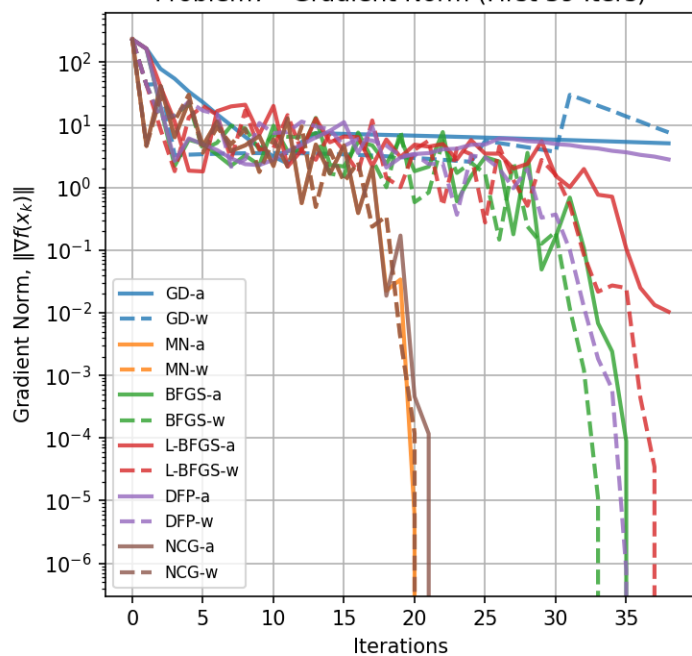|	Iters	Time	Convergence	Func Evals	Grad Evals	Hess Evals
GD-a	8	0.00267839	Converged. Gradient norm is below tolerance.	102	17	0
GD-w	7	0.00273061	Converged. Gradient norm is below tolerance.	132	8	0
MN-a	31	0.0324914	Converged. Gradient norm is below tolerance.	63	64	32
MN-w	31	0.0327451	Converged. Gradient norm is below tolerance.	63	33	32
BFGS-a	26	0.00672174	Converged. Gradient norm is below tolerance.	113	80	0
BFGS-w	24	0.00698209	Converged. Gradient norm is below tolerance.	129	50	0
L-BFGS-a	15	0.00412393	Converged. Gradient norm is below tolerance.	47	46	0
L-BFGS-w	14	0.00403428	Converged. Gradient norm is below tolerance.	50	29	0
DFP-a	142	0.028933	Converged. Gradient norm is below tolerance.	344	428	0
DFP-w	393	0.0922611	Converged. Gradient norm is below tolerance.	867	788	0
NCG-a	57	0.0401502	Converged. Gradient norm is below tolerance.	611	116	58
NCG-w	59	0.0449107	Converged. Gradient norm is below tolerance.	898	61	60

Problem6 - Gradient Norm (All Iterations)

Problem6 - Gradient Norm (First 10 Iters)

Problem6 - Function Value (All Iterations)

Problem6 - Function Value (First 10 Iters)
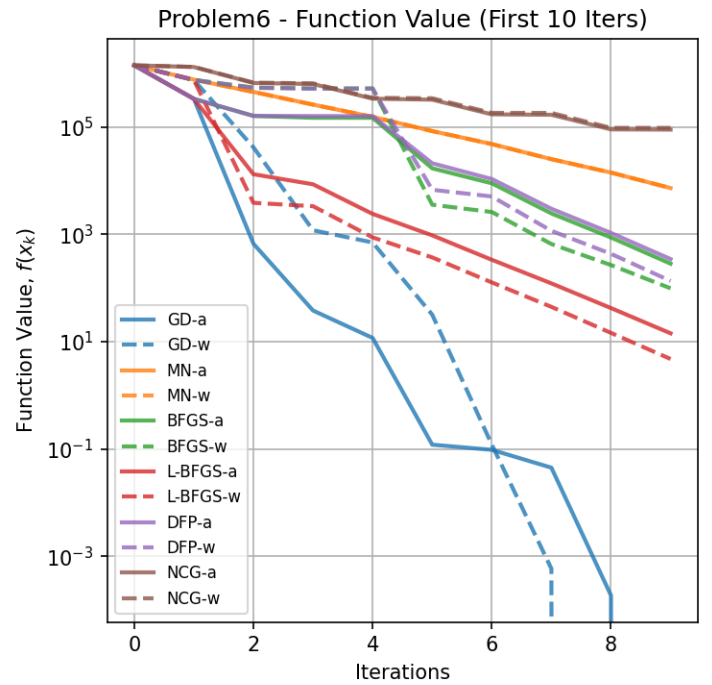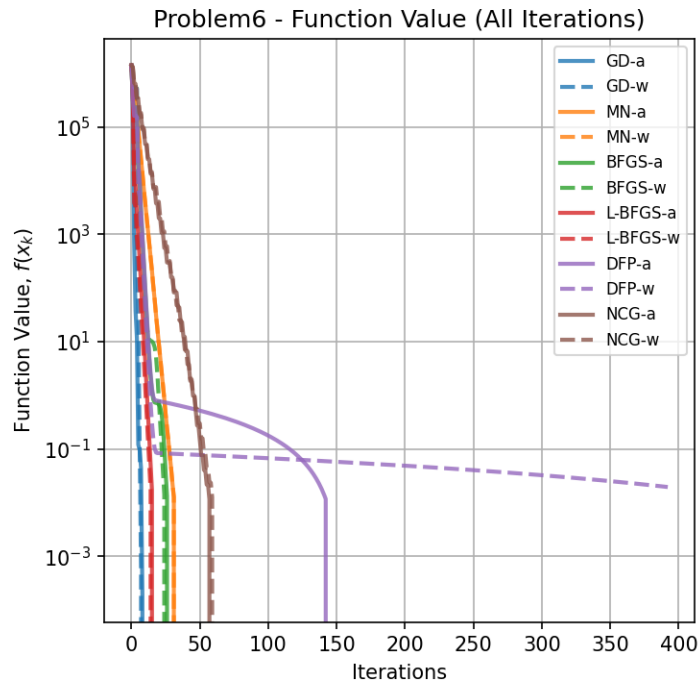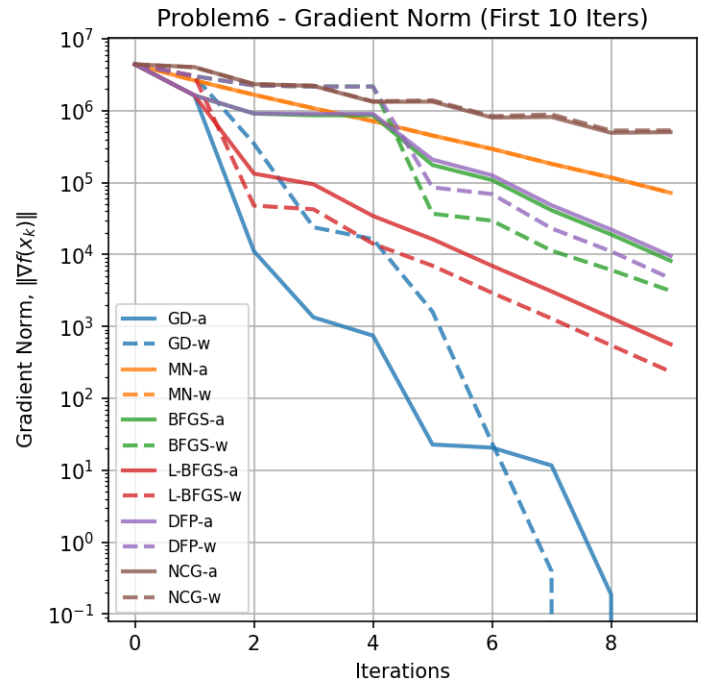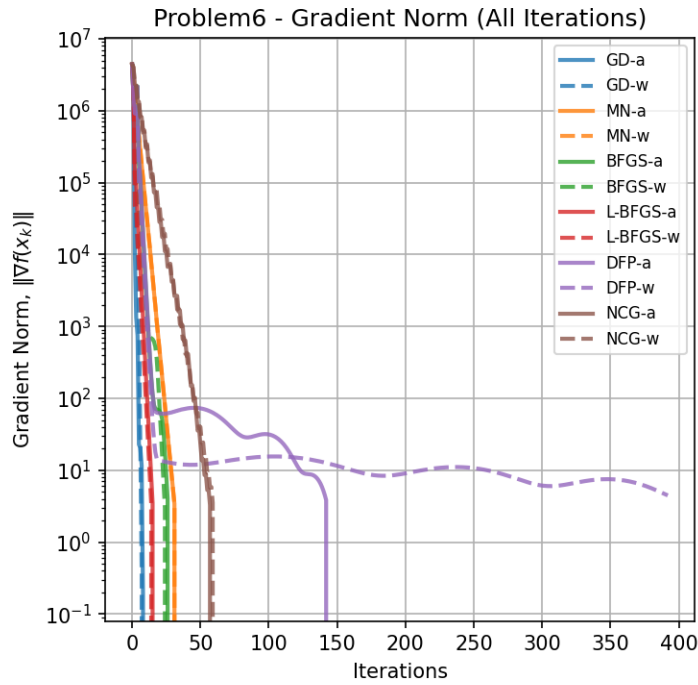
Problem 7

