

Contents & Tables Only

Probability & Statistics
for Engineers & Scientists

NINTH EDITION



WALPOLE | MYERS | MYERS | YE

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Appendix A

Statistical Tables and Proofs

Table A.1 Binomial Probability Sums $\sum_{x=0}^r b(x; n, p)$

Table A.1 (continued) Binomial Probability Sums $\sum_{x=0}^r b(x; n, p)$

<i>n</i>	<i>r</i>	<i>p</i>									
		0.10	0.20	0.25	0.30	0.40	0.50	0.60	0.70	0.80	0.90
8	0	0.4305	0.1678	0.1001	0.0576	0.0168	0.0039	0.0007	0.0001	0.0000	
	1	0.8131	0.5033	0.3671	0.2553	0.1064	0.0352	0.0085	0.0013	0.0001	
	2	0.9619	0.7969	0.6785	0.5518	0.3154	0.1445	0.0498	0.0113	0.0012	0.0000
	3	0.9950	0.9437	0.8862	0.8059	0.5941	0.3633	0.1737	0.0580	0.0104	0.0004
	4	0.9996	0.9896	0.9727	0.9420	0.8263	0.6367	0.4059	0.1941	0.0563	0.0050
	5	1.0000	0.9988	0.9958	0.9887	0.9502	0.8555	0.6846	0.4482	0.2031	0.0381
	6		0.9999	0.9996	0.9987	0.9915	0.9648	0.8936	0.7447	0.4967	0.1869
	7		1.0000	1.0000	0.9999	0.9993	0.9961	0.9832	0.9424	0.8322	0.5695
	8			1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
9	0	0.3874	0.1342	0.0751	0.0404	0.0101	0.0020	0.0003	0.0000		
	1	0.7748	0.4362	0.3003	0.1960	0.0705	0.0195	0.0038	0.0004	0.0000	
	2	0.9470	0.7382	0.6007	0.4628	0.2318	0.0898	0.0250	0.0043	0.0003	0.0000
	3	0.9917	0.9144	0.8343	0.7297	0.4826	0.2539	0.0994	0.0253	0.0031	0.0001
	4	0.9991	0.9804	0.9511	0.9012	0.7334	0.5000	0.2666	0.0988	0.0196	0.0009
	5	0.9999	0.9969	0.9900	0.9747	0.9006	0.7461	0.5174	0.2703	0.0856	0.0083
	6	1.0000	0.9997	0.9987	0.9957	0.9750	0.9102	0.7682	0.5372	0.2618	0.0530
	7		1.0000	0.9999	0.9996	0.9962	0.9805	0.9295	0.8040	0.5638	0.2252
	8			1.0000	1.0000	0.9997	0.9980	0.9899	0.9596	0.8658	0.6126
	9				1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
10	0	0.3487	0.1074	0.0563	0.0282	0.0060	0.0010	0.0001	0.0000		
	1	0.7361	0.3758	0.2440	0.1493	0.0464	0.0107	0.0017	0.0001	0.0000	
	2	0.9298	0.6778	0.5256	0.3828	0.1673	0.0547	0.0123	0.0016	0.0001	
	3	0.9872	0.8791	0.7759	0.6496	0.3823	0.1719	0.0548	0.0106	0.0009	0.0000
	4	0.9984	0.9672	0.9219	0.8497	0.6331	0.3770	0.1662	0.0473	0.0064	0.0001
	5	0.9999	0.9936	0.9803	0.9527	0.8338	0.6230	0.3669	0.1503	0.0328	0.0016
	6	1.0000	0.9991	0.9965	0.9894	0.9452	0.8281	0.6177	0.3504	0.1209	0.0128
	7		0.9999	0.9996	0.9984	0.9877	0.9453	0.8327	0.6172	0.3222	0.0702
	8			1.0000	1.0000	0.9999	0.9983	0.9893	0.9536	0.8507	0.6242
	9				1.0000	0.9999	0.9990	0.9940	0.9718	0.8926	0.6513
	10					1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
11	0	0.3138	0.0859	0.0422	0.0198	0.0036	0.0005	0.0000			
	1	0.6974	0.3221	0.1971	0.1130	0.0302	0.0059	0.0007	0.0000		
	2	0.9104	0.6174	0.4552	0.3127	0.1189	0.0327	0.0059	0.0006	0.0000	
	3	0.9815	0.8389	0.7133	0.5696	0.2963	0.1133	0.0293	0.0043	0.0002	
	4	0.9972	0.9496	0.8854	0.7897	0.5328	0.2744	0.0994	0.0216	0.0020	0.0000
	5	0.9997	0.9883	0.9657	0.9218	0.7535	0.5000	0.2465	0.0782	0.0117	0.0003
	6	1.0000	0.9980	0.9924	0.9784	0.9006	0.7256	0.4672	0.2103	0.0504	0.0028
	7		0.9998	0.9988	0.9957	0.9707	0.8867	0.7037	0.4304	0.1611	0.0185
	8			1.0000	0.9999	0.9994	0.9941	0.9673	0.8811	0.6873	0.3826
	9				1.0000	1.0000	0.9993	0.9941	0.9698	0.8870	0.6779
	10					1.0000	0.9995	0.9964	0.9802	0.9141	0.6862
	11						1.0000	1.0000	1.0000	1.0000	1.0000

Table A.1 (continued) Binomial Probability Sums $\sum_{x=0}^r b(x; n, p)$

<i>n</i>	<i>r</i>	<i>p</i>									
		0.10	0.20	0.25	0.30	0.40	0.50	0.60	0.70	0.80	0.90
12	0	0.2824	0.0687	0.0317	0.0138	0.0022	0.0002	0.0000			
	1	0.6590	0.2749	0.1584	0.0850	0.0196	0.0032	0.0003	0.0000		
	2	0.8891	0.5583	0.3907	0.2528	0.0834	0.0193	0.0028	0.0002	0.0000	
	3	0.9744	0.7946	0.6488	0.4925	0.2253	0.0730	0.0153	0.0017	0.0001	
	4	0.9957	0.9274	0.8424	0.7237	0.4382	0.1938	0.0573	0.0095	0.0006	0.0000
	5	0.9995	0.9806	0.9456	0.8822	0.6652	0.3872	0.1582	0.0386	0.0039	0.0001
	6	0.9999	0.9961	0.9857	0.9614	0.8418	0.6128	0.3348	0.1178	0.0194	0.0005
	7	1.0000	0.9994	0.9972	0.9905	0.9427	0.8062	0.5618	0.2763	0.0726	0.0043
	8		0.9999	0.9996	0.9983	0.9847	0.9270	0.7747	0.5075	0.2054	0.0256
	9		1.0000	1.0000	0.9998	0.9972	0.9807	0.9166	0.7472	0.4417	0.1109
	10			1.0000	0.9997	0.9968	0.9804	0.9150	0.7251	0.3410	
	11				1.0000	0.9998	0.9978	0.9862	0.9313	0.7176	
	12					1.0000	1.0000	1.0000	1.0000	1.0000	
13	0	0.2542	0.0550	0.0238	0.0097	0.0013	0.0001	0.0000			
	1	0.6213	0.2336	0.1267	0.0637	0.0126	0.0017	0.0001	0.0000		
	2	0.8661	0.5017	0.3326	0.2025	0.0579	0.0112	0.0013	0.0001		
	3	0.9658	0.7473	0.5843	0.4206	0.1686	0.0461	0.0078	0.0007	0.0000	
	4	0.9935	0.9009	0.7940	0.6543	0.3530	0.1334	0.0321	0.0040	0.0002	
	5	0.9991	0.9700	0.9198	0.8346	0.5744	0.2905	0.0977	0.0182	0.0012	0.0000
	6	0.9999	0.9930	0.9757	0.9376	0.7712	0.5000	0.2288	0.0624	0.0070	0.0001
	7	1.0000	0.9988	0.9944	0.9818	0.9023	0.7095	0.4256	0.1654	0.0300	0.0009
	8		0.9998	0.9990	0.9960	0.9679	0.8666	0.6470	0.3457	0.0991	0.0065
	9		1.0000	0.9999	0.9993	0.9922	0.9539	0.8314	0.5794	0.2527	0.0342
	10			1.0000	0.9999	0.9987	0.9888	0.9421	0.7975	0.4983	0.1339
	11				1.0000	0.9999	0.9983	0.9874	0.9363	0.7664	0.3787
	12					1.0000	0.9999	0.9987	0.9903	0.9450	0.7458
	13						1.0000	1.0000	1.0000	1.0000	
14	0	0.2288	0.0440	0.0178	0.0068	0.0008	0.0001	0.0000			
	1	0.5846	0.1979	0.1010	0.0475	0.0081	0.0009	0.0001			
	2	0.8416	0.4481	0.2811	0.1608	0.0398	0.0065	0.0006	0.0000		
	3	0.9559	0.6982	0.5213	0.3552	0.1243	0.0287	0.0039	0.0002		
	4	0.9908	0.8702	0.7415	0.5842	0.2793	0.0898	0.0175	0.0017	0.0000	
	5	0.9985	0.9561	0.8883	0.7805	0.4859	0.2120	0.0583	0.0083	0.0004	
	6	0.9998	0.9884	0.9617	0.9067	0.6925	0.3953	0.1501	0.0315	0.0024	0.0000
	7	1.0000	0.9976	0.9897	0.9685	0.8499	0.6047	0.3075	0.0933	0.0116	0.0002
	8		0.9996	0.9978	0.9917	0.9417	0.7880	0.5141	0.2195	0.0439	0.0015
	9		1.0000	0.9997	0.9983	0.9825	0.9102	0.7207	0.4158	0.1298	0.0092
	10			1.0000	0.9998	0.9961	0.9713	0.8757	0.6448	0.3018	0.0441
	11				1.0000	0.9994	0.9935	0.9602	0.8392	0.5519	0.1584
	12					0.9999	0.9991	0.9919	0.9525	0.8021	0.4154
	13					1.0000	0.9999	0.9992	0.9932	0.9560	0.7712
	14						1.0000	1.0000	1.0000	1.0000	

Table A.1 (continued) Binomial Probability Sums $\sum_{x=0}^r b(x; n, p)$

<i>n</i>	<i>r</i>	<i>p</i>											
		0.10	0.20	0.25	0.30	0.40	0.50	0.60	0.70	0.80	0.90		
15	0	0.2059	0.0352	0.0134	0.0047	0.0005	0.0000						
	1	0.5490	0.1671	0.0802	0.0353	0.0052	0.0005	0.0000					
	2	0.8159	0.3980	0.2361	0.1268	0.0271	0.0037	0.0003	0.0000				
	3	0.9444	0.6482	0.4613	0.2969	0.0905	0.0176	0.0019	0.0001				
	4	0.9873	0.8358	0.6865	0.5155	0.2173	0.0592	0.0093	0.0007	0.0000			
	5	0.9978	0.9389	0.8516	0.7216	0.4032	0.1509	0.0338	0.0037	0.0001			
	6	0.9997	0.9819	0.9434	0.8689	0.6098	0.3036	0.0950	0.0152	0.0008			
	7	1.0000	0.9958	0.9827	0.9500	0.7869	0.5000	0.2131	0.0500	0.0042	0.0000		
	8		0.9992	0.9958	0.9848	0.9050	0.6964	0.3902	0.1311	0.0181	0.0003		
	9			0.9999	0.9992	0.9963	0.9662	0.8491	0.5968	0.2784	0.0611	0.0022	
	10				1.0000	0.9999	0.9993	0.9907	0.9408	0.7827	0.4845	0.1642	0.0127
	11					1.0000	0.9999	0.9981	0.9824	0.9095	0.7031	0.3518	0.0556
	12						1.0000	0.9997	0.9963	0.9729	0.8732	0.6020	0.1841
	13							1.0000	0.9995	0.9948	0.9647	0.8329	0.4510
	14								1.0000	0.9995	0.9953	0.9648	0.7941
	15									1.0000	1.0000	1.0000	
16	0	0.1853	0.0281	0.0100	0.0033	0.0003	0.0000						
	1	0.5147	0.1407	0.0635	0.0261	0.0033	0.0003	0.0000					
	2	0.7892	0.3518	0.1971	0.0994	0.0183	0.0021	0.0001					
	3	0.9316	0.5981	0.4050	0.2459	0.0651	0.0106	0.0009	0.0000				
	4	0.9830	0.7982	0.6302	0.4499	0.1666	0.0384	0.0049	0.0003				
	5	0.9967	0.9183	0.8103	0.6598	0.3288	0.1051	0.0191	0.0016	0.0000			
	6	0.9995	0.9733	0.9204	0.8247	0.5272	0.2272	0.0583	0.0071	0.0002			
	7	0.9999	0.9930	0.9729	0.9256	0.7161	0.4018	0.1423	0.0257	0.0015	0.0000		
	8	1.0000	0.9985	0.9925	0.9743	0.8577	0.5982	0.2839	0.0744	0.0070	0.0001		
	9		0.9998	0.9984	0.9929	0.9417	0.7728	0.4728	0.1753	0.0267	0.0005		
	10			1.0000	0.9997	0.9984	0.9809	0.8949	0.6712	0.3402	0.0817	0.0033	
	11				1.0000	0.9997	0.9951	0.9616	0.8334	0.5501	0.2018	0.0170	
	12					1.0000	0.9991	0.9894	0.9349	0.7541	0.4019	0.0684	
	13						0.9999	0.9979	0.9817	0.9006	0.6482	0.2108	
	14							1.0000	0.9997	0.9967	0.9739	0.8593	0.4853
	15								1.0000	0.9997	0.9967	0.9719	0.8147
	16									1.0000	1.0000	1.0000	

