ALGEBRA Y GEOMETRÍA

PRIMER CUATRIMESTRE DE 2020

PRÁCTICA 1

1. Sean $\mathbf{u} = (1, -2)$, $\mathbf{v} = (3, 4)$, $\mathbf{w} = (-2, 5)$.

a) Graficar $\mathbf{u}$, $\mathbf{v}$ y $\mathbf{w}$.

b) Efectuar las siguientes operaciones y graficar

i) $\mathbf{u} + \mathbf{v}$, ii) $3\mathbf{u}$, iii) $-\mathbf{u}$, iv) $\frac{1}{2}\mathbf{u}$, v) $3\mathbf{u} + 2\mathbf{v}$,

vi) $\mathbf{u} - 2\mathbf{w}$, vii) $2\mathbf{u} + \mathbf{w}$, viii) $\mathbf{u} - (\mathbf{v} - 3\mathbf{w})$, ix) $-\mathbf{u} + 2(\mathbf{v} + 4\mathbf{w})$.

2. Sea $\mathbf{v} = (1, 2)$. Graficar los siguientes conjuntos de $\mathbb{R}^2$

a) $\{\alpha\mathbf{v} \,:\, \alpha \in \mathbb{R}\}$

b) $\{\beta\mathbf{v} \,:\, \beta \in \mathbb{R}_{\geq 0}\}$

c) $\{t\mathbf{v} \,:\, t \in [-1, 3]\}$

3. Sean $\mathbf{u} = (1, 2, 3)$, $\mathbf{v} = (1, 0, 2)$ y $\mathbf{w} = (2, 2, 0)$.

a) Graficar $\mathbf{u}$, $\mathbf{v}$ y $\mathbf{w}$.

b) Efectuar las siguientes operaciones:

i) $\mathbf{u} + \mathbf{w}$, ii) $3\mathbf{w}$, iii) $-\mathbf{u} + \mathbf{w}$, iv) $5\mathbf{u} - \frac{1}{2}\mathbf{v}$,

v) $-2\mathbf{u} + 3\mathbf{v} - 5\mathbf{w}$, vi) $-\mathbf{u} - 2(7\mathbf{v} + \mathbf{w})$, vii) $(\mathbf{u} - \mathbf{w}) + (\mathbf{v} - \mathbf{w})$.

4. Calcular la longitud (o norma) de los siguientes vectores:

a) $(3, 0)$, $(2, 1)$, $(-3, 4)$, $(\frac{3}{5}, \frac{4}{5})$,

b) $(1, 1, -1)$, $(1, 1, -2) + (3, 5, 6)$, $(2, -1, 3)$, $-2(2, -1, 3)$, $2(2, -1, 3)$.

5. a) Hallar un vector unitario que tenga la misma dirección y sentido que el vector $\mathbf{v} = (-3, 4)$.

b) Hallar un vector unitario que tenga la misma dirección y sentido contrario que el vector $\mathbf{v} = (1, -2, -1)$.

c) Normalizar los vectores $\mathbf{v} = (2, -3)$, $\mathbf{w} = (6, 8)$ y $\mathbf{u} = (2, -1, 3)$.

6. Hallar y graficar un vector $\mathbf{v}$ equivalente al vector $\overrightarrow{AB}$ en cada uno de los siguientes casos

a) $A = (1, 3)$, $B = (4, 1)$.

b) $A = (4, 1)$, $B = (1, 3)$.

c) $A = (1, 2, -1)$, $B = (2, 3, 4)$ (Hacer el gráfico con *geogebra*)

7. Calcular la distancia entre A y B si

a) $A = (1, -3)$, $B = (0, 0)$. b) $A = (1, -3)$, $B = (4, 1)$.

c) $A = (4, -2, 6)$, $B = (3, -4, 4)$. d) $A = (1, 2, -3)$, $B = (0, 3, 1)$.

8. Graficar los siguientes conjuntos de puntos del plano

a) $S_1 = \{A \in \mathbb{R}^2 : \|A\| = 1\}$.

b) $S_2 = \{A \in \mathbb{R}^2 : \|A\| > 2\}$.

c) $S_3 = \{A \in \mathbb{R}^2 : 1 \leq \|A\| \leq 3\}$.

d) $S_4 = \{A \in \mathbb{R}^2 : \|A - B\| \leq 2\}$, con $B = (1, 3)$.

9. Determinar todos los valores de k tales que:

a) $\|\mathbf{u}\| = 5$ si $\mathbf{u} = (4, k)$.

b) $\|\mathbf{u}\| = 2$ si $\mathbf{u} = (1, k, 0)$.

c) $d(A, B) = 2$ si $A = (1, 1, 1)$ y $B = (k, -k, 2)$.

d) $\|\mathbf{u}\| = 2$ si $\mathbf{u} = (1, -2, k)$.

e) $\|\mathbf{u}\| = 1$ si $\mathbf{u} = k(2, 2, 1)$.

10. a) Sean $\mathbf{u}_1 = (1, 2)$; $\mathbf{u}_2 = (-1, -2)$; $\mathbf{u}_3 = (-2, 1)$; $\mathbf{u}_4 = (1, 0)$ y $\mathbf{u}_5 = (0, 0)$
Calcular $\mathbf{u}_1 \cdot \mathbf{u}_2$, $\mathbf{u}_1 \cdot \mathbf{u}_3$, $\mathbf{u}_1 \cdot \mathbf{u}_5$, $\mathbf{u}_2 \cdot \mathbf{u}_3$, $\mathbf{u}_2 \cdot (\mathbf{u}_3 + \mathbf{u}_4)$, $(\mathbf{u}_4 - \mathbf{u}_3) \cdot \mathbf{u}_1$.

b) Sean $\mathbf{u}_1 = (1, 1, 1)$; $\mathbf{u}_2 = (1, -1, 0)$; $\mathbf{u}_3 = (2, -1, -1)$ y $\mathbf{u}_4 = (2, 3, -1)$
Calcular $\mathbf{u}_1 \cdot \mathbf{u}_2$, $\mathbf{u}_1 \cdot \mathbf{u}_3$, $\mathbf{u}_1 \cdot (\mathbf{u}_2 + \mathbf{u}_3)$, $\mathbf{u}_1 \cdot (2\mathbf{u}_2 - 3\mathbf{u}_3)$, $\mathbf{u}_1 \cdot \mathbf{u}_4$, $(\mathbf{u}_1 - 2\mathbf{u}_2) \cdot \mathbf{u}_3$.

11. Hallar el ángulo que forman $\mathbf{u}$ y $\mathbf{v}$ en cada uno de los siguientes casos

a) $\mathbf{u} = (-1, 0)$, $\mathbf{v} = (-1, -1)$.

b) $\mathbf{u} = (3, 1)$, $\mathbf{v} = (-1, 3)$.

c) $\mathbf{u} = (\sqrt{3}, 1)$, $\mathbf{v} = (2\sqrt{3}, -2)$.

d) $\mathbf{u} = (1, 1, 2)$, $\mathbf{v} = (-1, 2, 1)$.

12. Determinar si $\mathbf{u}$ y $\mathbf{v}$ son perpendiculares

a) $\mathbf{u} = (1, 1)$, $\mathbf{v} = (-2, 2)$.

b) $\mathbf{u} = (2, -3)$, $\mathbf{v} = (0, 0)$.

c) $\mathbf{u} = (1, 1, 1)$, $\mathbf{v} = (1, 0, 1)$.

d) $\mathbf{u} = (1, -2, 4)$, $\mathbf{v} = (-2, 1, 1)$.

13. a) Hallar 3 vectores del plano que sean ortogonales al $(-2, 3)$ ¿Qué relación encuentra entre los vectores hallados? Graficar.

b) Hallar todos los vectores perpendiculares al $(1, 1)$ que tengan norma 1. Graficar.

c) Hallar 3 vectores del espacio que sean perpendiculares al $(-1, 2, 1)$. Graficar con *Geogebra.*

d) Hallar un vector ortogonal al $(-1, 0, 2)$ que tenga norma igual a 8.

14. Sea $\mathbf{v} = (1, 1)$. Hallar un vector $\mathbf{w}$ tal que $\|\mathbf{w}\| = 2$ y $\text{áng}\,(\mathbf{w}, \mathbf{v}) = \frac{\pi}{4}$. ¿Es único?

15. Sean $\mathbf{v}, \mathbf{w} \in \mathbb{R}^n$ tales que $\mathbf{v} - \mathbf{w}$ es ortogonal a $\mathbf{v}$, $\|\mathbf{v}\| = 3$ y $\text{áng}\,(\mathbf{v}, \mathbf{w}) = \dfrac{\pi}{4}$. Determinar $\|\mathbf{w}\|$.

16. Dados los vectores $\mathbf{u} = (2, 0, -1)$, $\mathbf{v} = (-2, 4, 4)$. Calcular:

a) $\mathbf{u} \cdot \mathbf{v}$ b) $\text{áng}\,(\mathbf{u}, \mathbf{v})$ c) $\text{proy}_{\,\mathbf{u}}\mathbf{v}$ d) $\text{proy}_{\,\mathbf{v}}\mathbf{u}$.

17. Hallar la proyección de $\mathbf{u}$ sobre $\mathbf{v}$ en los casos

a) $\mathbf{u} = (-1, 1, 1)$, $\mathbf{v} = (2, 1, -3)$.

b) $\mathbf{u} = (2, 1, -3)$, $\mathbf{v} = (-1, 1, 1)$.

18. Sean $\mathbf{v} = (1, k, 2)$, $\mathbf{w} = (-3, 4, 0)$. Encontrar todos los valores de $k \in \mathbb{R}$ tales que $\|\,\text{proy}_{\,w}(v)\| = 1$.

19. Calcular los siguientes productos vectoriales:

a) $(1, 3, 5) \times (1, 3, 5)$.

b) $(1, 3, -4) \times (2, 1, 3)$.

c) $(2, 1, 3) \times (1, 3, -4)$.

d) $(1, 3, 4) \times (0, 0, 0)$.

e) $(2, 0, 0) \times (0, 0, 3)$.

20. Sean $\mathbf{u} = (2, 1, -3)$, $\mathbf{v} = (1, -2, 1)$ y $\mathbf{w} = (1, 2, 2)$.

a) Calcular $\mathbf{u} \times \mathbf{v}$ y $\mathbf{v} \times \mathbf{u}$.

b) Verificar que $\mathbf{u} \times \mathbf{v}$ es ortogonal a $\mathbf{u}$ y a $\mathbf{v}$.

c) Calcular $(\mathbf{u} \times \mathbf{v}) \times \mathbf{w}$ y $\mathbf{u} \times (\mathbf{v} \times \mathbf{w})$.

21. Encontrar un vector no nulo que sea ortogonal a $(1, 2, -3)$ y a $(-1, 5, -2)$ ¿Cúantos hay?

22. Encontrar todos los vectores $\mathbf{w}$ de norma igual a 2 que sean simultáneamente ortogonales a $\mathbf{u}$ y a $\mathbf{v}$ en los siguientes casos:

a) $\mathbf{u} = (1, 1, 1)$ y $\mathbf{v} = (2, 3, -1)$.

b) $\mathbf{u} = (2, -3, 4)$ y $\mathbf{v} = (-1, 5, 7)$.

23. a) Calcular el área del paralelogramo de vértices 0, A, B y $A + B$ donde $A = (1, 2, 4)$ y $B = (2, 3, -7)$.

b) Calcular el área del triángulo de vértices $A = (-1, 2, 4)$, $B = (1, 0, 5)$ y $C = (1, -1, 3)$.

24. Calcular el volumen del paralelepípedo con vértices

a) $O = (0, 0, 0)$, $A = (2, 1, -1)$, $B = (5, 0, -3)$ y $C = (1, -2, 1)$.

