Programming Assignment **3**

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Methods: Broyden-Fletcher-Goldfarb-Shanno (BFGS), Low-memory BFGS (L-BFGS), Newton-CG.

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| **Problem 1** | Final f(x) value | N. of Iterations | CPU time  Seconds | Termination Criteria | Modifications relevant? | Convergence rate  (start, middle, end) |
| BFGS | 2.75e-17 | 35 | 1.65e-02 | Convergence |  | Linear, then superlinear |
| L-BFGS | 6.55e-26 | 41 | 2.65e-02 | Convergence | Yes, although more iterations were required, it achieved a lower f(x) value | Linear, then superlinear |
| Newton-CG | 2.20e-22 | 23 | 1.33e-02 | Convergence | Significantly faster than BFGS variants | Linear, then superlinear |
| **Problem 2** |  |  |  |  |  |  |
| BFGS | 1.12e-17 | 47 | 1.45e-02 | Convergence |  | Linear, superlinear, linear |
| L-BFGS | 2.43e-16 | 54 | 1.65e-02 | Convergence | Similar performance | Linear, superlinear, linear |
| Newton-CG | 6.97e-17 | 45 | 1.09-02 | Convergence | Yes better performance in terms of convergence | Linear, superlinear, linear |
| **Problem 3** |  |  |  |  |  |  |
| BFGS | 3.99 | 87 | 2.523 | Convergence | Got stuck on a worse local minima | Superlinear, sublinear |
| L-BFGS | 3.99 | 86 | 3.36e-02 | Convergence | No, although **significantly** faster, got stuck on a worse local minima | Superlinear, sublinear |
| Newton-CG | 6.06e-17 | 23 | 7.91e-03 | Convergence | Yes, faster and found global minima | Linear |
| **Problem 4** |  |  |  |  |  |  |
| BFGS | 9.04e-17 | 210 | 1.61e-01 | Convergence |  | Sublinear, superlinear |
| L-BFGS | 2.49e-16 | 74 | 4.02e-02 | Convergence | Significantly faster | Linear |
| Newton-CG | 1.39e-16 | 172 | 1.09e-01 | Convergence |  | Sublinear, quadratic |
| **Problem 5** |  |  |  |  |  |  |
| BFGS | 2.55e-16 | 440 | 2.66e-01 | Max iterations |  | Sublinear, superlinear |
| L-BFGS | 3.91e-15 | 543 | 1.78e-01 | Convergence | Faster | Sublinear, superlinear |
| Newton-CG | 1.62e-16 | 20 | 1.30e-02 | Max Iterations |  | linear |
| **Problem 6** |  |  |  |  |  |  |
| BFGS | 8.73e+02 | 1206 | 50 | CPU time exceeded | Failed to minimize | Linear, sublinear |
| L-BFGS | 3.99 | 5020 | 2.375e+01 | CPU time exceeded | No, still failed to minimize | Linear, superlinear |
| Newton-CG | 1.80e-16 | 19 | 5.615e-01 | Convergence | Yes, Faster and achieved local minima | Linear, sublinear, linear |
| **Problem 7** |  |  |  |  |  |  |
| BFGS | 2.36e+05 | 4 | 6.441e+01 | CPU time exceeded | Failed to minimize | sublinear |
| L-BFGS | 9.85e+03 | 245 | 5.018e+01 | CPU time exceeded | Failed to minimize | Sublinear |
| Newton-CG | 5.56e-17 | 18 | 4.176e+01 | Convergence | Achieved local minima | linear |
| **Problem 8** |  |  |  |  |  |  |
| BFGS | 6.09e-18 | 16 | 8.791e-03 | Convergence |  | Superlinear |
| L-BFGS | 1.11e-17 | 20 | 4.502e-03 | Convergence |  | Superlinear |
| Newton-CG | 4.26e-23 | 9 | 5.727e-03 | Convergence | Similar performance, however less iterations and closer to local minima | Superlinear |
| **Problem 9** |  |  |  |  |  |  |
| BFGS | 1.46e-16 | 17 | 4.33e-03 | Convergence |  | Linear, superlinear |
| L-BFGS | 1.26e-17 | 20 | 3.608e-03 | Convergence | Faster | Linear, sublinear, linear |
| Newton-CG | 1.32e-18 | 8 | 1.682e-03 | Convergence |  | superlinear |
| **Problem 10** |  |  |  |  |  |  |
| BFGS | 5.63e-09 | 72 | 1.934e-02 | Convergence |  | Linear, sublinear, linear |
| L-BFGS | 8.43e-10 | 55 | 1.403e-02 | Convergence | Faster | linear |
| Newton-CG | 2.77e-08 | 18 | 4.454e-03 | Convergence |  | linear |

It can be observed that all Quasi-Newton methods have much better performance than the steepest descent method, as expected. Although the Newton method offers quadratic convergence close to the local minima, by using quasi-newton methods, we gain speed in terms of computation per iteration. Newton-CG seems to perform best for this set of problems and manages to solve all the problems, especially 6 and 7, which were not possible for the Newton method alone. Adding CG search solves the plain Newton step, achieving a better performance than the modified Newton, as expected. This can be observed just by looking at the number of iterations and computation time.