DS223_HW3

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```
if (!requireNamespace("pacman", quietly = TRUE)) install.packages("pacman")
# a cool library i learnt about not too long ago, it manages installation and
# loading of packages automatically
pacman::p_load(survival, survminer, flexsurv, dplyr, ggplot2,
broom, tibble, stringr, wesanderson)
```

```
telco <- read.csv("telco.csv", stringsAsFactors = FALSE)
head(telco)
```

```
##   ID region tenure age   marital address income                       ed
## 1  1 Zone 2     13  44   Married       9     64           College degree
## 2  2 Zone 3     11  33   Married       7    136    Post-undergraduate degree
## 3  3 Zone 3     68  52   Married      24    116 Did not complete high school
## 4  4 Zone 2     33  33 Unmarried      12     33          High school degree
## 5  5 Zone 2     23  30   Married       9     30 Did not complete high school
## 6  6 Zone 2     41  39 Unmarried      17     78          High school degree
##   retire gender voice internet forward       custcat churn
## 1     No   Male    No       No     Yes Basic service   Yes
## 2     No   Male   Yes       No     Yes Total service   Yes
## 3     No Female    No       No      No  Plus service    No
## 4     No Female    No       No      No Basic service   Yes
## 5     No   Male    No       No     Yes  Plus service    No
## 6     No Female    No       No      No  Plus service    No
```

```
telco$churn  <- ifelse(telco$churn %in% c(1,"Yes","yes","TRUE",TRUE), 1, 0)
```

Factorising the categories:

```
telco$region  <- factor(telco$region)
telco$marital <- factor(telco$marital)
telco$ed <- factor(telco$ed)
telco$retire <- factor(telco$retire)
telco$gender  <- factor(telco$gender)
telco$voice  <- factor(telco$voice)
telco$internet  <- factor(telco$internet)
telco$forward <- factor(telco$forward)
telco$custcat <- factor (telco$custcat)

surv_obj <- Surv(time = telco$tenure, event = telco$churn)
```

Finding available distributions

```r
names(survreg.distributions)
```

```
##  [1] "extreme"     "logistic"    "gaussian"    "weibull"     "exponential"
##  [6] "rayleigh"    "loggaussian" "lognormal"   "loglogistic" "t"
```

```r
aft_formula <- surv_obj ~ region + age + marital + address + income + ed +
  retire + gender + voice + internet + forward + custcat

reg_extreme     <- try(survreg(aft_formula, data = telco, dist = "extreme"))
reg_logistic    <- try(survreg(aft_formula, data = telco, dist = "logistic"))
reg_gaussian    <- try(survreg(aft_formula, data = telco, dist = "gaussian"))
reg_weibull     <- try(survreg(aft_formula, data = telco, dist = "weibull"))
reg_exponential <- try(survreg(aft_formula, data = telco, dist = "exponential"))
reg_rayleigh    <- try(survreg(aft_formula, data = telco, dist = "rayleigh"))
reg_loggaussian <- try(survreg(aft_formula, data = telco, dist = "loggaussian"))
reg_lognormal   <- try(survreg(aft_formula, data = telco, dist = "lognormal"))
reg_loglogistic <- try(survreg(aft_formula, data = telco, dist = "loglogistic"))
reg_t           <- try(survreg(aft_formula, data = telco, dist = "t"))
```

```r
dists_vec <- c("extreme","logistic","gaussian","weibull","exponential",
               "rayleigh","loggaussian","lognormal","loglogistic","t")

fits_vec  <- list(reg_extreme,reg_logistic,reg_gaussian,reg_weibull,reg_exponential,
                  reg_rayleigh,reg_loggaussian,reg_lognormal,reg_loglogistic,reg_t)

AIC_vec <- rep(NA_real_, length(fits_vec))
BIC_vec <- rep(NA_real_, length(fits_vec))
LL_vec  <- rep(NA_real_, length(fits_vec))
n_sig   <- rep(NA_integer_, length(fits_vec))
ok_vec  <- rep(FALSE, length(fits_vec))

for (i in seq_along(fits_vec)) {
  f <- fits_vec[[i]]
  if (!inherits(f, "try-error")) {
    ok_vec[i]  <- TRUE
    AIC_vec[i] <- AIC(f)
    BIC_vec[i] <- BIC(f)
    LL_vec[i]  <- as.numeric(logLik(f))
    st <- summary(f)$table
    if (!is.null(st) && "p" %in% colnames(st)) {
      # counting how many p < 0.05 excluding the intercept
      pvals <- st[, "p"]
      if (length(pvals) > 1) n_sig[i] <- sum(pvals[-1] < 0.05, na.rm = TRUE)
    }
  }
}

comparison <- data.frame(
  dist = dists_vec, ok = ok_vec,
  AIC = AIC_vec, BIC = BIC_vec, logLik = LL_vec, n_sig = n_sig,
  row.names = NULL)

comparison[order(comparison$AIC), ]
```

```
##           dist   ok      AIC      BIC    logLik n_sig
## 7  loggaussian TRUE 2954.024 3052.179 -1457.012     9
## 8    lognormal TRUE 2954.024 3052.179 -1457.012     9
## 9  loglogistic TRUE 2956.206 3054.361 -1458.103    11
## 4      weibull TRUE 2964.343 3062.498 -1462.172    11
## 5  exponential TRUE 2973.195 3066.442 -1467.598    10
## 6     rayleigh TRUE 3092.877 3186.124 -1527.438    12
## 3     gaussian TRUE 3135.221 3233.376 -1547.611    12
## 2     logistic TRUE 3149.896 3248.051 -1554.948    10
## 10           t TRUE 3165.914 3264.069 -1562.957    10
## 1      extreme TRUE 3182.381 3280.536 -1571.191    10
```

```r
ref <- telco[1, , drop = FALSE]
for (nm in names(ref)) {
  if (is.numeric(telco[[nm]])) {
    ref[[nm]] <- median(telco[[nm]], na.rm = TRUE)
  } else if (is.factor(telco[[nm]])) {
    ref[[nm]] <- names(sort(table(telco[[nm]]), decreasing = TRUE))[1]}}

surv_levels <- seq(0.9, 0.1, by = -0.1)
curve_df <- data.frame()
```

Weibull

```r
if (!inherits(reg_weibull, "try-error")) {
  qt_weib <- predict(reg_weibull, type = "quantile", p = 1 - surv_levels, newdata = ref)
  df_weib <- data.frame(dist = "weibull", Time = as.numeric(qt_weib), Survival = surv_levels)
  curve_df <- rbind(curve_df, df_weib)}
```

Lognormal

```r
if (!inherits(reg_lognormal, "try-error")) {
  qt_lnorm <- predict(reg_lognormal, type = "quantile", p = 1 - surv_levels, newdata = ref)
  df_lnorm <- data.frame(dist = "lognormal", Time = as.numeric(qt_lnorm), Survival = surv_levels)
  curve_df <- rbind(curve_df, df_lnorm)}
```

LogLogistic

```r
if (!inherits(reg_loglogistic, "try-error")) {
  qt_llog <- predict(reg_loglogistic, type = "quantile", p = 1 - surv_levels, newdata = ref)
  df_llog <- data.frame(dist = "loglogistic", Time = as.numeric(qt_llog), Survival = surv_levels)
  curve_df <- rbind(curve_df, df_llog)}
```

Extreme

```r
if (!inherits(reg_extreme, "try-error")) {
  qt_llog <- predict(reg_extreme, type = "quantile", p = 1 - surv_levels, newdata = ref)
  df_llog <- data.frame(dist = "extreme", Time = as.numeric(qt_llog), Survival = surv_levels)
  curve_df <- rbind(curve_df, df_llog)}
```

Logistic

```r
if (!inherits(reg_logistic, "try-error")) {
  qt_llog <- predict(reg_logistic, type = "quantile", p = 1 - surv_levels, newdata = ref)
  df_llog <- data.frame(dist = "logistic", Time = as.numeric(qt_llog), Survival = surv_levels)
  curve_df <- rbind(curve_df, df_llog)}
```

Gaussian

```r
if (!inherits(reg_gaussian, "try-error")) {
  qt_llog <- predict(reg_gaussian, type = "quantile", p = 1 - surv_levels, newdata = ref)
  df_llog <- data.frame(dist = "gaussian", Time = as.numeric(qt_llog), Survival = surv_levels)
  curve_df <- rbind(curve_df, df_llog)}
```

Exponential

```r
if (!inherits(reg_exponential, "try-error")) {
  qt_llog <- predict(reg_exponential, type = "quantile", p = 1 - surv_levels, newdata = ref)
  df_llog <- data.frame(dist = "exponential", Time = as.numeric(qt_llog), Survival = surv_levels)
  curve_df <- rbind(curve_df, df_llog)}
```

Rayleigh