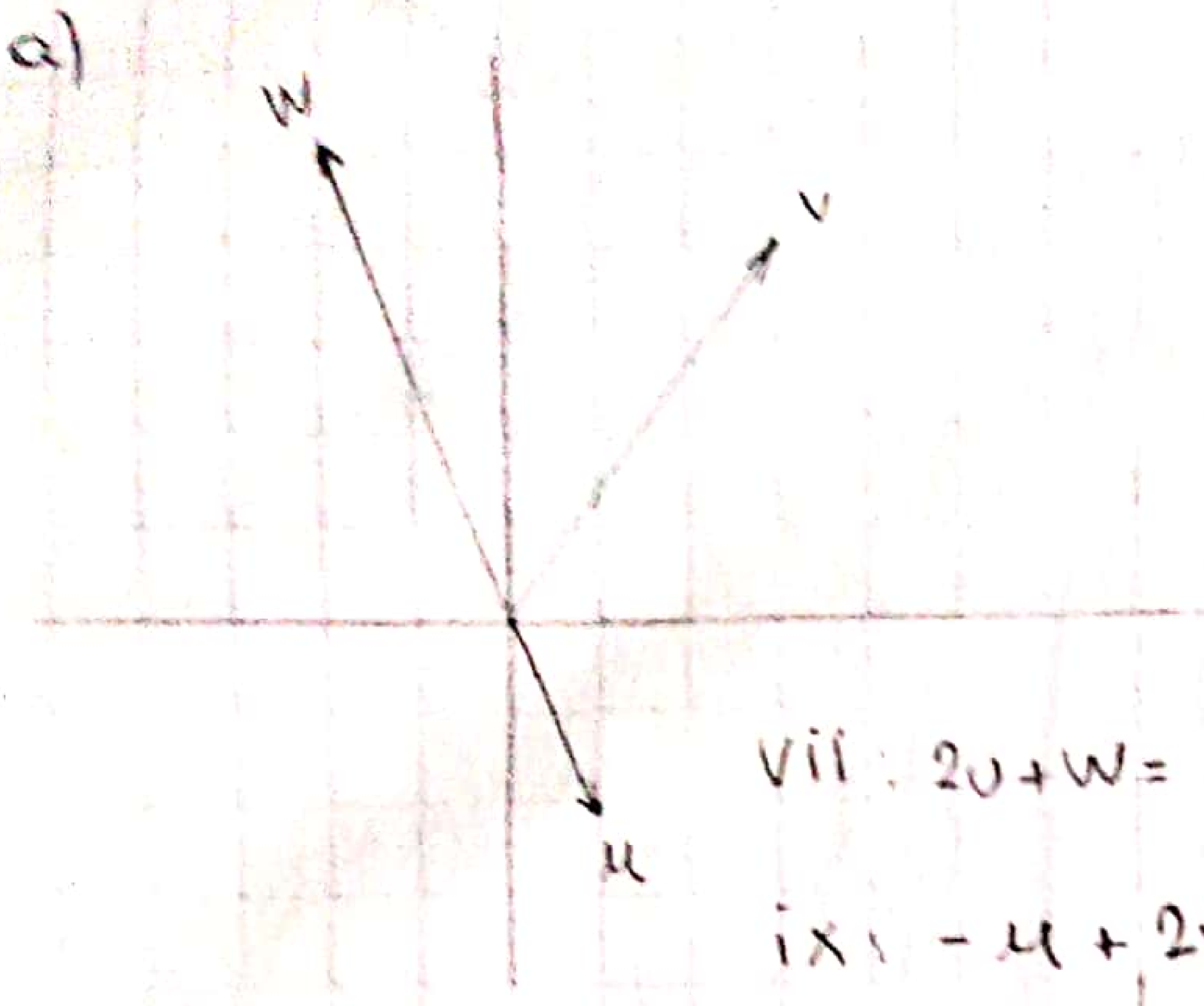
b) $O = (0, 0, 0)$, $A = (1, 0, 0)$, $B = (0, 3, -1)$, $C = (4, 2, -1)$.

c) $A = (0, 1, 0)$, $B = (1, 1, 1)$, $C = (0, 2, 0))$, $D = (3, 1, 2)$.

25. Determinar todos los valores de $k \in \mathbb{R}$ de modo que el volumen del paralelepípedo de vértices $A = (0, 1, 1)$, $B = (2, k, 0)$, $C = (1, -1, 0)$ y $D = (3, 0, -1)$ sea 21.

Práctica I

① a)



b) i: $u + v = (4,2)$ ii: $3u = (3,-6)$

iii: $-u = (-1,2)$ iv: $\frac{1}{2}u = (\frac{1}{2}, -1)$

v: $3u + 2v = (3,-6) + (6,8) = (9,2)$

vi: $u - 2w = (1,-2) - (-4,10) = (5,-12)$

vii: $2u + w = (2,-4) + (-2,5) = (0,1)$ viii: $u - v + 3w = (-8,9)$

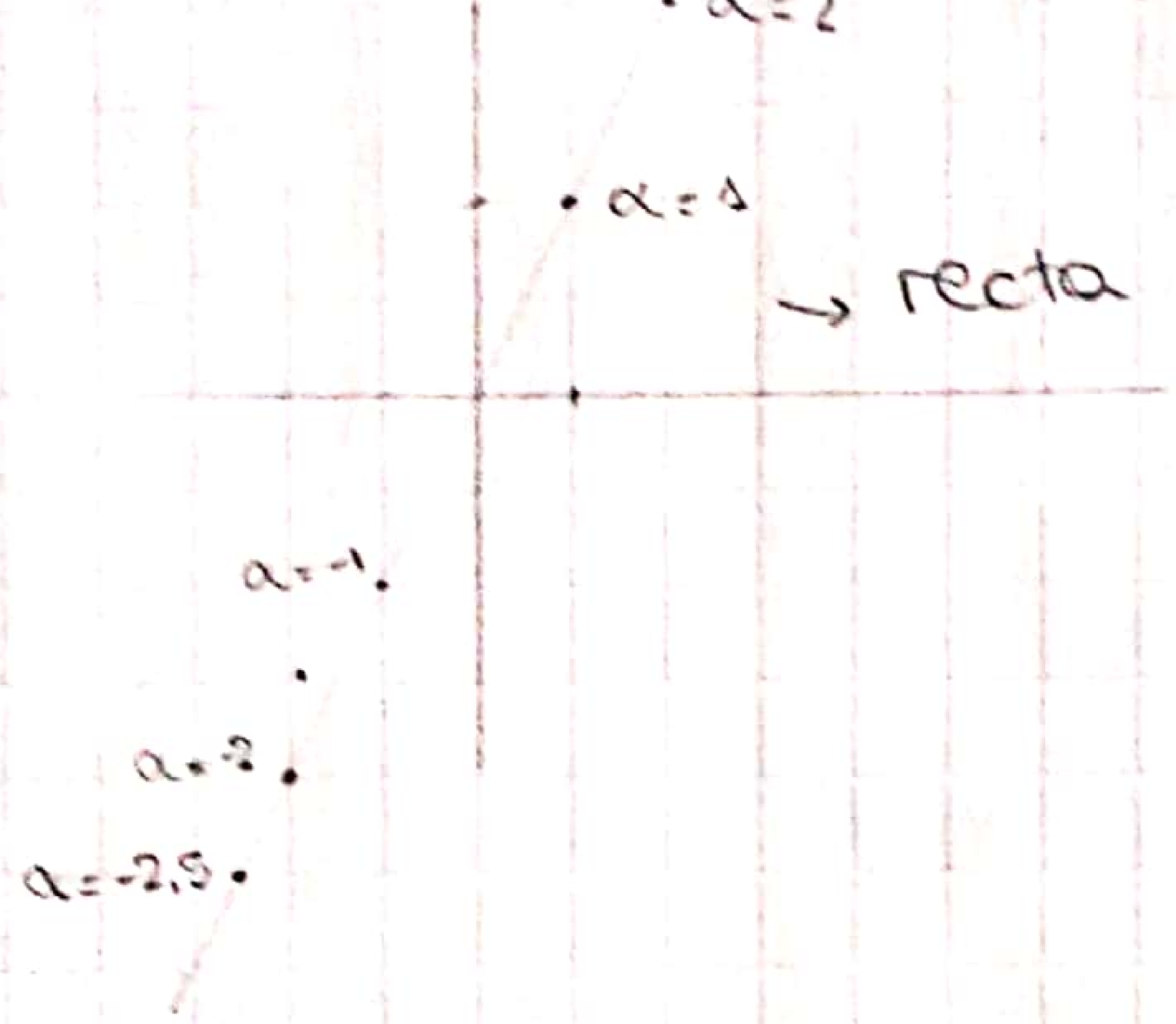
ix: $-u + 2v + 8w = (-11;45)$

② $v = (1,2)$

$(x,y) = (a,b) + \alpha(1,2)$

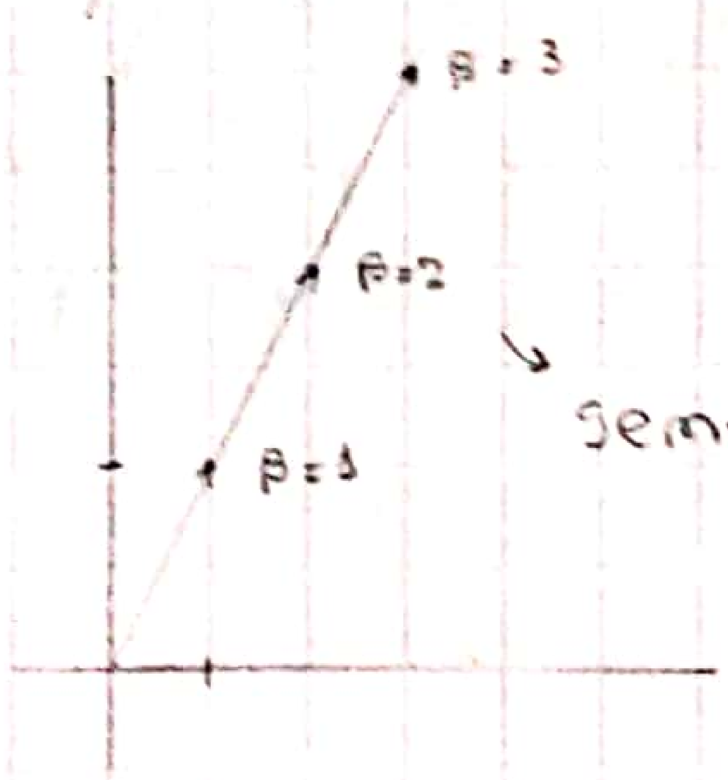
puede variar

a) $\{\alpha v : \alpha \in \mathbb{R}\}$



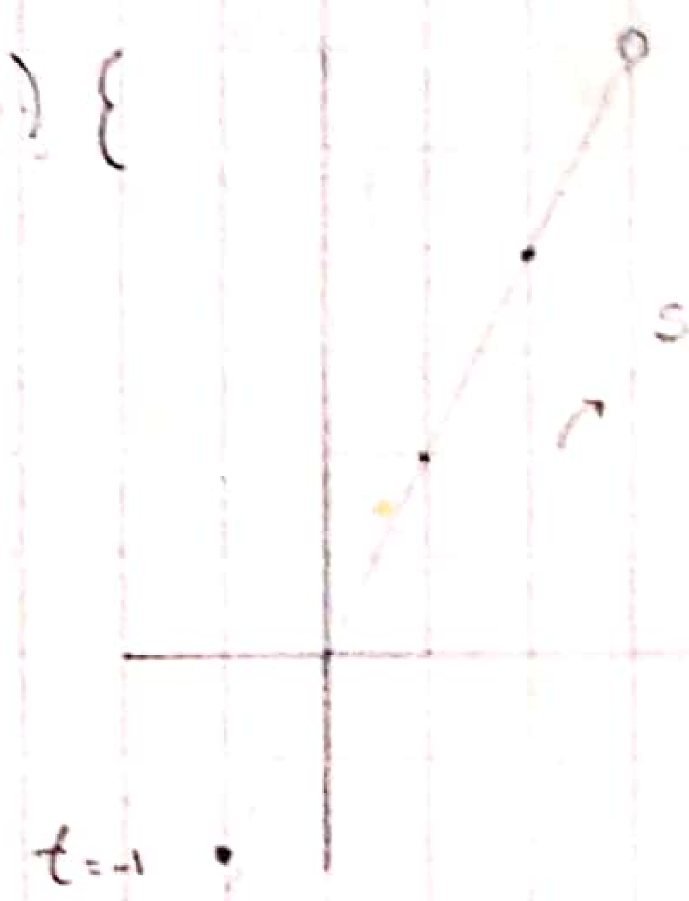
→ recta

b) $\{\beta u : \beta \in \mathbb{R}_{\geq 0}\}$



→ semirrecta

c) $\{tv : t \in [-1,3)\}$



segmento

$\vec{v} = (a,b)$ norma $= \sqrt{a^2+b^2}$

$\vec{v} = (a,b,c)$ norma $= \sqrt{a^2+b^2+c^2}$

$\vec{v} = (a,b,\dots,n)$ norma $= \sqrt{a^2+b^2+\dots+n^2}$
$\|\vec{v}\|$

Para que $\|\vec{v}\| = 1$ a) $\frac{u}{\|u\|}$ $\frac{(2,4)}{2\sqrt{5}}$ b) $u = \alpha(2,4)$

$u = (2\alpha, 4\alpha)$

$\|u\| = \sqrt{(2\alpha)^2 + (4\alpha)^2} = 1$ $20\alpha^2 = 1$

$\alpha = \frac{1}{2\sqrt{5}}$

$v = \frac{(2,4)}{2\sqrt{5}}$

__Distancia__ (entre 2 vectores)

$A = (2,4)$ $\vec{AB} = (2,1)$ $d(AB) = \|\vec{AB}\| = \|\vec{B} - \vec{A}\| =$
$B = (4,5)$

