

Algorithms for Non-Linear Programming

This repository contains implementations of various algorithms for solving non-linear programming problems.

Getting Started

Install Python 3.10

```
sudo apt-get install python3.10-full
```

Install Venv Package

```
sudo apt-get install python3-venv
```

Create a virtual environment

```
python3.10 -m venv env
```

Activate the virtual environment

```
source env/bin/activate
```

Install the required packages

```
pip install -r requirements.txt
```

How to Use

Run the specific problem script, for example:

```
python problem1.py
```

or run the main script to execute all problems:

```
python run_all_problems.py
```

To convert the markdown file to a PDF, use the following command:

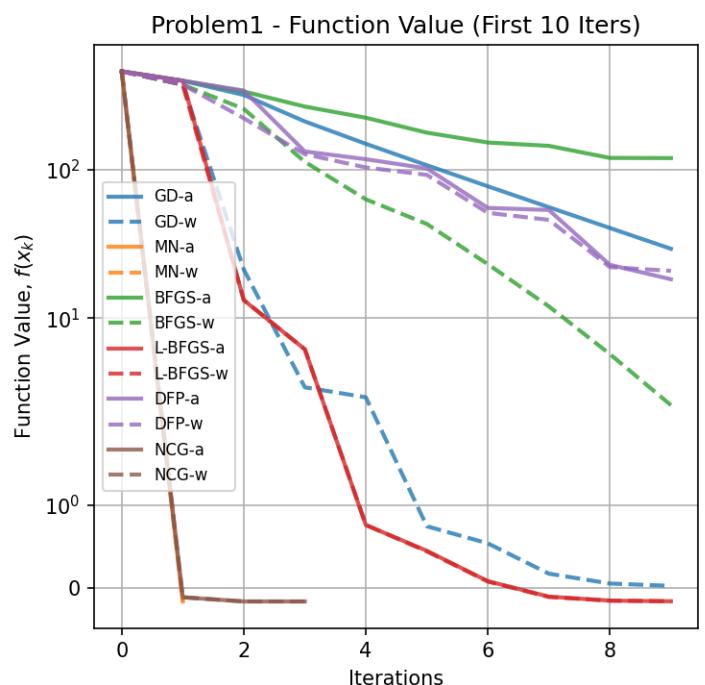
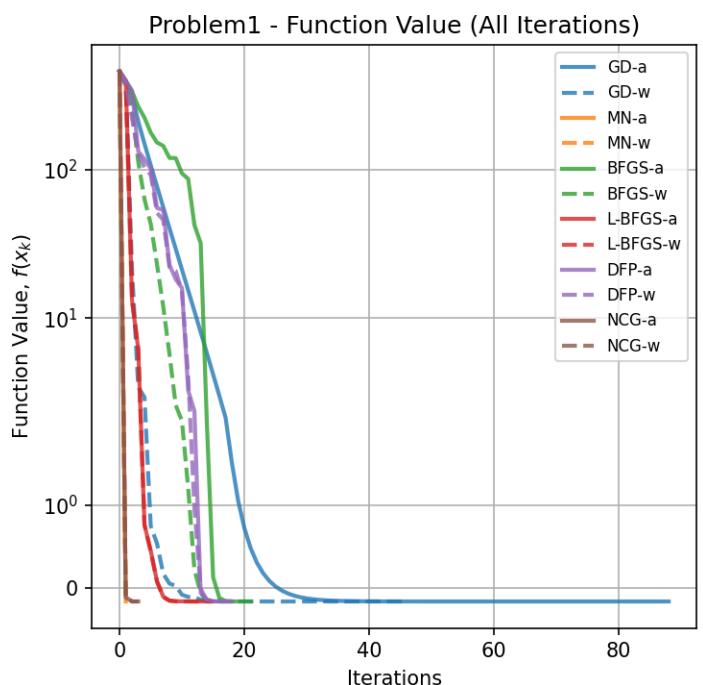
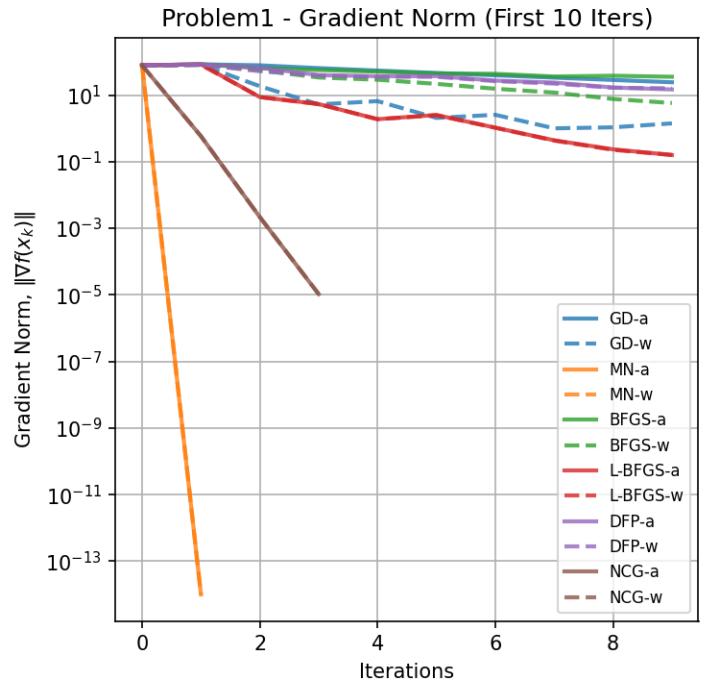
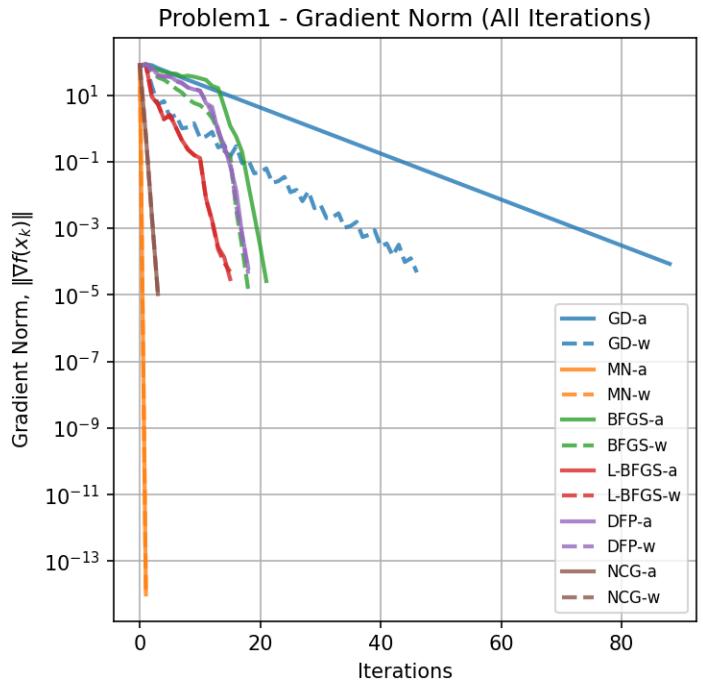
```
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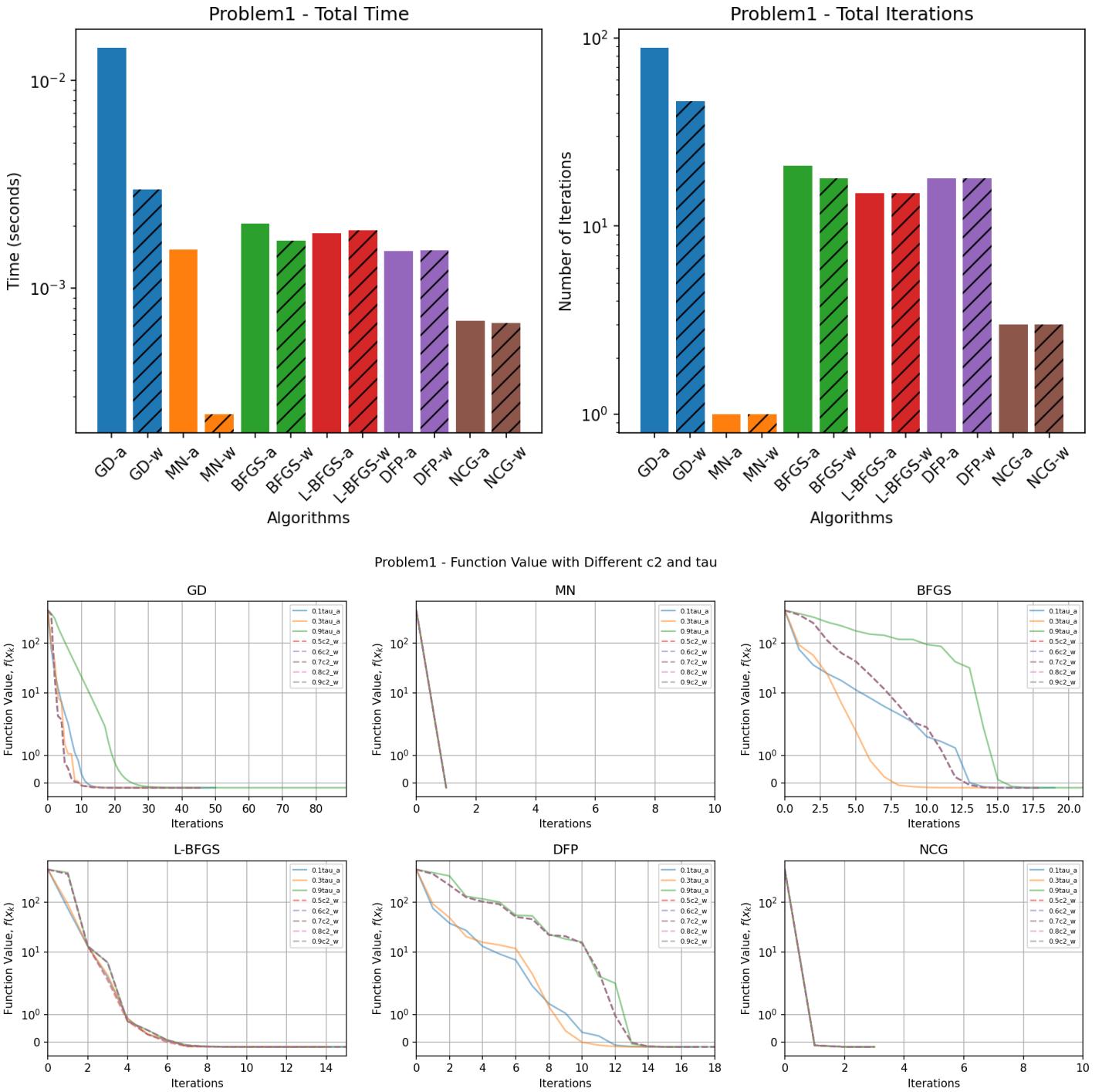
Results

Plots for each problem will be saved in the `figures` directory. Data for each problem will be saved in the `data` directory.

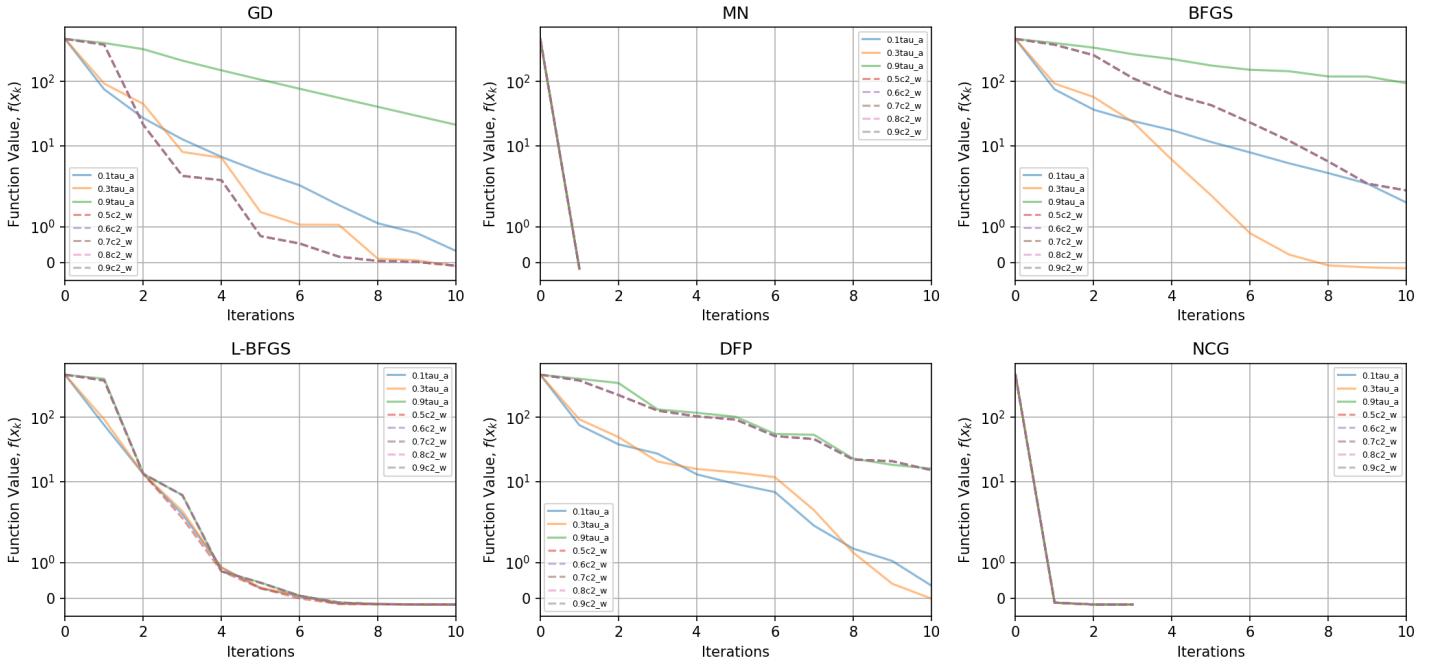
Problem 1

	Iters	Time	Convergence	Func Evals	Grad Evals	Hess Evals
GD-a	89	0.0144293	Converged. Gradient norm is below tolerance.	1599	179	0
GD-w	46	0.00298548	Converged. Gradient norm is below tolerance.	202	47	0
MN-a	1	0.00153804	Converged. Gradient norm is below tolerance.	3	4	2
MN-w	1	0.000247002	Converged. Gradient norm is below tolerance.	3	3	2
BFGS-a	21	0.00204563	Converged. Gradient norm is below tolerance.	92	65	0
BFGS-w	18	0.00169015	Converged. Gradient norm is below tolerance.	49	38	0
L-BFGS-a	15	0.00184393	Converged. Gradient norm is below tolerance.	44	46	0
L-BFGS-w	15	0.001894	Converged. Gradient norm is below tolerance.	33	31	0
DFP-a	18	0.00151396	Converged. Gradient norm is below tolerance.	58	56	0
DFP-w	18	0.00152302	Converged. Gradient norm is below tolerance.	41	38	0
NCG-a	3	0.000693083	Converged. Gradient norm is below tolerance.	7	8	4
NCG-w	3	0.000681162	Converged. Gradient norm is below tolerance.	7	5	4

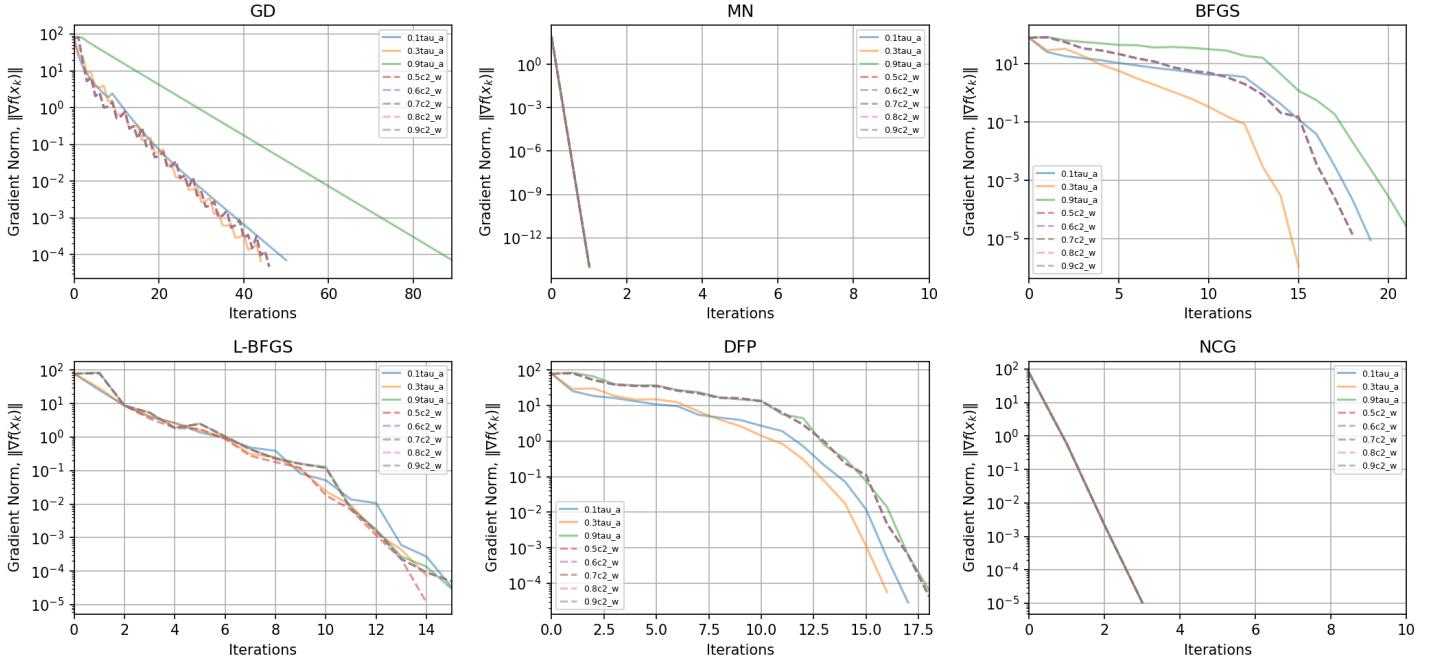




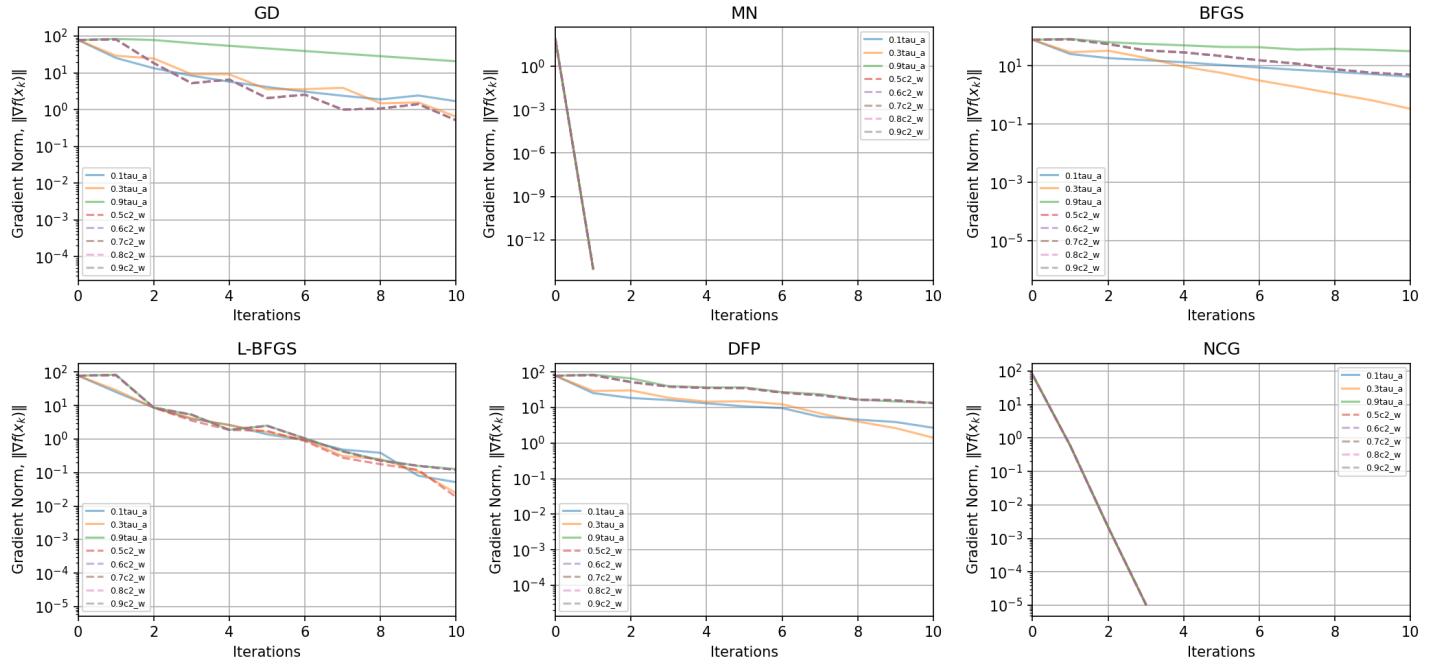
Problem1 - Function Value with Different c_2 and τ (zoom)



Problem1 - Gradient Norm with Different c_2 and tau

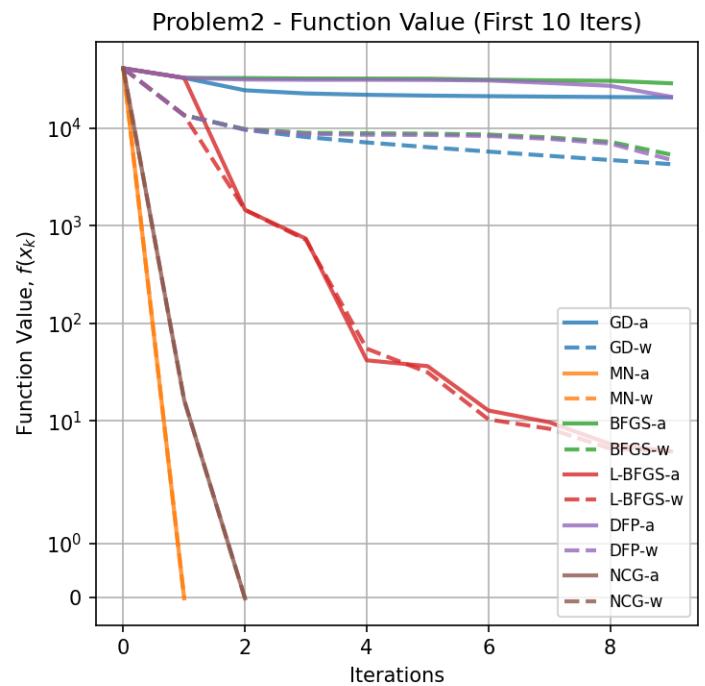
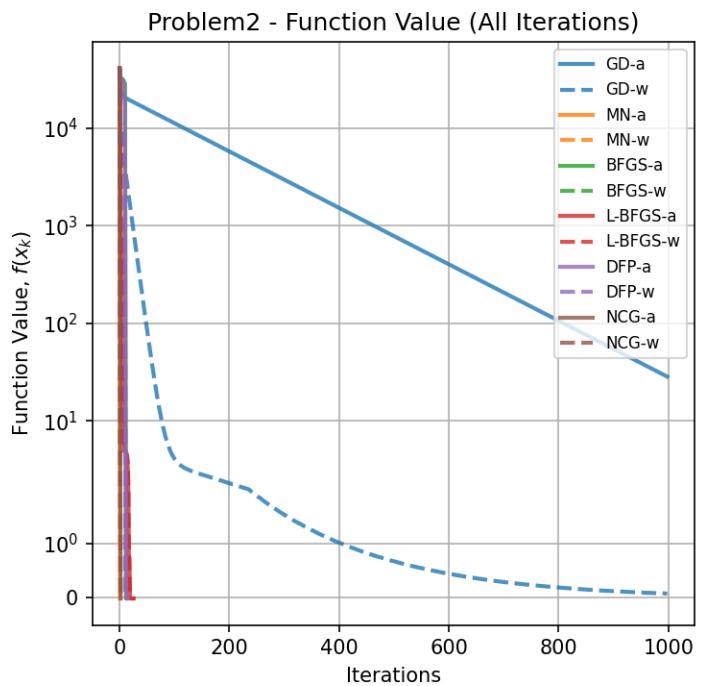
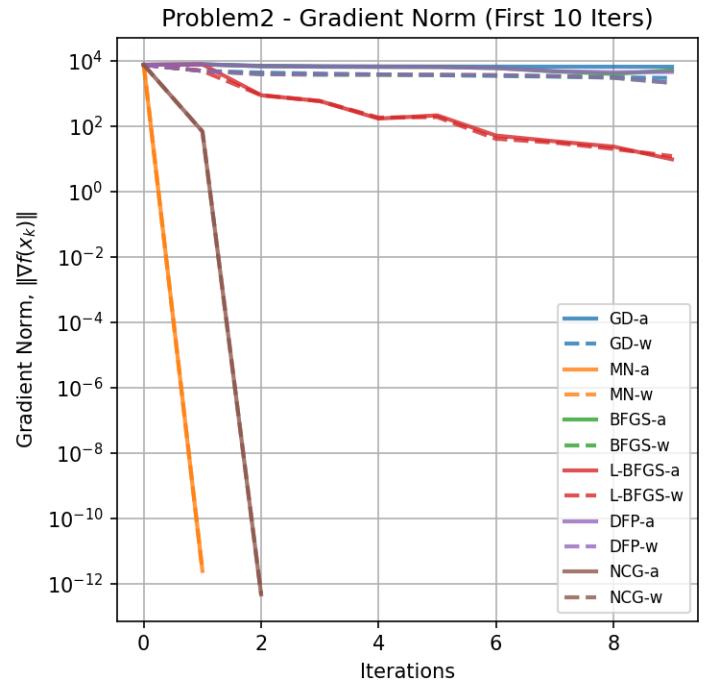
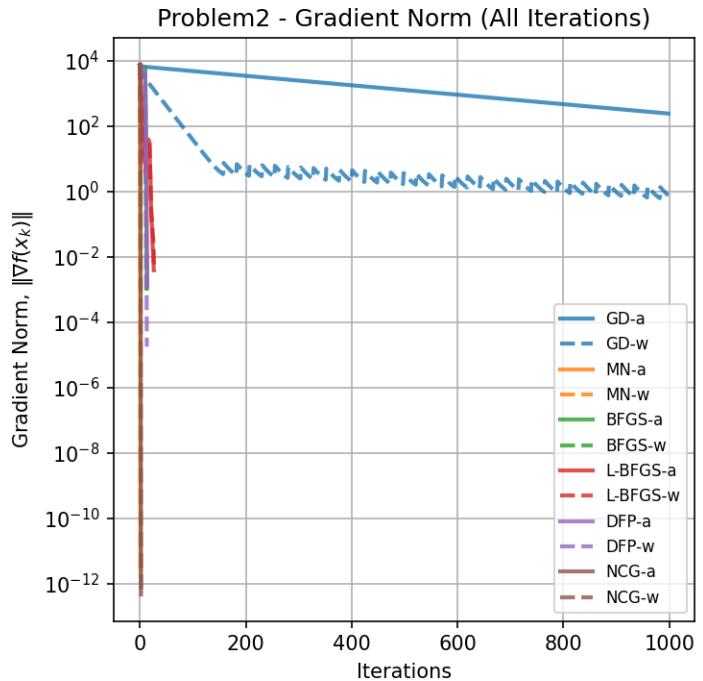


Problem1 - Gradient Norm with Different c_2 and τ (zoom)

