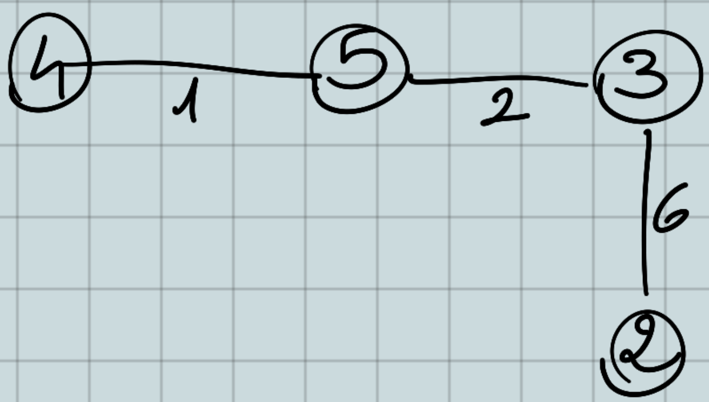
TSP - SIMMETRICO (B&B)

$G(V,E)$:

$$c_{ij} \quad \begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5 \\ \hline 1 & X & & & & \\ 2 & 5 & X & & & \\ 3 & 2 & 6 & X & & \\ 4 & 3 & 7 & 8 & X & \\ 5 & 8 & 10 & 2 & 1 & X \end{array}$$

$n = 1 \rightarrow \boxed{\text{1-TREE}}$:

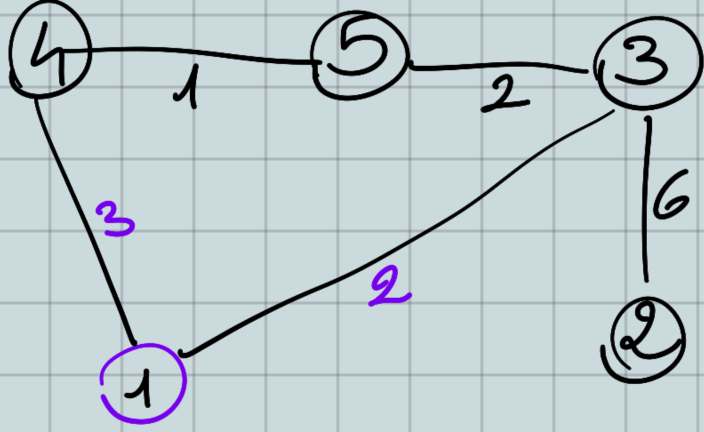
1) MST sui $\underbrace{V - \{1\}}_{= \{2,3,4,5\}}$ vertici:



2) Aggiungo i 2 archi incidenti su 1 di costo minimo:

UPPER BOUND

- NODO 1:



$\boxed{z^*} = +\infty$

$LB = 3+1+$
$+6+2+2 = 14$

$\boxed{1 - \text{TREE}}$ → CIRCUITO HAMILTONIANO ? NO ✗

NON TUTTI i VERTICI HANNO GRADO 2 (V_2)

⇓

BRANCH! (su 1)

INDIVIDUAZIONE SOTTOCIRCUITO:

$(1 - 3 - 5 - 4)$ CON ARCHI:

$$\left\{ \underset{①}{(1,3)},\ \underset{②}{(3,5)},\ \underset{③}{(5,4)},\ \underset{④}{(4,1)} \right\}$$

⇓

GENERO 4 NODI (SUBPROBLEMI) FIGLI:

- NODO 2:
$$\begin{cases} E_0 = (1,3) \\ E_1 = \emptyset \end{cases}$$

- NODO 3: $\begin{cases} E_0 = (3,5) \\ E_1 = (1,3) \end{cases}$

- NODO 4: $\begin{cases} E_0 : (5,4) \\ E_1 = \{(1,3),\ (3,5)\} \end{cases}$

- NODO 5: $\begin{cases} E_0 = (4,1) \\ E_1 = \{(1,3),\ (3,5),\ (5,4)\} \end{cases}$

