Algebra y Geometría

Primer Cuatrimestre de 2020

Práctica 3

1. En cada uno de los siguientes casos, construir la matriz correspondiente:

a) $A \in \mathbb{R}^{3\times 3}$, $a_{ij} = (-1)^{i+j}$.

b) $A \in \mathbb{R}^{2\times 3}$, $a_{ij} = (-1)^{i}(i+j)$.

c) $A \in \mathbb{R}^{4\times 4}$, $a_{ij} = \begin{cases} 1, & \text{si } i \leq j, \\ 0, & \text{en otro caso.} \end{cases}$

d) $A \in \mathbb{R}^{4\times 3}$, $a_{ij} = \begin{cases} ij, & \text{si } i = j, \\ 0, & \text{si } i \neq j. \end{cases}$

2. Calcular las matrices traspuestas de

$$A = \begin{pmatrix} 3 & 4 \\ 2 & 1 \\ 5 & 0 \end{pmatrix}, \qquad B = \begin{pmatrix} 2 & 5 & 7 \\ 1 & 2 & 3 \end{pmatrix} \quad \text{y} \quad C = \begin{pmatrix} 2 & 5 & 7 \\ 6 & -1 & 1 \\ 5 & 1 & 4 \end{pmatrix}.$$

3. Dadas las matrices $A = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$; $B = \begin{pmatrix} 1 & 0 \\ 3 & 1 \end{pmatrix}$ y $C = \begin{pmatrix} -1 & 0 \\ -1 & 0 \end{pmatrix}$, calcular

a) $3A - 2B + C$ b) $A - 3(B - C)$ c) $A - B + 2C^t$ d) $A - (B - 2C)^t$.

4. Sean $A \in \mathbb{R}^{4\times 5}$, $B \in \mathbb{R}^{5\times 7}$, $C \in \mathbb{R}^{4\times 5}$ y $D \in \mathbb{R}^{7\times 5}$. Indicar cuáles de las siguientes operaciones son posibles. En el caso afirmativo, indicar el tamaño (número de filas y número de columnas) de la matriz resultado.

a) AB, b) BA, c) AC, d) AC^t,

e) CB f) BDA, g) ABD h) $B(CD^t)^t$.

5. Cuando sea posible, calcular AB y BA ¿Vale la igualdad entre estos productos?

a) $A = \begin{pmatrix} 2 & 3 \\ 1 & -4 \end{pmatrix}$, $B = \begin{pmatrix} 3 & -2 & 2 \\ 1 & 0 & -1 \end{pmatrix}$.

b) $A = \begin{pmatrix} 1 & 2 & 3 \end{pmatrix}$, $B = \begin{pmatrix} -2 \\ 4 \\ 1 \end{pmatrix}$.

c) $A = \begin{pmatrix} 3 & 4 & 0 \\ -1 & 0 & 2 \end{pmatrix}$, $B = \begin{pmatrix} 6 & -1 & 2 \\ 0 & 4 & 5 \\ -1 & 5 & 4 \end{pmatrix}$.

d) $A = \begin{pmatrix} 1 & 0 & 0 \\ 2 & -1 & 2 \\ 2 & 2 & 1 \end{pmatrix}$, $B = \begin{pmatrix} 2 & 1 & 4 \\ 3 & 0 & -1 \\ 4 & -1 & 5 \end{pmatrix}$.

6. Sean $A = \begin{pmatrix} 3 & -4 \\ -5 & 1 \end{pmatrix}$ y $B = \begin{pmatrix} 7 & 4 \\ 5 & k \end{pmatrix}$.

Hallar todos los valores de $k \in \mathbb{R}$ para los cuales $A.B = B.A$.

7. Sean

$$A = \begin{pmatrix} 3 & 4 \\ -2 & 1 \\ 5 & 0 \end{pmatrix}, \qquad B = \begin{pmatrix} 2 & -5 & 7 \\ -1 & 2 & 3 \end{pmatrix}$$

Calcular

a) A^t y B^t b) $(A^t)^t$ c) $(AB)^t$ y B^tA^t.

8. Dadas las matrices

$$A = \begin{pmatrix} 3 & 4 & 0 \\ -1 & 0 & 2 \\ 1 & 2 & 0 \end{pmatrix} \quad \text{y} \quad B = \begin{pmatrix} 6 & -1 & 2 \\ 0 & 4 & 5 \\ -1 & 5 & 4 \end{pmatrix}$$

Hallar todas las $X \in \mathbb{R}^{3\times 3}$ que verifican $3X - 2A = 5B$.

9. Dado el sistema

$$S: \begin{cases} 2x_1 - 5x_2 + 3x_3 - 8x_4 = -2 \\ x_1 + 3x_2 - x_3 - 4x_4 = -6 \\ -x_1 + 2x_2 + x_3 - 6x_4 = 9 \end{cases}$$

a) Rescribir el sistema como producto de matrices (notación matricial).

b) Idem a) para el sistema homogéneo asociado.

10. En cada uno de los siguientes casos, rescribir, exhibiendo las ecuaciones, el sistema lineal $A\mathbf{x} = \mathbf{b}$ y hallar la solución.

a) $A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 2 & 2 \\ 1 & -1 & 0 \end{pmatrix}$, $b = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$, $b = \begin{pmatrix} 1 \\ 0 \\ 4 \end{pmatrix}$ $b = \begin{pmatrix} 16 \\ 12 \\ -5 \end{pmatrix}$.

b) $A = \begin{pmatrix} 1 & 1 & 2 & -1 \\ 2 & 1 & 1 & 0 \\ -1 & 1 & 2 & -1 \\ 0 & 2 & 4 & -2 \end{pmatrix}$, $b = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$, $b = \begin{pmatrix} 1 \\ 2 \\ 1 \\ 2 \end{pmatrix}$ $b = \begin{pmatrix} 2 \\ 0 \\ -1 \\ 3 \end{pmatrix}$.

11. Calcular, si es posible, la matriz inversa de cada una de las siguientes matrices. Verificar que la matriz hallada es efectivamente la inversa.

a) $\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$ b) $\begin{pmatrix} 3 & 4 \\ 0 & 0 \end{pmatrix}$ c) $\begin{pmatrix} 1 & 2 \\ -1 & 3 \end{pmatrix}$

d) $\begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 1 & 2 & 4 \end{pmatrix}$ e) $\begin{pmatrix} 1 & -2 & 1 \\ 0 & 1 & 2 \\ 1 & 1 & -1 \end{pmatrix}$ f) $\begin{pmatrix} 1 & 2 & -1 \\ -1 & 0 & 2 \\ 0 & 2 & 1 \end{pmatrix}$

g) $\begin{pmatrix} 1 & 2 & 1 \\ -1 & 1 & 0 \\ 1 & 0 & -1 \end{pmatrix}$ h) $\begin{pmatrix} 2 & 1 & 1 \\ 0 & 1 & 1 \\ 3 & 1 & -1 \end{pmatrix}$ i) $\begin{pmatrix} 2 & 1 & 1 \\ 0 & 1 & 1 \\ 2 & 0 & 0 \end{pmatrix}$

12. Verificar que $A = \begin{pmatrix} 1 & 2 \\ 1 & -1 \end{pmatrix}$ y $B = \begin{pmatrix} 3 & -1 \\ -2 & 1 \end{pmatrix}$ son inversibles y calcular

a) A^{-1}, B^{-1}. b) $(A^{-1})^{-1}$. c) $(A^t)^{-1}$.

d) $(A^{-1})^t$ e) $(AB)^{-1}$. f) $B^{-1}A^{-1}$.

13. Sean $A = \begin{pmatrix} 0 & -1 \\ -1 & 1 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 0 \\ 1 & 2 \end{pmatrix}$ y $C = \begin{pmatrix} -2 & 4 \\ -1 & 6 \end{pmatrix}$.

Hallar **todas** las $X \in \mathbb{R}^{2\times 2}$ que verifican $AX + 2X = B^tX + \frac{1}{2}C$.

14. Sean $A = \begin{pmatrix} -1 & -1 \\ -2 & 0 \end{pmatrix}$ y $B = \begin{pmatrix} 3 & -1 \\ -6 & 2 \end{pmatrix}$.

Hallar **todas** las $X \in \mathbb{R}^{2\times 2}$ que verifican $AX = -2X + B$.