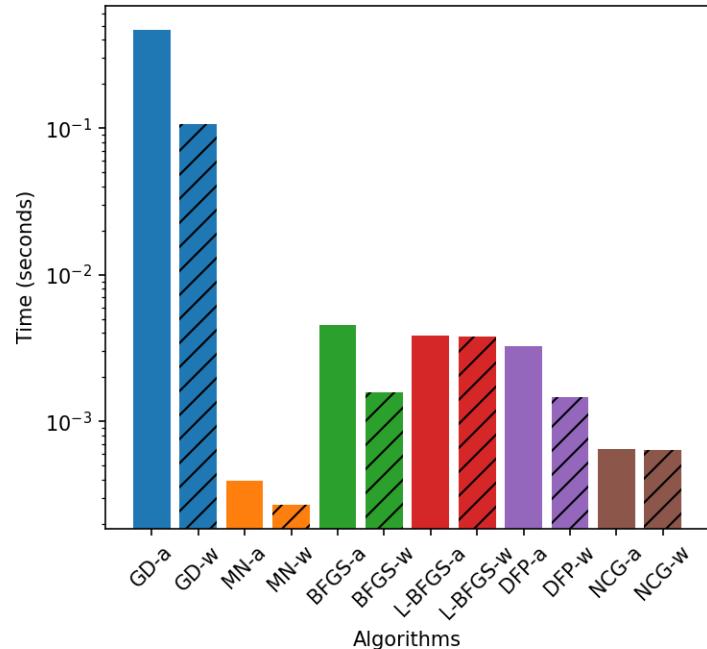


Problem 2

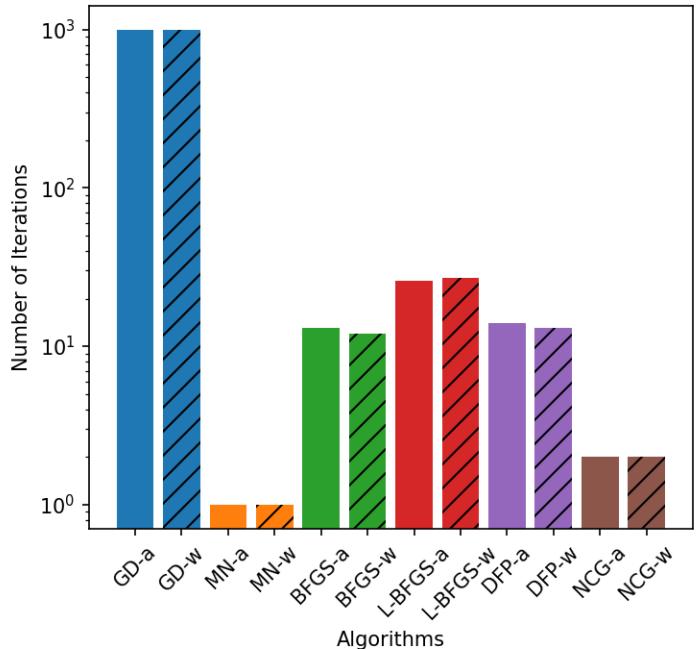
	Iters	Time	Convergence	Func Evals	Grad Evals	Hess Evals
GD-a	999	0.46963	Failed. Maximum iterations reached.	60998	2000	0
GD-w	999	0.106984	Failed. Maximum iterations reached.	10965	1000	0
MN-a	1	0.000394821	Converged. Gradient norm is below tolerance.	3	4	2
MN-w	1	0.000270605	Converged. Gradient norm is below tolerance.	3	3	2
BFGS-a	13	0.00450945	Converged. Gradient norm is below tolerance.	489	41	0
BFGS-w	12	0.00158501	Converged. Gradient norm is below tolerance.	100	26	0
L-BFGS-a	26	0.00386119	Converged. Gradient norm is below tolerance.	114	79	0
L-BFGS-w	27	0.00379038	Converged. Gradient norm is below tolerance.	67	55	0
DFP-a	14	0.00326848	Converged. Gradient norm is below tolerance.	321	44	0
DFP-w	13	0.00146627	Converged. Gradient norm is below tolerance.	81	28	0
NCG-a	2	0.000643015	Converged. Gradient norm is below tolerance.	5	6	3
NCG-w	2	0.000637531	Converged. Gradient norm is below tolerance.	5	4	3



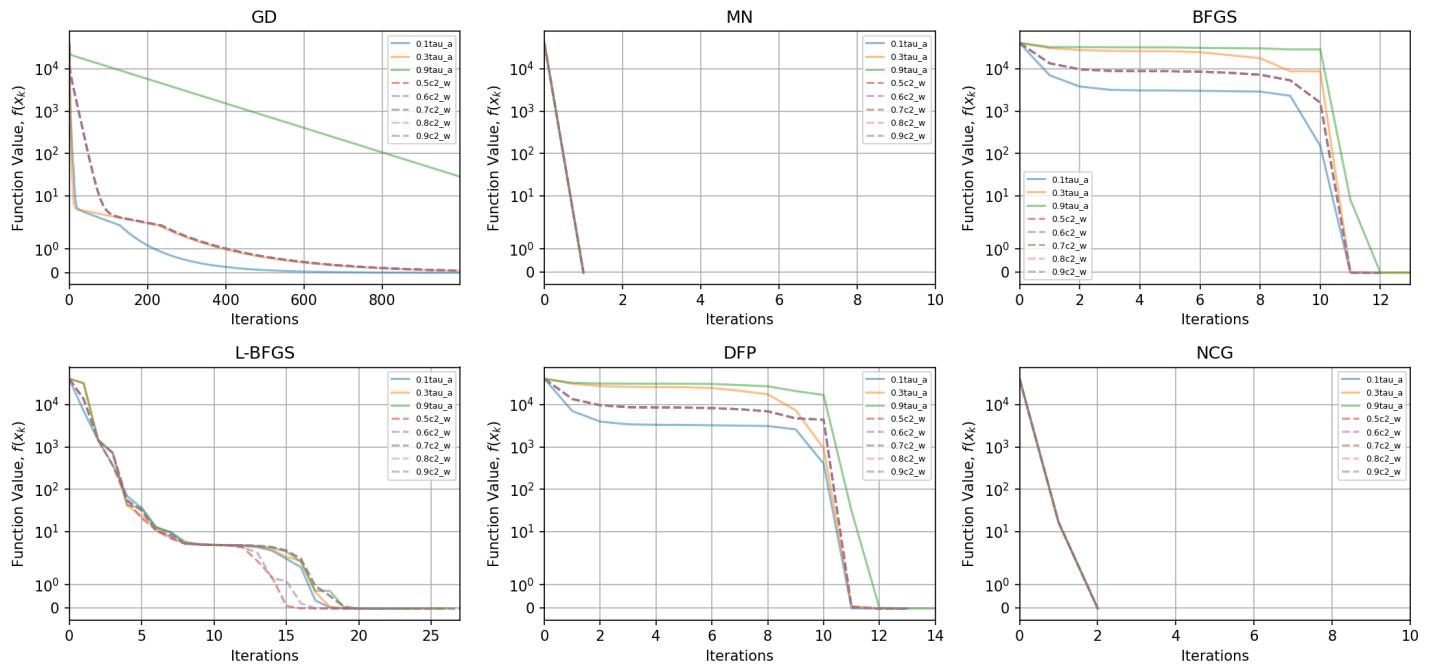
Problem2 - Total Time



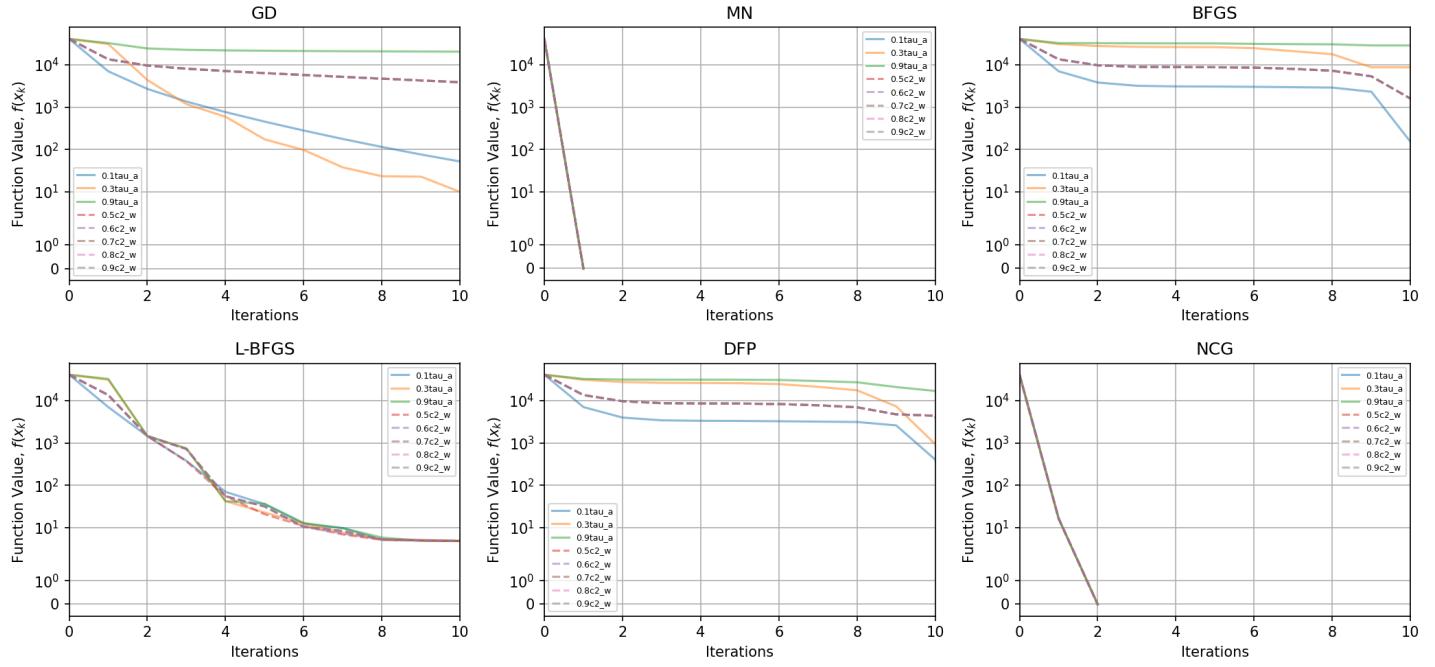
Problem2 - Total Iterations



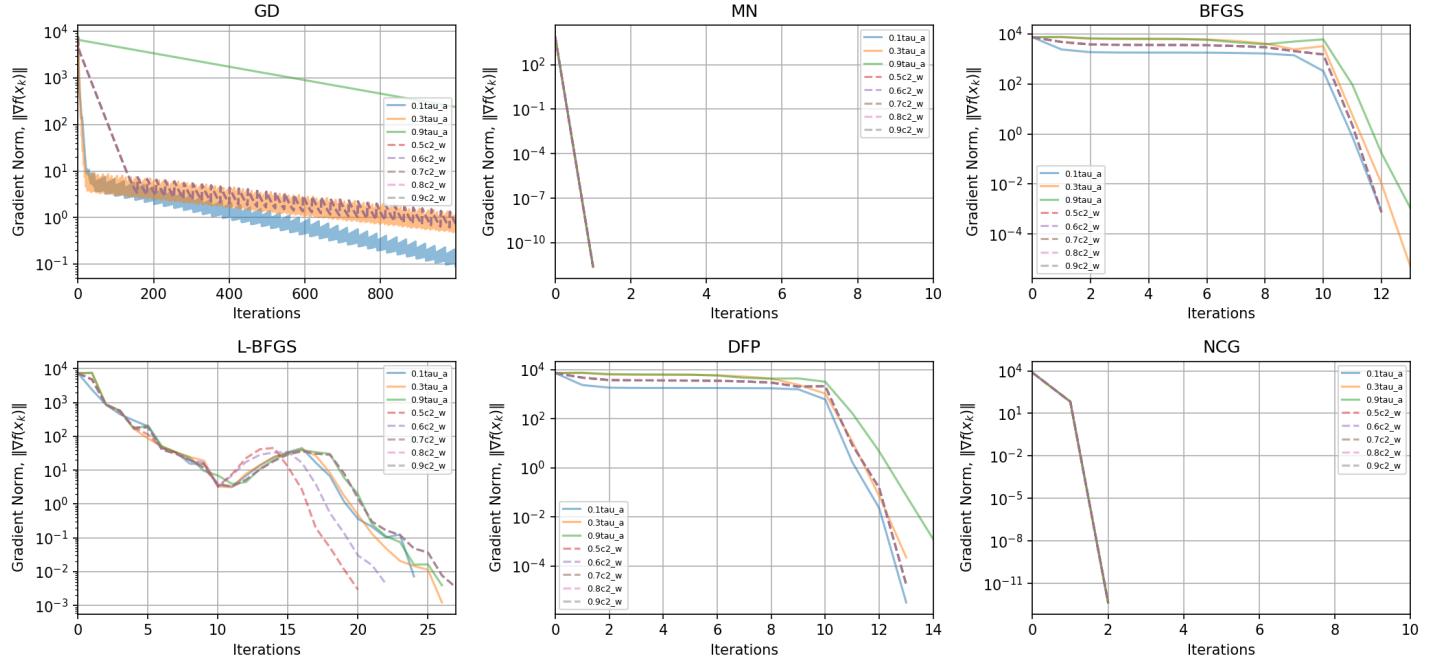
Problem2 - Function Value with Different c2 and tau

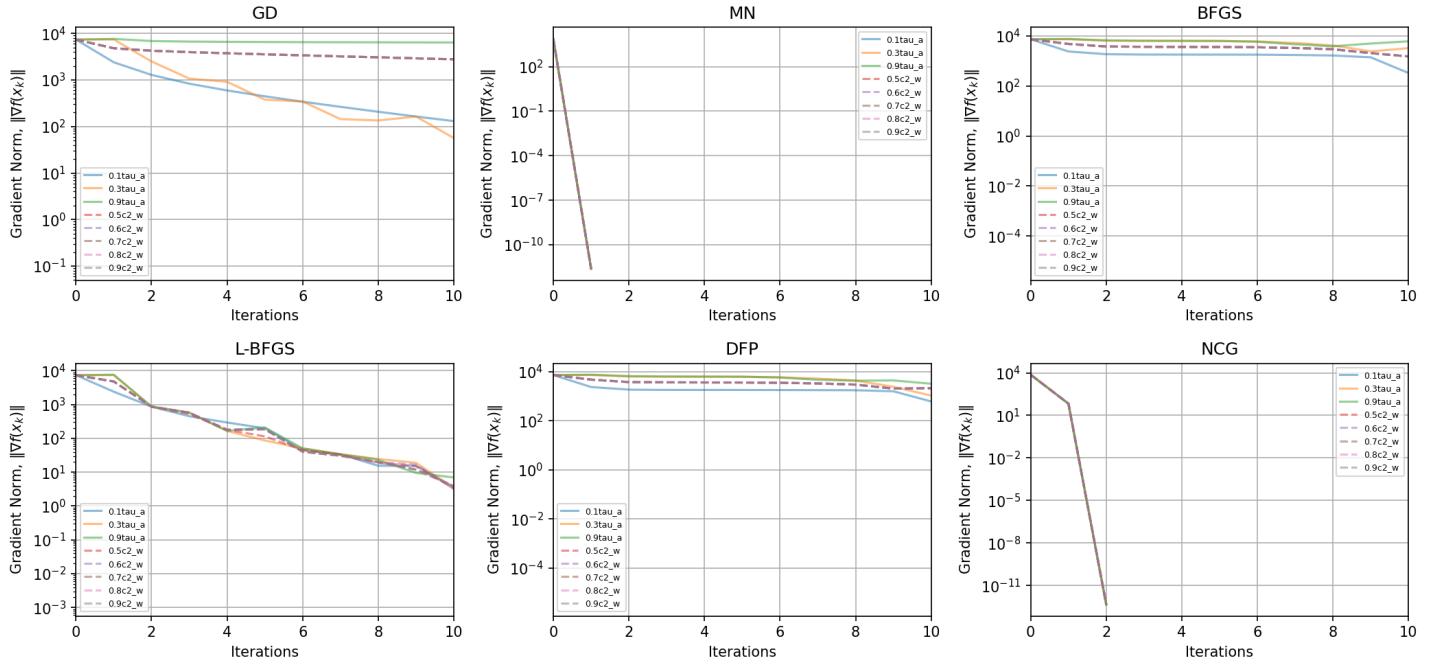


Problem2 - Function Value with Different c_2 and τ (zoom)



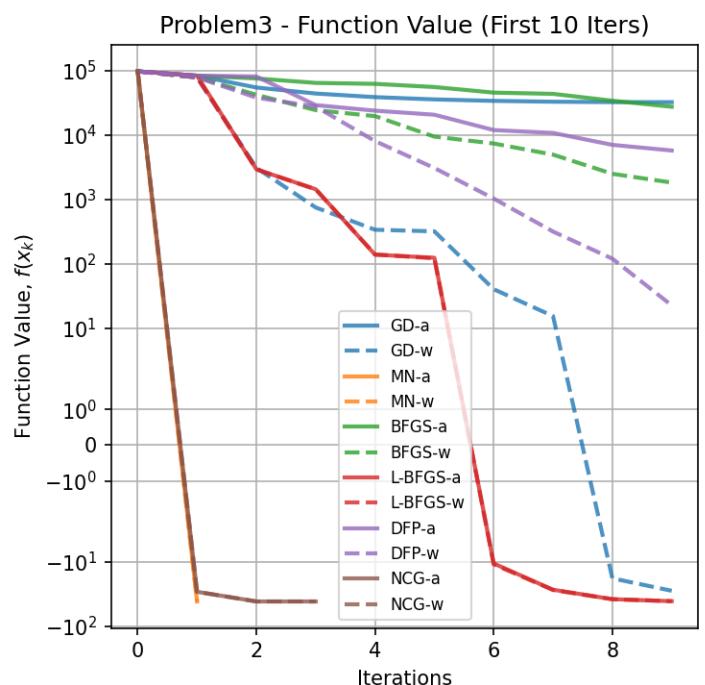
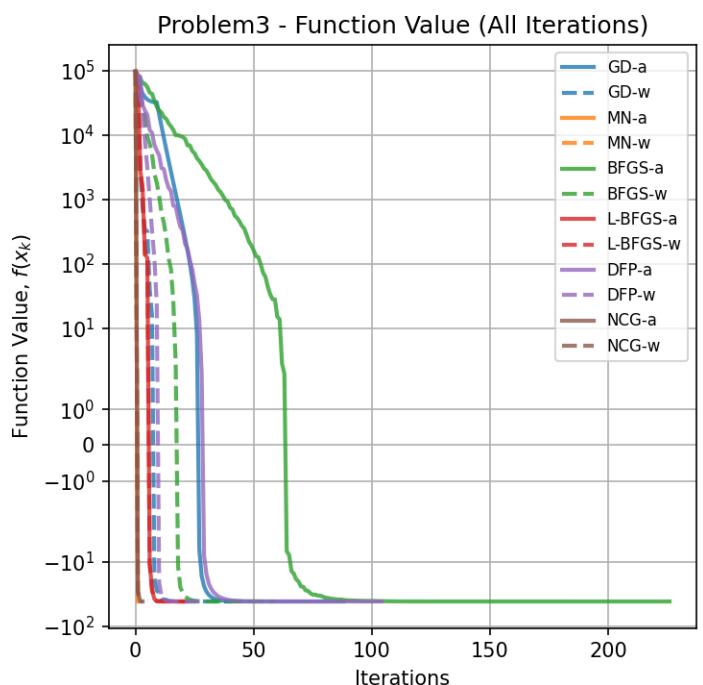
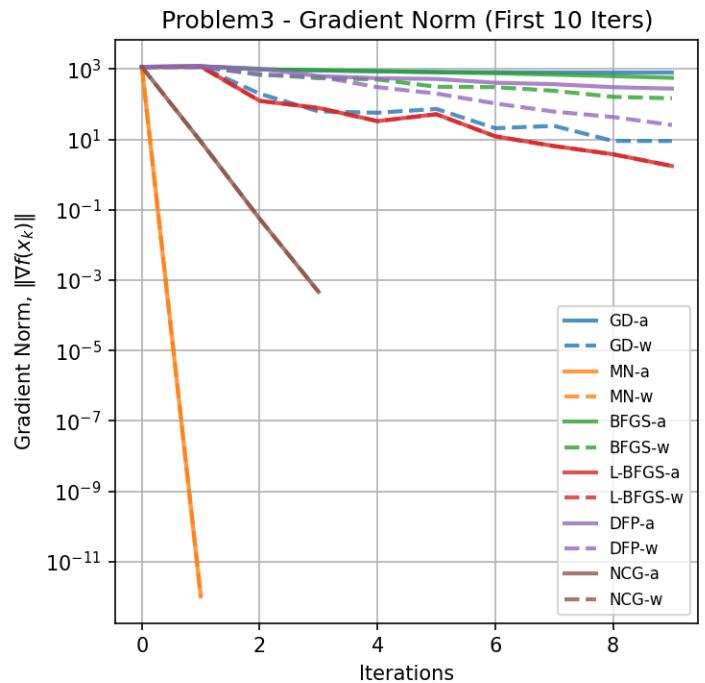
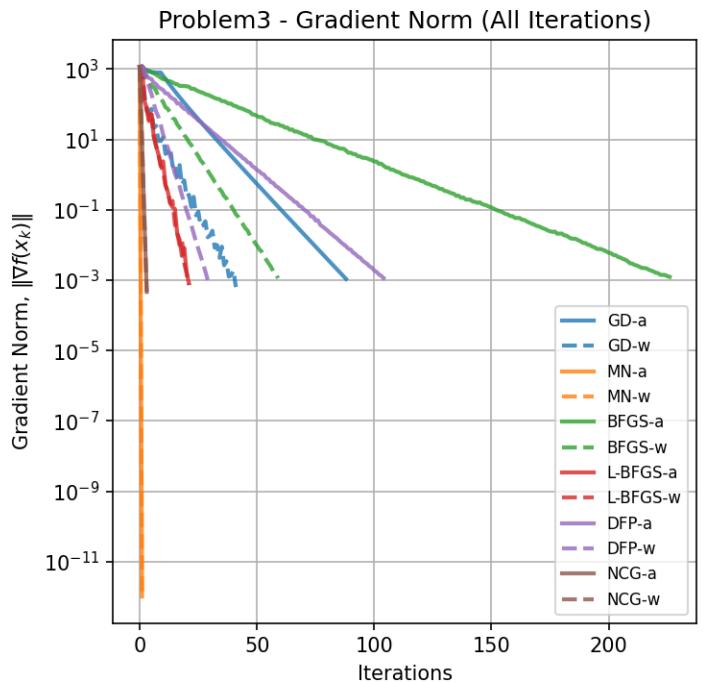
Problem2 - Gradient Norm with Different c_2 and tau

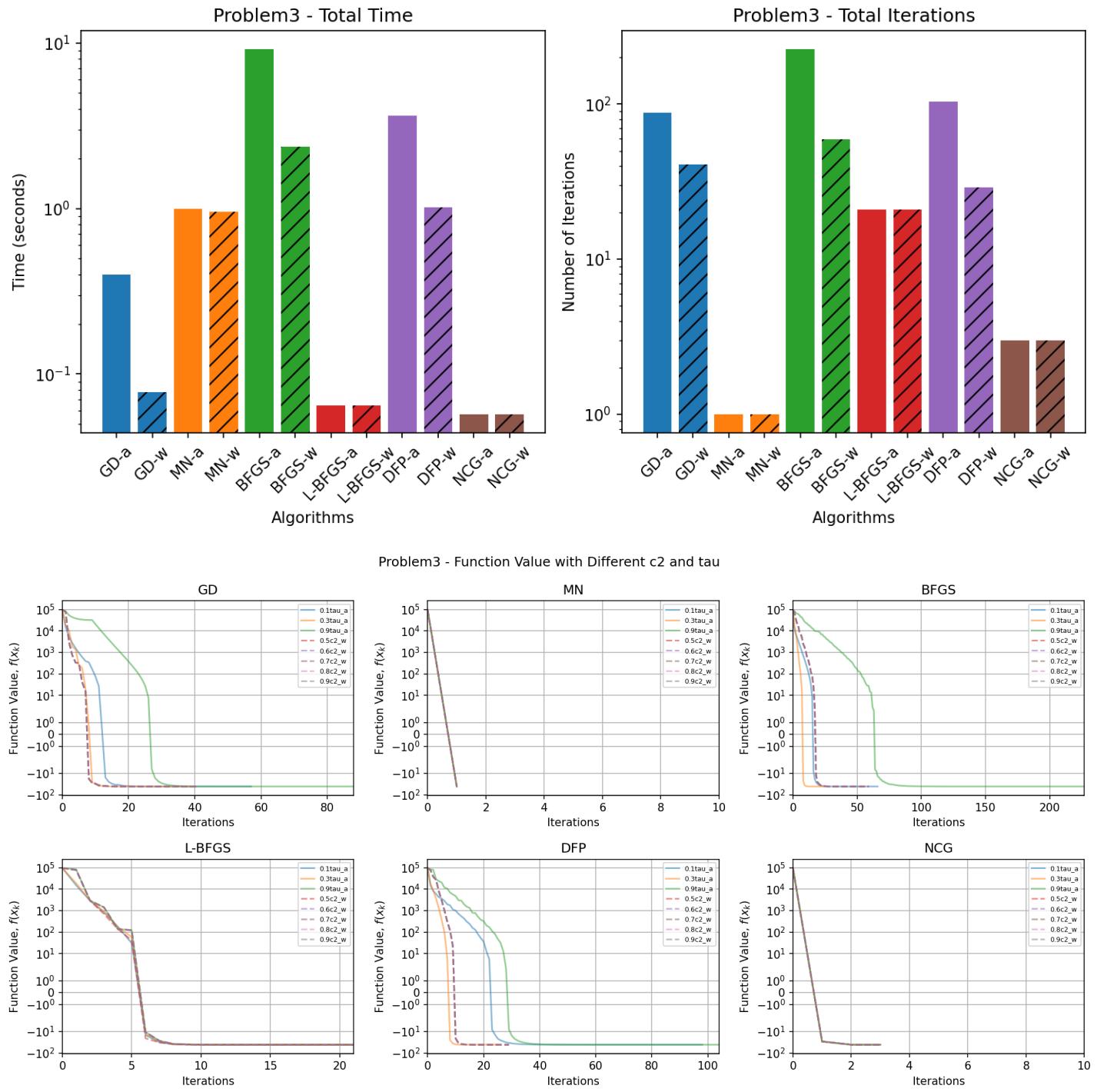


Problem2 - Gradient Norm with Different c_2 and τ (zoom)

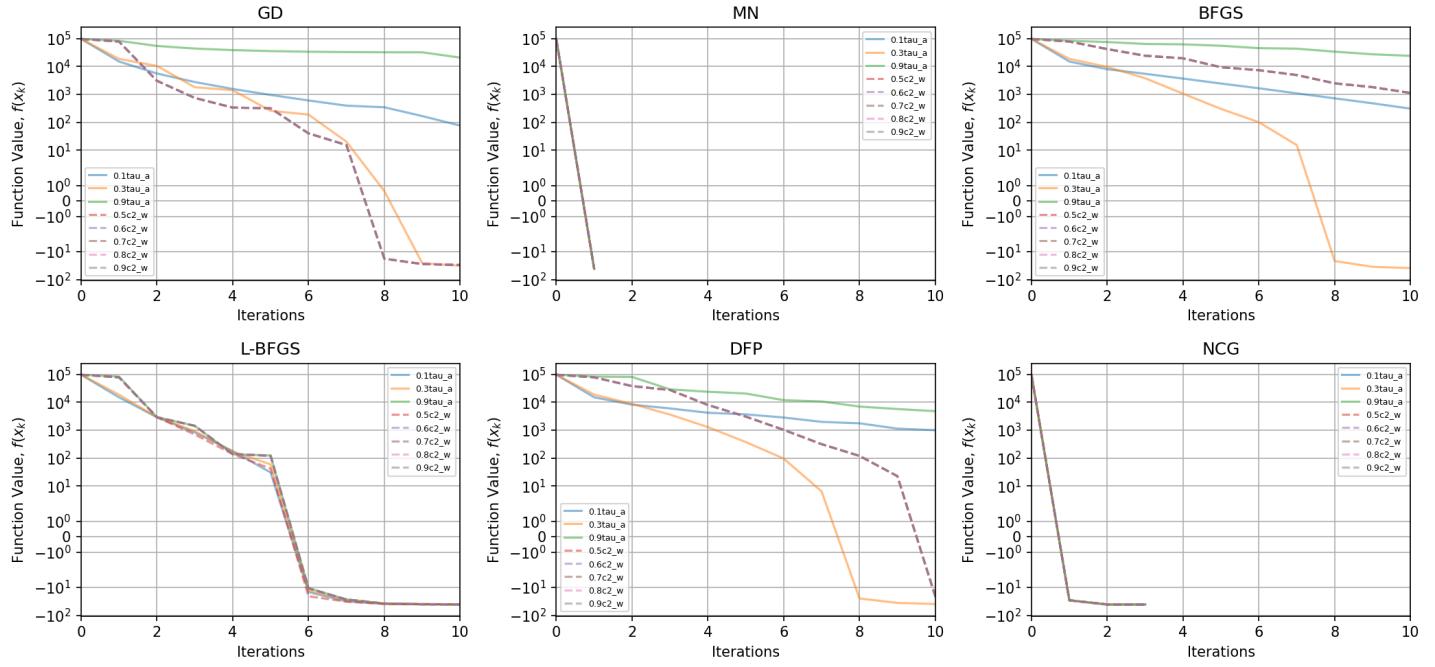
Problem 3

	Iters	Time	Convergence	Func Evals	Grad Evals	Hess Evals
GD-a	88	0.3963777	Converged. Gradient norm is below tolerance.	1574	177	0
GD-w	41	0.0774596	Converged. Gradient norm is below tolerance.	180	42	0
MN-a	1	0.998708	Converged. Gradient norm is below tolerance.	3	4	2
MN-w	1	0.958103	Converged. Gradient norm is below tolerance.	3	3	2
BFGS-a	227	9.21601	Converged. Gradient norm is below tolerance.	1361	683	0
BFGS-w	59	2.37059	Converged. Gradient norm is below tolerance.	179	120	0
L-BFGS-a	21	0.0642378	Converged. Gradient norm is below tolerance.	59	64	0
L-BFGS-w	21	0.0641718	Converged. Gradient norm is below tolerance.	46	43	0
DFP-a	104	3.65116	Converged. Gradient norm is below tolerance.	229	314	0
DFP-w	29	1.01067	Converged. Gradient norm is below tolerance.	88	60	0
NCG-a	3	0.0570514	Converged. Gradient norm is below tolerance.	7	8	4