```r
if (!inherits(reg_rayleigh, "try-error")) {
  qt_llog <- predict(reg_rayleigh, type = "quantile", p = 1 - surv_levels, newdata = ref)
  df_llog <- data.frame(dist = "rayleigh", Time = as.numeric(qt_llog), Survival = surv_levels)
  curve_df <- rbind(curve_df, df_llog)}
```

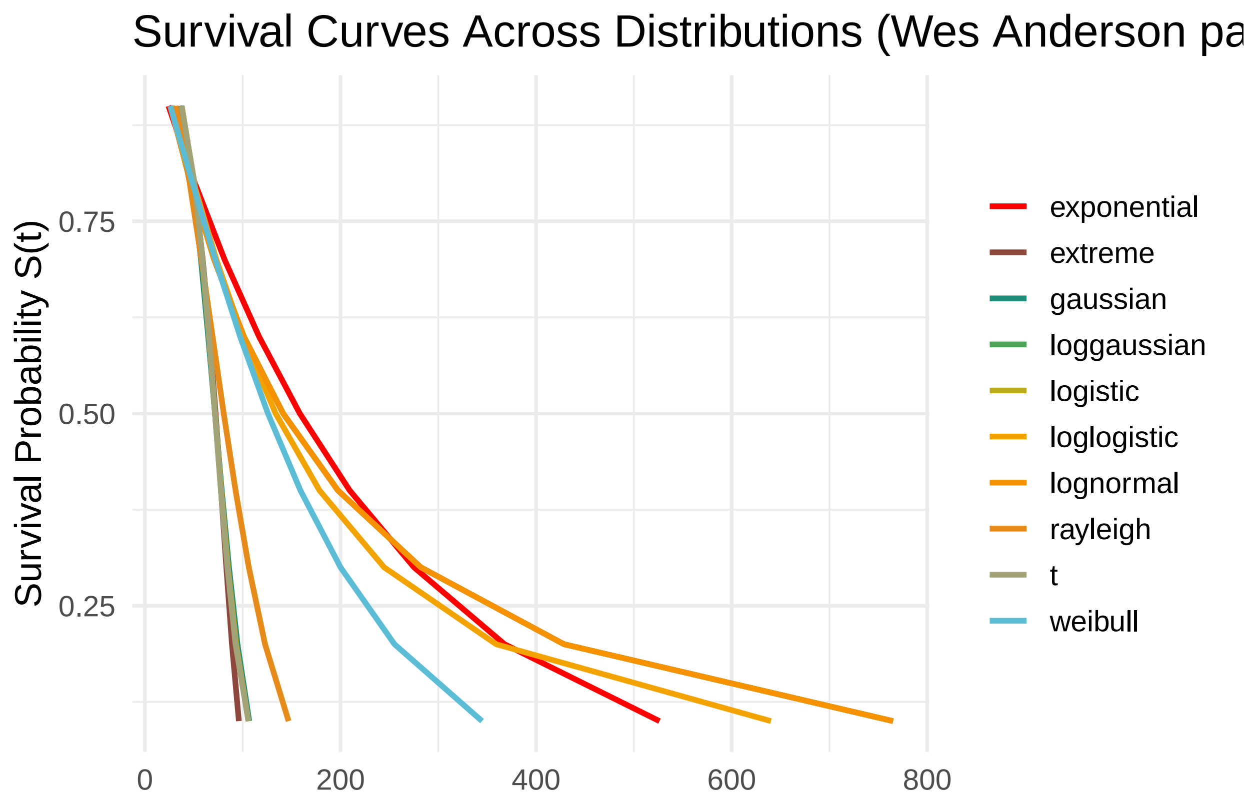
LogGaussian

```r
if (!inherits(reg_loggaussian, "try-error")) {
  qt_llog <- predict(reg_loggaussian, type = "quantile", p = 1 - surv_levels, newdata = ref)
  df_llog <- data.frame(dist = "loggaussian", Time = as.numeric(qt_llog), Survival = surv_levels)
  curve_df <- rbind(curve_df, df_llog)}
```

T

```r
if (!inherits(reg_t, "try-error")) {
  qt_llog <- predict(reg_t, type = "quantile", p = 1 - surv_levels, newdata = ref)
  df_llog <- data.frame(dist = "t", Time = as.numeric(qt_llog), Survival = surv_levels)
  curve_df <- rbind(curve_df, df_llog)}
```

```r
n_models <- length(unique(curve_df$dist))
wes_cols <- wes_palette("Darjeeling1", n_models, type = "continuous")

ggplot(curve_df, aes(x = Time, y = Survival, color = dist)) +
  geom_line(linewidth = 1) +
  scale_color_manual(values = wes_cols) +
  labs(title = "Survival Curves Across Distributions (Wes Anderson palette)",
       x = "", y = "Survival Probability S(t)") +
  theme_minimal(base_size = 14) +
  theme(
    legend.title = element_blank(),
    legend.position = "right")
```



Going back to the comparison table, we may see that the lowest AIC is yielded by the LogNormal and the LogGaussian, however for the CLV analysis, some literature suggested using the LogNormal method is better.

```
summary(reg_lognormal)
```

```
##
## Call:
## survreg(formula = aft_formula, data = telco, dist = "lognormal")
##                                    Value Std. Error     z       p
## (Intercept)                      2.36227    0.29263  8.07 6.9e-16
## regionZone 2                    -0.09704    0.14277 -0.68   0.497
## regionZone 3                     0.04822    0.14154  0.34   0.733
## age                              0.03267    0.00725  4.50 6.7e-06
## maritalUnmarried                -0.45515    0.11543 -3.94 8.0e-05
## address                          0.04254    0.00890  4.78 1.8e-06
## income                           0.00140    0.00092  1.52   0.129
## edDid not complete high school   0.37361    0.20159  1.85   0.064
## edHigh school degree             0.31593    0.16318  1.94   0.053
## edPost-undergraduate degree     -0.03436    0.22317 -0.15   0.878
## edSome college                   0.27232    0.16535  1.65   0.100
## retireYes                        0.02248    0.44407  0.05   0.960
## genderMale                       0.05188    0.11429  0.45   0.650
## voiceYes                        -0.43379    0.16895 -2.57   0.010
## internetYes                     -0.77150    0.14348 -5.38 7.6e-08
## forwardYes                      -0.19813    0.18004 -1.10   0.271
```

```
## custcatE-service                1.06642    0.17053  6.25 4.0e-10
## custcatPlus service             0.92495    0.21575  4.29 1.8e-05
## custcatTotal service            1.19860    0.25045  4.79 1.7e-06
## Log(scale)                      0.27577    0.04600  6.00 2.0e-09
##
## Scale= 1.32
##
## Log Normal distribution
## Loglik(model)= -1457   Loglik(intercept only)= -1602.5
##  Chisq= 291.01 on 18 degrees of freedom, p= 3.4e-51
## Number of Newton-Raphson Iterations: 5
## n= 1000
```

Here we are selecting the most appropriate features with the lowest yileded p-values.

```r
final_formula <- surv_obj ~ age + marital + address + voice + internet + custcat
final_dist    <- "lognormal"
final_model   <- survreg(final_formula, data = telco, dist = final_dist)
summary(final_model)
```

```
##
## Call:
## survreg(formula = final_formula, data = telco, dist = final_dist)
##                         Value Std. Error     z       p
## (Intercept)           2.53488    0.24261 10.45 < 2e-16
## age                   0.03683    0.00640  5.75 8.7e-09
## maritalUnmarried     -0.44732    0.11447 -3.91 9.3e-05
## address               0.04282    0.00885  4.84 1.3e-06
## voiceYes             -0.46350    0.16677 -2.78  0.0054
## internetYes          -0.84054    0.13826 -6.08 1.2e-09
## custcatE-service      1.02582    0.16905  6.07 1.3e-09
## custcatPlus service   0.82250    0.16942  4.85 1.2e-06
## custcatTotal service  1.01326    0.20958  4.83 1.3e-06
## Log(scale)            0.28303    0.04602  6.15 7.7e-10
##
## Scale= 1.33
##
## Log Normal distribution
## Loglik(model)= -1462.1   Loglik(intercept only)= -1602.5
##  Chisq= 280.83 on 8 degrees of freedom, p= 4.9e-56
## Number of Newton-Raphson Iterations: 5
## n= 1000
```

```r
fs_fit <- flexsurv::flexsurvreg(final_formula, data = telco, dist = final_dist)
```

```r
times <- 1:24
MM <- 1300
r <- 0.10
disc <- 1 / (1 + r/12)^(times - 1)

n <- nrow(telco)
S_mat <- matrix(NA_real_, nrow = n, ncol = length(times))
pb <- txtProgressBar(min = 0, max = n, style = 3)
```

```
## |                                                                         |
```

```
for (i in seq_len(n)) {
  s_obj <- try(
    summary(fs_fit,
            newdata = telco[i, , drop = FALSE],
            type = "survival", t = times),
    silent = TRUE)
  row_vals <- rep(NA_real_, length(times))
  if (!inherits(s_obj, "try-error") && !is.null(s_obj)) {
    if (is.list(s_obj)) {
      row_vals <- vapply(s_obj, function(d) d$est[1], numeric(1))
      } else if (is.data.frame(s_obj) && "est" %in% names(s_obj)) {
        row_vals <- s_obj$est
        }
    }
  if (length(row_vals) < length(times)) {
    row_vals <- c(row_vals, rep(NA_real_, length(times) - length(row_vals)))}
  row_vals[is.na(row_vals)] <- 0
  row_vals[row_vals < 0] <- 0
  row_vals[row_vals > 1] <- 1
  S_mat[i, ] <- row_vals
  setTxtProgressBar(pb, i)
  }
```

```
## |                                                                         |
```