15. Sean $A = \begin{pmatrix} 2 & 2 & -1 \\ 3 & 3 & 2 \\ 1 & 0 & 3 \end{pmatrix}$ y $B = \begin{pmatrix} 1 & 0 & -1 \\ 3 & 1 & 0 \\ 1 & -2 & -3 \end{pmatrix}$.

Hallar **todas** las $X \in \mathbb{R}^{3\times 3}$ que verifican $AX = 2X + B^t$.

16. Sea $A \in \mathbb{R}^{3\times 3}$ tal que $(A-3I)^{-1} = \begin{pmatrix} 1 & 1 & -1 \\ 0 & -2 & 1 \\ 0 & 0 & 1 \end{pmatrix}$.

Sean $B = \begin{pmatrix} -1 & 2 & 0 \\ 2 & 1 & 3 \\ 1 & 0 & 1 \end{pmatrix}$ y $b = \begin{pmatrix} -1 \\ 1 \\ -1 \end{pmatrix}$.

Hallar todas las $X \in \mathbb{R}^{3\times 1}$ para las cuales $ABX = 3BX + b$.

Matrices

$A \in \mathbb{R}^{n \times m}$ $\quad 1 \leq i \leq n$ $\quad a_{ij} = i + j$
$\quad\quad 1 \leq j \leq m$

(Fila, columna)

$$A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1m} \\ a_{21} & \cdots & \cdots & a_{2m} \\ \cdots & \cdots & & \\ a_{n1} & a_{n2} & \cdots & a_{nm} \end{pmatrix}$$

$A = (a_{ij})$

$A \in \mathbb{R}^{3 \times 4}$ $\quad A = \begin{pmatrix} 2 & 3 & 4 & 5 \\ 3 & 4 & 5 & 6 \\ 4 & 5 & 6 & 7 \end{pmatrix}$ $\quad 1 \leq i \leq 3$, $1 \leq j \leq 4$

$a_{11} = 1+1$
$a_{12} = 1+2$

$B \in \mathbb{R}^{3 \times 3}$

$$b_{ij} = \begin{cases} 1 & \text{si } i > j \\ 0 & \text{si } i \leq j \end{cases} \quad B = \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 1 & 0 \end{pmatrix}$$

Matriz traspuesta

$A \in \mathbb{R}^{n \times m} \longrightarrow A^T \in {}^{m \times n}$ $\quad \underset{(a^T{}_{ij})}{A^T} = \left(\text{en } (i,j) \text{ tiene a } (j,i) \text{ de la matriz original} \right)$

$$A = \begin{pmatrix} 1 & 4 \\ 2 & 3 \\ 1 & 5 \end{pmatrix} \quad A^T = \begin{pmatrix} 1 & 2 & 1 \\ 4 & 3 & 5 \end{pmatrix}$$

$A \in \mathbb{R}^{3 \times 2}$ $\quad A^T \in \mathbb{R}^{2 \times 3}$

Suma de matrices

$$\begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1m} \\ \cdots & \cdots & & a_{2m} \\ a_{n1} & & & a_{nm} \end{pmatrix} + \begin{pmatrix} b_{11} & b_{12} & \cdots & \\ \cdots & \cdots & & \\ \cdots & \cdots & & b_{nm} \end{pmatrix} = \begin{pmatrix} a_{11}+b_{11}, & \cdots & , a_{1m}+b_{1m} \\ \cdots & \cdots & \cdots \\ \cdots & \cdots & , a_{nm}+b_{nm} \end{pmatrix}$$

$$\begin{pmatrix} 1 & 3 & -1 \\ 0 & 6 & 3 \\ 0 & 0 & 2 \end{pmatrix} + \begin{pmatrix} 5 & -1 & 0 \\ 1 & 1 & 2 \\ 2 & 3 & 4 \end{pmatrix} = \begin{pmatrix} 6 & 2 & -1 \\ 1 & 7 & 5 \\ 2 & 3 & 6 \end{pmatrix}$$

Producto de matriz por escalar

$$\alpha \begin{pmatrix} a_{11} & a_{12} & \cdots \\ \cdots & \cdots & \cdots \\ \cdots & \cdots & a_{nm} \end{pmatrix} = \begin{pmatrix} \alpha a_{11} & \alpha a_{12} & \cdots \\ \cdots & \cdots & \cdots \\ \cdots & \cdots & \alpha a_{nm} \end{pmatrix} \quad 2\begin{pmatrix} -4 & 0 & 2 \\ 5 & 1 & 3 \end{pmatrix} = \begin{pmatrix} -8 & 0 & 4 \\ 10 & 2 & 6 \end{pmatrix}$$

$\alpha \in \mathbb{R}$, $A \in \mathbb{R}^{n \times m}$ $\quad A = (a_{ij}) \rightarrow \alpha \in \mathbb{R}$ $\quad \alpha A = (\alpha a_{ij})$

Propiedades

$A, B, C \in \mathbb{R}^{n \times m}$ $\quad \alpha_1, \alpha_2 \in \mathbb{R}$

a) $A + (B + C) = (A + B) + C$
b) $A + B = B + A$
c) $\alpha_1 (A + B) = \alpha_1 A + \alpha_1 B$
d) $(\alpha_1 + \alpha_2) A = \alpha_1 A + \alpha_2 A$
e) $A + 0 = A$
f) $A + (-A) = 0$
g) $0 A = 0$
h) $(A + B)^T = A^T + B^T$

Producto de matrices

$A \in \mathbb{R}^{n \times s}$ $\quad A = (a_{ij})$ $\quad C = (c_{ij})$ $\quad c_{ij}$ p. esc. de la fila i de A por la columna j de B.

$B \in \mathbb{R}^{s \times t}$ $\quad B = (B_{ij})$

$C \in \mathbb{R}^{n \times t}$

$A = \begin{pmatrix} 1 & 3 & -1 \end{pmatrix}$ $\quad B = \begin{pmatrix} 4 \\ -1 \\ 2 \end{pmatrix}$ $\quad C = A . B = \begin{pmatrix} -1 \end{pmatrix}$

$$A.B \qquad \begin{pmatrix} 2 & 3 & 1 \\ 3 & 0 & 2 \\ -1 & 1 & 0 \end{pmatrix} \begin{pmatrix} 2 & 0 \\ -1 & 2 \\ 4 & 1 \end{pmatrix} = \begin{pmatrix} 5 & 7 \\ 14 & 2 \\ -3 & 2 \end{pmatrix}$$

$$E.F: \quad \begin{pmatrix} -1 & 1 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} 2 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} -1 & -1 \\ 5 & 2 \end{pmatrix} \qquad F.E: \quad \begin{pmatrix} 2 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} -1 & 1 \\ 2 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 3 \\ -1 & 1 \end{pmatrix}$$

$E.F \neq F.E$

$G \begin{pmatrix} 3 & 1 \\ 1 & 1 \end{pmatrix}$ $\quad H \begin{pmatrix} 2 & 1 \\ 1 & 0 \end{pmatrix}$ $\quad G.H = \begin{pmatrix} 7 & 3 \\ 3 & 1 \end{pmatrix}$ $\quad H.G = \begin{pmatrix} 7 & 3 \\ 3 & 1 \end{pmatrix}$ $\quad GH = HG$

"El producto de matrices generalmente no es conmutativo"

Matriz Identidad (I)

$I \in \mathbb{R}^{n \times n}$

$$I = \begin{pmatrix} 1 & 0 & 0 & \cdots & \\ 0 & 1 & 0 & \cdots & \\ 0 & 0 & 1 & \cdots & \\ \cdots & & & 1 & 0 \\ \cdots & & & 0 & 1 \end{pmatrix} \qquad I_2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$\boxed{AI = IA = A}$

Propiedades del producto de matrices

$A \in \mathbb{R}^{m \times n}$ $\quad B \in \mathbb{R}^{n \times k}$ $\quad C \in {}^{k \times h}$

$A(B.C) = (A.B).C$

$A \in \mathbb{R}^{m \times n}$ $\quad B, C \in \mathbb{R}^{n \times k}$

$A(B+C) = AB + AC$

$A \in \mathbb{R}^{n \times k}$ $\quad \alpha_1, \alpha_2 \in \mathbb{R}$

$(\alpha_1 . \alpha_2) A = \alpha_1 (\alpha_2 A)$

$A \in \mathbb{R}^{n \times k}$ $\quad B \in \mathbb{R}^{k \times m}$ $\quad k \in \mathbb{R}$