NCG-w	3	0.0571702	Converged. Gradient norm is below tolerance.	7	5	4

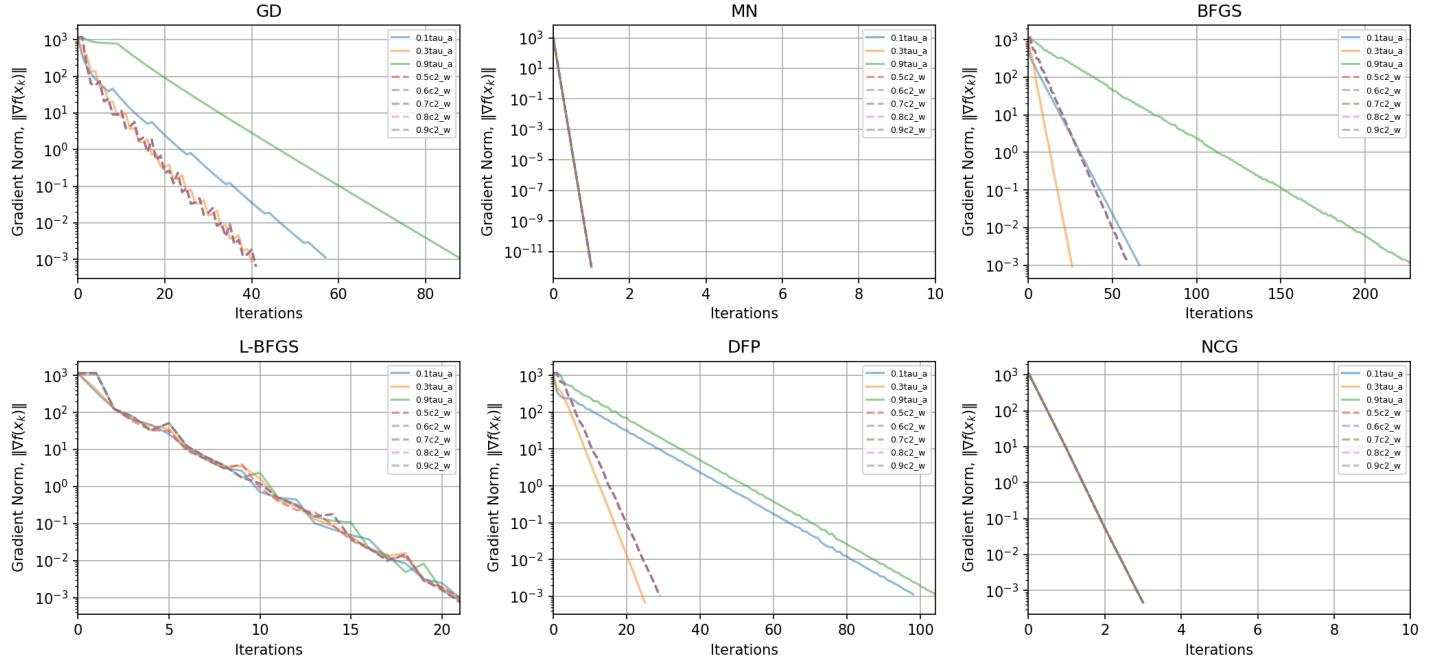




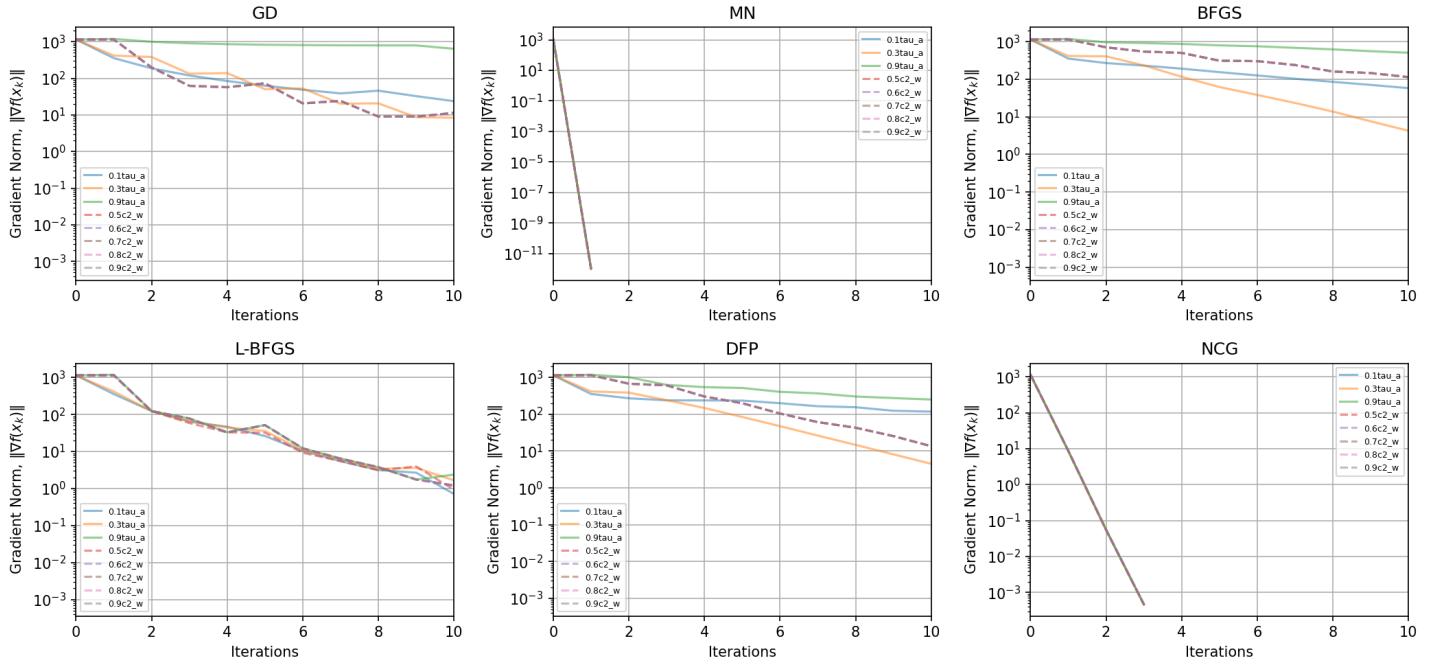
Problem3 - Function Value with Different c_2 and τ (zoom)



Problem3 - Gradient Norm with Different c_2 and τ

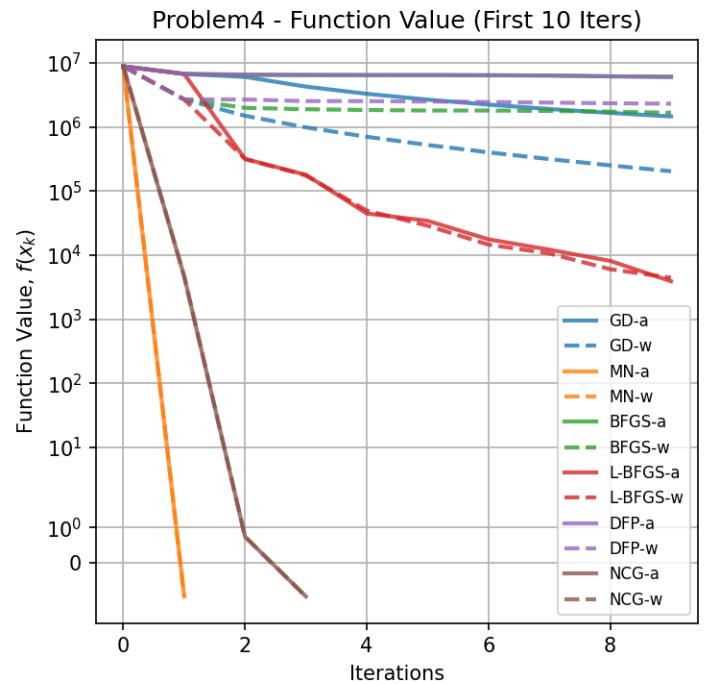
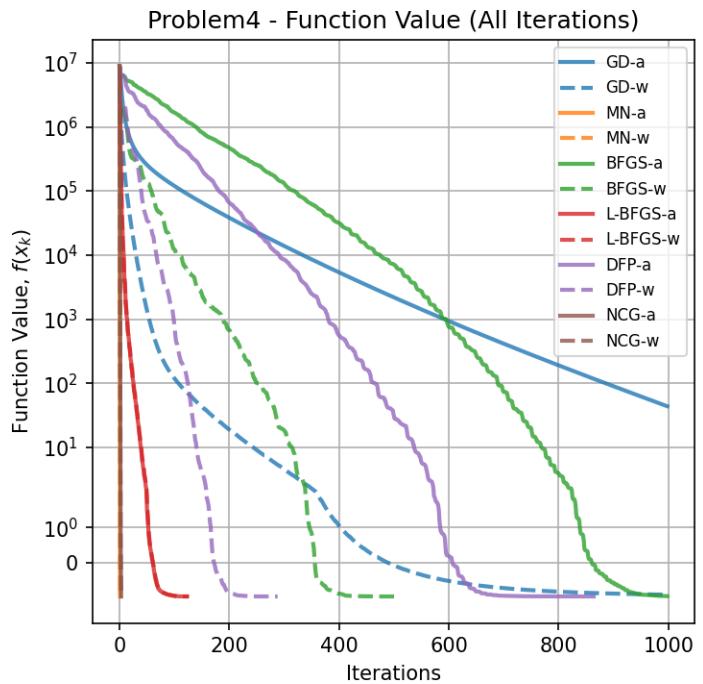
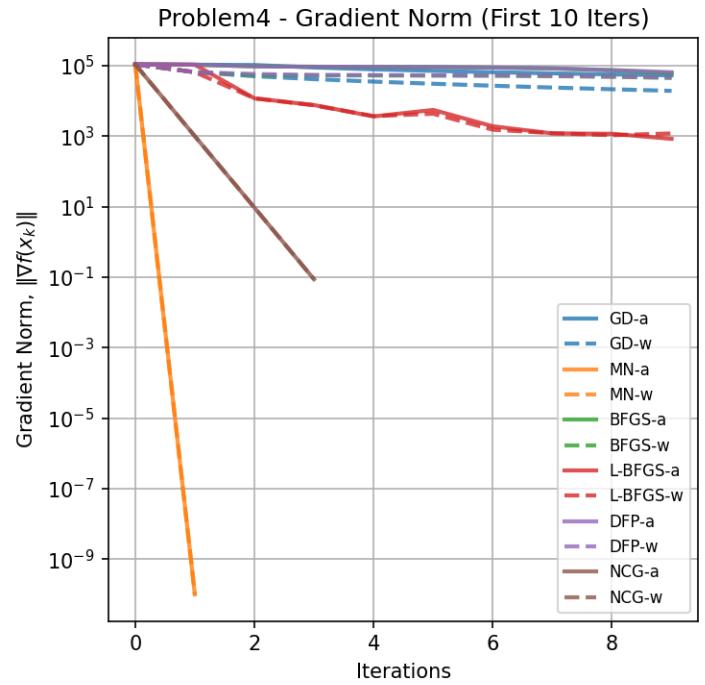
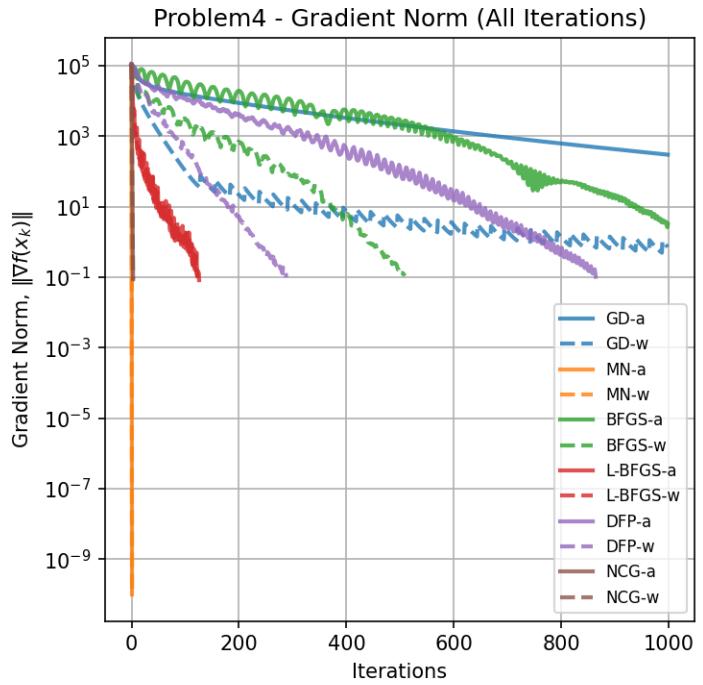


Problem3 - Gradient Norm with Different c_2 and τ (zoom)

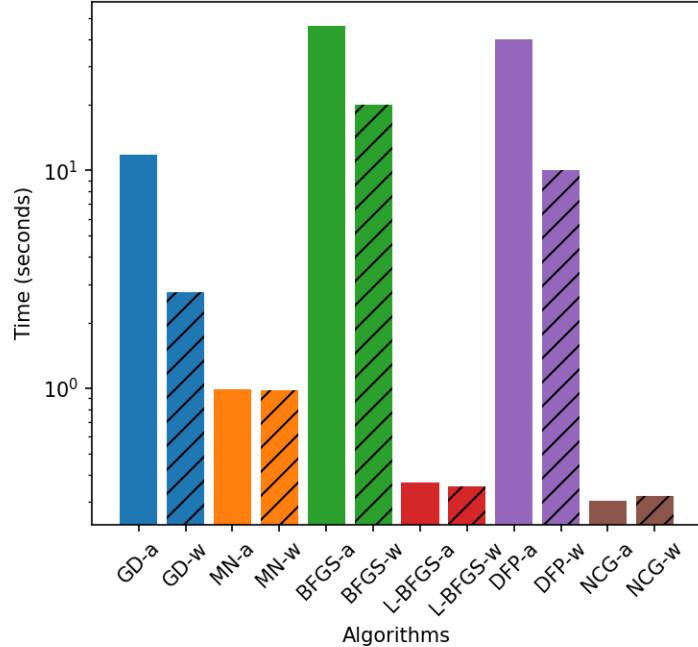


Problem 4

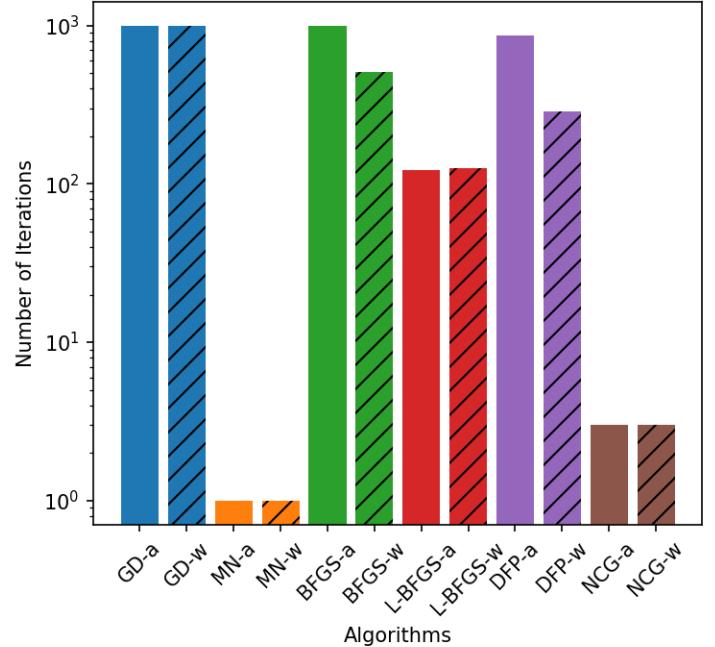
	Iters	Time	Convergence	Func Evals	Grad Evals	Hess Evals
GD-a	999	11.7596	Failed. Maximum iterations reached.	60997	2000	0
GD-w	999	2.74733	Failed. Maximum iterations reached.	10965	1000	0
MN-a	1	0.992127	Converged. Gradient norm is below tolerance.	3	4	2
MN-w	1	0.982068	Converged. Gradient norm is below tolerance.	3	3	2
BFGS-a	999	46.23	Failed. Maximum iterations reached.	47127	3001	0
BFGS-w	508	20.0932	Converged. Gradient norm is below tolerance.	4779	1018	0
L-BFGS-a	123	0.367558	Converged. Gradient norm is below tolerance.	316	370	0
L-BFGS-w	126	0.354212	Converged. Gradient norm is below tolerance.	267	253	0
DFP-a	864	39.9289	Converged. Gradient norm is below tolerance.	19411	2594	0
DFP-w	288	10.0012	Converged. Gradient norm is below tolerance.	1750	578	0
NCG-a	3	0.304365	Converged. Gradient norm is below tolerance.	7	8	4
NCG-w	3	0.319217	Converged. Gradient norm is below tolerance.	7	5	4



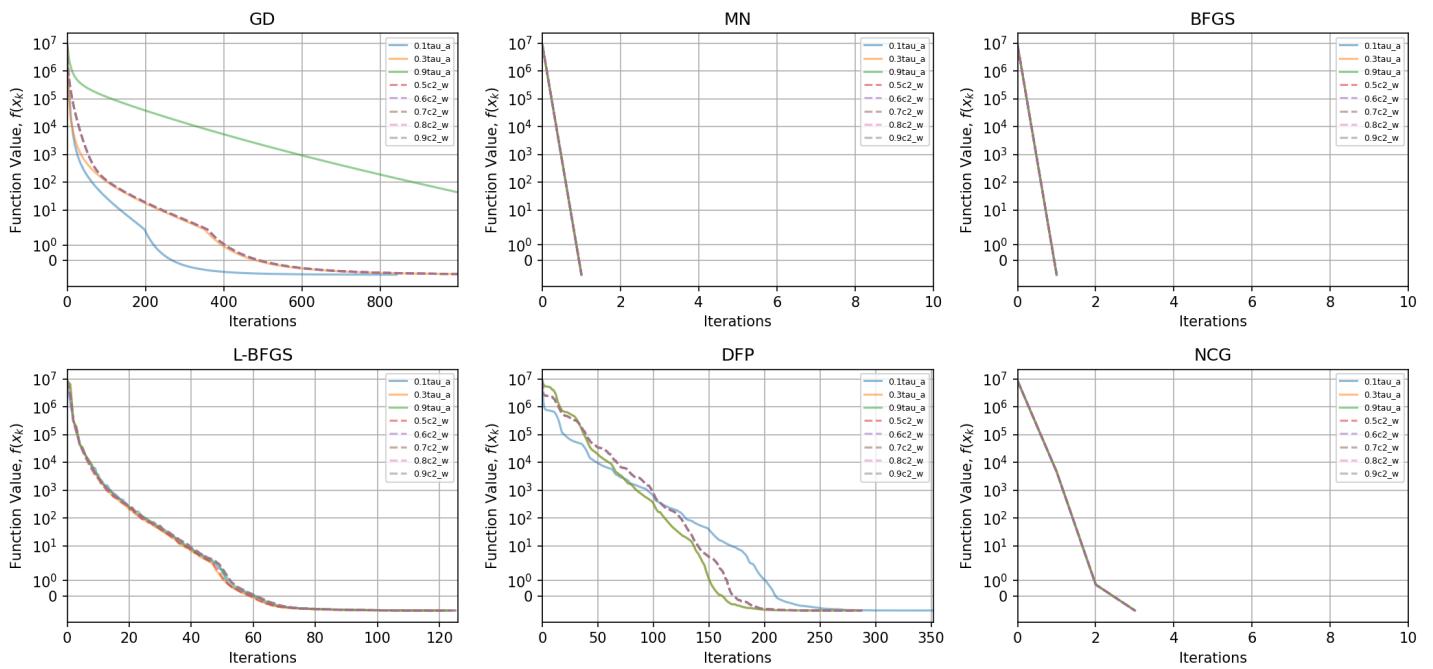
Problem4 - Total Time



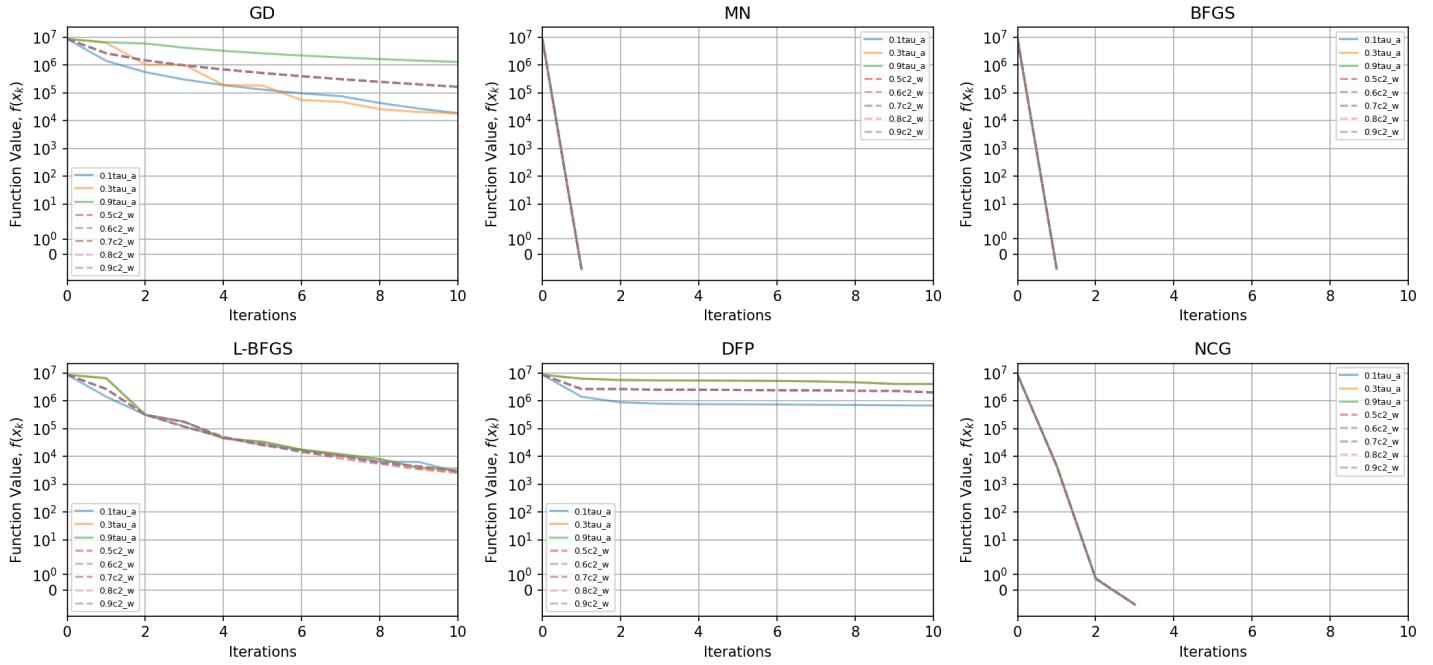
Problem4 - Total Iterations



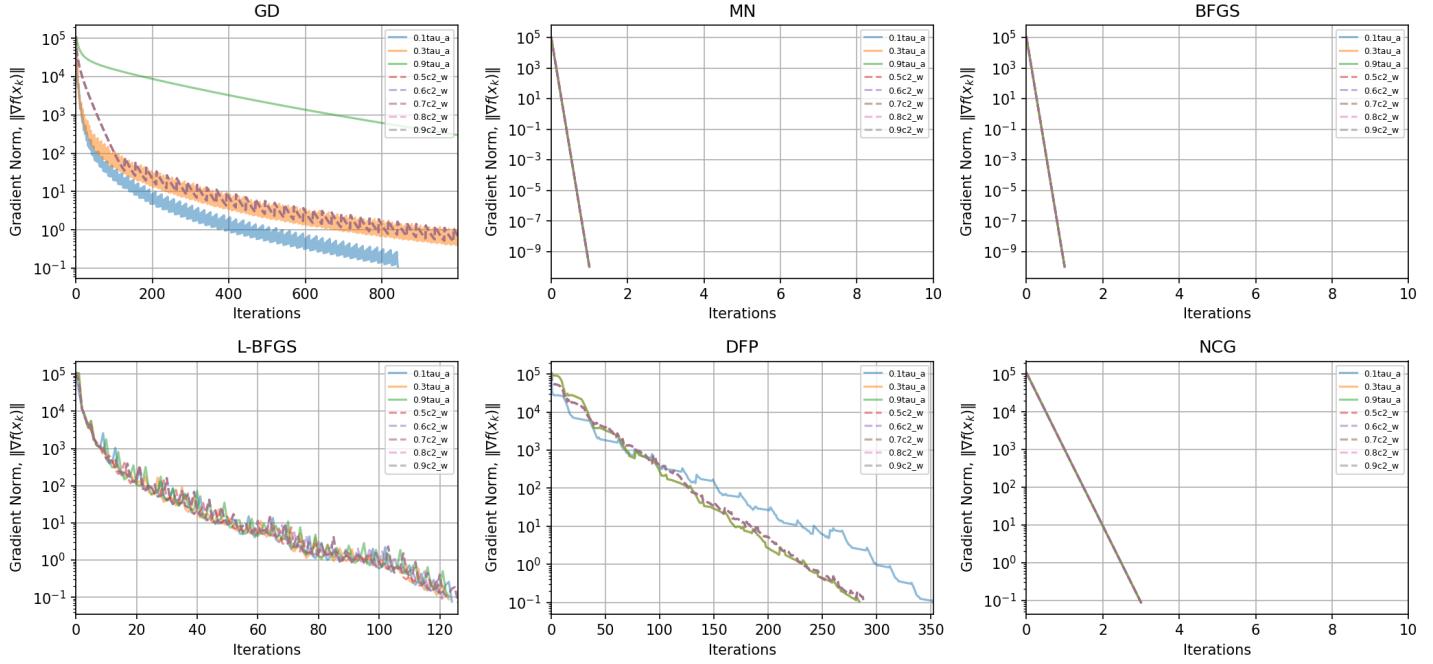
Problem4 - Function Value with Different c2 and tau



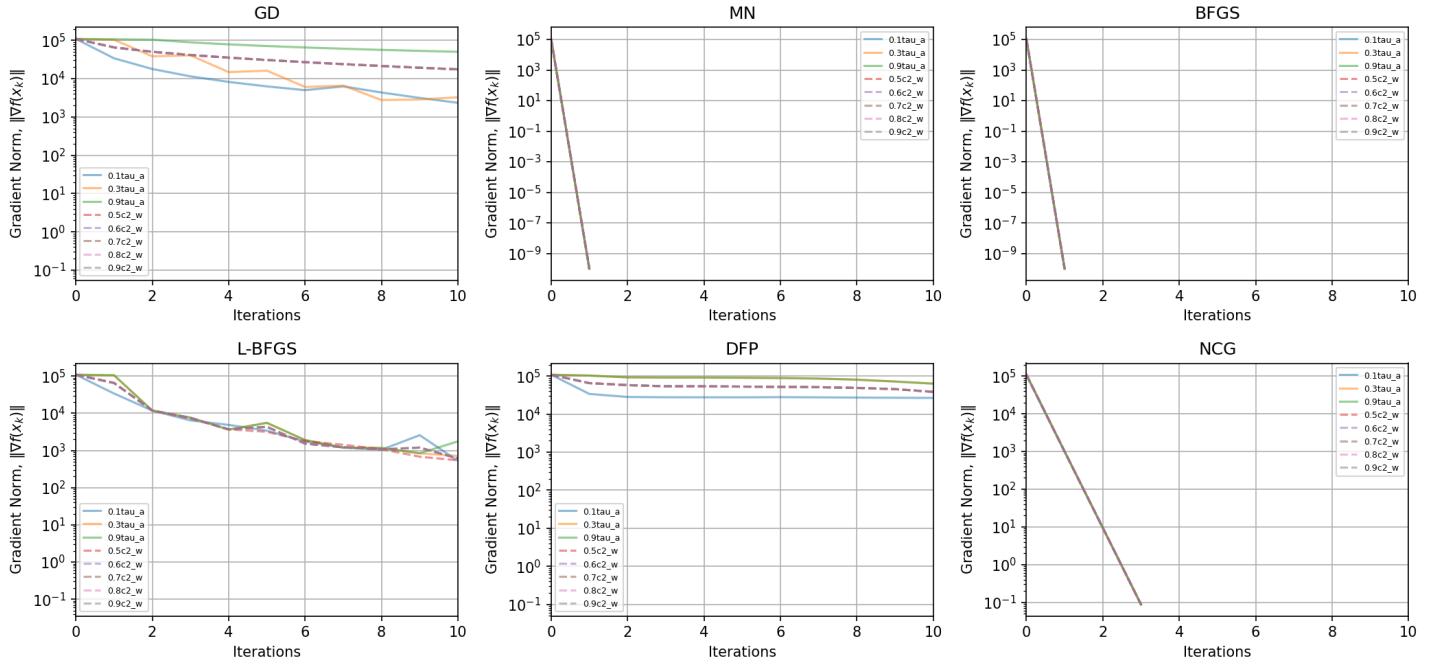
Problem4 - Function Value with Different c_2 and τ (zoom)



Problem4 - Gradient Norm with Different c_2 and τ



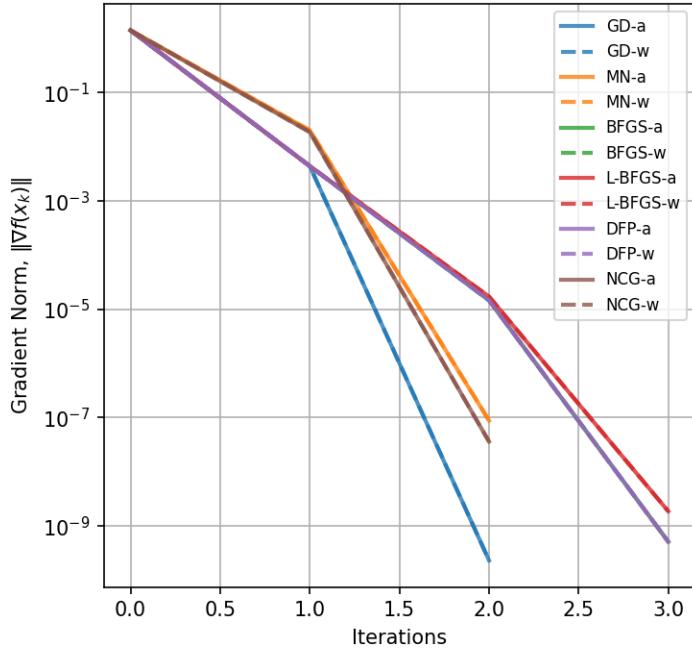
Problem4 - Gradient Norm with Different c_2 and τ (zoom)



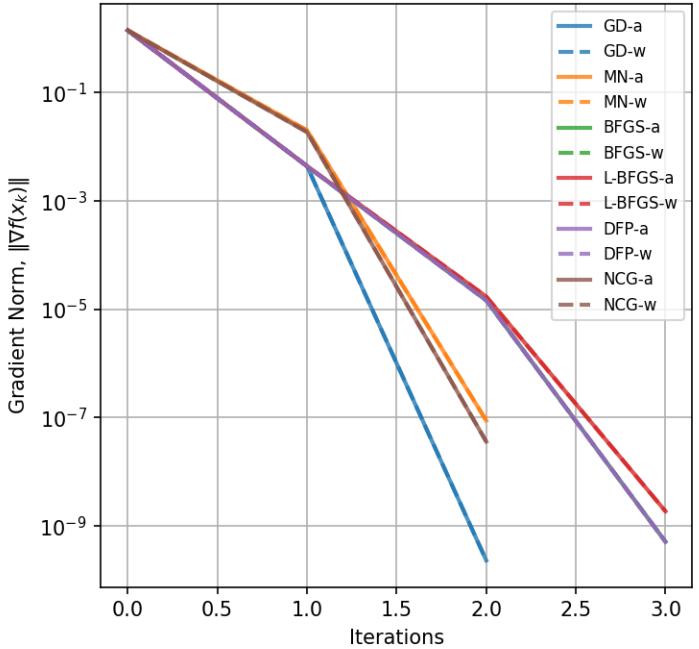
Problem 5

	Iters	Time	Convergence	Func Evals	Grad Evals	Hess Evals
GD-a	2	0.000328779	Converged. Gradient norm is below tolerance.	5	5	0
GD-w	2	0.000269413	Converged. Gradient norm is below tolerance.	5	3	0
MN-a	2	0.00072217	Converged. Gradient norm is below tolerance.	5	6	3
MN-w	2	0.000583649	Converged. Gradient norm is below tolerance.	5	4	3
BFGS-a	3	0.000513315	Converged. Gradient norm is below tolerance.	7	11	0
BFGS-w	3	0.00051403	Converged. Gradient norm is below tolerance.	7	8	0