Table A.1 (continued) Binomial Probability Sums $\sum_{x=0}^r b(x; n, p)$

<i>n</i>	<i>r</i>	<i>p</i>									
		0.10	0.20	0.25	0.30	0.40	0.50	0.60	0.70	0.80	0.90
17	0	0.1668	0.0225	0.0075	0.0023	0.0002	0.0000				
	1	0.4818	0.1182	0.0501	0.0193	0.0021	0.0001	0.0000			
	2	0.7618	0.3096	0.1637	0.0774	0.0123	0.0012	0.0001			
	3	0.9174	0.5489	0.3530	0.2019	0.0464	0.0064	0.0005	0.0000		
	4	0.9779	0.7582	0.5739	0.3887	0.1260	0.0245	0.0025	0.0001		
	5	0.9953	0.8943	0.7653	0.5968	0.2639	0.0717	0.0106	0.0007	0.0000	
	6	0.9992	0.9623	0.8929	0.7752	0.4478	0.1662	0.0348	0.0032	0.0001	
	7	0.9999	0.9891	0.9598	0.8954	0.6405	0.3145	0.0919	0.0127	0.0005	
	8	1.0000	0.9974	0.9876	0.9597	0.8011	0.5000	0.1989	0.0403	0.0026	0.0000
	9		0.9995	0.9969	0.9873	0.9081	0.6855	0.3595	0.1046	0.0109	0.0001
	10		0.9999	0.9994	0.9968	0.9652	0.8338	0.5522	0.2248	0.0377	0.0008
	11		1.0000	0.9999	0.9993	0.9894	0.9283	0.7361	0.4032	0.1057	0.0047
	12			1.0000	0.9999	0.9975	0.9755	0.8740	0.6113	0.2418	0.0221
	13				1.0000	0.9995	0.9936	0.9536	0.7981	0.4511	0.0826
	14					0.9999	0.9988	0.9877	0.9226	0.6904	0.2382
	15						1.0000	0.9999	0.9979	0.9807	0.8818
	16							1.0000	0.9998	0.9977	0.9775
	17								1.0000	1.0000	0.8332
18	0	0.1501	0.0180	0.0056	0.0016	0.0001	0.0000				
	1	0.4503	0.0991	0.0395	0.0142	0.0013	0.0001				
	2	0.7338	0.2713	0.1353	0.0600	0.0082	0.0007	0.0000			
	3	0.9018	0.5010	0.3057	0.1646	0.0328	0.0038	0.0002			
	4	0.9718	0.7164	0.5187	0.3327	0.0942	0.0154	0.0013	0.0000		
	5	0.9936	0.8671	0.7175	0.5344	0.2088	0.0481	0.0058	0.0003		
	6	0.9988	0.9487	0.8610	0.7217	0.3743	0.1189	0.0203	0.0014	0.0000	
	7	0.9998	0.9837	0.9431	0.8593	0.5634	0.2403	0.0576	0.0061	0.0002	
	8	1.0000	0.9957	0.9807	0.9404	0.7368	0.4073	0.1347	0.0210	0.0009	
	9		0.9991	0.9946	0.9790	0.8653	0.5927	0.2632	0.0596	0.0043	0.0000
	10		0.9998	0.9988	0.9939	0.9424	0.7597	0.4366	0.1407	0.0163	0.0002
	11		1.0000	0.9998	0.9986	0.9797	0.8811	0.6257	0.2783	0.0513	0.0012
	12			1.0000	0.9997	0.9942	0.9519	0.7912	0.4656	0.1329	0.0064
	13				1.0000	0.9987	0.9846	0.9058	0.6673	0.2836	0.0282
	14					0.9998	0.9962	0.9672	0.8354	0.4990	0.0982
	15						1.0000	0.9993	0.9918	0.9400	0.7287
	16							0.9999	0.9987	0.9858	0.9009
	17								1.0000	0.9999	0.9984
	18									1.0000	0.8499

Table A.1 (continued) Binomial Probability Sums $\sum_{x=0}^r b(x; n, p)$

<i>n</i>	<i>r</i>	<i>p</i>									
		0.10	0.20	0.25	0.30	0.40	0.50	0.60	0.70	0.80	0.90
19	0	0.1351	0.0144	0.0042	0.0011	0.0001					
	1	0.4203	0.0829	0.0310	0.0104	0.0008	0.0000				
	2	0.7054	0.2369	0.1113	0.0462	0.0055	0.0004	0.0000			
	3	0.8850	0.4551	0.2631	0.1332	0.0230	0.0022	0.0001			
	4	0.9648	0.6733	0.4654	0.2822	0.0696	0.0096	0.0006	0.0000		
	5	0.9914	0.8369	0.6678	0.4739	0.1629	0.0318	0.0031	0.0001		
	6	0.9983	0.9324	0.8251	0.6655	0.3081	0.0835	0.0116	0.0006		
	7	0.9997	0.9767	0.9225	0.8180	0.4878	0.1796	0.0352	0.0028	0.0000	
	8	1.0000	0.9933	0.9713	0.9161	0.6675	0.3238	0.0885	0.0105	0.0003	
	9		0.9984	0.9911	0.9674	0.8139	0.5000	0.1861	0.0326	0.0016	
10			0.9997	0.9977	0.9895	0.9115	0.6762	0.3325	0.0839	0.0067	0.0000
11			1.0000	0.9995	0.9972	0.9648	0.8204	0.5122	0.1820	0.0233	0.0003
12				0.9999	0.9994	0.9884	0.9165	0.6919	0.3345	0.0676	0.0017
13				1.0000	0.9999	0.9969	0.9682	0.8371	0.5261	0.1631	0.0086
14					1.0000	0.9994	0.9904	0.9304	0.7178	0.3267	0.0352
15						0.9999	0.9978	0.9770	0.8668	0.5449	0.1150
16						1.0000	0.9996	0.9945	0.9538	0.7631	0.2946
17							1.0000	0.9992	0.9896	0.9171	0.5797
18								0.9999	0.9989	0.9856	0.8649
19								1.0000	1.0000	1.0000	1.0000
20	0	0.1216	0.0115	0.0032	0.0008	0.0000					
	1	0.3917	0.0692	0.0243	0.0076	0.0005	0.0000				
	2	0.6769	0.2061	0.0913	0.0355	0.0036	0.0002				
	3	0.8670	0.4114	0.2252	0.1071	0.0160	0.0013	0.0000			
	4	0.9568	0.6296	0.4148	0.2375	0.0510	0.0059	0.0003			
	5	0.9887	0.8042	0.6172	0.4164	0.1256	0.0207	0.0016	0.0000		
	6	0.9976	0.9133	0.7858	0.6080	0.2500	0.0577	0.0065	0.0003		
	7	0.9996	0.9679	0.8982	0.7723	0.4159	0.1316	0.0210	0.0013	0.0000	
	8	0.9999	0.9900	0.9591	0.8867	0.5956	0.2517	0.0565	0.0051	0.0001	
	9	1.0000	0.9974	0.9861	0.9520	0.7553	0.4119	0.1275	0.0171	0.0006	
10			0.9994	0.9961	0.9829	0.8725	0.5881	0.2447	0.0480	0.0026	0.0000
11			0.9999	0.9991	0.9949	0.9435	0.7483	0.4044	0.1133	0.0100	0.0001
12			1.0000	0.9998	0.9987	0.9790	0.8684	0.5841	0.2277	0.0321	0.0004
13				1.0000	0.9997	0.9935	0.9423	0.7500	0.3920	0.0867	0.0024
14					1.0000	0.9984	0.9793	0.8744	0.5836	0.1958	0.0113
15						0.9997	0.9941	0.9490	0.7625	0.3704	0.0432
16						1.0000	0.9987	0.9840	0.8929	0.5886	0.1330
17							0.9998	0.9964	0.9645	0.7939	0.3231
18							1.0000	0.9995	0.9924	0.9308	0.6083
19								1.0000	0.9992	0.9885	0.8784
20									1.0000	1.0000	1.0000

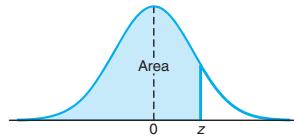
Table A.2 Poisson Probability Sums $\sum_{x=0}^r p(x; \mu)$

r	μ								
	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0	0.9048	0.8187	0.7408	0.6703	0.6065	0.5488	0.4966	0.4493	0.4066
1	0.9953	0.9825	0.9631	0.9384	0.9098	0.8781	0.8442	0.8088	0.7725
2	0.9998	0.9989	0.9964	0.9921	0.9856	0.9769	0.9659	0.9526	0.9371
3	1.0000	0.9999	0.9997	0.9992	0.9982	0.9966	0.9942	0.9909	0.9865
4		1.0000	1.0000	0.9999	0.9998	0.9996	0.9992	0.9986	0.9977
5			1.0000	1.0000	1.0000	0.9999	0.9998	0.9997	
6						1.0000	1.0000	1.0000	1.0000

Table A.2 (continued) Poisson Probability Sums

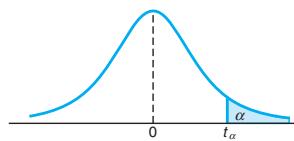
Table A.2 (continued) Poisson Probability Sums $\sum_{x=0}^r p(x; \mu)$

Table A.3 Normal Probability Table

**Table A.3** Areas under the Normal Curve

<i>z</i>	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
-3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
-3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
-3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
-3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
-3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
-2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
-2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
-2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
-2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
-2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
-2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
-2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
-2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
-2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
-2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
-1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
-1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
-1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
-1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
-1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
-1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
-1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
-1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
-1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
-1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
-0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
-0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
-0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
-0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
-0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
-0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
-0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
-0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
-0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
-0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641

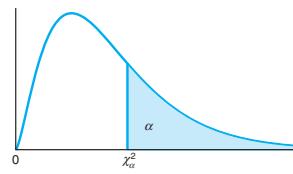
Table A.3 (continued) Areas under the Normal Curve

Table A.4 Critical Values of the *t*-Distribution

<i>v</i>	α						
	0.40	0.30	0.20	0.15	0.10	0.05	0.025
1	0.325	0.727	1.376	1.963	3.078	6.314	12.706
2	0.289	0.617	1.061	1.386	1.886	2.920	4.303
3	0.277	0.584	0.978	1.250	1.638	2.353	3.182
4	0.271	0.569	0.941	1.190	1.533	2.132	2.776
5	0.267	0.559	0.920	1.156	1.476	2.015	2.571
6	0.265	0.553	0.906	1.134	1.440	1.943	2.447
7	0.263	0.549	0.896	1.119	1.415	1.895	2.365
8	0.262	0.546	0.889	1.108	1.397	1.860	2.306
9	0.261	0.543	0.883	1.100	1.383	1.833	2.262
10	0.260	0.542	0.879	1.093	1.372	1.812	2.228
11	0.260	0.540	0.876	1.088	1.363	1.796	2.201
12	0.259	0.539	0.873	1.083	1.356	1.782	2.179
13	0.259	0.538	0.870	1.079	1.350	1.771	2.160
14	0.258	0.537	0.868	1.076	1.345	1.761	2.145
15	0.258	0.536	0.866	1.074	1.341	1.753	2.131
16	0.258	0.535	0.865	1.071	1.337	1.746	2.120
17	0.257	0.534	0.863	1.069	1.333	1.740	2.110
18	0.257	0.534	0.862	1.067	1.330	1.734	2.101
19	0.257	0.533	0.861	1.066	1.328	1.729	2.093
20	0.257	0.533	0.860	1.064	1.325	1.725	2.086
21	0.257	0.532	0.859	1.063	1.323	1.721	2.080
22	0.256	0.532	0.858	1.061	1.321	1.717	2.074
23	0.256	0.532	0.858	1.060	1.319	1.714	2.069
24	0.256	0.531	0.857	1.059	1.318	1.711	2.064
25	0.256	0.531	0.856	1.058	1.316	1.708	2.060
26	0.256	0.531	0.856	1.058	1.315	1.706	2.056
27	0.256	0.531	0.855	1.057	1.314	1.703	2.052
28	0.256	0.530	0.855	1.056	1.313	1.701	2.048
29	0.256	0.530	0.854	1.055	1.311	1.699	2.045
30	0.256	0.530	0.854	1.055	1.310	1.697	2.042
40	0.255	0.529	0.851	1.050	1.303	1.684	2.021
60	0.254	0.527	0.848	1.045	1.296	1.671	2.000
120	0.254	0.526	0.845	1.041	1.289	1.658	1.980
∞	0.253	0.524	0.842	1.036	1.282	1.645	1.960

Table A.4 (continued) Critical Values of the *t*-Distribution

<i>v</i>	α						
	0.02	0.015	0.01	0.0075	0.005	0.0025	0.0005
1	15.894	21.205	31.821	42.433	63.656	127.321	636.578
2	4.849	5.643	6.965	8.073	9.925	14.089	31.600
3	3.482	3.896	4.541	5.047	5.841	7.453	12.924
4	2.999	3.298	3.747	4.088	4.604	5.598	8.610
5	2.757	3.003	3.365	3.634	4.032	4.773	6.869
6	2.612	2.829	3.143	3.372	3.707	4.317	5.959
7	2.517	2.715	2.998	3.203	3.499	4.029	5.408
8	2.449	2.634	2.896	3.085	3.355	3.833	5.041
9	2.398	2.574	2.821	2.998	3.250	3.690	4.781
10	2.359	2.527	2.764	2.932	3.169	3.581	4.587
11	2.328	2.491	2.718	2.879	3.106	3.497	4.437
12	2.303	2.461	2.681	2.836	3.055	3.428	4.318
13	2.282	2.436	2.650	2.801	3.012	3.372	4.221
14	2.264	2.415	2.624	2.771	2.977	3.326	4.140
15	2.249	2.397	2.602	2.746	2.947	3.286	4.073
16	2.235	2.382	2.583	2.724	2.921	3.252	4.015
17	2.224	2.368	2.567	2.706	2.898	3.222	3.965
18	2.214	2.356	2.552	2.689	2.878	3.197	3.922
19	2.205	2.346	2.539	2.674	2.861	3.174	3.883
20	2.197	2.336	2.528	2.661	2.845	3.153	3.850
21	2.189	2.328	2.518	2.649	2.831	3.135	3.819
22	2.183	2.320	2.508	2.639	2.819	3.119	3.792
23	2.177	2.313	2.500	2.629	2.807	3.104	3.768
24	2.172	2.307	2.492	2.620	2.797	3.091	3.745
25	2.167	2.301	2.485	2.612	2.787	3.078	3.725
26	2.162	2.296	2.479	2.605	2.779	3.067	3.707
27	2.158	2.291	2.473	2.598	2.771	3.057	3.689
28	2.154	2.286	2.467	2.592	2.763	3.047	3.674
29	2.150	2.282	2.462	2.586	2.756	3.038	3.660
30	2.147	2.278	2.457	2.581	2.750	3.030	3.646
40	2.123	2.250	2.423	2.542	2.704	2.971	3.551
60	2.099	2.223	2.390	2.504	2.660	2.915	3.460
120	2.076	2.196	2.358	2.468	2.617	2.860	3.373
∞	2.054	2.170	2.326	2.432	2.576	2.807	3.290