```
close(pb)
```

```
# CLV_i = MM * sigma_t S_i(t) * discount_t
CLV <- numeric(n)
for (i in seq_len(n)) {
  CLV[i] <- MM * sum(S_mat[i, ] * disc)
  }
telco$CLV <- CLV
summary(telco$CLV)
```

```
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##    1145    1297    1300    1296    1300    1300
```

```
# mean CLV by customer category and internet
tab_cust_internet <- aggregate(CLV ~ custcat + internet, data = telco, FUN = mean, na.rm = TRUE)
tab_gender        <- aggregate(CLV ~ gender,             data = telco, FUN = mean, na.rm = TRUE)

tab_cust_internet
```

```
##         custcat internet      CLV
## 1 Basic service       No 1296.519
## 2     E-service       No 1299.799
## 3  Plus service       No 1299.566
## 4 Total service       No 1299.629
## 5 Basic service      Yes 1273.443
```

```
## 6     E-service      Yes 1297.807
## 7  Plus service      Yes 1297.349
## 8 Total service      Yes 1294.773
```

```r
tab_gender
```

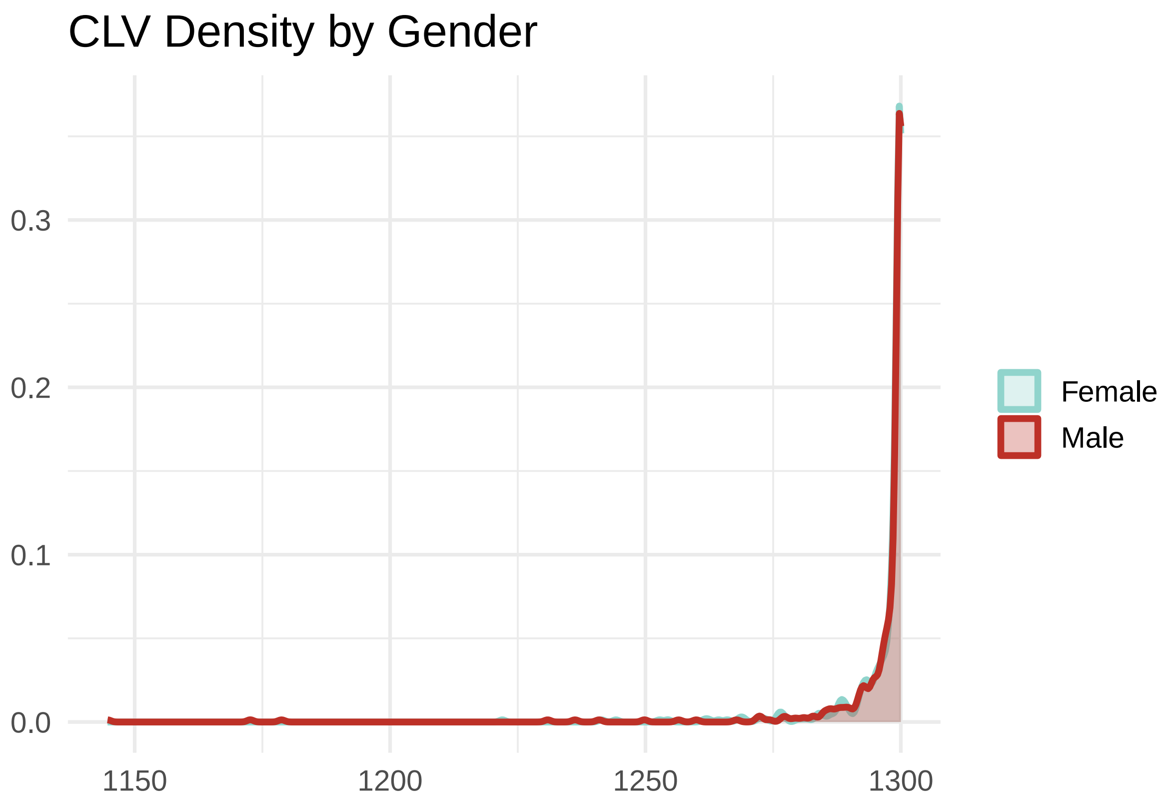
```
##   gender      CLV
## 1 Female 1296.548
## 2   Male 1295.950
```

the CLV is very similar across service categories — the differences are quite small, meaning service type alone does not strongly determine lifetime value. And as for gender, it does not meaningfully affect customer value either (no discrimination :D)

```r
aggregate(CLV ~ gender, data = telco, FUN = mean)
```

```
##   gender      CLV
## 1 Female 1296.548
## 2   Male 1295.950
```

```r
ggplot(telco, aes(x = CLV, color = gender, fill = gender)) +
  geom_density(alpha = 0.3, linewidth = 1.2) +
  scale_color_manual(values = wes_palette("FrenchDispatch", 2)) +
  scale_fill_manual(values = wes_palette("FrenchDispatch", 2)) +
  labs(title = "CLV Density by Gender", x = "", y = "") +
  theme_minimal(base_size = 14) +
  theme(legend.title = element_blank())
```



Again, this proves that gender has no meaningful impact on the CLV.

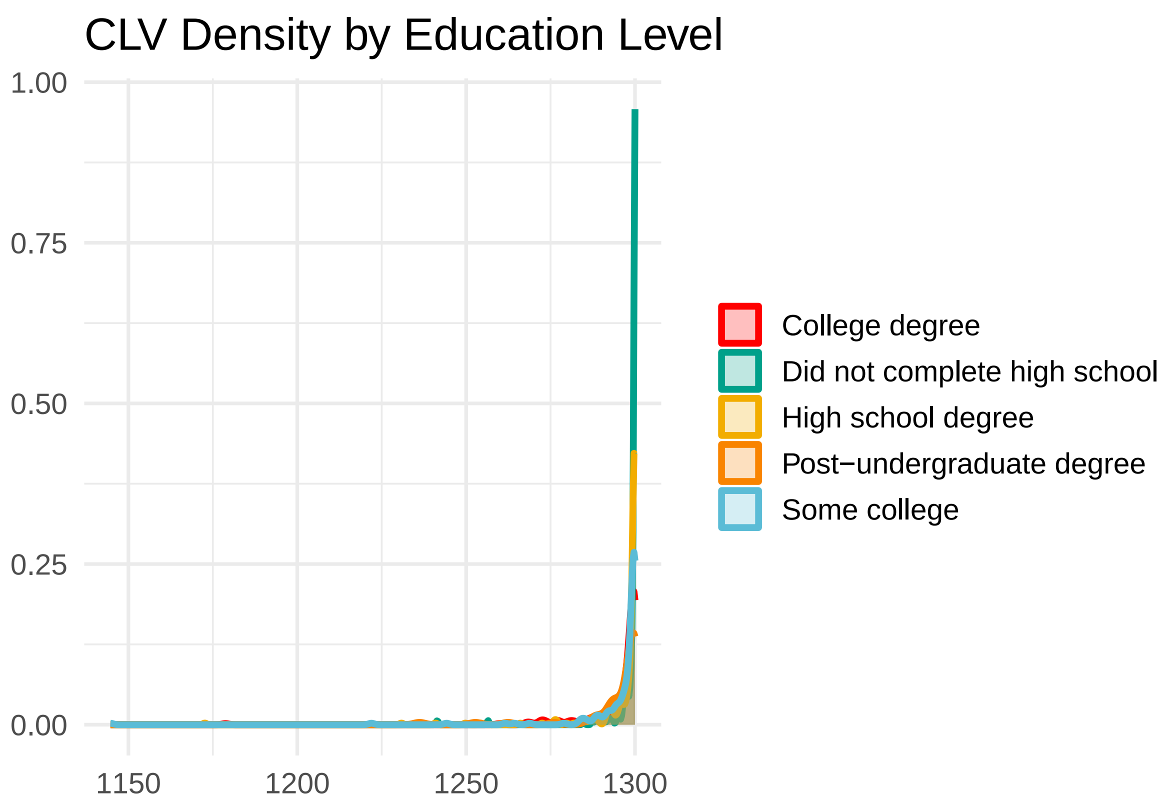
```
wes_auto <- function(x, palette = "Darjeeling1") {
  n <- nlevels(x)
  if (n <= length(wesanderson::wes_palettes[[palette]])) {
    wes_palette(palette, n, type = "discrete")
  } else {
    wes_palette(palette, n, type = "continuous")
  }}
```

```
aggregate(CLV ~ ed, data = telco, FUN = mean)
```

```
##                             ed      CLV
## 1               College degree 1295.199
## 2 Did not complete high school 1298.386
## 3           High school degree 1296.397
## 4    Post-undergraduate degree 1294.860
## 5                 Some college 1295.622
```