L-BFGS-a	3	0.000490189	Converged. Gradient norm is below tolerance.	7	10	0
L-BFGS-w	3	0.000533819	Converged. Gradient norm is below tolerance.	7	7	0
DFP-a	3	0.000446558	Converged. Gradient norm is below tolerance.	7	11	0
DFP-w	3	0.000475168	Converged. Gradient norm is below tolerance.	7	8	0
NCG-a	2	0.000369072	Converged. Gradient norm is below tolerance.	5	6	3
NCG-w	2	0.00038147	Converged. Gradient norm is below tolerance.	5	4	3

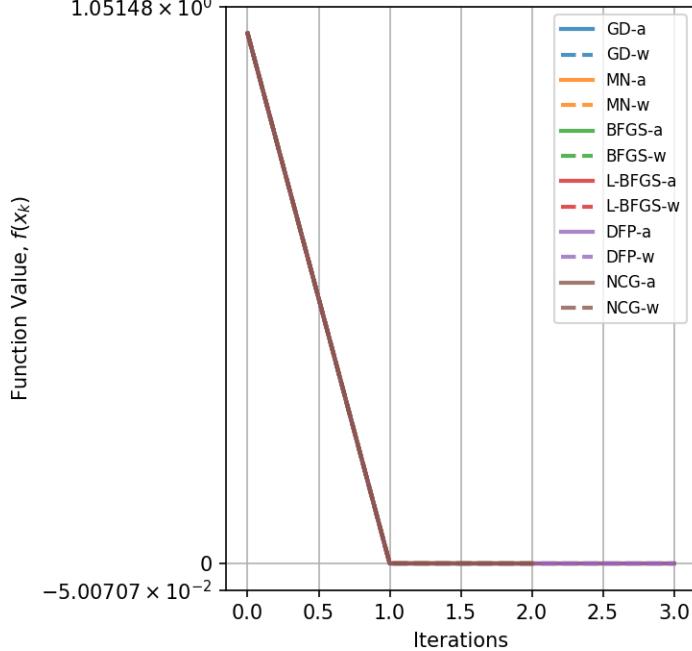
Problem5 - Gradient Norm (All Iterations)



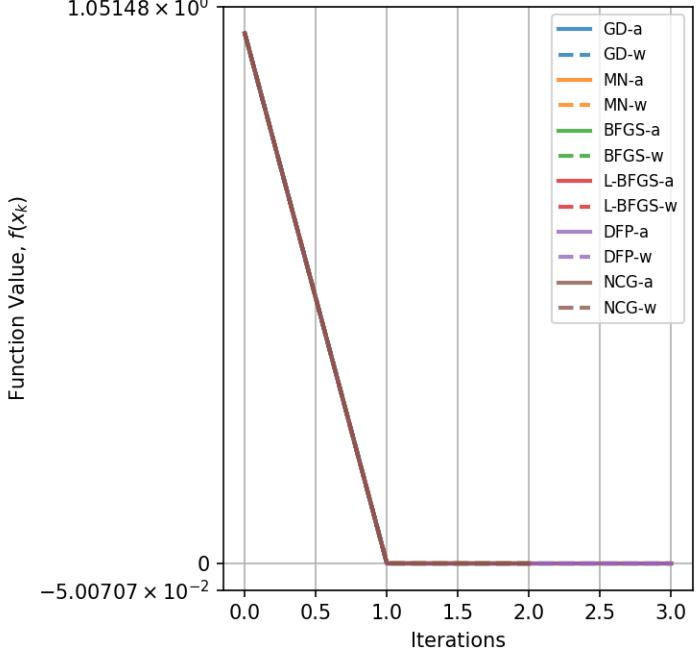
Problem5 - Gradient Norm (First 10 Iters)

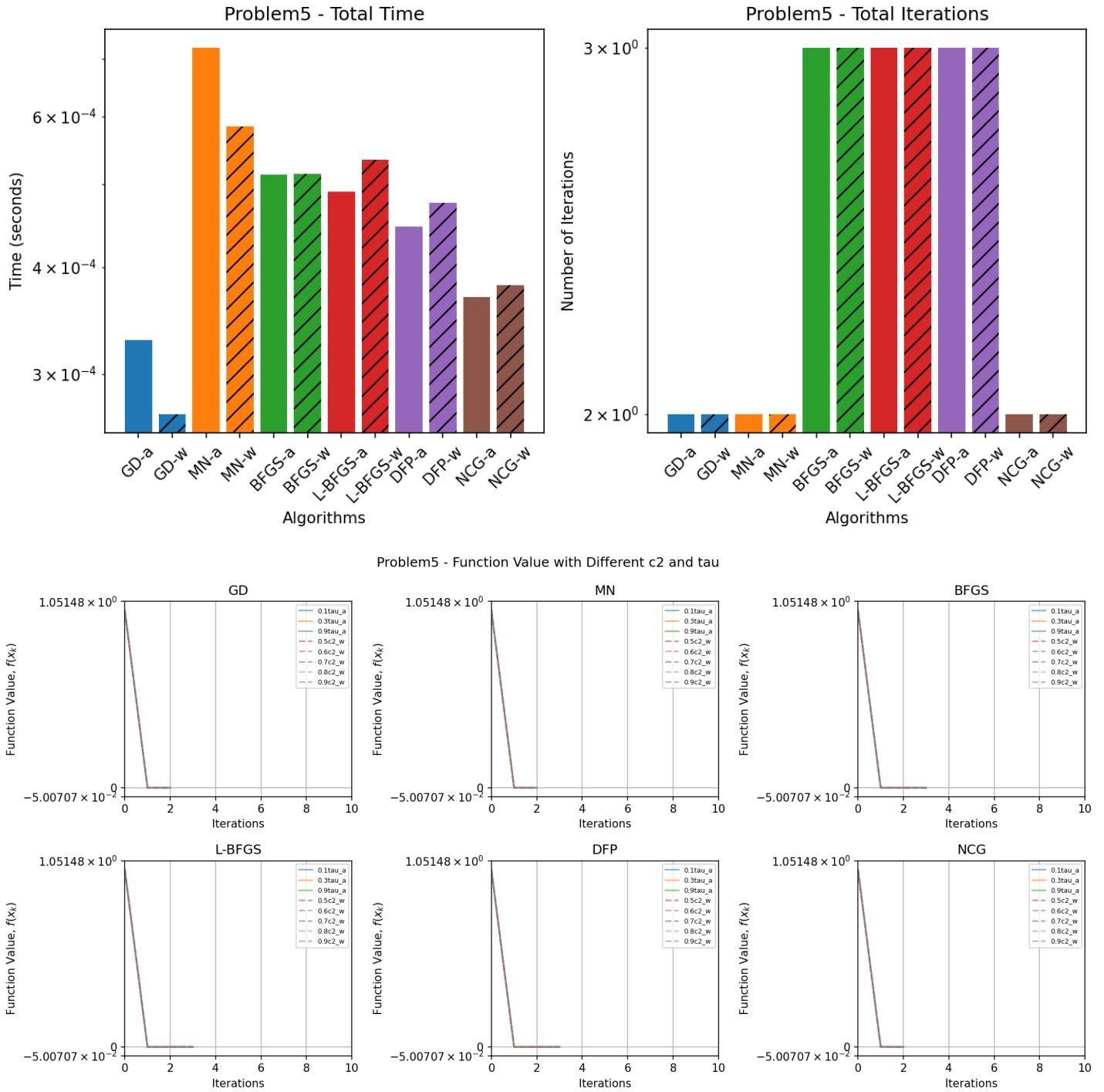


Problem5 - Function Value (All Iterations)

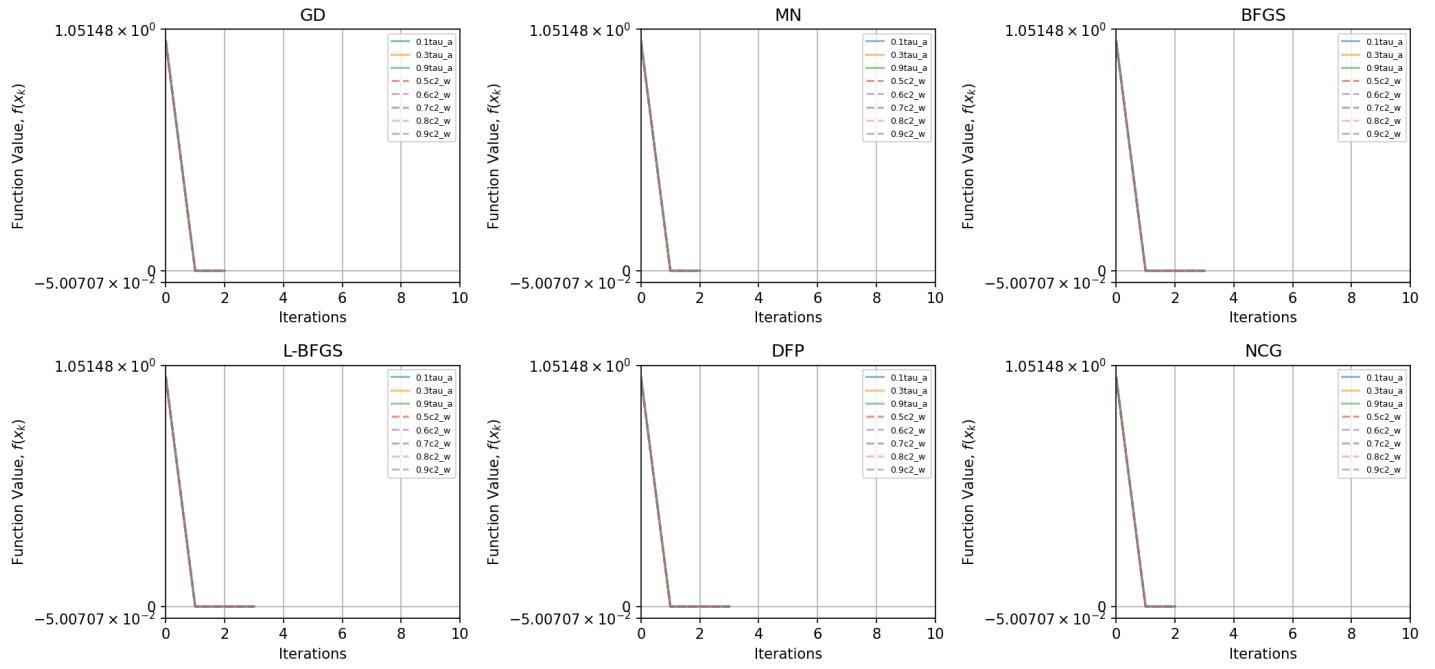


Problem5 - Function Value (First 10 Iters)

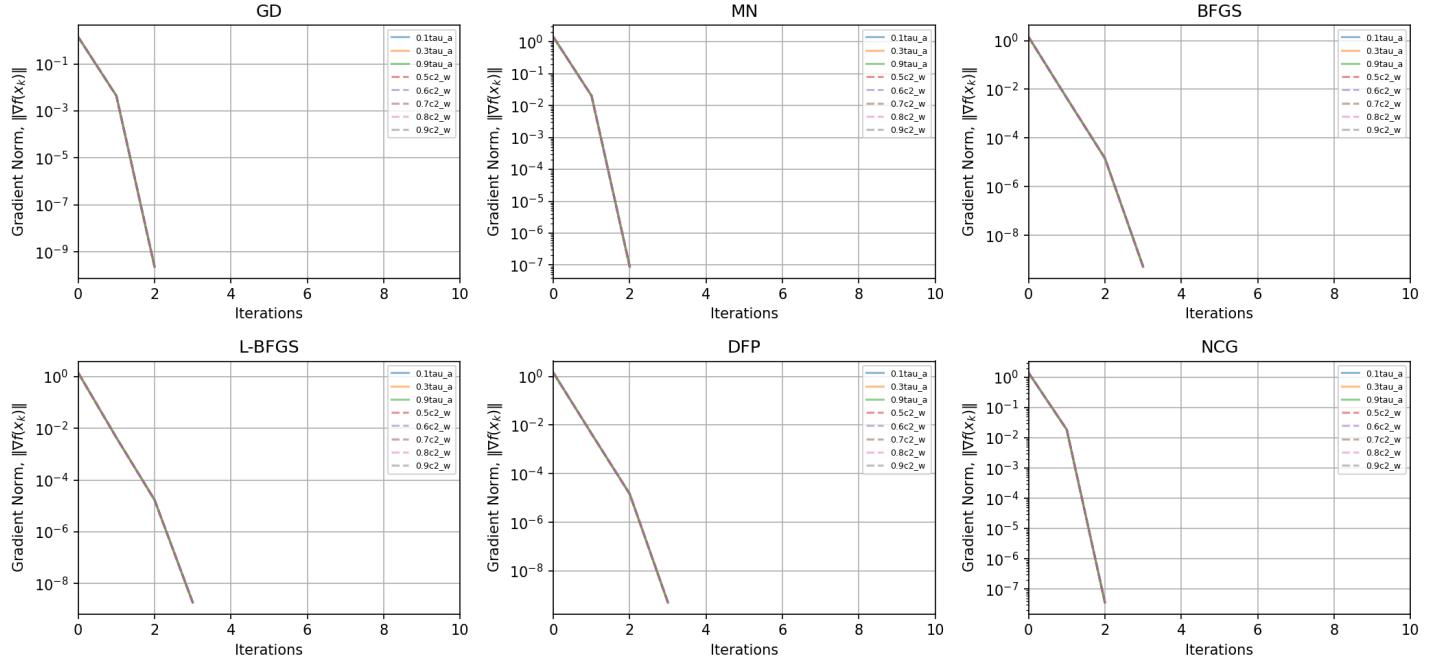




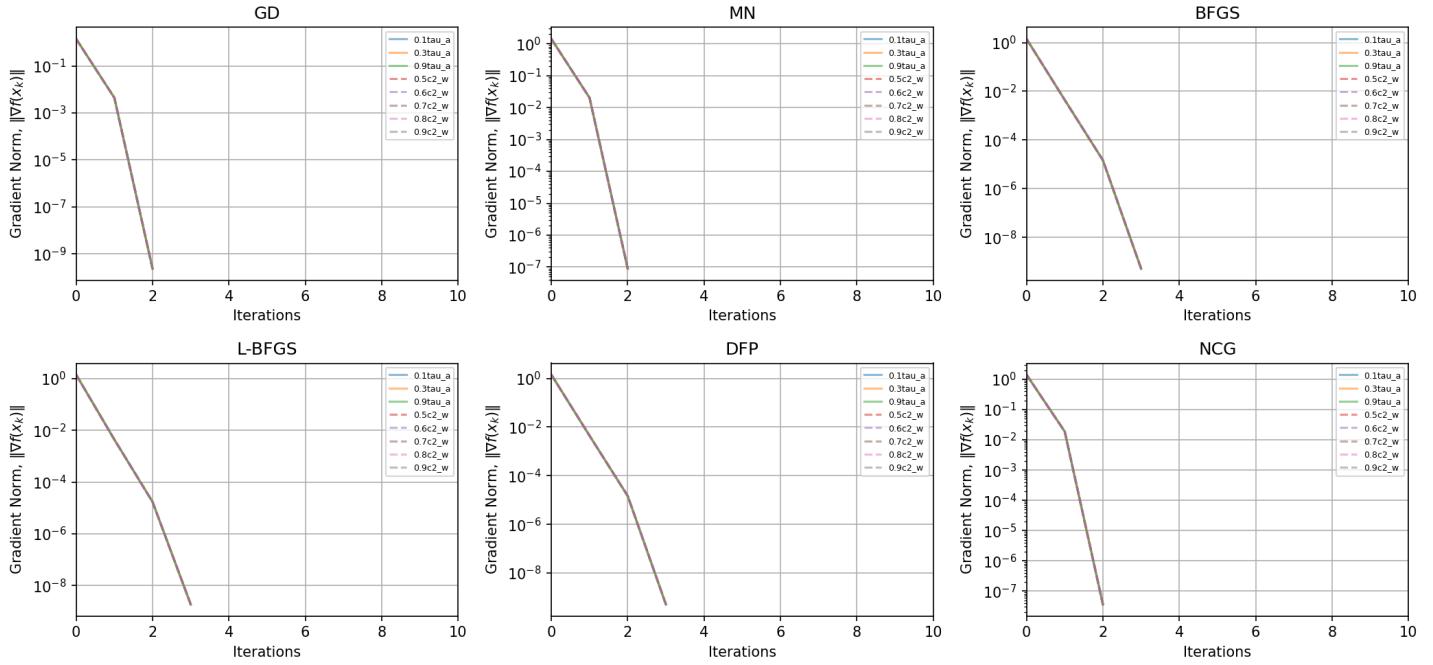
Problem5 - Function Value with Different c2 and tau (zoom)



Problem5 - Gradient Norm with Different c2 and tau



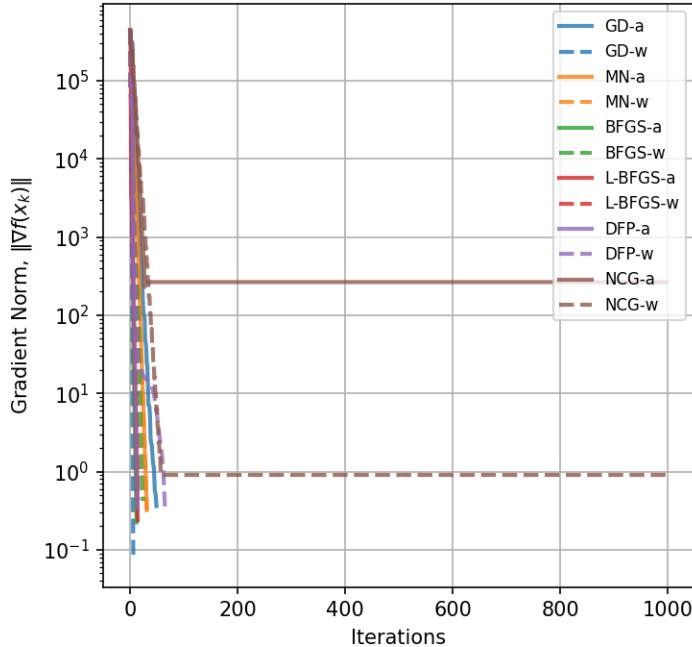
Problem5 - Gradient Norm with Different c_2 and τ (zoom)



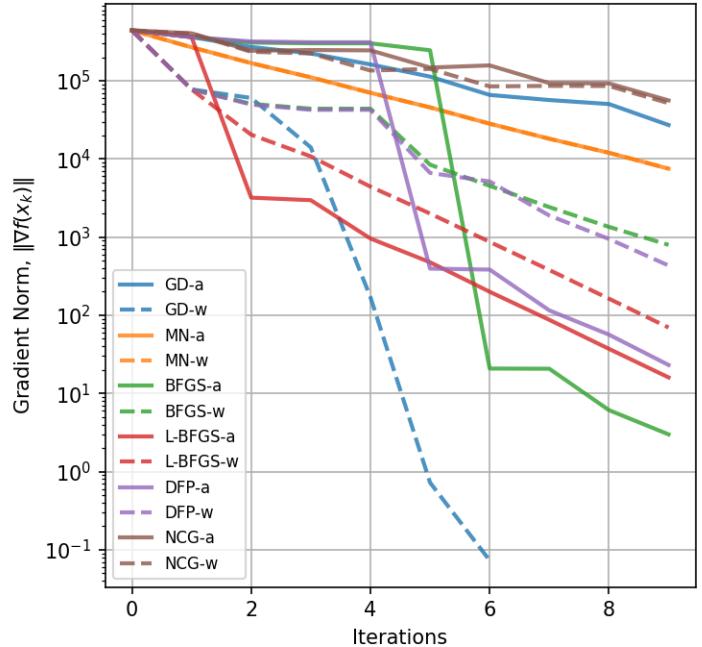
Problem 6

	Iters	Time	Convergence	Func Evals	Grad Evals	Hess Evals
GD-a	49	0.0399251	Converged. Gradient norm is below tolerance.	3463	99	0
GD-w	6	0.00139093	Converged. Gradient norm is below tolerance.	94	7	0
MN-a	31	0.0176444	Converged. Gradient norm is below tolerance.	63	64	32
MN-w	31	0.0169587	Converged. Gradient norm is below tolerance.	63	33	32
BFGS-a	12	0.00631166	Converged. Gradient norm is below tolerance.	458	38	0
BFGS-w	24	0.00440431	Converged. Gradient norm is below tolerance.	111	50	0
L-BFGS-a	14	0.00376868	Converged. Gradient norm is below tolerance.	145	43	0
L-BFGS-w	15	0.00306416	Converged. Gradient norm is below tolerance.	49	31	0
DFP-a	14	0.00632954	Converged. Gradient norm is below tolerance.	451	44	0
DFP-w	65	0.010287	Converged. Gradient norm is below tolerance.	192	132	0
NCG-a	999	4.09934	Failed. Maximum iterations reached.	349443	2001	1000
NCG-w	999	1.02456	Failed. Maximum iterations reached.	71398	1001	1000

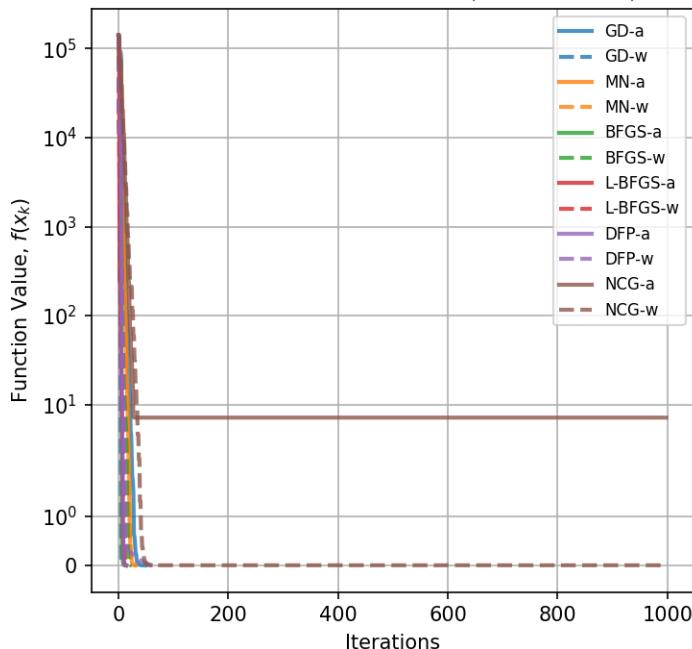
Problem6 - Gradient Norm (All Iterations)



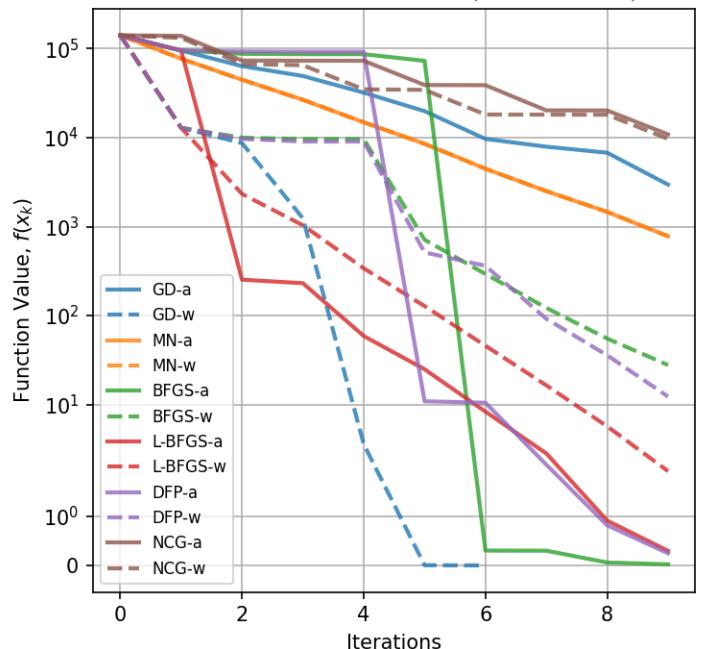
Problem6 - Gradient Norm (First 10 Iters)



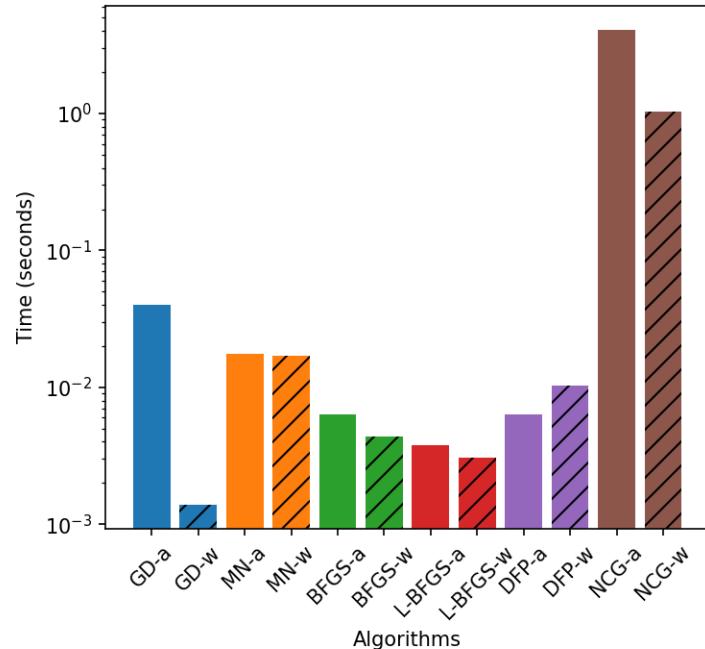
Problem6 - Function Value (All Iterations)



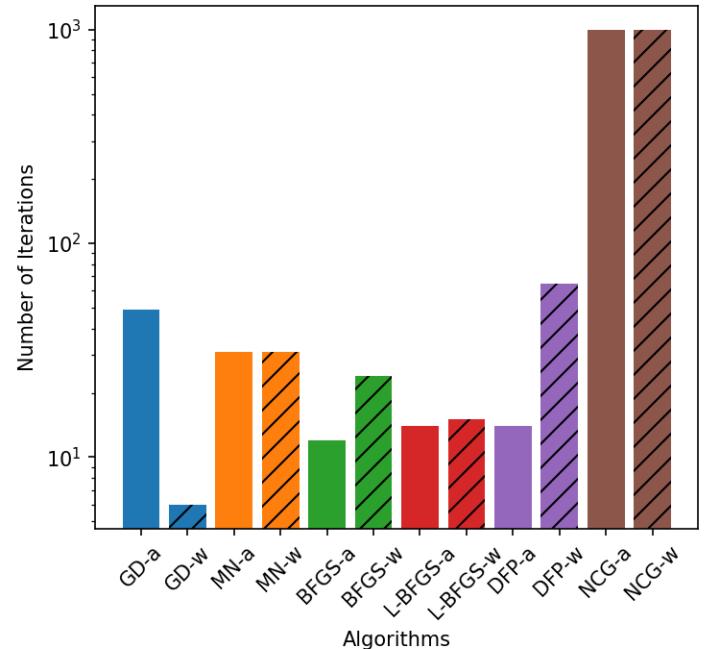
Problem6 - Function Value (First 10 Iters)



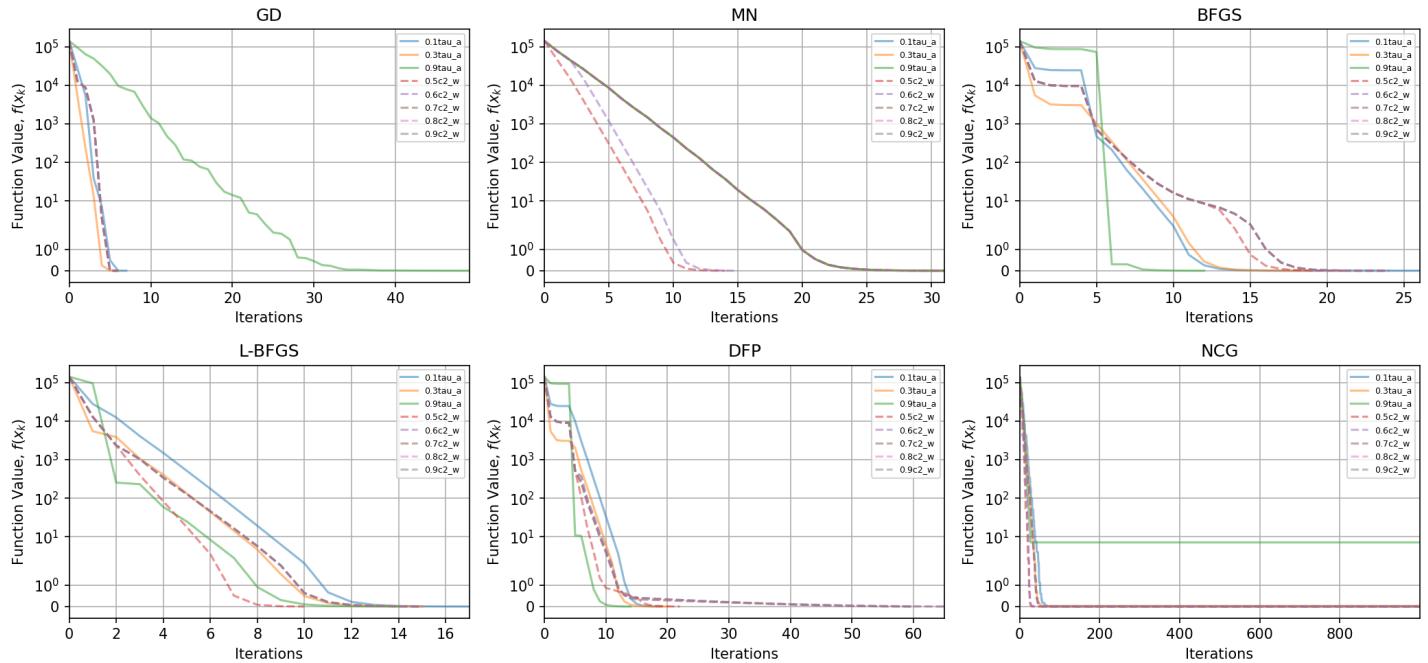
Problem6 - Total Time



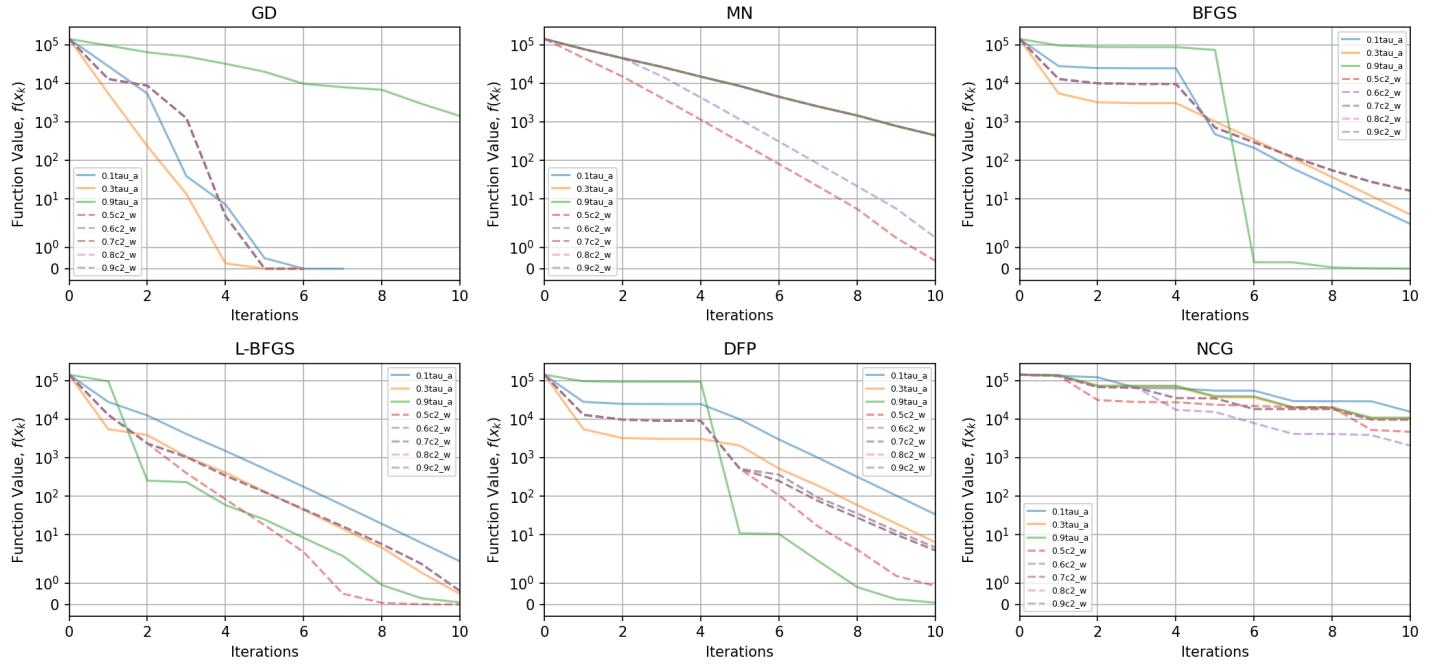
Problem6 - Total Iterations



