Electric Power Market with Intermittent Power Generators and Stability Contraints

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***Abstract*— We are discussing a stochastic market clearing problem to deal with uncertainty in electric power generation due to renewable generators. These fluctuations arise due to several imperfections in weather forecast data as well as multiple transmission line constraints. The forecast errors can give rise to imbalances in demand and supply. Inertial response and Primary Frequency Response (PFR) decline when conventional generation is displaced by renewable energy, that is not providing the needed control capabilities. The high variability of wind and solar imposes additional regulation burden on remaining conventional generation which motivates a need for new resources to provide Primary Frequency Response (PFR), Secondary Frequency Response (SFR) and Tertiary Frequency Response (TFR). In this paper, we formulate an Optimal power flow model with stability constraints which are frequency stability and voltage stability for a 3-bus system.**

Abbreviations: Primary Frequency Response (PFR), Secondary Frequency Response (SFR),Tertiary Frequency Response (TFR), Real Time(RT), Day Ahead(DA), AVRs (automatic voltage regulator), OXL (over excitation limiter), Optimal power flow (OPF), Under Frequency Load Shedding (UFLS), Short-term ramp rate (STRR),System Operator (SO).

I. Introduction

Several years back, when electric power generation was solely a product of conventional sources, a combination of inflexible and flexible generators was used to supply the load. As a result, the Day-Ahead commitment was balanced using the real time analysis. But, in the modern world, the focus has pivoted from the conventional generators to the renewable generators such as wind power generators and solar power panels. These sources of power generations are environment friendly and financially stable. But due to the unpredictable weather conditions, the power of these sources cannot be exactly predicted and controlled which makes the scheduling and dispatch of generators a very difficult task for the system operator.

In DA (day ahead), the calculations are performed in advance by predicting the expected load demand in RT(real time). Dispatching and scheduling of the generation is carried out to get the power flow while minimizing the cost of generation as well maintaining the system security. By Real Time (RT), we implement balancing the demand and supply of electricity during actual operation. The predicted day ahead and real time load and generation differ due to multiple reasons such as incoming of a new load demand or loss of a transmission line or due to tripping of any of our conventional generator or may be due to sunny or stormy weather condition. Such conditions not only create a problem of rescheduling of the generators to meet the load demand but also arises the problem of system reliability.

The frequency and voltage response have an important influence on the power system reliability. It is aimed to maintain the frequency of the system within a certain band around 60Hz and average voltage within the permissible limit of 110V at distribution end. Failure to maintain the frequency and voltage within the permissible limits causes financial penalties motivated by the desire to avoid load shedding or generator tripping and may even lead to Grid Failure.

A large load connection or a sudden loss of generation creates an imbalance in generation and load which causes frequency deviations. Such frequency deviation is arrested by the initial frequency response of the generators and using the primary frequency response (PFR) to reach the steady state frequency. The error resulting between the new steady state frequency and the desired nominal frequency is taken care of by the secondary frequency response (SFR). Then the generation is rescheduled, security margins and economic set point is restored using the tertiary frequency response (TFR). The procurement of the needed three type of frequency response is determined by the ex-ante reserve market, making it necessary to account for the required reserve of generation to provide the required frequency related services in the market dispatch model. High penetration of wind and solar energy requires increased dependency on PFR & SFR. Any resulting shortage of SFR will adversely affect PFR sufficiency resulting in unsatisfactory frequency response.

The voltage stability of a power system is related to the ability of a power system to maintain acceptable voltage at all buses of the power system under normal conditions and after being subjected to a disturbance. A system enters the state of voltage instability when a disturbance, intermittent generation or change in system conditions causes a progressive and uncontrollable decline in voltage. The factors contributing to voltage collapse are generator reactive/active control limits, load characteristics and characteristics of reactive compensation devices. Various methods are being used to

control the voltage at the buses such as AVRs (automatic voltage regulator), OXL (over excitation limiter).
A number of papers have already been published on incorporating the frequency constraints into a market dispatch model including those which do so within the economic dispatch but none of them accounts for the frequency control due to the intermittent wind generation in probabilistic scenarios as well accounts for the voltage stability in the same paper. In this paper, a comprehensive OPF model with both the PFR and SFR constraints with probabilistic wind scenarios and voltage stability is proposed and tested.

II. Problem Statement

As indicated in our abstract, we are designing an optimal power flow (OPF) with stability constraints, day ahead and real time computations of the renewable and non-renewable sources. Stability constraints like frequency response increase our total system cost which is further analyzed and depicted using our optimization model.

In the model presented in this paper, computations are carried out using weighted average for day ahead market to minimize the deviation between the day ahead and real time markets.

$$Weighted\ Average = \sum \mathrm{P(i)} * \text{Power Expected}(i)$$

where P(i) is the probability of i^{th} power generation

whereas the real time power is realized in the live scenario (5-60 minutes) ahead.

