The physical design of the wind turbine consisted of a central hub attached to three cone-like blades as shown in *Figure 1.* A 62 NC hole was drilled and threaded to accommodate a screw that would attach the hub more securely to the motor and therefore help increase efficiency.

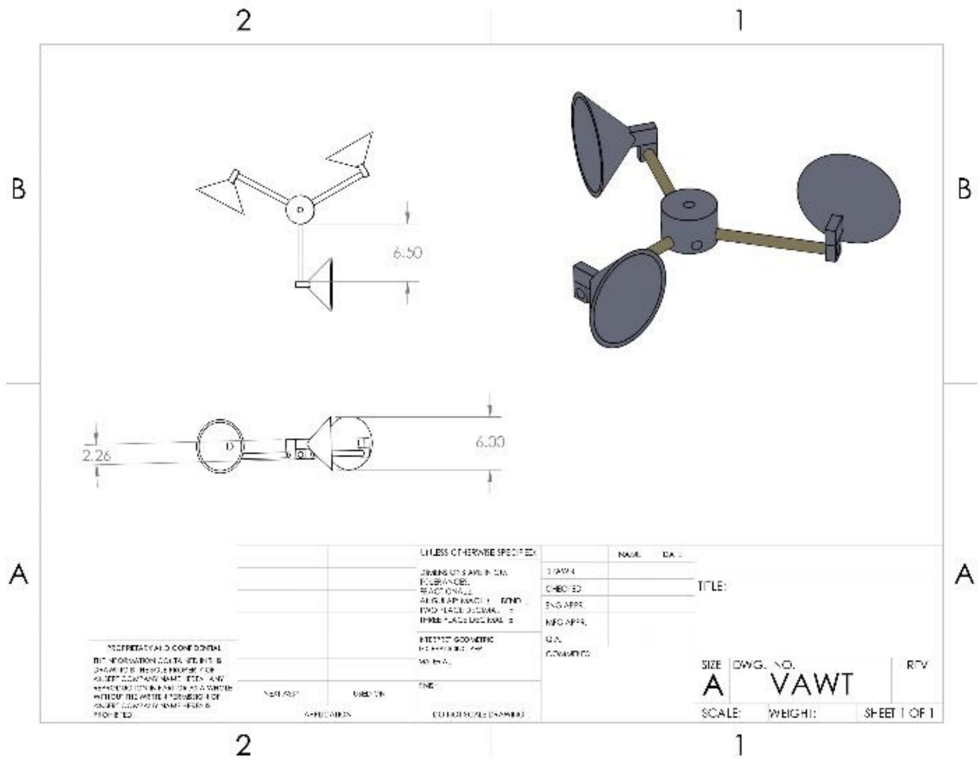

Figure 1: SolidWorks drawing of VAWT design

The initial design of the VAWT was 3 helical blades attached to the central hub, however, the design failed due to the blades not having a large enough drag coefficient which led to them not spinning. The final designed VAWT spun freely as each blade had a relatively a large drag coefficient, allowing the turbine to spin.

The VAWT was parametrized with 2 components: the blade length, and the cone diameter, as shown in figure 2.

The parameters were used in a factorial test matrix to evaluate the impact of the parameters on the total efficiency of the VAWT. Table 1 shows the results of the test matrix, where the resistance was kept constant at 25 Ohms, and the runs are completely randomized to ensure that no other factor or error were involved in the runs.

Table 1:Test matrix with efficiency results

Run number	Blade Length (cm)	Diameter of cones (cm)	Run Order	Efficiency (%)
1	5.5	4	3	1.491
2	6	4	1	0.866
3	6	6	2	0.713
4	5.5	6	4	1.187

Cross-sectional area of the turbine was a rectangular shape and was therefore calculated using Equation (1) where L was the length and W was the width.

$$A = L * W \tag{1}$$

The maximum possible power that could have been produced by the wind pass through the turbine, $\dot{W}_{wind}$, was calculated using Equation (2) where ρ was the density of the air, 1.225 kg/m^3, A_C was the cross-sectional area of the turbine and $\tilde{V}$ was the velocity of the air entering the turbine, 2.58 m/s.