**Table A.5** Critical Values of the Chi-Squared Distribution

v	α									
	0.995	0.99	0.98	0.975	0.95	0.90	0.80	0.75	0.70	0.50
1	0.0 ⁴ 393	0.0 ³ 157	0.0 ³ 628	0.0 ³ 982	0.00393	0.0158	0.0642	0.102	0.148	0.455
2	0.0100	0.0201	0.0404	0.0506	0.103	0.211	0.446	0.575	0.713	1.386
3	0.0717	0.115	0.185	0.216	0.352	0.584	1.005	1.213	1.424	2.366
4	0.207	0.297	0.429	0.484	0.711	1.064	1.649	1.923	2.195	3.357
5	0.412	0.554	0.752	0.831	1.145	1.610	2.343	2.675	3.000	4.351
6	0.676	0.872	1.134	1.237	1.635	2.204	3.070	3.455	3.828	5.348
7	0.989	1.239	1.564	1.690	2.167	2.833	3.822	4.255	4.671	6.346
8	1.344	1.647	2.032	2.180	2.733	3.490	4.594	5.071	5.527	7.344
9	1.735	2.088	2.532	2.700	3.325	4.168	5.380	5.899	6.393	8.343
10	2.156	2.558	3.059	3.247	3.940	4.865	6.179	6.737	7.267	9.342
11	2.603	3.053	3.609	3.816	4.575	5.578	6.989	7.584	8.148	10.341
12	3.074	3.571	4.178	4.404	5.226	6.304	7.807	8.438	9.034	11.340
13	3.565	4.107	4.765	5.009	5.892	7.041	8.634	9.299	9.926	12.340
14	4.075	4.660	5.368	5.629	6.571	7.790	9.467	10.165	10.821	13.339
15	4.601	5.229	5.985	6.262	7.261	8.547	10.307	11.037	11.721	14.339
16	5.142	5.812	6.614	6.908	7.962	9.312	11.152	11.912	12.624	15.338
17	5.697	6.408	7.255	7.564	8.672	10.085	12.002	12.792	13.531	16.338
18	6.265	7.015	7.906	8.231	9.390	10.865	12.857	13.675	14.440	17.338
19	6.844	7.633	8.567	8.907	10.117	11.651	13.716	14.562	15.352	18.338
20	7.434	8.260	9.237	9.591	10.851	12.443	14.578	15.452	16.266	19.337
21	8.034	8.897	9.915	10.283	11.591	13.240	15.445	16.344	17.182	20.337
22	8.643	9.542	10.600	10.982	12.338	14.041	16.314	17.240	18.101	21.337
23	9.260	10.196	11.293	11.689	13.091	14.848	17.187	18.137	19.021	22.337
24	9.886	10.856	11.992	12.401	13.848	15.659	18.062	19.037	19.943	23.337
25	10.520	11.524	12.697	13.120	14.611	16.473	18.940	19.939	20.867	24.337
26	11.160	12.198	13.409	13.844	15.379	17.292	19.820	20.843	21.792	25.336
27	11.808	12.878	14.125	14.573	16.151	18.114	20.703	21.749	22.719	26.336
28	12.461	13.565	14.847	15.308	16.928	18.939	21.588	22.657	23.647	27.336
29	13.121	14.256	15.574	16.047	17.708	19.768	22.475	23.567	24.577	28.336
30	13.787	14.953	16.306	16.791	18.493	20.599	23.364	24.478	25.508	29.336
40	20.707	22.164	23.838	24.433	26.509	29.051	32.345	33.66	34.872	39.335
50	27.991	29.707	31.664	32.357	34.764	37.689	41.449	42.942	44.313	49.335
60	35.534	37.485	39.699	40.482	43.188	46.459	50.641	52.294	53.809	59.335

Table A.5 (continued) Critical Values of the Chi-Squared Distribution

v	α									
	0.30	0.25	0.20	0.10	0.05	0.025	0.02	0.01	0.005	0.001
1	1.074	1.323	1.642	2.706	3.841	5.024	5.412	6.635	7.879	10.827
2	2.408	2.773	3.219	4.605	5.991	7.378	7.824	9.210	10.597	13.815
3	3.665	4.108	4.642	6.251	7.815	9.348	9.837	11.345	12.838	16.266
4	4.878	5.385	5.989	7.779	9.488	11.143	11.668	13.277	14.860	18.466
5	6.064	6.626	7.289	9.236	11.070	12.832	13.388	15.086	16.750	20.515
6	7.231	7.841	8.558	10.645	12.592	14.449	15.033	16.812	18.548	22.457
7	8.383	9.037	9.803	12.017	14.067	16.013	16.622	18.475	20.278	24.321
8	9.524	10.219	11.030	13.362	15.507	17.535	18.168	20.090	21.955	26.124
9	10.656	11.389	12.242	14.684	16.919	19.023	19.679	21.666	23.589	27.877
10	11.781	12.549	13.442	15.987	18.307	20.483	21.161	23.209	25.188	29.588
11	12.899	13.701	14.631	17.275	19.675	21.920	22.618	24.725	26.757	31.264
12	14.011	14.845	15.812	18.549	21.026	23.337	24.054	26.217	28.300	32.909
13	15.119	15.984	16.985	19.812	22.362	24.736	25.471	27.688	29.819	34.527
14	16.222	17.117	18.151	21.064	23.685	26.119	26.873	29.141	31.319	36.124
15	17.322	18.245	19.311	22.307	24.996	27.488	28.259	30.578	32.801	37.698
16	18.418	19.369	20.465	23.542	26.296	28.845	29.633	32.000	34.267	39.252
17	19.511	20.489	21.615	24.769	27.587	30.191	30.995	33.409	35.718	40.791
18	20.601	21.605	22.760	25.989	28.869	31.526	32.346	34.805	37.156	42.312
19	21.689	22.718	23.900	27.204	30.144	32.852	33.687	36.191	38.582	43.819
20	22.775	23.828	25.038	28.412	31.410	34.170	35.020	37.566	39.997	45.314
21	23.858	24.935	26.171	29.615	32.671	35.479	36.343	38.932	41.401	46.796
22	24.939	26.039	27.301	30.813	33.924	36.781	37.659	40.289	42.796	48.268
23	26.018	27.141	28.429	32.007	35.172	38.076	38.968	41.638	44.181	49.728
24	27.096	28.241	29.553	33.196	36.415	39.364	40.270	42.980	45.558	51.179
25	28.172	29.339	30.675	34.382	37.652	40.646	41.566	44.314	46.928	52.619
26	29.246	30.435	31.795	35.563	38.885	41.923	42.856	45.642	48.290	54.051
27	30.319	31.528	32.912	36.741	40.113	43.195	44.140	46.963	49.645	55.475
28	31.391	32.620	34.027	37.916	41.337	44.461	45.419	48.278	50.994	56.892
29	32.461	33.711	35.139	39.087	42.557	45.722	46.693	49.588	52.335	58.301
30	33.530	34.800	36.250	40.256	43.773	46.979	47.962	50.892	53.672	59.702
40	44.165	45.616	47.269	51.805	55.758	59.342	60.436	63.691	66.766	73.403
50	54.723	56.334	58.164	63.167	67.505	71.420	72.613	76.154	79.490	86.660
60	65.226	66.981	68.972	74.397	79.082	83.298	84.58	88.379	91.952	99.608

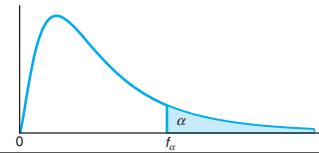


Table A.6 Critical Values of the F-Distribution

v_2	$f_{0.05}(v_1, v_2)$								
	v_1								
1	2	3	4	5	6	7	8	9	
1	161.45	199.50	215.71	224.58	230.16	233.99	236.77	238.88	240.54
2	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30
25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28
26	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27
27	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.25
28	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24
29	4.18	3.33	2.93	2.70	2.55	2.43	2.35	2.28	2.22
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12
60	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04
120	3.92	3.07	2.68	2.45	2.29	2.18	2.09	2.02	1.96
∞	3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88

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Table A.6 (continued) Critical Values of the *F*-Distribution

v_2	$f_{0.05}(v_1, v_2)$									
	v_1									
	10	12	15	20	24	30	40	60	120	∞
1	241.88	243.91	245.95	248.01	249.05	250.10	251.14	252.20	253.25	254.31
2	19.40	19.41	19.43	19.45	19.45	19.46	19.47	19.48	19.49	19.50
3	8.79	8.74	8.70	8.66	8.64	8.62	8.59	8.57	8.55	8.53
4	5.96	5.91	5.86	5.80	5.77	5.75	5.72	5.69	5.66	5.63
5	4.74	4.68	4.62	4.56	4.53	4.50	4.46	4.43	4.40	4.36
6	4.06	4.00	3.94	3.87	3.84	3.81	3.77	3.74	3.70	3.67
7	3.64	3.57	3.51	3.44	3.41	3.38	3.34	3.30	3.27	3.23
8	3.35	3.28	3.22	3.15	3.12	3.08	3.04	3.01	2.97	2.93
9	3.14	3.07	3.01	2.94	2.90	2.86	2.83	2.79	2.75	2.71
10	2.98	2.91	2.85	2.77	2.74	2.70	2.66	2.62	2.58	2.54
11	2.85	2.79	2.72	2.65	2.61	2.57	2.53	2.49	2.45	2.40
12	2.75	2.69	2.62	2.54	2.51	2.47	2.43	2.38	2.34	2.30
13	2.67	2.60	2.53	2.46	2.42	2.38	2.34	2.30	2.25	2.21
14	2.60	2.53	2.46	2.39	2.35	2.31	2.27	2.22	2.18	2.13
15	2.54	2.48	2.40	2.33	2.29	2.25	2.20	2.16	2.11	2.07
16	2.49	2.42	2.35	2.28	2.24	2.19	2.15	2.11	2.06	2.01
17	2.45	2.38	2.31	2.23	2.19	2.15	2.10	2.06	2.01	1.96
18	2.41	2.34	2.27	2.19	2.15	2.11	2.06	2.02	1.97	1.92
19	2.38	2.31	2.23	2.16	2.11	2.07	2.03	1.98	1.93	1.88
20	2.35	2.28	2.20	2.12	2.08	2.04	1.99	1.95	1.90	1.84
21	2.32	2.25	2.18	2.10	2.05	2.01	1.96	1.92	1.87	1.81
22	2.30	2.23	2.15	2.07	2.03	1.98	1.94	1.89	1.84	1.78
23	2.27	2.20	2.13	2.05	2.01	1.96	1.91	1.86	1.81	1.76
24	2.25	2.18	2.11	2.03	1.98	1.94	1.89	1.84	1.79	1.73
25	2.24	2.16	2.09	2.01	1.96	1.92	1.87	1.82	1.77	1.71
26	2.22	2.15	2.07	1.99	1.95	1.90	1.85	1.80	1.75	1.69
27	2.20	2.13	2.06	1.97	1.93	1.88	1.84	1.79	1.73	1.67
28	2.19	2.12	2.04	1.96	1.91	1.87	1.82	1.77	1.71	1.65
29	2.18	2.10	2.03	1.94	1.90	1.85	1.81	1.75	1.70	1.64
30	2.16	2.09	2.01	1.93	1.89	1.84	1.79	1.74	1.68	1.62
40	2.08	2.00	1.92	1.84	1.79	1.74	1.69	1.64	1.58	1.51
60	1.99	1.92	1.84	1.75	1.70	1.65	1.59	1.53	1.47	1.39
120	1.91	1.83	1.75	1.66	1.61	1.55	1.50	1.43	1.35	1.25
∞	1.83	1.75	1.67	1.57	1.52	1.46	1.39	1.32	1.22	1.00

Table A.6 (continued) Critical Values of the F-Distribution

v_2	$f_{0.01}(v_1, v_2)$								
	v_1								
1	2	3	4	5	6	7	8	9	
1	4052.18	4999.50	5403.35	5624.58	5763.65	5858.99	5928.36	5981.07	6022.47
2	98.50	99.00	99.17	99.25	99.30	99.33	99.36	99.37	99.39
3	34.12	30.82	29.46	28.71	28.24	27.91	27.67	27.49	27.35
4	21.20	18.00	16.69	15.98	15.52	15.21	14.98	14.80	14.66
5	16.26	13.27	12.06	11.39	10.97	10.67	10.46	10.29	10.16
6	13.75	10.92	9.78	9.15	8.75	8.47	8.26	8.10	7.98
7	12.25	9.55	8.45	7.85	7.46	7.19	6.99	6.84	6.72
8	11.26	8.65	7.59	7.01	6.63	6.37	6.18	6.03	5.91
9	10.56	8.02	6.99	6.42	6.06	5.80	5.61	5.47	5.35
10	10.04	7.56	6.55	5.99	5.64	5.39	5.20	5.06	4.94
11	9.65	7.21	6.22	5.67	5.32	5.07	4.89	4.74	4.63
12	9.33	6.93	5.95	5.41	5.06	4.82	4.64	4.50	4.39
13	9.07	6.70	5.74	5.21	4.86	4.62	4.44	4.30	4.19
14	8.86	6.51	5.56	5.04	4.69	4.46	4.28	4.14	4.03
15	8.68	6.36	5.42	4.89	4.56	4.32	4.14	4.00	3.89
16	8.53	6.23	5.29	4.77	4.44	4.20	4.03	3.89	3.78
17	8.40	6.11	5.18	4.67	4.34	4.10	3.93	3.79	3.68
18	8.29	6.01	5.09	4.58	4.25	4.01	3.84	3.71	3.60
19	8.18	5.93	5.01	4.50	4.17	3.94	3.77	3.63	3.52
20	8.10	5.85	4.94	4.43	4.10	3.87	3.70	3.56	3.46
21	8.02	5.78	4.87	4.37	4.04	3.81	3.64	3.51	3.40
22	7.95	5.72	4.82	4.31	3.99	3.76	3.59	3.45	3.35
23	7.88	5.66	4.76	4.26	3.94	3.71	3.54	3.41	3.30
24	7.82	5.61	4.72	4.22	3.90	3.67	3.50	3.36	3.26
25	7.77	5.57	4.68	4.18	3.85	3.63	3.46	3.32	3.22
26	7.72	5.53	4.64	4.14	3.82	3.59	3.42	3.29	3.18
27	7.68	5.49	4.60	4.11	3.78	3.56	3.39	3.26	3.15
28	7.64	5.45	4.57	4.07	3.75	3.53	3.36	3.23	3.12
29	7.60	5.42	4.54	4.04	3.73	3.50	3.33	3.20	3.09
30	7.56	5.39	4.51	4.02	3.70	3.47	3.30	3.17	3.07
40	7.31	5.18	4.31	3.83	3.51	3.29	3.12	2.99	2.89
60	7.08	4.98	4.13	3.65	3.34	3.12	2.95	2.82	2.72
120	6.85	4.79	3.95	3.48	3.17	2.96	2.79	2.66	2.56
∞	6.63	4.61	3.78	3.32	3.02	2.80	2.64	2.51	2.41