- NODO 2: $\begin{cases} E_0 = \{(1,3)\} \\ E_1 = \emptyset \end{cases}$

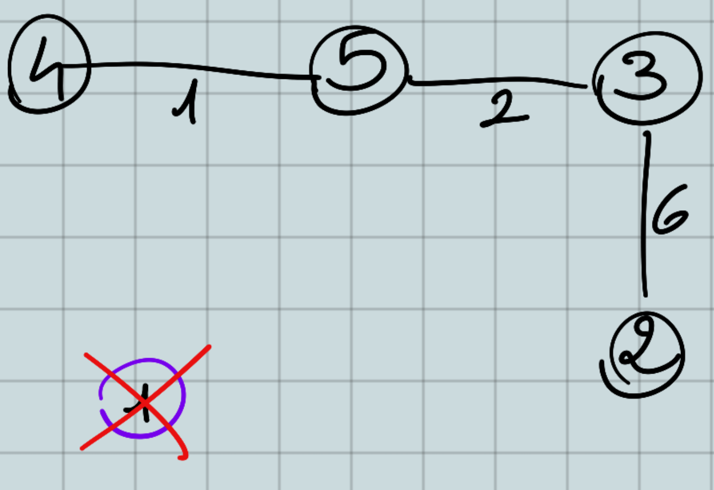
↓

$\boxed{\text{1-TREE}}$

1) MST sui $\underbrace{V - \{1\}}_{= \{2,3,4,5\}}$ vertici:

$\{\cancel{(1,3)}\} =$

*(NON PRENDENDO IN CONSIDERAZ. GLI ARCHI IN E_0))

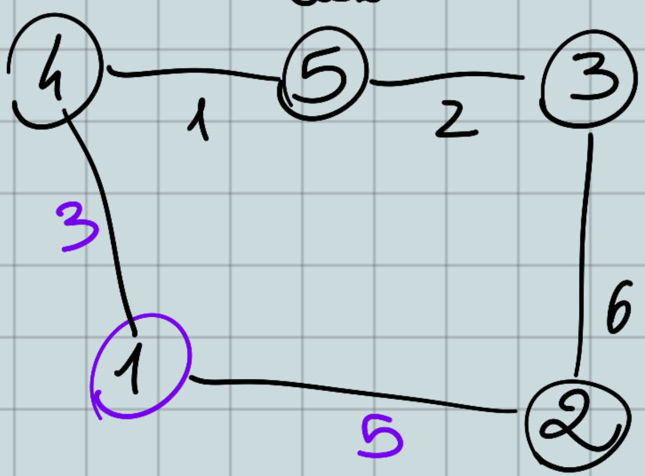


E inizializzando l'insieme di archi in soluz. con gli archi in $\boxed{E_1}$ non

INCIDENTI SU 1

$$
\begin{array}{c|ccccc}
c_{ij} & 1 & 2 & 3 & 4 & 5 \\
\hline
1 & \times & & & & \\
2 & 5 & \times & & & \\
3 & \cancel{2}\ (E_0) & 6 & \times & & \\
4 & 3 & 7 & 8 & \times & \\
5 & 8 & 10 & 2 & 1 & \times
\end{array}
$$

2) Aggiungo i 2 archi incidenti su 1 di costo minimo (★):