A. Frequency Response

Frequency control process can be of two types namely contingency control mode and normal control mode. In normal control mode the fluctuations in load are relatively small hence only SFR is utilized by Automatic Generation Control (AGC) to compensate for the resulting imbalance. The normal control mode maintains the frequency around the nominal frequency. In the case when there is a huge loss of generation, contingency control mode is activated which spans from a few seconds to several minutes. The speed governor does not respond to the frequency deviation immediately. Usually a dead band of a few MHz is set to avoid the unnecessary governor reaction to small frequency deviations. Under conditions where frequency deviation exceeds the deadband, the speed governor starts to adjust the valve position to arrest the further frequency drop. After a few seconds, following initial action of the generation, a new generation/load balance is reached, and the frequency dips to its nadir. If the nadir falls below the any Under Frequency Load Shedding (UFLS) relays trigger frequency, load will be shed to achieve new power balance. Sufficient PFR, with enough ramping capability, can prevent activation of UFLS relays. Following the point in time when the frequency reaches nadir point, the frequency will oscillate for several seconds and finally settle to a steady state which is closer to the nominal frequency. After the settling point, the SFR will correct the frequency deviation, bringing the frequency back to its nominal value.

B. Estimation of Nadir Time

When there is a major loss of generation or change in load, for the first few seconds there is no significant influence of PFR. The frequency continues to drop until the frequency deadband (f_{db}) is crossed after which the primary frequency response is triggered. the PFR will take action to decrease the rate of change of frequency until it becomes zero at the nadir. Based on the power balance swing equation, the initial slope of frequency dip is computed as S_0 which is given as follows:

$$\frac{2HP_0}{f_0} \cdot \frac{df}{dt} = \Delta P_m - \Delta P_e$$

$$S_0 = \frac{-P_{loss} \cdot f_0}{2H \cdot P_0}$$

where P_{loss} is the loss of generation in MW which is considered as power spilled in our model, H is the system inertia in seconds, f_0 is the nominal frequency and P_0 is the base MVA

When the frequency crosses the deadband, all the generators with speed governors respond to this frequency deviation at their maximum short-term ramp rate (STRR) until the time the frequency reaches the nadir point. For a given loss of generation or change in load, the minimum requirement on the ramping capability could also be equivalent to the maximum allowable time t_{nadir} for arresting the frequency dip which is expressed as follows

$$t_{nadir} \leq \frac{2H \cdot P_0 \left(\Delta f_{UFLS} - \Delta f_{db}\right)}{f_0 \cdot P_{loss}} + \frac{2H \cdot P_0 \cdot \Delta f_{db}}{f_0 \cdot P_{loss}}$$

where f_{UFLS} is the under-frequency load shedding frequency

In this paper this t_{nadir} is assumed to be 30 seconds as per our calculations. It can also be observed that the smaller the inertia is, and the larger the loss of generation P_{loss} is, the shorter time is required to prevent the UFLS trigger frequency.

$$y_i \leq rr_i \cdot t_{nadir}$$

$$rr_i \leq 2c_i(2H(f_0 - f_{min} - f_{db})/P_{loss}$$

$$\sum_i y(i) \geq P_{loss}$$

where y_i is the PFR capacity, rr_i is the PFR ramp rate, c_i is generator i governor ramp rate and f_{min} is the minimum acceptable frequency

Hence the primary frequency response (PFR) should be the amount that can be fully deployed before tnadir to arrest the frequency deviation.

C. Analysis of Secondary Frequnecy Response (SFR)

After the primary frequency response is exhausted, PFR begins to reduce and hence the frequency begins to drop again. The

SFR action begins to correct the steady-state error and move the frequency back to its nominal value. A control area is required to return the Area Control Error (ACE) to zero within 10 minutes as per NECR B1 criterion after the contingency. For secondary frequency response, the steady state frequency can be calculated using the following equation:

$$\Delta f_{ss} = -P_{loss}\frac{f_0 \cdot R}{P_0}$$

where R is the droop of the governor and P_o is the base MVA

The steady state Area Control Error (ACE) is as follows:

$$ACE_{ss} = \Delta f_{ss} \cdot \frac{1}{R}$$

The SFR requires enough SFR capacity r_i to cover the total ACE subject to the long-term ramp rate RR_i.

$$r_i \leq 10RR_i$$

D. Frequency Constrained OPF Model

The traditional OPF model is modified to include the PFR and SFR adequacy constraints to obtain an improved frequency performance. The proposed frequency constrained power flow model (FC-OPF) is formulated as follows:

minimize

$\sum C_i(g_i) + \sum P(s_i)*[\sum C_i*(P_{adjustement\_RT}) + \sum C_{iPFR}*(y_i) + \sum C_{iSFR}*(r_i) + \sum C_{iloadshed}*(b_i)]$

where C_i is the cost of the generator, s_i is the probability of scenarios, C_{iPFR} is cost of PFR, C_{iSFR} is cost of SFR, $C_{iloadshed}$ is cost of load shed and y_i is PFR, r_i is SFR, b_i is Load shed.

