Assignment cum practical evaluation

1 (a) Among 100 students, 32 study mathematics, 20 study physics, 45 study biology, 15 study mathematics and biology, 7 study mathematics and physics , 10 study physics and biology and 30 do not study any of the three subjects.

(i) Find the number of students studying all three subjects.

(ii) Find the number of students studying exactly one of three subjects.

(b) Let **R** be a binary relation on the set of all positive integers such that

R={ (a , b) | a - b is an odd positive integer }

Explain which of the following relations are true on R and why.

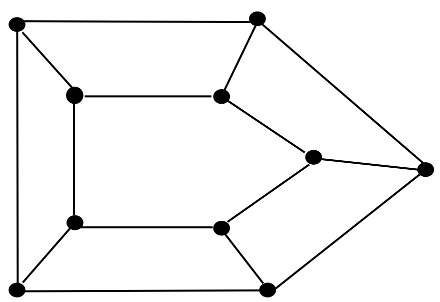
(i) Reflexive

(ii) Symmetric

(iii) Antisymmetric

(iv) Transitive

(c) (i) Derive an expression for the chromatic number of $C_{n,n}$ where n >= 3. $C_{n,n}$ is a graph with two concentric cycles and n vertices connected as shown below:



(ii) Prove that if a connected planar simple graph has e edges and v vertices with $v \geq 3$ and no circuits of length less than 3, then $e \leq 2v - 4$.

(d) (i) Find numeric function for the given generating function:

$$A(Z) = \frac{Z^4}{1-2Z}$$

(ii) Let a_r, b_r, c_r be three numeric functions such that $a_r * b_r = c_r$, Determine b_r.

$$a_r = \begin{cases} 1 & r = 0 \\ 2 & r = 1 \\ 0 & r \geq 2 \end{cases} \qquad c_r = \begin{cases} 1 & r = 0 \\ 0 & r \geq 1 \end{cases}$$

(e) Solve the following using Master's theorem if applicable. Give reasons if Master theorem is not applicable.

(i) $T(n) = 125\,T\left(\frac{n}{5}\right) + n^3$

(ii) $T(n) = 2T\left(\frac{n}{2}\right) + n\,log_2\,n$

(f) Use insertion sort to sort 7, 4, 6, 3, 2, 5, 9, 8 in ascending order showing the list obtained after each step.

(g) "If there was a ball game, then travelling was difficult. If they arrived on time, then travelling was not difficult. They arrived on time. Therefore, there was no ball game." Show that these statements constitute a valid argument.

2 (a) Determine whether the function f is a bijection from R to R . Find $f\ o\ g$ and $g\ o\ f$ for the function f and g where

$$f(x) = 2\,x^2 + 3 \text{ and } g(x) = x + 1.$$

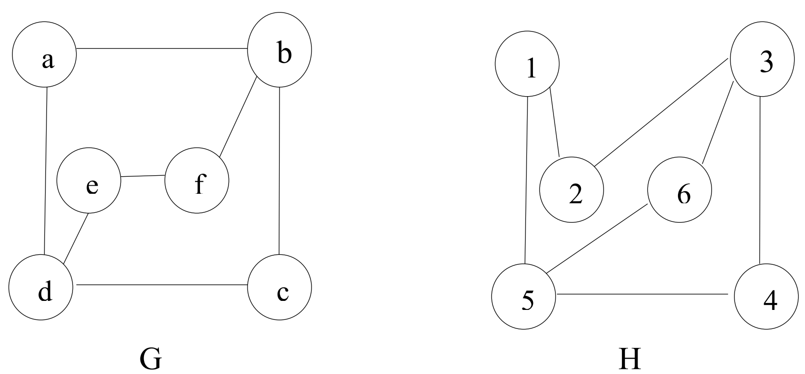
Is $f\ o\ g = g\ o\ f$?

(b) Eight chairs are numbered from 1 to 8. Two women and three men are to occupy one chair each. First the women choose the chairs from amongst the chairs 1 to 4 and then men select from remaining chairs. Find the number of possible arrangements.

3 Evaluate the sum

$$\sum_{k=1}^{\infty}(2k+1)x^{2k}$$

4 Determine whether the graphs G and H are isomorphic or not?



5 (a) (i) Prove that if an m-ary tree of height h has l leaves, then

$$h \geq \lceil \log_m l \rceil$$

(ii) If a tree has 2n vertices of degree-1, 3n vertices of degree-2 and n vertices of degree-3, find the number of vertices and edges in the tree.

(b) Is $2^{n+1} = O(2^n)$? Is $2^{2n} = O(2^n)$? Justify your answer.

6 (a) Write a program to find GCD of two numbers using recursion.

(b) Write a program to represent graphs using adjacency matrices and check if it has Euler path.

(c) Write a program to implement Insertion sort. Find the number of comparison during each pass and display the intermediate result.