$\boxed{\text{1-TREE}}$ è anche CIRCUITO HAMILTON. ✓

$\Downarrow$

NODO $\boxed{2}$ CHIUSO PER OTTIMALITÀ

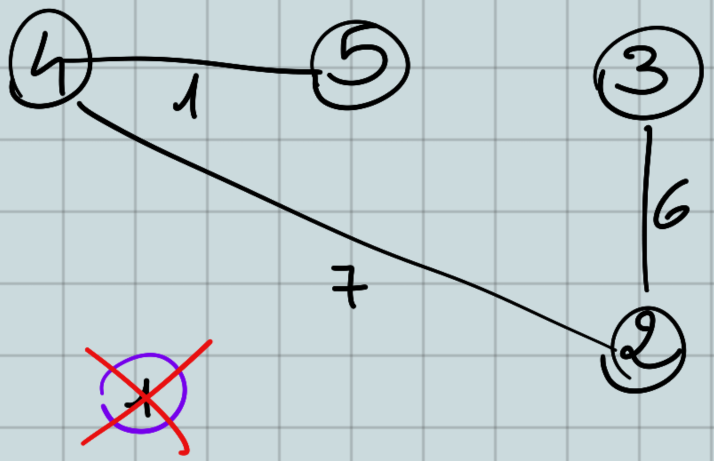
$\boxed{z^*} = LB = 17$

- NODO 3 : $\begin{cases} E_0 : \{(3,5)\} \\ E_1 : \{(1,3)\} \end{cases}$

$\downarrow$

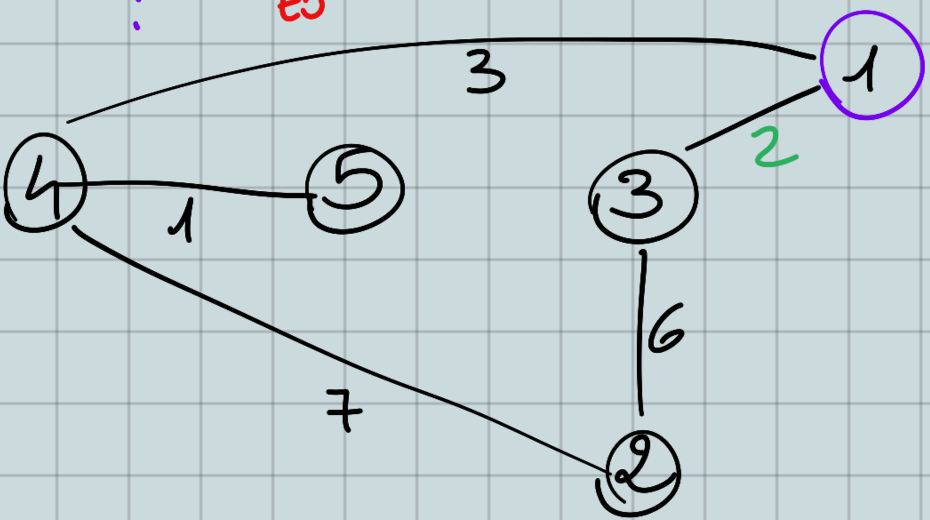
$\boxed{\text{1-TREE}}$

1) MST sui $\underbrace{\{V - \overset{n}{1}\}}_{\{2,3,4,5\}}$ vertici : (*)



c_{ij}	1	2	3	4	5
1	X				
2	5	X			
3	$\boxed{2}$	6	X		
4	3	7	8	X	
5	8	10	$\cancel{2}$ 10	1	X

2) (*)



$z^* = 17$

$(LB) = \underline{19} \geq \underset{17}{z} \Rightarrow$ NODO $\boxed{3}$ CHIUSO PER BOUND

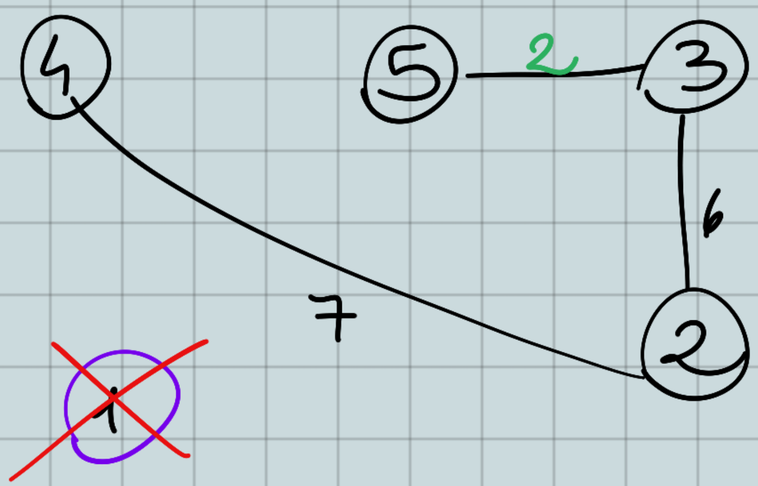
- NODO (4) :

$$\begin{cases} E_0 = \{\cancel{5,4}\} \\ E_1 = \{(4,3), (3,5)\} \end{cases}$$

$\downarrow$

1-TREE

1) MST sui $\underbrace{\{V - \underset{m}{(1)}\}}_{\{2,3,4,5\}}$ vertici (4) :