Subject to

$$\sum g_i + \sum w_i = d$$

$$\sum g_{i(DA)} + \sum w_{i(DA)} - \sum L(d) - \sum PF_{(DA)} = 0$$

$$P_{min} + \text{Reserve} \leq P_{gi} \leq P_{max} - \text{Reserve}$$

$$P_{wmin} \leq P_{wi} \leq P_{wmax}$$

$$s_{(i)} * \left(\sum g_{(iRT)} + \sum w_{(iRT)} - \sum w_{(DA)} - P_{(spill)}\right) + s_{(i)} * \left(\sum P_{(loadshed)}\right) - \sum PF_{(RT)} = 0$$

$$y_i \leq rr_i \cdot t_{nadir}$$

$$\sum_i y(i) \geq P_{loss}$$

$$r_i \leq 10RR_i$$

$$\sum r_i \geq -ACE_{ss}$$

The above equation is a cost minimization proposition with nodal constrains on both DA commitment and RT Power Flow, Generators are governed between its PFR limits and line limits In case of frequency stability there are namely three scenarios Under generation, Over generation and Optimal generation. To tackle the under-generation part the relays shed the load to make demand equal to generation maintaining the frequency stability in the system. Basically, we need to maintain this equation throughout

$$P_{consumed} = P_{Generated}$$

In Overgeneration case SO need to decrease the output of the flexible generation or the frequency will suffer and incur the cost of stability.

III. Voltage Stablity

The Analysis of voltage stability required us to import the data from our GAMS simulation to MATLAB. We use the PV curve to estimate maximum power flow of transmission . It illustrates the relationship between the receiving end voltage as a function of the power delivered through a transmission line for any given load angle. The relationship between the sending end voltage E, the receiving end voltage V, the active and reactive power P and Q and the reactance X, in power line is given by the following equation

$$V = \sqrt{-XQ + \frac{E^2}{2} \pm \sqrt{\frac{E^4}{4} - X^2P^2 - E^2XQ}}$$

For maximum deliverable power for a given sending end voltage this term needs to equal zero

$$0 = \sqrt{\frac{E^4}{4} - X^2P^2 - E^2XP\tan(\Phi)}$$

By solving the Quadratic, we get

$$P_{max} = \frac{-(\sin(\phi) - 1) \cdot E^2}{2 \cdot \cos(\phi) \cdot X}$$

The other parameters are expressed, analyzing a simple radial transmission line

$$I = \frac{1}{\sqrt{F}}\frac{E}{Z_{LN}} \qquad V = \frac{1}{\sqrt{F}}\frac{Z_{LD}}{Z_{LN}}E$$

$$P = \frac{Z_{LD}}{F}\left(\frac{E}{Z_{LN}}\right)^2 \cos\phi$$

Where I and V is receiving side and F is

$$F = 1 + \left(\frac{Z_{LD}}{Z_{LN}}\right)^2 + 2\left(\frac{Z_{LD}}{Z_{LN}}\right)\cos(\theta - \phi)$$

Z_{LD} Load Impedance and Z_{LN} is the transmission line impedance.Using the values obtain from our Optimization Problem we plot the PV curve and the see the maximum power delivered at different load angle displayed in Figure 1.

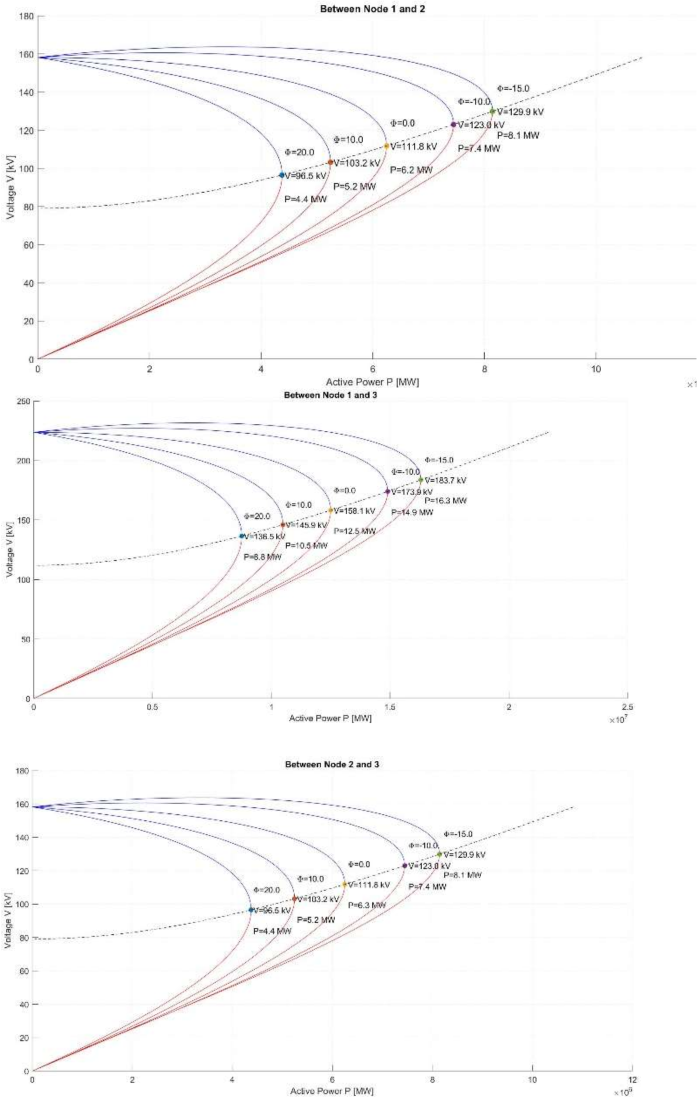

Figure1. PV curve (voltage Stability)