$\alpha(A.B) = (\alpha A)B = A(\alpha B)$

$A \in \mathbb{R}^{n \times k}$ $\quad B \in \mathbb{R}^{k \times n}$

$(A.B)^T = B^T . A^T$

Matriz inversa

$A \in \mathbb{R}^{n \times n}$, hay matriz inversa, si $\exists$ $B \in \mathbb{R}^{n \times n}$

$A.B = B.A = I$ $\qquad$ B es la matriz inversa de A

$B \in \mathbb{R}^{n \times n}$

$A = \begin{pmatrix} 8 & 3 \\ 5 & 2 \end{pmatrix}$ Buscar $B \,/\, A.B = I$

$B = \begin{pmatrix} x_1 & x_2 \\ y_1 & y_2 \end{pmatrix}$ $\begin{cases} 8x_1 + 3y_1 = 1 \\ 5x_1 + 2y_1 = 0 \\ 8x_2 + 3y_2 = 0 \\ 5x_2 + 2y_2 = 1 \end{cases}$ $\left(\begin{array}{cc|c} 8 & 3 & 1 \\ 5 & 2 & 0 \end{array}\right)$ $\left(\begin{array}{cc|c} 8 & 3 & 0 \\ 5 & 2 & 1 \end{array}\right)$

A

A

$\left(\begin{array}{cc|cc} 8 & 3 & 1 & 0 \\ 5 & 2 & 0 & 1 \end{array}\right)$ $\frac{1}{8}F_1 = F_1'$

$\left(\begin{array}{cc|cc} 1 & 3/8 & 1/8 & 0 \\ 5 & 2 & 0 & 1 \end{array}\right)$ $F_2' - 5F_1' = F_2''$

$\left(\begin{array}{cc|cc} 1 & 3/8 & 1/8 & 0 \\ 0 & 1/8 & -5/8 & 1 \end{array}\right)$ $F_1'' - 3F_2'' = F_1'''$

$\left(\begin{array}{cc|cc} 1 & 0 & 2 & -3 \\ 0 & 1/8 & -5/8 & 1 \end{array}\right)$ $8F_2''' = F_2^{IV}$

$\left(\begin{array}{cc|cc} 1 & 0 & 2 & -3 \\ 0 & 1 & -5 & 8 \end{array}\right)$

Verificar $\leftarrow B = \begin{pmatrix} 2 & -3 \\ -5 & 8 \end{pmatrix} \leftarrow$ $x_1 = 2$ $x_2 = -3$ $y_1 = -5$ $y_2 = 8$

$\begin{pmatrix} 2 & -3 \\ -5 & 8 \end{pmatrix}$

$\left(\begin{array}{cc|cc} 8 & 3 & 1 & 0 \\ 5 & 2 & 0 & 1 \end{array}\right)$

$A = \begin{pmatrix} 4 & 2 \\ 2 & 1 \end{pmatrix} \to \exists A^{-1}$?

$\left(\begin{array}{cc|cc} 4 & 2 & 1 & 0 \\ 2 & 1 & 0 & 1 \end{array}\right)$

$\left(\begin{array}{cc|cc} 1 & 1/2 & 1/4 & 0 \\ 2 & 1 & 0 & 1 \end{array}\right)$

$\left(\begin{array}{cc|cc} 1 & 1/2 & 1/4 & 0 \\ 0 & 0 & -1/2 & 1 \end{array}\right)$ SI: $\exists A^{-1}$

$\left(\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 2 & 2 & 1 & 0 & 1 & 0 \\ 3 & 1 & 2 & 0 & 0 & 1 \end{array}\right)$ $F_2 - 2F_1 = F_2'$, $F_3 - 3F_1 = F_3'$

$\left(\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 2 & -1 & -2 & 1 & 0 \\ 0 & 1 & -1 & -3 & 0 & 1 \end{array}\right)$ $F_2' : 2 = F_2''$

$\left(\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & -1/2 & -1 & 1/2 & 0 \\ 0 & 1 & -1 & -3 & 0 & 1 \end{array}\right)$ $F_3'' - F_2'' = F_3'''$

$\left(\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & -1/2 & -1 & 1/2 & 0 \\ 0 & 0 & -1/2 & -2 & -1/2 & 1 \end{array}\right)$ $2F_3''' = F_3^{IV}$

$\left(\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & -1/2 & -1 & 1/2 & 0 \\ 0 & 0 & 1 & 4 & 1 & -2 \end{array}\right)$ $F_1^V - F_3^V = F_1^{VI}$, $F_2^V + \frac{1}{2}F_3^V = F_2^{VI}$

$\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -3 & -1 & 2 \\ 0 & 1 & 0 & 1 & 1 & -1 \\ 0 & 0 & 1 & 4 & 1 & -2 \end{array}\right)$

$\longrightarrow B = \begin{pmatrix} -3 & -1 & 2 \\ 1 & 1 & -1 \\ 4 & 1 & -2 \end{pmatrix} = A^{-1}$

Propiedades de matriz inversa

- $(A^{-1})^{-1} = A$
- $(A.B)^{-1} = B^{-1} . A^{-1}$.
- A^t es inversible $(A^t)^{-1} = (A^{-1})^t$

Sistemas expresados en forma matricial

$\begin{cases} a_{11}x_1 + a_{12}x_2 \ldots + a_{1n}x_n = b_1 \\ \ldots \qquad\qquad\qquad\quad \ldots = b_2 \\ a_{m1}x_1 + a_{m2}x_2 \ldots + a_{mn}x_n = b_m \end{cases}$

$A.X = B$

$A \in \mathbb{R}^{m \times n}$ A = matriz de los coeficientes

$\begin{pmatrix} a_{11} & a_{12} & \ldots & a_{1n} \\ a_{21} & \ldots & \ldots & \ldots \\ \ldots & & & a_{mn} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_n \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \\ b_m \end{pmatrix}$

$X \in \mathbb{R}^{m \times 1}$ X = matriz de las incógnitas

$B \in \mathbb{R}^{m \times 1}$ B = matriz de los t. indep'tes

NOTA

FECHA

Resolución matricial de sistemas

$$AX = B$$

$$A^{-1}(AX) = A^{-1}.B \quad \text{siempre que } \exists A^{-1}$$

$$I.X = A^{-1}.B$$

$$X = \boxed{A^{-1}B}$$

$$\left\{ \begin{array}{l} A \in \mathbb{R}^{n\times n} \wedge \exists A^{-1} \\ \Rightarrow AX = B \text{ tiene} \\ \text{única solución} \\ \text{(SCD)} \end{array} \right\} \to \text{para cualquier } B$$

$$\downarrow$$

$AX = 0$ tiene única solución la trivial

$$\begin{cases} x_1 - x_3 = 3 \\ 2x_1 + 2x_2 + x_3 = 1 \\ 3x_1 + x_2 + 2x_3 = 0 \end{cases} \qquad \overbrace{\begin{pmatrix} 1 & 0 & -1 \\ 2 & 2 & 1 \\ 3 & 1 & 2 \end{pmatrix}}^{A} \cdot \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 3 \\ 1 \\ 0 \end{pmatrix}$$

$$A \longrightarrow \exists A^{-1} = \begin{pmatrix} -3 & -1 & 2 \\ 1 & 1 & -1 \\ 4 & 1 & -2 \end{pmatrix}$$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} -3 & -1 & 2 \\ 1 & 1 & -1 \\ 4 & 1 & -2 \end{pmatrix} \begin{pmatrix} 3 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} -10 \\ 4 \\ 13 \end{pmatrix}$$

Práctica 4

① a) $\begin{pmatrix} 1 & -1 & 1 \\ -1 & 1 & -1 \\ 1 & -1 & 1 \end{pmatrix}$ b) $\begin{pmatrix} -2 & -3 & -4 \\ 3 & 4 & 5 \end{pmatrix}$ c) $\begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$ d) $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 9 \\ 0 & 0 & 0 \end{pmatrix}$

② $A^T = \begin{pmatrix} 3 & 2 & 5 \\ 4 & 1 & 0 \end{pmatrix}$ $B^T \begin{pmatrix} 2 & 1 \\ 5 & 2 \\ 7 & 3 \end{pmatrix}$ $C^T \begin{pmatrix} 2 & 6 & 5 \\ 5 & -1 & 1 \\ 7 & 1 & 4 \end{pmatrix}$

