[2] Triola, 11th edition, example 2

EXAMPLE 2 **Movie Lengths** Data Set 9 in Appendix B includes lengths (in minutes) of randomly selected movies. Let's consider only the first 5 movie lengths: 110, 96, 170, 125, 119. With only 5 values, a histogram will not be very helpful in revealing the distribution of the data. Instead, construct a normal quantile plot for these 5 values and determine whether they appear to come from a population that is normally distributed.

SOLUTION The following steps correspond to those listed in the above procedure for constructing a normal quantile plot.

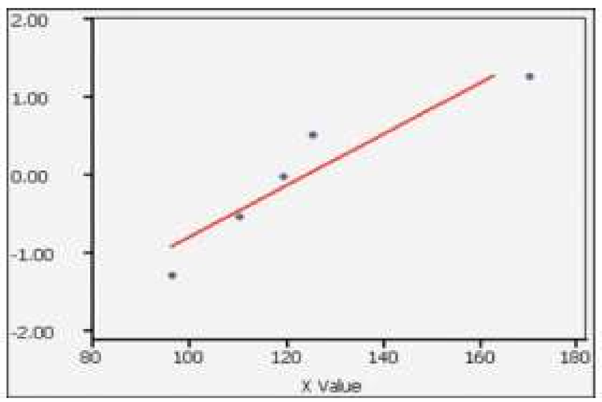
Step 1. First, sort the data by arranging them in order. We get 96, 110, 119, 125, 170.

Step 2. With a sample of size $n = 5$, each value represents a proportion of 1/5 of the sample, so we proceed to identify the cumulative areas to the left of the corresponding sample values. The cumulative left areas, which are expressed in general as $1/2n$, $3/2n$, $5/2n$, $7/2n$, and so on, become these specific areas for this example with $n = 5$: 1/10, 3/10, 5/10, 7/10, and 9/10. The cumulative left areas expressed in decimal form are 0.1, 0.3, 0.5, 0.7, and 0.9.

Step 3. We now search in the body of Table A-2 for the cumulative left areas of 0.1000, 0.3000, 0.5000, 0.7000, and 0.9000 to find these corresponding z scores: −1.28, −0.52, 0, 0.52, and 1.28.

Step 4. We now pair the original sorted movie lengths with their corresponding z scores. We get these (x, y) coordinates which are plotted in the accompanying STATDISK display: (96, −1.28), (110, −0.52), (119, 0), (125, 0.52), and (170, 1.28).

STATDISK



INTERPRETATION We examine the normal quantile plot in the STATDISK display. Because the points appear to lie reasonably close to a straight line and there does not appear to be a systematic pattern that is not a straight-line pattern, we conclude that the sample of five movie lengths appears to come from a normally distributed population.

Example 4, Chp8 Triola p.416

EXAMPLE 4 **Finding the Value of the Test Statistic** Let's again consider the claim that the XSORT method of gender selection increases the likelihood of having a baby girl. Preliminary results from a test of the XSORT method of gender selection involved 14 couples who gave birth to 13 girls and 1 boy. Use the given claim and the preliminary results to calculate the value of the test statistic. Use the format of the test statistic given above, so that a normal distribution is used to approximate a binomial distribution. (There are other exact methods that do not use the normal approximation.)

SOLUTION From Figure 8-2 and the example displayed next to it, the claim that the XSORT method of gender selection increases the likelihood of having a baby girl results in the following null and alternative hypotheses: H_0: $p = 0.5$ and H_1: $p > 0.5$. We work under the assumption that the null hypothesis is true with $p = 0.5$. The sample proportion of 13 girls in 14 births results in $\hat{p} = 13/14 = 0.929$. Using $p = 0.5$, $\hat{p} = 0.929$, and $n = 14$, we find the value of the test statistic as follows:

$$z = \frac{\hat{p} - p}{\sqrt{\dfrac{pq}{n}}} = \frac{0.929 - 0.5}{\sqrt{\dfrac{(0.5)(0.5)}{14}}} = 3.21$$

INTERPRETATION We know from previous chapters that a z score of 3.21 is "unusual" (because it is greater than 2). It appears that in addition to being greater than 0.5, the sample proportion of 13/14 or 0.929 is *significantly* greater than 0.5. Figure 8-3 shows that the sample proportion of 0.929 does fall within the range of values considered to be significant because they are so far above 0.5 that they are not likely to occur by chance (assuming that the population proportion is $p = 0.5$).