$= \sqrt{2^2+1^2} = \sqrt{5}$

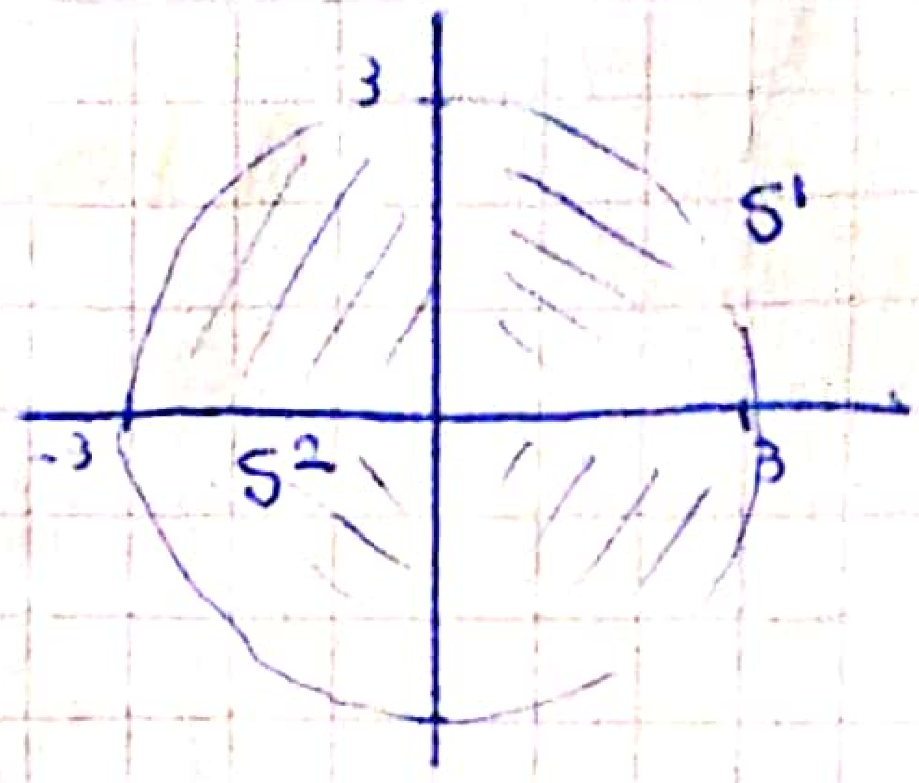
hola como te va

$$\lim_{x \to 0} \frac{x^2 - 3x + 2e^{-2x}}{x+2}$$

Graficar:

$S_1 = \} A \in \mathbb{R}^2 : \|A\| = 3 \{$

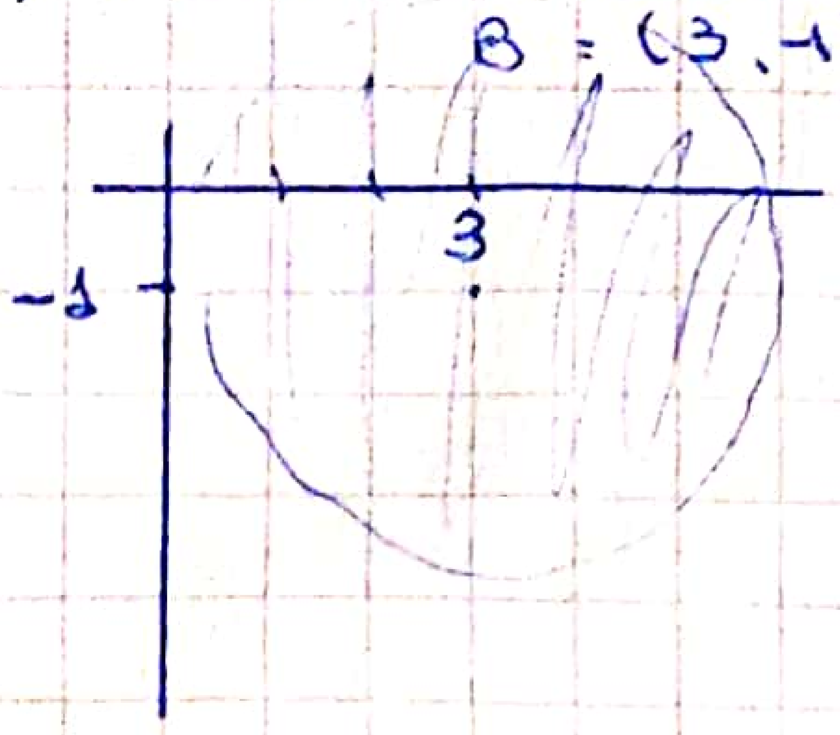
$S_2 . \} A \in \mathbb{R}^2 : \|A\| \leq 3 \{$



$S_5 \} A \in \mathbb{R}^2 : \|A - B\| < 3 \{$

$B = (3, -1)$

S_5



Determinar los valores de K / $d(AB) = 2$ si $A = (1\,1\,1)$ y $B = (k, -k, 2)$

$d\,AB = \|B - A\| = \|(k-1), (-k-1), 1\| = \sqrt{(k-1)^2 + (-k-1)^2 + 1} = 2$

$k^2 \cancel{-2k} + 1 + k^2 \cancel{+2k} + 1 + 1 = 4$

$2k^2 + 3 = 4$

$2k^2 = 1$

$k^2 = \frac{1}{2} \quad |k| = \frac{\sqrt{2}}{2} \quad k = \pm \frac{\sqrt{2}}{2}$

$\vec{u} \cdot \vec{v}$ producto escalar en $\mathbb{R}^n$

$\vec{u} \times \vec{v}$ producto vectorial en $\mathbb{R}^3$

$u = (a\,b\,c) \quad \vec{u} \cdot \vec{v} = ad + be + cf$ (un número)

$v = (d\,e\,f) \quad \vec{u} \times \vec{v}$

$u = (1\,2\,3)$

$v = (2\,1\,-3)$

$$\begin{array}{ccc} \oplus & \ominus & \oplus \\ i & j & k \\ 1 & 2 & 3 \\ 2 & 1 & -3 \end{array} \qquad \Delta i - \Delta j + \Delta k = -9i + 9j - 3k$$

Ángulo: $\cos\alpha = \dfrac{\vec{u} \cdot \vec{v}}{\|u\| . \|v\|} \qquad \vec{u} \cdot \vec{v} = 0 \Leftrightarrow u \perp v$

$\|u\|^2 = u . u$

$\text{proy}_v\, u = \dfrac{u . v}{v . v}\, v = \alpha . v$

Hallar 3 vectores del plano ortogonales al $(-1, 5)$

$u \perp v \Leftrightarrow u . v = 0 \qquad (a\,b)\,(-1\,5) = 0$

$-a + 5b = 0$

$a = 5b$

$\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 5b \\ b \end{pmatrix} = b \begin{pmatrix} 5 \\ 1 \end{pmatrix} \qquad V_1 : \begin{pmatrix} 5 \\ 1 \end{pmatrix}$

$V_2 = \begin{pmatrix} 10 \\ 2 \end{pmatrix}$

$V_3 = \begin{pmatrix} -5 \\ -1 \end{pmatrix}$

FECHA

Hallar un vector ortogonal al $(-4,0,3)$ de norma 5

$(-4\ 0\ 3)(a,b,c) = 0 \qquad -4a + 3c = 0 \rightarrow$ elijo $a = c = 0$

$V = (0,5,0)$

$V, W \in \mathbb{R}^n$

$(V-W) \perp V$

$\|V\| = 3$

$\alpha(V,W) = \frac{\pi}{4}$

Determ. $\|W\|$

Propiedades $(U, V, W \in \mathbb{R}^n)\ (\alpha \in \mathbb{R})$

$u.v = v.u$

$u(v \pm w) = u.v + u.w$

$\alpha(u.v) = (\alpha.u).v = u(\alpha.v)$

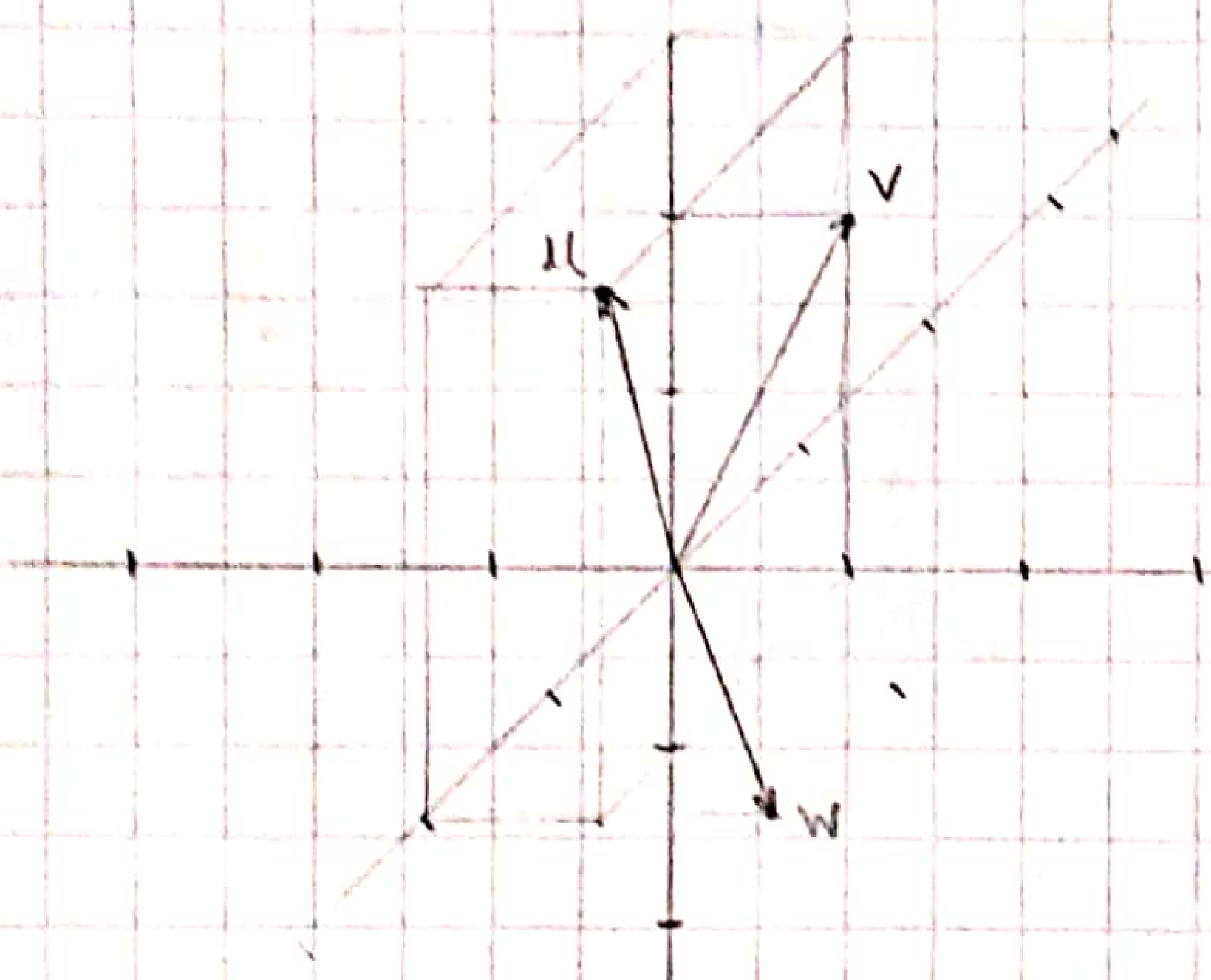
$V(V-W) = 0$
$V.V - V.W = 0$
$\|V\|^2 - V W = 0$

$\frac{V.W}{\|V\|\ \|W\|} = \frac{\sqrt{2}}{2} \qquad \|W\| = \frac{2}{\sqrt{2}} \frac{V.W}{\|V\|} = \|W\|$

$\frac{2}{\sqrt{2}} \frac{\|V\|^2}{\|V\|} = \|W\| = \frac{6}{\sqrt{2}}$

$\hookrightarrow \|V\| \neq 0$

③ a)



b) i) $u + w = (3,4,3)$ ii: $3w = (6,6,0)$

iii: $-u + w = (1,0,-3)$ iv: $-2u + 3v - 5w = (-9,-14,0)$

v: $5u - \frac{1}{2}v = (9/2, 10, 14)$

vi: $-u - 14v - 2w = (-19,-6,-31)$

vii: $u - 2w + v = (-2,-2,-1)$

④ a) norma: $3;\ \sqrt{5};\ 5;\ 1$

b) norma $= \sqrt{3},\ 2\sqrt{17},\ \sqrt{14},\ 2\sqrt{14},\ 2\sqrt{14}$

⑤ a) $\vec{U} = \alpha(-3,4) \quad \|U\| = 1 = \sqrt{(-3\alpha)^2 + (4\alpha)^2}$