③ a) $3A - 2B + C: \begin{pmatrix} 3 & 6 \\ 0 & 3 \end{pmatrix} - \begin{pmatrix} 2 & 0 \\ 6 & 2 \end{pmatrix} + \begin{pmatrix} -1 & 0 \\ -1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 6 \\ -7 & 1 \end{pmatrix}$

b) $A - 3(B - C) = A - 3B + 3C = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix} - \begin{pmatrix} 3 & 0 \\ 9 & 3 \end{pmatrix} + \begin{pmatrix} -3 & 0 \\ -3 & 0 \end{pmatrix} = \begin{pmatrix} -5 & 2 \\ -12 & -2 \end{pmatrix}$

c) $A - B + 2C^t = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix} - \begin{pmatrix} 1 & 0 \\ 3 & 1 \end{pmatrix} + 2\begin{pmatrix} -1 & -1 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 2 \\ -3 & 0 \end{pmatrix} + \begin{pmatrix} -2 & -2 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} -2 & 0 \\ -3 & 0 \end{pmatrix}$

d) $A - (B - 2C)^t = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix} - \left[\begin{pmatrix} 1 & 0 \\ 3 & 1 \end{pmatrix} - \begin{pmatrix} -2 & 0 \\ -2 & 0 \end{pmatrix}\right]^t = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix} - \begin{pmatrix} 3 & 5 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} -2 & -3 \\ 0 & 0 \end{pmatrix}$

④ $A \in \mathbb{R}^{4\times5}$, $B \in \mathbb{R}^{5\times7}$, $C \in \mathbb{R}^{4\times5}$, $D \in \mathbb{R}^{7\times5}$

a) $AB \in \mathbb{R}^{4\times7}$ b) $BA \nexists$ c) $AC \nexists$ d) $AC^t \in \mathbb{R}^{4\times4}$ e) $CB \in \mathbb{R}^{4\times7}$

f) $BDA \nexists$ g) $ABD \in \mathbb{R}^{4\times5}$ h) $CD^t \in \mathbb{R}^{4\times7} \to (CD^t)^t \in \mathbb{R}^{7\times4} \to B(CD^t)^t \in \mathbb{R}^{5\times4}$

⑤ a)

$A = \begin{pmatrix} 2 & 3 \\ 1 & -4 \end{pmatrix}$ $B = \begin{pmatrix} 3 & -2 & 2 \\ 1 & 0 & -1 \end{pmatrix}$ $AB = \begin{pmatrix} 9 & -4 & 1 \\ -1 & -2 & 6 \end{pmatrix}$ $BA \nexists$

b) $A = \begin{pmatrix} 1 & 2 & 3 \end{pmatrix}$ $B = \begin{pmatrix} -2 \\ 4 \\ 1 \end{pmatrix}$ $AB = \begin{pmatrix} 9 \end{pmatrix}$ $BA = \begin{pmatrix} -2 & -4 & -6 \\ 4 & 8 & 12 \\ 1 & 2 & 3 \end{pmatrix}$

c) $A = \begin{pmatrix} 3 & 4 & 0 \\ -1 & 0 & 2 \end{pmatrix}$ $B = \begin{pmatrix} 6 & -1 & 2 \\ 0 & 4 & 5 \\ -1 & 5 & 4 \end{pmatrix}$ $AB = \begin{pmatrix} 18 & 13 & 26 \\ -8 & 11 & 6 \end{pmatrix}$ $B.A \nexists$

d) $A = \begin{pmatrix} 1 & 0 & 0 \\ 2 & -1 & 2 \\ 2 & 2 & 1 \end{pmatrix}$ $B = \begin{pmatrix} 2 & 1 & 4 \\ 3 & 0 & -1 \\ 4 & -1 & 5 \end{pmatrix}$ $AB = \begin{pmatrix} 2 & 1 & 4 \\ 9 & 0 & 19 \\ 14 & 1 & 11 \end{pmatrix}$ $BA = \begin{pmatrix} 12 & 7 & 6 \\ 1 & -2 & -1 \\ 12 & 11 & 3 \end{pmatrix}$

(6) $A = \begin{pmatrix} 3 & -4 \\ -5 & 1 \end{pmatrix}$ $B = \begin{pmatrix} 7 & 4 \\ 5 & k \end{pmatrix}$ $AB = \begin{pmatrix} 1 & 12-4k \\ -30 & -20+k \end{pmatrix}$ $BA = \begin{pmatrix} 1 & -24 \\ 15-5k & -20+k \end{pmatrix}$ $\begin{cases} 12-4k = -24 \\ 15-5k = -30 \\ -20+k = -20+k \end{cases}$ $\boxed{k=9}$

* (9) $\begin{pmatrix} 1 & 2 \\ 2 & 4 \end{pmatrix} \cdot \begin{pmatrix} 0 & 2 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \rightarrow$ ni A ni B = 0

* (10) $A = \begin{pmatrix} 3 & 4 & 0 \\ -1 & 0 & 2 \\ 1 & 2 & 0 \end{pmatrix}$, $B = \begin{pmatrix} 6 & -1 & 2 \\ 0 & 4 & 5 \\ -1 & 5 & 4 \end{pmatrix}$, $X \in \mathbb{R}^{3\times 3}$, $3X - 2A = 5B \rightarrow 3X = 5B + 2A$

$X = \frac{5}{3}B + \frac{2}{3}A$

$$\frac{5}{3}B + \frac{2}{3}A = \begin{pmatrix} 10 & -5/3 & 10/3 \\ 0 & 20/3 & 25/3 \\ -5/3 & 25/3 & 20/3 \end{pmatrix} + \begin{pmatrix} 2 & 8/3 & 0 \\ -2/3 & 0 & 4/3 \\ 2/3 & 4/3 & 0 \end{pmatrix} = \begin{pmatrix} 12 & 1 & 10/3 \\ -2/3 & 20/3 & 29/3 \\ -1 & 29/3 & 20/3 \end{pmatrix} = X$$

(7) $A = \begin{pmatrix} 3 & 4 \\ -2 & 1 \\ 5 & 0 \end{pmatrix}$ $B = \begin{pmatrix} 2 & -5 & 7 \\ -1 & 2 & 3 \end{pmatrix}$ a) $A^t = \begin{pmatrix} 3 & -2 & 5 \\ 4 & 1 & 0 \end{pmatrix}$, $B^t = \begin{pmatrix} 2 & -1 \\ -5 & 2 \\ 7 & 3 \end{pmatrix}$

b) $(A^t)^t = A$ c) $(AB)^t = \begin{pmatrix} 2 & -7 & 33 \\ -5 & 12 & -11 \\ 10 & -25 & 35 \end{pmatrix}^t = \begin{pmatrix} 2 & -5 & 10 \\ -7 & 12 & -25 \\ 33 & -11 & 35 \end{pmatrix}$

$B^t A^t = \begin{pmatrix} 2 & -1 \\ -5 & 2 \\ 7 & 3 \end{pmatrix} \cdot \begin{pmatrix} 3 & -2 & 5 \\ 4 & 1 & 0 \end{pmatrix} = \begin{pmatrix} 2 & -5 & 10 \\ -7 & 12 & -25 \\ 33 & -11 & 35 \end{pmatrix}$ =

* (8) $A = \begin{pmatrix} 1 & 2 \\ 3 & 6 \end{pmatrix}$ $B = \begin{pmatrix} 3 & -8 \\ 2 & 3 \end{pmatrix}$ $C = \begin{pmatrix} 5 & 2 \\ 1 & -2 \end{pmatrix}$ $AB = \begin{pmatrix} 7 & -2 \\ 21 & -6 \end{pmatrix} = AC = \begin{pmatrix} 7 & -2 \\ 21 & -6 \end{pmatrix}$

$B \neq C \Leftrightarrow AB = AC$ pero $\nexists$ $\frac{\cancel{A}}{\cancel{A}} B = C$ ya que no se puede dividir matrices

(11) a) $\begin{pmatrix} 2 & -5 & 3 & -8 \\ 1 & 3 & -1 & -4 \\ -1 & 2 & 1 & -6 \end{pmatrix} \cdot \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} -2 \\ -6 \\ 9 \end{pmatrix}$ b) $\begin{pmatrix} 2 & -5 & 3 & -8 \\ 1 & 3 & -1 & -4 \\ -1 & 2 & 1 & -6 \end{pmatrix} \cdot \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} -2 \\ -6 \\ 9 \end{pmatrix}$

(12) a) $\begin{pmatrix} 1 & 2 & 3 \\ 2 & 2 & 2 \\ 1 & -1 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \left(\begin{array}{c|c|c} 0 & 1 & 16 \\ 0 & 0 & 12 \\ 0 & 4 & -5 \end{array}\right)$