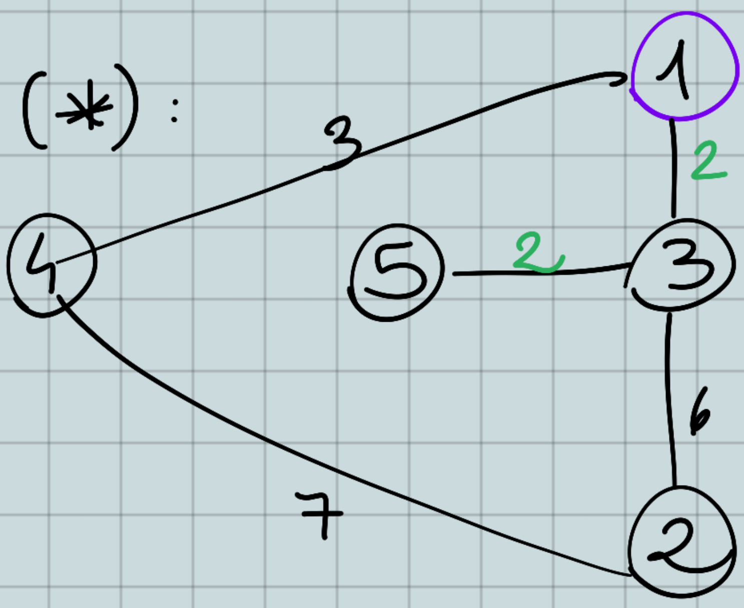
c_{ij}	1	2	3	4	5
1	X				
2	5	X			
3	3	6	X		

3 2 6 X

4 3 7 8 X

5 8 10 $\boxed{2}$ ~~1~~ X

2) (*) :



$z = 17$

$LB = 20 \geq z^* = 17 \Rightarrow$ NODO $\boxed{4}$ CHIUSO PER BOUND

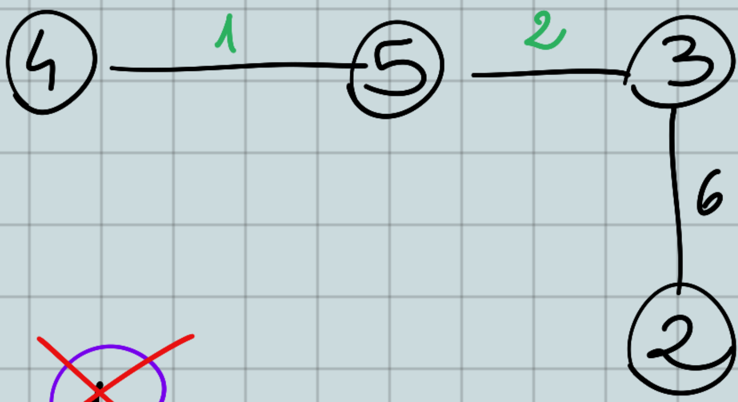
- NODO 5 : $E_0 = \{(4,1)\}$, $E_1 = \{(1,3), (3,5), (5,4)\}$

$\downarrow$

$\boxed{1\text{-TREE}}$

1) MST sui $\{V - \overset{n}{1}\}$ vertici (*) :