$1 = 9\alpha^2 + 16\alpha^2 \qquad \vec{U} = \frac{1}{5}(-3,4)$

$\frac{1}{25} = \alpha^2 \Rightarrow \alpha = \frac{1}{5}$

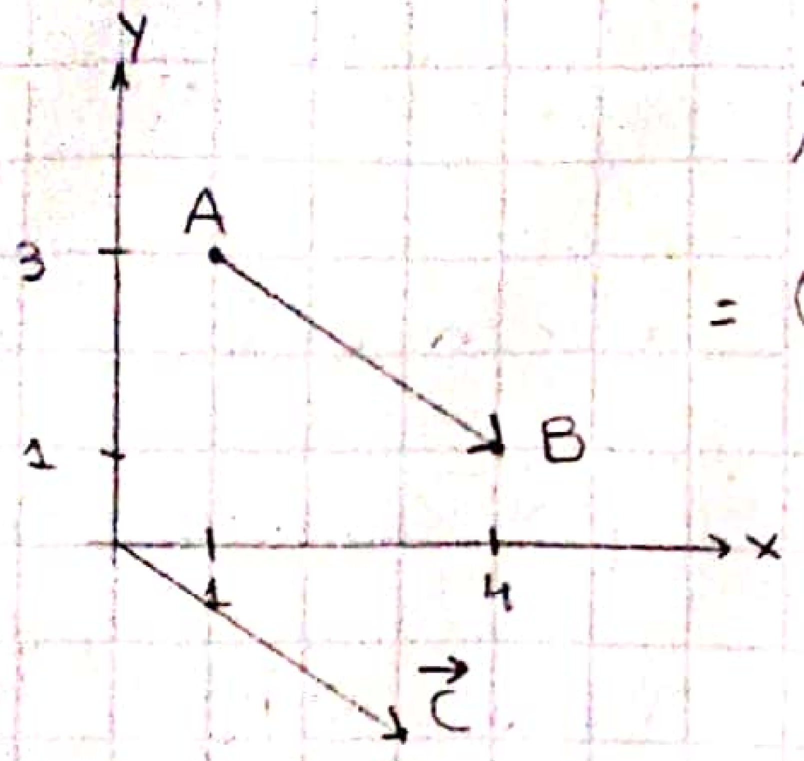
b) $\vec{U} = \beta(-1;2;1) \quad \|U\| = 1 = \sqrt{(-\beta)^2 + (2\beta)^2 + \beta^2}$

$1 = \beta^2 + 4\beta^2 + \beta^2$

$\frac{1}{6} = \beta^2 \Rightarrow \beta = \frac{1}{\sqrt{6}} = \frac{\sqrt{6}}{6} \qquad \vec{U} = \frac{\sqrt{6}}{6}(-1;2;1)$

c) $\frac{\vec{V}}{\|V\|} = \frac{(2;-3)}{\sqrt{13}} \qquad \frac{\vec{W}}{\|W\|} = \frac{(6,8)}{10} \qquad \frac{\vec{U}}{\|U\|} = \frac{(2,-1,3)}{\sqrt{14}}$

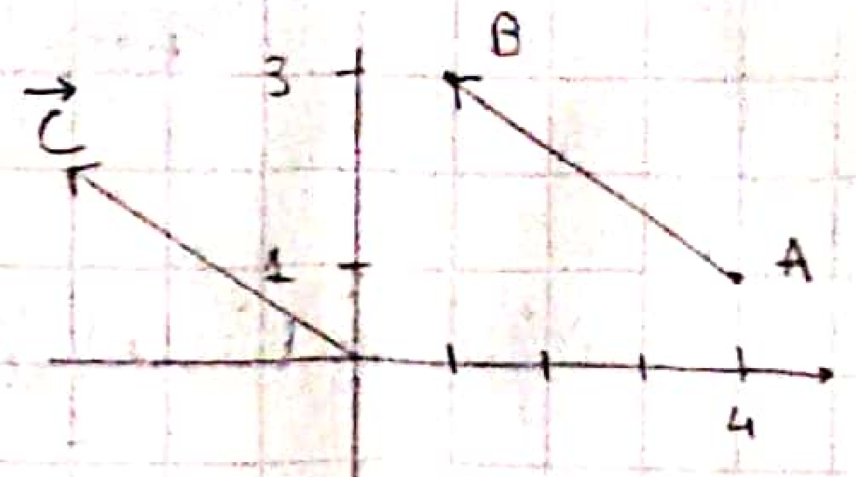
⑥ a)



$\vec{AB} = \vec{C} =$

$= (3; -2)$

b)



$\vec{AB} = \vec{C} =$

$= (-3; 2)$

c) $\vec{C} = \vec{B} - \vec{A} = (1, 1, 5)$

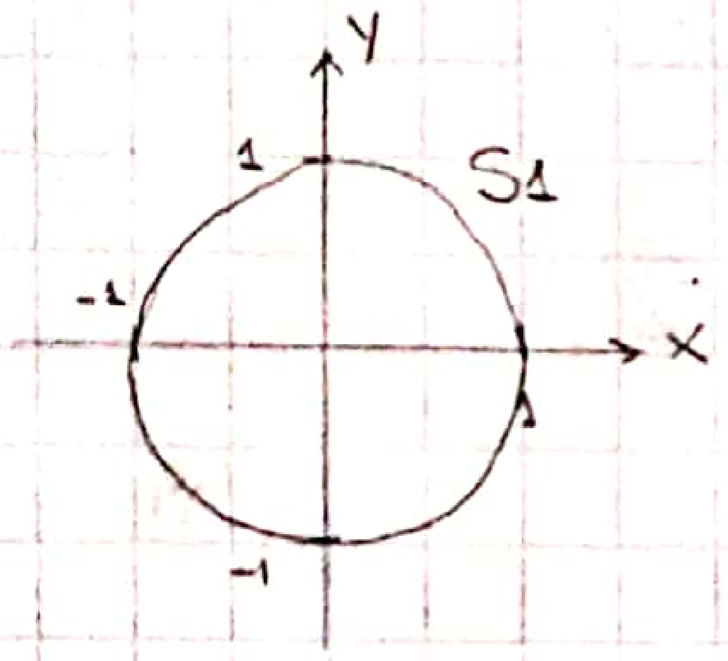
⑦ a) $dAB = \|B - A\| = \sqrt{(-1)^2 + 3^2} = \sqrt{10}$

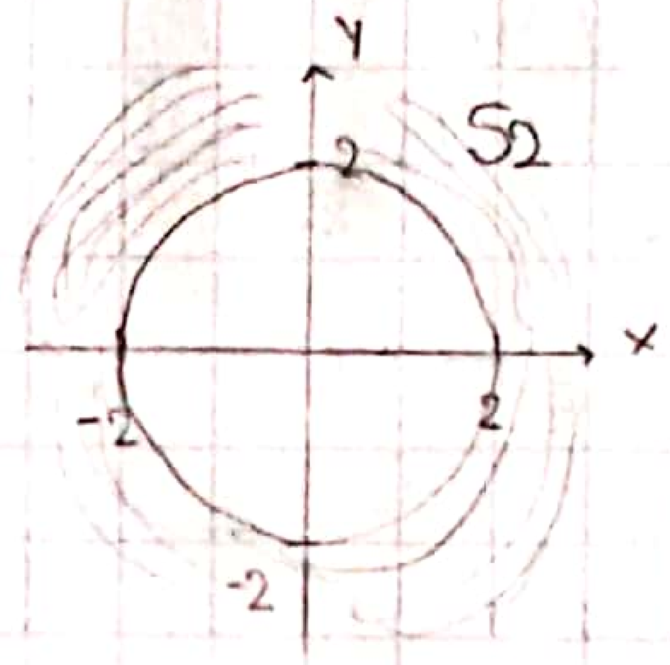
b) $dAB = \|B - A\| = \sqrt{3^2 + 4^2} = 5$

c) $dAB = \|B - \hat{A}\| = \sqrt{(-1)^2 + (-2)^2 + (-2)^2} = 3$

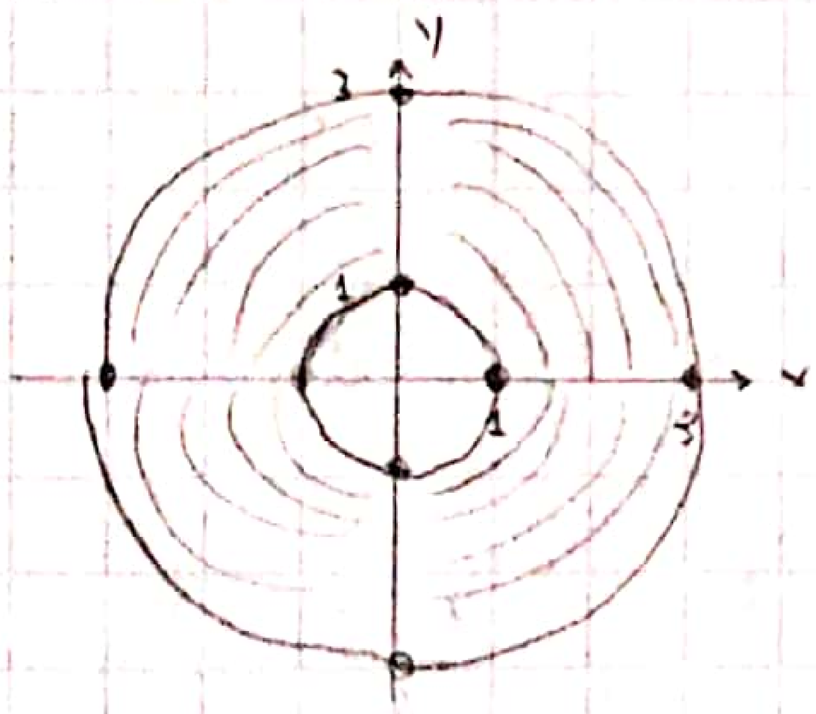
d) $dAB = \|B - A\| = \sqrt{(-1)^2 + 1^2 + 4^2} = 3\sqrt{2}$

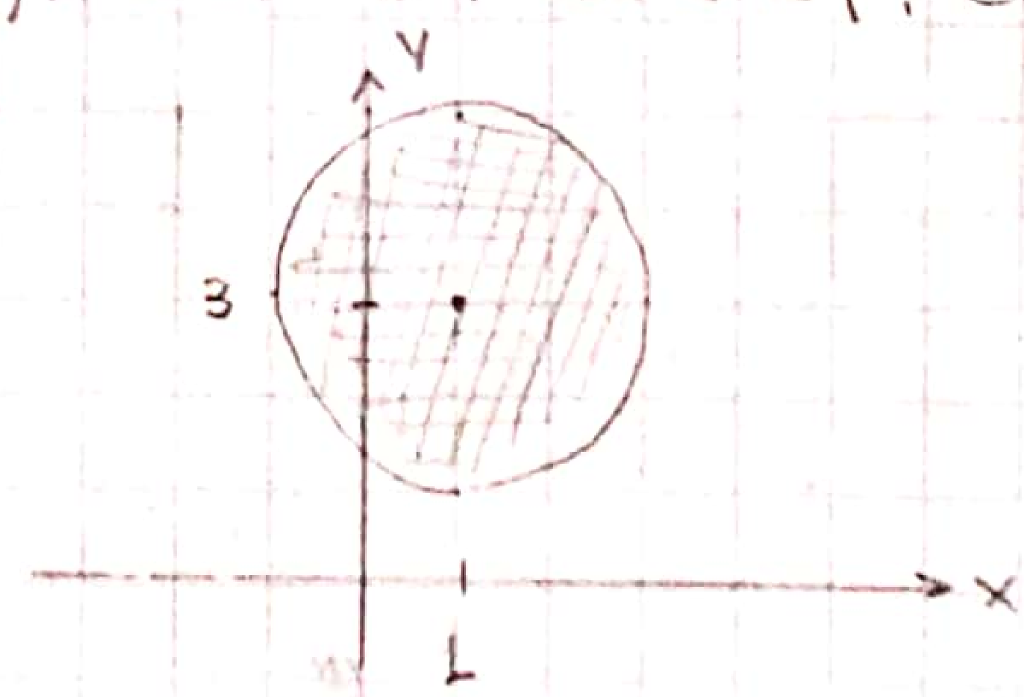
⑧ a) $S_1 = \} A \in \mathbb{R}^2 : \|A\| = 1 \{$ b) $S_2 = \} A \in \mathbb{R}^2 : \|A\| > 2 \{$





c) $S_3 = \} A \in \mathbb{R}^2 : 1 \leq \|A\| \leq 3 \{$ d) $S_4 = \} A \in \mathbb{R}^2 : \|A - B\| \leq 2 \{$, con $B = (1,3)$





⑨ a) $5^2 = 4^2 + k^2$

$9 = k^2$

$3 = |k|$

$k_1 = 3, \ k_2 = -3$

b) $2^2 = 1 + k^2$

$\sqrt{4-1} = |k|$

$k_1 = \sqrt{3}, \ k_2 = -\sqrt{3}$

c) $\|B - A\| = 2$

$4 = (k-1)^2 + (-k-1)^2 + 1^2$

$3 = k^2 - \cancel{2k} + 1 + k^2 + \cancel{2k} + 1$

$1 = 2k^2$

$\frac{1}{2} = k^2 \Rightarrow |k| = \frac{\sqrt{2}}{2} \quad k_1 = \frac{\sqrt{2}}{2}, \ k_2 = \frac{\sqrt{2}}{2}$

d) $2^2 = 1 + (-2)^2 + k^2$

$3 - 4 = k^2$

$-1 = k^2 \quad \nexists$ en $\mathbb{R}$

e) $1 = (2k)^2 + (2k)^2 + k^2$

$1 = 8k^2 + k^2$

$\frac{1}{9} = k^2 \Rightarrow |k| = \frac{1}{3}$

$k_1 = \frac{1}{3}, \ k_2 = -\frac{1}{3}$

⑩ a) $U_1 . U_2 = -1 - 4 = -5$ $\quad U_1 . U_5 = 0$ $\quad U_2 (U_3 + U_4) = U_2 . (-1, 1) = 1 - 2 = -1$