$S_1 \begin{cases} x+2y+3z=0 \\ -2y-4z=0 \\ -6z=0 \end{cases}$ $(x,y,z) = (0,0,0), (1,-3,2), (-1,4,3)$

$S_2 \begin{cases} x+2y+3z=1 & x=1 \\ -2y-4z=-2 & y=-3 \\ -6z=-12 \rightarrow & z=2 \end{cases}$

$\left(\begin{array}{ccc|c|c|c} 1 & 2 & 3 & 0 & 1 & 16 \\ 2 & 2 & 2 & 0 & 0 & 12 \\ 1 & -1 & 0 & 0 & 4 & -5 \end{array}\right)$ $F_2 - 2F_1 = F_2'$, $F_3 - F_1 = F_3'$ $\left(\begin{array}{ccc|c|c|c} 1 & 2 & 3 & 0 & 1 & 16 \\ 0 & -2 & -4 & 0 & -2 & -20 \\ 0 & 0 & -6 & 0 & -12 & -18 \end{array}\right)$

$\left(\begin{array}{cccccc} 1 & 2 & 3 & 0 & 1 & 16 \\ 0 & -2 & -4 & 0 & -2 & -20 \\ 0 & -3 & -3 & 0 & 3 & -21 \end{array}\right)$ $F_3'' = 3F_2' - 2F_3'$

$S_3 \begin{cases} x+2y+3z=16 & x=-1 \\ -2y-4z=-20 & y=4 \\ -6z=-18 & z=3 \end{cases}$

b) $\begin{pmatrix} 1 & 1 & 2 & -1 \\ 2 & 1 & 1 & 0 \\ -1 & 1 & 2 & -1 \\ 0 & 2 & 4 & -2 \end{pmatrix} \cdot \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \left(\begin{array}{c|c|c} 0 & 1 & 2 \\ 0 & 2 & 0 \\ 0 & 1 & -1 \\ 0 & 2 & 3 \end{array}\right)$

$S1: (x_1, x_2, x_3, x_4) = (0, -x_3, x_3, x_3) = x_3(0,-1,1,1) = \alpha(0,-1,1,1)$

$S2: (x_1, x_2, x_3, x_4) = (0, -x_3+2, x_3, 1+x_3) = \alpha(0,-1,1,1) + (0,2,0,1)$

$\left(\begin{array}{cccc|c|c|c} 1 & 1 & 2 & -1 & 0 & 1 & 2 \\ 2 & 1 & 1 & 0 & 0 & 2 & 0 \\ -1 & 1 & 2 & -1 & 0 & 1 & -1 \\ 0 & 2 & 4 & -2 & 0 & 2 & 3 \end{array}\right)$ $F_2 - 2F_1 = F_2'$, $F_3 + F_1 = F_3'$

$\left(\begin{array}{cccc|ccc} 1 & 1 & 2 & -1 & 0 & 1 & 2 \\ 0 & -1 & -3 & 2 & 0 & 0 & -4 \\ 0 & 2 & 4 & -2 & 0 & 2 & 1 \\ 0 & 2 & 4 & -2 & 0 & 2 & 3 \end{array}\right)$ $F_3' + 2F_2' = F_3''$, $2F_2' + F_4' = F_4''$

$\left(\begin{array}{cccc|ccc} 1 & 1 & 2 & -1 & 0 & 1 & 2 \\ 0 & -1 & -3 & 2 & 0 & 0 & -4 \\ 0 & 0 & -2 & 2 & 0 & 2 & -7 \\ 0 & 0 & -2 & 2 & 0 & 2 & -5 \end{array}\right)$ $F_4'' - F_3'' = F_4'''$

$S3$: SI

$\left(\begin{array}{cccc|ccc} 1 & 1 & 2 & -1 & 0 & 1 & 2 \\ 0 & -1 & -3 & 2 & 0 & 0 & -4 \\ 0 & 0 & -2 & 2 & 0 & 2 & -7 \\ 0 & 0 & 0 & 0 & 0 & 0 & 2 \end{array}\right)$

$$\begin{array}{l} S_1, \\ S_2, \\ S_3, \\ S_4 \end{array} \begin{cases} x_1 + x_2 + 2x_3 - x_4 = 0 = 1 = 2 \\ -x_2 - 3x_3 + 2x_4 = 0 = 0 = -4 \\ -2x_3 + 2x_4 = 0 = 2 = -7 \\ 0 = 0 = 0 = 2 \end{cases}$$

$S1$: SCI: $x_4 = x_3$ $\quad x_2 = -3x_3 + 2x_3 = -x_3$

$x_1 = x_3 - 2x_3 + x_3 = 0$

$S2$: SCI: $2x_4 = 2 + 2x_3$ $\quad x_2 = -3x_3 + 2 + 2x_3 = -x_3 + 2$

$x_4 = 1 + x_3$ $\quad x_1 = \cancel{1} + x_3 \cancel{-2} - 2x_3 + \cancel{1} + x_3$

$x_1 = 0$

(13) a) $\left(\begin{array}{cc|cc} 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{array}\right)$ $F_1 - F_2 = F_1'$

$\left(\begin{array}{cc|cc} 1 & 0 & 1 & -1 \\ 0 & 1 & 0 & 1 \end{array}\right)$

$A^{-1} = \begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix}$ $\quad \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ ✓

b) No tiene inversa

c) $\left(\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ -1 & 3 & 0 & 1 \end{array}\right)$ $F_2 + F_1 = F_2'$

$\left(\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 0 & 5 & 1 & 1 \end{array}\right)$ $F_2' \cdot \frac{1}{5} = F_2''$

$\left(\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 0 & 1 & \frac{1}{5} & \frac{1}{5} \end{array}\right)$ $F_1'' - 2F_2'' = F_1'''$

$\left(\begin{array}{cc|cc} 1 & 0 & \frac{3}{5} & -\frac{2}{5} \\ 0 & 1 & \frac{1}{5} & \frac{1}{5} \end{array}\right)$

$A \cdot A^{-1} = \begin{pmatrix} 1 & 2 \\ -1 & 3 \end{pmatrix} \cdot \begin{pmatrix} \frac{3}{5} & -\frac{2}{5} \\ \frac{1}{5} & \frac{1}{5} \end{pmatrix}$

$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ ✓

d) $\left(\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 & 1 & 0 \\ 1 & 2 & 4 & 0 & 0 & 1 \end{array}\right)$ $F_3 - F_1 = F_3'$

$\left(\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 & 1 & 0 \\ 0 & 0 & 1 & -1 & 0 & 1 \end{array}\right)$ $F_1' - 2F_2' = F_1''$

$\left(\begin{array}{ccc|ccc} 1 & 0 & -1 & 1 & -2 & 0 \\ 0 & 1 & 2 & 0 & 1 & 0 \\ 0 & 0 & 1 & -1 & 0 & 1 \end{array}\right)$ $F_1'' + F_3'' = F_1'''$, $F_2'' - 2F_3'' = F_2'''$

$\left.\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & -2 & 1 \\ 0 & 1 & 0 & 2 & 1 & -2 \\ 0 & 0 & 1 & -1 & 0 & 1 \end{array}\right)\right\}$ $A^{-1} = \begin{pmatrix} 0 & -2 & 1 \\ 2 & 1 & -2 \\ -1 & 0 & 1 \end{pmatrix}$

$A \cdot A^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

e) $\left(\begin{array}{ccc|ccc} 1 & -2 & 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 & 1 & 0 \\ 1 & 1 & -1 & 0 & 0 & 1 \end{array}\right)$ $F_3 - F_1 = F_3'$

$\left(\begin{array}{ccc|ccc} 1 & -2 & 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 & 1 & 0 \\ 0 & 3 & -2 & -1 & 0 & 1 \end{array}\right)$ $F_1' + 2F_2' = F_1''$, $F_3' - 3F_2' = F_3''$

$\left(\begin{array}{ccc|ccc} 1 & 0 & 5 & 1 & 2 & 0 \\ 0 & 1 & 2 & 0 & 1 & 0 \\ 0 & 0 & -8 & -1 & -3 & 1 \end{array}\right)$ $F_3'' \cdot -\frac{1}{8} = F_3'''$