Figure 8-3 shows the test statistic of $z = 3.21$, and other components in Figure 8-3 are described as follows.

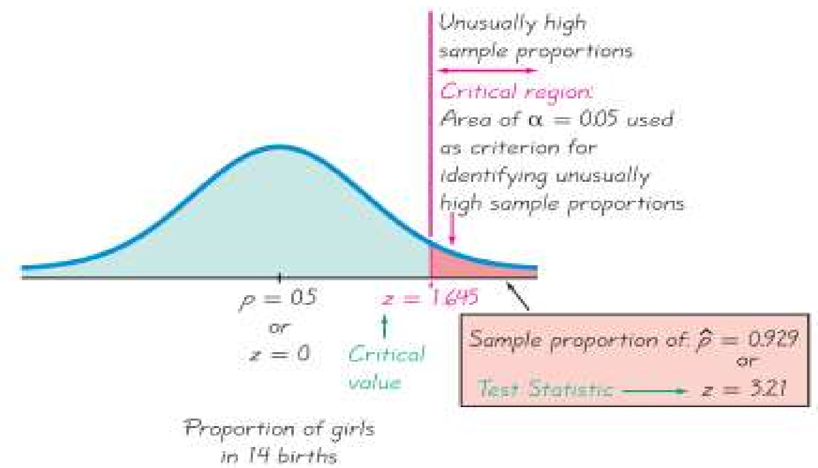


Table A.3 (continued) Areas under the N

z	.00	.01	.02	.03
0.0	0.5000	0.5040	0.5080	0.5120
0.1	0.5398	0.5438	0.5478	0.5517
0.2	0.5793	0.5832	0.5871	0.5910
0.3	0.6179	0.6217	0.6255	0.6293
0.4	0.6554	0.6591	0.6628	0.6664
0.5	0.6915	0.6950	0.6985	0.7019
0.6	0.7257	0.7291	0.7324	0.7357
0.7	0.7580	0.7611	0.7642	0.7673
0.8	0.7881	0.7910	0.7939	0.7967
0.9	0.8159	0.8186	0.8212	0.8238
1.0	0.8413	0.8438	0.8461	0.8485
1.1	0.8643	0.8665	0.8686	0.8708
1.2	0.8849	0.8869	0.8888	0.8907
1.3	0.9032	0.9049	0.9066	0.9082
1.4	0.9192	0.9207	0.9222	0.9236
1.5	0.9332	0.9345	0.9357	0.9370
1.6	0.9452	0.9463	0.9474	0.9484
1.7	0.9554	0.9564	0.9573	0.9582
1.8	0.9641	0.9649	0.9656	0.9664
1.9	0.9713	0.9719	0.9726	0.9732

between 0.9495 and 0.
this gives z = 1.645

EXAMPLE 8 **Finding a P-Value for a Critical Region in Two Tails** Consider the claim that with the XSORT method of gender selection, the likelihood of having a baby girl is different from $p = 0.5$, and use the test statistic $z = 3.21$ found from 13 girls in 14 births. First determine whether the given conditions result in a critical region in the right tail, left tail, or two tails, then use Figure 8-5 to find the P-value. Interpret the P-value.

SOLUTION The claim that the likelihood of having a baby girl is different from $p = 0.5$ can be expressed as $p \neq 0.5$, so the critical region is in two tails (as in Figure 8-4(a)). Using Figure 8-5 to find the P-value for a two-tailed test, we see that the P-value is *twice* the area to the right of the test statistic $z = 3.21$. We refer to Table A-2 (or use technology) to find that the area to the *right* of $z = 3.21$ is 0.0007. In this case, the P-value is twice the area to the right of the test statistic, so we have:

$$P\text{-value} = 2 \times 0.0007 = 0.0014$$

INTERPRETATION The P-value is 0.0014 (or 0.0013 if greater precision is used for the calculations). The small P-value of 0.0014 shows that there is a very small chance of getting the sample results that led to a test statistic of $z = 3.21$. This suggests that with the XSORT method of gender selection, the likelihood of having a baby girl is different from 0.5.