$U_1 . U_3 = -2 + 2 = 0$ $\quad U_2 . U_3 = 2 - 2 = 0$ $\quad (U_4 - U_3) U_1 = (3, -1) U_1 = 3 - 2 = 1$

b) $U_1 . U_2 = 1 - 1 = 0$ $\quad U_1 . (U_2 + U_3) = 3 - 2 - 1 = 0$ $\quad U_1 . U_4 = 2 + 3 - 1 = 4$

$U_1 . U_3 = 2 - 1 - 1 = 0$ $\quad U_1 (2U_2 - 3U_3) = -4 + 1 + 3 = 0$ $\quad (U_1 - 2U_2) U_3 = (-1, 3, 1) U_3 =$

$= -2 - 3 - 1 = -6$

(11) a) $\cos\alpha = \dfrac{u \cdot v}{\|u\| \cdot \|v\|}$

$\alpha = \arccos \dfrac{1}{\sqrt{2}} = \boxed{45°}$

b) $\cos\alpha = \dfrac{0}{(\sqrt{10})^2}$

$\alpha = \arccos 0 = \boxed{90°}$

c) $\cos\alpha = \dfrac{4}{2 \cdot 4}$

$\alpha = \arccos \dfrac{1}{2} = \boxed{60°}$

d) $\cos\alpha = \dfrac{3}{\sqrt{6} \cdot \sqrt{6}}$ $\qquad \alpha = \arccos\left(\dfrac{1}{2}\right) = \boxed{60°}$

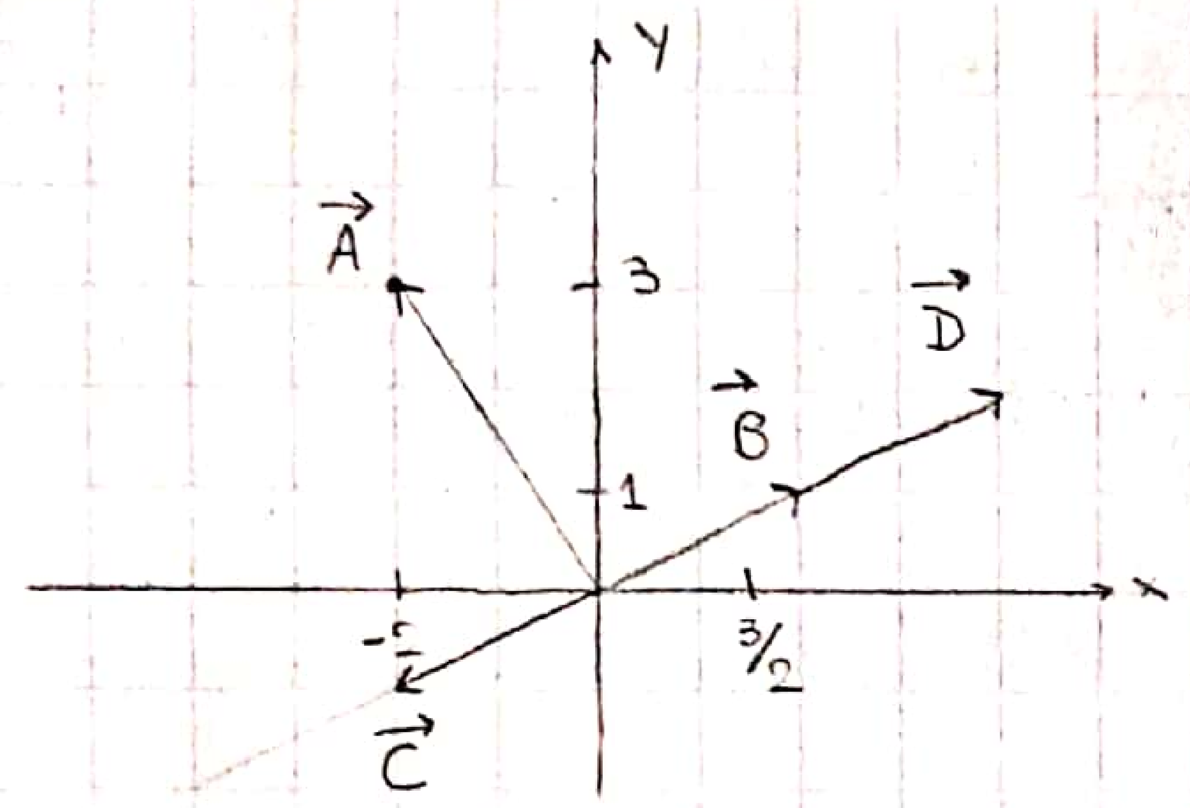
(12) a) $\bar{u} \cdot \bar{v} = -2 + 2 = 0 \leftrightarrow \bar{u} \perp \bar{v}$

d) $\bar{u} \cdot \bar{v} = -2 - 2 + 4 = 0 \leftrightarrow \bar{u} \perp \bar{v}$

b) $\bar{u} \cdot \bar{v} = 0 + 0 = 0 \leftrightarrow \bar{u} \perp \bar{v}$

c) $\bar{u} \cdot \bar{v} = 1 + 0 + 1 = 2 \Rightarrow \bar{u}$ no es $\perp$ a $\bar{v}$

(13) a)



$\vec{A} \perp \vec{B} \Leftrightarrow \vec{A} \cdot \vec{B} = 0$

$(-2,3) \cdot (a,b) = 0$

$-2a + 3b = 0$

$a = \dfrac{3}{2} b$

$(a,b) = \left(\dfrac{3}{2}b, b\right) = b\left(\dfrac{3}{2}, 1\right), \ b \neq 0$

Para $\vec{B}$, $b = 1$

" $\vec{C}$, $b = -1$

" $\vec{D}$, $b = 2$

b) $(1,1) \perp (a,b) \Leftrightarrow (1,1) \cdot (a,b) = 0$

$a + b = 0$

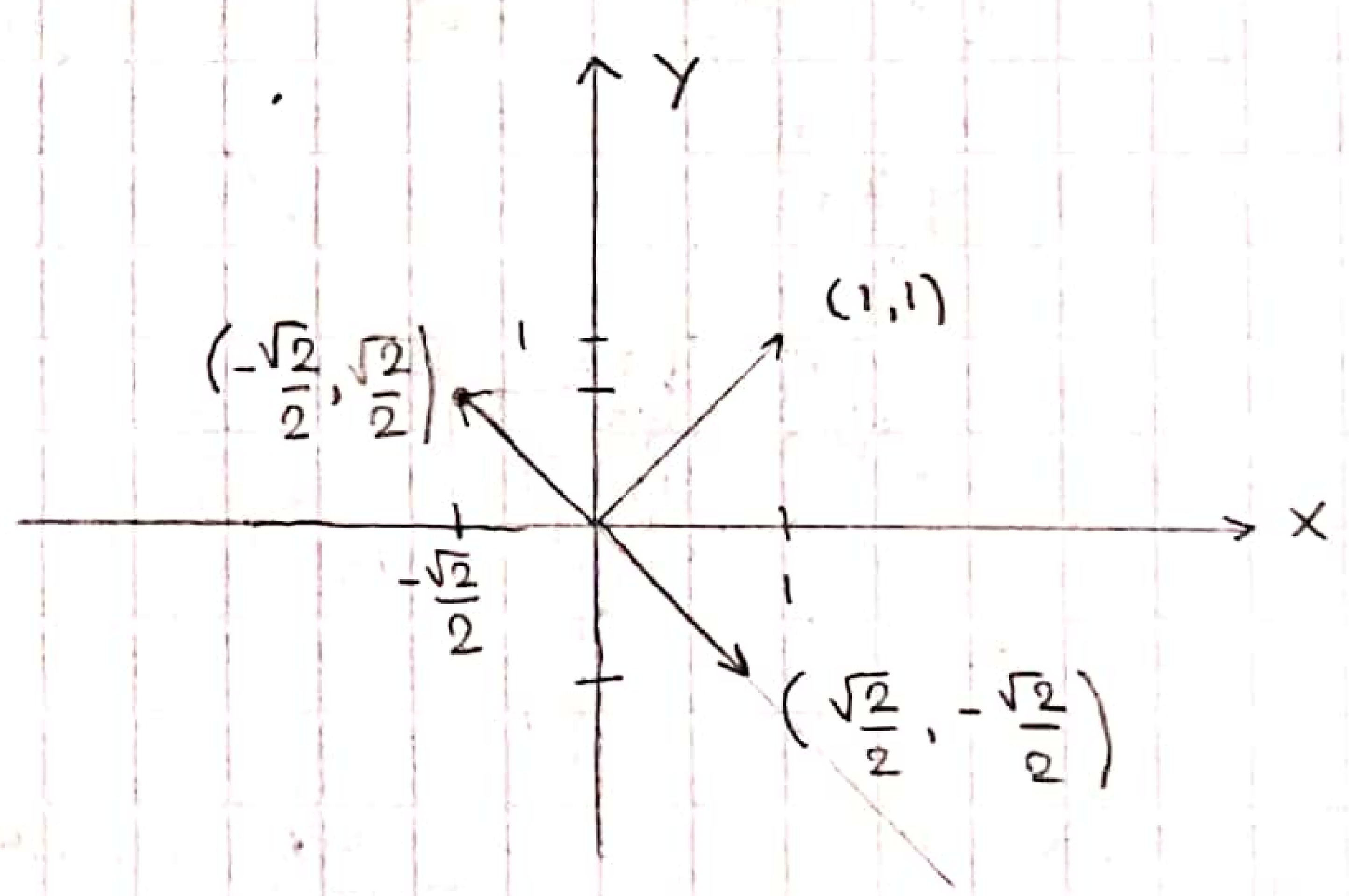
$(a,-a) = \vec{A}$

$\|A\| = 1 \Rightarrow 1 = a^2 + (-a)^2$

$1 = 2a^2$

$\dfrac{\sqrt{2}}{2} = |a| \Rightarrow a_1 = \dfrac{\sqrt{2}}{2}$

$a_2 = -\dfrac{\sqrt{2}}{2}$



c) $(-1,2,1) \perp (a,b,c) \Leftrightarrow (1,2,1) \cdot (a,b,c) = 0$

$a + 2b + c = 0$

Elijo: $a = -2$, $b = 1$, $c = 0$

$\alpha(-2,1,0) \perp (-1,2,1) \Rightarrow$ Rta: $(-2,1,0)$

$2(-2,1,0) = (-4,2,0)$

$3(-2,1,0) = (-6,3,0)$

d) $(-1,0,2) \perp (a,b,c) \Leftrightarrow (-1,0,2) \cdot (a,b,c) = 0$

$-a + 2c = 0$

$a = 2c$

$(a,b,c) = (2c,b,c)$

$8^2 = (2c)^2 + b^2 + c^2$

$64 = 4c^2 + b^2 + c^2$

$64 = 3c^2 + b^2$

Elijo $c = 0 \Rightarrow 64 = b^2$

$8^2 = b^2$

$|b| = 8$ $\quad b_1 = 8$, $b_2 = -8$

Rta: $(0,8,0)$

(14) $V = (1,1)$ $W = (a,b)$

$\|W\| = 2$

$\cos 45° = \dfrac{V \cdot W}{\|W\|\|V\|}$

$\dfrac{\sqrt{2}}{\cancel{2}} = \dfrac{a+b}{\cancel{2}\cdot\sqrt{2}}$

$2 = a+b$

$a = 2-b$

$W = (2-b, b)$

$2^2 = (2-b)^2 + b^2$

$4 = 4 - 4b + 2b^2$

$0 = 2b^2 - 4b$

$b_1 = 2$ $b_2 = 0$

Rta: $W_1 = (0,2)$ $W_2 = (2,0)$

(15) $V, W \in \mathbb{R}^n$

$(V - W) \perp V$

$\|V\| = 3$

$\cos 45° = \dfrac{V \cdot W}{\|V\|\|W\|}$

$\|W\| = ?$

$\dfrac{\sqrt{2}}{2} = \dfrac{V \cdot W}{3\|W\|}$

$(V-W)V = 0$

$V \cdot V - W \cdot V = 0$

$V \cdot V = W \cdot V = \|V\|^2 = 9$

$\dfrac{\sqrt{2}}{2} = \dfrac{9}{3\|W\|}$

$3\sqrt{2}\,\|W\| = 18$

$\|W\| = \dfrac{18}{3\sqrt{2}} = 3\sqrt{2}$

(16) $\mu = (2,0,-1)$, $V = (-2,4,4)$

a) $\mu \cdot V = -4 + 0 - 4 = -8$

b) $\cos\alpha = \dfrac{\mu \cdot V}{\|\mu\| \cdot \|V\|}$

$\alpha = \arccos \dfrac{-8}{\sqrt{5}\cdot 6} = \underline{126° 36'}$

c) $\text{proy}_\mu V = \dfrac{\mu \cdot V}{\|\mu\|^2} \cdot \mu = -\dfrac{8}{5}(2,0,-1)$

d) $\text{proy}_V \mu = \dfrac{V \cdot \mu}{\|V\|^2} \cdot V = -\dfrac{2}{9}(-2,4,4)$