The voltage line in RED shows the unstable region, How ever its never operated at the peak power value to avoid disturbance pushing it into instable region. Also, Maximum power is achieved when

For P_{max} $Z_{LD}=Z_{LN}$

IV. Model analysis

The system designed in this paper constitutes of two conventional generators G1 and G2 where G1 is the inflexible generator whereas G2 is the flexible generator. There are two renewable sources of power generation K1 (Wind Power Generator) and K2 (Solar Power Generator). The power generated in DA and RT from these renewable and nonrenewable sources of generation is used to feed the inelastic and inflexible demand D1 and D2. The specifications of the model elements are mentioned in the Table 1.

S. No	Elements	Type	Capacity (MW)
1	G1	Inflexible	600
2	G2	Flexible	400 ± 80
3	K1	Unpredictable	400
4	K2	Unpredictable	400
5	D1	Inelastic	400
6	D2	Inelastic	600

Table 1. Specifications of Generation and Load

V. Case Study

A. 3 BUS SYSTEM FREQUENCY STABILITY

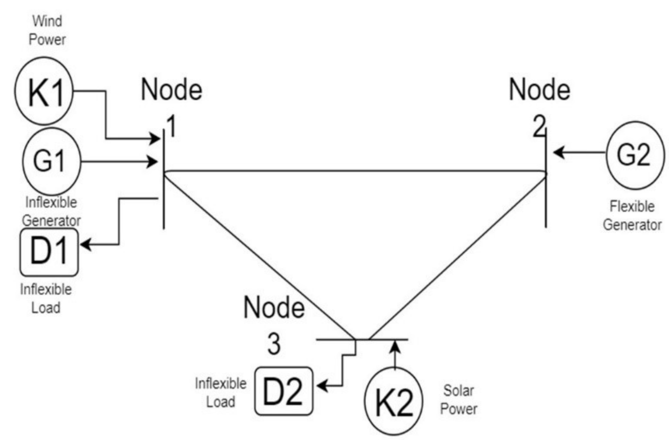

Figure 2: Three Node System

A 3-bus system is simulated to evaluate the performance of the proposed model. The system data is provided in Table 2 where the bidding prices for energy and reserve are both given. The reserve price does not only reflect the reserve opportunity cost but also the ramping capability. STTR and LRTT are also provided for each generator in units MW/sec and MW/min respectively. Transmission topology is plotted in Figure 2. The FC-OPF model is developed in GAMS and solved by KNITRO solver. The contingency event is whenever the generation of power from the renewable generators is below or more than the day ahead calculated value. Whenever, there is under generation, we need to deploy the reserve to check the frequency drop though when there is over generation by our renewable generator the contingency is easier to tackle as when always have the option to do spillage but shedding load in the case of supply shortage has a more negative impact on the system as a whole as it comes with a heavy penalty. The system total inertia is pre-known since the unit ON/OFF status is fixed. The governor deadband is set at 0.03Hz and the first UFLS is set at 0.4Hz. Some of the key worthy definitions for understanding this paper are:

1.) Frequency Nadir: The Frequency Nadir is the point where the frequency reaches its maximum point.
2.) Frequency Nadir Time: Frequency Nadir Time is the time when the frequency reaches its minimum point.
3.) Nadir based Frequency Response: The Nadir based frequency Response is the size of the contingency divided by the change of frequency at nadir. The unit is normalized to MW/0.1Hz.
4.) Settling Frequency: The Settling Frequency is the frequency measured to steady state point, usually around 50-60 seconds. It is assumed that before the settling point, only PFR responds to the frequency deviation, after then, the SFR began to act and return the frequency to nominal.
5.) Settling Time: Settling time is the time when the frequency reaches the steady state point.

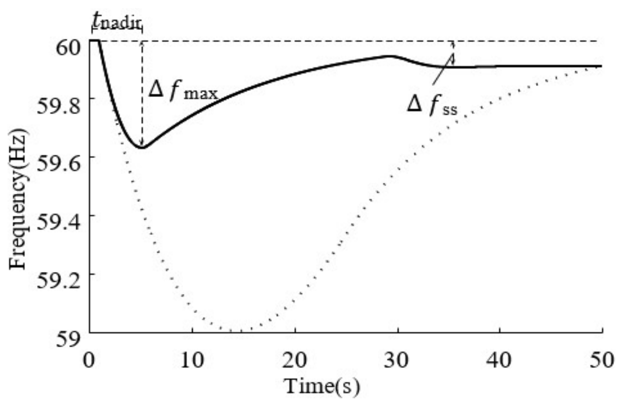

Figure 3: . Frequency performance between OPF (dashed) and FC-OPF (solid)