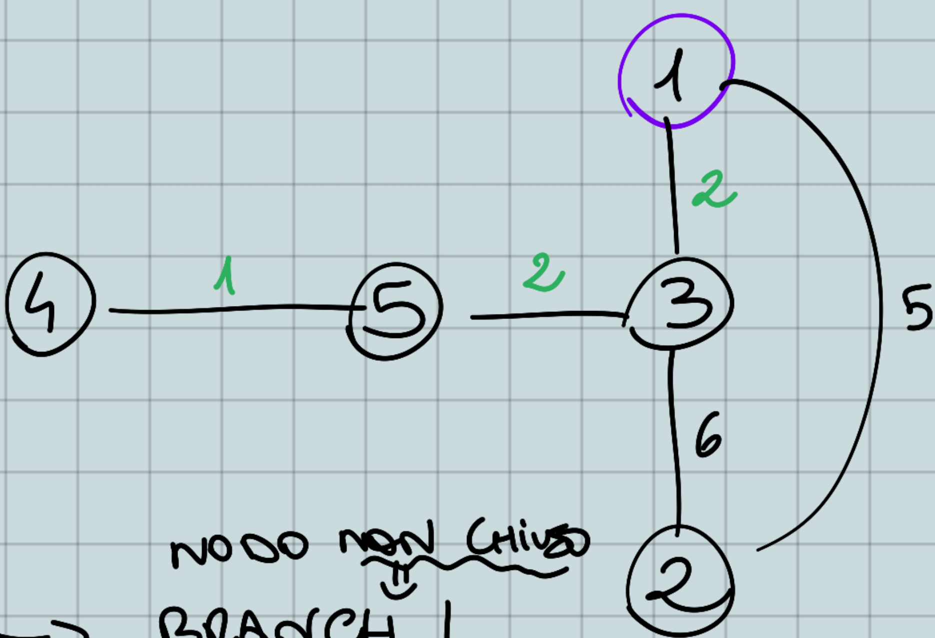
$$\{2, 3, 4, 5\}$$



c_{ij}

	1	2	3	4	5
1	X				
2	5	X			
3	$\boxed{2}$	6	X		
4	~~3~~	7	8	X	
5	8	10	$\boxed{2}$	$\boxed{1}$	X

2) (*) :



$\hat{z} = 17$ NODO NON CHIUSO

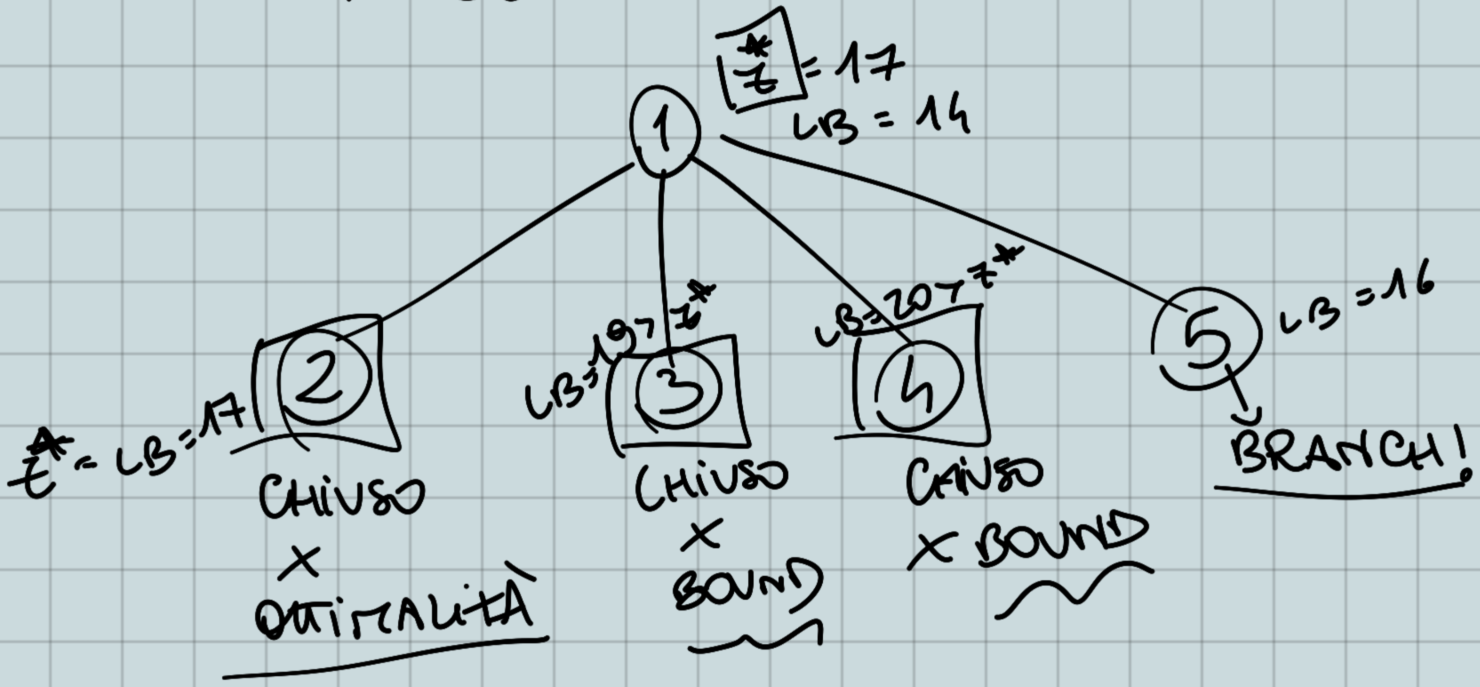
$LB = 16 \longrightarrow$ BRANCH !