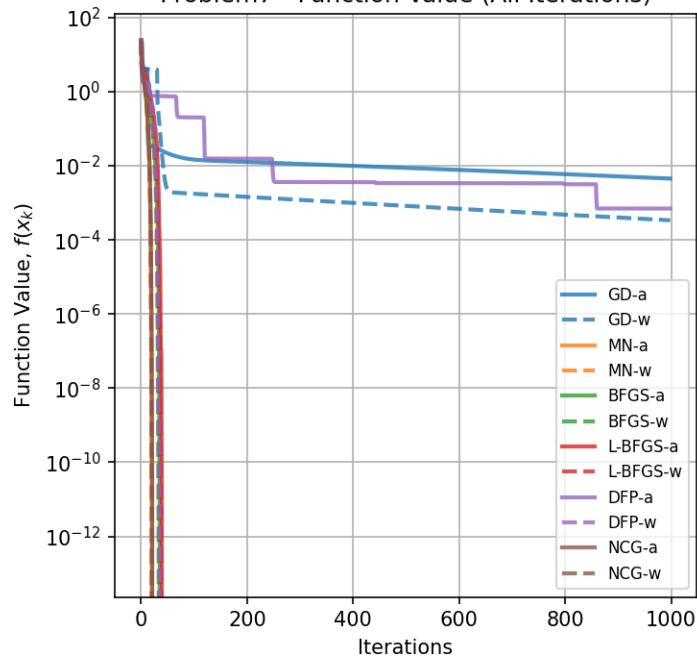|	Iters	Time	Convergence	Func Evals	Grad Evals	Hess Evals
GD-a	999	0.144972	Failed. Maximum iterations reached.	8923	2000	0
GD-w	999	0.16291	Failed. Maximum iterations reached.	10881	1001	0
MN-a	20	0.0048604	Converged. Gradient norm is below tolerance.	47	42	21
MN-w	20	0.00508499	Converged. Gradient norm is below tolerance.	48	22	21
BFGS-a	35	0.00653625	Converged. Gradient norm is below tolerance.	90	107	0
BFGS-w	33	0.0062995	Converged. Gradient norm is below tolerance.	86	68	0
L-BFGS-a	39	0.00668573	Converged. Gradient norm is below tolerance.	108	118	0
L-BFGS-w	37	0.0077796	Converged. Gradient norm is below tolerance.	95	83	0
DFP-a	999	0.170804	Failed. Maximum iterations reached.	2031	3001	0
DFP-w	35	0.00734663	Converged. Gradient norm is below tolerance.	88	76	0
NCG-a	21	0.00394392	Converged. Gradient norm is below tolerance.	49	44	22
NCG-w	20	0.00429821	Converged. Gradient norm is below tolerance.	48	22	21

Problem7 - Gradient Norm (All Iterations)
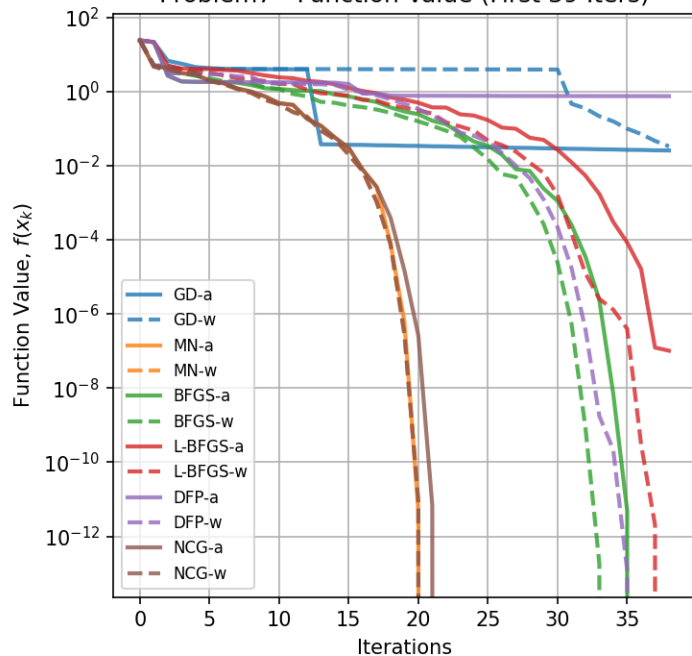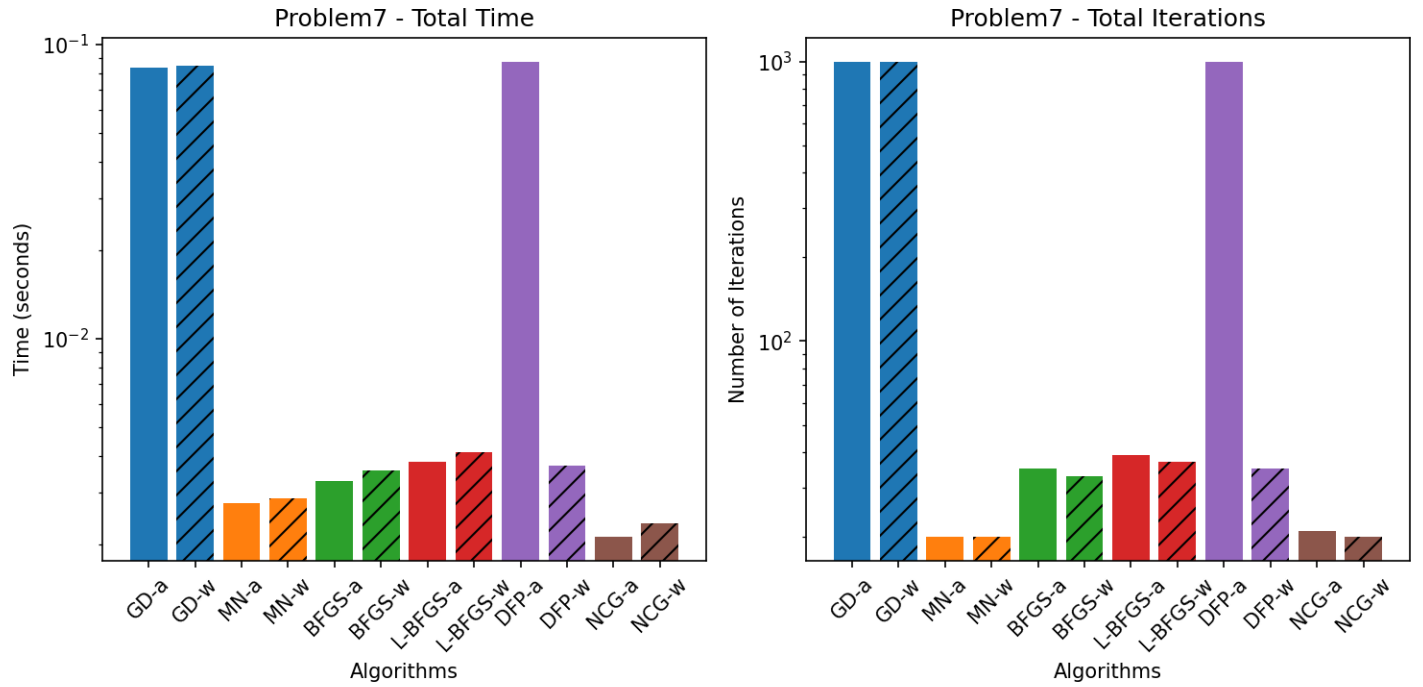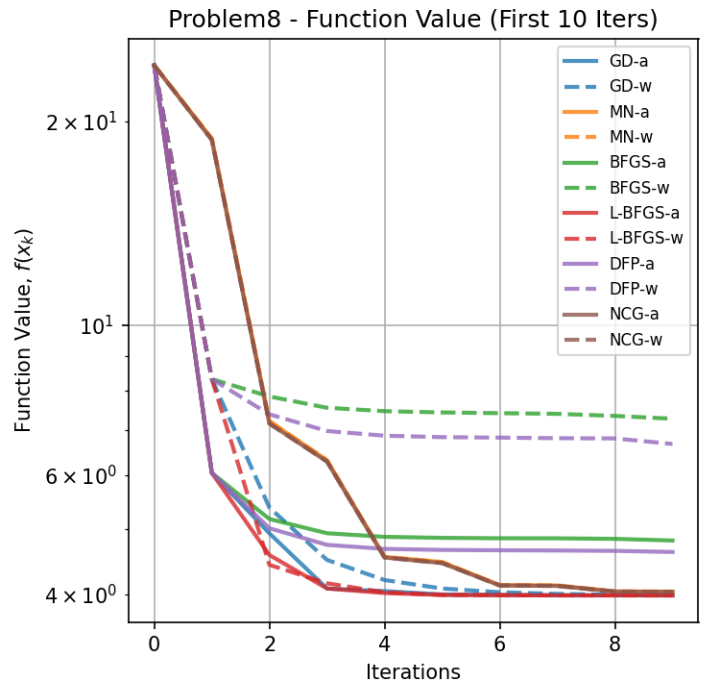
Problem7 - Gradient Norm (First 39 Iters)