	S1	S2	S3	S4	S5
W1	400	225	220	150	250
W2	400	225	220	150	250

Table 2. Wind Parameter in different Scenarios

I	P_{MAX}	C	R_{MAX}	CPRF	CSRF	LTRR	STRR	Res	Droop
1	600	25	0	10	10	5	0.5	80	0.05
2	400	36	80	2	2	1	0.25	40	0.04

Table 3. Generator Parameter

The frequency dynamics within 50 seconds is plotted in Fig.2. In our simulations, during scenario S1 when there is excess of generation of power through the renewable sources, there is an increase of frequency and to tackle that situation, the excess generation is spilled to maintain the frequency stability at the expense of the spillage cost incurred. In scenario, S5, the generation of power from the renewable resources is same as we predicted in our day ahead case, so there is no imbalance of supply and demand, so there is no need to activate the frequency response and there is neither any spillage of excess power nor any load shedding due to contingency. In scenario S2, there is less generation of power, but the deficit is balanced through the reserve, so there is no loss of load but at the expense of higher cost in maintaining the frequency with in the permissible limits. But, in case S3 and S4 when the generation of power through renewable sources is less than expected in the day ahead scenario, there is load curtailment in order to maintain the balance between the load and supply as first we apply the reserve to restore the balance but since the demand is much more which we aren't able to meet even applying the reserve, so the system operator doesn't have any other option than load shedding in order to maintain the frequency within limits and generation-demand equation.

VI. Conclusion and future scope

In our model, we create an electric market which employs renewable generators along with conventional generators and discuss about the OPF and FC-OPF which is a result of the intermittent generation of power by renewable sources. In our analysis, we found that FC-OPF increases the cost of electricity to the customer but at the same time gives a better frequency performance in terms of various frequency matrices such as nadir frequency, nadir time, nadir-based frequency response and settling time. And while implementing voltage stability in our model, we found the maximum value of power transfer at a point known as the critical point in our transmission line at different phase angles of voltage above which the system becomes unstable, so we need to make sure that the voltage in our system is always above that point. In the future, we will explore the possibility of incentivizing renewable energy, storage and demand response to provide the fast frequency response. Moreover, we just focused on the intermittent generation of renewable resources keeping the load constant, but variable demand load can also be implemented into the model. Currently, renewable energy, storage and demand response are all technically qualified for the high-quality reserve. But there is a lack of market mechanism to encourage them to provide the PFR and SFR. The approach proposed in this paper could be a possible market clearing model to incentivize these emerging resources to more actively participate in the reserve market.

VII. References

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VIII. APPENDIX A

MATLAB 3 BUS SYSTEM DEFINATION

```
function mpc = mybus
%MYBUS    Power flow data for 3 bus, 2 generator and 1 Wind case.
%   Please see CASEFORMAT (MATPOWER) for details on the case file format.
%
%   Based on data from PSOC Final Project DATA.

%   MATPOWER
%   Created by ASHISH UPADHYAY with Help from AYUSH AND YASH for Final Project in PSOC

%% MATPOWER Case Format : Version 2
mpc.version = '2';

%%-----  Power Flow Data  -----%%
%% system MVA base
mpc.baseMVA = 100;

%% bus data
%   bus_i   type   Pd    Qd   Gs   Bs   area   Vm   Va   baseKV   zone   Vmax   Vmin
mpc.bus = [
    1   3   400   0   0   0   1   1   0   100   1   1.1   0.9;
    2   2   0  0   0   0   2   1   -2.69   100   2   1.1   0.9;
    3   1   600   0   0   0   3   1   -2.137   100   3   1.1   0.9;
];

%% generator data
%   bus   Pg   Qg   Qmax   Qmin   Vg   mBase   status   Pmax   Pmin   Pc1   Pc2   Qc1min   Qc1max   Qc2min   Qc2max   ramp_agc   ramp_10   ramp_30   ramp_q   apf
mpc.gen = [
    1   600   0   0   0   1   100   1   600   0   0   0   0   0   0   0   0   3   0.3   0   1;
    1   400   0   0   0   1   100   1   400   0   0   0   0   0   0   0   0   0   0   0   1;
    2   400   0   0   0   1   100   1   400   0   0   0   0   0   0   0   0   3   0.3   0   1;
    3   400   0   0   0   1   100   1   400   0   0   0   0   0   0   0   0   0   0   0   1;
];

%% branch data
%   fbus   tbus   r   x   b   rateA   rateB   rateC   ratio   angle   status   angmin   angmax
mpc.branch = [
    1   2   0   0.002   0   200   200   200   0   0   1   -360   360;
    1   3   0   0.002   0   200   200   200   0   0   1   -360   360;
    2   3   0   0.002   0   200   200   200   0   0   1   -360   360;
];

%%-----  OPF Data  -----%%
%% area data
%   area   refbus
mpc.areas = [
    1   5;
```

```
];

%% generator cost data
%     1          startup   shutdown          n          x1         y1         ...          xn          yn
%     2          startup   shutdown          n          c(n-1)     ...        c0
mpc.gencost = [
      2          100                 0          2          25  25;
      2          0                   0          2          0   0;
      2          100                 0          2          36  36;
      2          0                   0          2          0   0;
];
```