$\left(\begin{array}{ccc|ccc} 1 & 0 & 5 & 1 & 2 & 0 \\ 0 & 1 & 2 & 0 & 1 & 0 \\ 0 & 0 & 1 & \frac{1}{8} & \frac{3}{8} & -\frac{1}{8} \end{array}\right)$ $F_1''' - 5F_3''' = F_1^{IV}$, $F_2''' - 2F_3''' = F_2^{IV}$

$\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{3}{8} & \frac{1}{8} & \frac{5}{8} \\ 0 & 1 & 0 & -\frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & 1 & \frac{1}{8} & \frac{3}{8} & -\frac{1}{8} \end{array}\right)$ $A^{-1} = \begin{pmatrix} \frac{3}{8} & \frac{1}{8} & \frac{5}{8} \\ -\frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{8} & \frac{3}{8} & -\frac{1}{8} \end{pmatrix}$

$A \cdot A^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

f) $\left(\begin{array}{ccc|ccc} 1&2&-1&1&0&0\\ -1&0&2&0&1&0\\ 0&2&1&0&0&1\end{array}\right)$ $F_1+F_2=F_2'$

$\left(\begin{array}{ccc|ccc} 1&2&-1&1&0&0\\ 0&2&1&1&1&0\\ 0&2&1&0&0&1\end{array}\right)$ $\frac{1}{2}F_2'=F_2''$

$\left(\begin{array}{ccc|ccc} 1&2&-1&1&0&0\\ 0&1&\frac{1}{2}&\frac{1}{2}&\frac{1}{2}&0\\ 0&2&1&0&0&1\end{array}\right)$ $F_1''-2F_1''=F_1'''$, $F_3''-2F_2''=F_3'''$

$\left(\begin{array}{ccc|ccc} 1&0&-2&0&-1&0\\ 0&1&\frac{1}{2}&\frac{1}{2}&\frac{1}{2}&0\\ 0&0&0&-1&-1&1\end{array}\right)$ → no tiene inversa

g) $\left(\begin{array}{ccc|ccc} 1&2&1&1&0&0\\ -1&1&0&0&1&0\\ 1&0&-1&0&0&1\end{array}\right)$ $F_2+F_1+F_2'$, $F_3-F_1=F_3'$

$\left(\begin{array}{ccc|ccc} 1&2&1&1&0&0\\ 0&3&1&1&1&0\\ 0&-2&-2&-1&0&1\end{array}\right)$ $\frac{1}{3}F_2'=F_2''$

$\left(\begin{array}{ccc|ccc} 1&2&1&1&0&0\\ 0&1&\frac{1}{3}&\frac{1}{3}&\frac{1}{3}&0\\ 0&-2&-2&-1&0&1\end{array}\right)$ $F_1''+F_3''=F_1'''$, $F_3''+2F_2''=F_3'''$

$\left(\begin{array}{ccc|ccc} 1&0&-1&0&0&1\\ 0&1&\frac{1}{3}&\frac{1}{3}&\frac{1}{3}&0\\ 0&0&-\frac{4}{3}&-\frac{1}{3}&\frac{2}{3}&1\end{array}\right)$ $-\frac{3}{4}F_3'''=F_3^{IV}$

$\left(\begin{array}{ccc|ccc} 1&0&-1&0&0&1\\ 0&1&\frac{1}{3}&\frac{1}{3}&\frac{1}{3}&0\\ 0&0&1&\frac{1}{4}&-\frac{1}{2}&-\frac{3}{4}\end{array}\right)$ $F_1^{IV}+F_3^{IV}=F_1^{V}$, $F_2^{IV}-\frac{1}{3}F_3^{IV}=F_2^{V}$

$\left(\begin{array}{ccc|ccc} 1&0&0&\frac{1}{4}&-\frac{1}{2}&\frac{1}{4}\\ 0&1&0&\frac{1}{4}&\frac{1}{2}&\frac{1}{4}\\ 0&0&1&\frac{1}{4}&-\frac{1}{2}&-\frac{3}{4}\end{array}\right)$

$A^{-1}=\begin{pmatrix} \frac{1}{4}&-\frac{1}{2}&\frac{1}{4}\\ \frac{1}{4}&\frac{1}{2}&\frac{1}{4}\\ \frac{1}{4}&-\frac{1}{2}&-\frac{3}{4}\end{pmatrix}$

$A\cdot A^{-1}=I_3$

h) $\left(\begin{array}{ccc|ccc} 1&\frac{1}{2}&\frac{1}{2}&\frac{1}{2}&0&0\\ 0&1&1&0&1&0\\ 3&1&-1&0&0&1\end{array}\right)$ $F_3-3F_1=F_3'$

$\left(\begin{array}{ccc|ccc} 1&\frac{1}{2}&\frac{1}{2}&\frac{1}{2}&0&0\\ 0&1&1&0&1&0\\ 0&-\frac{1}{2}&-\frac{5}{2}&-\frac{3}{2}&0&1\end{array}\right)$ $F_1'-\frac{1}{2}F_2'=F_1''$, $F_3'+\frac{1}{2}F_2'=F_3''$

$\left(\begin{array}{ccc|ccc} 1&0&0&\frac{1}{2}&-\frac{1}{2}&0\\ 0&1&1&0&1&0\\ 0&0&-2&-\frac{3}{2}&\frac{1}{2}&1\end{array}\right)$ $-\frac{1}{2}F_3''=F_3'''$

$\left(\begin{array}{ccc|ccc} 1&0&0&\frac{1}{2}&-\frac{1}{2}&0\\ 0&1&1&0&1&0\\ 0&0&1&\frac{3}{4}&-\frac{1}{4}&-\frac{1}{2}\end{array}\right)$ $F_2'''-F_3'''=F_2^{IV}$

$A^{-1}A=I_3$ ✓

$\left(\begin{array}{ccc|ccc} 1&0&0&\frac{1}{2}&-\frac{1}{2}&0\\ 0&1&0&-\frac{3}{4}&\frac{5}{4}&\frac{1}{2}\\ 0&0&1&\frac{3}{4}&-\frac{1}{4}&-\frac{1}{2}\end{array}\right)$ $A^{-1}=\begin{pmatrix} \frac{1}{2}&-\frac{1}{2}&0\\ -\frac{3}{4}&\frac{5}{4}&\frac{1}{2}\\ \frac{3}{4}&-\frac{1}{4}&-\frac{1}{2}\end{pmatrix}$

i) $\left(\begin{array}{ccc|ccc} 1&\frac{1}{2}&\frac{1}{2}&\frac{1}{2}&0&0\\ 0&1&1&0&1&0\\ 2&0&0&0&0&1\end{array}\right)$ $F_3-2F_1=F_3'$

$\left(\begin{array}{ccc|ccc} 1&\frac{1}{2}&\frac{1}{2}&\frac{1}{2}&0&0\\ 0&1&1&0&1&0\\ 0&-1&-1&-1&0&1\end{array}\right)$ $F_1'-\frac{1}{2}F_2'=F_1''$, $F_3'+F_2'=F_3''$

$\left(\begin{array}{ccc|ccc} 1&0&0&\frac{1}{2}&-\frac{1}{2}&0\\ 0&1&1&0&1&0\\ 0&0&0&-1&1&1\end{array}\right)$ No hay A^{-1} posible

(14) $\left(\begin{array}{cc|cc} 1&2&1&0\\ 1&-1&0&1\end{array}\right)$ $F_2-F_1=F_2'$

$\left(\begin{array}{cc|cc} 1&2&1&0\\ 0&-3&-1&1\end{array}\right)$ $-\frac{1}{3}F_2'=F_2''$

$\left(\begin{array}{cc|cc} 1&2&1&0\\ 0&1&\frac{1}{3}&-\frac{1}{3}\end{array}\right)$ $F_1''-2F_2''=F_1'''$

$\left(\begin{array}{cc|cc} 1&0&\frac{1}{3}&\frac{2}{3}\\ 0&1&\frac{1}{3}&-\frac{1}{3}\end{array}\right)$ ✓

$\left(\begin{array}{cc|cc} 3&-1&1&0\\ -2&1&0&1\end{array}\right)$ $\frac{1}{3}F_1=F_1'$

$\left(\begin{array}{cc|cc} 1&-\frac{1}{3}&\frac{1}{3}&0\\ -2&1&0&1\end{array}\right)$ $F_2'+2F_1'=F_2''$

$\left(\begin{array}{cc|cc} 1&-\frac{1}{3}&\frac{1}{3}&0\\ 0&\frac{1}{3}&\frac{2}{3}&1\end{array}\right)$ $F_1''+F_2''=F_1'''$, $3F_2''=F_2'''$