(su 5)

c_{ij}

	1	2	3	4	5
1	X				
2	5	X			
3	$\boxed{2}$	6	X		
4	~~3~~	7	8	X	
5	8	10	$\boxed{2}$	$\boxed{1}$	X

ALBERO DI BRANCH



- BRANCA (SU NODO $\tilde{5}$) :

$$\begin{cases} E_0 = \{(4,1)\} \\ E_1 = \{(1,3),(3,5),(5,4)\} \end{cases}$$

INDIVIDUAZIONE SOTTOCIRCUITO:

$(1 - 2 - 3)$ CON ARCHI:

$\{(1,2), (2,3), \cancel{(3,1)}\}$

① ② ~~③~~

$\Downarrow$

GENERO 2 NODI (SUBPROBLEMI) FIGLI:

- NODO 6: $\begin{cases} E_0 : \{(4,1), (1,2)\} \\ E_1 = \{(1,3), (3,5), (5,4)\} \end{cases}$

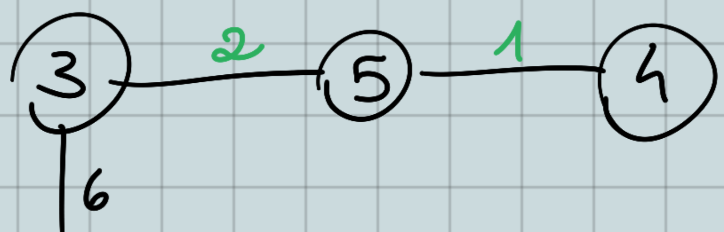
- NODO 7: $\begin{cases} E_0 = \{(4,1), (2,3)\} \\ E_1 = \{(1,3), (3,5), (5,4), (1,2)\} \end{cases}$

- NODO 6: $\begin{cases} E_0 = \{(4,\cancel{1}), (1,\cancel{2})\} \\ E_1 = \{(1,3), (3,5), (5,4)\} \end{cases}$

$\downarrow$

$\boxed{\text{1-TREE}}$:

1) MST sui $\underbrace{\{V - \overset{n}{1}\}}_{= \{2,3,4,5\}}$ vertici (*):

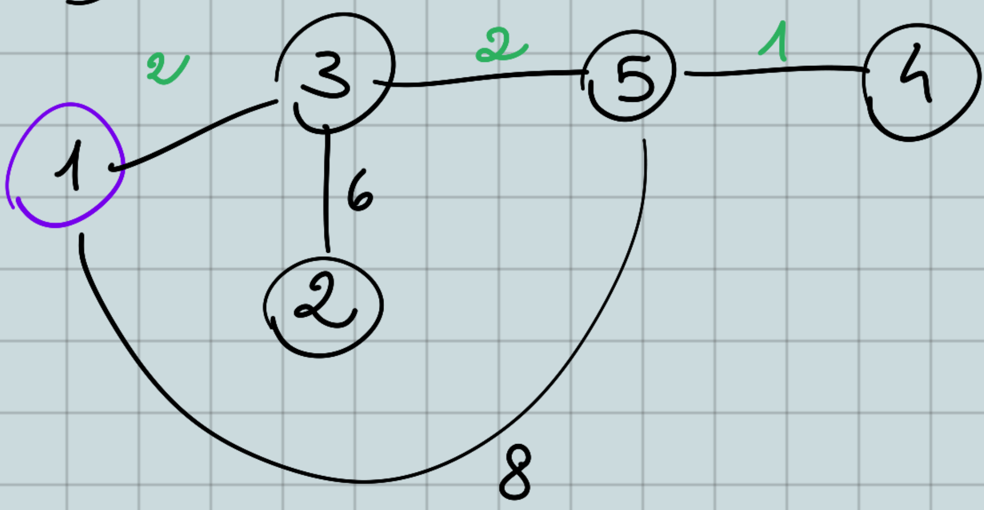


2

1

c_{ij}	1	2	3	4	5
1	x	-	-	-	-
2	~~5~~	x			
3	2	6	x		
4	~~3~~	7	8	x	
5	8	10	$\boxed{2}$	$\boxed{1}$	x

2) (*):



$LB = 19 \geq z_1^* = 17$

$\Downarrow$

NODO 6

CHIUSO PER BOUND!