```
pal_ed <- wes_auto(telco$ed)

ggplot(telco, aes(x = CLV, color = ed, fill = ed)) +
  geom_density(alpha = 0.25, linewidth = 1.2) +
  scale_color_manual(values = pal_ed) +
  scale_fill_manual(values = pal_ed) +
  labs(title = "CLV Density by Education Level", x = "", y = "") +
```

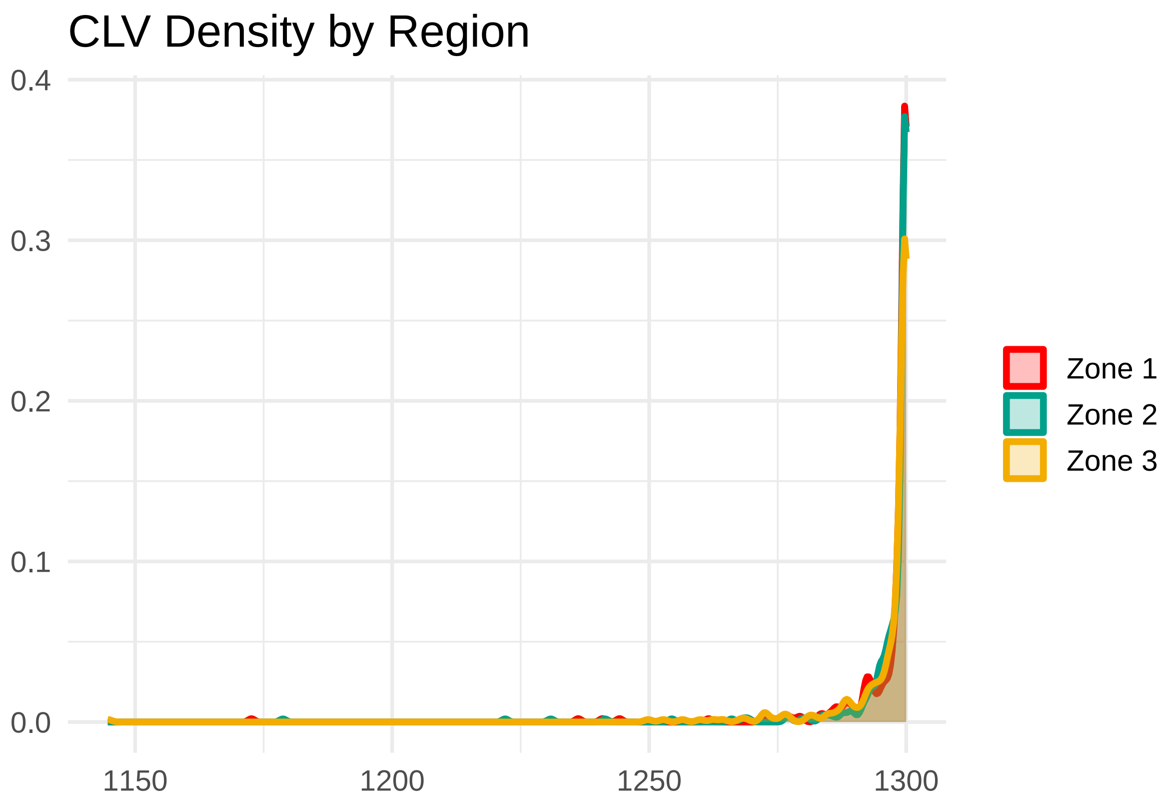
```r
  theme_minimal(base_size = 14) +
  theme(legend.title = element_blank())
```




```r
aggregate(CLV ~ region, data = telco, FUN = mean)
```

```
##   region      CLV
## 1 Zone 1 1296.381
## 2 Zone 2 1296.609
## 3 Zone 3 1295.805
```

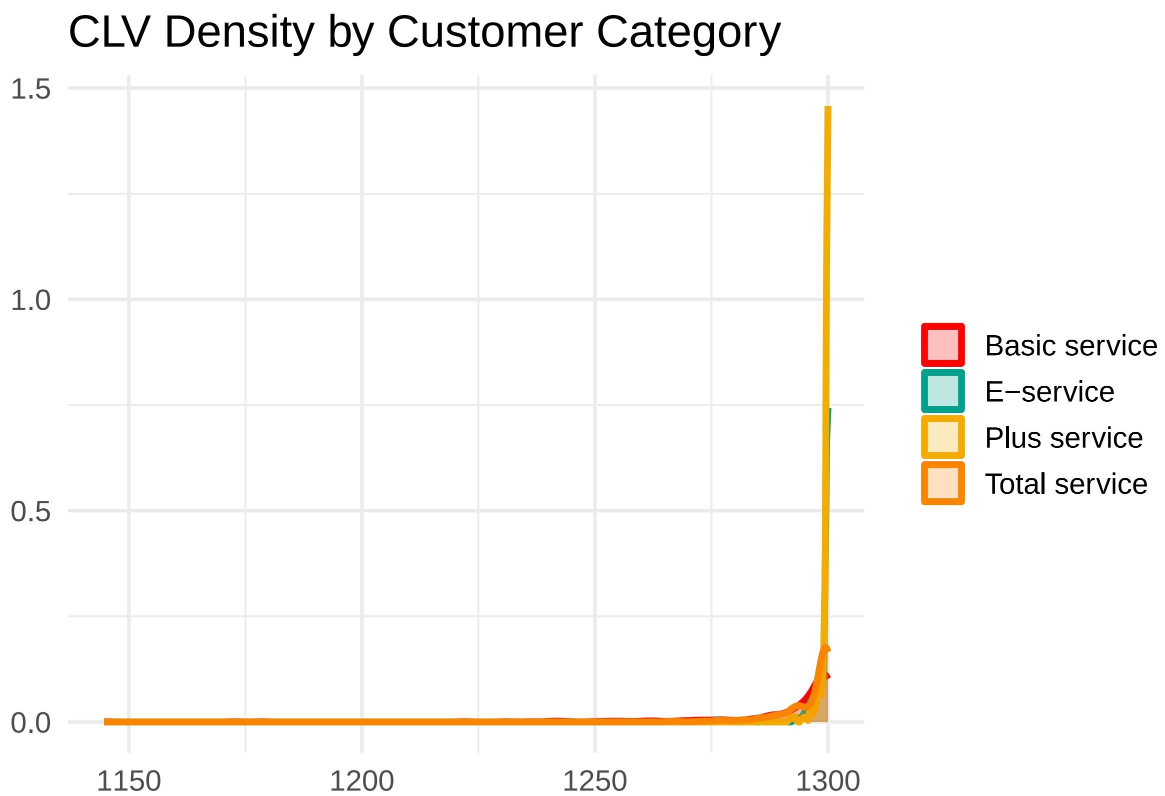
```r
pal_region <- wes_auto(telco$region)

ggplot(telco, aes(x = CLV, color = region, fill = region)) +
  geom_density(alpha = 0.25, linewidth = 1.2) +
  scale_color_manual(values = pal_region) +
  scale_fill_manual(values = pal_region) +
  labs(title = "CLV Density by Region", x = "", y = "") +
  theme_minimal(base_size = 14) +
  theme(legend.title = element_blank())
```




```r
aggregate(CLV ~ custcat, data = telco, FUN = mean)
```

```
##         custcat      CLV
## 1 Basic service 1291.054
## 2     E-service 1298.790
## 3  Plus service 1299.392
## 4 Total service 1296.069
```

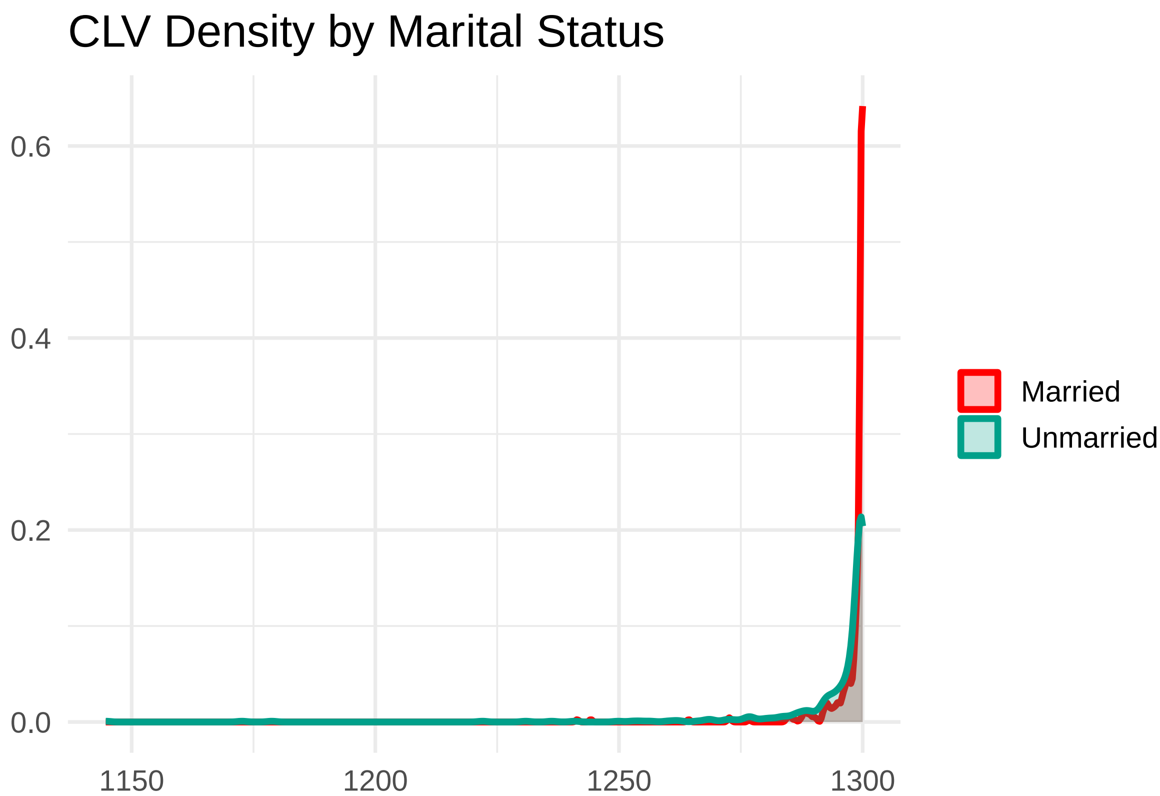
```r
pal_cust <- wes_auto(telco$custcat)

ggplot(telco, aes(x = CLV, color = custcat, fill = custcat)) +
  geom_density(alpha = 0.25, linewidth = 1.2) +
  scale_color_manual(values = pal_cust) +
  scale_fill_manual(values = pal_cust) +
  labs(title = "CLV Density by Customer Category", x = "", y = "") +
  theme_minimal(base_size = 14) +
  theme(legend.title = element_blank())
```