[2] p.419

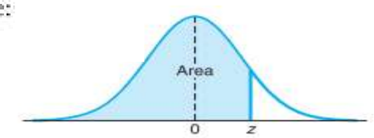


Table A.3 Areas under the Normal Curve

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998

A1	B	C	D	E	F	G	H	I	J	K	L
2	1	2	3	4	5	1 6	Row-sum	Row-ave	Row-stdv	Row-sum-squared	1
3	18	17.9	18.7	12.9	18.1	17.9	103.50	17.25	2.15	10712.25	=H3*H3
4	19	18.2	18.9	12.5	16.9	18.2	103.70	17.28	2.46	10753.69	
5	20	21.7	18.6	12.7	17.2	21.7	111.90	18.65	3.40	12521.61	
6	18.8	18.4	19.1	11.5	17.7	18.4	103.90	17.32	2.89	10795.21	
7	Data Extracted		2				grand-ave	17.63		44782.76	=SUM(K3:K6)
8	tolerance	0.05		=SUM(B3:G6)	X.. =	423.00	=AVERAGE(B3:G6)		Row-stdv	7463.79	=K7/C10
9	I(row)=	4.00		=G8*G8	sqr(X..) =	178929.00			=STDEV(B3:G3)		
10	J(column)=	6.00	=G9/(C9*C10) =	corr factor =	sqr(X..)/IJ =	7455.38					
11	H0 : mu1 = mu2 = mu3 = m4			all means are equal					=K8-G10	8.42	SSTr=SST-SSE
12	H1 : At least one of the means is different from the others.				3				=K11/(C9-1)	2.81	MSTr=SSTr/(I-1)
13	324.00	320.41	349.69	166.41	327.61	320.41					
14	361.00	331.24	357.21	156.25	285.61	331.24					
15	400.00	470.89	345.96	161.29	295.84	470.89					=H18-K11
16	353.44	338.56	364.81	132.25	313.29	338.56				SSE	153.07
17				=SUM(B13:G16)		Sum (Xij)*(Xij)	7616.86				
18				=H17-G10		SST=H17-G10	161.49		f calc	=K12/H19	
19				=L16/(C9*(C10-1))		MSE=SSE/I(J-1)	7.65		f=MSTr/MSE	0.37	
20											
21											

M–R
1 A software house wished to study the effect of four different methods of teaching on the proficiency of individuals creating tax return reports (how fast users can make their reports), ie. by using their tax return software application. The methods used were a) verbal cues b) verbal cues and video taped feed back c) video taped feed back d) the control treatment (neither verbal cues nor video taped feed back). A group of 24 first-timers (who had never made a tax return report), was randomly divided into four groups of six subjects each. Each group of subjects was taught using one of the four methods, and at the end of the course, each subject was timed while creating a certain tax return report (use 5% tolerance). The data is in the following table (in minutes). Does the data indicate that different teaching methods affect the true average time to finish making the report ? Also, please fill in the given empty table.

A1	B	C	D	E	F	G	H	I	J	K	L
22	1	2	3	4	5	6					
23	18.3	21.1	18.7	19.5	18.1	17.9	F(table) 4	3.1			Should one continue to Tukey test ? Why ? Do it Or Why not ?
24	17.5	17.6	18.9	20	16.9	18.2	F(0.05,3,20)		F(alpha, I-1, I(J-1))		Since Ha is rejected no method is significantly different
25	18	20.3	18.6	19.7	17.2	21.7	Hence f calc >=	3.1	is untrue	10	Ha is rejected in other words, ways of teaching had
26	17.5	18.9	19.1	18.8	17.7	18.4	10				no effect on making the reports
27											

A1	Source		d.f.		Sum of squares		Mean square	f(calc)
29	Methods	I-1=	3	SSTr=	8.42	MSTr=	2.81	0.37
30	Error	I(J-1)=	20	SSE=	153.07	MSE=	7.65	
31	Total		23	SST=	161.49			