Problem7 - Function Value (All Iterations)

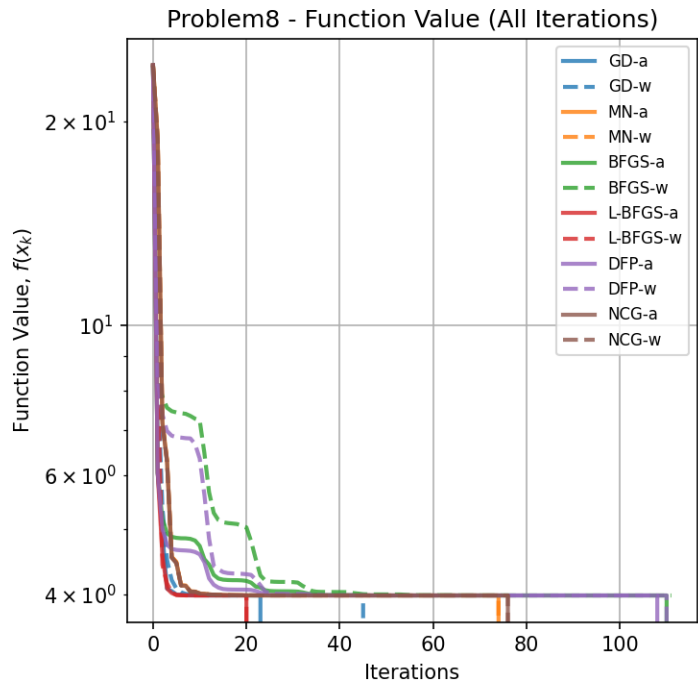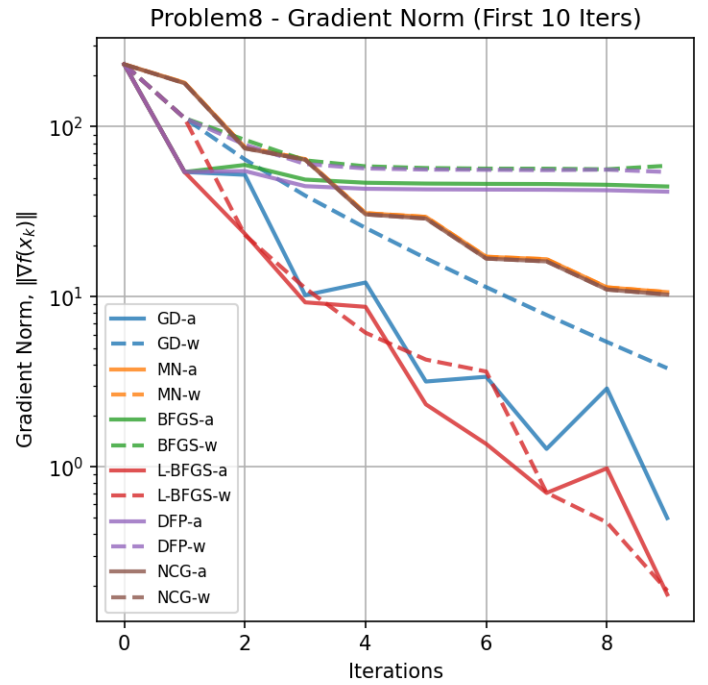Problem7 - Function Value (First 39 Iters)

Problem7 - Total Time

Problem7 - Total Iterations

Problem 8

	Iters	Time	Convergence	Func Evals	Grad Evals	Hess Evals
GD-a	23	0.0225849	Converged. Gradient norm is below tolerance.	218	47	0
GD-w	45	0.0581281	Converged. Gradient norm is below tolerance.	540	46	0
MN-a	74	2.09037	Converged. Gradient norm is below tolerance.	149	150	75
MN-w	74	2.42142	Converged. Gradient norm is below tolerance.	149	76	75
BFGS-a	110	0.236356	Converged. Gradient norm is below tolerance.	924	332	0
BFGS-w	112	0.25547	Converged. Gradient norm is below tolerance.	1124	226	0
L-BFGS-a	20	0.0263257	Converged. Gradient norm is below tolerance.	50	61	0
L-BFGS-w	20	0.0368845	Converged. Gradient norm is below tolerance.	53	41	0
DFP-a	108	0.194573	Converged. Gradient norm is below tolerance.	623	326	0
DFP-w	110	0.224674	Converged. Gradient norm is below tolerance.	730	222	0
NCG-a	76	0.134506	Converged. Gradient norm is below tolerance.	153	154	77
NCG-w	76	0.158	Converged. Gradient norm is below tolerance.	153	78	77

Problem8 - Gradient Norm (All Iterations)

Problem8 - Gradient Norm (First 10 Iters)

Problem8 - Function Value (All Iterations)