```r
aggregate(CLV ~ marital, data = telco, FUN = mean)
```

```
##     marital      CLV
## 1   Married 1298.123
## 2 Unmarried 1294.432
```

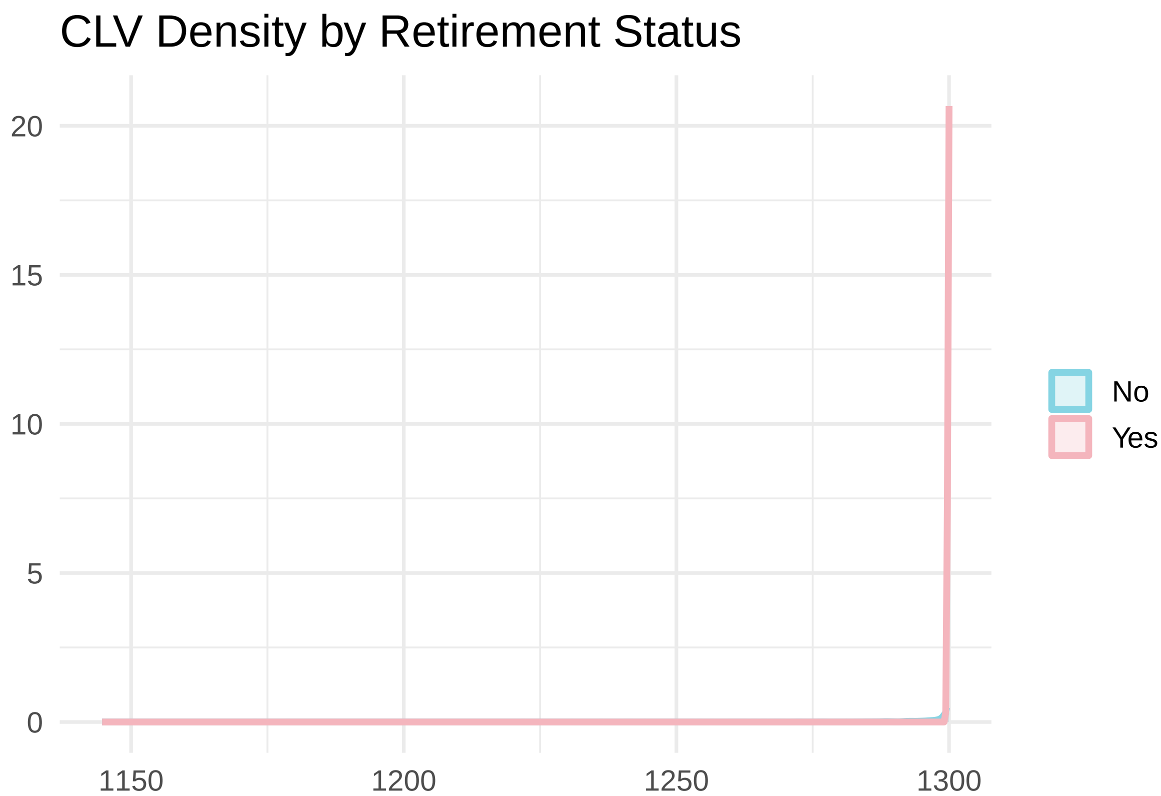
```r
pal_marital <- wes_auto(telco$marital)

ggplot(telco, aes(x = CLV, color = marital, fill = marital)) +
  geom_density(alpha = 0.25, linewidth = 1.2) +
  scale_color_manual(values = pal_marital) +
  scale_fill_manual(values = pal_marital) +
  labs(title = "CLV Density by Marital Status", x = "", y = "") +
  theme_minimal(base_size = 14) +
  theme(legend.title = element_blank())
```




```r
aggregate(CLV ~ retire, data = telco, FUN = mean)
```

```
##   retire      CLV
## 1     No 1296.077
## 2    Yes 1299.960
```

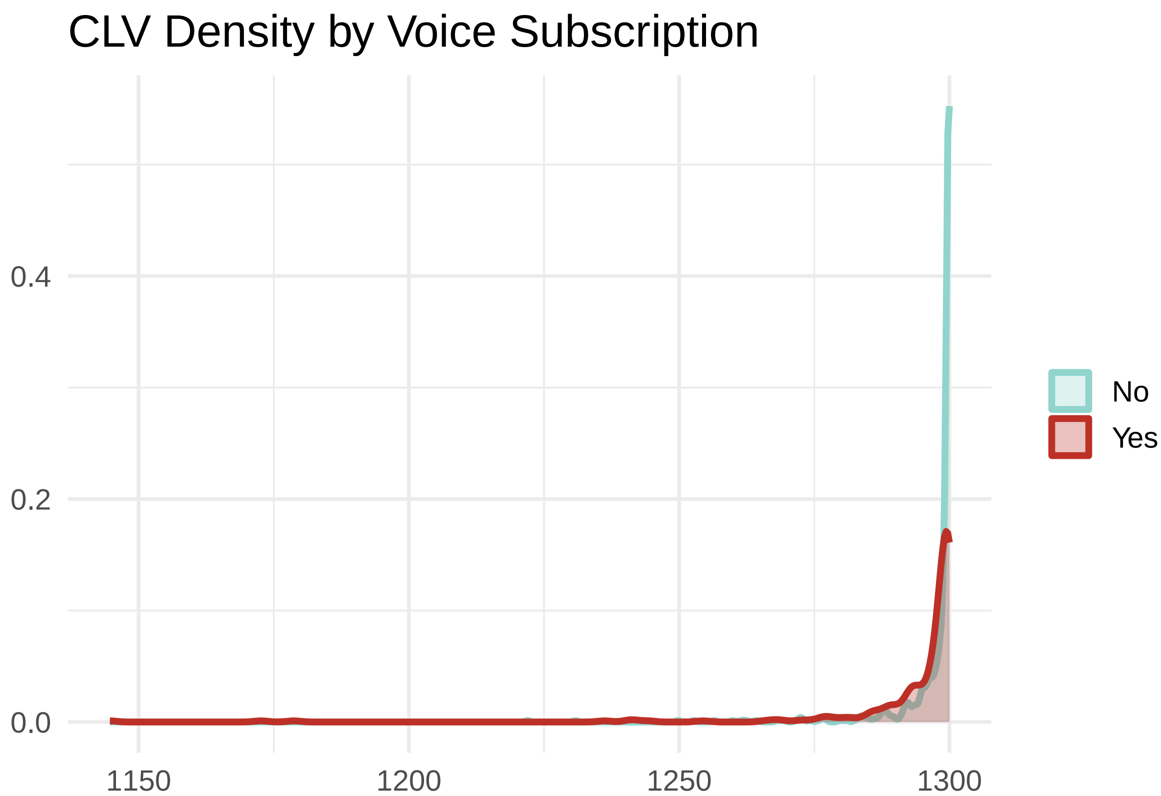
```r
pal_retire <- wes_auto(telco$retire, palette = "Moonrise3")

ggplot(telco, aes(x = CLV, color = retire, fill = retire)) +
  geom_density(alpha = 0.25, linewidth = 1.2) +
  scale_color_manual(values = pal_retire) +
  scale_fill_manual(values = pal_retire) +
  labs(title = "CLV Density by Retirement Status", x = "", y = "") +
  theme_minimal(base_size = 14) +
  theme(legend.title = element_blank())
```