Table A.6 (continued) Critical Values of the *F*-Distribution

v_2	$f_{0.01}(v_1, v_2)$									
	v_1									
10	12	15	20	24	30	40	60	120	∞	
1	6055.85	6106.32	6157.28	6208.73	6234.63	6260.65	6286.78	6313.03	6339.39	6365.86
2	99.40	99.42	99.43	99.45	99.46	99.47	99.47	99.48	99.49	99.50
3	27.23	27.05	26.87	26.69	26.60	26.50	26.41	26.32	26.22	26.13
4	14.55	14.37	14.20	14.02	13.93	13.84	13.75	13.65	13.56	13.46
5	10.05	9.89	9.72	9.55	9.47	9.38	9.29	9.20	9.11	9.02
6	7.87	7.72	7.56	7.40	7.31	7.23	7.14	7.06	6.97	6.88
7	6.62	6.47	6.31	6.16	6.07	5.99	5.91	5.82	5.74	5.65
8	5.81	5.67	5.52	5.36	5.28	5.20	5.12	5.03	4.95	4.86
9	5.26	5.11	4.96	4.81	4.73	4.65	4.57	4.48	4.40	4.31
10	4.85	4.71	4.56	4.41	4.33	4.25	4.17	4.08	4.00	3.91
11	4.54	4.40	4.25	4.10	4.02	3.94	3.86	3.78	3.69	3.60
12	4.30	4.16	4.01	3.86	3.78	3.70	3.62	3.54	3.45	3.36
13	4.10	3.96	3.82	3.66	3.59	3.51	3.43	3.34	3.25	3.17
14	3.94	3.80	3.66	3.51	3.43	3.35	3.27	3.18	3.09	3.00
15	3.80	3.67	3.52	3.37	3.29	3.21	3.13	3.05	2.96	2.87
16	3.69	3.55	3.41	3.26	3.18	3.10	3.02	2.93	2.84	2.75
17	3.59	3.46	3.31	3.16	3.08	3.00	2.92	2.83	2.75	2.65
18	3.51	3.37	3.23	3.08	3.00	2.92	2.84	2.75	2.66	2.57
19	3.43	3.30	3.15	3.00	2.92	2.84	2.76	2.67	2.58	2.49
20	3.37	3.23	3.09	2.94	2.86	2.78	2.69	2.61	2.52	2.42
21	3.31	3.17	3.03	2.88	2.80	2.72	2.64	2.55	2.46	2.36
22	3.26	3.12	2.98	2.83	2.75	2.67	2.58	2.50	2.40	2.31
23	3.21	3.07	2.93	2.78	2.70	2.62	2.54	2.45	2.35	2.26
24	3.17	3.03	2.89	2.74	2.66	2.58	2.49	2.40	2.31	2.21
25	3.13	2.99	2.85	2.70	2.62	2.54	2.45	2.36	2.27	2.17
26	3.09	2.96	2.81	2.66	2.58	2.50	2.42	2.33	2.23	2.13
27	3.06	2.93	2.78	2.63	2.55	2.47	2.38	2.29	2.20	2.10
28	3.03	2.90	2.75	2.60	2.52	2.44	2.35	2.26	2.17	2.06
29	3.00	2.87	2.73	2.57	2.49	2.41	2.33	2.23	2.14	2.03
30	2.98	2.84	2.70	2.55	2.47	2.39	2.30	2.21	2.11	2.01
40	2.80	2.66	2.52	2.37	2.29	2.20	2.11	2.02	1.92	1.80
60	2.63	2.50	2.35	2.20	2.12	2.03	1.94	1.84	1.73	1.60
120	2.47	2.34	2.19	2.03	1.95	1.86	1.76	1.66	1.53	1.38
∞	2.32	2.18	2.04	1.88	1.79	1.70	1.59	1.47	1.32	1.00

Table A.7 Tolerance Factors for Normal Distributions

n	Two-Sided Intervals						One-Sided Intervals							
	$\gamma = 0.05$			$\gamma = 0.01$			$\gamma = 0.05$			$\gamma = 0.01$				
	$1 - \alpha$		$\gamma = 0.90$	$\gamma = 0.95$	$\gamma = 0.99$	$1 - \alpha$		$\gamma = 0.90$	$\gamma = 0.95$	$\gamma = 0.99$	$1 - \alpha$		$\gamma = 0.90$	$\gamma = 0.95$
2	32.019	37.674	48.430	160.193	188.491	242.300	20.581	26.260	37.094	103.029	131.426	185.617		
3	8.380	9.916	12.861	18.930	22.401	29.055	6.156	7.656	10.553	13.995	17.170	23.896		
4	5.369	6.370	8.299	9.398	11.150	14.527	4.162	5.144	7.042	7.380	9.083	12.387		
5	4.275	5.079	6.634	6.612	7.855	10.260	3.407	4.203	5.741	5.362	6.578	8.939		
6	3.712	4.414	5.775	5.337	6.345	8.301	3.006	3.708	5.062	4.411	5.406	7.335		
7	3.369	4.007	5.248	4.613	5.488	7.187	2.756	3.400	4.642	3.859	4.728	6.412		
8	3.136	3.732	4.891	4.147	4.936	6.468	2.582	3.187	4.354	3.497	4.285	5.812		
9	2.967	3.532	4.631	3.822	4.550	5.966	2.454	3.031	4.143	3.241	3.972	5.389		
10	2.839	3.379	4.433	3.582	4.265	5.594	2.355	2.911	3.981	3.048	3.738	5.074		
11	2.737	3.259	4.277	3.397	4.045	5.308	2.275	2.815	3.852	2.898	3.556	4.829		
12	2.655	3.162	4.150	3.250	3.870	5.079	2.210	2.736	3.747	2.777	3.410	4.633		
13	2.587	3.081	4.044	3.130	3.727	4.893	2.155	2.671	3.659	2.677	3.290	4.472		
14	2.529	3.012	3.955	3.029	3.608	4.737	2.109	2.615	3.585	2.593	1.189	4.337		
15	2.480	2.954	3.878	2.945	3.507	4.605	2.068	2.566	3.520	2.522	3.102	4.222		
16	2.437	2.903	3.812	2.872	3.421	4.492	2.033	2.524	3.464	2.460	3.028	4.123		
17	2.400	2.858	3.754	2.808	3.345	4.393	2.002	2.486	3.414	2.405	2.963	4.037		
18	2.366	2.819	3.702	2.753	3.279	4.307	1.974	2.453	3.370	2.357	2.905	3.960		
19	2.337	2.784	3.656	2.703	3.221	4.230	1.949	2.423	3.331	2.314	2.854	3.892		
20	2.310	2.752	3.615	2.659	3.168	4.161	1.926	2.396	3.295	2.276	2.808	1.832		
25	2.208	2.631	3.457	2.494	2.972	3.904	1.838	2.292	3.158	2.129	2.633	3.001		
30	2.140	2.549	3.350	2.385	2.841	3.733	1.777	2.220	3.064	2.030	2.516	3.447		
35	2.090	2.490	3.272	2.306	2.748	3.611	1.732	2.167	2.995	1.957	2.430	3.334		
40	2.052	2.445	3.213	2.247	2.677	3.518	1.697	2.126	2.941	1.902	2.364	3.249		
45	2.021	2.408	3.165	2.200	2.621	3.444	1.669	2.092	2.898	1.857	2.312	3.180		
50	1.996	2.379	3.126	2.162	2.576	3.385	1.646	2.065	2.863	1.821	2.269	3.125		
60	1.958	2.333	3.066	2.103	2.506	3.293	1.609	2.022	2.807	1.764	2.202	3.038		
70	1.929	2.299	3.021	2.060	2.454	3.225	1.581	1.990	2.765	1.722	2.153	2.974		
80	1.907	2.272	2.986	2.026	2.414	3.173	1.559	1.965	2.733	1.688	2.114	2.924		
90	1.889	2.251	2.958	1.999	2.382	3.130	1.542	1.944	2.706	1.661	2.082	2.883		
100	1.874	2.233	2.934	1.977	2.355	3.096	1.527	1.927	2.684	1.639	2.056	2.850		
150	1.825	2.175	2.859	1.905	2.270	2.983	1.478	1.870	2.611	1.566	1.971	2.741		
200	1.798	2.143	2.816	1.865	2.222	2.921	1.450	1.837	2.570	1.524	1.923	2.679		
250	1.780	2.121	2.788	1.839	2.191	2.880	1.431	1.815	2.542	1.496	1.891	2.638		
300	1.767	2.106	2.767	1.820	2.169	2.850	1.417	1.800	2.522	1.476	1.868	2.608		
∞	1.645	1.960	2.576	1.645	1.960	2.576	1.282	1.645	2.326	1.282	1.645	2.326		

Adapted from C. Eisenhart, M. W. Hastay, and W. A. Wallis, *Techniques of Statistical Analysis*, Chapter 2, McGraw-Hill Book Company, New York, 1947. Used with permission of McGraw-Hill Book Company.

Table A.8 Sample Size for the *t*-Test of the Mean

Single-Sided Test	Double-Sided Test	Level of <i>t</i> -Test																											
		$\alpha = 0.005$				$\alpha = 0.01$				$\alpha = 0.025$				$\alpha = 0.05$															
		$\beta = 0.1$		$\alpha = 0.01$	$\alpha = 0.05$	$\alpha = 0.01$		$\alpha = 0.05$	$\beta = 0.2$	$\beta = 0.5$	$\beta = 0.05$	$\beta = 0.1$	$\beta = 0.2$	$\beta = 0.5$	$\beta = 0.05$	$\beta = 0.1$	$\beta = 0.2$	$\beta = 0.5$											
		0.05																											
		0.10																											
		0.15																122											
		0.20																70											
		0.25				110				90				128	64		139	101	45										
		0.30				134	78			115	63			119	90	45	122	97	71	32									
		0.35				125	99	58		109	85	47		109	88	67	34	90	72	52	24								
		0.40				115	97	77	45	101	85	66	37	117	84	68	51	26	101	70	55	40	19						
		0.45				92	77	62	37	110	81	68	53	30	93	67	54	41	21	80	55	44	33	15					
		0.50				100	75	63	51	30	90	66	55	43	25	76	54	44	34	18	65	45	36	27	13				
		0.55				83	63	53	42	26	75	55	46	36	21	63	45	37	28	15	54	38	30	22	11				
		0.60				71	53	45	36	22	63	47	39	31	18	53	38	32	24	13	46	32	26	19	9				
		0.65				61	46	39	31	20	55	41	34	27	16	46	33	27	21	12	39	28	22	17	8				
		0.70				53	40	34	28	17	47	35	30	24	14	40	29	24	19	10	34	24	19	15	8				
		0.75				47	36	30	25	16	42	31	27	21	13	35	26	21	16	9	30	21	17	13	7				
		0.80				41	32	27	22	14	37	28	24	19	12	31	22	19	15	9	27	19	15	12	6				
		0.85				37	29	24	20	13	33	25	21	17	11	28	21	17	13	8	24	17	14	11	6				
		0.90				34	26	22	18	12	29	23	19	16	10	25	19	16	12	7	21	15	13	10	5				
Value of Δ/σ	0.95					31	24	20	17	11	27	21	18	14	9	23	17	14	11	7	19	14	11	9	5				
	1.00					28	22	19	16	10	25	19	16	13	9	21	16	13	10	6	18	13	11	8	5				
	1.1					24	19	16	14	9	21	16	14	12	8	18	13	11	9	6		15	11	9	7				
	1.2					21	16	14	12	8	18	14	12	10	7	15	12	10	8	5		13	10	8	6				
	1.3					18	15	13	11	8	16	13	11	9	6		14	10	9	7		11	8	7	6				
	1.4					16	13	12	10	7	14	11	10	9	6	12	9	8	7	10	8	7	5						
	1.5					15	12	11	9	7	13	10	9	8	6	11	8	7	6		9	7	6						
	1.6					13	11	10	8	6	12	10	9	7	5		10	8	7	6		8	6	6					
	1.7					12	10	9	8	6		11	9	8	7		9	7	6	5		8	6	5					
	1.8					12	10	9	8	6		10	8	7	7		8	7	6			7	6						
	1.9					11	9	8	7	6		10	8	7	6		8	6	6			7	5						
	2.0					10	8	8	7	5		9	7	7	6		7	6	5			6							
	2.1					10	8	7	7			8	7	6	6			7	6				6						
	2.2					9	8	7	6			8	7	6	5			7	6				6						
	2.3					9	7	7	6			8	6	6				6	5				5						
	2.4					8	7	7	6			7	6	6				6					6						
	2.5					8	7	6	6			7	6	6				6					6						
	3.0					7	6	6	5			6	5	5					5										
	3.5					6	5	5						5															
	4.0					6																							

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Table A.9 Sample Size for the *t*-Test of the Difference between Two Means