Proposition: Let $\bar{X}_{i\cdot}$ and S_i^2 $(i = 1, \ldots, I)$ denote the sample mean and variance of the ith sample. Define the **between-samples estimator** $\hat\sigma_B^2$ by

$$\hat\sigma_B^2 = JS_{\bar X}^2 = \frac{J\sum_{i=1}^{I}(\bar X_{i\cdot} - \bar X_{\cdot\cdot})^2}{I-1} = \frac{\sum_{i=1}^{I}\sum_{j=1}^{J}(\bar X_{i\cdot} - \bar X_{\cdot\cdot})^2}{I-1} \quad (10.1)$$

and the **within-sample estimator** $\hat\sigma_W^2$ by

$$\hat\sigma_W^2 = \frac{\sum_{i=1}^{I} S_i^2}{I} = \frac{1}{I}\left[\sum_{i=1}^{I}\frac{1}{J-1}\sum_{j=1}^{J}(X_{ij} - \bar X_{i\cdot})^2\right] = \frac{\sum_{i=1}^{I}\sum_{j=1}^{J}(X_{ij} - \bar X_{i\cdot})^2}{I(J-1)} \quad (10.2)$$

Then $\hat\sigma_B^2$ is an unbiased estimator of σ^2 when H_0 is true, but $E(\hat\sigma_B^2) > \sigma^2$ when H_0 is false, while $\hat\sigma_W^2$ is unbiased for σ^2 whether or not H_0 is true.

$$F = \frac{\hat\sigma_B^2}{\hat\sigma_W^2} = \frac{J\sum(\bar X_{i\cdot} - \bar X_{\cdot\cdot})^2/(I-1)}{\sum\sum(X_{ij} - \bar X_{i\cdot})^2/I(J-1)} \quad (10.3)$$

Definition: The **total sum of squares** (*SST*), **treatment sum of squares** (*SSTr*), and **error sum of squares** (*SSE*) are given by

$$SST = \sum_{i=1}^{I}\sum_{j=1}^{J}(X_{ij} - \bar X_{\cdot\cdot})^2 = \sum_{i=1}^{I}\sum_{j=1}^{J}X_{ij}^2 - \frac{1}{IJ}X_{\cdot\cdot}^2$$

5

$$SSTr = \sum_{i=1}^{I}\sum_{j=1}^{J}(\bar X_{i\cdot} - \bar X_{\cdot\cdot})^2 = \frac{1}{J}\sum_{i=1}^{I}X_{i\cdot}^2 - \frac{1}{IJ}X_{\cdot\cdot}^2 \quad (10.5)$$

$$SSE = \sum_{i=1}^{I}\sum_{j=1}^{J}(X_{ij} - \bar X_{i\cdot})^2, \text{ where } X_{i\cdot} = \sum_{j=1}^{J}X_{ij},\ X_{\cdot\cdot} = \sum_{i=1}^{I}\sum_{j=1}^{J}X_{ij}$$

The fundamental identity of single-factor ANOVA:

6

$$SST = SSTr + SSE \quad (10.6)$$

7 8 A 9

$$MSTr = \frac{SSTr}{I-1}, \quad MSE = \frac{SSE}{I(J-1)}, \quad F = \frac{MSTr}{MSE} \quad (10.9)$$

Empty Table

Source	d.f.	Sum of squares	Mean square	F
Methods				
Error				
Total				

Table A.7 Critical Values F_{α, ν_1, ν_2} for the F Distribution $\alpha = .05$ $\alpha = .05$

ν_2 \ ν_1	1	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40	60	120	∞
1	161.4	199.5	215.7	224.6	230.2	234.0	236.8	238.9	240.5	241.9	243.9	245.9	248.0	249.1	250.1	251.1	252.2	253.3	254.3
2	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38	19.40	19.41	19.43	19.45	19.45	19.46	19.47	19.48	19.49	19.50
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79	8.74	8.70	8.66	8.64	8.62	8.59	8.57	8.55	8.53
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.91	5.86	5.80	5.77	5.75	5.72	5.69	5.66	5.63
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.68	4.62	4.56	4.53	4.50	4.46	4.43	4.40	4.36
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	4.00	3.94	3.87	3.84	3.81	3.77	3.74	3.70	3.67
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.57	3.51	3.44	3.41	3.38	3.34	3.30	3.27	3.23
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35	3.28	3.22	3.15	3.12	3.08	3.04	3.01	2.97	2.93
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	3.07	3.01	2.94	2.90	2.86	2.83	2.79	2.75	2.71
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.91	2.85	2.77	2.74	2.70	2.66	2.62	2.58	2.54