Problem8 - Function Value (First 10 Iters)

Problem8 - Total Time
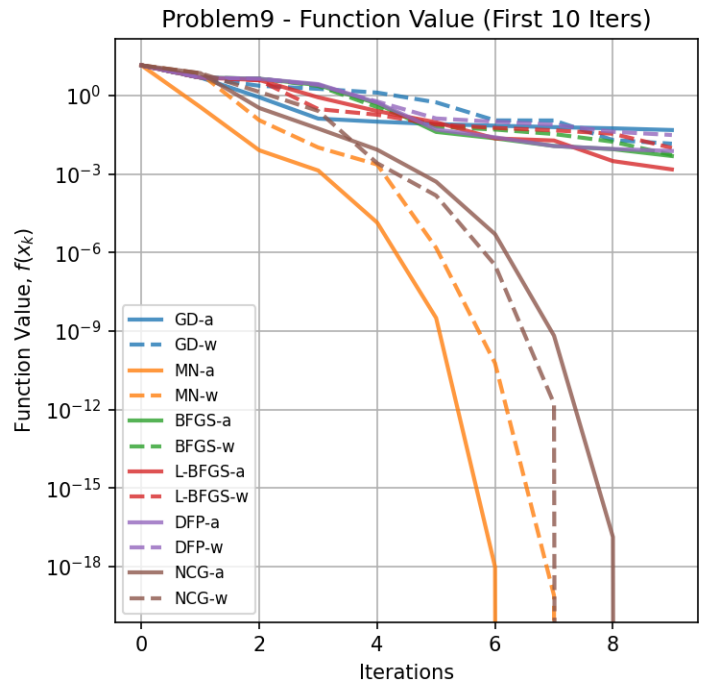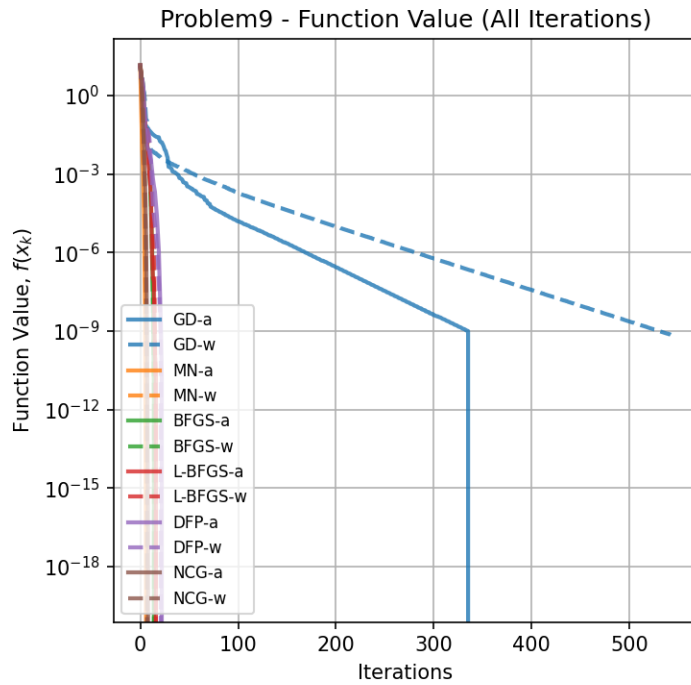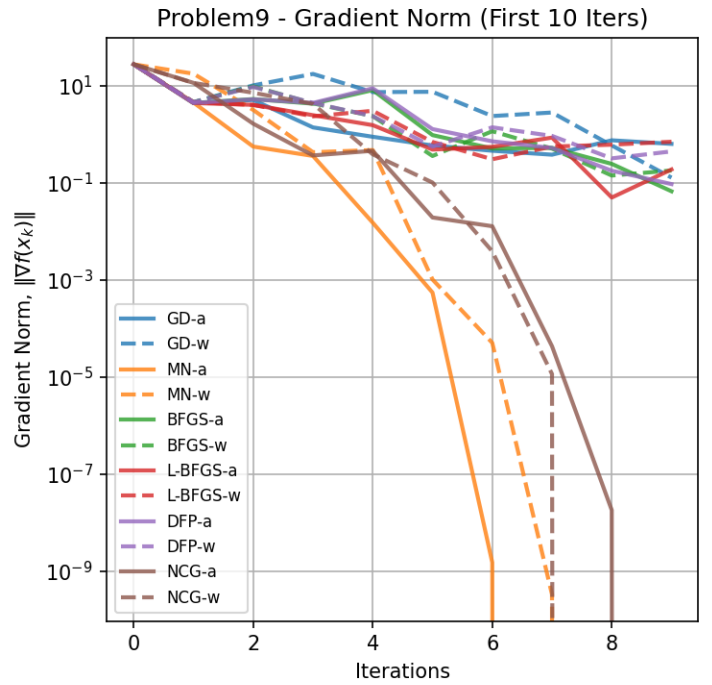

Problem8 - Total Iterations
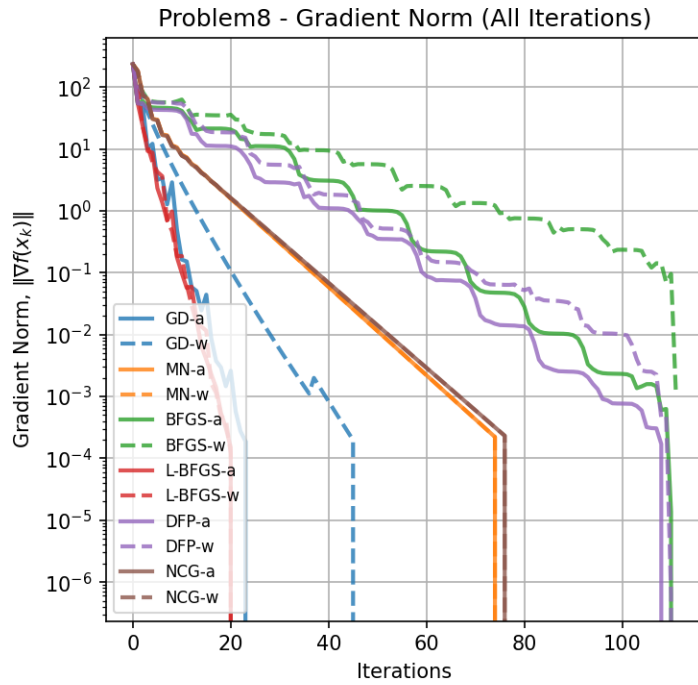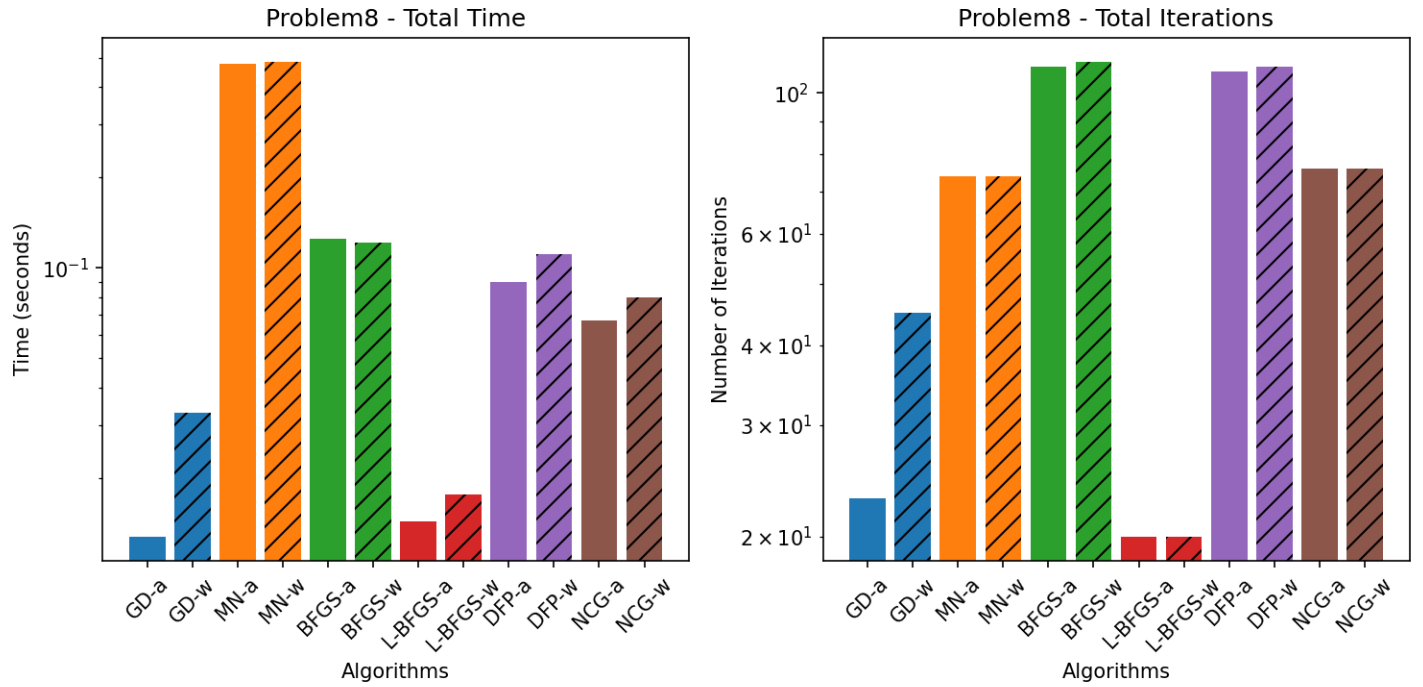
Problem 9