```
aggregate(CLV ~ voice, data = telco, FUN = mean)
```

```
##   voice      CLV
## 1    No 1297.271
## 2   Yes 1293.944
```

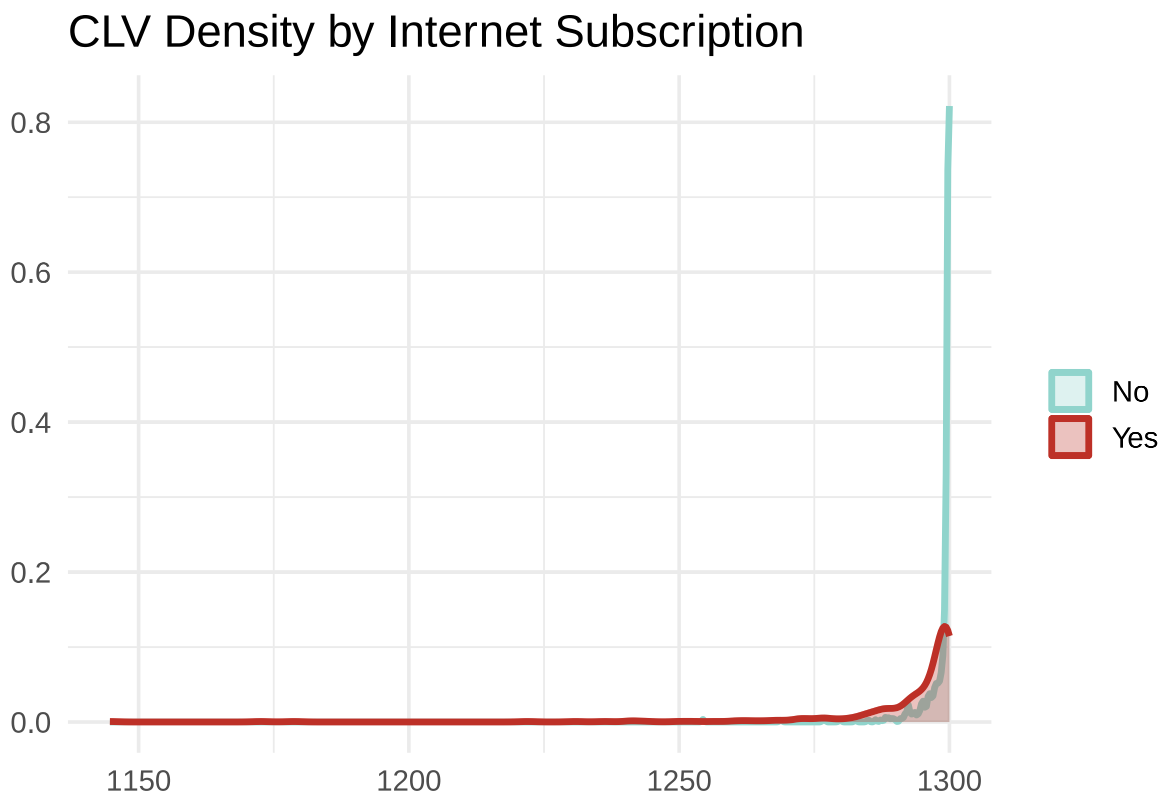
```
ggplot(telco, aes(x = CLV, color = voice, fill = voice)) +
  geom_density(alpha = 0.3, linewidth = 1.2) +
  scale_color_manual(values = wes_palette("FrenchDispatch", 2)) +
  scale_fill_manual(values = wes_palette("FrenchDispatch", 2)) +
  labs(title = "CLV Density by Voice Subscription", x = "", y = "") +
  theme_minimal(base_size = 14) +
  theme(legend.title = element_blank())
```



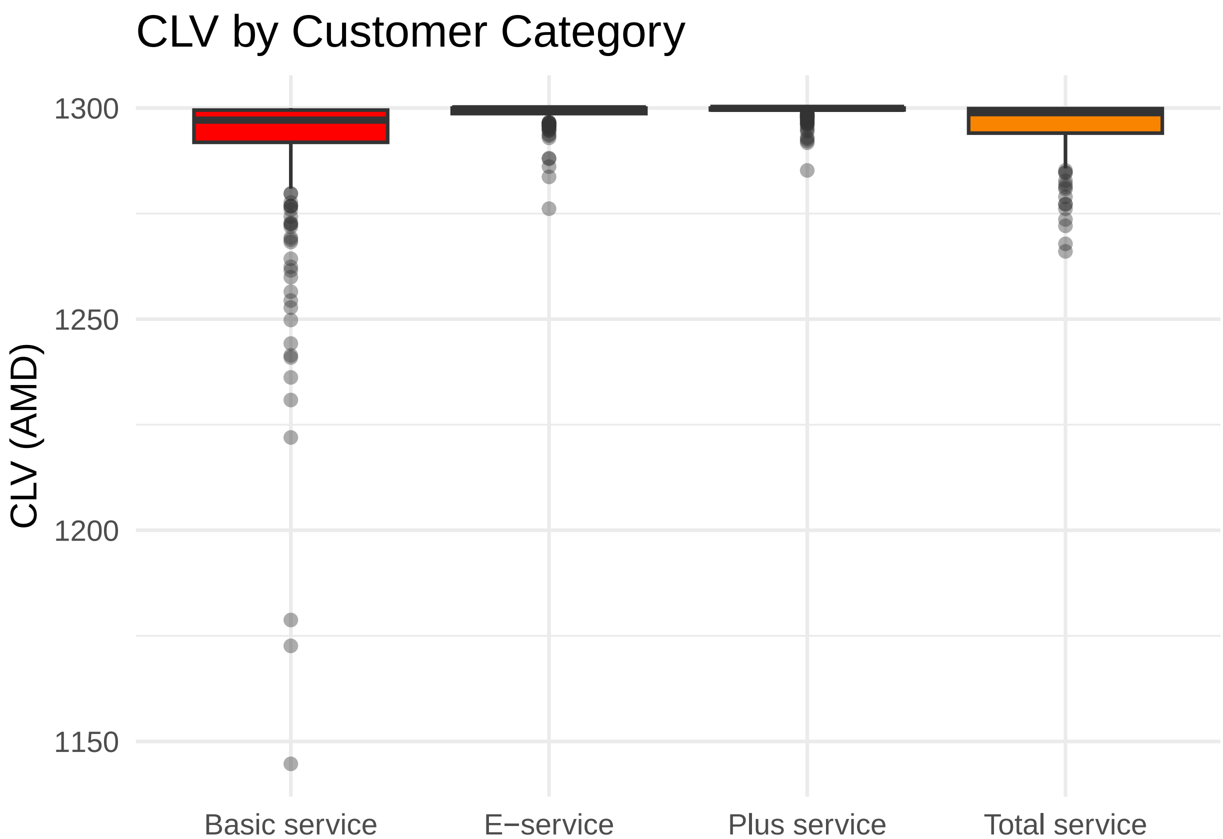

```r
aggregate(CLV ~ internet, data = telco, FUN = mean)
```

```
##   internet      CLV
## 1       No 1298.633
## 2      Yes 1292.182
```

```r
ggplot(telco, aes(x = CLV, color = internet, fill = internet)) +
  geom_density(alpha = 0.3, linewidth = 1.2) +
  scale_color_manual(values = wes_palette("FrenchDispatch", 2)) +
  scale_fill_manual(values = wes_palette("FrenchDispatch", 2)) +
  labs(title = "CLV Density by Internet Subscription", x = "", y = "") +
  theme_minimal(base_size = 14) +
  theme(legend.title = element_blank())
```




```
telco$custcat <- factor(telco$custcat)
telco$custcat <- droplevels(telco$custcat)

n_levels <- nlevels(telco$custcat)
stopifnot(n_levels > 0)
pal_name <- "Darjeeling1"

if (n_levels <= length(wesanderson::wes_palettes[[pal_name]])) {
  pal_cat <- wes_palette(pal_name, n_levels, type = "discrete")
} else {
  pal_cat <- wes_palette(pal_name, n_levels, type = "continuous")
}
ggplot(telco, aes(x = custcat, y = CLV, fill = custcat)) +
  geom_boxplot(outlier.alpha = 0.4) +
  scale_fill_manual(values = pal_cat) +
  labs(title = "CLV by Customer Category", x = NULL, y = "CLV (AMD)") +
  theme_minimal(base_size = 14) +
  theme(legend.position = "none")
```



Higher service plams like the E-service, Plus, Total service have higher and more stable CLV, suggesting stronger loyalty and lower churn risk. In contrast, Basic service customers show noticeably lower and more variable CLV, indicating higher churn likelihood and inconsistent engagement.

```
n <- nrow(telco)

get_est <- function(out) {
  if (is.list(out) && !is.data.frame(out)) {
    vapply(out, function(x) x$est[1], numeric(1))
  } else if (is.data.frame(out) && "est" %in% names(out)) {
    out$est
  } else numeric(0)
}

times12 <- 1:12
disc12  <- (1 + r/12)^(-(times12 - 1))

S12   <- numeric(n)
CLV12 <- numeric(n)

for (i in seq_len(n)) {
  s12 <- summary(fs_fit, newdata = telco[i, , drop = FALSE],
                 type = "survival", t = 12)
  e12 <- get_est(s12)
  S12[i] <- if (length(e12)) pmin(pmax(e12[1], 0), 1) else 0
  s_list <- summary(fs_fit, newdata = telco[i, , drop = FALSE],
                    type = "survival", t = times12)
```

```r
  Si <- get_est(s_list)

  if (length(Si) < length(times12)) {
    pad <- if (length(Si)) rep(tail(Si, 1), length(times12) - length(Si)) else rep(0, length(times12))
    Si <- c(Si, pad)
  } else if (length(Si) > length(times12)) {
    Si <- Si[seq_along(times12)]
  }

  Si <- pmin(pmax(Si, 0), 1)
  CLV12[i] <- MM * sum(Si * disc12)
}

telco$S12         <- S12
telco$PrChurn_12m <- 1 - S12
telco$at_risk_12m <- telco$S12 <= 0.5
telco$CLV12       <- CLV12
```

```r
expected_loss <- sum(telco$PrChurn_12m * telco$CLV12, na.rm = TRUE)
budget_15pct  <- 0.15 * expected_loss
budget_20pct  <- 0.20 * expected_loss
format(round(budget_15pct, 0), big.mark = ",")
```