IX. APPENDIX B

GAMS CODE FOR OPTIMIZATION

```
*Set OutPut Location of gdx File generated
$setglobal location G:\PSOC\Project\PSOC Project\FINAL\

*Naming of file and parameters to be written
*$set matout "'TEst.gdx', cost,P_W,P, returnStat ";
$set matout  "fin.gdx"
$set DMULT 0.5

*Defination of Parameters
sets
i      conventional generators /i1*i2/
d      inelastic loads /d1*d2/
n      buses /n1*n3/
s      scenarios /s1/
k      Renewable power generators /k1*k2/

slack(n)         /n1/
Mapi (i,n)      /i1.n1 , i2.n2/
Mapd(d,n)      /d1.n1 , d2.n3/
Mapnm(n,n)    /n1.n2 , n2.n1, n2.n3, n1.n3, n3.n1, n3.n2/
Mapk(k,n)    /k1.n1 , k2.n3/
alias (n,m);
scalar
F0      Nominal Frequency /60/
Fmin    nadir Frequency /59/
f_db    Deadband Freq   /0.03/
td       governer response /1/
tnadir  nadir time /0.5/
Stiff    stifness /2/;
parameters
W_max(k) Renewables installed capacity
                         /k1 = 400, k2 = 400/;

parameters
Prob(s)                  probability of scenarios /
s1  1
*s2  0.20
*s3  0.20
*s4  0.20
*s5  0.20
*s6  0.1
*s7  0.1
```

```
*s8  0.1
*s9  0.1
*s10 0.1
 /

L(d) Load level /d1 600
                d2 400/

V(d) value of lost load /d1 00
                        d2 0  /

TABLE Cost_Excess(k,s)   value associated with spillage
        s1
 k1     0
 k2     0        ;



*In Case of Spot market uncomment this and Comment out above table

$ontext
TABLE Cost_Excess(k,s)   value associated with spillage
        s1      s2     s3     s4     s5
 k1     -5      -5     -5     -5     -5
 k2     -5      -5     -5     -5     -5    ;

$offtext


TABLE Generation_Para (i,*)  generators input data
        Pmax     C      Rmax   CPFR    CSFR    LTRR     STRR   ramp_rate inertia reserveMax droop
i1      600      25      0      10      5       2       0.5    3         3       0       .05
i2      400      36      80     2       1       1       0.25   3         3.5     0       .04;


 Table Renewable_Para(k,s)  Renewable realization under different scenarios
         s1
k1       400
k2       400    ;
Table Line_Limit(n,n)  Transmission lines capacity
         n1      n2     n3
n1       0       200    200
n2       200     0      200
n3       200     200    0               ;
Table Susceptance(n,n)    Tranmission lines susceptance
         n1      n2     n3
n1       0       500   500
n2       500     0     500
n3       500     500    0                 ;

variables
cost               Total expected system cost including Day Ahead and Real Time
theta_DA(n)        Voltage angles in DA
Pflow_DA(n,m)      Power flows in DA
Pflow_RT(n,m,s)    Power flows in RT
theta_RT(n,s)      Voltage angles in RT
PowerAdj(i,s)      Power adjustment of generator i in RT under scenario s
```

```
SFR(i,s)
delf(i,s)
PFR(i,s)
;
positive variables
Load_shed(d,s)     Curtailed load
Pgen_DA(i)         DA dispatch of generators
Rnew_DA(k)         Renewable dispatch in DA
Power_spill(k,s)   Wind Spillage ;
*[sum(i,Generation_Para(i,'CSFR')*SFR(i,s))]
*Equations Of Optimization
equations
ObjectiveFunction,nodalEq_DA,Pmax,Rmax,flow_DA,flow_max_DA,node_RT,UpperLmt,LowerLmt,generation_min,
generation_max,flow_RT,flow_max_RT,shedding,spillage,slack_RT,slack_DA,PFreqResponsemin,PFreqResponsemax,delafreq,PFRresv,SecFreqResup,SecFreqRes,spillageup,Sheddingup;
ObjectiveFunction.. cost=e=sum(s,Prob(s)*{[sum(i,Generation_Para(i,'C')*PowerAdj(i,s))+[sum(i,Generation_Para(i,'CPFR')*PFR(i,s))]
                                          +[sum(i,Generation_Para(i,'CSFR')*SFR(i,s))]]+[sum(d,V(d)*Load_shed(d,s))]})
                                          +[sum(i,Generation_Para(i,'C')*Pgen_DA(i))];
*Day Ahead Constrains (subjected to)
nodalEq_DA(n)..                        sum(i$Mapi(i,n),Pgen_DA(i))+sum(k$Mapk(k,n),Rnew_DA(k))-sum(d$Mapd(d,n),L(d))