$$\dot{W}_{wind} = \frac{1}{2}\rho A_C \tilde{V}^3 \tag{2}$$

The average electrical power, $\bar{\dot{W}}_{elec}$, was given by Equation (3) where V_{rms} was the rms voltage and R was the resistance which was kept at a constant 25Ω.

$$\bar{\dot{W}}_{elec} = \frac{V_{rms}^2}{R} \tag{3}$$

The total efficiency of the wind turbine was given by Equation (4) where $\bar{\dot{W}}_{elec}$ was the average electrical power calculated by Equation (3) and $\dot{W}_{wind}$ was the maximum wind power calculated by Equation (2).

$$n_{total} = \frac{\bar{\dot{W}}_{elec}}{\dot{W}_{wind}} \tag{4}$$

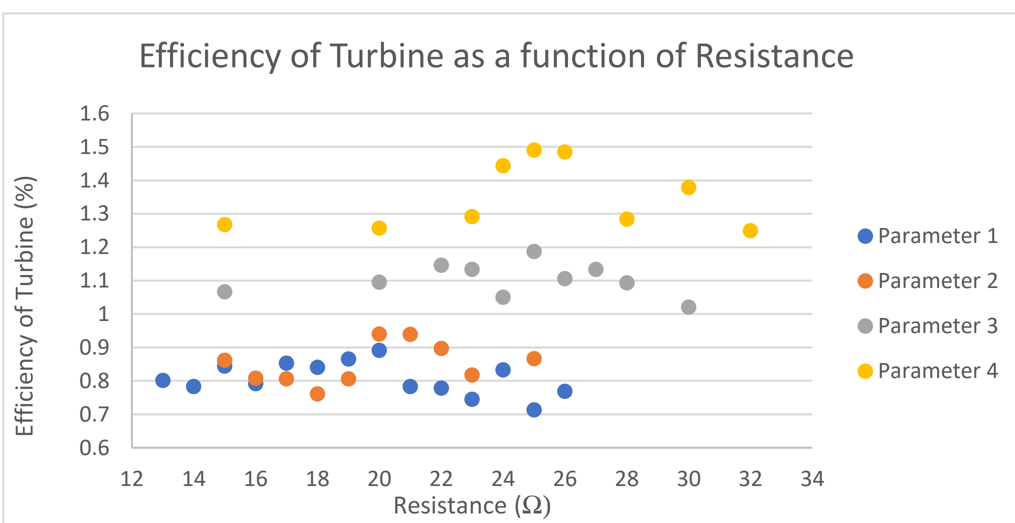

Figure 2: Efficiency of Turbine as a function of Resistance

All of the uncertainties in this experiment were calculated using the propagation of uncertainty theory and were calculated for the design with 5.5cm blades and 4cm diameter cones. The uncertainty of the Cross-sectional area, δA, was calculated using Equation (5) where δL was the uncertainty in length and δW was the uncertainty in width. Both the length and width were measured using a ruler with an uncertainty of 0.001m. The uncertainty in cross-sectional area was found to be 0.0064^2.

$$\delta A = \sqrt{[W * \delta L]^2 + [L * \delta W]^2} \tag{5}$$

The uncertainty of the maximum possible wind power, δA, was calculated by Equation (6) where $\delta \rho$ was the uncertainty in air density which was assumed to be 0, δA_C was the uncertainty in Cross-sectional area and $\delta \tilde{V}$ was the uncertainty in the velocity of the air entering the turbine which was 0.05 m/s. The uncertainty in wind power was found to be 0.0043 Watts.

$$\delta \dot{W}_{wind} = \sqrt{\left[\frac{1}{2} A_C \tilde{V}^3 * \delta \rho\right]^2 + \left[\frac{1}{2} \rho \tilde{V}^3 * \delta A_C\right]^2 + \left[\frac{3}{2} \rho A_C \tilde{V}^2 * \delta \tilde{V}\right]^2} \tag{6}$$

The uncertainty of the average electrical power, $\delta \bar{\dot{W}}_{elec}$, was calculated by Equation (7) where δV_{rms} was the uncertainty in the rms voltage, assumed to be 0.01V, and $\delta R$was the uncertainty in the resistance which was assumed to be 0.1Ω. The uncertainty in average electrical power was found to be 0.001 Watts.