$\left(\begin{array}{cc|cc} 1&0&1&1\\ 0&1&2&3\end{array}\right)$ ✓

a) $\begin{pmatrix} \frac{1}{3}&\frac{2}{3}\\ \frac{1}{3}&-\frac{1}{3}\end{pmatrix}\cdot\begin{pmatrix} 1&1\\ 2&3\end{pmatrix}=\begin{pmatrix} \frac{5}{3}&\frac{7}{3}\\ -\frac{1}{3}&-\frac{2}{3}\end{pmatrix}$

b) $(A^{-1})^{-1}=A$

c) $\left(\begin{array}{cc|cc} 1&1&1&0\\ 2&-1&0&1\end{array}\right)$ $F_2-2F_1=F_2'$

$\left(\begin{array}{cc|cc} 1&1&1&0\\ 0&-3&-2&1\end{array}\right)$ $F_1'+\frac{1}{3}F_2'=F_1''$, $-\frac{1}{3}F_2'=F_2''$

$\left(\begin{array}{cc|cc} 1&0&\frac{1}{3}&\frac{1}{3}\\ 0&1&\frac{2}{3}&-\frac{1}{3}\end{array}\right)$ $(A^t)^{-1}=\begin{pmatrix} \frac{1}{3}&\frac{1}{3}\\ \frac{2}{3}&-\frac{1}{3}\end{pmatrix}$

d) $(A^{-1})^t=\begin{pmatrix} \frac{1}{3}&\frac{1}{3}\\ \frac{2}{3}&-\frac{1}{3}\end{pmatrix}$

e) $\left(\begin{array}{cc|cc} 1&-1&1&0\\ -2&5&0&1\end{array}\right)$ $F_2+2F_1=F_2'$

$\left(\begin{array}{cc|cc} 1&-1&1&0\\ 0&3&2&1\end{array}\right)$ $F_1'+\frac{1}{3}F_2'=F_1''$, $\frac{1}{3}F_2'=F_2''$

$\left(\begin{array}{cc|cc} 1&0&\frac{5}{3}&\frac{1}{3}\\ 0&1&\frac{2}{3}&\frac{1}{3}\end{array}\right)$

f) $B^{-1}A^{-1}=\begin{pmatrix} \frac{2}{3}&\frac{1}{3}\\ \frac{5}{3}&\frac{1}{3}\end{pmatrix}$

(15) a) $\begin{pmatrix} 2&-1\\ 0&1\end{pmatrix}\cdot\begin{pmatrix} a&b\\ c&d\end{pmatrix}=\begin{pmatrix} 5&1\\ 0&2\end{pmatrix}$

$\begin{cases} 2a-c=5\\ 2b-d=1\\ c=0\\ d=2\end{cases}$ $a=\frac{5}{2}$, $b=\frac{3}{2}$ $X=\begin{pmatrix} \frac{5}{2}&\frac{3}{2}\\ 0&2\end{pmatrix}$

b) $\begin{pmatrix} -2&1\\ 3&4\\ 2&5\end{pmatrix}\cdot\begin{pmatrix} a&b\\ c&d\end{pmatrix}=\begin{pmatrix} 1&3\\ 2&-4\\ 3&7\end{pmatrix}$

$\begin{cases} 2a+5c=3\\ -2a+c=1\\ -2b+d=3 \ \times\\ 3a+4c=2 \ \times\\ 3b+4d=-4 \ ✓\\ 2b+5d=7\end{cases}$ $\left.\begin{matrix}2a+5c=3\\-2a+c=1\end{matrix}\right\}+$ $6c=4$, $c=\frac{2}{3}$ $a=\frac{3-\frac{10}{3}}{2}=-\frac{1}{6}$

$\nexists$ ✗

$b=\frac{7-5d}{2}$, $b=\frac{-4-4d}{3}$ $\frac{7-5d}{2}=\frac{-4-4d}{3}$ $d=\frac{29}{7}$, $b=-\frac{48}{7}$

FECHA

c) $\begin{pmatrix} a & b \\ c & d \end{pmatrix} \cdot \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} a & b \\ c & d \end{pmatrix}$

$\cancel{a} = \cancel{a} + c \qquad c = 0$

$a + \cancel{b} = \cancel{b} + d \qquad a = d \qquad X = \begin{pmatrix} a & b \\ 0 & a \end{pmatrix}$

$\cancel{c = c}$

$c + \cancel{d} = \cancel{d} \qquad c = 0$

d) $\begin{pmatrix} -2 & 1 \\ 2 & -1 \end{pmatrix} \cdot \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \cdot \begin{pmatrix} -2 & 1 \\ 2 & -1 \end{pmatrix}$

$\cancel{-2a} + c = \cancel{-2a} + 2b \qquad 2a - c = -2c + 2d$

$-2b + d = a - b \qquad 2b - \cancel{d} = c - \cancel{d}$

$d = a + b \qquad 2a + c = 2d$

$c = 2b$

$d = a + b \qquad X = \begin{pmatrix} a & b \\ 2b & a+b \end{pmatrix}$

(16) $A = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \quad B = \begin{pmatrix} 5a & 2(b-3) \\ 2 & a+4 \end{pmatrix}$

$AB = BA$

$\cancel{5a} + 2 = \cancel{5a} + 2(b-3) \qquad b = 4$

$\cancel{2(b-3)} + a + 4 = 5a + \cancel{2(b-3)} \qquad a = 1$

$5a + \cancel{2} = \cancel{2} + a + 4 \qquad 5 = 5$

$2(b-3) \cancel{+ a + 4} = 2 \cancel{+ a + 4} \qquad 2 = 2$

$\boxed{a = 1,\ b = 4}$

(18) $AX = 2X + B^t$

$AX - 2IX = B^t$

$(A - 2I)X = B^t$

$\begin{pmatrix} 0 & 2 & -1 \\ 3 & 1 & 2 \\ 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} = \begin{pmatrix} 1 & 3 & 1 \\ 0 & 1 & -2 \\ -1 & 0 & -3 \end{pmatrix}$

$2d - g = 1 \qquad d = \frac{1}{2} + \frac{1}{2}g$

$2e - h = 3 \qquad e = \frac{3}{2} + \frac{1}{2}h$

$2f - i = 1 \qquad f = \frac{1}{2} + \frac{1}{2}i$

$3a + d + 2g = 0 \qquad -3 - 3g + \frac{1}{2} + \frac{1}{2}g + 2g = 0$

$3b + e + 2h = 1 \qquad -3h + \frac{3}{2} + \frac{1}{2}h + 2h = 1$

$3c + f + 2i = -2 \qquad -9 - 3i + \frac{1}{2} + \frac{1}{2}i + 2i = -2$

$a + g = -1 \qquad a = -1 - g$

$b + h = 0 \qquad b = -h$

$c + i = -3 \qquad c = -3 - i$

$g = -5 \quad a = 4 \quad d = -2$

$h = 1 \quad b = -1 \quad e = 2$

$i = -13 \quad f = -6 \quad c = 10$

$X = \begin{pmatrix} 4 & -1 & 10 \\ -2 & 2 & -6 \\ -5 & 1 & -13 \end{pmatrix}$

(17) $AX + 2X = B^t X + \frac{1}{2}C$

$(A + 2I - B^t)X = \frac{1}{2}C$

$A = \begin{pmatrix} 0 & -1 \\ -1 & 1 \end{pmatrix} \quad B = \begin{pmatrix} 1 & 0 \\ 1 & 2 \end{pmatrix} \quad C = \begin{pmatrix} -2 & 4 \\ -1 & 6 \end{pmatrix} \quad B^t = \begin{pmatrix} 1 & 1 \\ 0 & 2 \end{pmatrix} \quad 2I = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$

$A + 2I - B^t = \begin{pmatrix} 1 & -2 \\ -1 & 1 \end{pmatrix} \qquad \begin{pmatrix} 1 & -2 \\ -1 & 1 \end{pmatrix} \cdot \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} -1 & 2 \\ -\frac{1}{2} & 3 \end{pmatrix}$

$a - 2c = -1 \qquad -a + c = -\frac{1}{2}$

$b - 2d = 2 \qquad -b + d = 3$

$a = -1 + 2c \qquad 1 - 2c + c = -\frac{1}{2} \qquad c = \frac{3}{2}$

$b = 2 + 2d \qquad -2 - 2d + d = 3 \qquad d = -5$

$a = 2 \quad c = \frac{3}{2}$

$b = -8 \quad d = -5 \qquad X = \begin{pmatrix} 2 & -8 \\ \frac{3}{2} & -5 \end{pmatrix}$