		Level of <i>t</i> -Test																			
Single-Sided Test		$\alpha = 0.005$			$\alpha = 0.01$			$\alpha = 0.025$			$\alpha = 0.05$										
Double-Sided Test		$\alpha = 0.01$			$\alpha = 0.02$			$\alpha = 0.05$			$\alpha = 0.1$										
$\beta = 0.1$.01	.05	.1	.2	.5	.01	.05	.1	.2	.5	.01	.05	.1	.2	.5					
0.05																					
0.10																					
0.15																					
0.20															137						
0.25												124		88							
0.30								123				87		61							
0.35					110			90				64		102	45						
0.40					85			70			100	50		108	78	35					
0.45				118	68		101	55		105	79	39	108	86	62	28					
0.50			96	55		106	82	45	106	86	64	32	88	70	51	23					
0.55		101	79	46		106	88	68	38	87	71	53	27	112	73	58	42	19			
0.60		101	85	67	39		90	74	58	32	104	74	60	45	23	89	61	49	36	16	
0.65		87	73	57	34	104	77	64	49	27	88	63	51	39	20	76	52	42	30	14	
0.70		100	75	63	50	29	90	66	55	43	24	76	55	44	34	17	66	45	36	26	12
0.75		88	66	55	44	26	79	58	48	38	21	67	48	39	29	15	57	40	32	23	11
0.80		77	58	49	39	23	70	51	43	33	19	59	42	34	26	14	50	35	28	21	10
0.85		69	51	43	35	21	62	46	38	30	17	52	37	31	23	12	45	31	25	18	9
0.90		62	46	39	31	19	55	41	34	27	15	47	34	27	21	11	40	28	22	16	8
Value of 0.95		55	42	35	28	17	50	37	31	24	14	42	30	25	19	10	36	25	20	15	7
$\Delta = \delta /\sigma$ 1.00		50	38	32	26	15	45	33	28	22	13	38	27	23	17	9	33	23	18	14	7
1.1		42	32	27	22	13	38	28	23	19	11	32	23	19	14	8	27	19	15	12	6
1.2		36	27	23	18	11	32	24	20	16	9	27	20	16	12	7	23	16	13	10	5
1.3		31	23	20	16	10	28	21	17	14	8	23	17	14	11	6	20	14	11	9	5
1.4		27	20	17	14	9	24	18	15	12	8	20	15	12	10	6	17	12	10	8	4
1.5		24	18	15	13	8	21	16	14	11	7	18	13	11	9	5	15	11	9	7	4
1.6		21	16	14	11	7	19	14	12	10	6	16	12	10	8	5	14	10	8	6	4
1.7		19	15	13	10	7	17	13	11	9	6	14	11	9	7	4	12	9	7	6	3
1.8		17	13	71	10	6	15	12	10	8	5	13	10	8	6	4	11	8	7	5	
1.9		16	12	11	9	6	14	11	9	8	5	12	9	7	6	4	10	7	6	5	
2.0		14	11	10	8	6	13	10	9	7	5	11	8	7	6	4	9	7	6	4	
2.1		13	10	9	8	5	12	9	8	7	5	10	8	6	5	3	8	6	5	4	
2.2		12	10	8	7	5	11	9	7	6	4	9	7	6	5		8	6	5	4	
2.3		11	9	8	7	5	10	8	7	6	4	9	7	6	5		7	5	5	4	
2.4		11	9	8	6	5	10	8	7	6	48	6	5	4		7	5	4	4		
2.5		10	8	7	6	4	9	7	6	5	4	8	6	5	4		6	5	4	3	
3.0		8	6	6	5	4	7	6	5	4	3	6	5	4	4		5	4	3		
3.5		6	5	5	4	3	6	5	4	4	5	4	4	3		4	3				
4.0		6	5	4	4		5	4	4	3	4	4	4		4						

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Table A.10 Critical Values for Bartlett's Test

n	$b_k(0.01; n)$									
	Number of Populations, k									
2	3	4	5	6	7	8	9	10		
3	0.1411	0.1672								
4	0.2843	0.3165	0.3475	0.3729	0.3937	0.4110				
5	0.3984	0.4304	0.4607	0.4850	0.5046	0.5207	0.5343	0.5458	0.5558	
6	0.4850	0.5149	0.5430	0.5653	0.5832	0.5978	0.6100	0.6204	0.6293	
7	0.5512	0.5787	0.6045	0.6248	0.6410	0.6542	0.6652	0.6744	0.6824	
8	0.6031	0.6282	0.6518	0.6704	0.6851	0.6970	0.7069	0.7153	0.7225	
9	0.6445	0.6676	0.6892	0.7062	0.7197	0.7305	0.7395	0.7471	0.7536	
10	0.6783	0.6996	0.7195	0.7352	0.7475	0.7575	0.7657	0.7726	0.7786	
11	0.7063	0.7260	0.7445	0.7590	0.7703	0.7795	0.7871	0.7935	0.7990	
12	0.7299	0.7483	0.7654	0.7789	0.7894	0.7980	0.8050	0.8109	0.8160	
13	0.7501	0.7672	0.7832	0.7958	0.8056	0.8135	0.8201	0.8256	0.8303	
14	0.7674	0.7835	0.7985	0.8103	0.8195	0.8269	0.8330	0.8382	0.8426	
15	0.7825	0.7977	0.8118	0.8229	0.8315	0.8385	0.8443	0.8491	0.8532	
16	0.7958	0.8101	0.8235	0.8339	0.8421	0.8486	0.8541	0.8586	0.8625	
17	0.8076	0.8211	0.8338	0.8436	0.8514	0.8576	0.8627	0.8670	0.8707	
18	0.8181	0.8309	0.8429	0.8523	0.8596	0.8655	0.8704	0.8745	0.8780	
19	0.8275	0.8397	0.8512	0.8601	0.8670	0.8727	0.8773	0.8811	0.8845	
20	0.8360	0.8476	0.8586	0.8671	0.8737	0.8791	0.8835	0.8871	0.8903	
21	0.8437	0.8548	0.8653	0.8734	0.8797	0.8848	0.8890	0.8926	0.8956	
22	0.8507	0.8614	0.8714	0.8791	0.8852	0.8901	0.8941	0.8975	0.9004	
23	0.8571	0.8673	0.8769	0.8844	0.8902	0.8949	0.8988	0.9020	0.9047	
24	0.8630	0.8728	0.8820	0.8892	0.8948	0.8993	0.9030	0.9061	0.9087	
25	0.8684	0.8779	0.8867	0.8936	0.8990	0.9034	0.9069	0.9099	0.9124	
26	0.8734	0.8825	0.8911	0.8977	0.9029	0.9071	0.9105	0.9134	0.9158	
27	0.8781	0.8869	0.8951	0.9015	0.9065	0.9105	0.9138	0.9166	0.9190	
28	0.8824	0.8909	0.8988	0.9050	0.9099	0.9138	0.9169	0.9196	0.9219	
29	0.8864	0.8946	0.9023	0.9083	0.9130	0.9167	0.9198	0.9224	0.9246	
30	0.8902	0.8981	0.9056	0.9114	0.9159	0.9195	0.9225	0.9250	0.9271	
40	0.9175	0.9235	0.9291	0.9335	0.9370	0.9397	0.9420	0.9439	0.9455	
50	0.9339	0.9387	0.9433	0.9468	0.9496	0.9518	0.9536	0.9551	0.9564	
60	0.9449	0.9489	0.9527	0.9557	0.9580	0.9599	0.9614	0.9626	0.9637	
80	0.9586	0.9617	0.9646	0.9668	0.9685	0.9699	0.9711	0.9720	0.9728	
100	0.9669	0.9693	0.9716	0.9734	0.9748	0.9759	0.9769	0.9776	0.9783	

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Table A.10 (continued) Critical Values for Bartlett's Test

n	$b_k(0.05; n)$									
	Number of Populations, k									
2	3	4	5	6	7	8	9	10		
3	0.3123	0.3058	0.3173	0.3299						
4	0.4780	0.4699	0.4803	0.4921	0.5028	0.5122	0.5204	0.5277	0.5341	
5	0.5845	0.5762	0.5850	0.5952	0.6045	0.6126	0.6197	0.6260	0.6315	
6	0.6563	0.6483	0.6559	0.6646	0.6727	0.6798	0.6860	0.6914	0.6961	
7	0.7075	0.7000	0.7065	0.7142	0.7213	0.7275	0.7329	0.7376	0.7418	
8	0.7456	0.7387	0.7444	0.7512	0.7574	0.7629	0.7677	0.7719	0.7757	
9	0.7751	0.7686	0.7737	0.7798	0.7854	0.7903	0.7946	0.7984	0.8017	
10	0.7984	0.7924	0.7970	0.8025	0.8076	0.8121	0.8160	0.8194	0.8224	
11	0.8175	0.8118	0.8160	0.8210	0.8257	0.8298	0.8333	0.8365	0.8392	
12	0.8332	0.8280	0.8317	0.8364	0.8407	0.8444	0.8477	0.8506	0.8531	
13	0.8465	0.8415	0.8450	0.8493	0.8533	0.8568	0.8598	0.8625	0.8648	
14	0.8578	0.8532	0.8564	0.8604	0.8641	0.8673	0.8701	0.8726	0.8748	
15	0.8676	0.8632	0.8662	0.8699	0.8734	0.8764	0.8790	0.8814	0.8834	
16	0.8761	0.8719	0.8747	0.8782	0.8815	0.8843	0.8868	0.8890	0.8909	
17	0.8836	0.8796	0.8823	0.8856	0.8886	0.8913	0.8936	0.8957	0.8975	
18	0.8902	0.8865	0.8890	0.8921	0.8949	0.8975	0.8997	0.9016	0.9033	
19	0.8961	0.8926	0.8949	0.8979	0.9006	0.9030	0.9051	0.9069	0.9086	
20	0.9015	0.8980	0.9003	0.9031	0.9057	0.9080	0.9100	0.9117	0.9132	
21	0.9063	0.9030	0.9051	0.9078	0.9103	0.9124	0.9143	0.9160	0.9175	
22	0.9106	0.9075	0.9095	0.9120	0.9144	0.9165	0.9183	0.9199	0.9213	
23	0.9146	0.9116	0.9135	0.9159	0.9182	0.9202	0.9219	0.9235	0.9248	
24	0.9182	0.9153	0.9172	0.9195	0.9217	0.9236	0.9253	0.9267	0.9280	
25	0.9216	0.9187	0.9205	0.9228	0.9249	0.9267	0.9283	0.9297	0.9309	
26	0.9246	0.9219	0.9236	0.9258	0.9278	0.9296	0.9311	0.9325	0.9336	
27	0.9275	0.9249	0.9265	0.9286	0.9305	0.9322	0.9337	0.9350	0.9361	
28	0.9301	0.9276	0.9292	0.9312	0.9330	0.9347	0.9361	0.9374	0.9385	
29	0.9326	0.9301	0.9316	0.9336	0.9354	0.9370	0.9383	0.9396	0.9406	
30	0.9348	0.9325	0.9340	0.9358	0.9376	0.9391	0.9404	0.9416	0.9426	
40	0.9513	0.9495	0.9506	0.9520	0.9533	0.9545	0.9555	0.9564	0.9572	
50	0.9612	0.9597	0.9606	0.9617	0.9628	0.9637	0.9645	0.9652	0.9658	
60	0.9677	0.9665	0.9672	0.9681	0.9690	0.9698	0.9705	0.9710	0.9716	
80	0.9758	0.9749	0.9754	0.9761	0.9768	0.9774	0.9779	0.9783	0.9787	
100	0.9807	0.9799	0.9804	0.9809	0.9815	0.9819	0.9823	0.9827	0.9830	

Table A.11 Critical Values for Cochran's Test

k	$\alpha = 0.01$													
	n													
2	3	4	5	6	7	8	9	10	11	17	37	145	∞	
2	0.9999	0.9950	0.9794	0.9586	0.9373	0.9172	0.8988	0.8823	0.8674	0.8539	0.7949	0.7067	0.6062	0.5000
3	0.9933	0.9423	0.8831	0.8335	0.7933	0.7606	0.7335	0.7107	0.6912	0.6743	0.6059	0.5153	0.4230	0.3333
4	0.9676	0.8643	0.7814	0.7212	0.6761	0.6410	0.6129	0.5897	0.5702	0.5536	0.4884	0.4057	0.3251	0.2500
5	0.9279	0.7885	0.6957	0.6329	0.5875	0.5531	0.5259	0.5037	0.4854	0.4697	0.4094	0.3351	0.2644	0.2000
6	0.8828	0.7218	0.6258	0.5635	0.5195	0.4866	0.4608	0.4401	0.4229	0.4084	0.3529	0.2858	0.2229	0.1667
7	0.8376	0.6644	0.5685	0.5080	0.4659	0.4347	0.4105	0.3911	0.3751	0.3616	0.3105	0.2494	0.1929	0.1429
8	0.7945	0.6152	0.5209	0.4627	0.4226	0.3932	0.3704	0.3522	0.3373	0.3248	0.2779	0.2214	0.1700	0.1250
9	0.7544	0.5727	0.4810	0.4251	0.3870	0.3592	0.3378	0.3207	0.3067	0.2950	0.2514	0.1992	0.1521	0.1111
10	0.7175	0.5358	0.4469	0.3934	0.3572	0.3308	0.3106	0.2945	0.2813	0.2704	0.2297	0.1811	0.1376	0.1000
12	0.6528	0.4751	0.3919	0.3428	0.3099	0.2861	0.2680	0.2535	0.2419	0.2320	0.1961	0.1535	0.1157	0.0833
15	0.5747	0.4069	0.3317	0.2882	0.2593	0.2386	0.2228	0.2104	0.2002	0.1918	0.1612	0.1251	0.0934	0.0667
20	0.4799	0.3297	0.2654	0.2288	0.2048	0.1877	0.1748	0.1646	0.1567	0.1501	0.1248	0.0960	0.0709	0.0500
24	0.4247	0.2871	0.2295	0.1970	0.1759	0.1608	0.1495	0.1406	0.1338	0.1283	0.1060	0.0810	0.0595	0.0417
30	0.3632	0.2412	0.1913	0.1635	0.1454	0.1327	0.1232	0.1157	0.1100	0.1054	0.0867	0.0658	0.0480	0.0333
40	0.2940	0.1915	0.1508	0.1281	0.1135	0.1033	0.0957	0.0898	0.0853	0.0816	0.0668	0.0503	0.0363	0.0250
60	0.2151	0.1371	0.1069	0.0902	0.0796	0.0722	0.0668	0.0625	0.0594	0.0567	0.0461	0.0344	0.0245	0.0167
120	0.1225	0.0759	0.0585	0.0489	0.0429	0.0387	0.0357	0.0334	0.0316	0.0302	0.0242	0.0178	0.0125	0.0083
∞	0	0	0	0	0	0	0	0	0	0	0	0	0	0

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Table A.11 (continued) Critical Values for Cochran's Test