$$\delta \bar{\dot{W}}_{elec} = \sqrt{\left[2 \frac{V_{rms}}{R} * \delta V_{rms}\right]^2 + \left[-\frac{V_{rms}^2}{R^2} * \delta R\right]^2} \tag{7}$$

The uncertainty of the total efficiency of the wind turbine, δn_{total}, was calculated by Equation (8) where $\delta \bar{\dot{W}}_{elec}$ was the uncertainty in the average electrical power and $\delta \dot{W}_{wind}$ was the uncertainty in the maximum wind power. The uncertainty in total efficiency was found to be 0.104%.

$$\delta n_{total} = \sqrt{\left[\frac{1}{\dot{W}_{wind}} * \delta \bar{\dot{W}}_{elec}\right]^2 + \left[-\frac{\bar{\dot{W}}_{elec}}{\dot{W}_{wind}{}^2} * \delta \dot{W}_{wind}\right]^2} \tag{8}$$

The main effects of diameter of cones and radius of blades were calculated using Equations (9) and (10) respectively. The interaction between diameter of cones and radius of blades was calculated using Equation (11)

$$MED = \frac{-1.491 - 0.866 + 1.187 + 0.713}{4} = -0.11425\% \tag{9}$$

Run Number	D	L	DL	Efficiency %
1	-	-	+	1.491
2	-	+	-	0.866
3	+	-	-	1.187
4	+	+	+	0.713
Mean				1.06425

Figure 1: Results for test matrix

$$MEL = \frac{-1.491 + 0.866 - 1.187 + 0.713}{4} = -0.27475\%$$

(10)

$$IDL = \frac{1.491 - 0.866 - 1.187 + 0.713}{4} = 0.03775\%$$

(11)

$$Eff = \overline{Eff} + MED\ D + MEL\ L + IDL\ L\ D$$

(12)

$$D = \frac{Diameter - 5cm}{1cm}$$

(13)

$$L = \frac{Length - 5.75cm}{0.25cm}$$

(14)

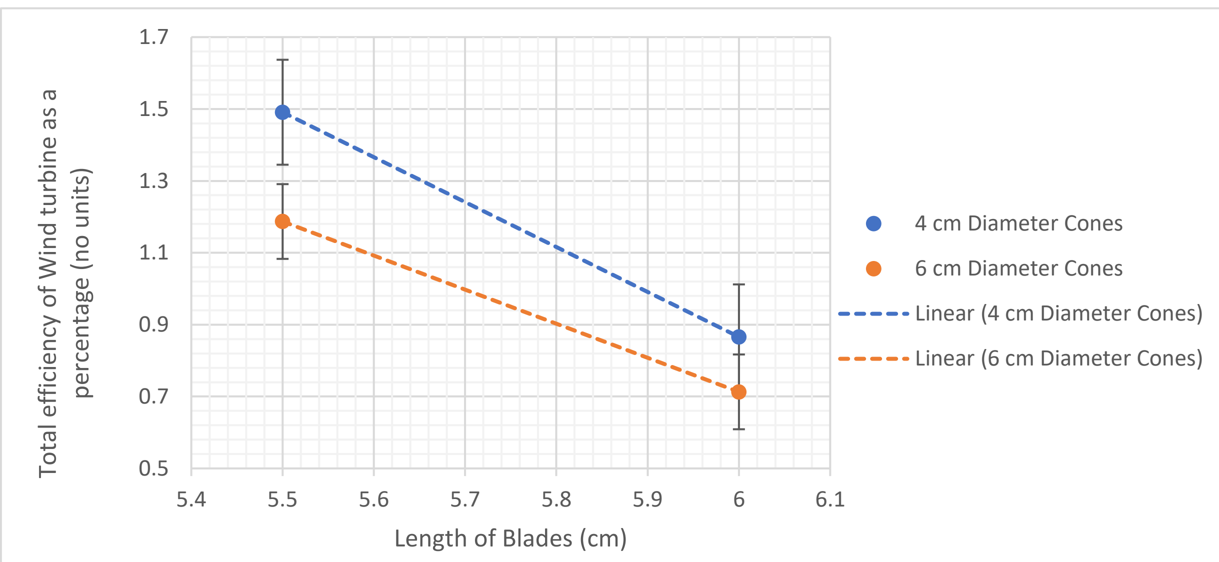

Figure 2: Efficiency as a function of length of blades for the 4 cm diameter cones and the 6cm diameter cones