Example from Devore

Anova example with Tukey

A biologist wished to study the effects of ethanol on sleep time. A sample of 20 rats, matched for age and other characteristics, was selected, and each rat was given an oral injection having a particular concentration of ethanol per body weight. The rapid eye movement (REM) sleep time for each rat was then recorded for a 24-hour period, with the following results: (Example 10-6 [1]). Does the data indicate that the true average REM sleep time depends on the concentration of ethanol? (This example is based on an experiment reported in "Relationship of Ethanol Blood Level to REM and Non-REM Sleep Time and Distribution in the Rat, "Life Sciences", 1978: 839–846.) . Use 95% confidence interval. If Ho is rejected then Use Tukey to find the method which has the most impact on REM.

The Table next (10.4 p425) [1] is a SAS ANOVA table. The last column gives the P-value as 0.0001. Using a significance level of .05, we reject the null hypothesis H_0: $\mu_1 = \mu_2 = \mu_3 = \mu_4$, since P-value = 0.0001 < .05 = α. True average REM sleep time does appear to depend on concentration level.

They $\bar{x}_{i.}$s differ rather substantially from one another, but there is also a great deal of variability within each sample, so to answer the question precisely we must carry out the ANOVA.

Type	Treatment (concentration of ethanol)					$x_{i.}$	$\bar{x}_{i.}$	Row-sum	Row-ave	Row-stdv	Row-sum-squared	
0 (control)	88.6	73.2	91.4	68	75.2	396.4	79.28	396.40	79.28	10.18	157132.96	=J18*J18
1 g/kg	63	53.9	69.2	50.1	71.5	307.7	61.54	307.70	61.54	9.34	94679.29	
2 g/kg	44.9	59.5	40.2	56.3	38.7	239.6	47.92	239.60	47.92	9.46	57408.16	
4 g/kg	31	39.6	45.3	25.2	22.7	163.8	32.76	163.80	32.76	9.56	26830.44	
						1107.5	**55.375**	grand-ave	55.38		336050.85	=SUM(M18:M21)
						$x_{..}$=	$\bar{x}_{..}$ =	=AVERAGE(C18:G21)		**Row-stdv**	67210.17	=M22/C27
										=STDEV(C18:G21)		
										19.69472		

Data Extracted

tolerance	0.05		=SUM(C18:G21)	**X.. =**	1107.50	$\Sigma\Sigma x^2_{ij}$
I(row)=	4.00		=G25*G25	**sqr(X..) =**	1226556.25	
J(column)=	5.00	=G26/(C26*C27) =	corr factor =	**sqr(X..)/IJ =**	61327.81	

=K8-G10	5882.36	**SSTr=SST-SSE**
=L26/(C26-1)	1960.79	**MSTr=SSTr/(I-1)**

7849.96	5358.24	8353.96	4624	5655.04
3969	2905.21	4788.64	2510.01	5112.25
2016.01	3540.25	1616.04	3169.69	1497.69
961	1568.16	2052.09	635.04	515.29

	=H18-K11
SSE	**1487.40**

=SUM(C29:G32)	Sum (Xij)*(Xij)	**68697.57**
=H17-G10	**SST=H17-G10**	7369.76
=L16/(C9*(C10-1))	**MSE=SSE/I(J-1)**	92.96

f calc	=L27/G36
f=MSTr/MSE	**21.09**

F(table) 3.24

F(0.05,3,16) F(alpha, I-1, I(J-1))

Hence f calc >= 3.24 is True

13.63 >= 3.24 is True

Should one continue to Tukey test ? YES W some effect on the REM

Since Ha is NOT rejected thus a/some methods are significantly different

Ha is NOT rejected in other words, ways of ethanol treatment HAVE some effect on the REM.