(17) a) $\text{proy}_V \mu = \dfrac{\mu \cdot V}{\|V\|^2} \cdot V = \dfrac{-2+1-3}{14} \cdot V = -\dfrac{2}{7}(2,1,-3)$

b) $\text{proy}_V \mu = \dfrac{-2+1-3}{3} \cdot V = -\dfrac{4}{3}(-1,1,1)$

(18) $V = (1, k, 2)$

$W = (-3,4,0)$

$\|\text{proy}_W V\| = 1$

$k \in \mathbb{R} = ?$

$\left\| \left(\dfrac{\bar{V} \cdot \bar{W}}{\|W\|^2} \right) \bar{W} \right\| = 1$

$\left\| \dfrac{-3+4k}{25} (-3,4,0) \right\| = 1$

$\dfrac{|-3+4k|}{25} \cdot \|-3,4,0\| = 1$

$\dfrac{|-3+4k| \cdot \cancel{5}}{\cancel{25}\,5} = 1$

$|-3+4k| = 5$

$-3+4k = 5 \quad \vee \quad -3+4k = -5$

$4k = 8 \quad \vee \quad 4k = -2$

$k = 2 \quad \vee \quad k = -\dfrac{1}{2}$

(19) a)
$\begin{matrix} i & j & k \\ 1 & 3 & 5 \\ 1 & 3 & 5 \end{matrix}$ $(1,3,5) \times (1,3,5) = 0i - 0j + 0k$

b)
$\begin{matrix} i & j & k \\ 1 & 3 & -4 \\ 2 & 1 & 3 \end{matrix}$ $(1,3,-4) \times (2,1,3) = (9+4)i - (3+8)j + (1-6)k = 13i - 11j - 5k$

c)
$\begin{matrix} i & j & k \\ 2 & 1 & 3 \\ 1 & 3 & -4 \end{matrix}$ $(2,1,3) \times (1,3,-4) = (-4-9)i - (-8-3)j + (6-1)k = -13i + 11j + 5k$

d)
$\begin{matrix} i & j & k \\ 1 & 3 & 4 \\ 0 & 0 & 0 \end{matrix}$ $(1,3,4) \times (0,0,0) = 0i + 0j + 0k$

e)
$\begin{matrix} i & j & k \\ 2 & 0 & 0 \\ 0 & 0 & 3 \end{matrix}$ $(2,0,0) \times (0,0,3) = 0i - 6j + 0k$

$\vec{u} \parallel \vec{v} \longleftrightarrow \vec{u} = \alpha . \vec{v} \quad \alpha \neq 0$

HOJA N

FECHA

(20) $\bar{u} = (2,1,-3)$, $\bar{v} = (1,-2,1)$, $\bar{w} = (1,2,2)$

a) $\begin{array}{cccl} i & j & k & \\ 2 & 1 & -3 & u \\ 1 & -2 & 1 & v \end{array}$ $\quad \bar{u} \times \bar{v} = (1-6)i - (2+3)j + (-4-1)k = -5i - 5j - 5k$

$\bar{v} \times \bar{u} = (6-1)i - (-3-2)j + (1+4)k = 5i + 5j + 5k$

b) $(\bar{u} \times \bar{v}) \cdot \bar{u} = (-5,-5,-5) \cdot (2,1,-3) = -10 - 5 + 15 = 0 \leftrightarrow (\bar{u} \times \bar{v}) \perp \bar{u}$

$(\bar{u} \times \bar{v}) \cdot \bar{v} = (-5,-5,-5) \cdot (1,-2,1) = -5 + 10 - 5 = 0 \leftrightarrow (\bar{u} \times \bar{v}) \perp \bar{v}$

c) $(u \times v) \times w \quad \begin{array}{ccc} i & j & k \\ -5 & -5 & -5 \\ 1 & 2 & 2 \end{array} \quad (-10+10)i - (-10+5)j + (-10+5)k = 0i + 5j - 5k$

$u \times (v \times w)$ $\quad 1°\ v \times w \quad \begin{array}{ccc} i & j & k \\ 1 & -2 & 1 \\ 1 & 2 & 2 \end{array} = (-4-2)i - (2-1)j + (2+2)k = -6i - j + 4k$

$2°\ u \times (v \times w) \quad \begin{array}{ccc} i & j & k \\ 2 & 1 & -3 \\ -6 & -1 & 4 \end{array} \quad (4-3)i - (8-18)j + (-2+6)k = i + 10j + 4k$

(21) $(1,2,-3) \times (-1,5,-2) = (-4+15, 3+2, 5+2) = (11,5,7)$

$\begin{array}{ccc} i & j & k \\ 1 & 2 & -3 \\ -1 & 5 & -2 \end{array}$ Hay ∞ posibilidades tomando $\alpha(11,5,7)$, con $\alpha \neq 0$

(22) $\|w\| = 2 \quad \perp u \wedge \perp v \quad w = (a,b,c)$

a) $\bar{u} \times \bar{v} = (-1-3, 2+1, 3-2) = (-4,3,1) \Rightarrow$ tomo $\alpha(-4,3,1)$

$\begin{array}{ccc} i & j & k \\ 1 & 1 & 1 \\ 2 & 3 & -1 \end{array}$ $\quad \left.\begin{array}{l} \|\alpha(-4,3,1)\| = 2 \\ |\alpha| . \sqrt{26} = 2 \\ |\alpha| = \frac{\sqrt{26}}{13} \end{array}\right\} \Rightarrow \begin{array}{l} w_1 = \frac{\sqrt{26}}{13}(-4,3,1) = \left(-\frac{4\sqrt{26}}{13}, \frac{3\sqrt{26}}{13}, \frac{\sqrt{26}}{13}\right) \\ w_2 = -\frac{\sqrt{26}}{13}(-4,3,1) = \left(\frac{4\sqrt{26}}{13}, -\frac{3\sqrt{26}}{13}, -\frac{\sqrt{26}}{13}\right) \end{array}$

b) $\bar{u} \times \bar{v} = (-21-20, -4-14, 10-3) = (-41,-18,7) \Rightarrow$ tomo $\alpha(-41,-18,7)$

$\begin{array}{ccc} i & j & k \\ 2 & -3 & 4 \\ -1 & 5 & 7 \end{array}$ $\quad \left.\begin{array}{l} \|\alpha(-41,-18,7)\| = 2 \\ |\alpha| \ \|(-41,-18,7)\| = 2 \\ |\alpha| = \frac{2}{\sqrt{2054}} \end{array}\right\} \begin{array}{l} w_1 = \frac{2}{\sqrt{2054}}(-41,-18,7) = \left(-\frac{82}{\sqrt{2054}}, -\frac{36}{\sqrt{2054}}, \frac{14}{\sqrt{2054}}\right) \\ w_2 = -\frac{2}{\sqrt{2054}}(-41,-18,7) = \left(\frac{82}{\sqrt{2054}}, \frac{36}{\sqrt{2054}}, -\frac{14}{\sqrt{2054}}\right) \end{array}$

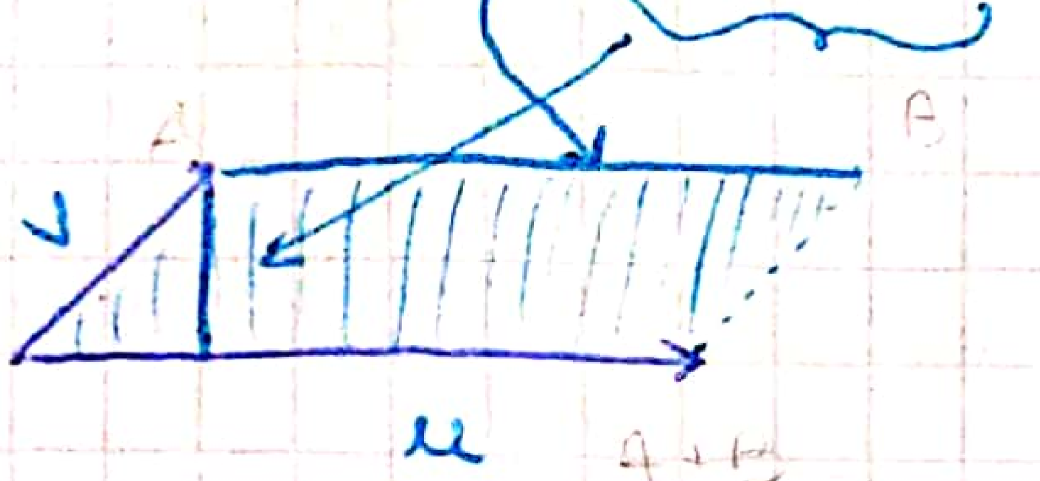
$\|proy_v u\| = \dfrac{|u \cdot v|}{\|v\|}$ $\quad$ $proy_v u = \dfrac{u \cdot v}{\|v\|^2} \cdot \vec{v}$

$u \cdot v = \|u\|\|v\| \cos \alpha$

$u \times v$, $u \in \mathbb{R}^3$ y $v \in \mathbb{R}^3$

$u \times v = w$, $w \in \mathbb{R}^3$

$\|u \times v\| = \|\vec{u}\| \cdot \|\vec{v}\| \operatorname{sen} \alpha$



$\|u \times v\| =$ Área paralelogramo

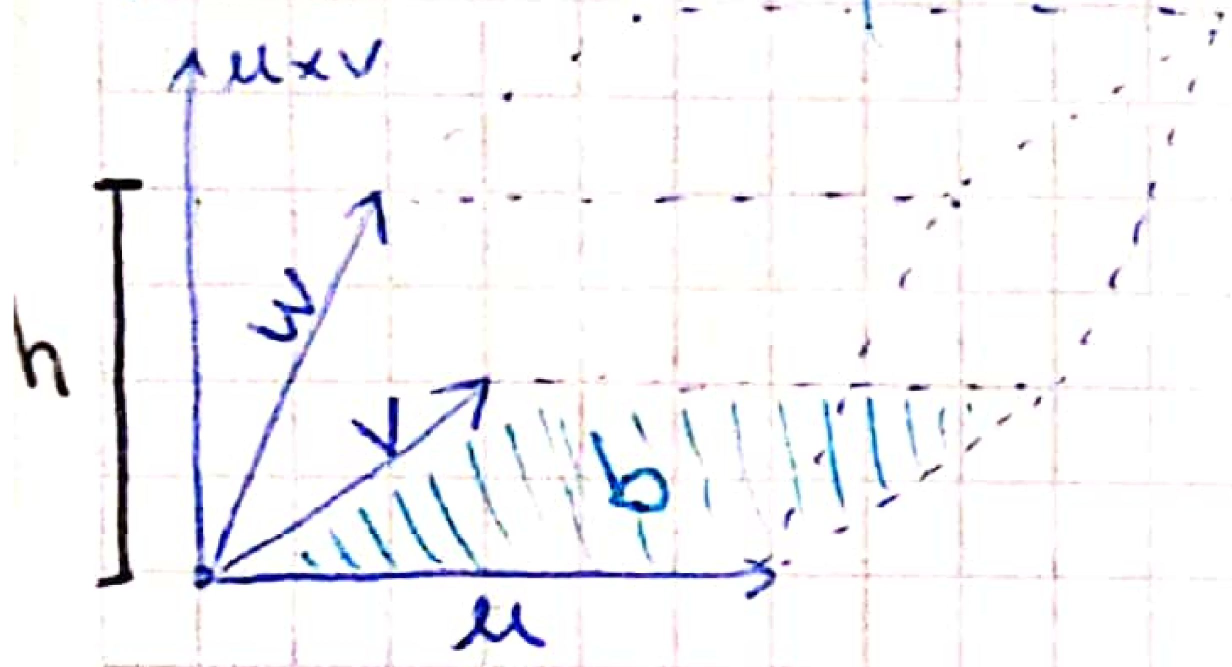
$\vec{u} \times \vec{v} = -\vec{v} \times \vec{u}$

$\vec{u} \times \vec{0} = \vec{0} \times \vec{a}$

$\vec{u} \times \vec{u} = \vec{0}$

$\alpha(\vec{u} \times \vec{v}) = \alpha \vec{u} \times \vec{v} = \vec{u} \times \alpha \vec{v}$