	Iters	Time	Convergence	Func Evals	Grad Evals	Hess Evals
GD-a	335	0.0457895	Converged. Gradient norm is below tolerance.	1740	671	0
GD-w	543	0.0870702	Converged. Gradient norm is below tolerance.	3523	544	0
MN-a	6	0.00235844	Converged. Gradient norm is below tolerance.	25	14	7
MN-w	7	0.00260925	Converged. Gradient norm is below tolerance.	55	9	8
BFGS-a	14	0.00322413	Converged. Gradient norm is below tolerance.	36	44	0
BFGS-w	14	0.00324917	Converged. Gradient norm is below tolerance.	37	30	0
L-BFGS-a	16	0.0034287	Converged. Gradient norm is below tolerance.	39	49	0
L-BFGS-w	16	0.00382686	Converged. Gradient norm is below tolerance.	44	33	0
DFP-a	22	0.00456572	Converged. Gradient norm is below tolerance.	52	68	0
DFP-w	21	0.0048666	Converged. Gradient norm is below tolerance.	51	44	0
NCG-a	8	0.00196147	Converged. Gradient norm is below tolerance.	19	18	9
NCG-w	7	0.0023942	Converged. Gradient norm is below tolerance.	17	9	8

Problem9 - Gradient Norm (All Iterations)

Problem9 - Gradient Norm (First 10 Iters)

Problem9 - Function Value (All Iterations)

Problem9 - Function Value (First 10 Iters)

Problem9 - Total Time / Problem9 - Total Iterations
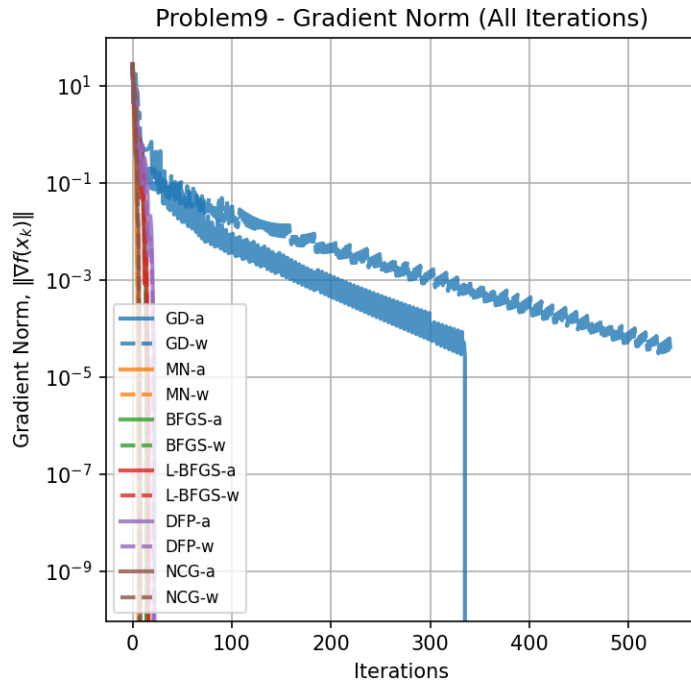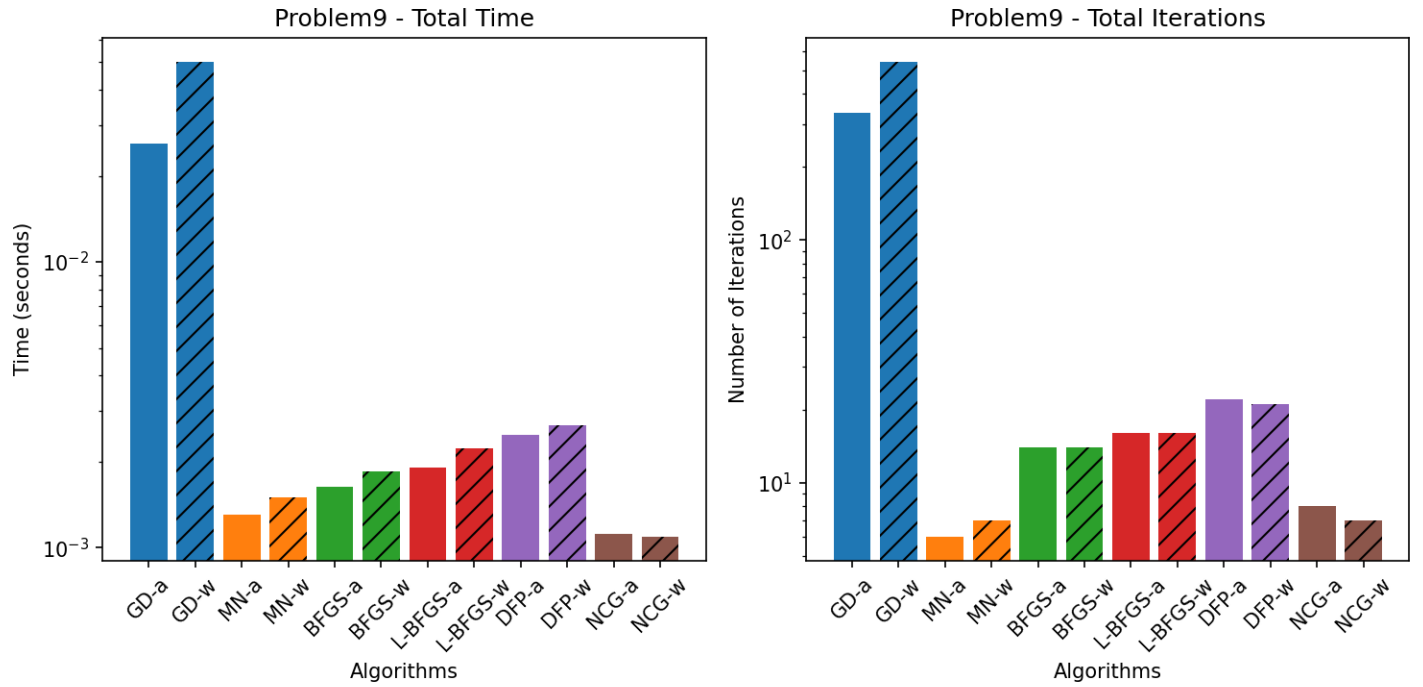
Problem 10