$w = Q_{[\alpha, I, I(J-1)]}\sqrt{MSE/J}$ → and get $Q_{[\alpha, I, I(J-1)]}$ from table

Tukey Method & Formula ------------->

There are I = 4 treatments and 16 df for error, from which $w = Q_{[\alpha, I, I(J-1)]}\sqrt{MSE/J}$ →giving

$Q_{(0.5,4,16)}$ = 4.05 and w = 4.05 $\sqrt{92.96/5}$ = 17.46. Ordering the means and underscoring yields

w = 17.46

w =4.05*SQRT(G36/C27)

$\bar{x}_{4.}$		$\bar{x}_{3.}$		$\bar{x}_{2.}$		$\bar{x}_{1.}$
79.28		61.54		47.92		32.76
	d3		d2		d1	
	=E51-F51		=F51-G51		=G51-H51	
	17.74		13.62		15.16	
17.74		>		17.46 --> ???		
					TRUE	

0 (control) has the most impact on REM due to ethanol

79.28
61.54
47.92
32.76

Table A.7 Critical Values F_{α,ν_1,ν_2} for the F Distribution $\alpha = .05$

ν_2 \ ν_1	1	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40	60	120	∞
1	161.4	199.5	215.7	224.6	230.2	234.0	236.8	238.9	240.5	241.9	243.9	245.9	248.0	249.1	250.1	251.1	252.2	253.3	254.3
2	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38	19.40	19.41	19.43	19.45	19.45	19.46	19.47	19.48	19.49	19.50
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79	8.74	8.70	8.66	8.64	8.62	8.59	8.57	8.55	8.53
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.91	5.86	5.80	5.77	5.75	5.72	5.69	5.66	5.63
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.68	4.62	4.56	4.53	4.50	4.46	4.43	4.40	4.36
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	4.00	3.94	3.87	3.84	3.81	3.77	3.74	3.70	3.67
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.57	3.51	3.44	3.41	3.38	3.34	3.30	3.27	3.23
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35	3.28	3.22	3.15	3.12	3.08	3.04	3.01	2.97	2.93
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	3.07	3.01	2.94	2.90	2.86	2.83	2.79	2.75	2.71
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.91	2.85	2.77	2.74	2.70	2.66	2.62	2.58	2.54
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85	2.79	2.72	2.65	2.61	2.57	2.53	2.49	2.45	2.40
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75	2.69	2.62	2.54	2.51	2.47	2.43	2.38	2.34	2.30
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67	2.60	2.53	2.46	2.42	2.38	2.34	2.30	2.25	2.21
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60	2.53	2.46	2.39	2.35	2.31	2.27	2.22	2.18	2.13
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54	2.48	2.40	2.33	2.29	2.25	2.20	2.16	2.11	2.07
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49	2.42	2.35	2.28	2.24	2.19	2.15	2.11	2.06	2.01
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	2.45	2.38	2.31	2.23	2.19	2.15	2.10	2.06	2.01	1.96
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41	2.34	2.27	2.19	2.15	2.11	2.06	2.02	1.97	1.92
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38	2.31	2.23	2.16	2.11	2.07	2.03	1.98	1.93	1.88
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35	2.28	2.20	2.12	2.08	2.04	1.99	1.95	1.90	1.84

Table A.8 Critical Values $Q_{\alpha,m,\nu}$ for the Studentized Range Distribution

