Anova example with Tukey

A biologist wished to study the effects of ethanol on sleep time. A sample of 20 rats, matched for age and other characteristics, was selected, and each rat was given an oral injection having a particular concentration of ethanol per body weight. The rapid eye movement (REM) sleep time for each rat was then recorded for a 24-hour period, with the following results: (Example 10-6 [1])

Type	Treatment (concentration of ethanol)					$x_{i.}$	$\bar{x}_{i.}$
0 (control)	68.0	73.2	91.4	88.6	75.2	396.4	$\bar{x}_{1.} = 79.28$
1 g/kg	50.1	53.9	69.2	63.0	71.5	307.7	$\bar{x}_{2.} = 61.54$
2 g/kg	56.3	59.5	40.2	11.9	38.7	239.6	$\bar{x}_{3.} = 47.92$
4 g/kg	25.2	39.6	45.3	31.0	22.7	163.8	$\bar{x}_{4.} = 32.76$
						$x_{..} = 1107.5$	**$\bar{x}_{..} = 55.375$**

Does the data indicate that the true average REM sleep time depends on the concentration of ethanol? (This example is based on an experiment reported in "Relationship of Ethanol Blood Level to REM and Non-REM Sleep Time and Distribution in the Rat, "*Life Sciences*", 1978: 839–846.) Their $\bar{x}_{i.}$s differ rather substantially from one another, but there is also a great deal of variability within each sample, so to answer the question precisely we must carry out the ANOVA.

With $\Sigma\Sigma\, x^2_{ij} = 68{,}697.6$ and correction factor $x^2_{..}(IJ) = (1107.5)^2/20 = 61{,}327.8$, the computing formulas yield:

SST = 68,697.6 - 61,327.8 = 7369.8

SSTr = $\frac{1}{5}[(396.40)^2 + (307.70)^2 + (239.60)^2 + (163.80)^2] - 61{,}327.8 = 67{,}210.2 - 61{,}327.8 = 5882.4$

SSE = 7369.8 - 5882.4 = 1487.4

The Table next (10.4 p425) [1] is a SAS ANOVA table. The last column gives the P-value as 0.0001. Using a significance level of .05, we reject the null hypothesis
H_0: $\mu_1 = \mu_2 = \mu_3 = \mu_4$, since P-value = 0.0001 < .05 = α. True average REM sleep time does appear to depend on concentration level of ethanol.
Also $F_{(0.5,3,16)} = 3.24$ → since F(calc) > $F_{(0.5,3,16)}$ is TRUE then reject H_0 or accept H_a and continue to Tukey's method to find out which concentration(s) level has/have the most impact.

Table 10.4 (10.4 p425) [1] SAS ANOVA Table

		Analysis of	Variance Procedure		
Dependent	Variable:	TIME			
Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	3	5882.35750	1960.78583	21.09	0.0001
Error	16	1487.40000	92.96250		
Corrected Total	19	7369.75750			

There are I = 4 treatments and 16 df for error, from which $w = Q_{[\alpha,\, I,\, I(J-1)]}\sqrt{MSE/J}$ → giving $Q_{(0.5,4,16)}$ = 4.05 and then w = 4.05 $\sqrt{93/5}$ = 17.47. Ordering the means and underscoring yields : (ordering the values of mean is a MUST – here, left to right, smallest to biggest value)

$\bar{x}_{4.}$	$\bar{x}_{3.}$	$\bar{x}_{2.}$	$\bar{x}_{1.}$
32.76	47.92	61.54	79.28

The interpretation of this underscoring must be done with care, since we seem to have concluded that treatments 2 and 3 do not differ, 3 and 4 do not differ, yet 2 and 4 do differ. The suggested way of expressing this is to say that although evidence allows us to conclude that treatments 2 and 4 differ from one another, neither has been shown to be significantly different from 3. **Treatment 1 has a significantly ($\bar{x}_{1.}$) higher true average REM sleep time than any of the other treatments.** Figure 10.4 shows SAS output from the application of Tukey's procedure.

Alpha = 0.05 df =16 MSE = 92. 9625
Critical Value of Studentized Range (Q) = 4.046
Minimum Significant Difference (w) = 17.47
Means with the same letter are not significantly different.

Tukey Grouping	Mean	N	TREATMENT
A	79.280	5	0 (control)
B	61.540	5	1 gm/kg
B			
C B	47.920	5	2 gm/kg
C			
C	32.760	5	4 gm/kg

Figure 10.4 Tukey's method using SAS (giving A as most significantly different mean = $\bar{x}_{1.}$)

Source		d.f.		Sum of squares		Mean square	f(calc)
Methods	I-1=	3	SSTr=	5882.36	MSTr=	1960.79	21.09
Error	I(J-1)=	16	SSE=	1487.40	MSE=	92.96	
Total		19	SST=	7369.76			

Reference

[1] Jay L. Devore, *Probability & Statistics for Engineering & the Sciences*. 2012.

Note:
See also other MS Excel examples from the Lecturer.