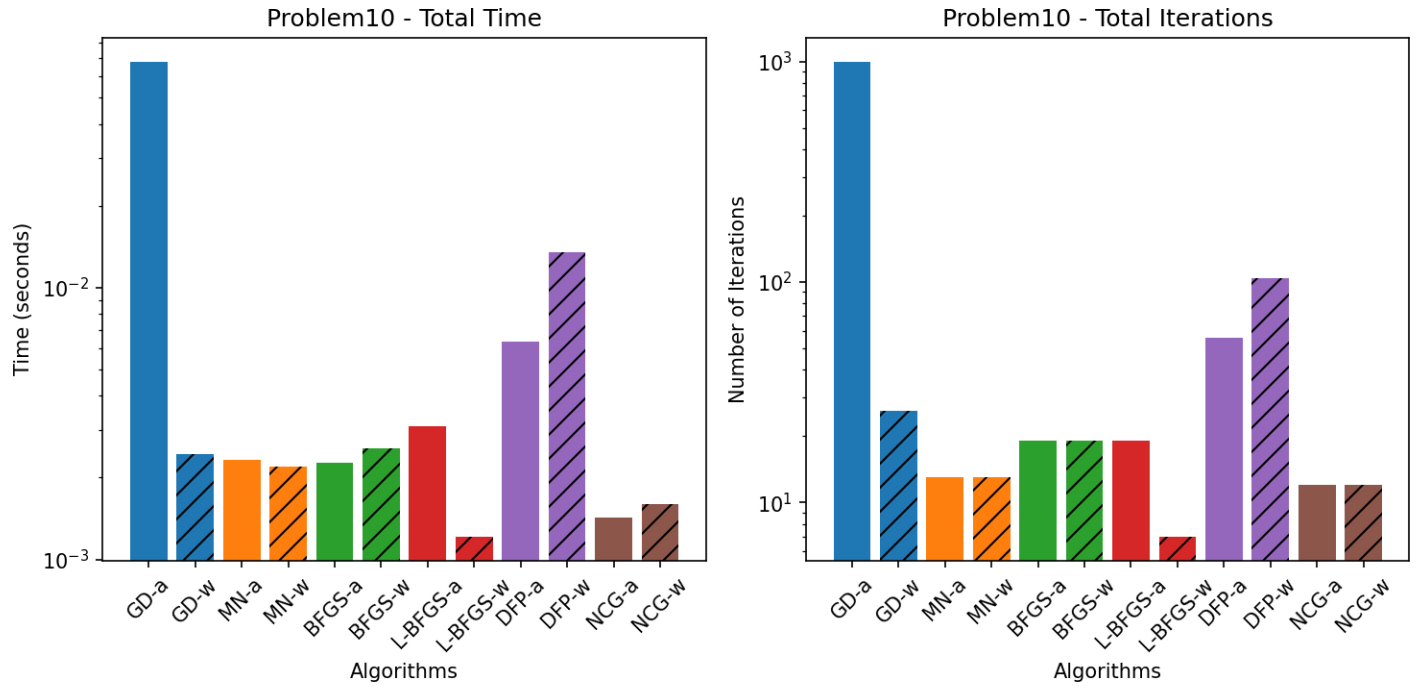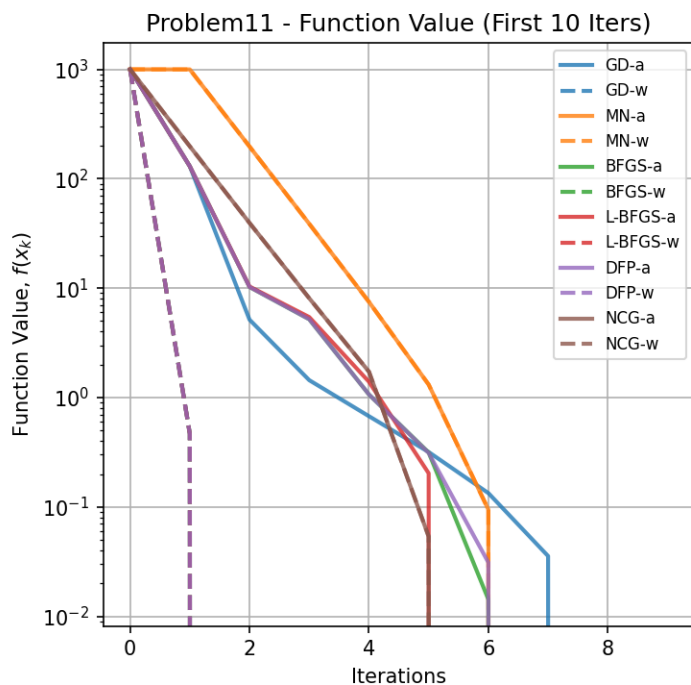|	Iters	Time	Convergence	Func Evals	Grad Evals	Hess Evals
GD-a	999	0.177505	Failed. Maximum iterations reached.	2001	2000	0
GD-w	26	0.00651288	Converged. Gradient norm is below tolerance.	67	27	0
MN-a	13	0.00596166	Converged. Gradient norm is below tolerance.	35	28	14
MN-w	13	0.00613093	Converged. Gradient norm is below tolerance.	37	15	14
BFGS-a	19	0.00570154	Converged. Gradient norm is below tolerance.	41	59	0
BFGS-w	19	0.00648093	Converged. Gradient norm is below tolerance.	42	41	0
L-BFGS-a	19	0.00757098	Converged. Gradient norm is below tolerance.	42	58	0
L-BFGS-w	7	0.0029273	Converged. Gradient norm is below tolerance.	18	19	0
DFP-a	56	0.0158935	Converged. Gradient norm is below tolerance.	118	170	0
DFP-w	104	0.0415792	Converged. Gradient norm is below tolerance.	215	222	0
NCG-a	12	0.00361443	Converged. Gradient norm is below tolerance.	25	26	13
NCG-w	12	0.00551796	Converged. Gradient norm is below tolerance.	25	14	13
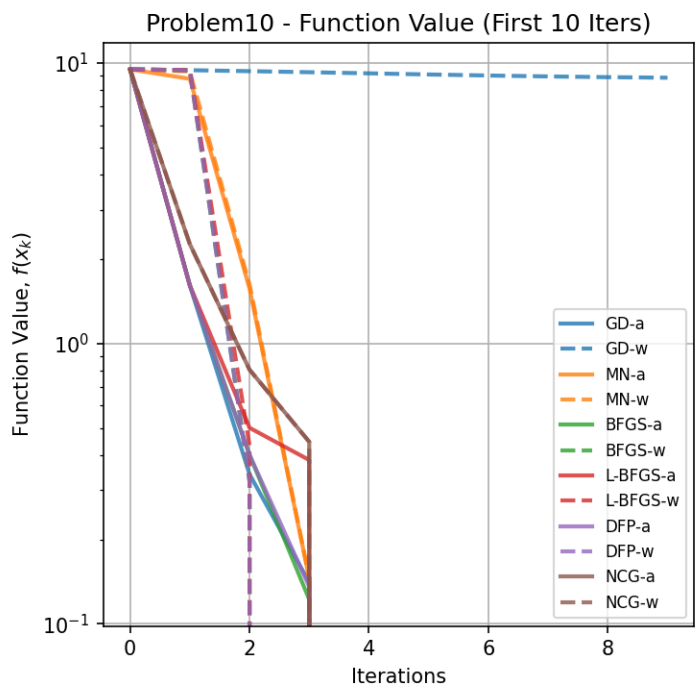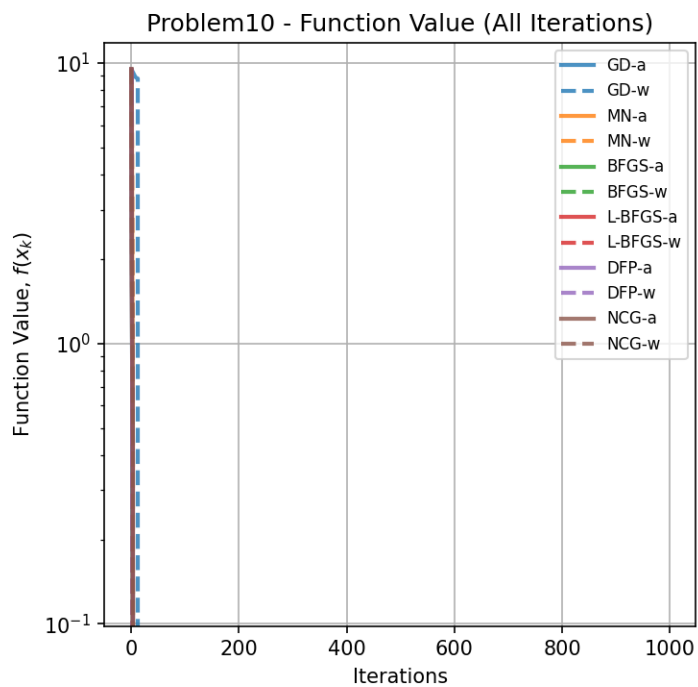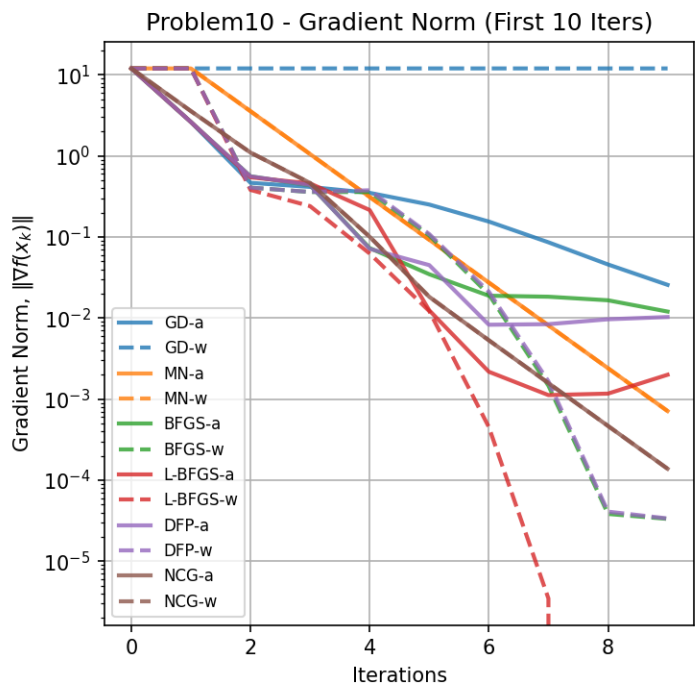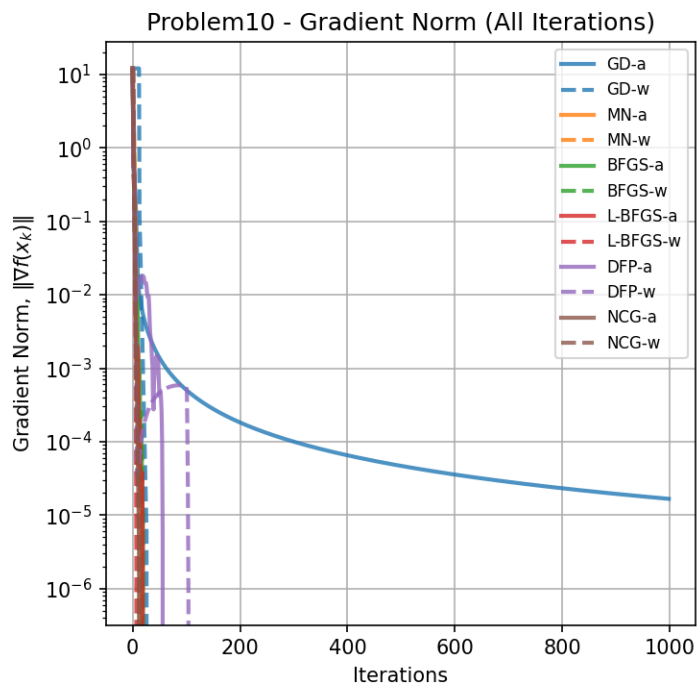
Problem 11