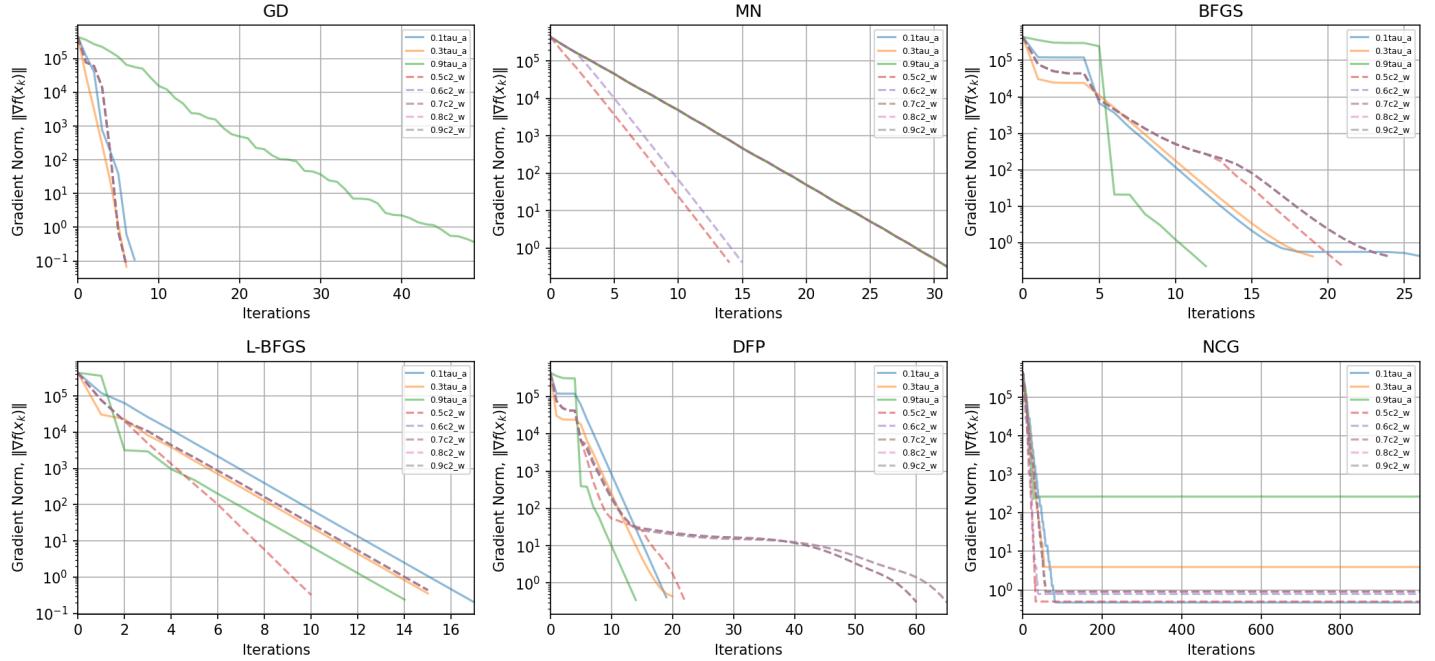
Problem6 - Function Value with Different c2 and tau

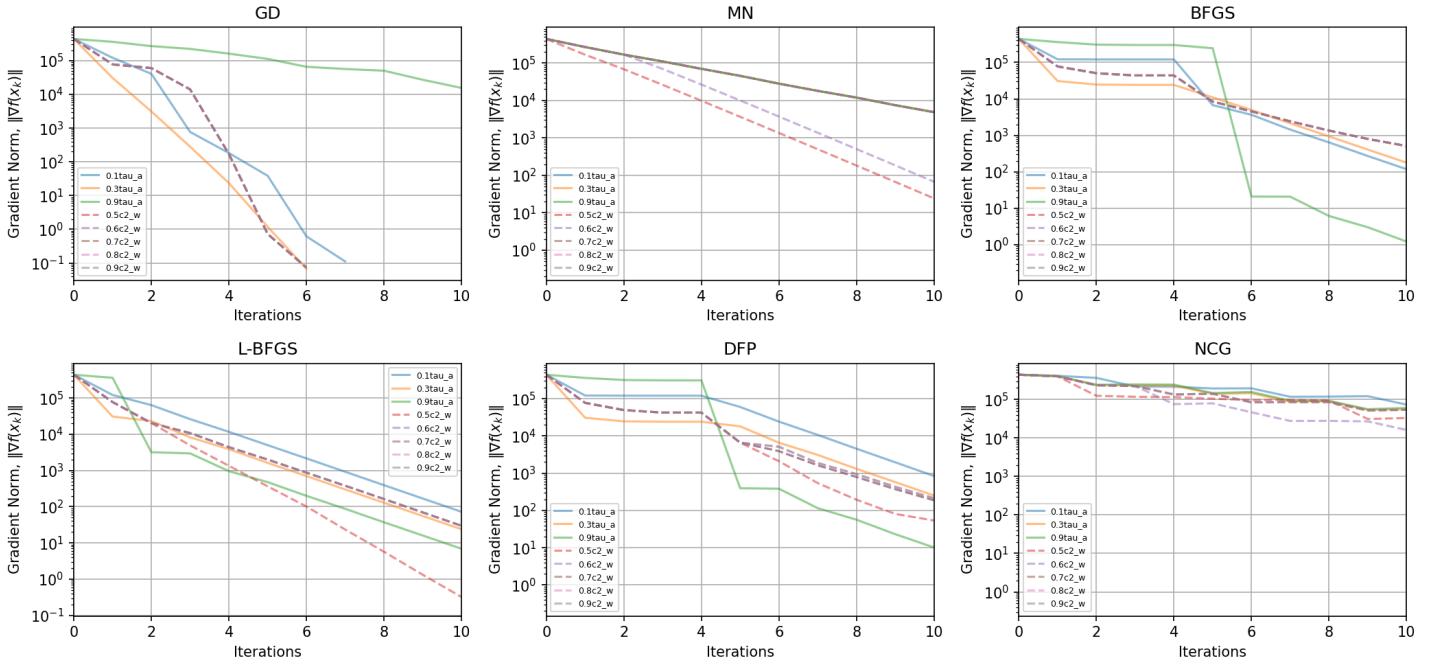


Problem6 - Function Value with Different c_2 and τ (zoom)



Problem6 - Gradient Norm with Different c_2 and τ

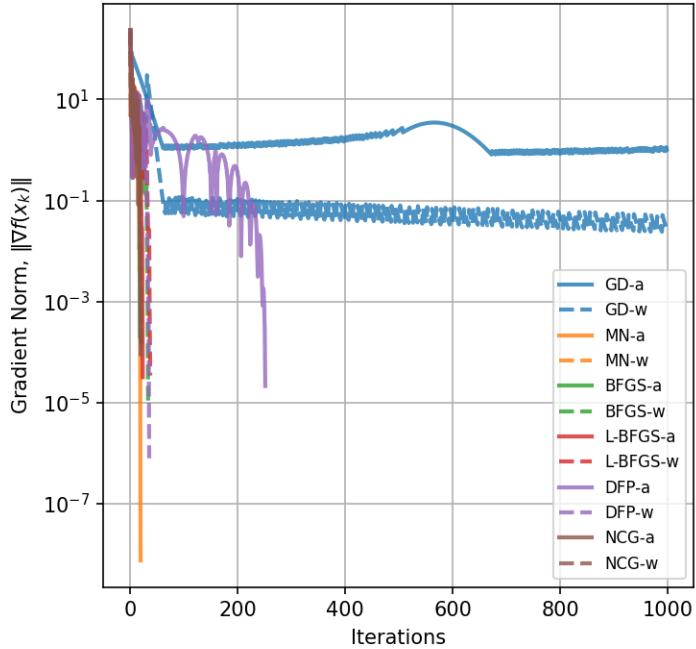


Problem6 - Gradient Norm with Different c_2 and τ (zoom)

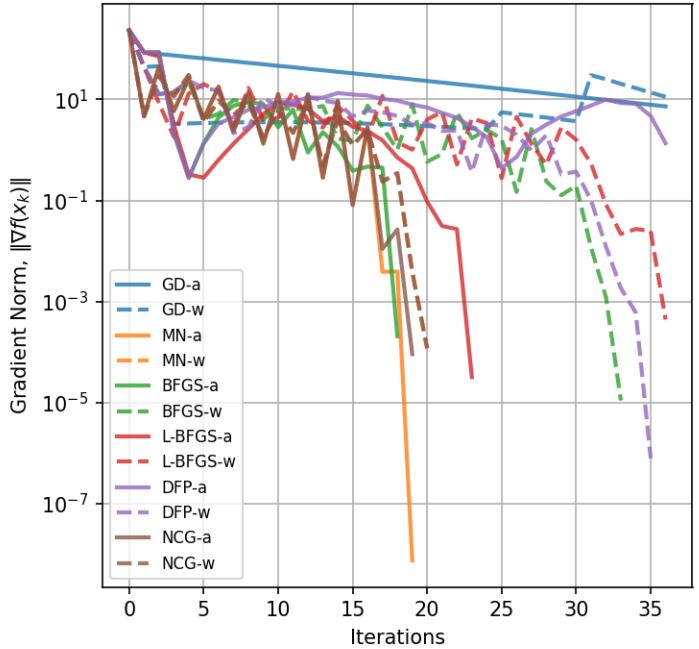
Problem 7

	Iters	Time	Convergence	Func Evals	Grad Evals	Hess Evals
GD-a	999	0.439839	Failed. Maximum iterations reached.	66070	2000	0
GD-w	999	0.0946846	Failed. Maximum iterations reached.	10881	1001	0
MN-a	19	0.00315094	Converged. Gradient norm is below tolerance.	95	40	20
MN-w	20	0.00304937	Converged. Gradient norm is below tolerance.	48	22	21
BFGS-a	18	0.00251579	Converged. Gradient norm is below tolerance.	143	56	0
BFGS-w	33	0.0038867	Converged. Gradient norm is below tolerance.	86	68	0
L-BFGS-a	23	0.00288868	Converged. Gradient norm is below tolerance.	129	70	0
L-BFGS-w	37	0.00453901	Converged. Gradient norm is below tolerance.	95	83	0
DFP-a	251	0.0238671	Converged. Gradient norm is below tolerance.	612	755	0
DFP-w	35	0.00393677	Converged. Gradient norm is below tolerance.	88	76	0
NCG-a	19	0.00245333	Converged. Gradient norm is below tolerance.	95	40	20
NCG-w	20	0.00254416	Converged. Gradient norm is below tolerance.	48	22	21

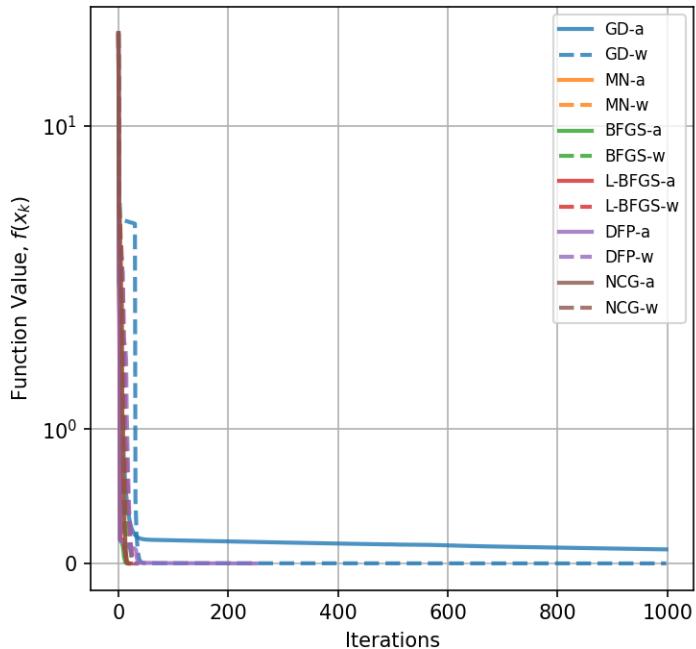
Problem7 - Gradient Norm (All Iterations)



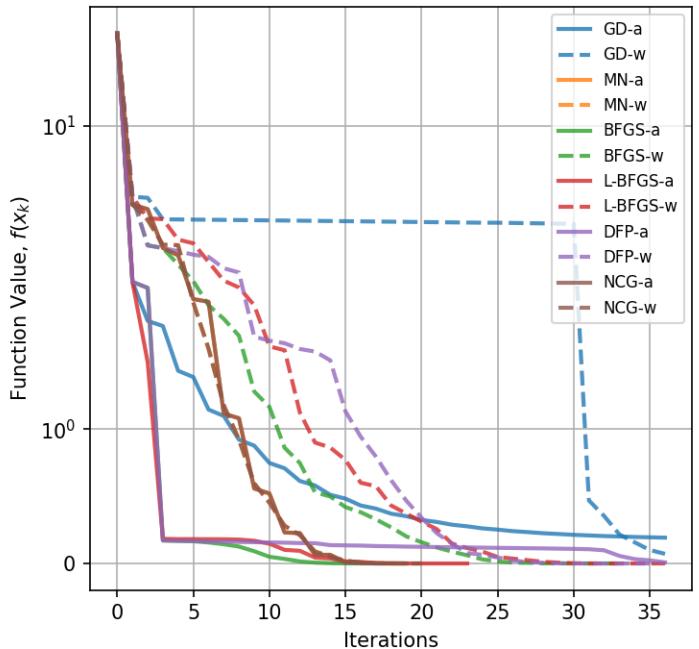
Problem7 - Gradient Norm (First 37 Iters)



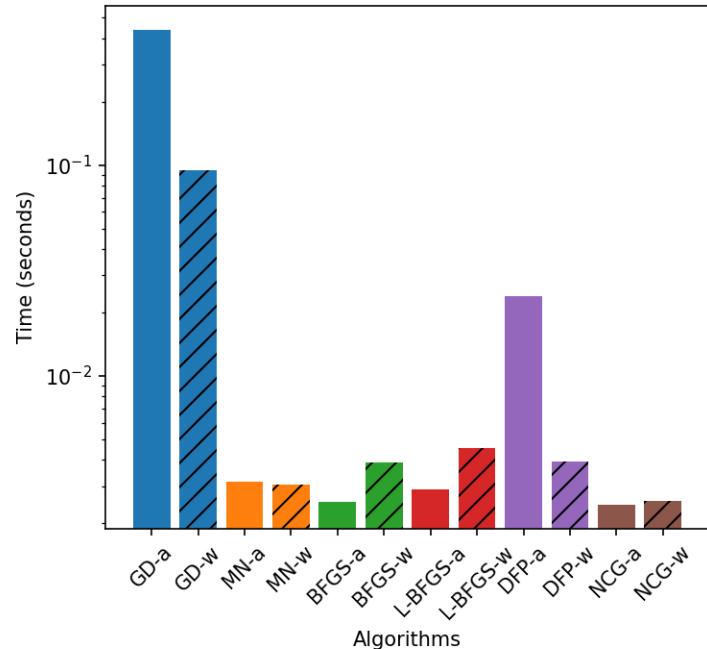
Problem7 - Function Value (All Iterations)



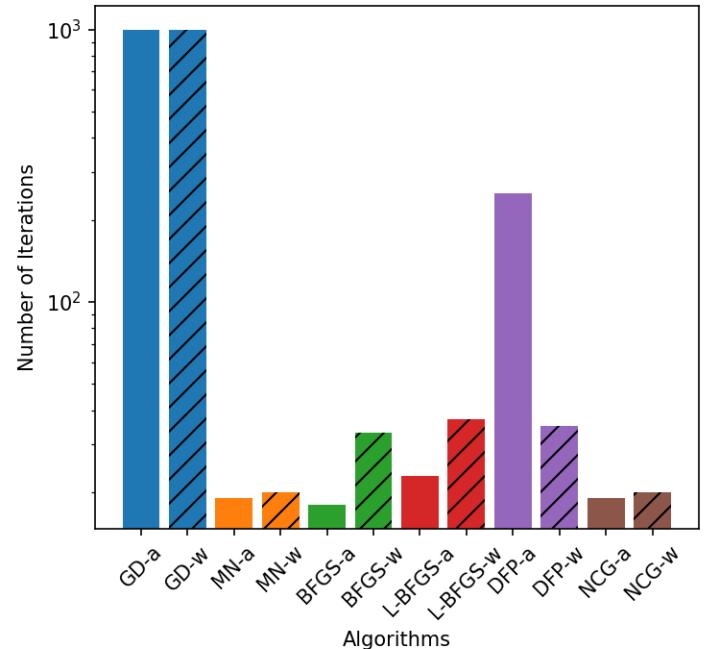
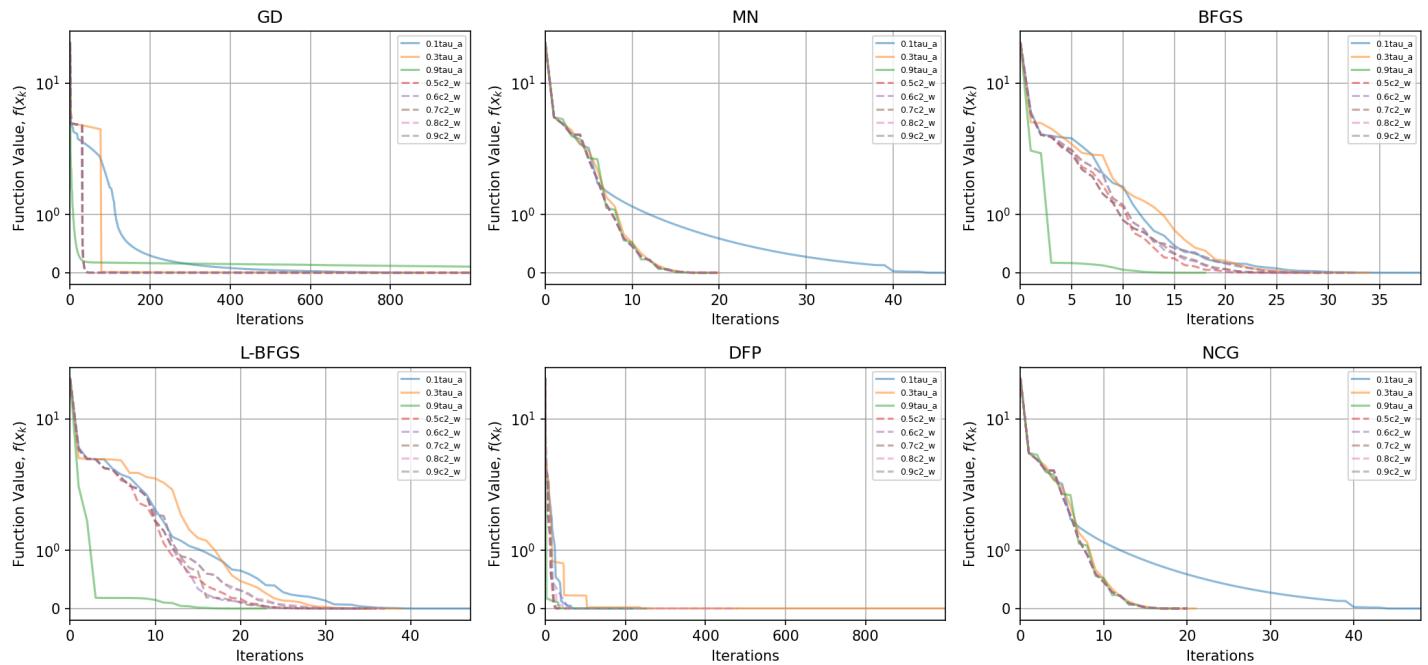
Problem7 - Function Value (First 37 Iters)



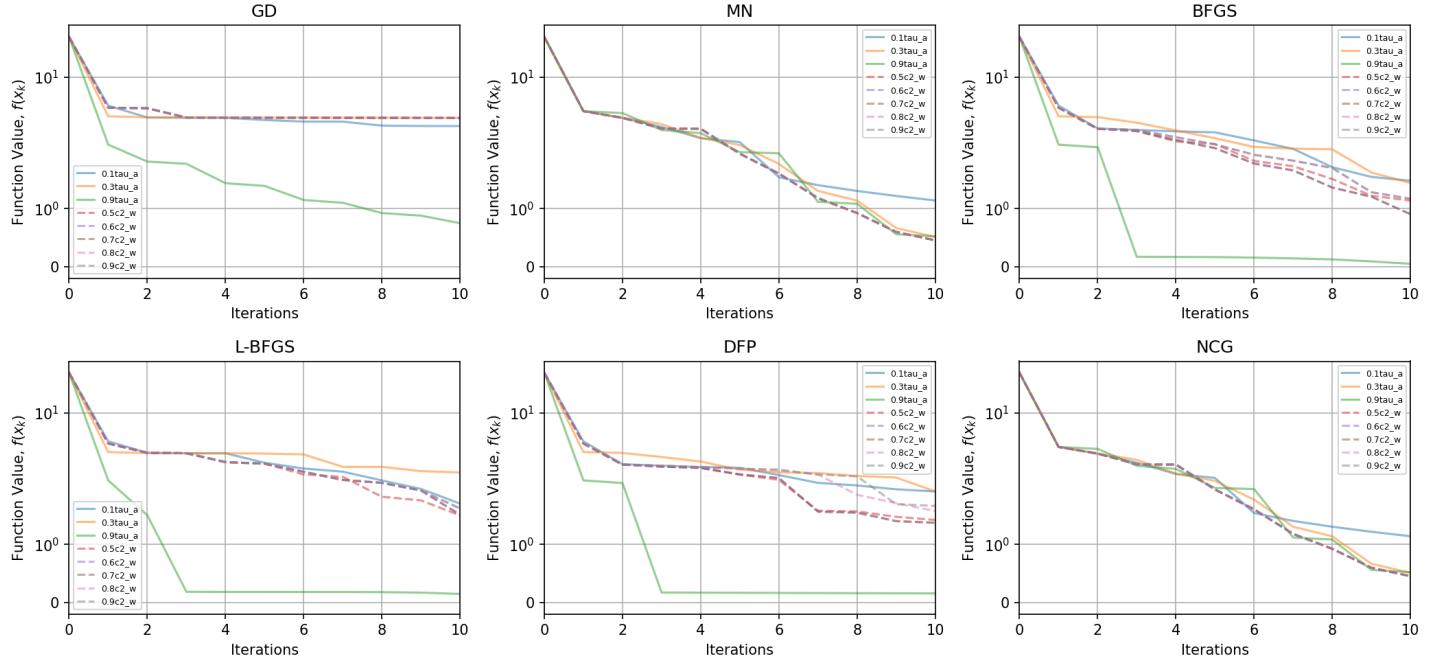
Problem7 - Total Time



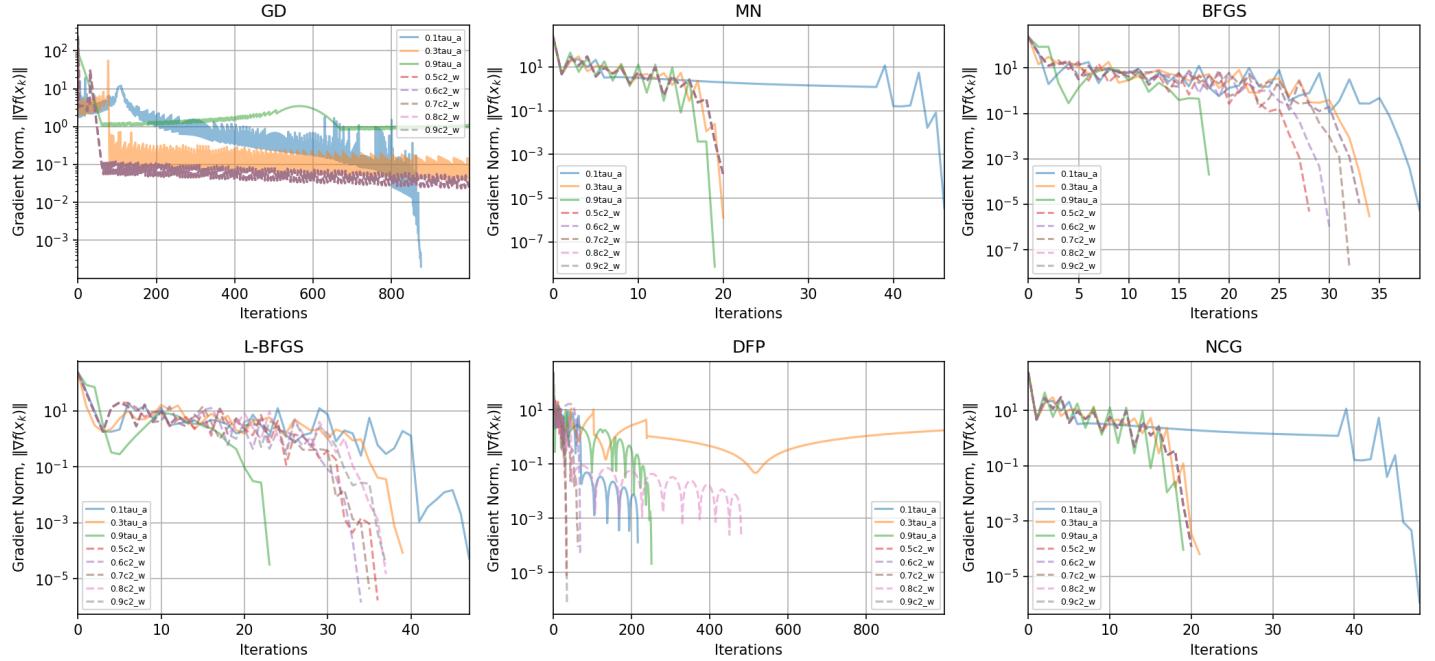
Problem7 - Total Iterations

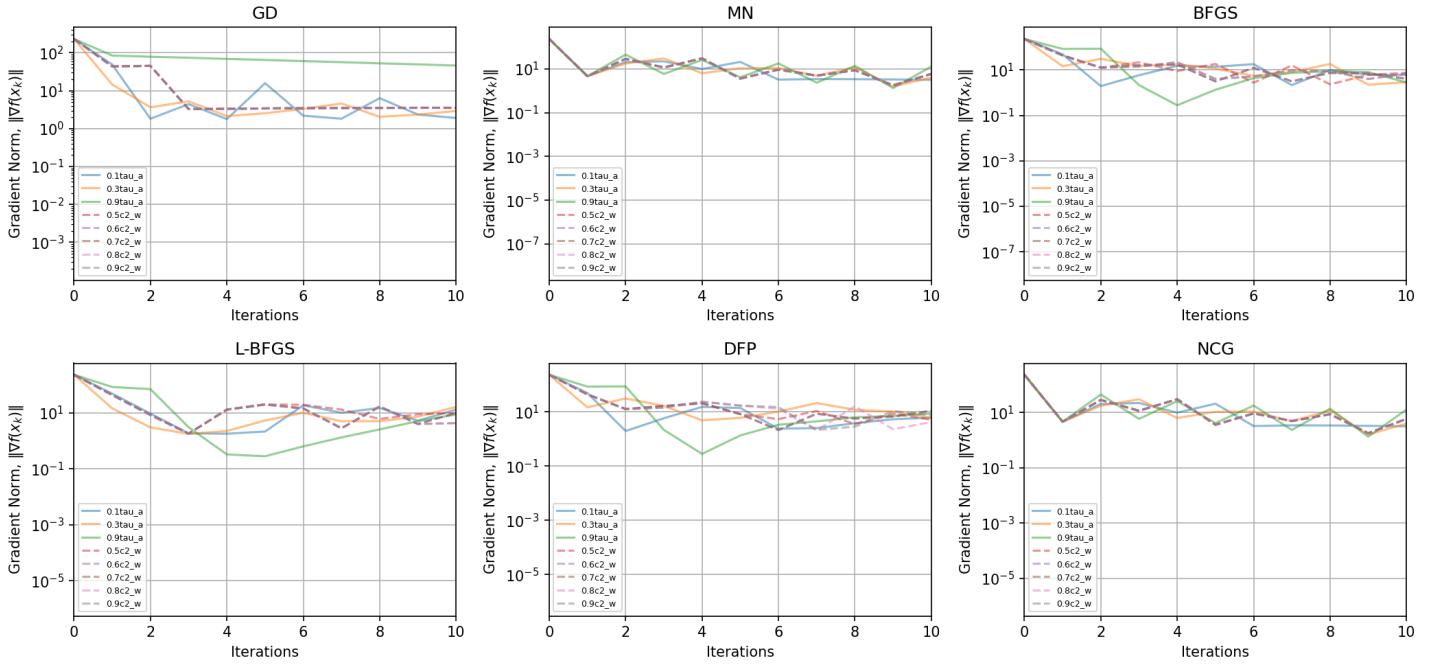
Problem7 - Function Value with Different c_2 and τ 

Problem7 - Function Value with Different c_2 and τ (zoom)



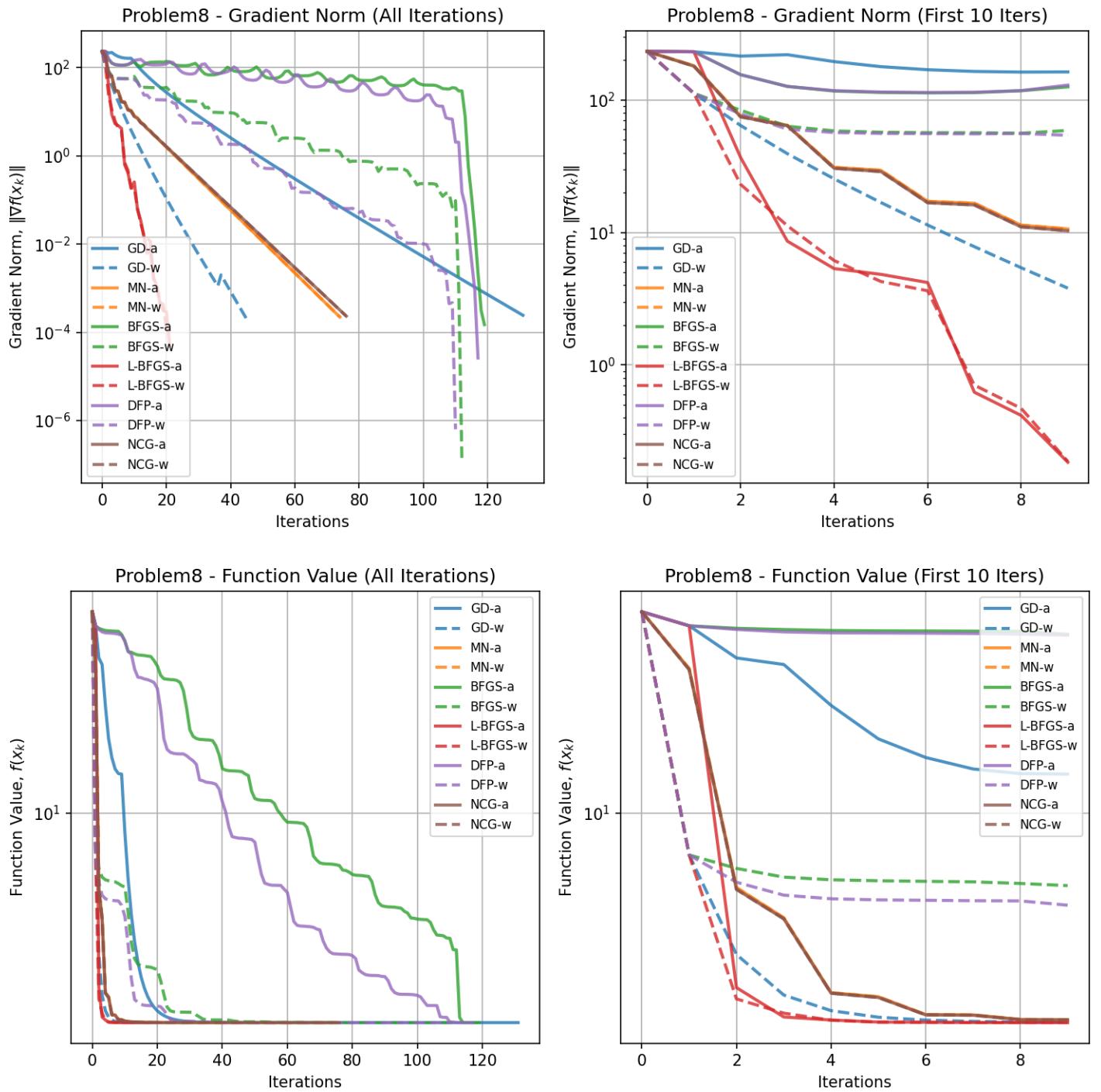
Problem7 - Gradient Norm with Different c_2 and tau



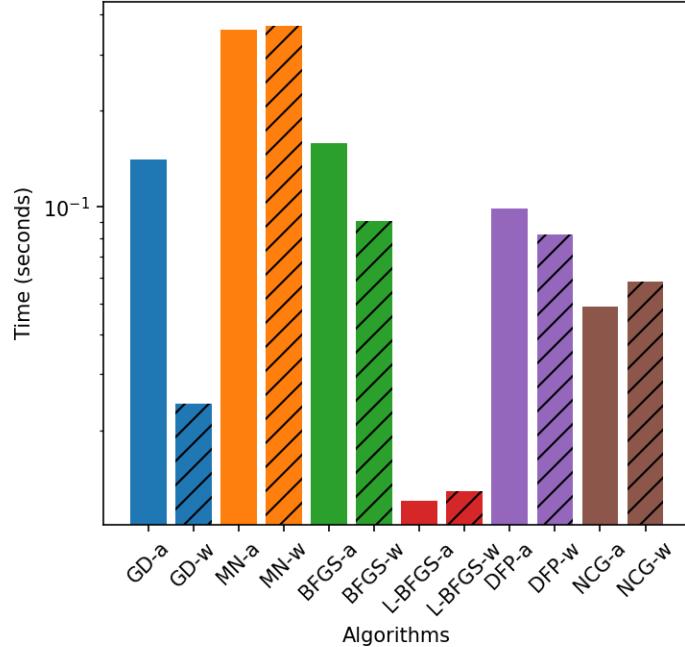
Problem7 - Gradient Norm with Different c_2 and τ (zoom)

Problem 8

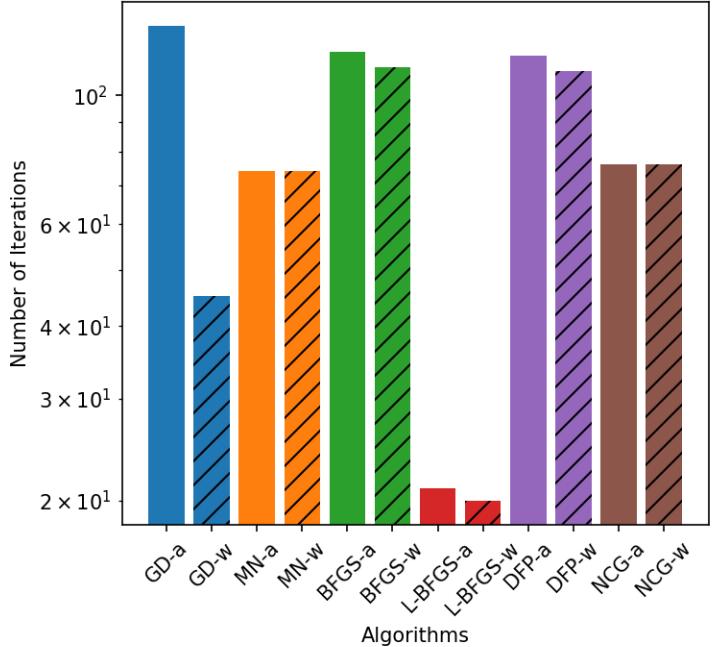
	Iters	Time	Convergence	Func Evals	Grad Evals	Hess Evals
GD-a	132	0.140508	Converged. Gradient norm is below tolerance.	8832	265	0
GD-w	45	0.0243149	Converged. Gradient norm is below tolerance.	540	46	0
MN-a	74	0.358855	Converged. Gradient norm is below tolerance.	149	150	75