$(\vec{u} \times \vec{v}) \cdot \vec{v}$



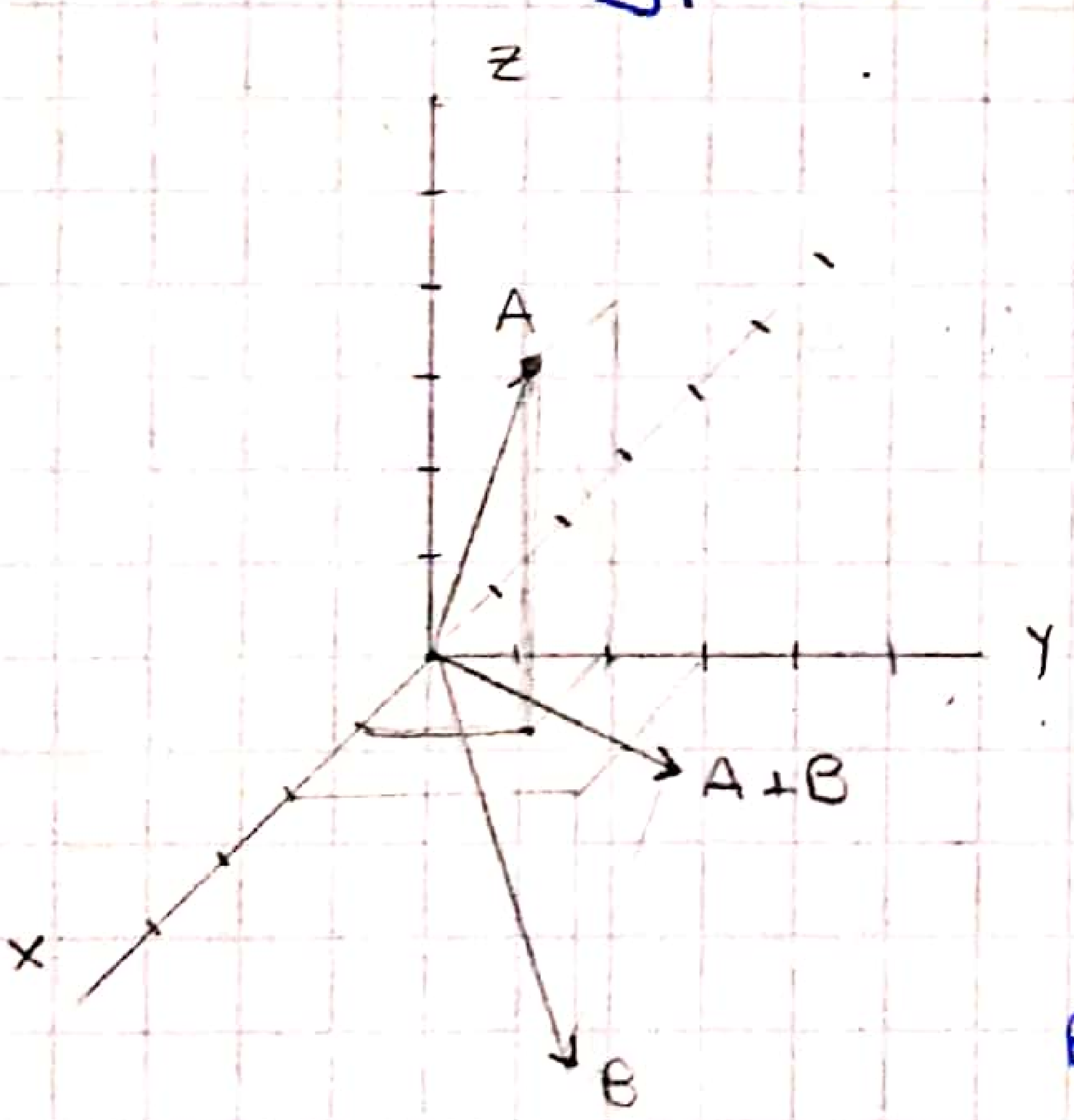
$Vol = \|u \times v\| \, |(\|W\| \cos \beta)| =$

$= |(\bar{u} \times \bar{v}) \cdot \vec{w}| = |(\bar{u} \times \bar{w}) \cdot \bar{v}| =$

$= |(\bar{v} \times \bar{w}) \cdot \bar{u}| = \ldots$

Si $Vol = 0 \Leftrightarrow \bar{u}, \bar{v}$ y $\bar{w}$ son coplanares

(23) a)

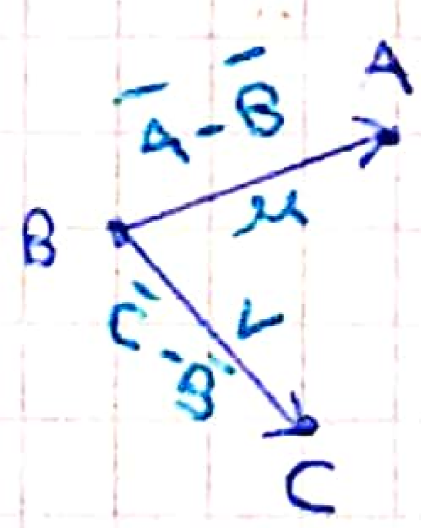


$\bar{A} = (1,2,4)$ $\quad$ $\bar{A} \times \bar{B} = (-14-12, 8+7, 3-4)$

$\bar{B} = (2,3,-7)$ $\quad$ $\|\bar{A} \times B\| = \sqrt{(-26)^2 + 15^2 + (-1)^2} =$

$= \sqrt{902}$

b) Área $\triangle$ = Área $\square$: 2



$\bar{u} = (-2,2,-1)$ $\quad$ $\bar{v} = (0,-1,-2)$

$u \times v = (-5,4,2)$

$\dfrac{\|u \times v\|}{2} = \dfrac{\sqrt{25+16+4}}{2} = \dfrac{3\sqrt{5}}{2}$

(24) a) Volumen $= |(\bar{A} \times \bar{B}) \cdot \bar{C}| = |(6,9,-2) \cdot (1,-1,3)| = |6-9-6| = 9$

$\begin{matrix} i & j & k \\ -1 & 2 & 4 \\ 1 & 0 & 5 \end{matrix}$

b) Volumen $= |(\bar{A} \times \bar{B}) \cdot \bar{C}| = |(0,1,3) \cdot (4,2,-1)| = |0+2-3| = 1$

$\begin{matrix} i & j & k \\ 1 & 0 & 0 \\ 0 & 3 & -1 \end{matrix}$

c) Volumen $= |[(\bar{B}-\bar{A}) \times (\bar{C}-\bar{A})] \cdot (\bar{D}-A)| = |(-1,0,1) \cdot (3,0,2)| = |-3+0+2| = 1$

$\begin{array}{c|ccc} & i & j & k \\ \bar{B}-\bar{A} & 1 & 0 & 1 \\ \bar{C}-\bar{A} & 0 & 1 & 0 \end{array}$

Ecuación de la recta en $\mathbb{R}^2$

Ecuación implícita

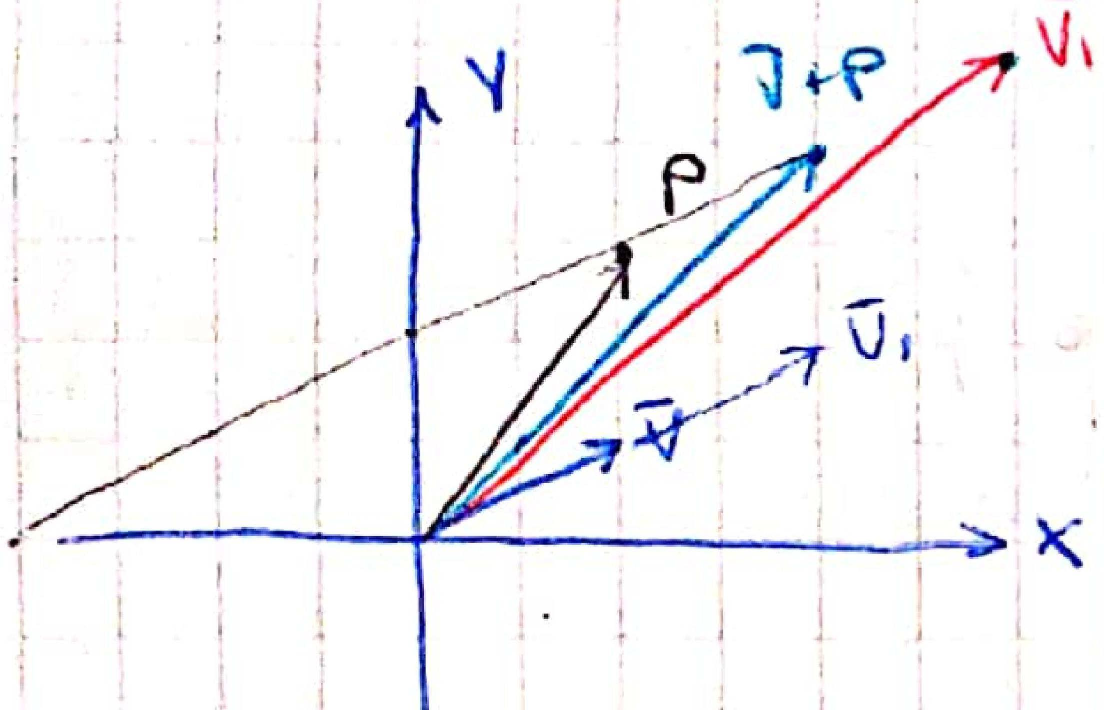
$Ax + By = C \quad (A,B) \neq (0,0) \quad 3x+2y=1 \rightarrow y = -\frac{3}{2}x + \frac{1}{2}$

$\underbrace{y = -\frac{A}{B}x + \frac{C}{B}}_{\text{forma explícita} \rightarrow \text{no admite recta vertical}}$ / $\begin{matrix} A=0 \\ B\neq 0 \end{matrix}$ $y = \frac{C}{B}$ (cte) / $\begin{matrix} A\neq 0 \\ B=0 \end{matrix}$ $x = \frac{C}{A}$ (vertical)

Forma vectorial o paramétrica de la recta en $\mathbb{R}^2$

$X = K.\bar{v} \quad \bar{v} = (v_1, v_2) \quad K \in \mathbb{R}$ → pasa por el origen de coordenadas

↳ vector director



Ecuación de una recta de dirección $\bar{v}$ que pase por P: $\boxed{X = K.\bar{v} + P \quad K \in \mathbb{R}}$

Ejemplo: hallar X si $\bar{v} = (1,2)$, $P = (3,1) \Rightarrow X = K(1,2) + (3,1)$

→ Llevar a la ecuación implícita $\begin{cases} x = K+3 \rightarrow K = x-3 \\ y = 2K+1 \rightarrow K = \frac{y-1}{2} \end{cases}$ $x-3 = \frac{y-1}{2}$; $2x - y = 5$

→ Llevar a la explícita → despejo y: $y = 2x - 5$

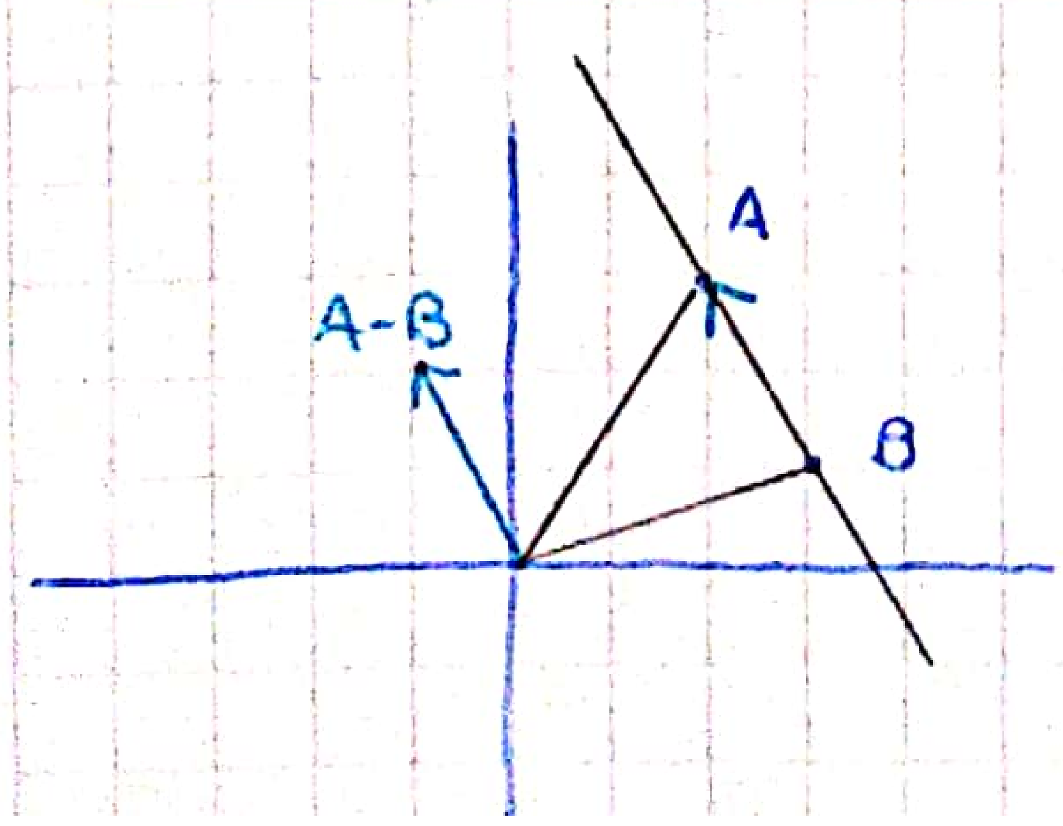
—o—

$L = 2x + 4y = 1 \longrightarrow y = \frac{-2x+1}{4} = -\frac{1}{2}x + \frac{1}{4} = \left(x, -\frac{1}{2}x + \frac{1}{4}\right) =$

$= \left(x, -\frac{1}{2}x\right) + \left(0, \frac{1}{4}\right) = x\left(1, -\frac{1}{2}\right) + \left(0, \frac{1}{4}\right) \quad x \in \mathbb{R}$

$(x,y) = K\left(1, -\frac{1}{2}\right) + \left(0, \frac{1}{4}\right) \quad K \in \mathbb{R}$

Hallar la ecuación de la recta que pase por 2 ptos.