(19) $ABX = 3BX + b$

$ABX - 3BX = b$

$(A - 3I)BX = b$

$\left(\begin{array}{ccc|ccc} 1 & 1 & -1 & 1 & 0 & 0 \\ 0 & -2 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right) \quad F_2 \cdot -\frac{1}{2} = F_2'$

$\left(\begin{array}{ccc|ccc} 1 & 1 & -1 & 1 & 0 & 0 \\ 0 & 1 & -\frac{1}{2} & 0 & -\frac{1}{2} & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right) \quad F_1' - F_2' = F_1''$

$\left(\begin{array}{ccc|ccc} 1 & 0 & -\frac{1}{2} & 1 & \frac{1}{2} & 0 \\ 0 & 1 & -\frac{1}{2} & 0 & -\frac{1}{2} & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right) \quad \begin{array}{l} F_1'' + \frac{1}{2}F_3'' = F_1''' \\ F_2'' + \frac{1}{2}F_3'' = F_2''' \end{array}$

$\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & \frac{1}{2} & \frac{1}{2} \\ 0 & 1 & 0 & 0 & -\frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right) \cdot \underbrace{\begin{pmatrix} -1 & 2 & 0 \\ 2 & 1 & 3 \\ 1 & 0 & 1 \end{pmatrix}}_{B} = \begin{pmatrix} \frac{1}{2} & \frac{5}{2} & 2 \\ -\frac{1}{2} & -\frac{1}{2} & -1 \\ 1 & 0 & 1 \end{pmatrix}$

$(A - 3I)$

$\begin{pmatrix} \frac{1}{2} & \frac{5}{2} & 2 \\ -\frac{1}{2} & -\frac{1}{2} & -1 \\ 1 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \\ -1 \end{pmatrix}$

$\frac{1}{2}x + \frac{5}{2}y + 2z = -1$

$-\frac{1}{2}x - \frac{1}{2}y - z = 1$

$x + z = -1$

$x = -1 - z$

$\frac{1}{2} + \frac{1}{2}z - \frac{1}{2}y - z = 1$

$-\frac{1}{2}z - \frac{1}{2} = \frac{1}{2}y$

$-z - 1 = y$

$\frac{5}{2}y = -\frac{5}{2}z - \frac{5}{2}$

$-\frac{1}{2} - \frac{1}{2}z - \frac{5}{2}z - \frac{5}{2} + 2z = -1$

$-z = 2$

$z = -2$

$x = 1$

$y = 1$

$\boxed{X = \begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix}}$

21) $(A+I)^{-1} = \begin{pmatrix}1&0&1\\2&1&-1\\0&1&3\end{pmatrix}$

a) $B = \begin{pmatrix}-1&2&-2\\1&-1&1\\-1&2&-1\end{pmatrix}$ $\quad \to F_1\cdot(-1) = F_1$

$\left(\begin{array}{ccc|ccc}1&-2&2&-1&0&0\\1&-1&1&0&1&0\\-1&2&-1&0&0&1\end{array}\right)$ $F_2-F_1=F_2'$, $F_3+F_1=F_3'$

$\left(\begin{array}{ccc|ccc}1&-2&2&-1&0&0\\0&1&-1&1&1&0\\0&0&1&-1&0&1\end{array}\right)$ $F_1'+2F_2'=F_1''$

$\left(\begin{array}{ccc|ccc}1&0&0&1&2&0\\0&1&-1&1&1&0\\0&0&1&-1&0&1\end{array}\right)$ $F_2''+F_3''=F_2'''$

$\left(\begin{array}{ccc|ccc}1&0&0&1&2&0\\0&1&0&0&1&1\\0&0&1&-1&0&1\end{array}\right)$ $B^{-1} = \begin{pmatrix}1&2&0\\0&1&1\\-1&0&1\end{pmatrix}$

b) $(B^tXA^t + B^tX)^t = B^2$

$(B^tXA^t)^t + (B^tX)^t = B^2$

$A(B^tX)^t + X^t.B = B^2$

$A.X^t.B + IX^tB = B^2$

$(A+I)(X^tB) = B^2$

$(A+I)X^t\cancel{B}\,\cancel{B^{-1}} = B^2B^{-1}$

$(A+I)X^t = B$

$\left(\begin{array}{ccc|ccc}1&0&1&1&0&0\\2&1&-1&0&1&0\\0&1&3&0&0&1\end{array}\right)$ $F_2-2F_1=F_2'$

$\left(\begin{array}{ccc|ccc}1&0&1&1&0&0\\0&1&-3&-2&1&0\\0&1&3&0&0&1\end{array}\right)$ $F_3'-F_2'=F_3''$ $\quad 6$

$\left(\begin{array}{ccc|ccc}1&0&1&1&0&0\\0&1&-3&-2&1&0\\0&0&1&1/3&-1/6&1/6\end{array}\right)$ $F_2''+3F_3''=F_2'''$, $F_1''-F_3''=F_1'''$

$\left(\begin{array}{ccc|ccc}1&0&0&2/3&1/6&-1/6\\0&1&0&-1&1/2&1/2\\0&0&1&1/3&-1/6&1/6\end{array}\right)$

$\underbrace{\qquad}_{A+I}$

$\begin{pmatrix}2/3&1/6&-1/6\\-1&1/2&1/2\\1/3&-1/6&1/6\end{pmatrix}\begin{pmatrix}A&D&G\\B&E&H\\C&F&I\end{pmatrix} = \left.\begin{pmatrix}2/3A+1/6B-1/6C & 2/3D+1/6E-1/6F & 2/3G+1/6H-1/6I\\-A+1/2B+1/2C & -D+1/2E+1/2F & -G+1/2H+1/2I\\1/3A-1/6B+1/6C & 1/3D-1/6E+1/6F & 1/3G-1/6H+1/6I\end{pmatrix}\right\} (A+I)\cdot X^t$

$\left.\begin{array}{l}2/3A+1/6B-1/6C=-1\\-A+1/2B+1/2C=1\\1/3A-1/6B+1/6C=-1\end{array}\right\}\begin{array}{l}A=-2\\B=0\\C=-2\end{array}$

$\left.\begin{array}{l}2/3D+1/6E-1/6F=2\\-D+1/2E+1/2F=-1\\1/3D-1/6E+1/6F=2\end{array}\right\}\begin{array}{l}D=4\\E=1\\F=5\end{array}$ $\quad \left.\begin{array}{l}2/3G+1/6H-1/6I=-2\\-G+1/2H+1/2I=1\\1/3G-1/6H+1/6I=-1\end{array}\right\}\begin{array}{l}G=-3\\H=-2\\I=-2\end{array}$

$X^t = \begin{pmatrix}-2&4&-3\\0&1&-2\\-2&5&-2\end{pmatrix} \longrightarrow X = \begin{pmatrix}-2&0&-2\\4&1&5\\-3&-2&-2\end{pmatrix}$

22) $2B^{-1}XC^{-1} - B^t = B^{-1}(XC^{-1}+A)$

$B^{-1}XC^{-1} = B^{-1}A + B^t$

$B^{-1}XC^{-1} = B^{-1}(A+BB^t)$

$\cancel{B}\,\cancel{B^{-1}}XC^{-1} = \cancel{B}\,\cancel{B^{-1}}(A+BB^t)$

$XC^{-1} = A+BB^t$

$\begin{pmatrix}A&B&C\\D&E&F\\G&H&I\end{pmatrix}\cdot\begin{pmatrix}1&1&-1\\0&-1&1\\0&0&1\end{pmatrix} = \begin{pmatrix}0&1&1\\0&0&1\\0&0&0\end{pmatrix}$

$\left.\begin{array}{l}A=0\\A-B=1\\-A+B+C=1\end{array}\right\}\begin{array}{l}B=-1\\C=2\\A=0\end{array}$ $\quad \left.\begin{array}{l}D=0\\D-E=0\\-D+E+F=1\end{array}\right\}\begin{array}{l}D=0\\E=0\\F=1\end{array}$ $\quad \left.\begin{array}{l}G=0\\G-H=0\\-G+H+I=0\end{array}\right\}\begin{array}{l}G=0\\H=0\\I=0\end{array}$ $\quad X = \begin{pmatrix}0&-1&2\\0&0&1\\0&0&0\end{pmatrix}$