```
## [1] "244,781"
```

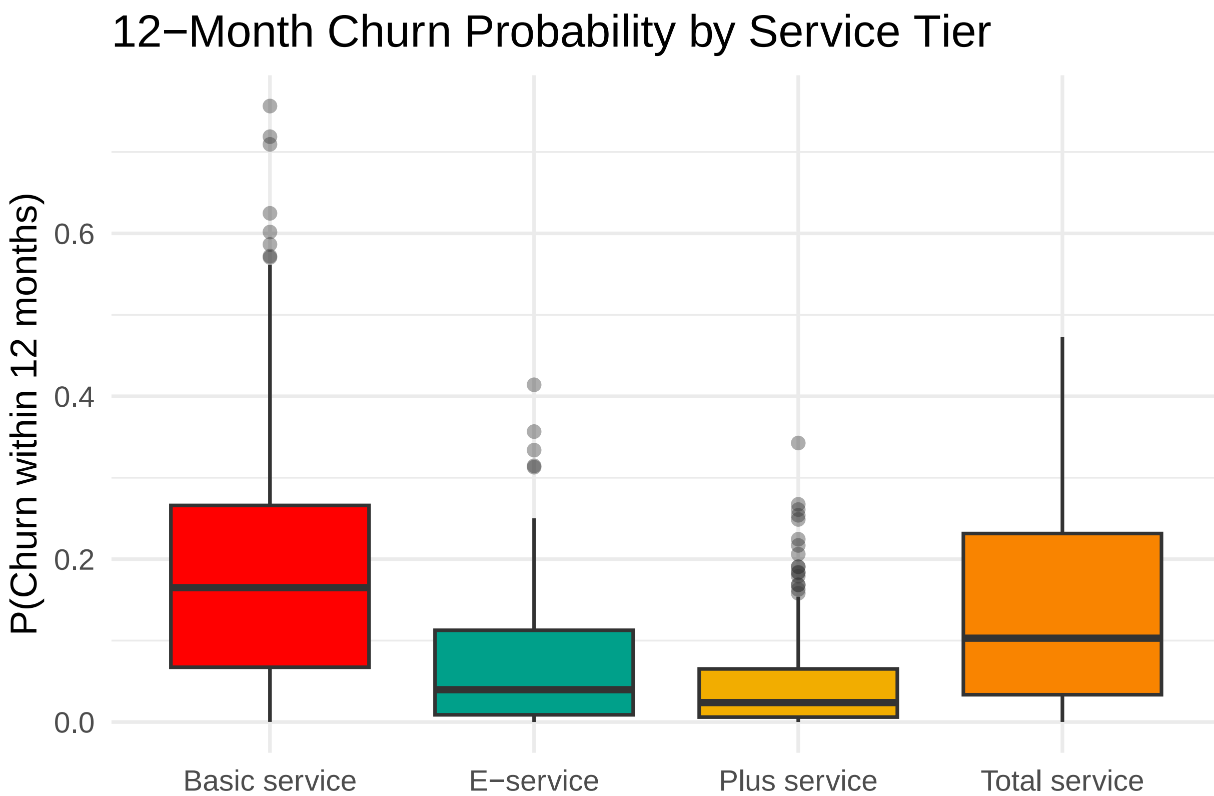
```r
format(round(budget_20pct, 0), big.mark = ",")
```

```
## [1] "326,374"
```

```r
at_risk <- telco$PrChurn_12m >= 0.5
expected_loss_risk <- sum(telco$CLV12[at_risk], na.rm = TRUE)
budget_risk_15pct  <- 0.15 * expected_loss_risk
format(round(budget_risk_15pct, 0), big.mark = ",")
```

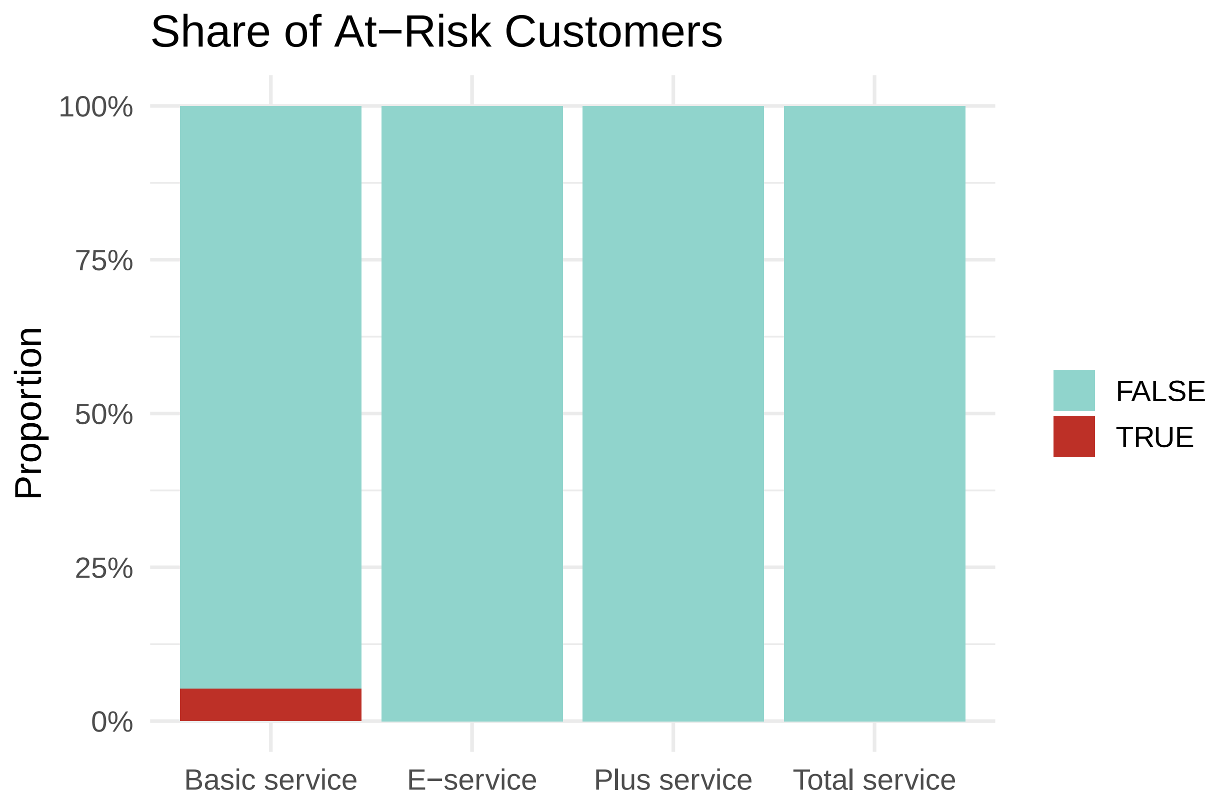
```
## [1] "29,565"
```

```r
ggplot(telco, aes(x = custcat, y = PrChurn_12m, fill = custcat)) +
  geom_boxplot(outlier.alpha = 0.4) +
  scale_fill_manual(values = pal_cat) +
  labs(title = "12-Month Churn Probability by Service Tier",
       x = "", y = "P(Churn within 12 months)") +
  theme_minimal(base_size = 14) +
  theme(legend.position = "none")
```



The risk of churn varies substantially by service tier. Basic service customers have the highest predicted churn probabilities (as expected based on the past analysis), with a median risk around 15–25% and some individuals exceeding 60% within a year. This indicates that customers with minimal service bundles are more price-sensitive and more likely to switch. In contrast, Plus and E-service customers show much lower churn risk, typically below 10%, reflecting stronger loyalty and perceived value in enhanced service features. Total service customers show a wider distribution, suggesting that while many of them are loyal, a proper count of them demonstrates churn risk—possibly due to higher expectations or pricing sensitivity.

```
risk_summary <- telco %>%
  group_by(custcat, at_risk_12m) %>%
  summarise(n = n(), .groups = "drop") %>%
  group_by(custcat) %>%
  mutate(prop = n / sum(n))

ggplot(risk_summary, aes(x = custcat, y = prop, fill = at_risk_12m)) +
  geom_col(position = "fill") +
  scale_fill_manual(values = wes_palette("FrenchDispatch", 2)) +
  labs(title = "Share of At-Risk Customers",
       x = "", y = "Proportion") +
  theme_minimal(base_size = 14) +
  scale_y_continuous(labels = scales::percent) +
  theme(legend.title = element_blank())
```



The customers most likely to churn within the next 12 months are the Basic service.