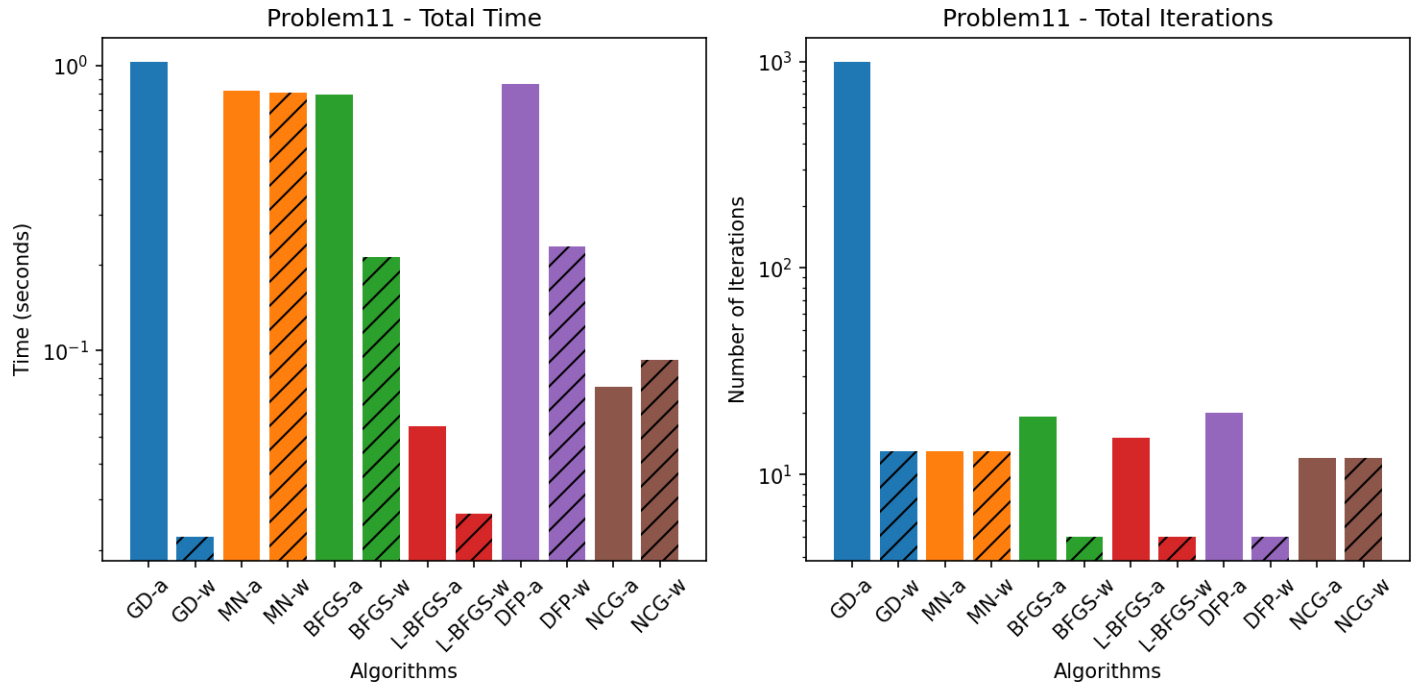|	Iters	Time	Convergence	Func Evals	Grad Evals	Hess Evals
GD-a	999	1.87948	Failed. Maximum iterations reached.	2001	2000	0
GD-w	13	0.0392365	Converged. Gradient norm is below tolerance.	29	16	0
MN-a	13	1.56481	Converged. Gradient norm is below tolerance.	35	28	14
MN-w	13	1.58117	Converged. Gradient norm is below tolerance.	37	15	14
BFGS-a	19	1.23579	Converged. Gradient norm is below tolerance.	40	59	0
BFGS-w	5	0.376815	Converged. Gradient norm is below tolerance.	16	14	0
L-BFGS-a	15	0.109908	Converged. Gradient norm is below tolerance.	32	46	0
L-BFGS-w	5	0.0544705	Converged. Gradient norm is below tolerance.	16	15	0
DFP-a	20	1.32766	Converged. Gradient norm is below tolerance.	42	62	0
DFP-w	5	0.352003	Converged. Gradient norm is below tolerance.	16	14	0
NCG-a	12	0.12774	Converged. Gradient norm is below tolerance.	25	26	13
NCG-w	12	0.199319	Converged. Gradient norm is below tolerance.	25	14	13