MN-w	74	0.36906	Converged. Gradient norm is below tolerance.	149	76	75
BFGS-a	119	0.158617	Converged. Gradient norm is below tolerance.	5847	359	0
BFGS-w	112	0.0903254	Converged. Gradient norm is below tolerance.	1124	226	0
L-BFGS-a	21	0.0120935	Converged. Gradient norm is below tolerance.	112	64	0
L-BFGS-w	20	0.0129056	Converged. Gradient norm is below tolerance.	53	41	0
DFP-a	117	0.0989053	Converged. Gradient norm is below tolerance.	2889	353	0
DFP-w	110	0.0820246	Converged. Gradient norm is below tolerance.	730	222	0
NCG-a	76	0.0487905	Converged. Gradient norm is below tolerance.	153	154	77
NCG-w	76	0.0585678	Converged. Gradient norm is below tolerance.	153	78	77



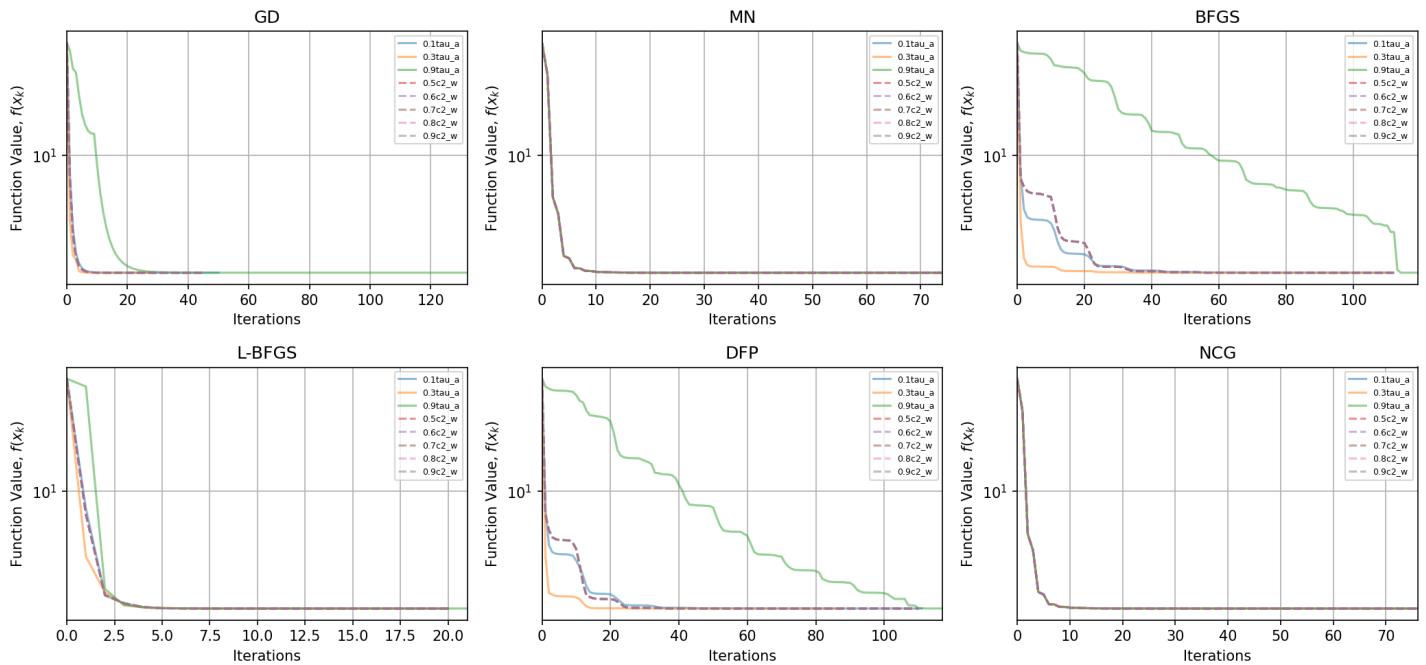
Problem8 - Total Time



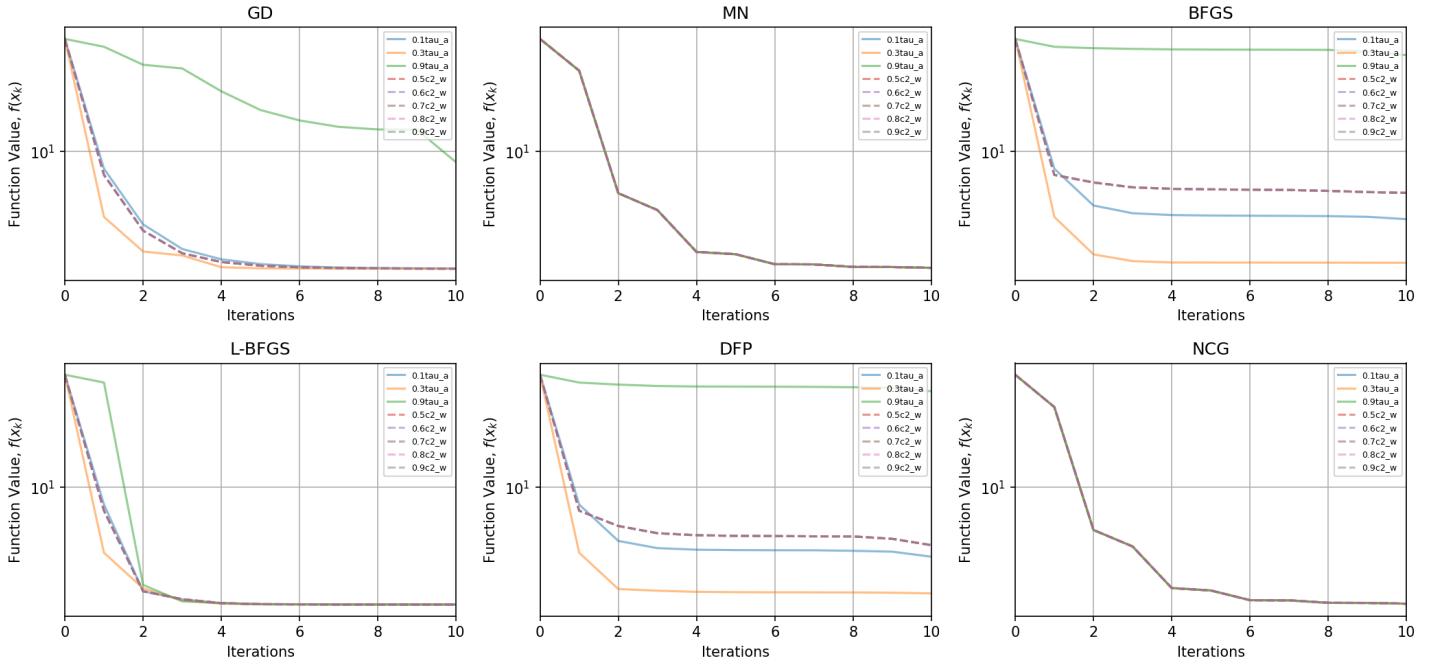
Problem8 - Total Iterations



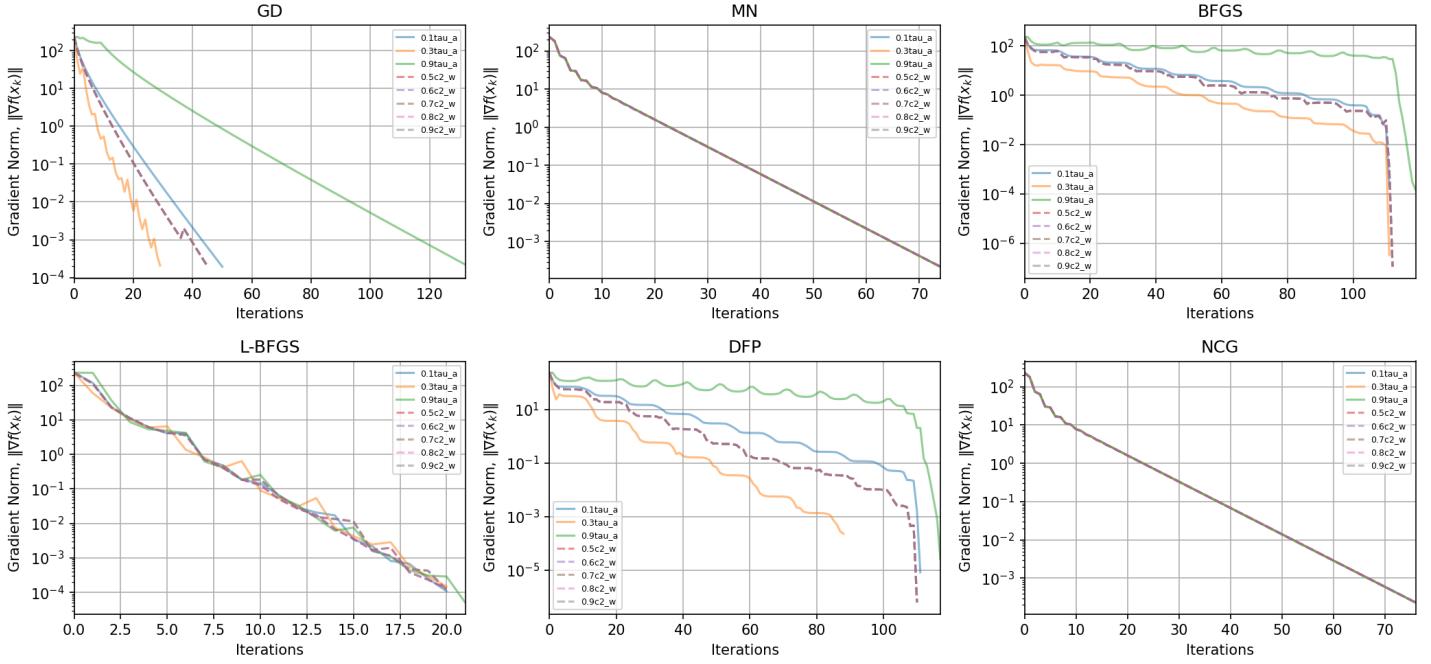
Problem8 - Function Value with Different c2 and tau



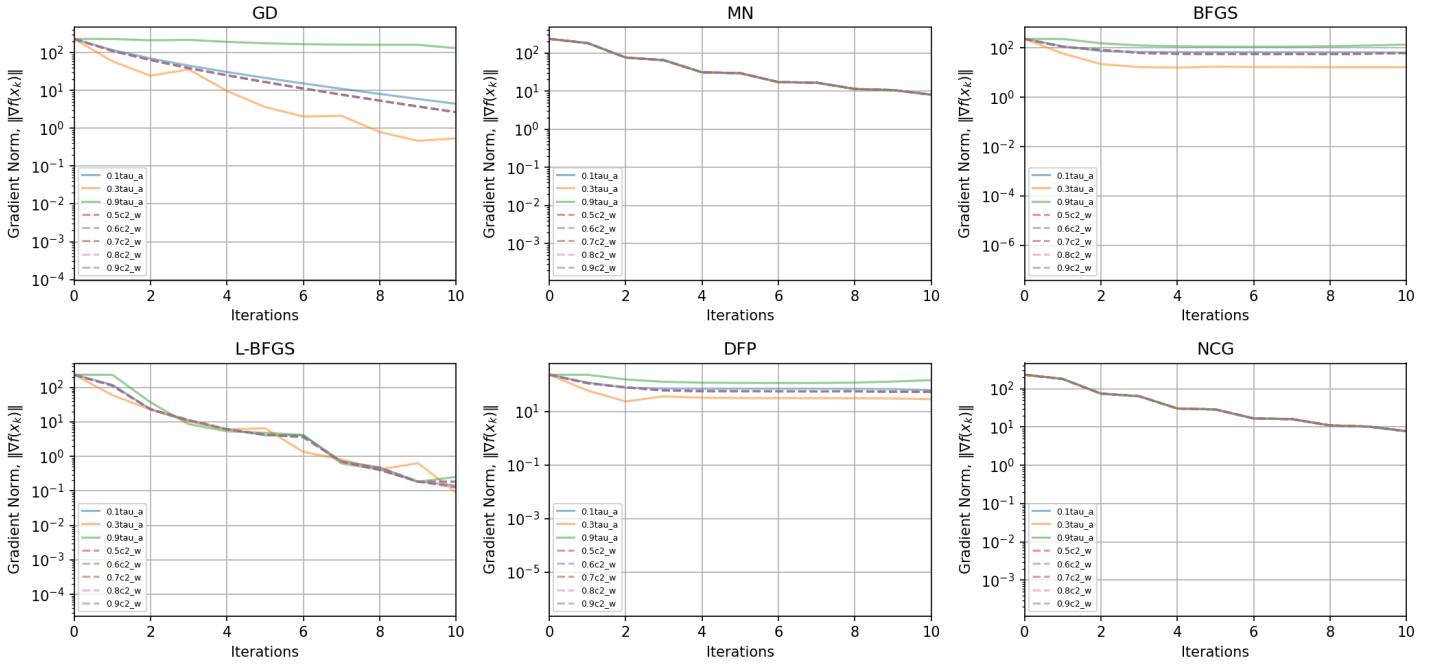
Problem8 - Function Value with Different c_2 and τ (zoom)



Problem8 - Gradient Norm with Different c_2 and tau

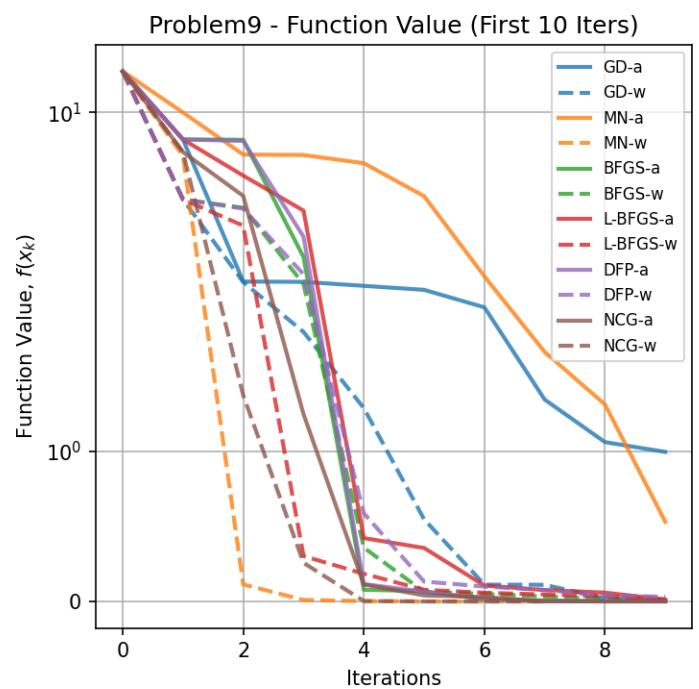
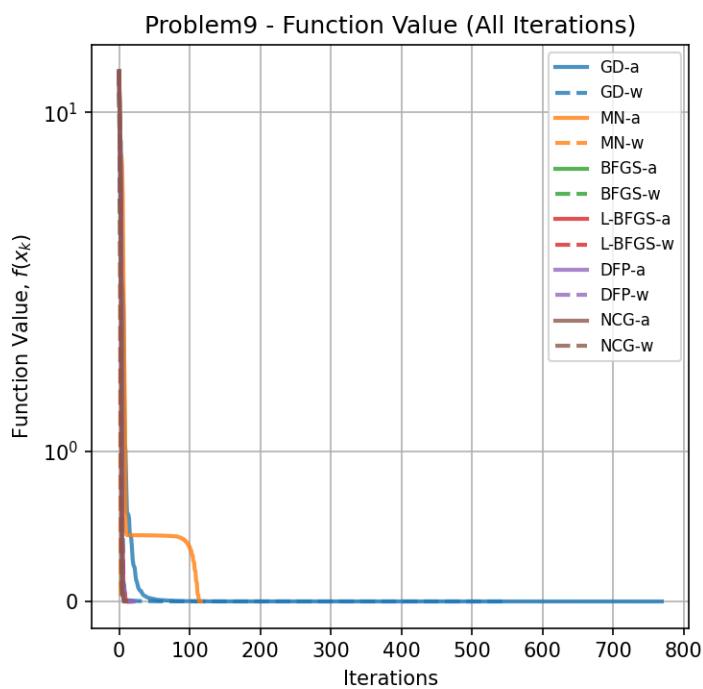
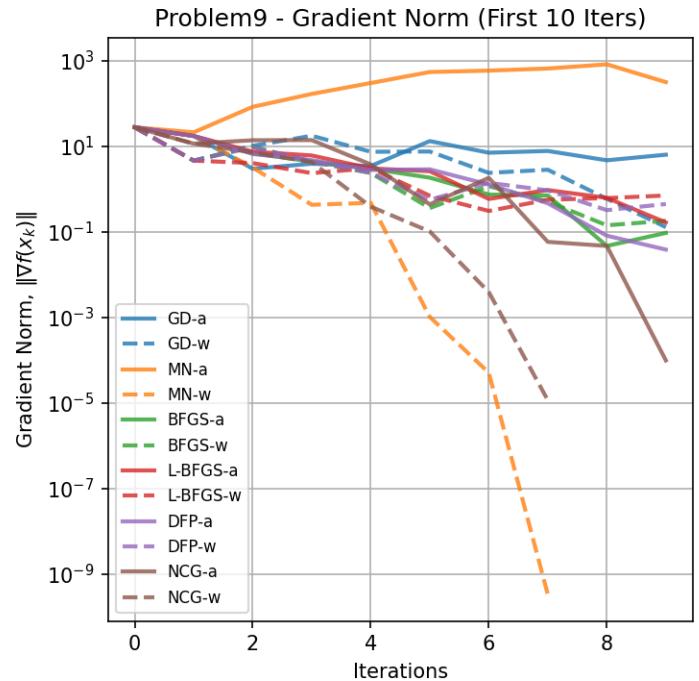
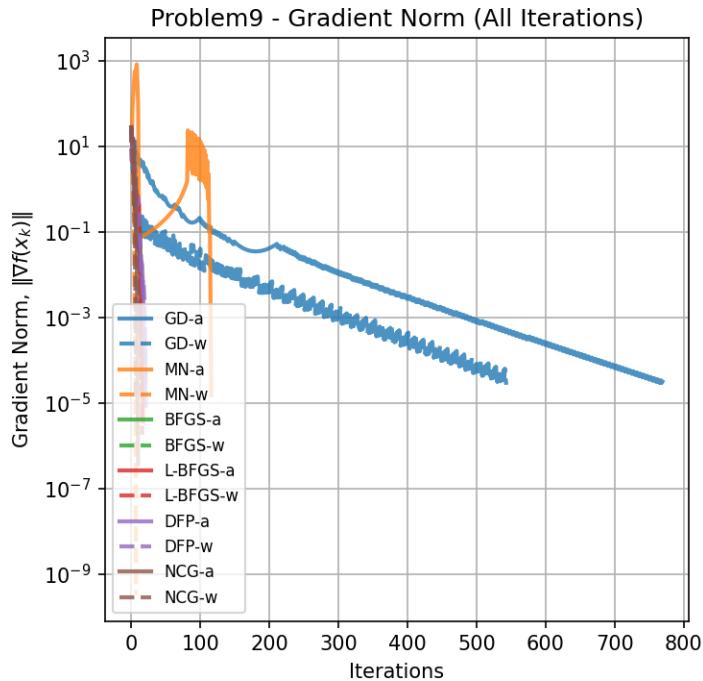


Problem8 - Gradient Norm with Different c_2 and τ (zoom)

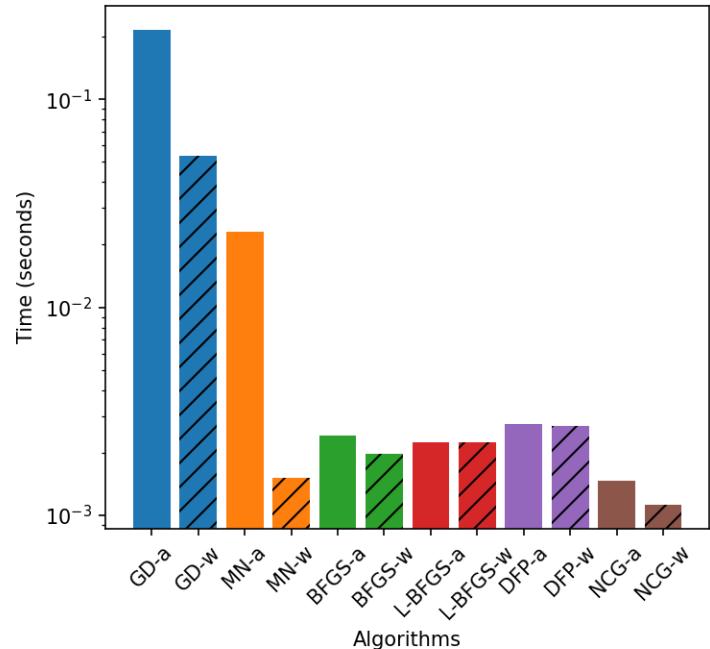


Problem 9

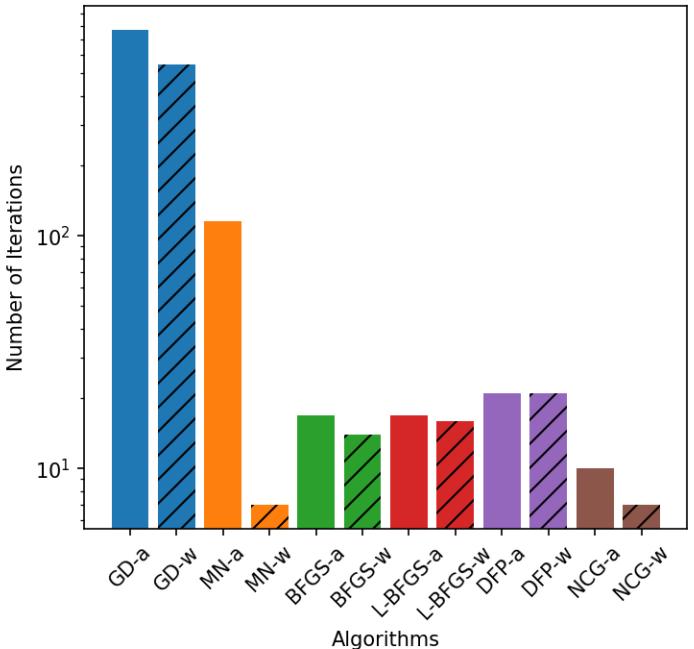
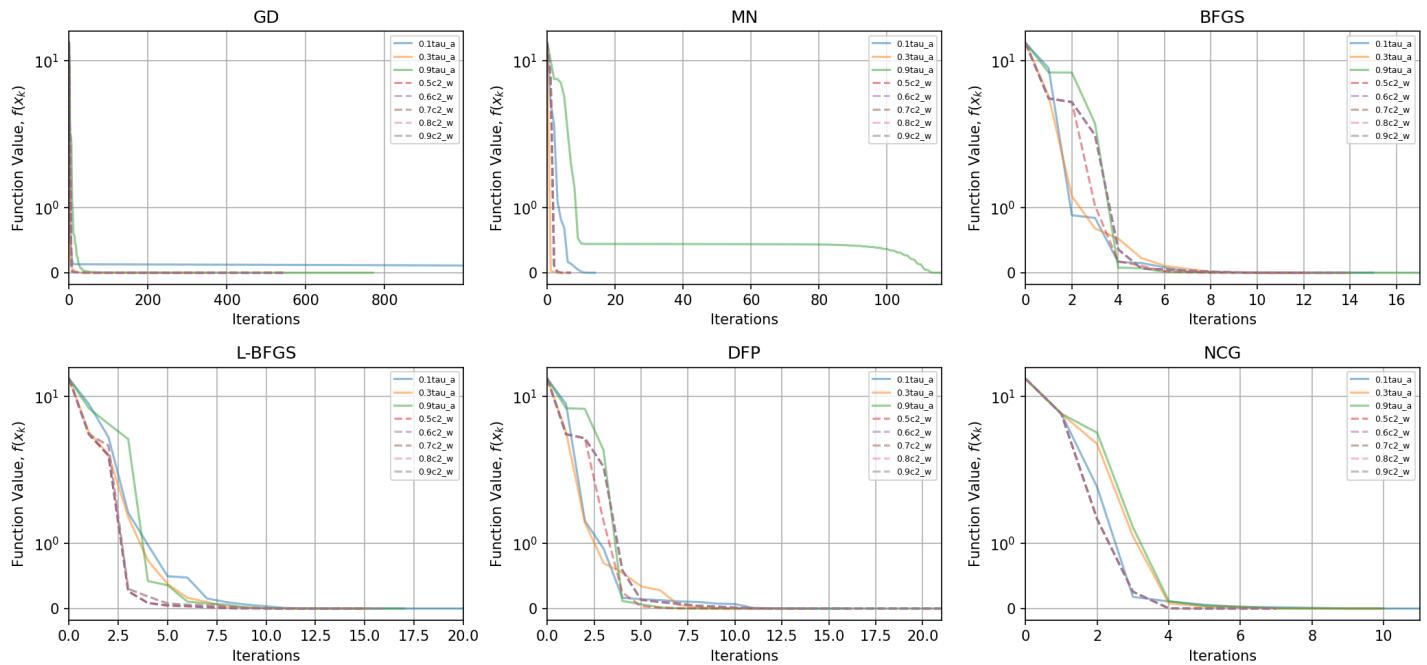
	Iters	Time	Convergence	Func Evals	Grad Evals	Hess Evals
GD-a	770	0.216622	Converged. Gradient norm is below tolerance.	24495	1541	0
GD-w	543	0.0536568	Converged. Gradient norm is below tolerance.	3523	544	0
MN-a	116	0.0230932	Converged. Gradient norm is below tolerance.	1008	234	117
MN-w	7	0.00151277	Converged. Gradient norm is below tolerance.	55	9	8
BFGS-a	17	0.00241065	Converged. Gradient norm is below tolerance.	84	53	0
BFGS-w	14	0.00196934	Converged. Gradient norm is below tolerance.	37	30	0
L-BFGS-a	17	0.00225377	Converged. Gradient norm is below tolerance.	67	52	0
L-BFGS-w	16	0.002249	Converged. Gradient norm is below tolerance.	44	33	0
DFP-a	21	0.00273633	Converged. Gradient norm is below tolerance.	93	65	0
DFP-w	21	0.00269604	Converged. Gradient norm is below tolerance.	51	44	0
NCG-a	10	0.00146031	Converged. Gradient norm is below tolerance.	39	22	11
NCG-w	7	0.0011251	Converged. Gradient norm is below tolerance.	17	9	8



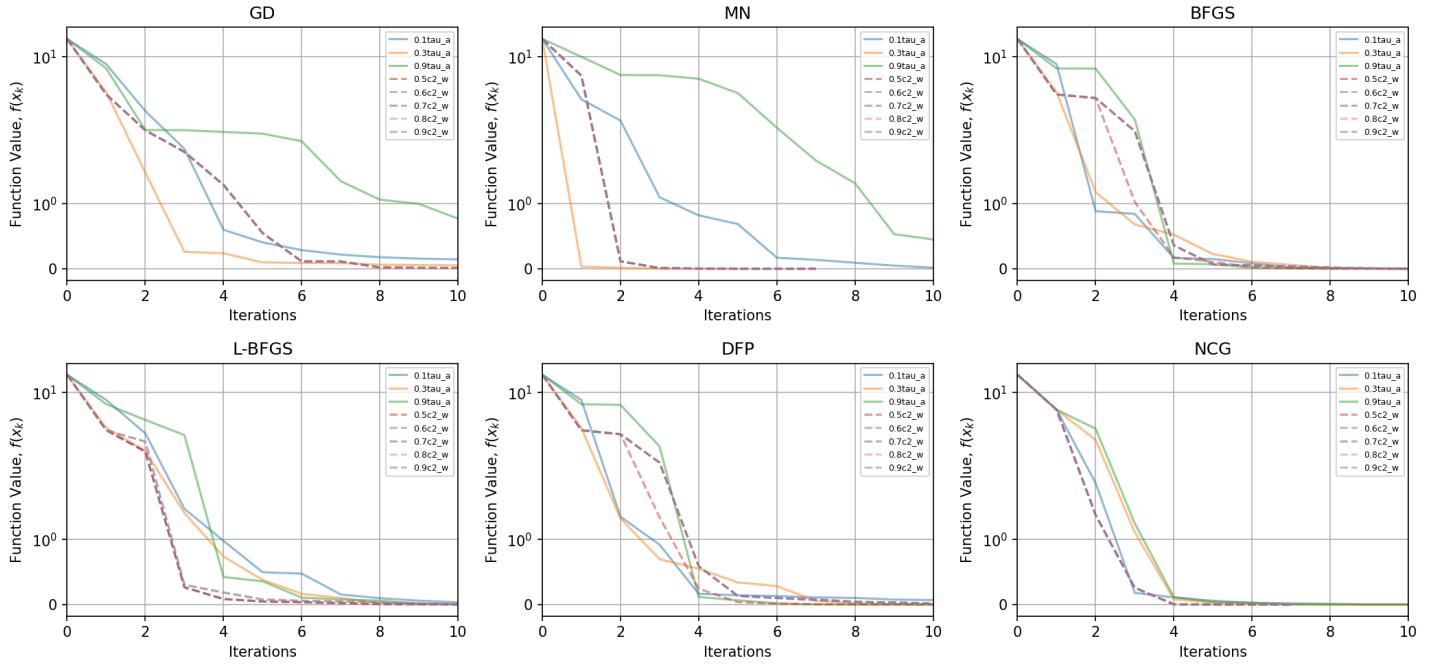
Problem9 - Total Time



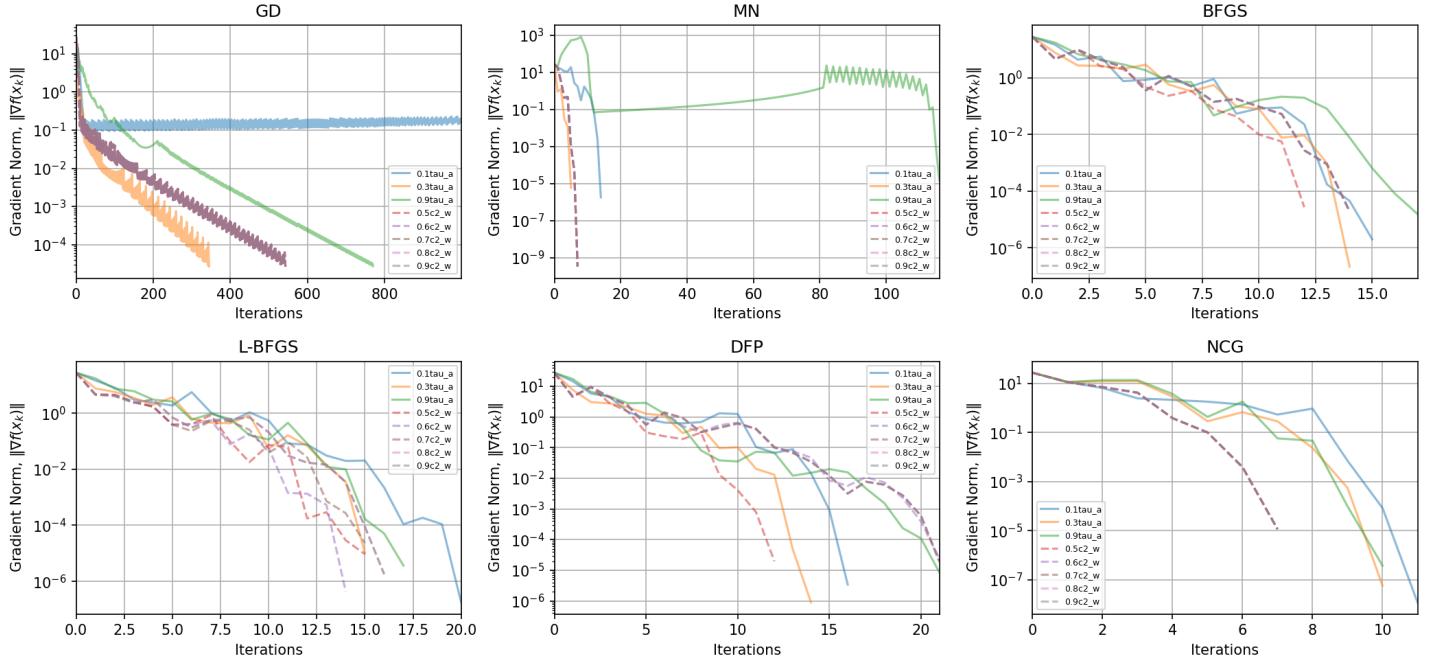
Problem9 - Total Iterations

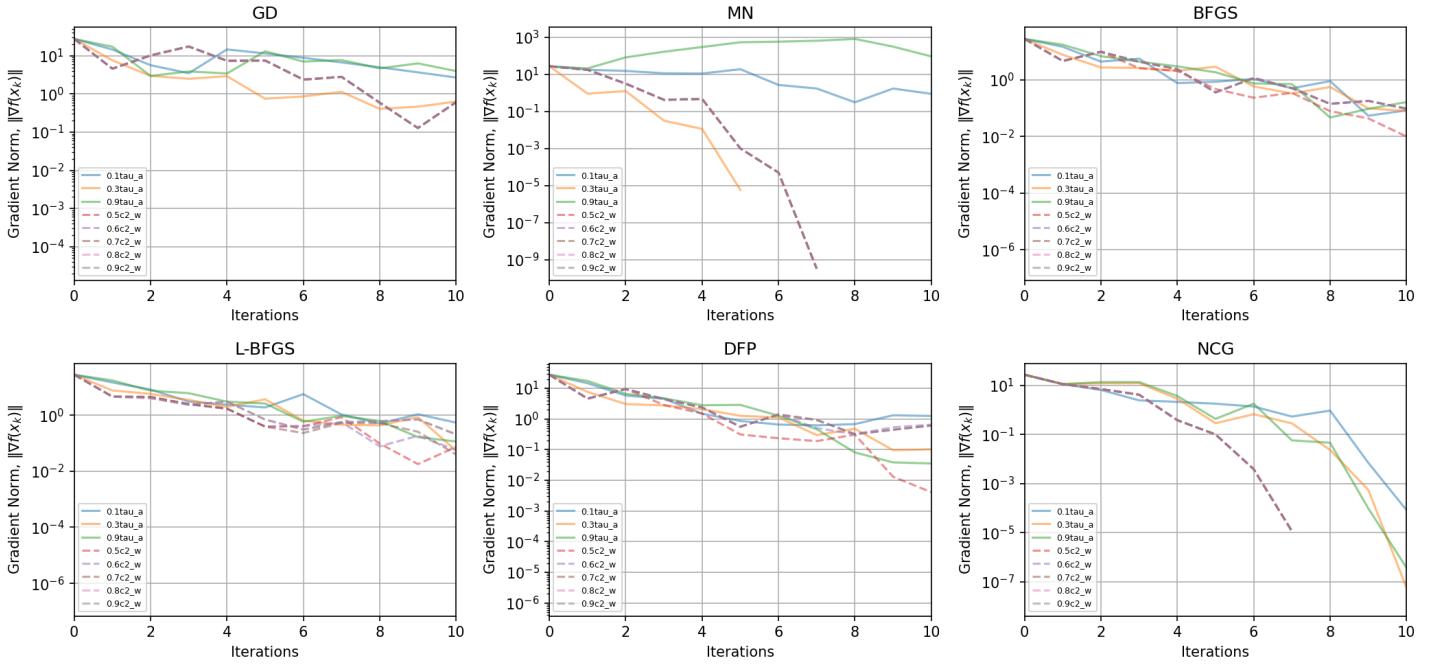