Table A.12 Upper Percentage Points of the Studentized Range Distribution: Values of $q(0.05; k, v)$

Degrees of Freedom, v	Number of Treatments k								
	2	3	4	5	6	7	8	9	10
1	18.0	27.0	32.8	37.2	40.5	43.1	15.1	47.1	49.1
2	6.09	5.33	9.80	10.89	11.73	12.43	13.03	13.54	13.99
3	4.50	5.91	6.83	7.51	8.04	8.47	8.85	9.18	9.46
4	3.93	5.04	5.76	6.29	6.71	7.06	7.35	7.60	7.83
5	3.64	4.60	5.22	5.67	6.03	6.33	6.58	6.80	6.99
6	3.46	4.34	4.90	5.31	5.63	5.89	6.12	6.32	6.49
7	3.34	4.16	4.68	5.06	5.35	5.59	5.80	5.99	6.15
8	3.26	4.04	4.53	4.89	5.17	5.40	5.60	5.77	5.92
9	3.20	3.95	4.42	4.76	5.02	5.24	5.43	5.60	5.74
10	3.15	3.88	4.33	4.66	4.91	5.12	5.30	5.46	5.60
11	3.11	3.82	4.26	4.58	4.82	5.03	5.20	5.35	5.49
12	3.08	3.77	4.20	4.51	4.75	4.95	5.12	5.27	5.40
13	3.06	3.73	4.15	4.46	4.69	4.88	5.05	5.19	5.32
14	3.03	3.70	4.11	4.41	4.65	4.83	4.99	5.13	5.25
15	3.01	3.67	4.08	4.37	4.59	4.78	4.94	5.08	5.20
16	3.00	3.65	4.05	4.34	4.56	4.74	4.90	5.03	5.05
17	2.98	3.62	4.02	4.31	4.52	4.70	4.86	4.99	5.11
18	2.97	3.61	4.00	4.28	4.49	4.67	4.83	4.96	5.07
19	2.96	3.59	3.98	4.26	4.47	4.64	4.79	4.92	5.04
20	2.95	3.58	3.96	4.24	4.45	4.62	4.77	4.90	5.01
24	2.92	3.53	3.90	4.17	4.37	4.54	4.68	4.81	4.92
30	2.89	3.48	3.84	4.11	4.30	4.46	4.60	4.72	4.83
40	2.86	3.44	3.79	4.04	4.23	4.39	4.52	4.63	4.74
60	2.83	3.40	3.74	3.98	4.16	4.31	4.44	4.55	4.65
120	2.80	3.36	3.69	3.92	4.10	4.24	4.36	4.47	4.56
∞	2.77	3.32	3.63	3.86	4.03	4.17	4.29	4.39	4.47

Table A.13 Least Significant Studentized Ranges $r_p(0.05; p, v)$

v	$\alpha = 0.05$									
	2	3	4	5	6	7	8	9	10	
1	17.97	17.97	17.97	17.97	17.97	17.97	17.97	17.97	17.97	17.97
2	6.085	6.085	6.085	6.085	6.085	6.085	6.085	6.085	6.085	6.085
3	4.501	4.516	4.516	4.516	4.516	4.516	4.516	4.516	4.516	4.516
4	3.927	4.013	4.033	4.033	4.033	4.033	4.033	4.033	4.033	4.033
5	3.635	3.749	3.797	3.814	3.814	3.814	3.814	3.814	3.814	3.814
6	3.461	3.587	3.649	3.68	3.694	3.697	3.697	3.697	3.697	3.697
7	3.344	3.477	3.548	3.588	3.611	3.622	3.626	3.626	3.626	3.626
8	3.261	3.399	3.475	3.521	3.549	3.566	3.575	3.579	3.579	
9	3.199	3.339	3.420	3.470	3.502	3.523	3.536	3.544	3.547	
10	3.151	3.293	3.376	3.430	3.465	3.489	3.505	3.516	3.522	
11	3.113	3.256	3.342	3.397	3.435	3.462	3.48	3.493	3.501	
12	3.082	3.225	3.313	3.370	3.410	3.439	3.459	3.474	3.484	
13	3.055	3.200	3.289	3.348	3.389	3.419	3.442	3.458	3.470	
14	3.033	3.178	3.268	3.329	3.372	3.403	3.426	3.444	3.457	
15	3.014	3.160	3.25	3.312	3.356	3.389	3.413	3.432	3.446	
16	2.998	3.144	3.235	3.298	3.343	3.376	3.402	3.422	3.437	
17	2.984	3.130	3.222	3.285	3.331	3.366	3.392	3.412	3.429	
18	2.971	3.118	3.210	3.274	3.321	3.356	3.383	3.405	3.421	
19	2.960	3.107	3.199	3.264	3.311	3.347	3.375	3.397	3.415	
20	2.950	3.097	3.190	3.255	3.303	3.339	3.368	3.391	3.409	
24	2.919	3.066	3.160	3.226	3.276	3.315	3.345	3.370	3.390	
30	2.888	3.035	3.131	3.199	3.250	3.290	3.322	3.349	3.371	
40	2.858	3.006	3.102	3.171	3.224	3.266	3.300	3.328	3.352	
60	2.829	2.976	3.073	3.143	3.198	3.241	3.277	3.307	3.333	
120	2.800	2.947	3.045	3.116	3.172	3.217	3.254	3.287	3.314	
∞	2.772	2.918	3.017	3.089	3.146	3.193	3.232	3.265	3.294	

Abridged from H. L. Harter, "Critical Values for Duncan's New Multiple Range Test," *Biometrics*, **16**, No. 4, 1960, by permission of the author and the editor.

Table A.13 (continued) Least Significant Studentized Ranges $r_p(0.01; p, v)$

v	$\alpha = 0.01$									
	p									
2	3	4	5	6	7	8	9	10		
1	90.03	90.03	90.03	90.03	90.03	90.03	90.03	90.03	90.03	90.03
2	14.04	14.04	14.04	14.04	14.04	14.04	14.04	14.04	14.04	14.04
3	8.261	8.321	8.321	8.321	8.321	8.321	8.321	8.321	8.321	8.321
4	6.512	6.677	6.740	6.756	6.756	6.756	6.756	6.756	6.756	6.756
5	5.702	5.893	5.989	6.040	6.065	6.074	6.074	6.074	6.074	6.074
6	5.243	5.439	5.549	5.614	5.655	5.680	5.694	5.701	5.703	
7	4.949	5.145	5.260	5.334	5.383	5.416	5.439	5.454	5.464	
8	4.746	4.939	5.057	5.135	5.189	5.227	5.256	5.276	5.291	
9	4.596	4.787	4.906	4.986	5.043	5.086	5.118	5.142	5.160	
10	4.482	4.671	4.790	4.871	4.931	4.975	5.010	5.037	5.058	
11	4.392	4.579	4.697	4.780	4.841	4.887	4.924	4.952	4.975	
12	4.320	4.504	4.622	4.706	4.767	4.815	4.852	4.883	4.907	
13	4.260	4.442	4.560	4.644	4.706	4.755	4.793	4.824	4.850	
14	4.210	4.391	4.508	4.591	4.654	4.704	4.743	4.775	4.802	
15	4.168	4.347	4.463	4.547	4.610	4.660	4.700	4.733	4.760	
16	4.131	4.309	4.425	4.509	4.572	4.622	4.663	4.696	4.724	
17	4.099	4.275	4.391	4.475	4.539	4.589	4.630	4.664	4.693	
18	4.071	4.246	4.362	4.445	4.509	4.560	4.601	4.635	4.664	
19	4.046	4.220	4.335	4.419	4.483	4.534	4.575	4.610	4.639	
20	4.024	4.197	4.312	4.395	4.459	4.510	4.552	4.587	4.617	
24	3.956	4.126	4.239	4.322	4.386	4.437	4.480	4.516	4.546	
30	3.889	4.056	4.168	4.250	4.314	4.366	4.409	4.445	4.477	
40	3.825	3.988	4.098	4.180	4.244	4.296	4.339	4.376	4.408	
60	3.762	3.922	4.031	4.111	4.174	4.226	4.270	4.307	4.340	
120	3.702	3.858	3.965	4.044	4.107	4.158	4.202	4.239	4.272	
∞	3.643	3.796	3.900	3.978	4.040	4.091	4.135	4.172	4.205	

Table A.14 Values of $d_{\alpha/2}(k, v)$ for Two-Sided Comparisons between k Treatments and a Control

v	$\alpha = 0.05$								
	$k = \text{Number of Treatment Means (excluding control)}$								
1	2	3	4	5	6	7	8	9	
5	2.57	3.03	3.29	3.48	3.62	3.73	3.82	3.90	3.97
6	2.45	2.86	3.10	3.26	3.39	3.49	3.57	3.64	3.71
7	2.36	2.75	2.97	3.12	3.24	3.33	3.41	3.47	3.53
8	2.31	2.67	2.88	3.02	3.13	3.22	3.29	3.35	3.41
9	2.26	2.61	2.81	2.95	3.05	3.14	3.20	3.26	3.32
10	2.23	2.57	2.76	2.89	2.99	3.07	3.14	3.19	3.24
11	2.20	2.53	2.72	2.84	2.94	3.02	3.08	3.14	3.19
12	2.18	2.50	2.68	2.81	2.90	2.98	3.04	3.09	3.14
13	2.16	2.48	2.65	2.78	2.87	2.94	3.00	3.06	3.10
14	2.14	2.46	2.63	2.75	2.84	2.91	2.97	3.02	3.07
15	2.13	2.44	2.61	2.73	2.82	2.89	2.95	3.00	3.04
16	2.12	2.42	2.59	2.71	2.80	2.87	2.92	2.97	3.02
17	2.11	2.41	2.58	2.69	2.78	2.85	2.90	2.95	3.00
18	2.10	2.40	2.56	2.68	2.76	2.83	2.89	2.94	2.98
19	2.09	2.39	2.55	2.66	2.75	2.81	2.87	2.92	2.96
20	2.09	2.38	2.54	2.65	2.73	2.80	2.86	2.90	2.95
24	2.06	2.35	2.51	2.61	2.70	2.76	2.81	2.86	2.90
30	2.04	2.32	2.47	2.58	2.66	2.72	2.77	2.82	2.86
40	2.02	2.29	2.44	2.54	2.62	2.68	2.73	2.77	2.81
60	2.00	2.27	2.41	2.51	2.58	2.64	2.69	2.73	2.77
120	1.98	2.24	2.38	2.47	2.55	2.60	2.65	2.69	2.73
∞	1.96	2.21	2.35	2.44	2.51	2.57	2.61	2.65	2.69

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Table A.14 (continued) Values of $d_{\alpha/2}(k, v)$ for Two-Sided Comparisons between k Treatments and a Control

v	$\alpha = 0.01$								
	$k = \text{Number of Treatment Means (excluding control)}$								
1	2	3	4	5	6	7	8	9	
5	4.03	4.63	4.98	5.22	5.41	5.56	5.69	5.80	5.89
6	3.71	4.21	4.51	4.71	4.87	5.00	5.10	5.20	5.28
7	3.50	3.95	4.21	4.39	4.53	4.64	4.74	4.82	4.89
8	3.36	3.77	4.00	4.17	4.29	4.40	4.48	4.56	4.62
9	3.25	3.63	3.85	4.01	4.12	4.22	4.30	4.37	4.43
10	3.17	3.53	3.74	3.88	3.99	4.08	4.16	4.22	4.28
11	3.11	3.45	3.65	3.79	3.89	3.98	4.05	4.11	4.16
12	3.05	3.39	3.58	3.71	3.81	3.89	3.96	4.02	4.07
13	3.01	3.33	3.52	3.65	3.74	3.82	3.89	3.94	3.99
14	2.98	3.29	3.47	3.59	3.69	3.76	3.83	3.88	3.93
15	2.95	3.25	3.43	3.55	3.64	3.71	3.78	3.83	3.88
16	2.92	3.22	3.39	3.51	3.60	3.67	3.73	3.78	3.83
17	2.90	3.19	3.36	3.47	3.56	3.63	3.69	3.74	3.79
18	2.88	3.17	3.33	3.44	3.53	3.60	3.66	3.71	3.75
19	2.86	3.15	3.31	3.42	3.50	3.57	3.63	3.68	3.72
20	2.85	3.13	3.29	3.40	3.48	3.55	3.60	3.65	3.69
24	2.80	3.07	3.22	3.32	3.40	3.47	3.52	3.57	3.61
30	2.75	3.01	3.15	3.25	3.33	3.39	3.44	3.49	3.52
40	2.70	2.95	3.09	3.19	3.26	3.32	3.37	3.41	3.44
60	2.66	2.90	3.03	3.12	3.19	3.25	3.29	3.33	3.37
120	2.62	2.85	2.97	3.06	3.12	3.18	3.22	3.26	3.29
∞	2.58	2.79	2.92	3.00	3.06	3.11	3.15	3.19	3.22

Table A.15 Values of $d_\alpha(k, v)$ for One-Sided Comparisons between k Treatments and a Control

v	$\alpha = 0.05$								
	$k = \text{Number of Treatment Means (excluding control)}$								
1	2	3	4	5	6	7	8	9	
5	2.02	2.44	2.68	2.85	2.98	3.08	3.16	3.24	3.30
6	1.94	2.34	2.56	2.71	2.83	2.92	3.00	3.07	3.12
7	1.89	2.27	2.48	2.62	2.73	2.82	2.89	2.95	3.01
8	1.86	2.22	2.42	2.55	2.66	2.74	2.81	2.87	2.92
9	1.83	2.18	2.37	2.50	2.60	2.68	2.75	2.81	2.86
10	1.81	2.15	2.34	2.47	2.56	2.64	2.70	2.76	2.81
11	1.80	2.13	2.31	2.44	2.53	2.60	2.67	2.72	2.77
12	1.78	2.11	2.29	2.41	2.50	2.58	2.64	2.69	2.74
13	1.77	2.09	2.27	2.39	2.48	2.55	2.61	2.66	2.71
14	1.76	2.08	2.25	2.37	2.46	2.53	2.59	2.64	2.69
15	1.75	2.07	2.24	2.36	2.44	2.51	2.57	2.62	2.67
16	1.75	2.06	2.23	2.34	2.43	2.50	2.56	2.61	2.65
17	1.74	2.05	2.22	2.33	2.42	2.49	2.54	2.59	2.64
18	1.73	2.04	2.21	2.32	2.41	2.48	2.53	2.58	2.62
19	1.73	2.03	2.20	2.31	2.40	2.47	2.52	2.57	2.61
20	1.72	2.03	2.19	2.30	2.39	2.46	2.51	2.56	2.60
24	1.71	2.01	2.17	2.28	2.36	2.43	2.48	2.53	2.57
30	1.70	1.99	2.15	2.25	2.33	2.40	2.45	2.50	2.54
40	1.68	1.97	2.13	2.23	2.31	2.37	2.42	2.47	2.51
60	1.67	1.95	2.10	2.21	2.28	2.35	2.39	2.44	2.48
120	1.66	1.93	2.08	2.18	2.26	2.32	2.37	2.41	2.45
∞	1.64	1.92	2.06	2.16	2.23	2.29	2.34	2.38	2.42

Reproduced from Charles W. Dunnett, "A Multiple Comparison Procedure for Comparing Several Treatments with a Control," *J. Am. Stat. Assoc.*, **50**, 1955, 1096–1121, by permission of the author and the editor.