                                   -sum(m$Mapnm(n,m),Pflow_DA(n,m))=e=0;
Pmax(i)..                              Pgen_DA(i)=l=Generation_Para(i,'Pmax');
*Rmax(k)..                             Rnew_DA(k)=l=Renewable_Para(k);
Rmax(k)..                              Rnew_DA(k)=e=Sum(s,Prob(s)*Renewable_Para(k,s));
flow_DA(n,m)$Mapnm(n,m)..              Pflow_DA(n,m)=e=Susceptance(n,m)*(theta_DA(n)-theta_DA(m));
flow_max_DA(n,m)$Mapnm(n,m)..          Pflow_DA(n,m)=l=Line_Limit(n,m);
slack_DA..                             theta_DA('n1')=e=0;
* Real Time Constrains
node_RT(n,s) ..                        sum(i$Mapi(i,n),PowerAdj(i,s))+sum(k$Mapk(k,n),Renewable_Para(k,s)-Rnew_DA(k)-Power_spill(k,s))
                                   +sum(d$Mapd(d,n),Load_shed(d,s))-sum(m$Mapnm(n,m),Pflow_RT(n,m,s)-Pflow_DA(n,m))=e=0;
UpperLmt(i,s)..                        PowerAdj(i,s)=l=+Generation_Para(i,'Rmax');
LowerLmt(i,s)..                        PowerAdj(i,s)=g=-Generation_Para(i,'Rmax');
*LowerLmt(i,s)..                       PowerAdj(i,s)=g=0;
generation_min(i,s) ..                 [Pgen_DA(i)+PowerAdj(i,s)-PFR(i,s)]=g=0;
```

```
generation_max(i,s) ..                  [Pgen_DA(i)+PowerAdj(i,s)+PFR(i,s)+SFR(i,s)]=l=Generation_Para(i,'Pmax');
PFRresv(i,s)..                          PFR(i,s) =l=  Generation_Para(i,'reserveMax') ;
*PFRresvmin(i,s)..                      PFR(i,s) =g=  0;
PFreqResponsemax(i,s)..                 PFR(i,s) =l=  Generation_Para(i,'STRR') * tnadir;
PFreqResponsemin(s)..                   sum(i,PFR(i,s))=g= sum(k,Power_spill(k,s)) + sum(d,Load_shed(d,s));
SecFreqRes(i,s)..                       SFR(i,s)=l= 10*Generation_Para(i,'Ramp_Rate');
delafreq(i,s)..                         delf(i,s)=e= -1*(sum(k,Power_spill(k,s)) - sum(d,Load_shed(d,s))) /( (sum(d,L(d))/f0)*((1/Generation_Para(i,'Droop')+(Stiff))) );
SecFreqResup(s)..                       sum(i,SFR(i,s)) =g= -1* Sum(i,(delf(i,s))*(1/Generation_Para(i,'Droop'))+(sum(k,Power_spill(k,s)) + sum(d,Load_shed(d,s))));
flow_RT(n,m,s)$Mapnm(n,m) ..            Pflow_RT(n,m,s)=e=Susceptance(n,m)*(theta_RT(n,s)-theta_RT(m,s));
flow_max_RT(n,m,s)$Mapnm(n,m) ..        Pflow_RT(n,m,s)=l=Line_Limit(n,m);
shedding(d,s)..                         Load_shed(d,s)=l=L(d);
sheddingup(d,s)..                       Load_shed(d,s)=g=0;
spillage(k,s)..                         Power_spill(k,s)=l=Renewable_Para(k,s);
spillageup(k,s)..                       Power_spill(k,s)=g=0;
slack_RT(s)..                           theta_RT('n1',s)=e=0;
*frequency Stablity
model Stochastic_clearing /all/  ;
$if exist matout.gms $include matout.gms
option MIP=CPLEX   ;
solve Stochastic_clearing using MIP minimizing cost ;
*time.l = Stochastic_clearing.resusd;
parameters
weightedcostspill                 Weighted cost spill
costspill                         Cost of spillage
LambdaN_DA(n)
spill_T(k,s)
```

```
costnew                          Cost After considering spillage cost
LambdaN_RT(n,s)        ;
LambdaN_DA(n)=nodalEq_DA.m(n)   ;
LambdaN_RT(n,s)=node_RT.m(n,s)/Prob(s);
Spill_T(k,s)=Power_spill.l(k,s);
*costnew = cost.l+sum((k,s),Spill_T(k,s)*sp(k,s));
costspill =  sum((k,s),Spill_T(k,s)*Cost_Excess(k,s));
weightedcostspill  =sum((k,s),Spill_T(k,s)*Cost_Excess(k,s)* Prob(s));
costnew = cost.l+weightedcostspill;
display
SFR.l,PFR.l,delf.l,cost.l,weightedcostspill, costnew, LambdaN_DA, LambdaN_RT, Rnew_DA.l, Pgen_DA.l, PowerAdj.l, Power_spill.l, Load_shed.l,Spill_T;
*$libinclude matout cost.l
*$libinclude matout P_W.l
*set stat /cost, P_W,P/;
*parameter returnStat(stat);
*returnStat('cost') = Stochastic_clearing.modelstat;
*returnStat('P_W') = Stochastic_clearing.modelstat;
*returnStat('P') = Stochastic_clearing.modelstat;
execute_unload "%location%%matout%";
```