Problem9 - Function Value with Different c_2 and τ_a 

Problem9 - Function Value with Different c_2 and τ (zoom)



Problem9 - Gradient Norm with Different c_2 and τ

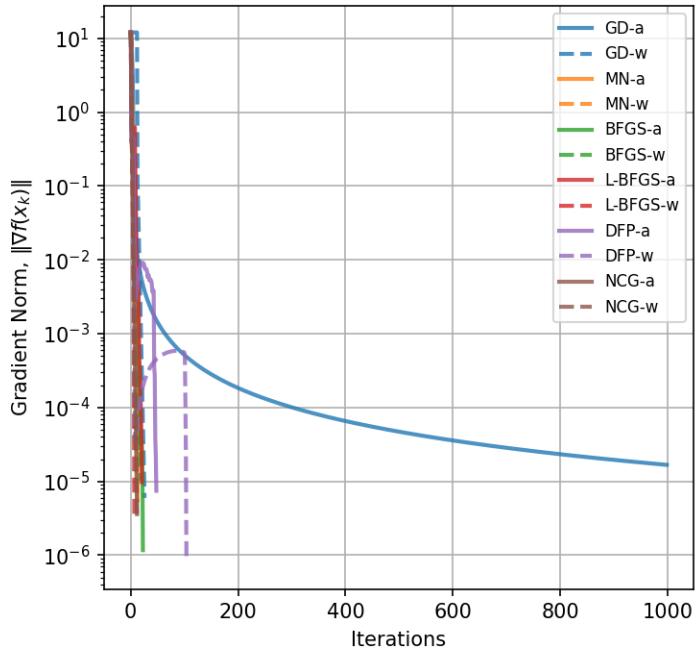


Problem9 - Gradient Norm with Different c_2 and τ (zoom)

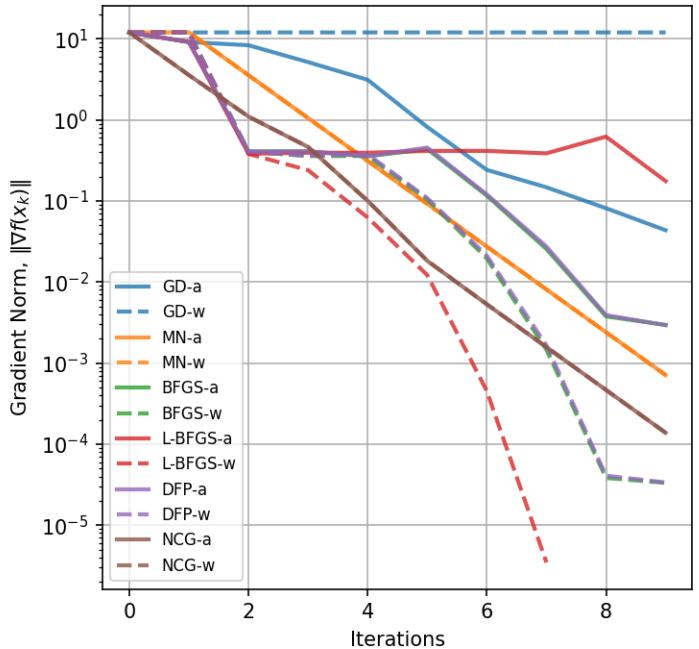
Problem 10

	Iters	Time	Convergence	Func Evals	Grad Evals	Hess Evals
GD-a	999	0.106802	Failed. Maximum iterations reached.	2019	2000	0
GD-w	26	0.00361109	Converged. Gradient norm is below tolerance.	67	27	0
MN-a	13	0.00429225	Converged. Gradient norm is below tolerance.	93	28	14
MN-w	13	0.00318384	Converged. Gradient norm is below tolerance.	37	15	14
BFGS-a	23	0.00413561	Converged. Gradient norm is below tolerance.	59	71	0
BFGS-w	19	0.00379848	Converged. Gradient norm is below tolerance.	42	41	0
L-BFGS-a	21	0.0173724	Converged. Gradient norm is below tolerance.	799	64	0
L-BFGS-w	7	0.00175238	Converged. Gradient norm is below tolerance.	18	19	0
DFP-a	48	0.00837111	Converged. Gradient norm is below tolerance.	137	146	0
DFP-w	104	0.0192931	Converged. Gradient norm is below tolerance.	215	222	0
NCG-a	12	0.00206304	Converged. Gradient norm is below tolerance.	25	26	13
NCG-w	12	0.00231838	Converged. Gradient norm is below tolerance.	25	14	13

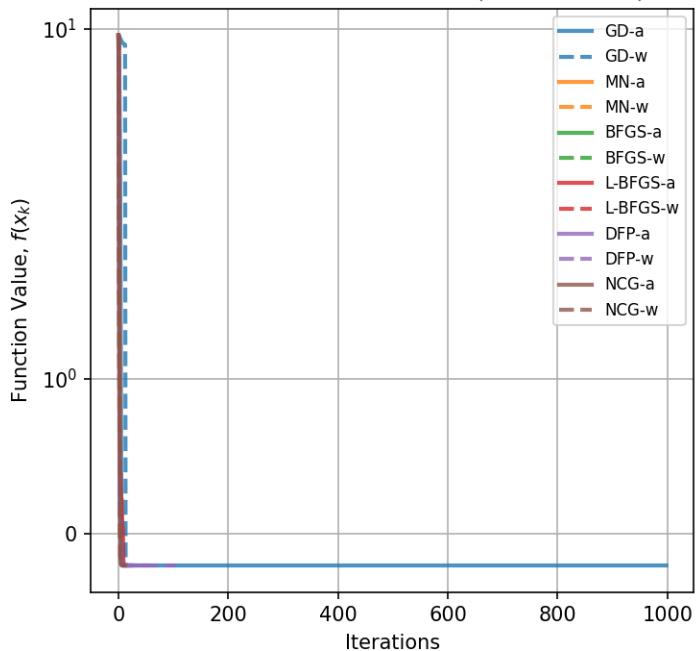
Problem10 - Gradient Norm (All Iterations)



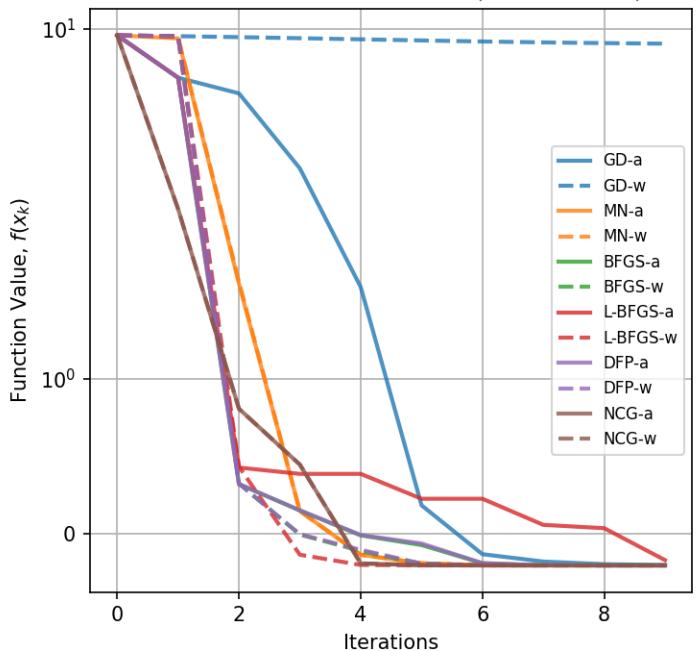
Problem10 - Gradient Norm (First 10 Iters)

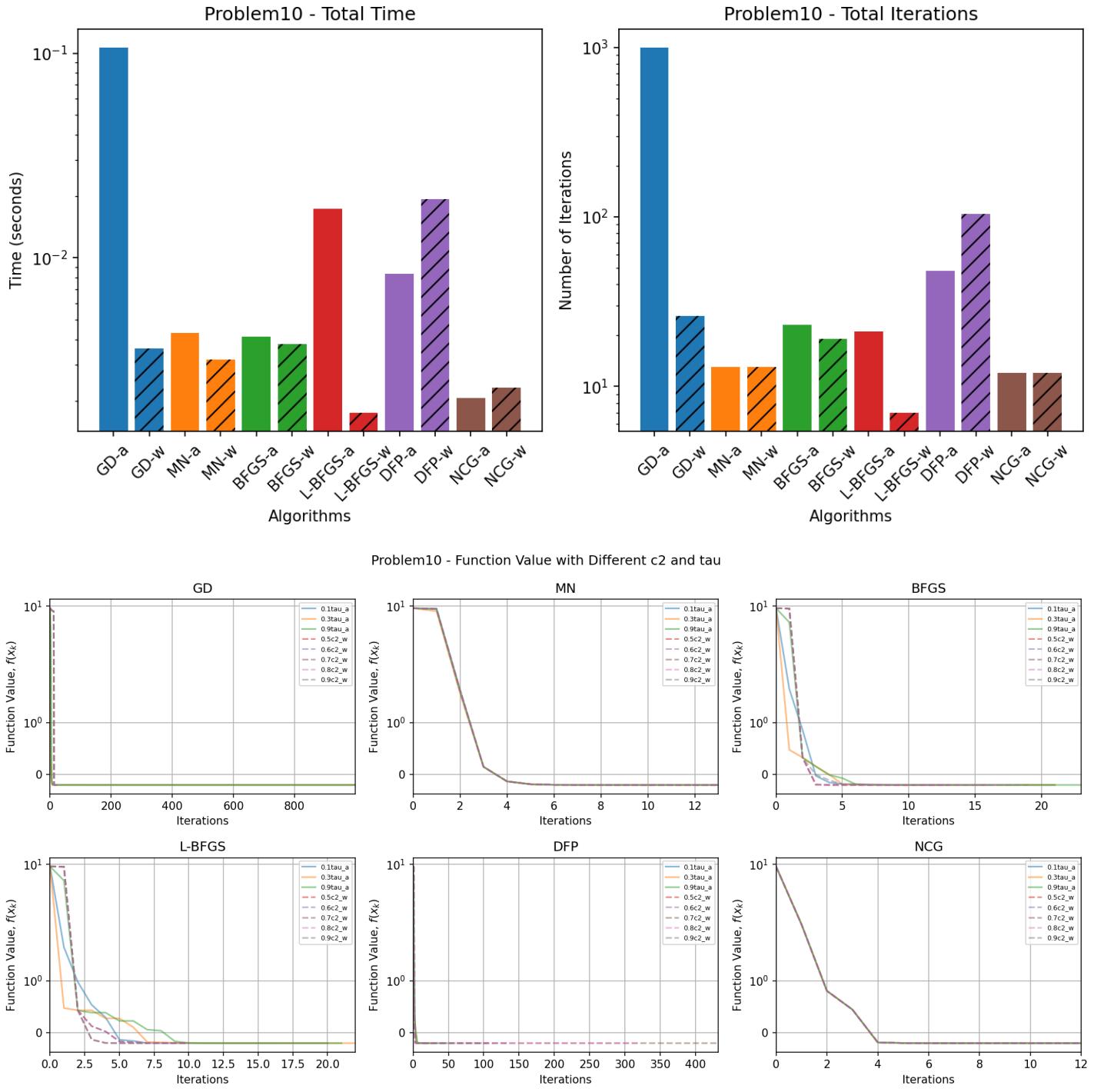


Problem10 - Function Value (All Iterations)

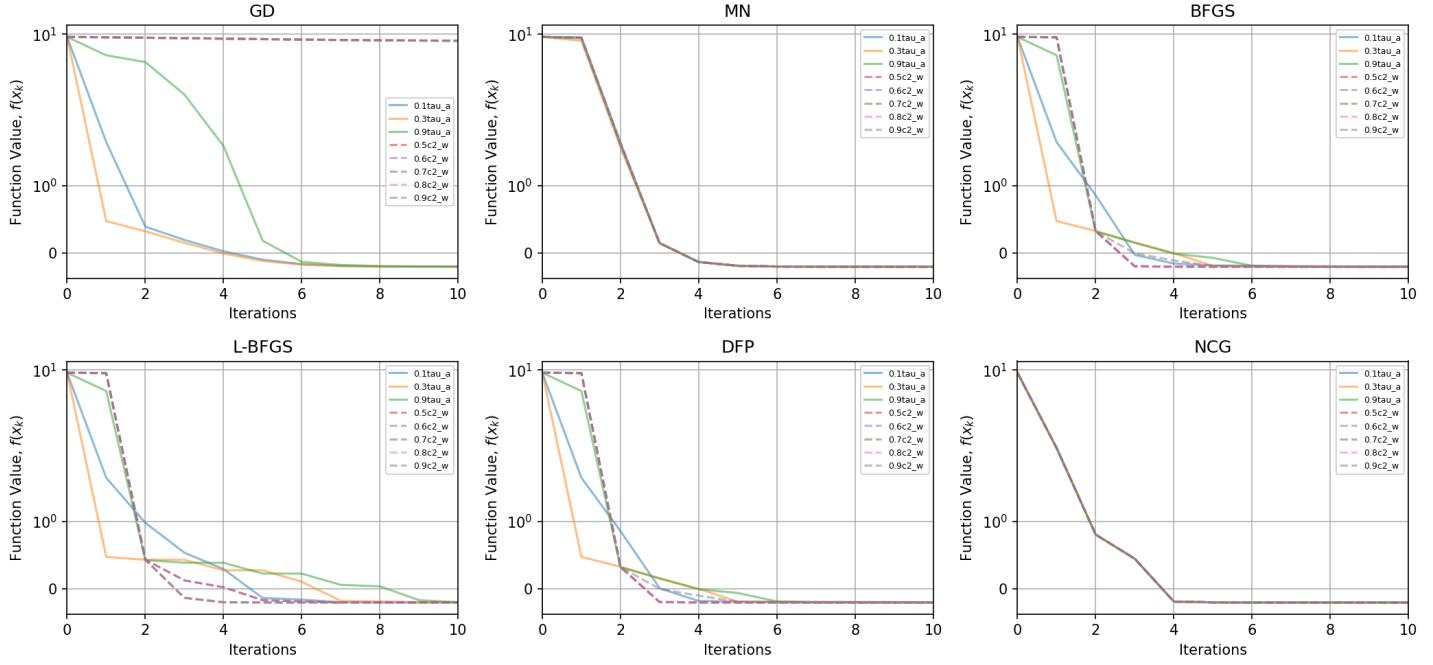


Problem10 - Function Value (First 10 Iters)

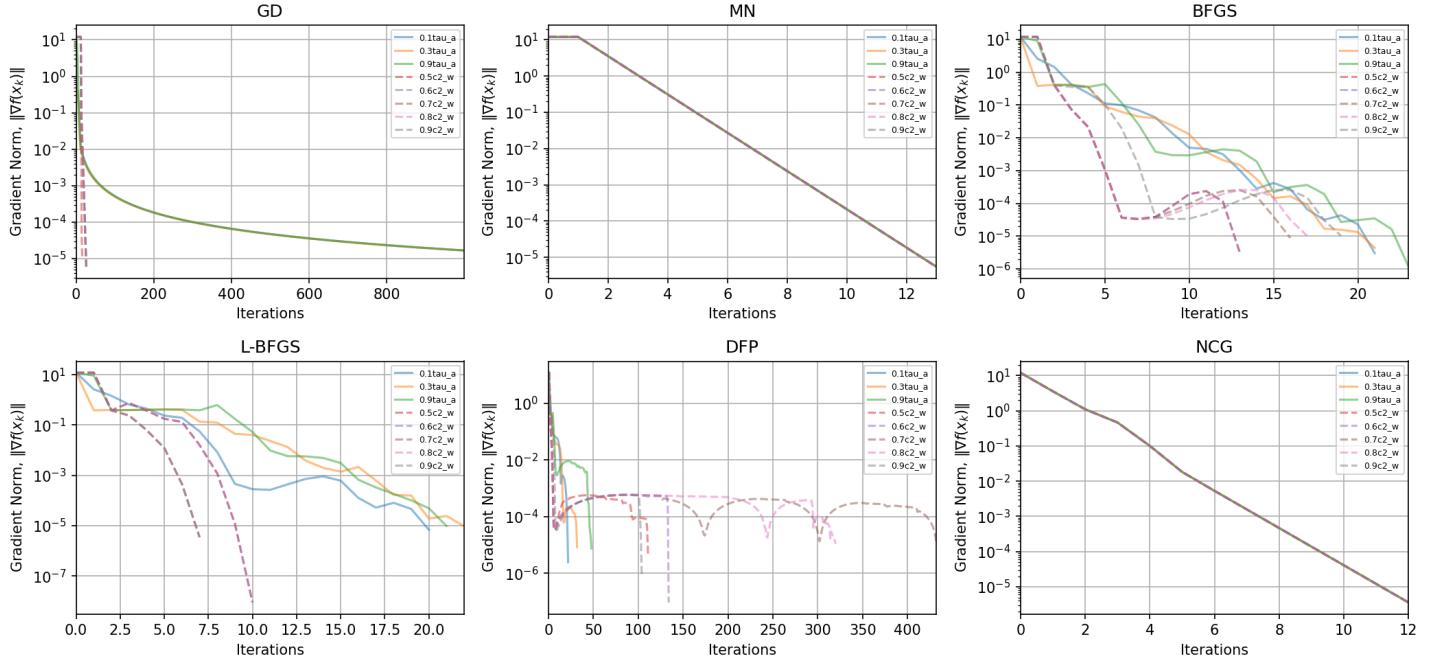


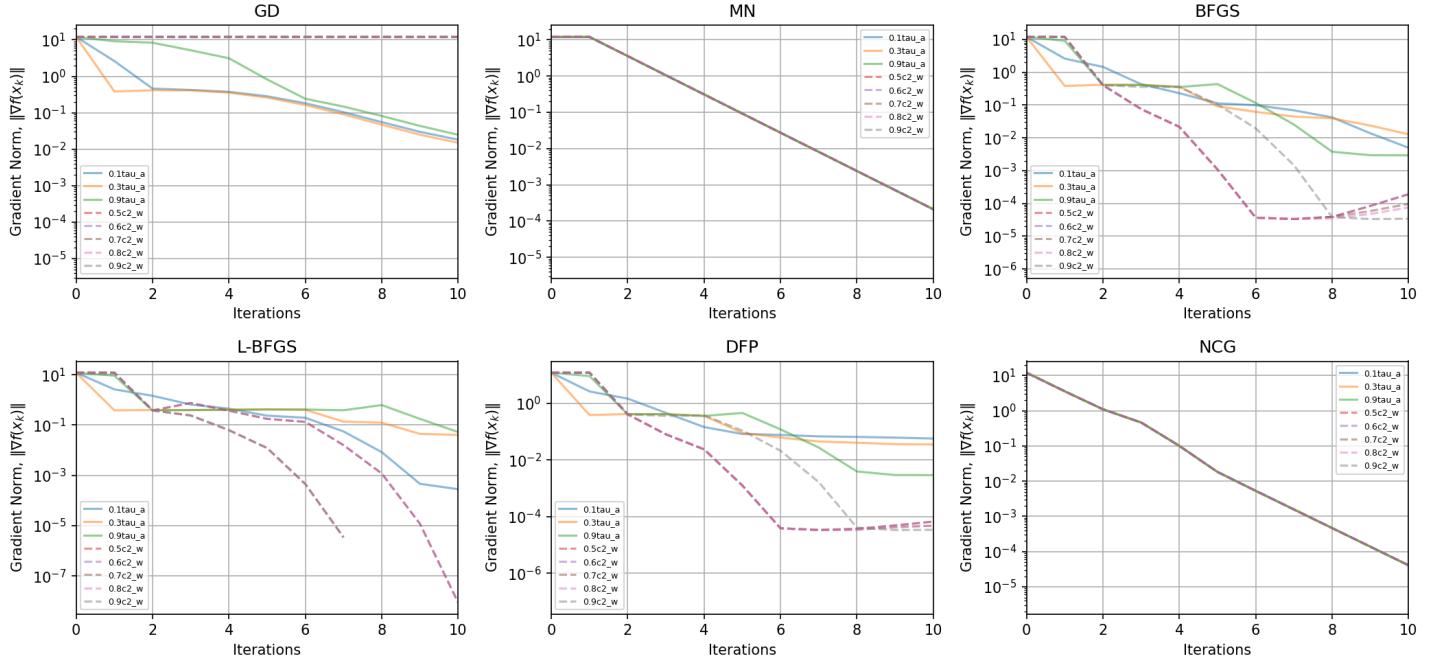


Problem10 - Function Value with Different c_2 and tau (zoom)



Problem10 - Gradient Norm with Different c_2 and tau

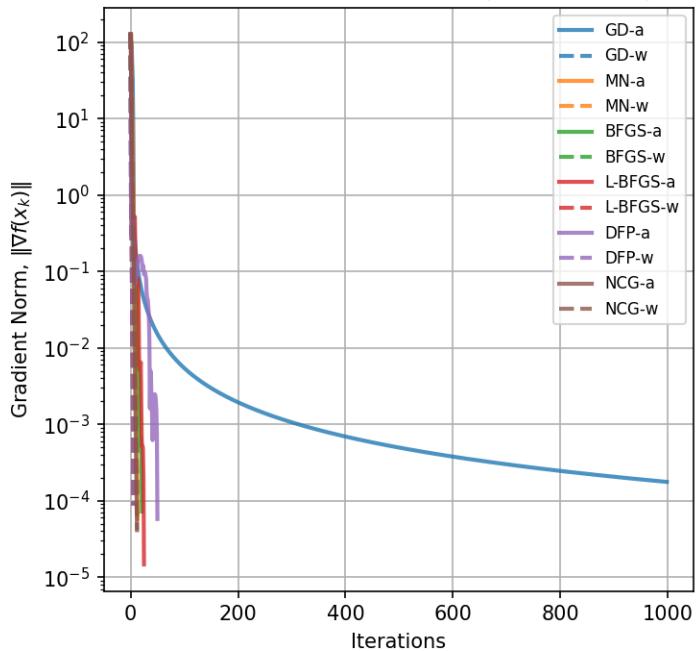


Problem10 - Gradient Norm with Different c_2 and tau (zoom)

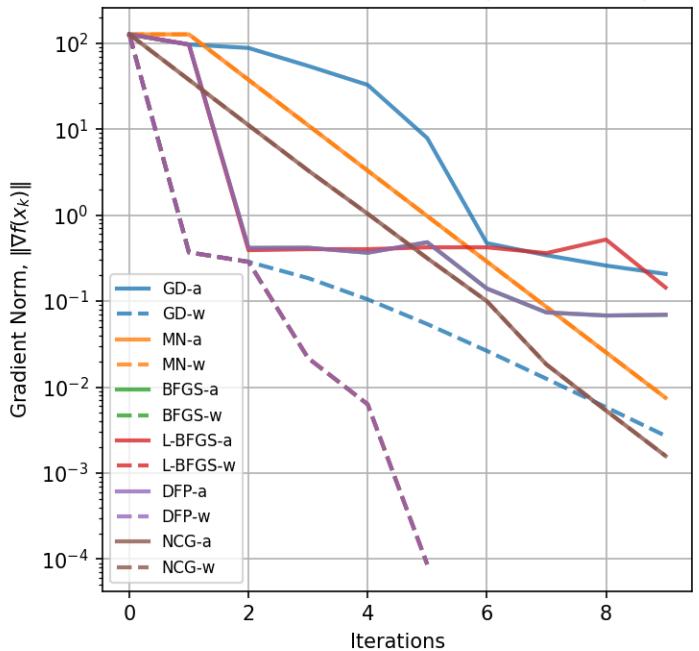
Problem 11

	Iters	Time	Convergence	Func Evals	Grad Evals	Hess Evals
GD-a	999	1.03768	Failed. Maximum iterations reached.	2019	2000	0
GD-w	13	0.0217977	Converged. Gradient norm is below tolerance.	29	16	0
MN-a	13	0.958231	Converged. Gradient norm is below tolerance.	91	28	14
MN-w	13	0.987752	Converged. Gradient norm is below tolerance.	37	15	14
BFGS-a	21	0.846363	Converged. Gradient norm is below tolerance.	55	65	0
BFGS-w	5	0.215343	Converged. Gradient norm is below tolerance.	16	14	0
L-BFGS-a	25	0.107871	Converged. Gradient norm is below tolerance.	793	76	0