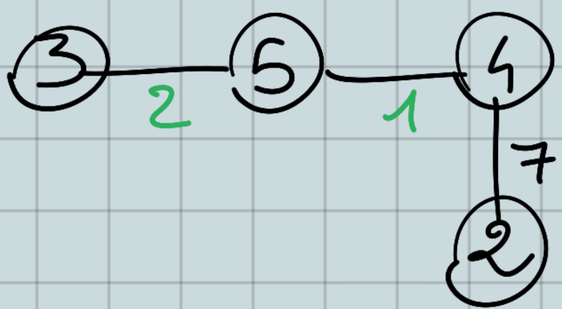
c_{ij}	1	2	3	4	5
1	x	-	-	-	-
2	~~5~~	x			
3	$\boxed{2}$	6	x		
4	~~3~~	7	8	x	
5	8	10	$\boxed{2}$	$\boxed{1}$	x

- NODO (7) : $\begin{cases} E_0 = \{ \cancel{(4,1)}, \ \cancel{(2,3)} \} \\ E_1 = \{ (1,3), (3,5), (5,4), (1,2) \} \end{cases}$

$\boxed{\text{1 - TREE}}$:

1) MST sui $\underbrace{\{ V - \overset{n}{\overset{\shortparallel}{(1)}} \}}_{\shortparallel}$ vertici (*) :

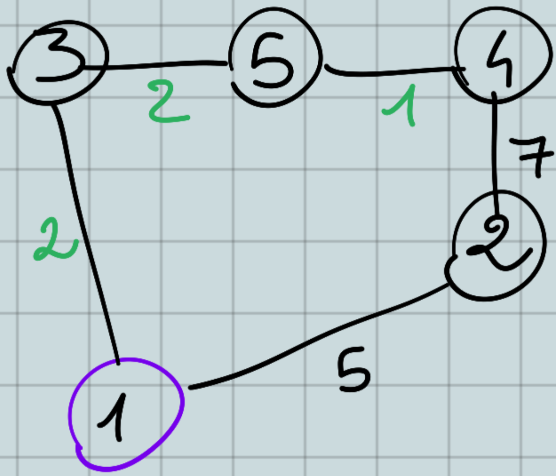
$\{ 2, 3, 4, 5 \}$



c_{ij}	1	2	3	4	5
1	x	-	-	-	-
2	5	X			
3	2	$\cancel{6}$	X		
4	[illegible]	[illegible]	[illegible]	X	

4	~~3~~	7	8	X
5	8	10	$\boxed{2}$	$\boxed{1}$	X

2) (*):



(UGUALE ALLO z^* ATTUALE → z^* NON AGGIORN...)

$z^* = LB = 17$

⇓

IL $\boxed{1\text{-TREE}}$ RISULTANTE È ANCHE UN CIRCUITO HAMILTON.

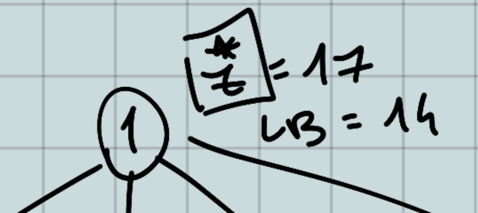
⇓

NODO 7 CHIUSO PER OTTIMALITÀ

c_{ij}	1	2	3	4	5
1	X	-	-	-	-
2	5	X			
3	$\boxed{2}$	~~6~~	X		
4	~~3~~	7	8	X	
5	8	10	$\boxed{2}$	$\boxed{1}$	X

⇒ ALBERO DI BRANCH (FINALE):

ALBERO DI BRANCH



$\Rightarrow$ SOL. DEL TSP: 2 POSSIBILI CIRCUITI HAMILTONIANI (DI COSTO UGUALE):

1) $\{(1,2), (2,3), (3,5), (5,4), (4,1)\} = \boxed{\text{NODO } 2}$

2) $\{(1,2), (2,4), (4,5), (5,3), (3,1)\} = \boxed{\text{NODO } 7}$