X. APPENDIX C

MATLAB IMPORT DATA

```
phi = [20 10 0 -10 -15];
s.name='Susceptance'
s.form='full'
s.compress=true
x =rgdx('fin',s)
Sus=x.val
Suels=x.uels
s.name='Pflow_RT'
x =rgdx('fin',s)
Pow=x.val
puels=x.uels
Z=1./Sus
E=Pow./Z
E = sqrt(E(:,:,5))
fig=1;
for i = 1:3
for j=1:3
if Pow(i,j,5)>0
figure (fig)
```

```
fig=fig+1;
PlotPV(Z(i,j),E(i,j),phi)
title(['Between Node ',num2str(i),' and ',num2str(j)]);
grid on

end
end
end
```

PLOT PV

```
%       Data imported from GAMS Optimized soluton
%   Created by ASHISH UPADHYAY with Help from AYUSH AND YASH for Final Project in PSOC
function PlotPV(X,E,phi)
 %clear all; clc;
%phi = [20 10 0 -10 -15];
 %X = 37.4; %Reactance of LINE
%E = 132; % Rated voltage of BUS
 pmax = @(phi) -((sind(phi)-1)*E.^2)/(2*cosd(phi)*X);

for k = 1:length(phi)% for each value of phi, do the following:
 % #1 - Determine maximum power, Pmax, for current value of phi.
Pmax = pmax(phi(k));

 % #2 - Generate a P vector going from zero to Pmax.
% Make the distance between points smaller at the tip...
 % on the curve, for example:
p = 0:Pmax/100:Pmax*9/10;
 p = [p,(Pmax*(9/10+1/1000):Pmax/1000:Pmax)];
% #3 - Calculate the upper and lower voltage solutions at each...
 % point in the P vector.
V_upper = sqrt(-X*tand(phi(k))*p+(E.^2)/2+sqrt...
 ((E.^4)/4-X.^2*p.^2-E.^2*X*tand(phi(k))*p));
V_lower = sqrt(-X*tand(phi(k))*p+(E.^2)/2-sqrt...
 ((E.^4)/4-X.^2*p.^2-E.^2*X*tand(phi(k))*p));
% #4 - Plot the voltage V as function of the power P
 hold on
plot(p,abs(V_upper),'b')
 plot(p,abs(V_lower),'r')
% #5 - add a label to the curve containing the current value of...
 % phi in degrees
str=sprintf('\\Phi=%.1f',phi(k));
str2=sprintf('\n\nV=%.1f kV',abs(V_upper(end)));
str3=sprintf('\n\nP=%.1f MW',max(p)/1000000);
 text(Pmax*1.01,V_upper(end),strcat(str,str2,str3),'HorizontalAlignment','left')
%stem(max(p),abs(V_upper(end)),'filled','o','MarkerSize',3)
scatter(max(p),abs(V_upper(end)),'filled');
end

 % Add axis labels and plot the locus of critical points
phi = -30:1:90;
 Pmax_vec = zeros(1,length(phi));
V_vec = zeros(1,length(phi));
 for k = 1:length(phi)
Pmax_vec(k) = pmax(phi(k));
 V_vec(k) = sqrt(-X*tand(phi(k))*Pmax_vec(k)+(E.^2)/2);
```

```
 
end
 grid on
ax = gca;
 ax.GridLineStyle = ':';
plot(Pmax_vec,abs(V_vec),'k--')
 str=sprintf('Locus of critical points');
text(Pmax_vec(end)*0.99,V_vec(end),str,'VerticalAlignment'...
 ,'middle','HorizontalAlignment','right')
%axes label
% xlim([0 350])
xlabel('Active Power P [MW]')
 ylabel('Voltage V [kV]')
print -depsc PLOTPV.eps
```

XI. APENDIX D

RECEIVING END VOLTAGE USING NR

```
% Newton Raphson Method to find the Recieving End Voltage
%P= E*B*Sin(Theta)
%Q= -E*B*Cos(Theta)
%Assuming Initial Condition of 1 and Angle 0
%Jacobian Matrix is
%
%J = [ V*B*Cos(theta)   B*sin(Theta);
%       V*B*sin(Theta)  -B*Cos(Theta);]
%Data imported from GAMS Optimized soluton
%   Created by ASHISH UPADHYAY, AYUSH AND YASH for Final Project in PSOC
clear all
format short

s.name='Susceptance';
s.form='full';
s.compress=true;

x =rgdx('fin',s);
Sus=x.val;
Suels=x.uels;

s.name='Pflow_RT';
x =rgdx('fin',s);
Pow=x.val;
puels=x.uels;
for a = 1:5

disp(['Calculation for Probablistic Senario # ',num2str(a)]);
for i = 1:length(Pow(:,:,a))
    for j = 1:length(Pow(:,:,a))
        if Pow(i,j,a)>0
B=5;
x=[0;1];
P=Pow(i,j,a)/100;

Q=0; %Enter Value of P and Q
% The Newton-Rapshson iterations starts here
del=1;
```

```
indx=0;
while del>1e-6
f=[(B*x(2)*sin(x(1)))-P; (B*x(2)*cos(x(1)))-Q];
J=[B*x(2)*cos(x(1)),B*sin(x(1)); -1*B*x(2)*sin(x(1)),cos(x(1))];
delx=-pinv(J)*f;
x=x+delx;
del=max(abs(f));
indx=indx+1;
end
disp(['Between Node ',num2str(i),' and ',num2str(j)]);
x
%figure(a);
%stem(a,x(2));
%disp(['P.F. Between Node ',num2str(i),' and ',num2str(j)]);
%cos(x(2))
%disp(['Recieving End Volt Between Node ',num2str(i),' and ',num2str(j)]);
%x(1)
        end
        end
    end
end
```