L-BFGS-w	5	0.0270112	Converged. Gradient norm is below tolerance.	16	15	0
DFP-a	50	1.87603	Converged. Gradient norm is below tolerance.	120	152	0
DFP-w	5	0.209583	Converged. Gradient norm is below tolerance.	16	14	0
NCG-a	12	0.0796263	Converged. Gradient norm is below tolerance.	25	26	13
NCG-w	12	0.0716743	Converged. Gradient norm is below tolerance.	25	14	13

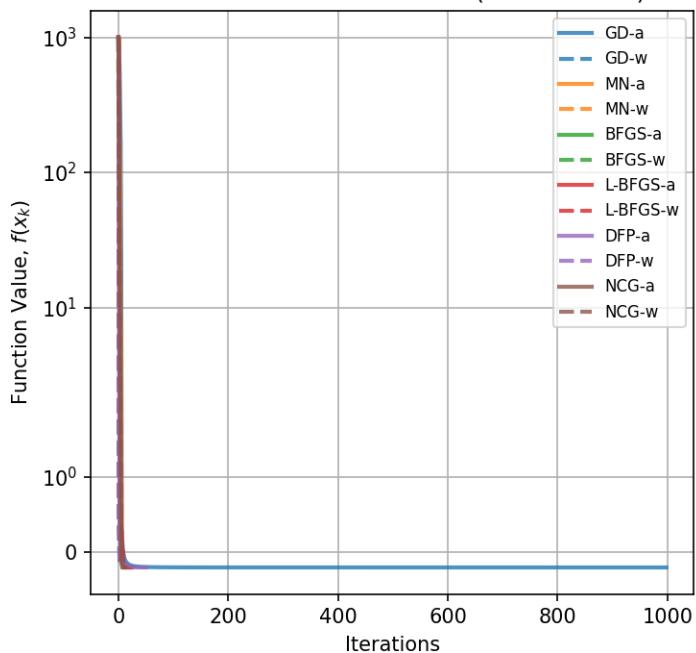
Problem11 - Gradient Norm (All Iterations)



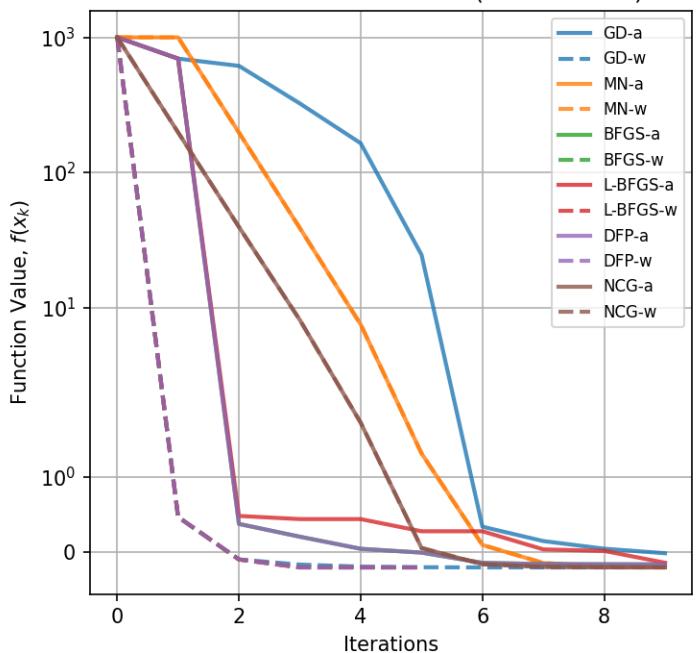
Problem11 - Gradient Norm (First 10 Iters)

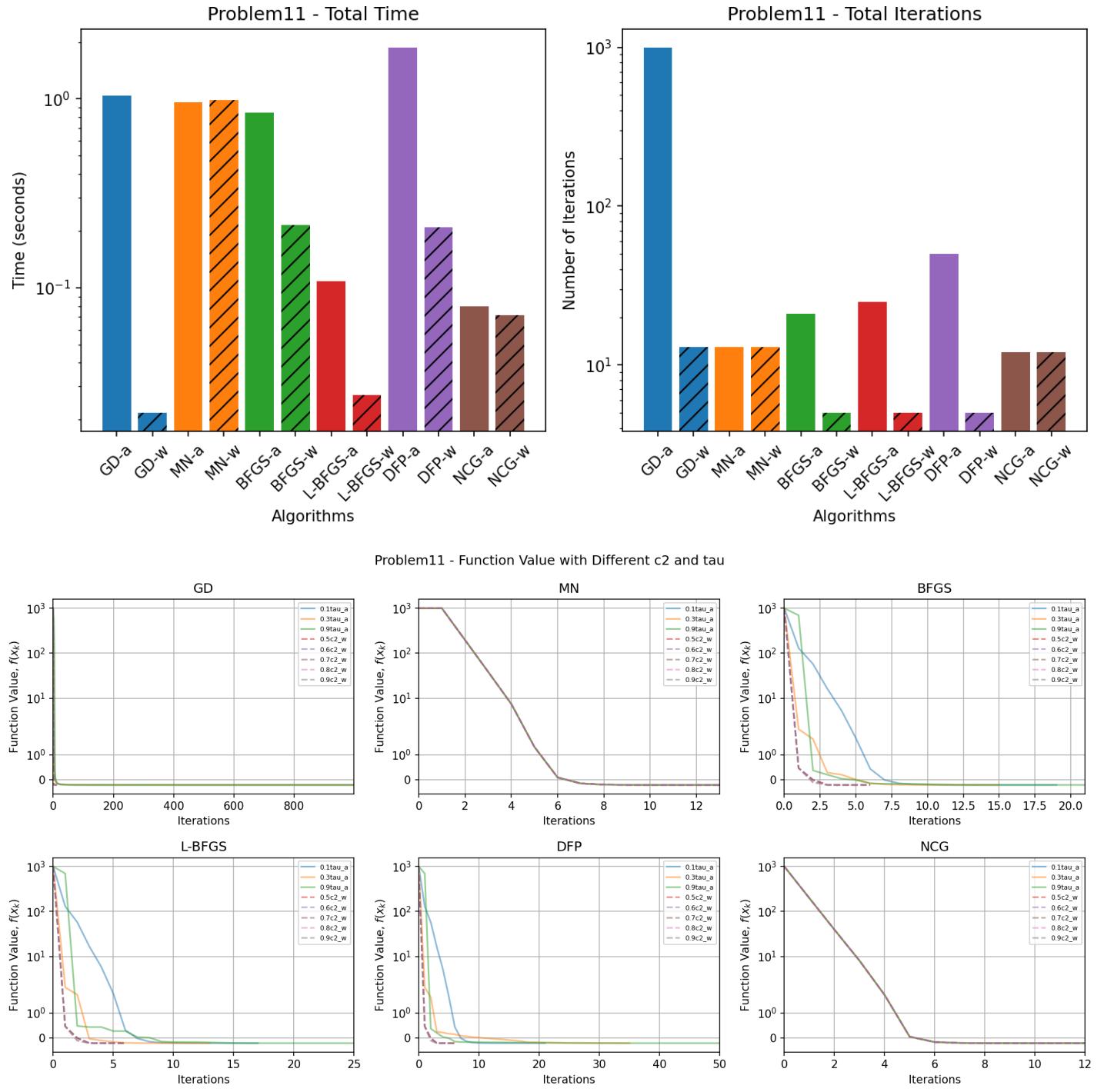


Problem11 - Function Value (All Iterations)

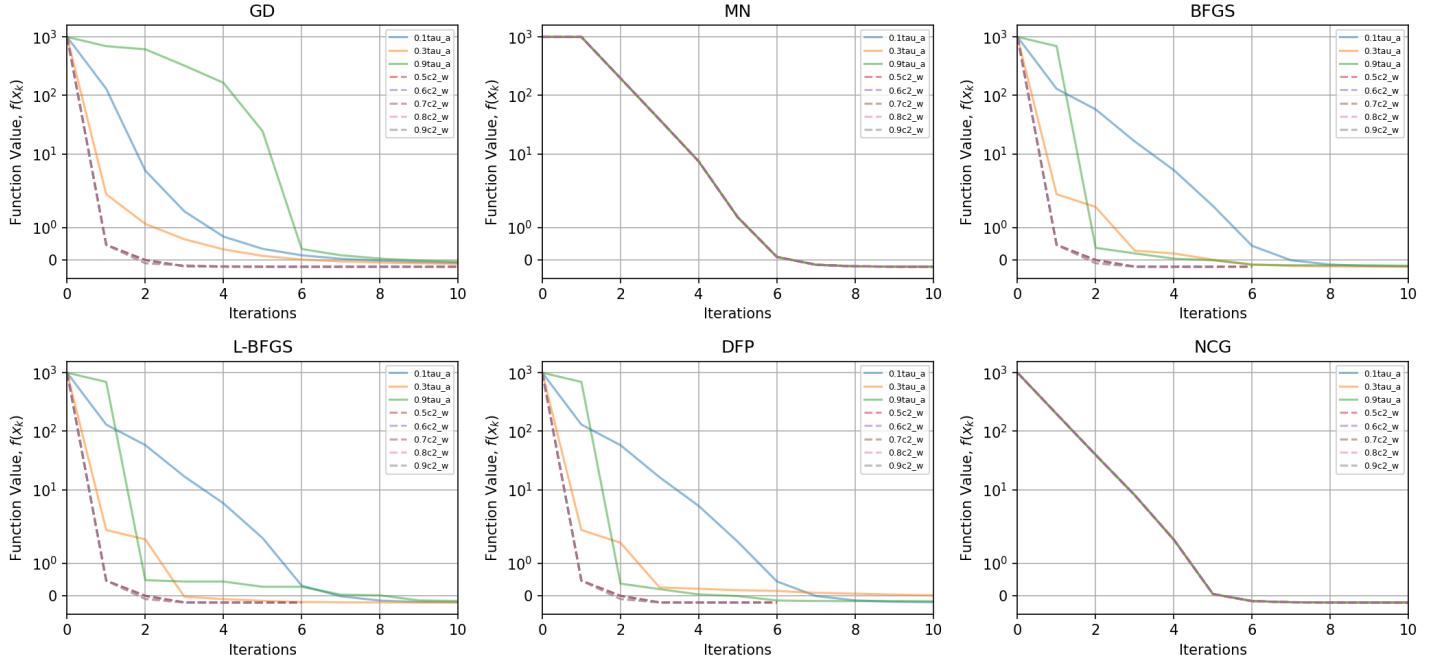


Problem11 - Function Value (First 10 Iters)

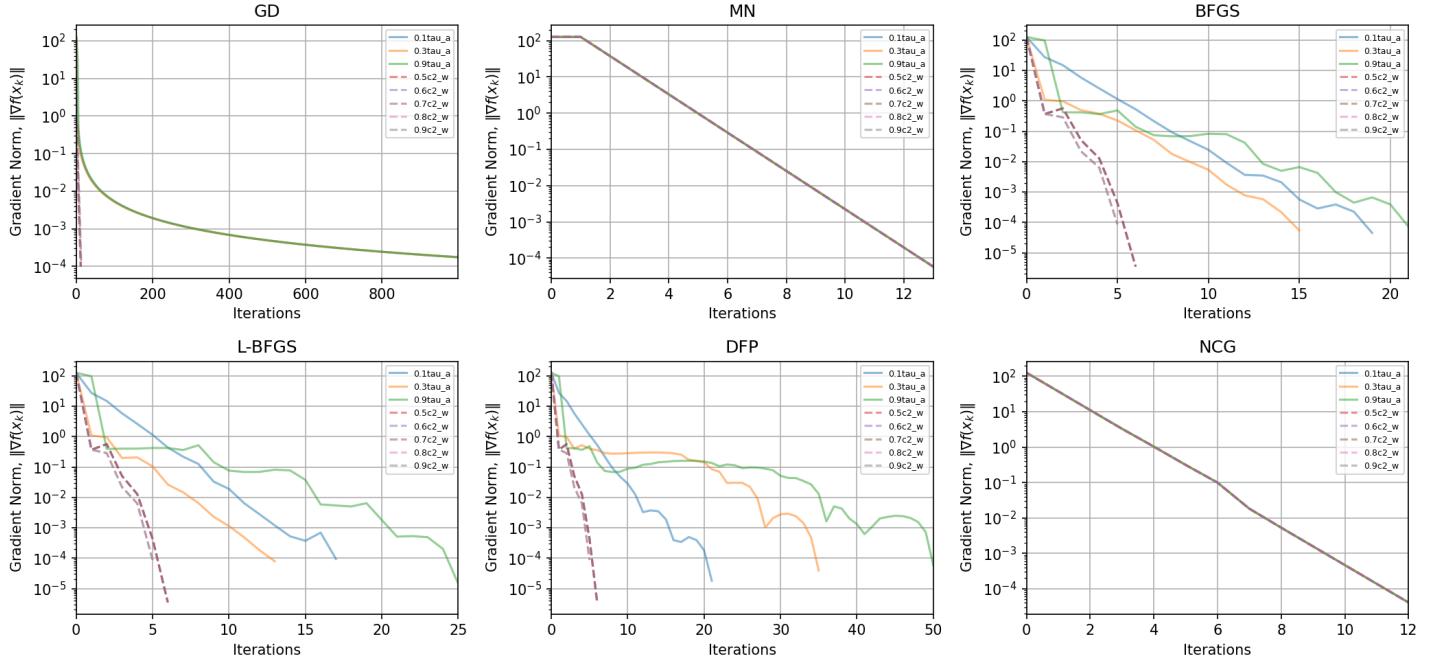




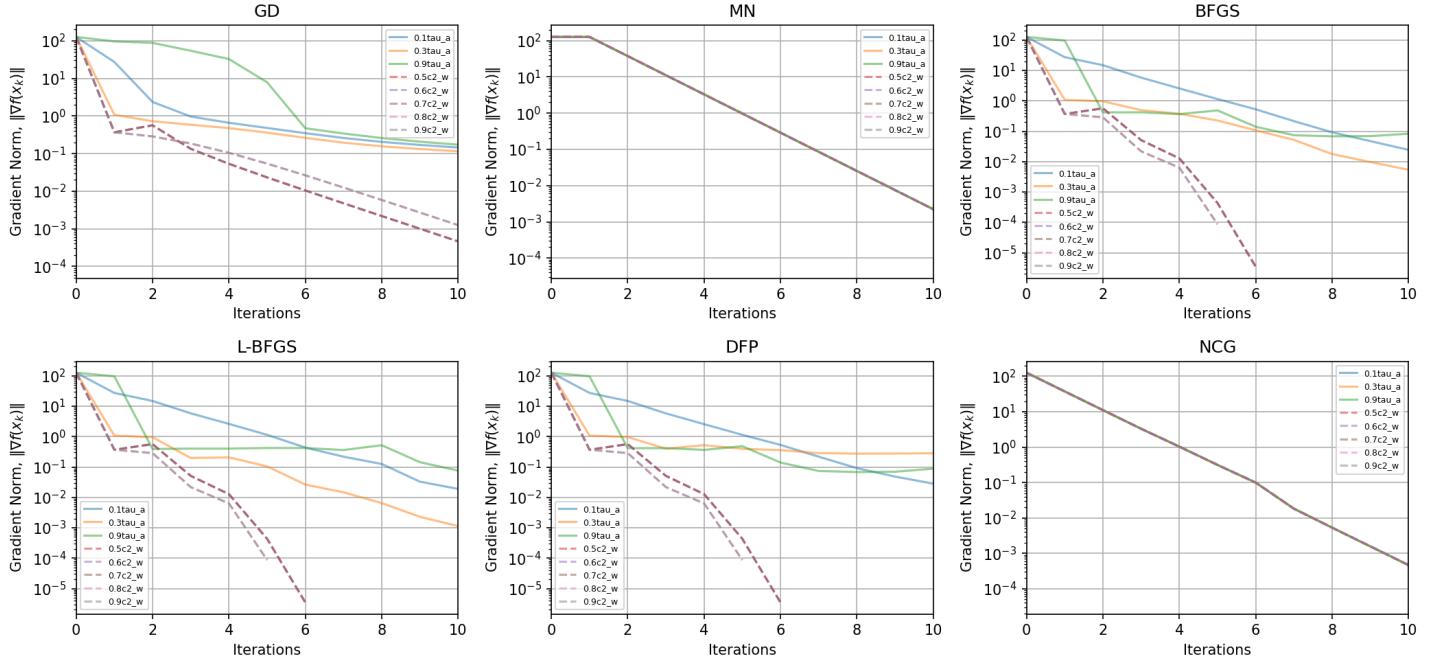
Problem11 - Function Value with Different c_2 and tau (zoom)



Problem11 - Gradient Norm with Different c_2 and tau



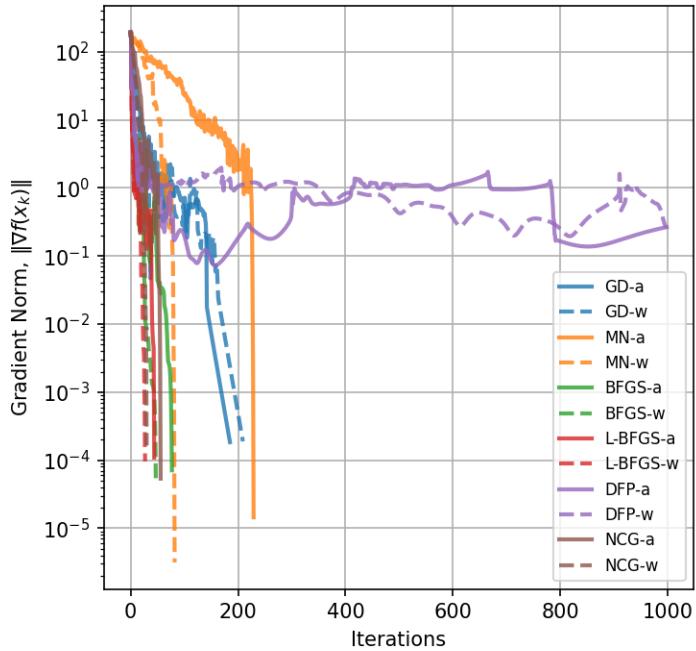
Problem11 - Gradient Norm with Different c_2 and tau (zoom)



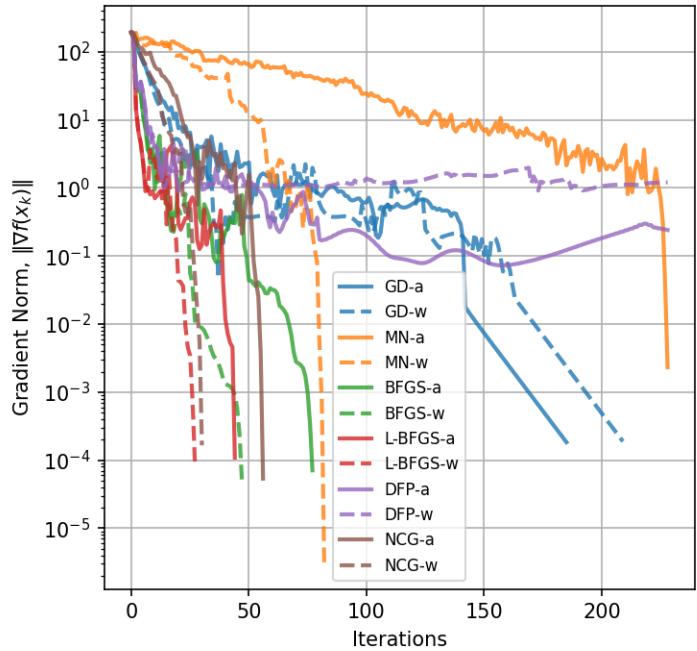
Problem 12

	Iters	Time	Convergence	Func Evals	Grad Evals	Hess Evals
GD-a	185	0.0526369	Converged. Gradient norm is below tolerance.	1492	371	0
GD-w	209	0.0488276	Converged. Gradient norm is below tolerance.	674	223	0
MN-a	229	0.363286	Converged. Gradient norm is below tolerance.	14903	460	230
MN-w	82	0.0427117	Converged. Gradient norm is below tolerance.	920	88	83
BFGS-a	77	0.0223153	Converged. Gradient norm is below tolerance.	328	233	0
BFGS-w	47	0.0141892	Converged. Gradient norm is below tolerance.	111	98	0
L-BFGS-a	44	0.0147493	Converged. Gradient norm is below tolerance.	221	133	0
L-BFGS-w	27	0.00890994	Converged. Gradient norm is below tolerance.	65	55	0
DFP-a	999	0.240519	Failed. Maximum iterations reached.	2259	3001	0
DFP-w	999	0.288047	Failed. Maximum iterations reached.	2036	2059	0
NCG-a	56	0.0172029	Converged. Gradient norm is below tolerance.	213	114	57
NCG-w	30	0.0103898	Converged. Gradient norm is below tolerance.	66	42	31

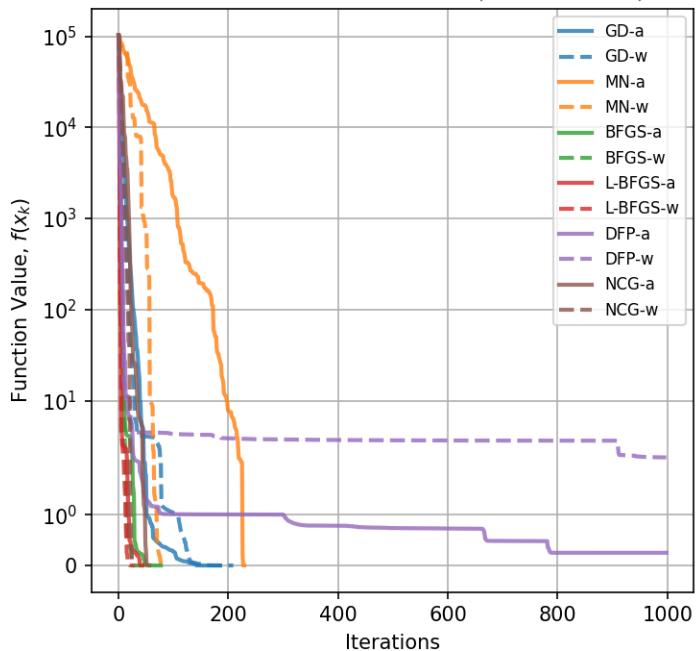
Problem12 - Gradient Norm (All Iterations)



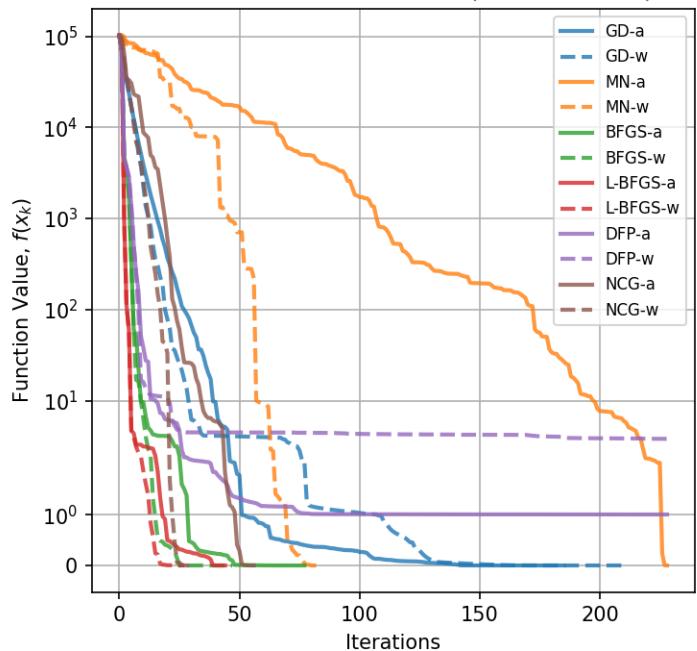
Problem12 - Gradient Norm (First 229 Iters)

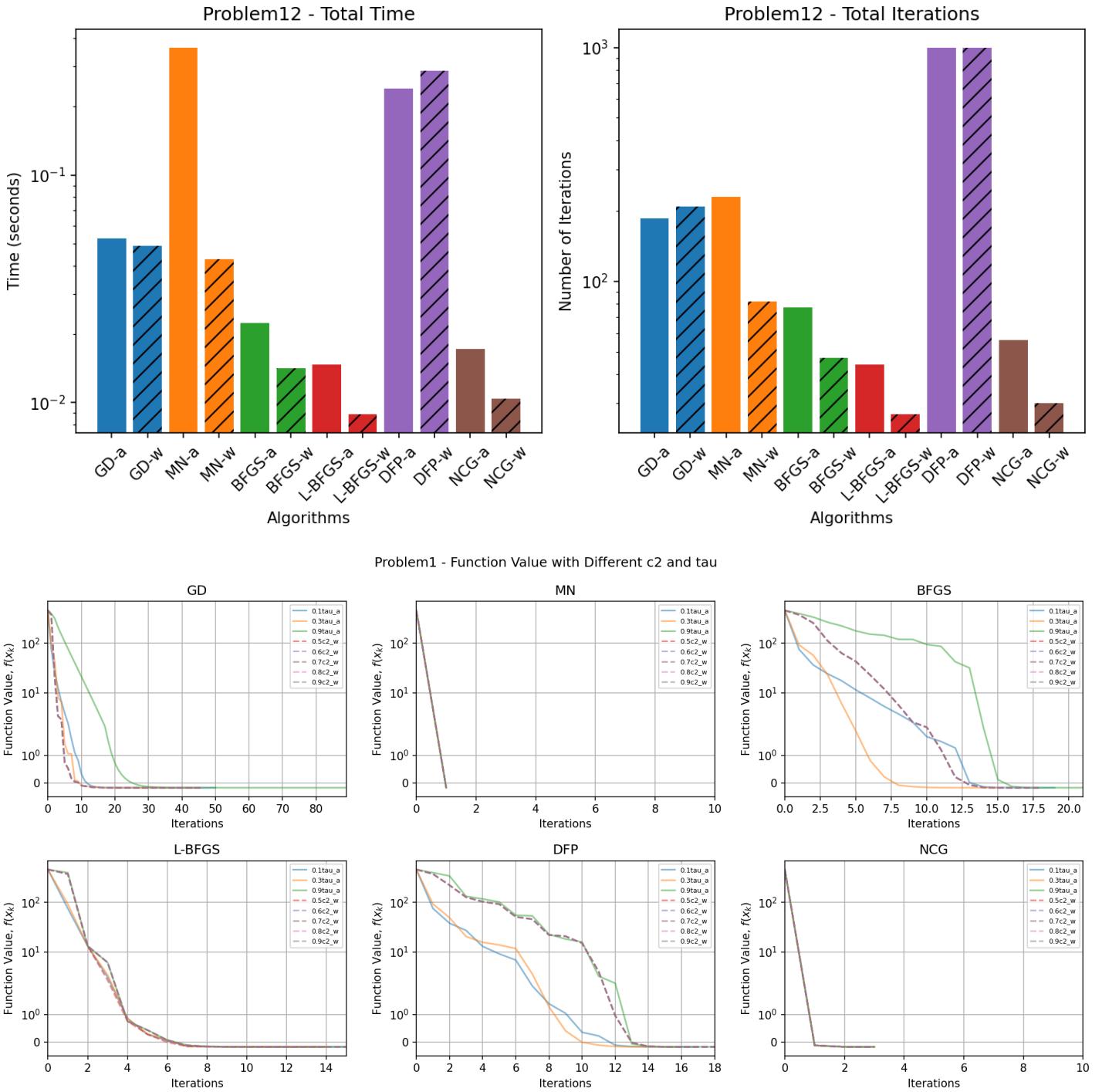


Problem12 - Function Value (All Iterations)

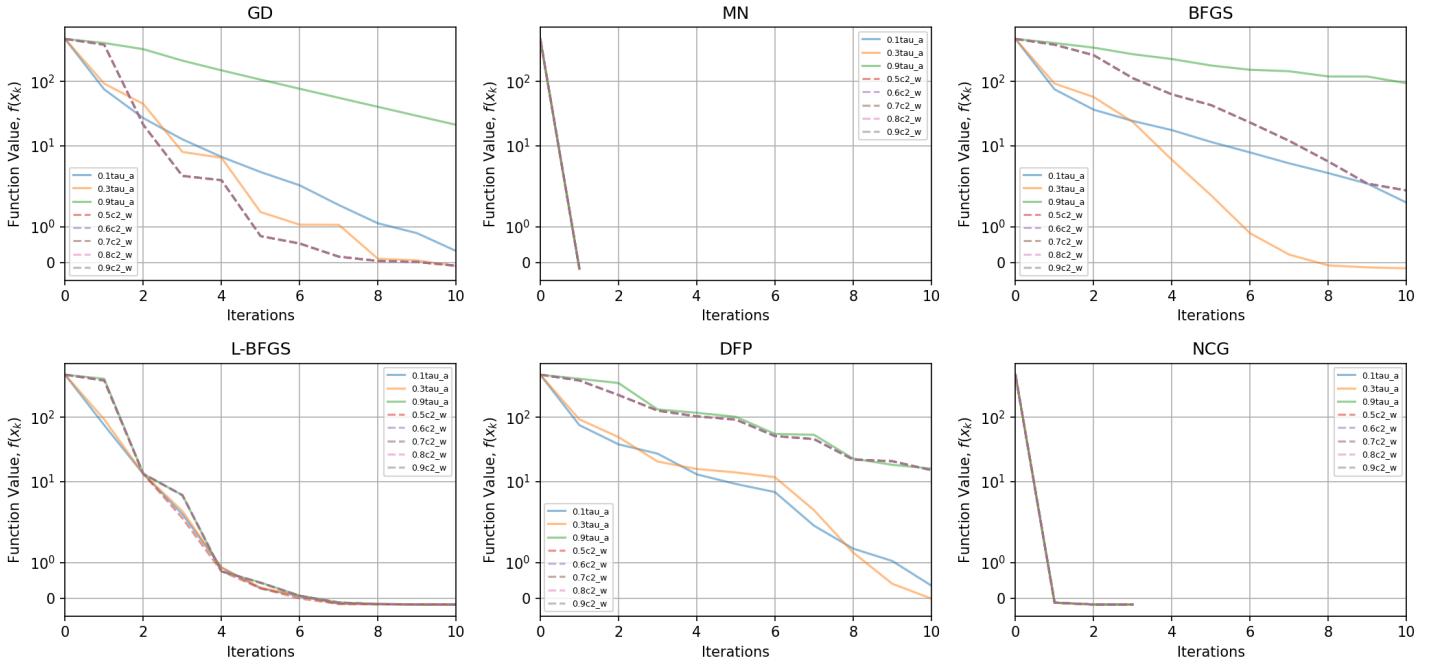


Problem12 - Function Value (First 229 Iters)

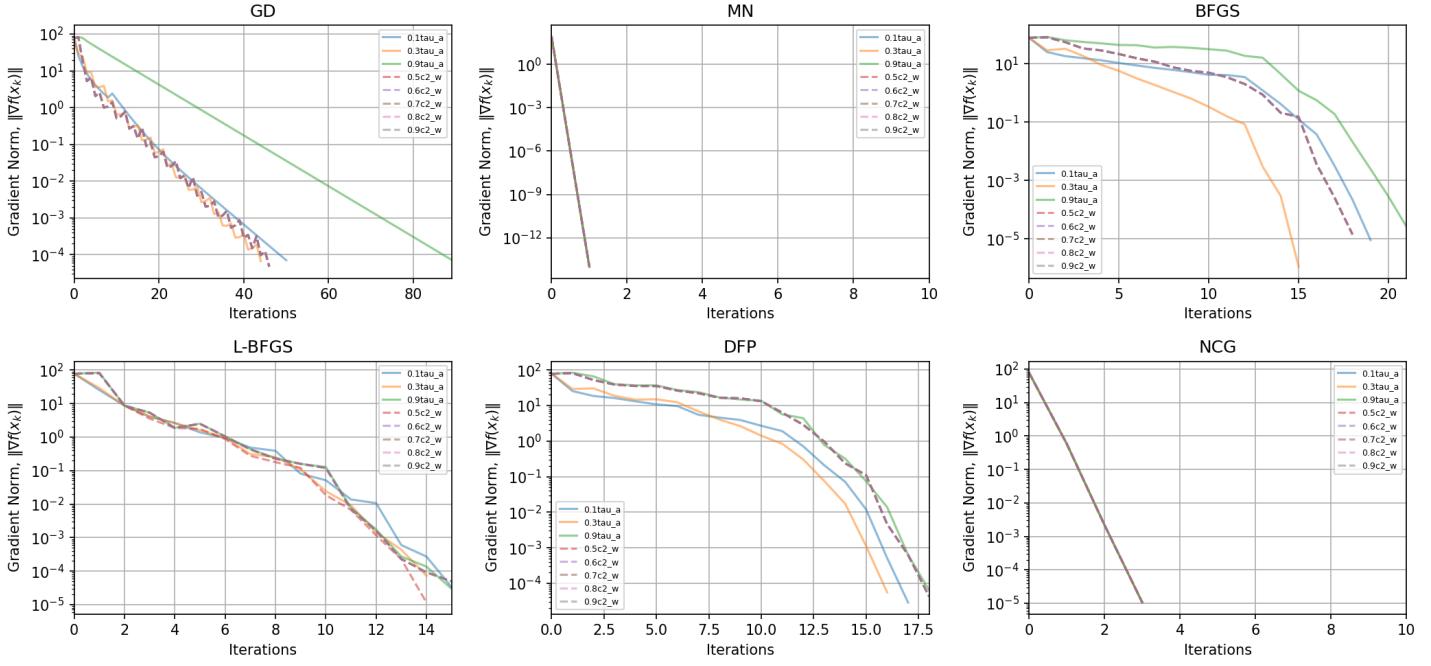




Problem1 - Function Value with Different c_2 and τ (zoom)



Problem1 - Gradient Norm with Different c_2 and tau



Problem1 - Gradient Norm with Different c_2 and τ (zoom)