$X = K\bar{v} + P \quad K \in \mathbb{R}$

$X = K(A-B) + A$ o B

o

$X = \lambda\,(B-A) + A$ o B

$A = (1,3) \quad B = (2,6)$

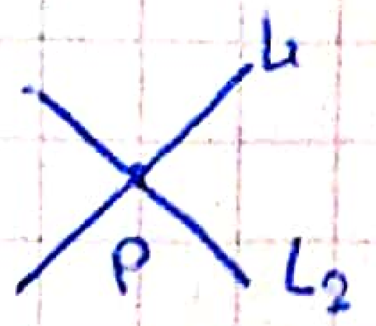
$X = K(-1,3) + (1,3) \quad K \in \mathbb{R}$

$L: (x,y) = t(1,2) + 0,5 \quad t \in \mathbb{R}$. Hallar $L_1 // L$ si $(-2,6) \in L_1$

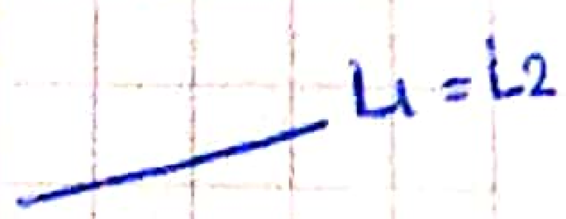
$$\left(\begin{array}{lll} L_1 // L_2 \leftrightarrow \bar{V}_1 // \bar{V}_2 \leftrightarrow \bar{V}_1 = \lambda \bar{V}_2 & \lambda \neq 0 \\ L_1 \perp L_1 \leftrightarrow \bar{V}_1 \cdot \bar{V}_2 = 0 & (m_1 \cdot m_2 = -1) \end{array} \right)$$

$L_1: X = s(1,2) + (-2,6)$

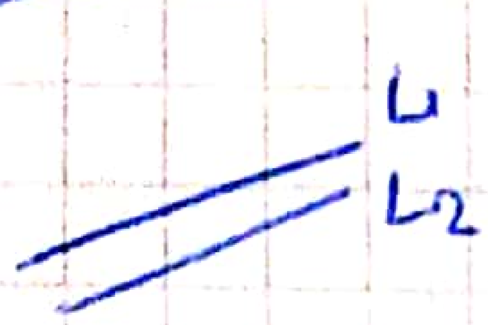
Posiciones relativas de dos rectas en $\mathbb{R}^2$



$L_1 \cap L_2 = \{P\}$ concurrentes



$L_1 \cap L_2 = L_1 = L_2$ coincidentes



$L_1 \cap L_2 = \emptyset$ paralelas

$$\left.\begin{array}{l} L_1: 2x + 6y = -4 \\ L_2: x + 3y = -2 \end{array}\right\} \begin{array}{l} \longrightarrow y = -\frac{2}{3} - \frac{1}{3}x \\ \longrightarrow y = -\frac{2}{3} - \frac{1}{3}x \end{array} \Big\} \; 0 = 0$$

$L_1 \cap L_2 = \{(x,y) / y = -\frac{2}{3} - \frac{1}{3}\}$

L_1 y L_2 son coincidentes

$L_1: 2x - 3y + 1 = 0$

$L_2: \underset{\downarrow \atop (x,y)}{X} = \beta(1,-3) + (2,-2) \longrightarrow \left\{ \begin{array}{l} 2x - 3y + 1 = 0 \\ x = 2 + \beta \\ y = -2 - 3\beta \end{array} \right\}$

$2(2+\beta) - 3(-2-3\beta) + 1 = 0$

$11\beta + 11 = 0 \quad \boxed{\beta = -1} \rightarrow a\ L_2$

$\left\{ \begin{array}{l} x = 1 \\ y = 1 \end{array} \right. \Rightarrow L_1 \cap L_2 = \{(1,1)\}$

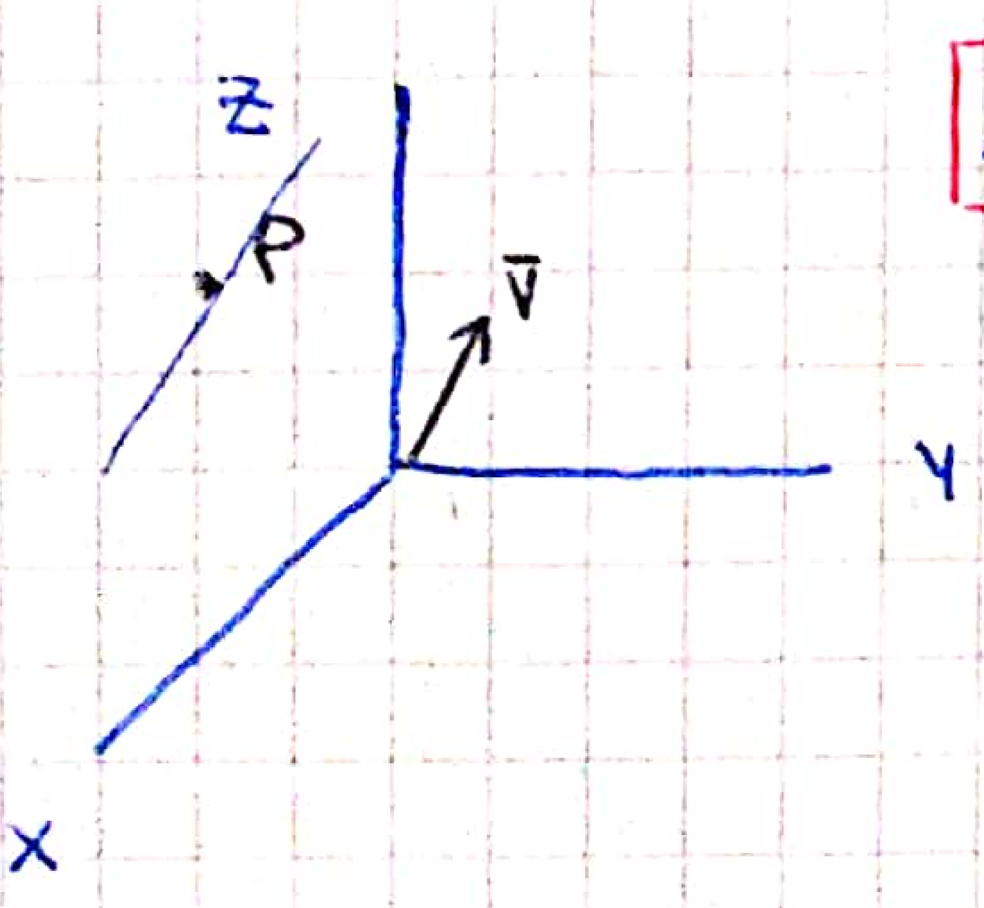
$L_1: X = \alpha(6,2) + (-1,3) \quad \left\{ \begin{array}{l} x = 6\alpha - 1 \\ y = 2\alpha + 3 \end{array} \right.$

$L_2: X = \beta(3,1) + (1,-3) \quad \left\{ \begin{array}{l} x = 3\beta + 1 \\ y = \beta - 3 \end{array} \right.$

$\longrightarrow \left\{ \begin{array}{l} 6\alpha - 1 = 3\beta + 1 \\ 2\alpha + 3 = \beta - 3 \end{array} \right. \rightarrow \boxed{0\alpha = 20}$ abs

$L_1 \cap L_2 = \emptyset$

Ecuación de la recta en $\mathbb{R}^3$



$\boxed{X = K \cdot \bar{v} + P}$

$P = (x_0, y_0, z_0)$

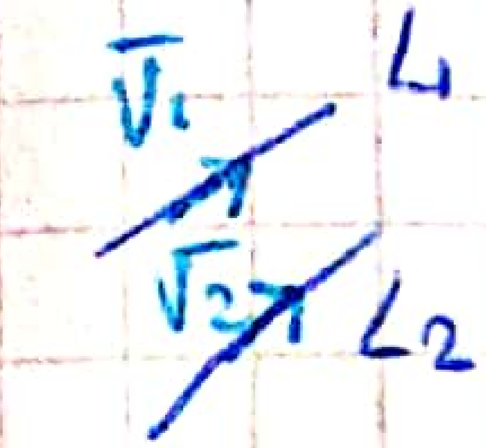
$\bar{v} = (v_1, v_2, v_3)$

$X = K(A - B) + A \quad K \in \mathbb{R}$

$X = \alpha(B - A) + A \quad \alpha \in \mathbb{R}$

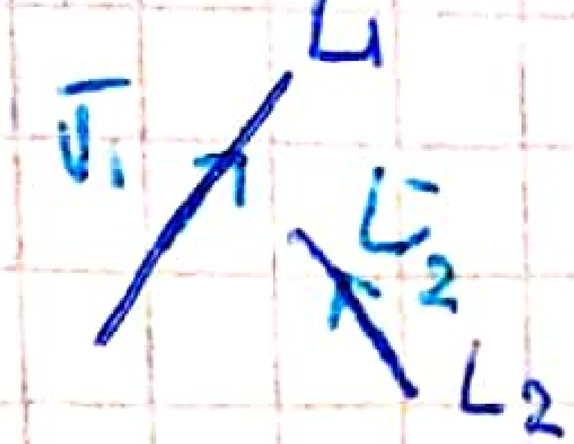
Posiciones relativas de 2 rectas en $\mathbb{R}^3$

Concurrentes, coincidentes, paralelas



$L_1 \cap L_2 = \emptyset \quad \bar{V}_1 \parallel \bar{V}_2$

o alabeadas



$L_1 \cap L_2 = \emptyset \ , \ \bar{V}_1 \not\parallel \bar{V}_2$

$L_1: X = \beta(1,-4,2) + (3,1,0)$ $\quad \begin{cases} x = \beta + 3 \\ y = 4\beta + 1 \\ z = 2\beta \end{cases}$

$L_2: X = \alpha(2,-1,1) + (-2,7,0)$ $\quad \begin{cases} x = 2\alpha - 2 \\ y = -\alpha + 7 \\ z = \alpha \end{cases}$

$\begin{cases} \beta + 3 = 2\alpha - 2 \\ 4\beta + 1 = -\alpha + 7 \\ 2\beta = \alpha \end{cases} \quad \begin{cases} \beta = -3 \\ \alpha = -6 \end{cases}$

$-3 + 3 = 2(-6) - 2$

$0 \neq -14$

Abs

$L_1 \cap L_2 = \emptyset \longrightarrow$ analizo los vectores directores

$k?$ $(1,-4,2) = k(2,-1,1)$ $\quad \begin{cases} 1 = 2k & k = 1/2 \\ -4 = -k \\ 2 = k & k = 2 \end{cases}$ $\rightarrow$ abs $\Rightarrow$ L_1 y L_2 son alabeadas

$L_1: X = \beta(1,-4,2) + (3,1,0)$ $\quad x = \beta + 3 \quad y = -4\beta + 1 \quad z = 2\beta$

$L_2: X = \alpha(2,-1,1) + (-3,-3,0)$ $\quad x = 2\alpha - 3 \quad y = -\alpha - 3 \quad z = \alpha$

$2\alpha - 3 = \beta + 3$
$-\alpha - 3 = -4\beta + 1$
$2\beta = \alpha$

$2(2\beta) - 3 = \beta + 3$
$4\beta - \beta = 6$
$3\beta = 6 \rightarrow \underline{\beta = 2}$

$\underline{\alpha = 4}$

$-4 - 3 = -8 + 1$
$-7 = -7$
$\underline{0 = 0}$

$L_1: X = (5,-7,4)$

$L_1 \cap L_2 = \{(5,-7,4)\} \rightarrow$ son concurrentes

Si fueran coincidentes, no encontrarías ni α ni β, pero tampoco encontrarías un absurdo

(25) $Vol = |A(B \times C)| = |(B-A)[(C-A)\times(D-A)]| = |(2,k-1,-1)[(1,-2,-1)\times(3,-1,-2)]| =$

$= |(2,k-1,-1)(3,-1,5)| = |6-k+1-5| = |2-k| = 21 \rightarrow k_1 = 23 \quad k_2 = -19$