Table A.15 (continued) Values of $d_\alpha(k, v)$ for One-Sided Comparisons between k Treatments and a Control

v	$\alpha = 0.01$								
	$k = \text{Number of Treatment Means (excluding control)}$								
1	2	3	4	5	6	7	8	9	
5	3.37	3.90	4.21	4.43	4.60	4.73	4.85	4.94	5.03
6	3.14	3.61	3.88	4.07	4.21	4.33	4.43	4.51	4.59
7	3.00	3.42	3.66	3.83	3.96	4.07	4.15	4.23	4.30
8	2.90	3.29	3.51	3.67	3.79	3.88	3.96	4.03	4.09
9	2.82	3.19	3.40	3.55	3.66	3.75	3.82	3.89	3.94
10	2.76	3.11	3.31	3.45	3.56	3.64	3.71	3.78	3.83
11	2.72	3.06	3.25	3.38	3.48	3.56	3.63	3.69	3.74
12	2.68	3.01	3.19	3.32	3.42	3.50	3.56	3.62	3.67
13	2.65	2.97	3.15	3.27	3.37	3.44	3.51	3.56	3.61
14	2.62	2.94	3.11	3.23	3.32	3.40	3.46	3.51	3.56
15	2.60	2.91	3.08	3.20	3.29	3.36	3.42	3.47	3.52
16	2.58	2.88	3.05	3.17	3.26	3.33	3.39	3.44	3.48
17	2.57	2.86	3.03	3.14	3.23	3.30	3.36	3.41	3.45
18	2.55	2.84	3.01	3.12	3.21	3.27	3.33	3.38	3.42
19	2.54	2.83	2.99	3.10	3.18	3.25	3.31	3.36	3.40
20	2.53	2.81	2.97	3.08	3.17	3.23	3.29	3.34	3.38
24	2.49	2.77	2.92	3.03	3.11	3.17	3.22	3.27	3.31
30	2.46	2.72	2.87	2.97	3.05	3.11	3.16	3.21	3.24
40	2.42	2.68	2.82	2.92	2.99	3.05	3.10	3.14	3.18
60	2.39	2.64	2.78	2.87	2.94	3.00	3.04	3.08	3.12
120	2.36	2.60	2.73	2.82	2.89	2.94	2.99	3.03	3.06
∞	2.33	2.56	2.68	2.77	2.84	2.89	2.93	2.97	3.00

Table A.16 Critical Values for the Signed-Rank Test

<i>n</i>	One-Sided $\alpha = 0.01$	One-Sided $\alpha = 0.025$	One-Sided $\alpha = 0.05$
	Two-Sided $\alpha = 0.02$	Two-Sided $\alpha = 0.05$	Two-Sided $\alpha = 0.1$
5			1
6		1	2
7	0	2	4
8	2	4	6
9	3	6	8
10	5	8	11
11	7	11	14
12	10	14	17
13	13	17	21
14	16	21	26
15	20	25	30
16	24	30	36
17	28	35	41
18	33	40	47
19	38	46	54
20	43	52	60
21	49	59	68
22	56	66	75
23	62	73	83
24	69	81	92
25	77	90	101
26	85	98	110
27	93	107	120
28	102	117	130
29	111	127	141
30	120	137	152

Reproduced from F. Wilcoxon and R. A. Wilcox, *Some Rapid Approximate Statistical Procedures*, American Cyanamid Company, Pearl River, N.Y., 1964, by permission of the American Cyanamid Company.

Table A.17 Critical Values for the Wilcoxon Rank-Sum Test

One-Tailed Test at $\alpha = 0.001$ or Two-Tailed Test at $\alpha = 0.002$															
n_1	n_2														
	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1															
2															
3															
4															
5	0	0	1	1	2	2	3	3	4	5	5	6	7	7	
6	0	1	2	2	3	4	4	5	6	7	8	9	10	11	12
7		2	3	3	5	6	7	8	9	10	11	13	14	15	16
8			5	5	6	8	9	11	12	14	15	17	18	20	21
9				7	8	10	12	14	15	17	19	21	23	25	26
10					10	12	14	17	19	21	23	25	27	29	32
11						15	17	20	22	24	27	29	32	34	37
12							20	23	25	28	31	34	37	40	42
13								26	29	32	35	38	42	45	48
14									32	36	39	43	46	50	54
15										40	43	47	51	55	59
16											48	52	56	60	65
17												57	61	66	70
18													66	71	76
19														77	82
20															88

One-Tailed Test at $\alpha = 0.01$ or Two-Tailed Test at $\alpha = 0.02$

n_1	n_2															
	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1																
2																
3		0	0	1	1	1	2	2	2	3	3	4	4	4	5	
4	0	1	1	2	3	3	4	5	5	6	7	7	8	9	10	
5	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
6	3	4	6	7	8	9	11	12	13	15	16	18	19	20	22	
7		6	8	9	11	12	14	16	17	19	21	23	24	26	28	
8			10	11	13	15	17	20	22	24	26	28	30	32	34	
9				14	16	18	21	23	26	28	31	33	36	38	40	
10					19	22	24	27	30	33	36	38	41	44	47	
11						25	28	31	34	37	41	44	47	50	53	
12							31	35	38	42	46	49	53	56	60	
13								39	43	47	51	55	59	63	67	
14									47	51	56	60	65	69	73	
15										56	61	66	70	75	80	
16											66	71	76	82	87	
17												77	82	88	93	
18													88	94	100	
19													101	107		
20															114	

Based in part on Tables 1, 3, 5, and 7 of D. Auble, "Extended Tables for the Mann-Whitney Statistic," *Bulletin of the Institute of Educational Research at Indiana University*, 1, No. 2, 1953, by permission of the director.

Table A.17 (continued) Critical Values for the Wilcoxon Rank-Sum Test

One-Tailed Test at $\alpha = 0.025$ or Two-Tailed Test at $\alpha = 0.05$																		
n_1	n_2																	
	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	
1																		
2					0	0	0	0	1	1	1	1	1	2	2	2	2	
3	0	1	1	2	2	3	3	4	4	5	5	5	6	6	7	7	8	
4	1	2	3	4	4	5	6	7	8	9	10	11	11	12	13	13	13	
5	2	3	5	6	7	8	9	11	12	13	14	15	17	18	19	19	20	
6		5	6	8	10	11	13	14	16	17	19	21	22	24	25	25	27	
7			8	10	12	14	16	18	20	22	24	26	28	30	32	32	34	
8				13	15	17	19	22	24	26	29	31	34	36	38	38	41	
9					17	20	23	26	28	31	34	37	39	42	45	45	48	
10						23	26	29	33	36	39	42	45	48	52	52	55	
11							30	33	37	40	44	47	51	55	58	58	62	
12								37	41	45	49	53	57	61	65	69		
13									45	50	54	59	63	67	72	76		
14										55	59	64	67	74	78	83		
15											64	70	75	80	85	90		
16												75	81	86	92	98		
17													87	93	99	105		
18														99	106	112		
19															113	119		
20																127		

One-Tailed Test at $\alpha = 0.05$ or Two-Tailed Test at $\alpha = 0.1$

One-Tailed Test at $\alpha = 0.05$ or Two-Tailed Test at $\alpha = 0.1$																		
n_1	n_2																	
	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1																	0	0
2		0	0	0	1	1	1	1	2	2	3	3	3	3	4	4	4	4
3	0	0	1	2	2	3	4	4	5	5	6	7	7	8	9	9	10	11
4	1	2	3	4	5	6	7	8	9	10	11	12	14	15	16	17	18	
5	4	5	6	8	9	11	12	13	15	16	18	19	20	22	23	25		
6		7	8	10	12	14	16	17	19	21	23	25	26	28	30	32		
7			11	13	15	17	19	21	24	26	28	30	33	35	37	39		
8				15	18	20	23	26	28	31	33	36	39	41	44	47		
9					21	24	27	30	33	36	39	42	45	48	51	54		
10						27	31	34	37	41	44	48	51	55	58	62		
11							34	38	42	46	50	54	57	61	65	69		
12								42	47	51	55	60	64	68	72	77		
13									51	56	61	65	70	75	80	84		
14										61	66	71	77	82	87	92		
15											72	77	83	88	94	100		
16												83	89	95	101	107		
17													96	102	109	115		
18														109	116	123		
19															123	130		
20																138		

Table A.18 $P(V \leq v^* \text{ when } H_0 \text{ is true})$ in the Runs Test

(n_1, n_2)	v^*									
	2	3	4	5	6	7	8	9	10	
(2, 3)	0.200	0.500	0.900	1.000						
(2, 4)	0.133	0.400	0.800	1.000						
(2, 5)	0.095	0.333	0.714	1.000						
(2, 6)	0.071	0.286	0.643	1.000						
(2, 7)	0.056	0.250	0.583	1.000						
(2, 8)	0.044	0.222	0.533	1.000						
(2, 9)	0.036	0.200	0.491	1.000						
(2, 10)	0.030	0.182	0.455	1.000						
(3, 3)	0.100	0.300	0.700	0.900	1.000					
(3, 4)	0.057	0.200	0.543	0.800	0.971	1.000				
(3, 5)	0.036	0.143	0.429	0.714	0.929	1.000				
(3, 6)	0.024	0.107	0.345	0.643	0.881	1.000				
(3, 7)	0.017	0.083	0.283	0.583	0.833	1.000				
(3, 8)	0.012	0.067	0.236	0.533	0.788	1.000				
(3, 9)	0.009	0.055	0.200	0.491	0.745	1.000				
(3, 10)	0.007	0.045	0.171	0.455	0.706	1.000				
(4, 4)	0.029	0.114	0.371	0.629	0.886	0.971	1.000			
(4, 5)	0.016	0.071	0.262	0.500	0.786	0.929	0.992	1.000		
(4, 6)	0.010	0.048	0.190	0.405	0.690	0.881	0.976	1.000		
(4, 7)	0.006	0.033	0.142	0.333	0.606	0.833	0.954	1.000		
(4, 8)	0.004	0.024	0.109	0.279	0.533	0.788	0.929	1.000		
(4, 9)	0.003	0.018	0.085	0.236	0.471	0.745	0.902	1.000		
(4, 10)	0.002	0.014	0.068	0.203	0.419	0.706	0.874	1.000		
(5, 5)	0.008	0.040	0.167	0.357	0.643	0.833	0.960	0.992	1.000	
(5, 6)	0.004	0.024	0.110	0.262	0.522	0.738	0.911	0.976	0.998	
(5, 7)	0.003	0.015	0.076	0.197	0.424	0.652	0.854	0.955	0.992	
(5, 8)	0.002	0.010	0.054	0.152	0.347	0.576	0.793	0.929	0.984	
(5, 9)	0.001	0.007	0.039	0.119	0.287	0.510	0.734	0.902	0.972	
(5, 10)	0.001	0.005	0.029	0.095	0.239	0.455	0.678	0.874	0.958	
(6, 6)	0.002	0.013	0.067	0.175	0.392	0.608	0.825	0.933	0.987	
(6, 7)	0.001	0.008	0.043	0.121	0.296	0.500	0.733	0.879	0.966	
(6, 8)	0.001	0.005	0.028	0.086	0.226	0.413	0.646	0.821	0.937	
(6, 9)	0.000	0.003	0.019	0.063	0.175	0.343	0.566	0.762	0.902	
(6, 10)	0.000	0.002	0.013	0.047	0.137	0.288	0.497	0.706	0.864	
(7, 7)	0.001	0.004	0.025	0.078	0.209	0.383	0.617	0.791	0.922	
(7, 8)	0.000	0.002	0.015	0.051	0.149	0.296	0.514	0.704	0.867	
(7, 9)	0.000	0.001	0.010	0.035	0.108	0.231	0.427	0.622	0.806	
(7, 10)	0.000	0.001	0.006	0.024	0.080	0.182	0.355	0.549	0.743	
(8, 8)	0.000	0.001	0.009	0.032	0.100	0.214	0.405	0.595	0.786	
(8, 9)	0.000	0.001	0.005	0.020	0.069	0.157	0.319	0.500	0.702	
(8, 10)	0.000	0.000	0.003	0.013	0.048	0.117	0.251	0.419	0.621	
(9, 9)	0.000	0.000	0.003	0.012	0.044	0.109	0.238	0.399	0.601	
(9, 10)	0.000	0.000	0.002	0.008	0.029	0.077	0.179	0.319	0.510	
(10, 10)	0.000	0.000	0.001	0.004	0.019	0.051	0.128	0.242	0.414	

Reproduced from C. Eisenhart and R. Swed, "Tables for Testing Randomness of Grouping in a Sequence of Alternatives," *Ann. Math. Stat.*, 14, 1943, by permission of the editor.