```r
seg <- telco |>
  dplyr::group_by(custcat, internet) |>
  dplyr::summarise(
    mean_CLV   = mean(CLV,   na.rm = TRUE),
    mean_CLV12= mean(CLV12, na.rm = TRUE),
    med_pchurn= median(PrChurn_12m, na.rm = TRUE),
    n = dplyr::n(),
    .groups = "drop"
  ) |>
  dplyr::arrange(dplyr::desc(mean_CLV))
seg
```

```
## # A tibble: 8 x 6
##   custcat       internet mean_CLV mean_CLV12 med_pchurn     n
##   <fct>         <fct>       <dbl>      <dbl>      <dbl> <int>
## 1 E-service     No          1300.      14908.    0.0131   107
## 2 Total service No          1300.      14906.    0.0225    63
## 3 Plus service  No          1300.      14905.    0.0204   259
## 4 E-service     Yes         1298.      14885.    0.0932   110
## 5 Plus service  Yes         1297.      14880.    0.0870    22
## 6 Basic service No          1297.      14870.    0.121    203
## 7 Total service Yes         1295.      14850.    0.143    173
## 8 Basic service Yes         1273.      14605.    0.350     63
```

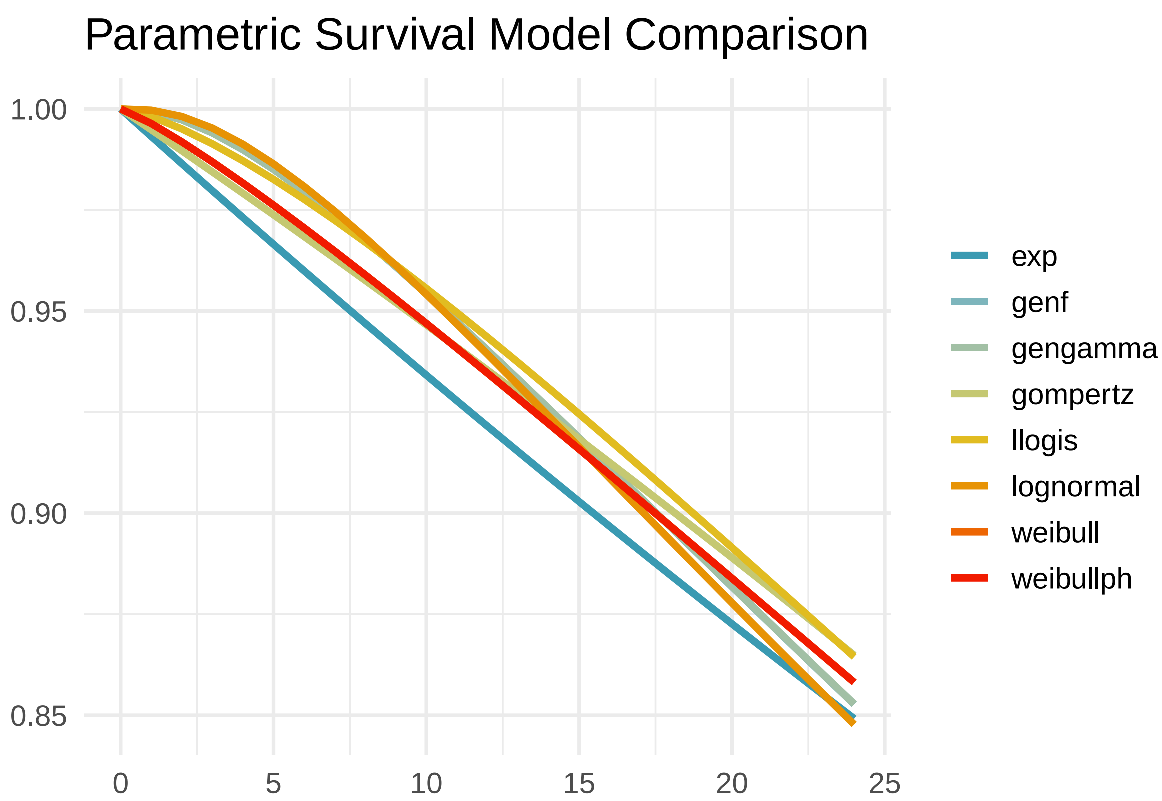
Up to this point, we studied the model - LogNormal as it had the lowest AIC, now let's consider all the other models available.

```r
base_formula <- Surv(tenure, churn == 1) ~ age + marital + address + voice + internet + custcat
dists <- c("exp","weibull","weibullph","gompertz","lognormal","llogis","gengamma","genf")

fits <- setNames(vector("list", length(dists)), dists)
for (i in seq_along(dists)) {
  fits[[i]] <- try(flexsurv::flexsurvreg(base_formula, data = telco, dist = dists[i]), silent = TRUE)}
fits <- fits[!vapply(fits, function(x) inherits(x, "try-error"), logical(1))]


cmp <- dplyr::bind_rows(lapply(seq_along(fits), function(i) {
  f <- fits[[i]]
  k <- nrow(f$res)                    # number of parameters
  tibble::tibble(
    dist   = dists[i],
    logLik = as.numeric(stats::logLik(f)),
    k      = k,
    AIC    = f$AIC,
    BIC    = f$AIC + log(nrow(telco)) * k
  )
})) %>% dplyr::arrange(AIC)
cmp
```

```
## # A tibble: 8 x 5
##   dist      logLik     k   AIC   BIC
##   <chr>      <dbl> <int> <dbl> <dbl>
## 1 lognormal -1462.    10 2944. 3013.
## 2 gengamma  -1462.    11 2945. 3021.
## 3 genf      -1462.    12 2947. 3030.
## 4 llogis    -1464.    10 2947. 3016.
## 5 gompertz  -1467.    10 2954. 3023.
## 6 weibull   -1468.    10 2956. 3025.
## 7 weibullph -1468.    10 2956. 3025.
## 8 exp       -1473.     9 2963. 3026.
```

```r
times  <- 0:24
newref <- telco[1, , drop = FALSE]

curves <- dplyr::bind_rows(lapply(dists, function(d) {
  s <- summary(fits[[d]], newdata = newref, t = times, type = "survival")
  data.frame(dist = d, t = times, S = s[[1]]$est)
}))



ggplot(curves, aes(t, S, color = dist)) +
  geom_line(linewidth = 1.3) +
  scale_color_manual(values = wes_palette("Zissou1", n_distinct(curves$dist),
                                          type = "continuous")) +
  labs(title = "Parametric Survival Model Comparison",
       x = "", y = "",
       color = "Distribution") +
```

```r
  theme_minimal(base_size = 14) +
  theme(legend.position = "right", legend.title = element_blank())
```




```r
coef_tab <- broom::tidy(final_model) |>
  dplyr::mutate(time_ratio = exp(estimate)) |>
  dplyr::select(term, estimate, std.error, statistic, p.value, time_ratio)
coef_tab
```

```
## # A tibble: 10 x 6
##    term                 estimate std.error statistic  p.value time_ratio
##    <chr>                   <dbl>     <dbl>     <dbl>    <dbl>      <dbl>
##  1 (Intercept)            2.53     0.243       10.4  1.49e-25     12.6
##  2 age                    0.0368   0.00640      5.75 8.69e- 9      1.04
##  3 maritalUnmarried      -0.447    0.114       -3.91 9.32e- 5      0.639
##  4 address                0.0428   0.00885      4.84 1.30e- 6      1.04
##  5 voiceYes              -0.463    0.167       -2.78 5.45e- 3      0.629
##  6 internetYes           -0.841    0.138       -6.08 1.21e- 9      0.431
##  7 custcatE-service       1.03     0.169        6.07 1.29e- 9      2.79
##  8 custcatPlus service    0.823    0.169        4.85 1.21e- 6      2.28
##  9 custcatTotal service   1.01     0.210        4.83 1.33e- 6      2.75
## 10 Log(scale)             0.283    0.0460       6.15 7.74e-10      1.33
```

Insights. To model churn behaviour, a series of Accelerated Failure Time (AFT) models were fitted using both survreg and flexsurvreg. The comparison across distributions showed that the LogNormal model achieved the lowest AIC among the parametric forms tested that yielded the values of AIC being approximately 2944, and BIC – 3013. The LogNormal specification was therefore selected as the final model. The final model identified age, marital status, years at current address, voice subscription, internet subscription, and customer category as statistically significant predictors (p-value being less than 0.05) of time-to-churn. Interpreting coefficients via time ratios (exp(beta)), subscriptions to voice and internet services increase expected lifetime (time ratios > 1), indicating slower churn hazard and higher retention stability. Basic service customers exhibit lower time ratios, implying faster churn, while higher-end users like Plus, E-service, and Total customers have longer survival times. The positive association between address duration and retention suggests that settled customers are more loyal, while certain marital categories reflect more stable usage patterns consistent with likely shared household plans. Customer Lifetime Value (CLV) was estimated from predicted survival curves using monthly discounting. Mean 12-month CLV ranged from roughly 14,605 AMD to 14,908 AMD across service tiers. While CLV was nearly identical across gender and education groups, it varied meaningfully across customer categories and internet segments. For example, E-service (No internet) customers exhibited the highest mean CLV12 of about 14,908 AMD and very low median churn probability about 1.3%, while Basic + Internet customers had the lowest CLV12 about 14,605 AMD and the highest median churn probability around 35%. This demonstrates that service tier and internet subscruption structure, rather than demographic traits, are the primary drivers of a customer's value. Using the model-derived churn probabilities, customers with S(12) <= 0.5 were labeled as being at-risk. These were concentrated overwhelmingly in the Basic category which was expected. The expected monetary loss from churn across the full base was estimated at around 244,781–326,374 AMD, depending on assumed intervention intensity, while targeting only at-risk subscribers reduces the required annual retention budget to approximately 29,565 AMD at a 15% incentive level.

To conclude, the most valuable customers are therefore the high-tier subscribers, especially those with both voice and internet services, as they exhibit higher CLV, lower churn risk, and more stable engagement. Most effective retention focus is: Basic service customers with high churn risk, especially those not enrolled in bundled offerings.

P.S. I apologise for the long report.