The fact that the two lines in Figure 2 are not parallel is an indicator that there is an interaction between the two factors. Both parameters have negative main effects between the length of the blades and the diameter of the cones, meaning that to increase efficiency, you must decrease both the length of the blades and the diameter of the cones. After optimizing this equation in EES, it was found that the maximum efficiency is when the diameter of the cones and the length of the blades is zero, however, that doesn't make sense, because at those dimensions, a wind turbine doesn't even exist. In conclusion, the shorter the length of the blades and the smaller the diameter of the cones, the more efficient it will be, as long as it exists.

Using Equations (2) and (4), a new Equation (15) was derived,

$$n_{total} = \frac{\bar{\dot{W}}_{elec}}{\frac{1}{2}\rho A_C \tilde{V}^3} \tag{15}$$

Where $\bar{\dot{W}}_{elec}$ was the electrical power that we wanted to generate: 200kW, ρ was the density of the air, 1.225 kg/m^3, $\tilde{V}$ was the velocity of the air entering the turbine, 6 m/s and n_{total} was the total efficiency of our wind turbine which was found to be 1.491%. Solving Equation (15), the cross-sectional area,A_C was found to be 101389 m^2.

From our most efficient design of our wind turbine, we had a cross sectional length of 0.16 m and a cross sectional width of 0.04 m. We then developed a ratio between the cross-sectional length and the cross-sectional width as shown in Equation (16).

$$Length - to - width\ ratio = \frac{L}{W} \tag{16}$$

The length to width ratio was found to be 4 and therefore a general form of the cross-sectional length, L, was written in the terms of the cross-sectional width, W, in Equation (17)

$$L = 4W \tag{17}$$

After plugging Equation (17), into Equation (1), the cross-sectional area of the wind turbine can be written in terms of the cross-sectional width only.

$$A_C = 4W * W \tag{18}$$

Using the cross-sectional area of 101389 m^2 found by Equation (15), the cross-sectional width was found to be 159.2m. Substituting W = 159.2 into Equation (17), the length was found to be 636.8 m.

A ratio between the blade velocity and the air velocity from our design was found using Equation (19)

$$BladeVelocity - to - AirVelocity = \frac{2\pi r f_{max}}{V_{air}} = 8.0464x10^{-4} \tag{19}$$

Where r was half the length of the cross-sectional area, 0.08m, f_{max} was the frequency which was found to be 4.13mHz and V_{air} was the velocity of the air coming out of the fan, 2.58 m/s. Using the ratio calculated in Equation (19), the length of the new wind turbine of

318.4m and the new air velocity of 6m/s, a new frequency was calculated using Equation (20).

$$8.0464x10^{-4} = \frac{2\pi r f_{max}}{V_{air}} \tag{20}$$

The new frequency was found to be $2.41323x10^{-6}\ Hz$. Equation (21) was then used to calculate the rotational speed of the wind turbine to be $1.51628x10^{-5}\ rad/s$.

$$\omega = 2\pi f \tag{21}$$

In conclusion, for our wind turbine to be able produce 200kW of electrical power with an air velocity of 6 m/s, we need to have blades with a length roughly 318.4 m, not taking into consideration the size of the hub that will be holding the blades to the generator. We would need to have cones with a diameter of 159.2m. This design would produce a rotational speed of roughly$1.52x10^{-5}\ rad/s$.

To calculate the cost of all expenses of the wind turbine, an excel spread sheet was created and Equation (2) was used to calculate the wind power produced at each speed and then using the wind turbines efficiency of 1.491%, a new calculated electrical power, generated in kW, was multiplied by the number of hours each velocity was kept at, to calculate the total electrical power produced in one year at each location. The resulting value then was multiplied by 1.5 to find the total amount of money produced by the wind turbine over the 10 years. It was assumed that electricity prices stay constant, the wind turbine never needs to be repaired and works ideally under all conditions. It was found that the total money generated in Madison would be $1,936,225 and the total money generated in Amarillo would be 3,880,318. Clearly Amarillo is the better location to have the wind turbine.

Figure 3 Electrical Power Generated as a function of Wind Velocity for Madison and Amarillo