		m									
ν	α	2	3	4	5	6	7	8	9	10	11
5	.05	3.64	4.60	5.22	5.67	6.03	6.33	6.58	6.80	6.99	7.17
	.01	5.70	6.98	7.80	8.42	8.91	9.32	9.67	9.97	10.24	10.48
6	.05	3.46	4.34	4.90	5.30	5.63	5.90	6.12	6.32	6.49	6.65
	.01	5.24	6.33	7.03	7.56	7.97	8.32	8.61	8.87	9.10	9.30
7	.05	3.34	4.16	4.68	5.06	5.36	5.61	5.82	6.00	6.16	6.30
	.01	4.95	5.92	6.54	7.01	7.37	7.68	7.94	8.17	8.37	8.55
8	.05	3.26	4.04	4.53	4.89	5.17	5.40	5.60	5.77	5.92	6.05
	.01	4.75	5.64	6.20	6.62	6.96	7.24	7.47	7.68	7.86	8.03
9	.05	3.20	3.95	4.41	4.76	5.02	5.24	5.43	5.59	5.74	5.87
	.01	4.60	5.43	5.96	6.35	6.66	6.91	7.13	7.33	7.49	7.65
10	.05	3.15	3.88	4.33	4.65	4.91	5.12	5.30	5.46	5.60	5.72
	.01	4.48	5.27	5.77	6.14	6.43	6.67	6.87	7.05	7.21	7.36
11	.05	3.11	3.82	4.26	4.57	4.82	5.03	5.20	5.35	5.49	5.61
	.01	4.39	5.15	5.62	5.97	6.25	6.48	6.67	6.84	6.99	7.13
12	.05	3.08	3.77	4.20	4.51	4.75	4.95	5.12	5.27	5.39	5.51
	.01	4.32	5.05	5.50	5.84	6.10	6.32	6.51	6.67	6.81	6.94
13	.05	3.06	3.73	4.15	4.45	4.69	4.88	5.05	5.19	5.32	5.43
	.01	4.26	4.96	5.40	5.73	5.98	6.19	6.37	6.53	6.67	6.79
14	.05	3.03	3.70	4.11	4.41	4.64	4.83	4.99	5.13	5.25	5.36
	.01	4.21	4.89	5.32	5.63	5.88	6.08	6.26	6.41	6.54	6.66
15	.05	3.01	3.67	4.08	4.37	4.59	4.78	4.94	5.08	5.20	5.31
	.01	4.17	4.84	5.25	5.56	5.80	5.99	6.16	6.31	6.44	6.55
16	.05	3.00	3.65	4.05	4.33	4.56	4.74	4.90	5.03	5.15	5.26
	.01	4.13	4.79	5.19	5.49	5.72	5.92	6.08	6.22	6.35	6.46

A1	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q

Example from Devore

Anova example with Tukey

A biologist wished to study the effects of ethanol on sleep time. A sample of 20 rats, matched for age and other characteristics, was selected, and each rat was given an oral injection having a particular concentration of ethanol per body weight. The rapid eye movement (REM) sleep time for each rat was then recorded for a 24-hour period, with the following results: (Example 10-6 [1]). Does the data indicate that the true average REM sleep time depends on the concentration of ethanol (Ha)? (This example is based on an experiment reported in “Relationship of Ethanol Blood Level to REM and Non-REM Sleep Time and Distribution in the Rat, “Life Sciences”, 1978: 839–846.) . Use 95% confidence of interval. If Ho is rejected then Use Tukey to find the method which has the most impact on REM.

They $\bar{x}_{i.}$s differ rather substantially from one another, but there is also a great deal of variability within each sample, so to answer the question precisely we must carry out the ANOVA.

Type	Treatment (concentration of ethanol)					$x_{i.}$	$\bar{x}_{i.}$
0 (control)	88.6	73.2	91.4	68	75.2	396.4	79.28
1 g/kg	63	53.9	69.2	50.1	71.5	307.7	61.54
2 g/kg	44.9	59.5	40.2	56.3	38.7	239.6	47.92
4 g/kg	31	39.6	45.3	25.2	22.7	163.8	32.76
						1107.5	**55.375**
						x..=	$\bar{x}_{..}$ =

Data Extracted

tolerance

I(row)=

J(column)=

Tukey Method & Formula ------------->

w = $Q_{[\alpha, I, I(J-1)]}\sqrt{MSE/J}$ → and get $Q_{[\alpha, I, I(J-1)]}$ from table

w = $Q_{[\alpha, I, I(J-1)]}\sqrt{MSE/J}$ → and get $Q_{[\alpha, I, I(J-1)]}$ from table

Table A.8 Critical Values $Q_{\alpha,m,\nu}$ for the Studentized Range Distribution