Problem11 - Total Time

Problem11 - Total Iterations
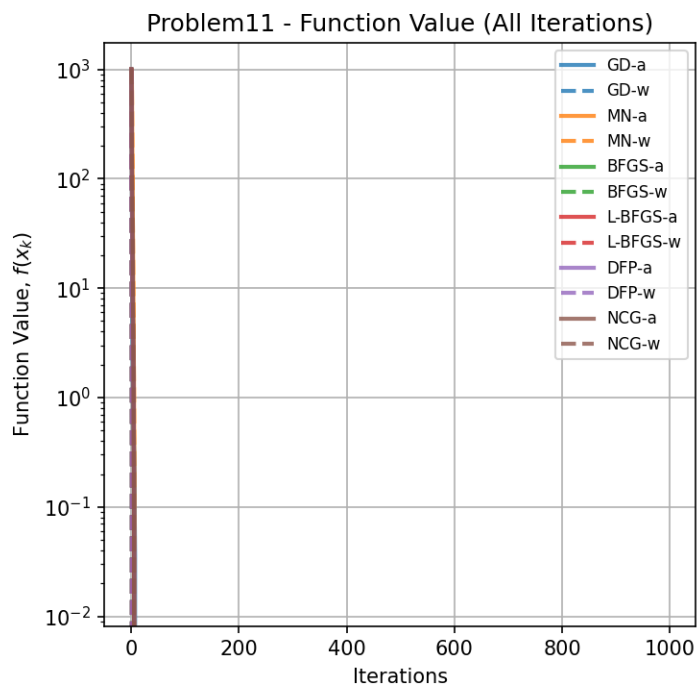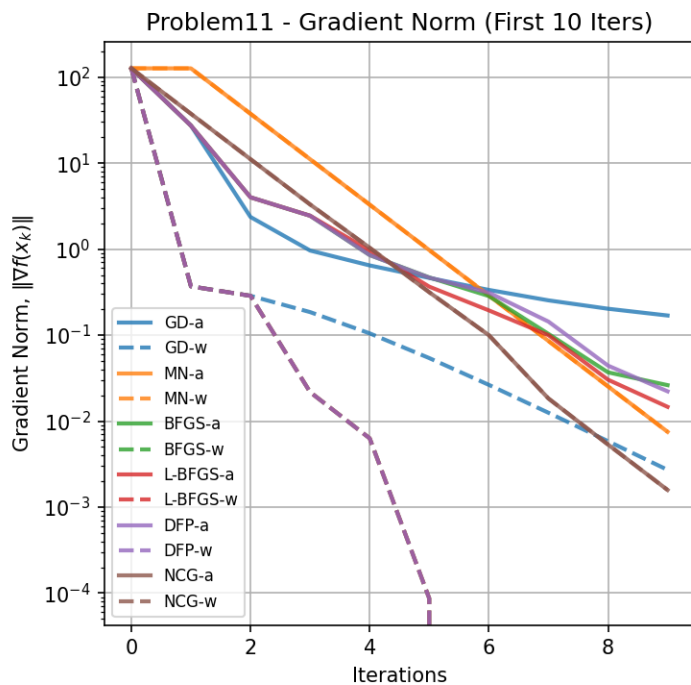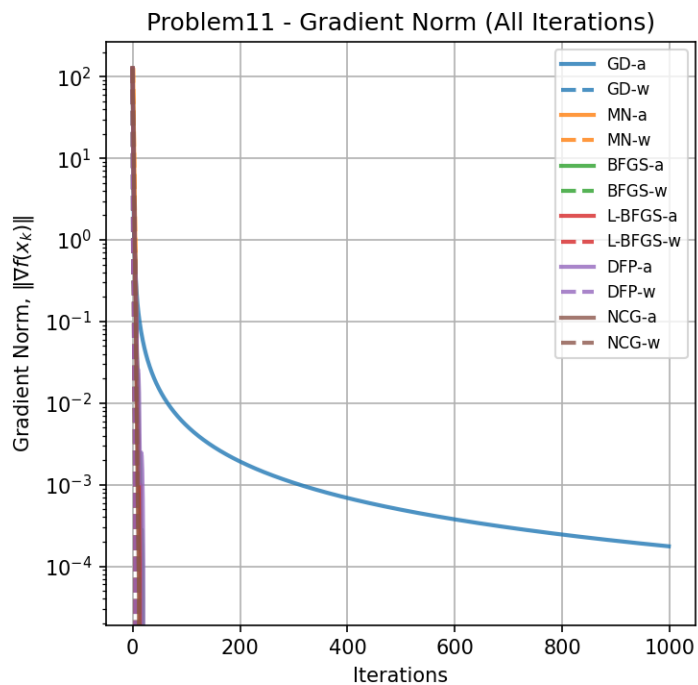
Problem 12

	Iters	Time	Convergence	Func Evals	Grad Evals	Hess Evals
GD-a	146	0.0412164	Converged. Gradient norm is below tolerance.	352	293	0
GD-w	209	0.0819199	Converged. Gradient norm is below tolerance.	674	223	0
MN-a	69	0.0552101	Converged. Gradient norm is below tolerance.	613	140	70
MN-w	82	0.0782318	Converged. Gradient norm is below tolerance.	920	88	83
BFGS-a	72	0.0310001	Converged. Gradient norm is below tolerance.	179	218	0
BFGS-w	47	0.024925	Converged. Gradient norm is below tolerance.	111	98	0
L-BFGS-a	26	0.0127645	Converged. Gradient norm is below tolerance.	64	79	0
L-BFGS-w	27	0.0155084	Converged. Gradient norm is below tolerance.	65	55	0
DFP-a	999	0.404262	Failed. Maximum iterations reached.	2025	3001	0
DFP-w	999	0.49196	Failed. Maximum iterations reached.	2029	2066	0
NCG-a	55	0.0252078	Converged. Gradient norm is below tolerance.	122	112	56
NCG-w	30	0.0177178	Converged. Gradient norm is below tolerance.	66	42	31

Problem12 - Total Time

Problem12 - Total Iterations