Table A.18 (continued) $P(V \leq v^* \text{ when } H_0 \text{ is true})$ in the Runs Test

(n_1, n_2)	v^*									
	11	12	13	14	15	16	17	18	19	20
(2, 3)										
(2, 4)										
(2, 5)										
(2, 6)										
(2, 7)										
(2, 8)										
(2, 9)										
(2, 10)										
(3, 3)										
(3, 4)										
(3, 5)										
(3, 6)										
(3, 7)										
(3, 8)										
(3, 9)										
(3, 10)										
(4, 4)										
(4, 5)										
(4, 6)										
(4, 7)										
(4, 8)										
(4, 9)										
(4, 10)										
(5, 5)										
(5, 6)	1.000									
(5, 7)	1.000									
(5, 8)	1.000									
(5, 9)	1.000									
(5, 10)	1.000									
(6, 6)	0.998	1.000								
(6, 7)	0.992	0.999	1.000							
(6, 8)	0.984	0.998	1.000							
(6, 9)	0.972	0.994	1.000							
(6, 10)	0.958	0.990	1.000							
(7, 7)	0.975	0.996	0.999	1.000						
(7, 8)	0.949	0.988	0.998	1.000	1.000					
(7, 9)	0.916	0.975	0.994	0.999	1.000					
(7, 10)	0.879	0.957	0.990	0.998	1.000					
(8, 8)	0.900	0.968	0.991	0.999	1.000	1.000				
(8, 9)	0.843	0.939	0.980	0.996	0.999	1.000	1.000			
(8, 10)	0.782	0.903	0.964	0.990	0.998	1.000	1.000			
(9, 9)	0.762	0.891	0.956	0.988	0.997	1.000	1.000	1.000		
(9, 10)	0.681	0.834	0.923	0.974	0.992	0.999	1.000	1.000	1.000	
(10, 10)	0.586	0.758	0.872	0.949	0.981	0.996	0.999	1.000	1.000	1.000

Table A.19 Sample Size for Two-Sided Nonparametric Tolerance Limits

$1 - \alpha$	$1 - \gamma$					
	0.50	0.70	0.90	0.95	0.99	0.995
0.995	336	488	777	947	1325	1483
0.99	168	244	388	473	662	740
0.95	34	49	77	93	130	146
0.90	17	24	38	46	64	72
0.85	11	16	25	30	42	47
0.80	9	12	18	22	31	34
0.75	7	10	15	18	24	27
0.70	6	8	12	14	20	22
0.60	4	6	9	10	14	16
0.50	3	5	7	8	11	12

Reproduced from Table A-25d of Wilfrid J. Dixon and Frank J. Massey, Jr.,

Introduction to Statistical Analysis, 3rd ed. McGraw-Hill, New York, 1969.

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Table A.20 Sample Size for One-Sided Nonparametric Tolerance Limits

$1 - \alpha$	$1 - \gamma$				
	0.50	0.70	0.95	0.99	0.995
0.995	139	241	598	919	1379
0.99	69	120	299	459	688
0.95	14	24	59	90	135
0.90	7	12	29	44	66
0.85	5	8	19	29	43
0.80	4	6	14	21	31
0.75	3	5	11	7	25
0.70	2	4	9	13	20
0.60	2	3	6	10	14
0.50	1	2	5	7	10

Reproduced from Table A-25e of Wilfrid J. Dixon and Frank J. Massey, Jr.,

Introduction to Statistical Analysis, 3rd ed. McGraw-Hill, New York, 1969.

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Table A.21 Critical Values for Spearman's Rank Correlation Coefficients

n	$\alpha = 0.05$	$\alpha = 0.025$	$\alpha = 0.01$	$\alpha = 0.005$
5	0.900			
6	0.829	0.886	0.943	
7	0.714	0.786	0.893	
8	0.643	0.738	0.833	0.881
9	0.600	0.683	0.783	0.833
10	0.564	0.648	0.745	0.794
11	0.523	0.623	0.736	0.818
12	0.497	0.591	0.703	0.780
13	0.475	0.566	0.673	0.745
14	0.457	0.545	0.646	0.716
15	0.441	0.525	0.623	0.689
16	0.425	0.507	0.601	0.666
17	0.412	0.490	0.582	0.645
18	0.399	0.476	0.564	0.625
19	0.388	0.462	0.549	0.608
20	0.377	0.450	0.534	0.591
21	0.368	0.438	0.521	0.576
22	0.359	0.428	0.508	0.562
23	0.351	0.418	0.496	0.549
24	0.343	0.409	0.485	0.537
25	0.336	0.400	0.475	0.526
26	0.329	0.392	0.465	0.515
27	0.323	0.385	0.456	0.505
28	0.317	0.377	0.448	0.496
29	0.311	0.370	0.440	0.487
30	0.305	0.364	0.432	0.478

Reproduced from E. G. Olds, "Distribution of Sums of Squares of Rank Differences for Small Samples," *Ann. Math. Stat.*, **9**, 1938, by permission of the editor.

Table A.22 Factors for Constructing Control Charts

Obs. in Sample	Chart for Averages				Chart for Standard Deviations				Chart for Ranges					
	Factors for Control Limits		Factors for Centerline		Factors for Control Limits				Factors for Centerline		Factors for Control Limits			
	<i>n</i>	<i>A</i> ₂	<i>A</i> ₃	<i>c</i> ₄	<i>1/c</i> ₄	<i>B</i> ₃	<i>B</i> ₄	<i>B</i> ₅	<i>B</i> ₆	<i>d</i> ₂	<i>1/d</i> ₂	<i>d</i> ₃	<i>D</i> ₃	<i>D</i> ₄
2	1.880	2.659	0.7979	1.2533	0	3.267	0	2.606	1.128	0.8865	0.853	0	3.267	
3	1.023	1.954	0.8862	1.1284	0	2.568	0	2.276	1.693	0.5907	0.888	0	2.574	
4	0.729	1.628	0.9213	1.0854	0	2.266	0	2.088	2.059	0.4857	0.880	0	2.282	
5	0.577	1.427	0.9400	1.0638	0	2.089	0	1.964	2.326	0.4299	0.864	0	2.114	
6	0.483	1.287	0.9515	1.0510	0.030	1.970	0.029	1.874	2.534	0.3946	0.848	0	2.004	
7	0.419	1.182	0.9594	1.0423	0.118	1.882	0.113	1.806	2.704	0.3698	0.833	0.076	1.924	
8	0.373	1.099	0.9650	1.0363	0.185	1.815	0.179	1.751	2.847	0.3512	0.820	0.136	1.864	
9	0.337	1.032	0.9693	1.0317	0.239	1.761	0.232	1.707	2.970	0.3367	0.808	0.184	1.816	
10	0.308	0.975	0.9727	1.0281	0.284	1.716	0.276	1.669	3.078	0.3249	0.797	0.223	1.777	
11	0.285	0.927	0.9754	1.0252	0.321	1.679	0.313	1.637	3.173	0.3152	0.787	0.256	1.744	
12	0.266	0.886	0.9776	1.0229	0.354	1.646	0.346	1.610	3.258	0.3069	0.778	0.283	1.717	
13	0.249	0.850	0.9794	1.0210	0.382	1.618	0.374	1.585	3.336	0.2998	0.770	0.307	1.693	
14	0.235	0.817	0.9810	1.0194	0.406	1.594	0.399	1.563	3.407	0.2935	0.763	0.328	1.672	
15	0.223	0.789	0.9823	1.0180	0.428	1.572	0.421	1.544	3.472	0.2880	0.756	0.347	1.653	
16	0.212	0.763	0.9835	1.0168	0.448	1.552	0.440	1.526	3.532	0.2831	0.750	0.363	1.637	
17	0.203	0.739	0.9845	1.0157	0.466	1.534	0.458	1.511	3.588	0.2787	0.744	0.378	1.622	
18	0.194	0.718	0.9854	1.0148	0.482	1.518	0.475	1.496	3.640	0.2747	0.739	0.391	1.608	
19	0.187	0.698	0.9862	1.0140	0.497	1.503	0.490	1.483	3.689	0.2711	0.734	0.403	1.597	
20	0.180	0.680	0.9869	1.0133	0.510	1.490	0.504	1.470	3.735	0.2677	0.729	0.415	1.585	
21	0.173	0.663	0.9876	1.0126	0.523	1.477	0.516	1.459	3.778	0.2647	0.724	0.425	1.575	
22	0.167	0.647	0.9882	1.0119	0.534	1.466	0.528	1.448	3.819	0.2618	0.720	0.434	1.566	
23	0.162	0.633	0.9887	1.0114	0.545	1.455	0.539	1.438	3.858	0.2592	0.716	0.443	1.557	
24	0.157	0.619	0.9892	1.0109	0.555	1.445	0.549	1.429	3.895	0.2567	0.712	0.451	1.548	
25	0.153	0.606	0.9896	1.0105	0.565	1.435	0.559	1.420	3.931	0.2544	0.708	0.459	4.541	

Table A.23 The Incomplete Gamma Function: $F(x; \alpha) = \int_0^x \frac{1}{\Gamma(\alpha)} y^{\alpha-1} e^{-y} dy$

x	α									
	1	2	3	4	5	6	7	8	9	10
1	0.6320	0.2640	0.0800	0.0190	0.0040	0.0010	0.0000	0.0000	0.0000	0.0000
2	0.8650	0.5940	0.3230	0.1430	0.0530	0.0170	0.0050	0.0010	0.0000	0.0000
3	0.9500	0.8010	0.5770	0.3530	0.1850	0.0840	0.0340	0.0120	0.0040	0.0010
4	0.9820	0.9080	0.7620	0.5670	0.3710	0.2150	0.1110	0.0510	0.0210	0.0080
5	0.9930	0.9600	0.8750	0.7350	0.5600	0.3840	0.2380	0.1330	0.0680	0.0320
6	0.9980	0.9830	0.9380	0.8490	0.7150	0.5540	0.3940	0.2560	0.1530	0.0840
7	0.9990	0.9930	0.9700	0.9180	0.8270	0.6990	0.5500	0.4010	0.2710	0.1700
8	1.0000	0.9970	0.9860	0.9580	0.9000	0.8090	0.6870	0.5470	0.4070	0.2830
9		0.9990	0.9940	0.9790	0.9450	0.8840	0.7930	0.6760	0.5440	0.4130
10	1.0000	0.9970	0.9900	0.9710	0.9330	0.8700	0.7800	0.6670	0.5420	
11			0.9990	0.9950	0.9850	0.9620	0.9210	0.8570	0.7680	0.6590
12			1.0000	0.9980	0.9920	0.9800	0.9540	0.9110	0.8450	0.7580
13				0.9990	0.9960	0.9890	0.9740	0.9460	0.9000	0.8340
14				1.0000	0.9980	0.9940	0.9860	0.9680	0.9380	0.8910
15					0.9990	0.9970	0.9920	0.9820	0.9630	0.9300

A.24 Proof of Mean of the Hypergeometric Distribution

To find the mean of the hypergeometric distribution, we write

$$\begin{aligned} E(X) &= \sum_{x=0}^n x \frac{\binom{k}{x} \binom{N-k}{n-x}}{\binom{N}{n}} = k \sum_{x=1}^n \frac{(k-1)!}{(x-1)!(k-x)!} \cdot \frac{\binom{N-k}{n-x}}{\binom{N}{n}} \\ &= k \sum_{x=1}^n \frac{\binom{k-1}{x-1} \binom{N-k}{n-x}}{\binom{N}{n}}. \end{aligned}$$

Since

$$\binom{N-k}{n-1-y} = \binom{(N-1)-(k-1)}{n-1-y} \quad \text{and} \quad \binom{N}{n} = \frac{N!}{n!(N-n)!} = \frac{N}{n} \binom{N-1}{n-1},$$

letting $y = x - 1$, we obtain

$$\begin{aligned} E(X) &= k \sum_{y=0}^{n-1} \frac{\binom{k-1}{y} \binom{N-k}{n-1-y}}{\binom{N}{n}} \\ &= \frac{nk}{N} \sum_{y=0}^{n-1} \frac{\binom{k-1}{y} \binom{(N-1)-(k-1)}{n-1-y}}{\binom{N-1}{n-1}} = \frac{nk}{N}, \end{aligned}$$

since the summation represents the total of all probabilities in a hypergeometric experiment when $N-1$ items are selected at random from $N-1$, of which $k-1$ are labeled success.

A.25 Proof of Mean and Variance of the Poisson Distribution

Let $\mu = \lambda t$.

$$E(X) = \sum_{x=0}^{\infty} x \cdot \frac{e^{-\mu} \mu^x}{x!} = \sum_{x=1}^{\infty} x \cdot \frac{e^{-\mu} \mu^x}{x!} = \mu \sum_{x=1}^{\infty} \frac{e^{-\mu} \mu^{x-1}}{(x-1)!}.$$

Since the summation in the last term above is the total probability of a Poisson random variable with mean μ , which can be easily seen by letting $y = x - 1$, it equals 1. Therefore, $E(X) = \mu$. To calculate the variance of X , note that

$$E[X(X-1)] = \sum_{x=0}^{\infty} x(x-1) \frac{e^{-\mu} \mu^x}{x!} = \mu^2 \sum_{x=2}^{\infty} \frac{e^{-\mu} \mu^{x-2}}{(x-2)!}.$$

Again, letting $y = x - 2$, the summation in the last term above is the total probability of a Poisson random variable with mean μ . Hence, we obtain

$$\sigma^2 = E(X^2) - [E(X)]^2 = E[X(X-1)] + E(X) - [E(X)]^2 = \mu^2 + \mu - \mu^2 = \mu = \lambda t.$$

A.26 Proof of Mean and Variance of the Gamma Distribution

To find the mean and variance of the gamma distribution, we first calculate

$$E(X^k) = \frac{1}{\beta^\alpha \Gamma(\alpha)} \int_0^\infty x^{\alpha+k-1} e^{-x/\beta} dx = \frac{\beta^{k+\alpha} \Gamma(\alpha+k)}{\beta^\alpha \Gamma(\alpha)} \int_0^\infty \frac{x^{\alpha+k-1} e^{-x/\beta}}{\beta^{k+\alpha} \Gamma(\alpha+k)} dx,$$

for $k = 0, 1, 2, \dots$. Since the integrand in the last term above is a gamma density function with parameters $\alpha + k$ and β , it equals 1. Therefore,

$$E(X^k) = \beta^k \frac{\Gamma(k+\alpha)}{\Gamma(\alpha)}.$$

Using the recursion formula of the gamma function from page 194, we obtain

$$\mu = \beta \frac{\Gamma(\alpha+1)}{\Gamma(\alpha)} = \alpha\beta \quad \text{and} \quad \sigma^2 = E(X^2) - \mu^2 = \beta^2 \frac{\Gamma(\alpha+2)}{\Gamma(\alpha)} - \mu^2 = \beta^2 \alpha(\alpha+1) - (\alpha\beta)^2 = \alpha\beta^2.$$