		m									
ν	α	2	3	4	5	6	7	8	9	10	11
5	.05	3.64	4.60	5.22	5.67	6.03	6.33	6.58	6.80	6.99	7.17
	.01	5.70	6.98	7.80	8.42	8.91	9.32	9.67	9.97	10.24	10.48
6	.05	3.46	4.34	4.90	5.30	5.63	5.90	6.12	6.32	6.49	6.65
	.01	5.24	6.33	7.03	7.56	7.97	8.32	8.61	8.87	9.10	9.30
7	.05	3.34	4.16	4.68	5.06	5.36	5.61	5.82	6.00	6.16	6.30
	.01	4.95	5.92	6.54	7.01	7.37	7.68	7.94	8.17	8.37	8.55
8	.05	3.26	4.04	4.53	4.89	5.17	5.40	5.60	5.77	5.92	6.05
	.01	4.75	5.64	6.20	6.62	6.96	7.24	7.47	7.68	7.86	8.03
9	.05	3.20	3.95	4.41	4.76	5.02	5.24	5.43	5.59	5.74	5.87
	.01	4.60	5.43	5.96	6.35	6.66	6.91	7.13	7.33	7.49	7.65
10	.05	3.15	3.88	4.33	4.65	4.91	5.12	5.30	5.46	5.60	5.72
	.01	4.48	5.27	5.77	6.14	6.43	6.67	6.87	7.05	7.21	7.36
11	.05	3.11	3.82	4.26	4.57	4.82	5.03	5.20	5.35	5.49	5.61
	.01	4.39	5.15	5.62	5.97	6.25	6.48	6.67	6.84	6.99	7.13
12	.05	3.08	3.77	4.20	4.51	4.75	4.95	5.12	5.27	5.39	5.51
	.01	4.32	5.05	5.50	5.84	6.10	6.32	6.51	6.67	6.81	6.94
13	.05	3.06	3.73	4.15	4.45	4.69	4.88	5.05	5.19	5.32	5.43
	.01	4.26	4.96	5.40	5.73	5.98	6.19	6.37	6.53	6.67	6.79
14	.05	3.03	3.70	4.11	4.41	4.64	4.83	4.99	5.13	5.25	5.36
	.01	4.21	4.89	5.32	5.63	5.88	6.08	6.26	6.41	6.54	6.66
15	.05	3.01	3.67	4.08	4.37	4.59	4.78	4.94	5.08	5.20	5.31
	.01	4.17	4.84	5.25	5.56	5.80	5.99	6.16	6.31	6.44	6.55
16	.05	3.00	3.65	4.05	4.33	4.56	4.74	4.90	5.03	5.15	5.26
	.01	4.13	4.79	5.19	5.49	5.72	5.92	6.08	6.22	6.35	6.46

$\bar{x}_{4.}$ $\bar{x}_{3.}$ $\bar{x}_{2.}$ $\bar{x}_{1.}$

Table A.7 Critical Values F_{α,ν_1,ν_2} for the F Distribution

$\alpha = .05$

ν_2 \ ν_1	1	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40	60	120	∞
1	161.4	199.5	215.7	224.6	230.2	234.0	236.8	238.9	240.5	241.9	243.9	245.9	248.0	249.1	250.1	251.1	252.2	253.3	254.3
2	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38	19.40	19.41	19.43	19.45	19.45	19.46	19.47	19.48	19.49	19.50
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79	8.74	8.70	8.66	8.64	8.62	8.59	8.57	8.55	8.53
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.91	5.86	5.80	5.77	5.75	5.72	5.69	5.66	5.63
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.68	4.62	4.56	4.53	4.50	4.46	4.43	4.40	4.36
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	4.00	3.94	3.87	3.84	3.81	3.77	3.74	3.70	3.67
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.57	3.51	3.44	3.41	3.38	3.34	3.30	3.27	3.23
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35	3.28	3.22	3.15	3.12	3.08	3.04	3.01	2.97	2.93
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	3.07	3.01	2.94	2.90	2.86	2.83	2.79	2.75	2.71
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.91	2.85	2.77	2.74	2.70	2.66	2.62	2.58	2.54
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85	2.79	2.72	2.65	2.61	2.57	2.53	2.49	2.45	2.40
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75	2.69	2.62	2.54	2.51	2.47	2.43	2.38	2.34	2.30
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67	2.60	2.53	2.46	2.42	2.38	2.34	2.30	2.25	2.21
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60	2.53	2.46	2.39	2.35	2.31	2.27	2.22	2.18	2.13
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54	2.48	2.40	2.33	2.29	2.25	2.20	2.16	2.11	2.07
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49	2.42	2.35	2.28	2.24	2.19	2.15	2.11	2.06	2.01
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	2.45	2.38	2.31	2.23	2.19	2.15	2.10	2.06	2.01	1.96
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41	2.34	2.27	2.19	2.15	2.11	2.06	2.02	1.97	1.92
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38	2.31	2.23	2.16	2.11	2.07	2.03	1.98	1.93	1.88
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35	2.28	2.20	2.12	2.08	2.04	1.99	1.95	1.90	1.84